

Chapter 1

On the Potential for Improved Measurement in the Human and Social Sciences

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Abstract Geometry is the most ancient branch of physics. All linear measurement is essentially a form of practical geometry. Following Maxwell's method of drawing analogies from geometry, Rasch conceptualized measurement models as analogous to scientific laws. Rasch likely absorbed Maxwell's method via close and prolonged interactions with colleagues known for their use of it. Examination of the common form of the relationships posited in the Pythagorean theorem, multiplicative natural laws, and Rasch models leads to a new perspective on the potential unity of science. To be fully realized in the social sciences, Rasch's measurement ideas need to be dissociated from statistics and IRT, and instead rooted in the Maxwellian sources Rasch actually drew from. Following through on the method of analogy from geometry may make human and social measurement more intuitive and useful.

Keywords Geometry • Measurement • Scientific law • Rasch models

1.1 Introduction

All linear measurement makes use of the geometric figure of the line. For persons educated in basic scientific conventions, quantitative comparisons automatically bring images of a number line to mind. Despite these associations, most statistical methods in the social sciences do not require experimental tests of the hypothesis that any given numeric difference stands for a constant unit amount. Further, to many the very idea

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that geometry could provide a useful basis for measurement in the social sciences seems implausible. To what extent, however, might this implausibility be more a function of unexamined prejudices than careful reasoning? There may be more of value in this line of thinking than meets the eye.

1.2 Linear Measurement as Practical Geometry

In the natural sciences, the basis for quantitative units is established, in effect, via analogies from geometry. The Pythagoreans considered tonal proportions to be the geometry of motion, for instance, encompassing sound, celestial bodies, and the human soul in a comprehensive cosmology (Isacoff 2001, p. 38). Similarly, the essential question for Copernicus was not “Does the earth move?” but, rather, “. . . what motions should we attribute to the earth in order to obtain the simplest and most harmonious geometry of the heavens that will accord with the facts?” (Burt 1954, p. 39). Both Boscovich and Legendre based their contributions to the method of least squares in geometrical formulations (Stigler 1986, pp. 42, 46, 47, 57). Galileo “derived his rule relating time and distance using geometry” (Heilbron 1998, p. 129). Einstein (1922) considered geometry to be “the most ancient branch of physics,” according “special importance” to his view that “all linear measurement in physics is practical geometry,” “because without it I should have been unable to formulate the theory of relativity” (p. 14).

Though the method of least squares is foundational to contemporary statistical analysis, it was originally formulated by Boscovich, who “followed in a Newtonian tradition of giving geometric descriptions rather than analytic ones” (Stigler 1986, pp. 42–43, 51). Boscovich’s work was only later expressed analytically, by Laplace. Pledge (1939) makes the historical connection between geometry and natural law in the general point that

as the Greeks gave us the abstract ideas (point, line, etc.) with which to think of space, and the 17th century those (mass, acceleration, etc.) with which to think of mechanics, so Carnot gave us those needed in thinking of heat engines. In each case the ideas are so pervasive that we use them even to state that they never apply exactly to visible objects (p. 144).

Narens (2002) explicitly roots measurement theory in a Pythagorean sense of scientific definability focused on meaningfulness as invariance across transformations. Maxwell provides the clearest method for making linear measurement analogous with practical geometry (Black 1962; Nersessian 2002; Turner 1955). Inventing the contemporary concept of mathematical modeling (Hesse 1961, p. 206), Maxwell freed physics from the constraints of Newtonian mechanics via his concept of the abstract mathematical field (Rautio 2005, p. 53; McMullin 2002). His work still stands as one of the most productive examples of how to draw geometric analogies of phenomena (Klein 1974, p. 474; Rautio 2005).

To understand Maxwell’s method of analogy, it is important to know that, in the eighteenth and nineteenth centuries, scientists and philosophers in many fields employed Newton’s laws of motion as a framework for structuring investigations

of a wide range of different phenomena. Newton's theory of gravitation provided the form of a Standard Model adopted across the sciences of nature as the hallmark criterion of scientific method (Heilbron 1993, pp. 5–6).

Nersessian (2002) concurs, saying "After Newton, the inverse-square-law model of gravitational force served as a generic model of action-at-a-distance forces for those who tried to bring all forces into the scope of Newtonian mechanics" (p. 139). Maxwell learned the method of drawing analogies from the standard model from his colleague William Thomson (Lord Kelvin), and told him that he "intended to borrow it for a season. . .but applying it in a somewhat different way" (Nersessian 2002, p. 144).

The difference between Thomson's method and Maxwell's use of it is telling. Like Maxwell, Thomson constructed a number of analogies, such as between heat and electrostatics. But Thomson merely took existing equations describing a known physical system and changed the names of the parameters to match the system under investigation (Nersessian 2002, p. 144). This was the typical way in which the Standard Model was applied in research up to that time.

The superficiality of this method, however, made it vulnerable to two errors Maxwell (1965/1890, p. 155) sought to avoid, distraction by abstract mathematical analyses and by too-literal preconceptions of the physical phenomenon. As Maxwell put it,

By referring everything to the purely geometrical idea of the motion of an imaginary fluid, I hope to attain generality and precision, and to avoid the dangers arising from a premature theory professing to explain the cause of the phenomena. . . [so that one might in due course arrive at] a mature theory, in which physical facts will be physically explained (Maxwell 1965/1890, p. 159).

Maxwell (1965/1890, p. 155) considered a too-quick leap to mathematical analysis a distraction, saying purely mathematical simplifications are likely to cause the investigator "entirely lose sight of the phenomena to be explained; and though we may trace out the consequences of given laws, we can never obtain more extended views of the connexions of the subject." In the human and social sciences, little attention is paid to modeling constructs, though there are several significant exceptions (Burdick et al. 2010; Dawson et al. 2006; Stenner et al. 1983; Wilson 2005, 2008) that take up the challenge in ways analogous to the approach advocated by Maxwell, in terms of psychosocial explanations of psychosocial facts.

Maxwell, then, started from simple geometric ideas and built up an understanding of the construct via analogy (Black 1962; Nersessian 2002; Turner 1955). In so doing, he provided "the prototype for all the great triumphs of twentieth-century physics" (Dyson, in Rautio 2005, p. 53). Ludwig Boltzmann considered Maxwell's method of analogy as important as his scientific work (Boumans 2005, pp. 24, 28). Boltzmann's student, Ehrenfest, and Ehrenfest's student, Tinbergen, each employed Maxwell's approach to mathematical modeling and his method of analogy in their studies in economics (Boumans 2005, pp. 24, 28, 31, 41).

Rasch was, then, connected through his associations with Tinbergen, Frisch, and Koopmans (Frisch's and Tinbergen's student) with a direct line of intellectual descent from Maxwell (Fisher 2010). Rasch (1960, pp. 110–115) established a

basis for a Maxwellian Standard Model in the social sciences when he structured his models in the pattern of Maxwell's analysis of mass, force, and acceleration. Few researchers to date, however, have noted or expanded upon the connection Rasch drew between his models and Maxwell's analysis, in large part because Rasch himself did not effectively follow through to a full implementation of Maxwell's method. The quality of research using Rasch's models suffers for this loss.

Rasch presented his models in a manner similar to Thomson's method of merely substituting parameter names across the different phenomena studied, and this is, in effect, exactly how Rasch models are usually applied. Easily performed computer analyses disconnect statistical considerations from the conceptualization and evaluation of the construct (Stenner et al. 1983; Wilson 2013). The question then arises as to how a shift from Thomson's method to Maxwell's might be achieved in the human and social sciences.

Significant untapped potential for such a shift can be found in the shared mathematical formalism of the Pythagorean theorem, the multiplicative structure of natural laws, and Rasch models. These connections suggest much could be gained from closer study of Maxwell's reasoning process (Nersessian 2002) and the ways in which it is similar to and different from predictive construct models.

1.3 Geometry and Natural Law

Figure 1.1 illustrates a proof of the Pythagorean theorem, where the square of the hypotenuse of a right triangle is equal to the sum of the squares of the other two sides:

$$a^2 + b^2 = c^2$$

For Fig. 1.1, this works out as:

$$3^2 + 4^2 = 5^2 = 9 + 16 = 25$$

Most scientific laws are, however, written in a multiplicative form (which also includes equations involving division) (Crease 2004; Taagepera 2008; Burdick et al. 2006), like this:

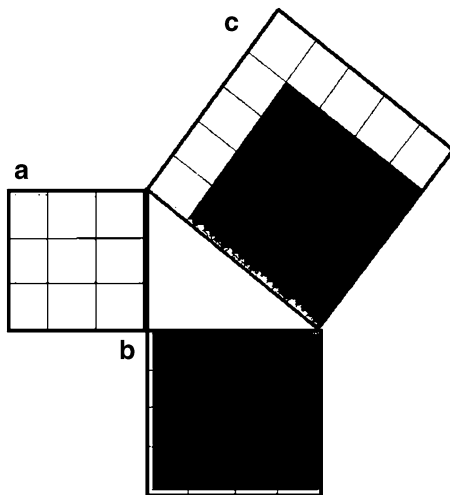
$$a = f/m$$

or

$$f = m * a$$

where the acceleration of an object can be estimated by dividing the applied force by the object's mass, or the force is estimated by multiplying the mass by the

Fig. 1.1 A proof of the Pythagorean theorem



acceleration. This, of course, is how Maxwell (1920/1876) presented Newton’s Second Law.

Other geometric relationships have the same multiplicative form as scientific laws, such as the definition of the circle as a closed arc equidistant from a single point, with the circumference equal to pi times the radius squared. The Pythagorean theorem can also be written in the form of a multiplicative law, by means of the number e (2.71828...) (Maor 1994):

$$e^9 * e^{16} = e^{25}$$

Substituting a for e^9 , b for e^{16} , and c for e^{25} in this description of the triangle in Fig. 1.1 gives:

$$a * b = c$$

and could be solved as

$$8103 * 8,886,015 \approx 72,003,378,611$$

Converting back to the additive form using the natural logarithm, the equation looks like this:

$$\ln(8,103) = \ln(72,003,378,611) - \ln(8,886,015)$$

and this

$$9 = 25 - 16.$$

Whether expressed in multiplicative or additive forms, Newton's Second Law and the Pythagorean theorem both define the way changes in one parameter in a mathematical model result in proportionate changes in the other parameters.

Furthermore, the empirical relational structure stays the same no matter what unit characterizes the numerical relational structure. Maxwell presented Newton's Second Law in this form:

$$A_{vj} = F_j / M_v.$$

Applying catapult j 's force F of 7.389 N (53.445 poundals) to object v 's mass M of 1.6487 kg (3.635 lb) results in an acceleration of 4.4817 m (14.70 ft) per second, per second. (That is, $7.389/1.6487 = 4.4817$, or $53.445/3.635 \approx 14.70$). The proportional relationships are constant no matter which units are used, satisfying the criterion of meaningfulness (Mundy 1986; Narens 2002; Rasch, 1961). In this context, Rasch (1960, 112–113; Burdick et al. 2006) noted that,

If for any two objects we find a certain ratio of their accelerations produced by one instrument, then the same ratio will be found for any other of the instruments. Or, in a slightly mathematized form: The accelerations are proportional.

Conversely, it is true that if for any two instruments we find a certain ratio of the accelerations produced for one object, then the same ratio will be found for any other objects.

Rasch's (1961, p. 322) model for measuring reading ability and text reading difficulty has the multiplicative form of

$$\varepsilon_{vi} = \theta_v \sigma_i$$

and the additive form (Rasch 1961, p. 333):

$$\varepsilon_{vi} = \theta_{v+} \sigma_i.$$

Rasch (1960, pp. 110–115) cites Maxwell's presentation of Newton's Second Law as his source for these formulations. This model takes reading comprehension ε as the product (or the sum) of person v 's reading ability θ and item i 's text complexity σ . The model is also often written as

$$\Pr \{X_{ni} = 1\} = e^{\beta n - \delta i} / 1 + e^{\beta n - \delta i}$$

or

$$P_{ni} = \exp(B_n - D_i) / [1 + \exp(B_n - D_i)]$$

or

$$\ln[P_{ni} / (1 - P_{ni})] = B_n - D_i$$

which is to say that the log-odds of a correct response from person n on item i is equal to the difference between the estimate B of person n 's ability and the estimate D of item i 's difficulty (Wright 1997; Wright and Stone 1979). Moving the effect of e from one side of the equation to the other makes the response odds equal to e taken to the power of the difference between B and D , divided by one plus e to that power.

In light of the proportionality obtained in these relationships, Rasch (Rasch 1960; also see his 1961, p. 325) formulated a separability theorem in terms that apply to both additive and multiplicative forms of the models, saying

It is possible to arrange the observational situation in such a way that from the responses of a number of persons to the set of tests or items in question we may derive two sets of quantities, the distributions of which depend only on the test or item parameters, and only on the personal parameters, respectively. Furthermore, the conditional distribution of the whole set of data for given values of the two sets of quantities does not depend on any of the parameters (p. 122).

The separability of the parameters is evident in the proportionality of the relationships expected by the model. As any one parameter is varied relative to a second parameter, values for the third are predictable. For example, for a person-item interaction in which there is a 0.82 likelihood of a correct response, the odds ratio of 4.556 (0.82/0.18) gives a log-odds (logit) difference of 1.5 between the person ability and item difficulty estimates (see Wright and Stone 1979, p. 16, for a table relating response probabilities to logit differences). Any ability measure that is 1.5 logits different from a difficulty calibration implies a 0.82 probability of a correct response.

If the 1.5 logit difference results from a comparison of a person measure of 2.0 and an item calibration of 0.5, then, to obtain the multiplicative form of the model,

$$\varepsilon_{vi} = \theta_v \sigma_i$$

we have, with the previous values entered

$$e^{2.0} = e^{1.5} * e^{0.5},$$

which is exactly the same equation as that previously used to illustrate Newton's Second Law: $7.389 = 4.4817 * 1.6487$.

1.4 Predictive Construct Modeling

Rasch (1960, 2010/1972) explained how the structure of Newton's second law of motion (relating force, mass, and acceleration) is analogous to the structure of a law relating reading ability, text complexity, and comprehension rates. Rasch held that,

Where this law can be applied it provides a principle of measurement on a ratio scale of both stimulus parameters and object parameters, the conceptual status of which is comparable to that of measuring mass and force. Thus...the reading accuracy of a child... can be measured with the same kind of objectivity as we may tell its weight... (p. 115)

Wright (1997, p. 44), a physicist who worked with Nobelists Townes and Mulliken before turning to psychology and collaborations with Rasch, concurs, saying, “Today there is no methodological reason why social science cannot become as stable, as reproducible, and hence as useful as physics.” Andrich (1988, p. 22) observes that “. . .when the key features of a statistical model relevant to the analysis of social science data are the same as those of the laws of physics, then those features are difficult to ignore.”

In his retirement speech, after describing multiple examples and elaborating the logic of the analogy in detail, as he also had in his book (Rasch 1960, pp. 110–115), Rasch (2010/1972) concluded that,

With all of this available to us, we will have an instrumentarium with which many kinds of problems in the social sciences can be formulated and handled with the same types of mathematical tools that physics has at its disposal—without it becoming a case of superficial analogies (p. 1272).

But nowhere in his book, retirement lecture, or other publications does Rasch provide a theory of a substantive construct behaving in accord with the structure of a lawful regularity. As Maxwell understood would happen, the convenient analytical formulation of Rasch’s models has caused us to lose sight of the phenomena to be explained, such that we “never obtain more extended views of the connexions of the subject” (Maxwell 1965/1890, p. 155). Rasch emphasized the positing and testing of invariances, but ignored the constitutive cause and effect relationships.

In asserting that “Thereby you can gradually reach a clarification of the field of validity of the law,” and in next taking “a closer look at the contents of the law,” Rasch (2010/1972, p. 1254) does not follow Maxwell’s process. Rasch does not try to explain individual-centered variation in a psychological or social phenomenon in psychological or social terms, as one would in investigations emulating Maxwell’s interest in explaining a physical phenomenon in physical terms. Instead, Rasch’s focus on the contents of the law is strictly mathematical. His concern is with the nature of the independence of the comparisons made in a context of infinite possibility. He shows how the frame of reference provides a means for defining all possible relevant observational situations, but he does not show, as does Maxwell for electromagnetism, what makes any given observation conform to the model in the way that it does.

In the wake of Rasch’s work and later large-scale studies equating high stakes reading tests (Jaeger 1973; Rentz and Bashaw 1977), however, Stenner and colleagues (Stenner 2001; Stenner et al. 2006) developed an effective and parsimonious predictive theory of what makes text easy or difficult to read. Others have similarly devised predictive models of other cognitive and behavioral constructs (Dawson et al. 2006; Embretson 1998; Fischer 1973; Fisher 2008; Green and Kluever 1992; Wilson 2008) with the aim of achieving the degree of control over the instrumentation needed for the reliable and highly efficient automated production of assessment items (Bejar et al. 2003; Stenner and Stone 2003).

Generalizing these accomplishments requires a systematic and methodical way of interweaving substantive qualitative content and abstract mathematical construct

issues. Various systems for assessing constructs (Embretson 1998; Stenner and Smith 1982; Stenner et al. 1983; Burdick et al. 2010; Wilson 2005) set the stage for fuller realizations of model-based reasoning in the psychosocial sciences by prioritizing theory development. In the context of these systems, hypotheses are formulated and tested by iterating through a sequence of moments in a method, any one of which may serve as a point of entry or exit. Building on the way in which data, instruments, and theory have each historically served to mediate each other's interrelations in the history of science (Ackermann 1985), and focusing on the predictive control of the construct, new horizons for qualitatively-informed quantitative social science can be envisioned.

References

- Ackermann, J. R. (1985). *Data, instruments, and theory: A dialectical approach to understanding science*. Princeton: Princeton University Press.
- Andrich, D. (1988). *Rasch models for measurement* (Sage University paper series on quantitative applications in the social sciences, Vol. 07–068). Beverly Hills: Sage Publications.
- Bejar, I., Lawless, R. R., Morley, M. E., Wagner, M. E., Bennett, R. E., & Revuelta, J. (2003). A feasibility study of on-the-fly item generation in adaptive testing. *The Journal of Technology, Learning, and Assessment*, 2(3), 1–29.
- Black, M. (1962). *Models and metaphors*. Ithaca: Cornell University Press.
- Boumans, M. (2005). *How economists model the world into numbers*. New York: Routledge.
- Burdick, D. S., Stone, M. H., & Stenner, A. J. (2006). The combined Gas Law and a Rasch Reading Law. *Rasch Measurement Transactions*, 20(2), 1059–1060.
- Burdick, D. S., Stenner, A. J., & Kyngdon, A. (2010, Summer). From model to measurement with dichotomous items. *Journal of Applied Measurement*, 11(2), 112–121.
- Burt, E. A. (1954). *The metaphysical foundations of modern physical science* (Rev. ed.). Garden City: Doubleday Anchor.
- Crease, R. (2004). The greatest equations ever. *Physics World*, 17(10), 19.
- Dawson, T. L., Fischer, K. W., & Stein, Z. (2006). Reconsidering qualitative and quantitative research approaches: A cognitive developmental perspective. *New Ideas in Psychology*, 24, 229–239.
- Einstein, A. (1922). Geometry and experience. In G. B. Jeffery, W. Perrett (Trans.), *Sidelights on relativity* (pp. 12–23). London: Methuen & Co., Ltd.
- Embretson, S. E. (1998). A cognitive design system approach to generating valid tests: Application to abstract reasoning. *Psychological Methods*, 3(3), 380–396.
- Fischer, G. H. (1973). The linear logistic test model as an instrument in educational research. *Acta Psychologica*, 37, 359–374.
- Fisher, W. P., Jr. (2008). *A predictive theory for the calibration of physical functioning patient survey items*. Presented at the second conference on patient reported outcome measurement information systems, Bethesda: NIH and NIAMS, March 2–5.
- Fisher, W. P., Jr. (2010). The standard model in the history of the natural sciences, econometrics, and the social sciences. *Journal of Physics: Conference Series*, 238(1). http://iopscience.iop.org/1742-6596/238/1/012016/pdf/1742-6596_238_1_012016.pdf.
- Green, K. E., & Kluever, R. C. (1992). Components of item difficulty of Raven's matrices. *The Journal of General Psychology*, 119, 189–199.

- Heilbron, J. L. (1993). Weighing imponderables and other quantitative science around 1800. *Historical Studies in the Physical and Biological Sciences*, 24(Supplement), Pt. I, 1–337.
- Heilbron, J. L. (1998). *Geometry civilized: History, culture, and technique*. Oxford: Clarendon.
- Hesse, M. (1961). *Forces and fields: A study of action at a distance in the history of physics*. London: Thomas Nelson and Sons.
- Isacoff, S. M. (2001). *Temperament: The idea that solved music's greatest riddle*. New York: Knopf.
- Jaeger, R. M. (1973). The national test equating study in reading (The Anchor Test Study). *Measurement in Education*, 4, 1–8.
- Klein, H. A. (1974). *The world of measurements: Masterpieces, mysteries and muddles of metrology*. New York: Simon & Schuster.
- Maor, E. (1994). *E: The story of a number*. Princeton: Princeton University Press.
- Maxwell, J. C. (1920/1876). *Matter and motion* (J. Larmor, Ed.). New York: The Macmillan Co.
- Maxwell, J. C. (1965/1890). *The scientific papers of James Clerk Maxwell* (W. D. Niven, Ed.). New York: Dover Publications.
- McMullin, E. (2002). The origins of the field concept in physics. *Physics in Perspective*, 4(1), 13–39.
- Mundy, B. (1986). On the general theory of meaningful representation. *Synthese*, 67(3), 391–437.
- Narens, L. (2002). A meaningful justification for the representational theory of measurement. *Journal of Mathematical Psychology*, 46(6), 746–768.
- Nersessian, N. J. (2002). Maxwell and “the method of physical analogy”: Model-based reasoning, generic abstraction, and conceptual change. In D. Malament (Ed.), *Essays in the history and philosophy of science and mathematics* (pp. 129–166). Lasalle: Open Court.
- Pledge, H. T. (1939). *Science since 1500: A short history of mathematics, physics, chemistry, biology*. London: His Majesty's Stationery Office.
- Rasch, G. (1960). *Probabilistic models for some intelligence and attainment tests* (Reprint, with Foreword and Afterword by B. D. Wright, Chicago: University of Chicago Press, 1980). Copenhagen: Danmarks Paedagogiske Institut.
- Rasch, G. (1961). On general laws and the meaning of measurement in psychology. In J. Neyman (Ed.), *Proceedings of the fourth Berkeley symposium on mathematical statistics and probability* (Contributions to biology and problems of medicine, Vol. IV, pp. 321–333). Berkeley: University of California Press.
- Rasch, G. (2010/1972). Retirement lecture of 9 March 1972: Objectivity in social sciences: A method problem (C. Kreiner, Trans.). *Rasch Measurement Transactions*, 24(1), 1252–1272.
- Rautio, J. C. (2005). Maxwell's legacy. *IEEE Microwave Magazine*, 6(2), 46–53.
- Rentz, R. R., & Bashaw, W. L. (1977). The National Reference Scale for Reading: An application of the Rasch model. *Journal of Educational Measurement*, 14(2), 161–179.
- Stenner, A. J. (2001). The Lexile Framework: A common metric for matching readers and texts. *California School Library Journal*, 25(1), 41–42.
- Stenner, A. J., & Stone, M. (2003). Item specification vs. item banking. *Rasch Measurement Transactions*, 17(3), 929–930.
- Stenner, A. J., & Smith, M., III. (1982). Testing construct theories. *Perceptual and Motor Skills*, 55, 415–426.
- Stenner, A. J., Smith, M., III, & Burdick, D. S. (1983, Winter). Toward a theory of construct definition. *Journal of Educational Measurement*, 20(4), 305–316.
- Stenner, A. J., Burdick, H., Sanford, E. E., & Burdick, D. S. (2006). How accurate are Lexile text measures? *Journal of Applied Measurement*, 7(3), 307–322.
- Stigler, S. (1986). *The history of statistics: The measurement of uncertainty before 1900*. Cambridge: Harvard University Press.
- Taagepera, R. (2008). *Making social sciences more scientific: The need for predictive models*. New York: Oxford University Press.
- Turner, J. (1955). Maxwell on the method of physical analogy. *The British Journal for the Philosophy of Science*, 6, 226–238.

- Wilson, M. (2005). *Constructing measures: An item response modeling approach*. Mahwah: Lawrence Erlbaum Associates.
- Wilson, M. (2008). Cognitive diagnosis using item response models. *Zeitschrift Für Psychologie/ Journal of Psychology*, 216(2), 74–88.
- Wilson, M. (2013). Seeking a balance between the statistical and scientific elements in psychometrics. *Psychometrika*, 78(2), 211–236.
- Wright, B. D. (1997). A history of social science measurement. *Educational Measurement: Issues and Practice*, 16(4), 33–45, 52.
- Wright, B. D., & Stone, M. H. (1979). *Best test design: Rasch measurement*. Chicago: MESA Press.