Chapter 9
The Arrival Passenger Flow Short-Term Forecasting of Urban Rail Transit Based on the Fractal Theory

Liangliang Zhang, Yuanhua Jia, Xihui Yin, and Zhong-hai Niu

Abstract According to fractal characteristics of the arrival passenger flow volume of the urban rail transit station, the paper established the arrival passenger flow volume short-term forecasting model based on the fractal theory. Then, the paper took the *AHQB* station of MRT Line Four in Beijing as an example and forecasted the short-term arrival passenger flow volume. Lastly, the paper gave a comparison between the predicted value and the actual value, and found that the maximum error was less than 7 %. It indicated that the model established could be used to forecast the short-term arrival passenger flow volume of the urban rail transit, and had better adaptability.

Keywords Urban rail transit • Arrival passenger flow volume • Fractal theory • Short-term forecasting

9.1 Introduction

Urban rail transit is the important part of the urban comprehensive traffic and transportation, and plays a great role in guiding and improving construction location of city. In recent years, urban rail transit has developed especially rapidly in China. In some big cities, the urban rail transit network has begun to form rail transit network, for example Beijing, Shanghai, Guangzhou and so on. Other cities are also actively prepared for the construction of urban rail transit. According to an urban rail transit planning, there are about 40 cities that have urban rail transit by 2020, in China, and the total mileage of the rail transit will be 7,000 km.

Not only planning and construction, but also the department organized the passenger flow when it had put into operation, did urban rail transit passenger

L. Zhang • Y. Jia (✉) • X. Yin • Z-h. Niu
Institute of Systems Engineering and Control, School of Traffic and Transportation, Beijing Jiaotong University, Beijing, China
e-mail: 10114212@bjtu.edu.cn

H. Xia and Y. Zhang (eds.), *The 2nd International Symposium on Rail Transit Comprehensive Development (ISRTCD) Proceedings*,
DOI 10.1007/978-3-642-37589-7_9, © Springer-Verlag Berlin Heidelberg 2014

flow volume forecasting be very significant. At present, there were many researches on urban rail transit passenger flow volume forecasting in domestic and foreign. They used Four-stage Model [1–3] and Disaggregate Model [4, 5] for forecasting urban rail passenger flow volume, but they mainly focused on the long-term forecasting. At the same time, domestic and foreign researcher used linear neural network [6–8] and other methods which used a historical data of some day to forecast passenger flow volume of other day. According to analyzing the history of their researches and practices, the paper found that current researches on urban passenger flow volume forecasting were more macro, and could not reflect the unbalanced passenger flow, especially the peak hours and off-peak hours. Therefore, in order to achieve short-term arrival urban rail station passenger flow volume forecasting research, the paper established the short-term arrival passenger flow volume forecasting model based to the fractal theory, and hoped the method and the forecasted result could be used for passenger flow organization and optical allocation facilities.

9.2 Fractal Theory

The fractal theory was founded by B.B. Mandelbrot, and could effectively describe a variety of complex graphics in nature and scientific research [9]. Since it was founded, the theory had been widely used in mathematics, physics, and so on. In recent years, it was used for many other fields (for example, the linguistics, economics, and social science), and became modern and an important branch of nonlinear science. Along with in-depth research and applications expanded in more and more fields, it contained several meanings [10].

1. Fractal could be geometry and also could be mathematical models that constructed by using the functions and information.
2. Fractal had self-similarity of three aspects including the morphology, function and information, also could have self-similarity of one aspect.
3. Fractal set has some form of self-similarity, and may be approximate self-similar or statistical self-similar.
4. Self-similarity of fractal had some variation in the level, and fractal has infinitely nested hierarchy in mathematics.
5. Fractal could contain complex detail of all elements in different scale, and also could give a quantitative description about fractal geometry of the complex nature.

9.3 Model Formulas

9.3.1 Fractal Model

The fractal dimension was a significant parameter that could be used to describe quantitatively characterization of fractal. Based on the definition of fractal theory, the fractal distribution could be defined a formula of power-exponent [11].

$$\mathrm{N} = \frac{\mathrm{c}}{\mathrm{r}^{\mathrm{D}}} \tag{9.1}$$

Where N which related r variable was a variable, r variable was characteristic length (time, length, etc), D is fractal dimension, and c was a constant.

Then the paper used natural logarithm (base e) to change the formula (9.1), and obtained the formula (9.2).

$$\ln N = \ln c - \mathrm{D}\ln r \tag{9.2}$$

In common, D which was fractal dimension was a constant, so that the formula (9.2) was an approximately straight line in the logarithmic coordinates. Therefore, according to two points ($\mathbf{N_i},\mathbf{r_i}$) and ($\mathbf{N_j},\mathbf{r_j}$) of the approximately straight line, the paper could calculate D and c.

$$\mathrm{D} = \frac{\ln\left(^{\mathrm{N_i}}/_{\mathrm{N_j}}\right)}{\ln\left(^{\mathrm{r_j}}/_{\mathrm{r_i}}\right)} \tag{9.3}$$

However, the raw data could only get some discrete points in practical application, and the function was unknown. In this case, the paper needed to use the transformed method, so that the data could be associated with a fractal distribution model when they were done a series of transformation. In prior researches, a commonly transformed method is CUSUM. The paper gave the detailed steps in following content.

(1) The paper layout the point ($\mathrm{N_i},\mathrm{r_i}$), (i = 1,2,3,..., n.) in double logarithmic coordinate. The raw data may be some discrete points in common. The paper must use transformed method if the raw data was discrete points.
(2) The paper established various orders CUSUM that was $\left\{\mathrm{S_i^{(n)'}}\right\}$: first order was $\mathrm{S_i'} = \mathrm{N_1} + \mathrm{N_2} + \cdots + \mathrm{N_i}$, i = 1, 2, 3,.., n; second order was $\mathrm{S_i''} = \mathrm{S_1'} + \mathrm{S_2'} + \cdots + \mathrm{S_i'}$;and n order $\mathrm{S_i^{(n)'}} = \mathrm{S_1^{(n-1)'}} + \mathrm{S_2^{(n-1)'}} + \cdots + \mathrm{S_i^{(n-1)'}}$, i = 1,2,3,.., n.
(3) On the basic of step (2), the paper established fractal model, calculated various orders CUSUM's fractal dimension, and determined the final result of D and c.

(4) Finally, the paper forecasted the result, $\mathbf{S}_{\mathbf{i+1}}^{(\mathbf{n})'}$. According to the formula $\left(S_{i+1}^{(n-1)'} = S_{i+1}^{(n)'} - S_{i}^{(n)'}\right)$, the paper obtained the N_i.

9.3.2 R/S Analysis

Nowadays, R/S analysis was used to analyze whether a time series has fractal property or not. In common, the progress of the R/S analysis contained four steps.

1. The paper assumed the existence of time series $x = x_1, x_2, \ldots, x_n$;
2. The paper calculated the mean value E(x) and standard deviation S(n);
3. The paper calculated accumulation's deviation $X(i,n) = \sum x_n - E(x)_n$;
4. The paper calculated ranges $R(n) = \max X(i,n) - \min X(i,n)$.

Hurst index was determined by formula $R(n)/S(n) = c * n^H$. According to Brownian movement, the fractal dimension could be a formula $D = 2 - H$. Hurst index could be calculated from R/S analysis, and judging the trend of development of the time series. When the H was 0.5, the time series was a common Brownian movement; the time series also had fractal dimension when the H was not 0.5.

9.4 Computational Experiments

The paper took the *AHQB* station of MRT Line Four in Beijing as an example. Due to the arrival passenger flow volume during peak hours was more significant that the station organize the passenger flow, the paper took the arrival passenger flow volume from 7:00 to 8:30 as object of research, and divided the one and half hours into 18 periods of time in cycle lasting 5 min.

For the convenience of calculation, the paper defined 7:00–7:05 as 1 period of time (called r_1), and the arrival passenger flow volume was N_1; the paper define 7:06–7:10 as 2 period of time (r_2), and the arrival passenger flow volume was N_2; and so on, the 8:26–8:30 as 18 period of time (call r_{18}), and the arrival passenger flow volume was N_{18}. Based on the definitions above all, the paper draw line graph of the actual passenger flow value, in Fig. 9.1.

According to the process of R/S analysis, the paper got the result of the Hurst index. In order to reduce error, the paper chose the some points. Finally, the Hurst index was 0.031, and the paper verified the arrival passenger flow of urban rail transit had fractal property. So the paper could use fractal model to forecast the arrival passenger flow.

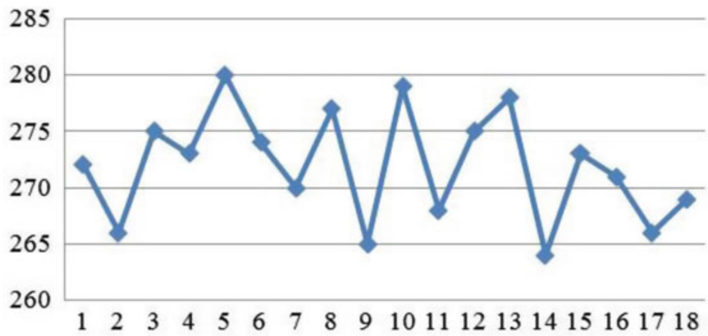

Fig. 9.1 The arrival passenger flow volume of *AHQB* Station 1–18 periods of time (x-axis was r_i; y-axis was N_i)

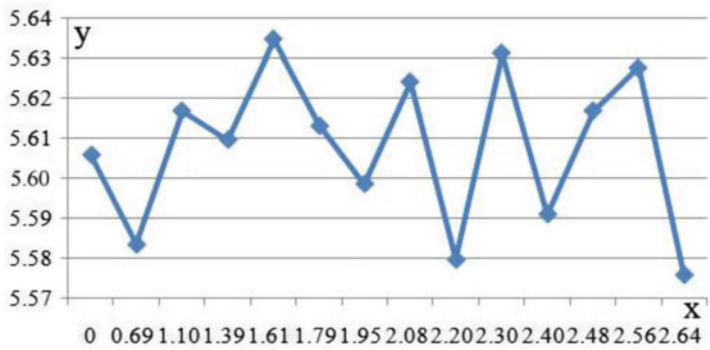

Fig. 9.2 The point (N_i, r_i) were layout in double logarithmic coordinate (x-axis was ln(r_i); y-axis was ln(N_i))

In the forecasted process, the paper used the first 14 period of time (i.e. 7:00–8:10) and lay out the point (N_i,r_i), ($i = 1, 2, 3,\ldots, n.$) in double logarithmic coordinate. Then, the paper obtained the Fig. 9.2.

From the Fig. 9.2, we could conclude that the raw data was discrete points. Therefore, the paper should use transformed method to obtain the known function. According to the formula (9.3), the paper calculated the results of the various orders CUSUM, and their fractal dimension. The final results of fractal dimension were shown in Table 9.1.

In the Table 9.1, DS represented the calculated results that used initial point (N_i,r_i) and point (N_{i+1},r_{i+1}) according to formula (9.3), DS′ represented the results that used first order CUSUM method, DS″ represented the results that used second order CUSUM method; DS‴ represented the results that used third order CUSUM method; and DS⁗ represented the results that used firth order CUSUM method.

From the Table 9.1, we could conclude that the results of the fractal dimensions were not only positive but also negative if calculation used the initial values.

Table 9.1 the results of fractal dimension which used the first 14 periods of time

	DS	DS′	DS″	DS‴	DS⁗
1	0.0322	−0.9840	−1.5743	−1.9920	−2.3155
2	−0.0821	−1.0183	−1.7141	−2.2599	−2.7077
3	0.0254	−1.0064	−1.7808	−2.4120	−2.9460
4	−0.1135	−1.0280	−1.8297	−2.5147	−3.1101
5	0.1188	−1.0027	−1.8551	−2.5857	−3.2294
6	0.0954	−0.9887	−1.8705	−2.6369	−3.3197
7	−0.1917	−1.0142	−1.8885	−2.6776	−3.3909
8	0.3760	−0.9711	−1.8938	−2.7082	−3.4480
9	−0.4886	−1.0228	−1.9082	−2.7347	−3.4952
10	0.4220	−0.9822	−1.9129	−2.7560	−3.5348
11	−0.2963	−1.0083	−1.9212	−2.7744	−3.5686
12	−0.1356	−1.0182	−1.9295	−2.7908	−3.5979
13	0.6973	−0.9674	1.9298	−2.8040	−3.6233

Table 9.2 The forecasting value compared with actual value and the relative error

	Actual value	Forecasting value	Relative error (%)
15	273	260	−4.76
16	271	259	−4.41
17	266	272	2.26
18	269	252	−6.32

However, all of the fractal dimensions were negative when the paper used the various CUSUM. By contrast, the results of the second order CUSUM were better than others', and the last two results' deviation was 0.0003.

So, the paper chose the arrival passenger flow volume of 13th and 14th period of time to calculate the fractal dimension D.

$$D = -1.9298.$$

Then, according to formula (9.2), the paper calculated the result of c.

$$c = 175.8387.$$

Lastly, according to formula (9.1), the paper calculated the results of S''_{15}, S''_{16}, S''_{17}, and S''_{18}. On the basis of result of S''_{15}, S''_{16}, S''_{17}, and S''_{18}, the paper used $S'_{i+1} = S''_{i+1} - S''_{i}$ to calculate the arrival passenger flow volume of the 15th, 16th, 17th and 18th period of time. The results were shown in Table 9.2. The paper compared the actual value of the 15th, 16th, 17th and 18th period of time with the forecasting value of the 15th, 16th, 17th and 18th period of time, and found that the maximum relative error was −6.32 %.

9.5 Conclusions

The arrival passenger flow volume, especially the arrival passenger flow volume during peak hours, was the base of urban rail transit station which organized passenger flow and optical allocation facilities. Therefore, the paper established the short-term arrival passenger flow volume forecasting model, took the AHQB station of the MRT Line Four in Beijing as an example. Lastly, the paper gave a comparison between the predicted value and the actual value, and found that the maximum error was less than 7 %. It indicated that the established model could be used for short-term arrival passenger flow volume forecasting However, due to the fractal dimension D and the constant c were approximate, and the relative error was increased if the expansion of the forecasting increased.

Acknowledgments Special thanks to Yuanhua Jia, my teacher, and Xihui Yin, Zhong-hai Niu, my classmates, for all information and advises.

References

1. Tan Ben-xuan (2009) Research on urban rail transit passenger flow forecasting technology [D]. Wuhan University Technology, Wuhan
2. Li Qing-hua (2011) The comparison of urban transit demand forecasting method [J]. Technol Econ Areas Commun 1(63):54–57
3. Lu S, Guo X-c (2001) Method of the URT's passenger flow forecast [J]. Syst Eng Theory Pract 21(8):106–110
4. Yan Ji, Shao Chun-fu (2001) Passenger flow forecast of urban rail line based on multi-logit method [J]. J Xi'an Highway Univ 3:68–72
5. He Ming, Guo Xiu-cheng (2010) Forecasting rail transit split with disaggregated MNL model [J]. J Transp Syst Eng Inf Technol 10(2):136–142
6. Ran Y (2010) Study on passenger flow forecasting and operation scheduling of urban rail transit [D]. Beijing Jiaotong University, Beijing
7. Wang Li-na (2011) Traffic forecasting and scheduling of the urban passenger rail network based on historical data [J]. Beijing Jiaotong University, Beijing
8. Mao Jing (2011) Urban rail traffic passenger flow short-term forecasting method and empirical research [D]. Beijing Jiaotong University, Beijing
9. Mandelbrot BB (1997) Fractals: form, chance and dimension [M]. W H Freeman, San Francisco
10. Lin Hong-yi (1992) Fractal theory-singularity discussed [M]. Beijing Institute of Technology Press, Beijing
11. Turcotte DL (1992) Fractals and chaos in geology and geophysics [M]. Cambridge University Press, Cambridge