Chapter 36
The Optimization Model of Urban Transit Departure Frequency

Yuan Li and Bingfeng Si

Abstract Transit operator is the key of public transport management. To enhance the level of public transport, departure frequency management is an important means of improving the service quality of urban public transport and attracting passengers. This paper analyzes urban transit departure frequency in terms of service quality and efficiency then builds transit service frequency optimal model, with multitarget optimization model to passengers′ travel cost as well as for the benefit of the transit company. The genetic algorithms is used to solve this model. Based on realistic transit service survey data, the model is applied in transit route and proved to be useful.

Keywords Transit operator • Departure frequency • Optimization model • Genetic algorithms

36.1 Introduction

Departure frequency optimization is an important part of the transit operators. Domestic and foreign scholars have conducted a lot of research, in which the majority through a supply or demand model to determine the departure frequency of the line [1–3]. Furth and Wilson [4] establish the model for assigning the frequencies in the given set of transit lines by optimizing the sum of the waiting time and travel time. Hasselstrom sought to determine simultaneously the itinerary and the frequency of transit lines on a general network. Stephanedes and Kwon consider the fare as a decision variable, as well as the line frequencies.

The departure frequency optimization directly affects the condition of urban transit. Because t frequency optimization is an important research subject that can

Y. Li (✉) • B. Si
School of Traffic and Transportation, Beijing Jiaotong University,
100044 Beijing, China
e-mail: 11120963@bjtu.edu.cn

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solve practical transit problems and can produce high benefits, it is very necessary to formulate more practical models and algorithms.

This paper aims at minimum passenger travel time costs and the economic benefits of transit corporation, establishes the transit frequency optimization model. Because the model is a transit scheduling problem, the practical application of the mode will be complexity with the increase of travel times, sections and site, which means very difficult to solve the model accurately. In this paper, an improved genetic algorithm is used to solve this problem.

36.2 Model Optimization

36.2.1 Assumptions

All vehicles are running along lines, not allowing more stations and overtaking [5]; The vehicles are running with the scheduling timetable, traffic jams and other accidents are not considered [6]; The parking time of the vehicle at each station is a fixed value; In the period of optimization, the demand flow is independent from departure time instant; The waiting time passengers accept is a fixed value, over the value passengers will generate dissatisfaction.

36.2.2 Variable Definition

f—departure frequency; $\lambda_i(t)$—passenger arrival rate of the i-th station at time t, people/min; t_{ij}—the time of the j-th vehicle arriving the i-th station; S_j^i—the parking time of the j-th vehicle at the i-th station; M_p^w—the time costs of passengers waiting at the station; M_p^t—the time costs of passengers running in the vehicle; l—passengers order; H—the waiting time passengers accept; ω—the number of seats in the vehicle; ρ_0—the satisfaction membership value of standing passengers when not crowded; x_1—the number of standing passengers in the vehicle; G_j^i—due to the limitations of the vehicle capacity, the number of passengers not boarding the vehicle and continue waiting when the j-th vehicle arrives the i-th station; $h_{\max} h_{\min}$—maximum/minimum departure interval of adjacent cars (min).

36.2.3 Total Passenger Costs

In addition to the fare, the main source of the passengers cost can be converted into the waiting time in the station and the in-vehicle time, multiplied with the corresponding time value, we can obtained the corresponding total passengers time cost.

36.2.3.1 The Waiting Time of Passengers at a Station

The waiting time of passengers at a station C_h includes the cost passengers waiting vehicle arriving c_ρ the cost passengers waiting boarding c_u and the remaining passengers extra time cost c_φ

$$C_h = c_p + c_u + c_\varphi. \tag{36.1}$$

Where

$$c_\rho\left(f, w_j^i\right) = \zeta_h M_p^w \sum_j \sum_i \sum_{i=1}^{w_j^i} \left(1/f + t + S_j^{i-1} - l/\int_{t_{ij}-H}^{t_{ij}} \lambda_i(t)dt\right). \tag{36.2}$$

ζ_h is the satisfaction coefficient of the passenger waiting, which is the satisfied passengers proportion of the total number. Considering the number of overtime passengers u_j^i corresponding to in the j-th vehicle in the i-th station, divided into three cases;

$$u_j^i = \begin{cases} 0 & P_j^i - \int_{t_{ij-H}}^{t_{ij}} \lambda_i(t)dt \le 0 \\ P_j^i - \int_{t_{ij-H}}^{t_{ij}} \lambda_i(t)dt & P_j^i - \int_{t_{ij-H}}^{t_{ij}} \lambda_i(t)dt > 0 \text{ and } P_{j-1}^i - \int_{t_{ij-H}}^{t_{ij}} \lambda_i(t)dt - U_{j-1}^i \le 0 \\ P_j^i - \int_{t_{ij-H}}^{t_{ij}} \lambda_i(t)dt - \left(P_{j-1}^i - \int_{t_{ij-H}}^{t_{ij}} \lambda_i(t)dt - W_{j-1}^i\right) & else \end{cases} \tag{36.3}$$

Given an Gi initial state, we can calculate the number of unsatisfied passengers vehicle-by-station are not. Therefore, the satisfaction of the passengers waiting meets

$$\zeta_h = \frac{\sum_{i=1}^m \int_{T_1}^{T_2} \lambda_i(t)dt - \sum_{i=1}^m \sum_{j=1}^n u_j^i}{\sum_{i=1}^m \int_{T_1}^{T_2} \lambda_i(t)dt} \tag{36.4}$$

The cost passengers waiting boarding c_u is the waiting time before vehicle arriving and the time passengers spent during the parking time,

$$c_u\left(W_j^i\right) = M_p^w \sum_j \sum_i \sum_{i=1}^{w_j^i} (l-1)\overline{u} \tag{36.5}$$

The remaining passengers extra time cost c_φ is the sum of the waiting time for the former vehicle, the waiting time for the present vehicle and the parking time of the present vehicle, i.e.

$$c_\varphi\left(f, G_j^i\right) = M_p^w \varphi \sum_j \sum_i \left(\sum_{i=w_j^i+1}^{w_j^i+G_j^i} \left(1/f + S_j^{i-1} - (l-1)/\int_{t_{ij-H}}^{t_{ij}} \lambda_i(t)dt\right) + G_j^i/f \right. \tag{36.6}$$

36.2.3.2 In-Vehicle Time Cost

The in-vehicle time cost $C_c^{'}$ including two parts: the in-vehicle time of the in vehicle passengers $c_{t1}^{'}$ and the in-vehicle time of the passengers getting off at the i-th station c_{t2}, i.e.

$$C_c^{'} = c_{t1}^{'} + c_{t2} \tag{36.7}$$

Introducing the passenger comfort ζ_c adjust the in-vehicle time proportion in the total cost, i.e.

$$c_{t1}^{'} = \zeta_c M_p^t \sum_j \sum_i \left(A_j^i t_0 + S_j^i A_j^i (1 - k_i)\right) \tag{36.8}$$

The density of the passengers in-vehicle can effectively reflect ζ_c, expressed as a function of the degree of membership. The maximum of membership is 1, becoming smaller with the increasing in-vehicle congestion. The satisfaction membership of the standing passengers in-vehicle is expressed by $\zeta(x_1)(0, V_j - w)$, i.e.

$$\zeta(x_1) = \begin{cases} \rho_0 & 0 < B_j^i - w < \delta_0 \\ \dfrac{V_j - w - x_1}{V_j - w - \delta_0}\rho_0 & \text{else} \end{cases} \tag{36.9}$$

Thus

$$\zeta_c = \frac{\sum_{j=1} \sum_{i=1} b_j^i}{\sum_{j=1}^{n} \sum_{i=1}^{m-1} B_j^i} \tag{36.10}$$

The passengers getting off the vehicle at the i-th station time cost is

$$c_{t2} = M_p^t \sum_j \sum_i \left(\sum_{i=1}^{w_j^i} \left(S_j^i - (l-1)\overline{u} \right) + \sum_{l=1}^{A_j^i q_i} \overline{d} n \right) \tag{36.11}$$

36.2.3.3 Total Passenger Costs

The Total passenger costs is the sum of waiting time and in-vehicle time, i.e.

$$R_p^z = w_h C_h + w_z C_c^{'} \tag{36.12}$$

36.2.4 Transit Company Income

36.2.4.1 Operating Income

Operating income main depends on the flow, that the product of the total passenger and fare, i.e.

$$N_s = C_p^r \sum_j \sum_i w_j^i \tag{36.13}$$

36.2.4.2 Operating Cost

The operating cost includes the vehicle cost and the variable cost, i.e.

$$N_c = \varepsilon_v + \varepsilon_0 \tag{36.14}$$

the vehicle cost

$$\varepsilon_v = C_c^v T/f \tag{36.15}$$

the variable cost

$$\varepsilon_0 = C_c^0 \sum_i D_i T/f \tag{36.16}$$

36.2.4.3 Total Income

The total income equals the operating income minusing the operating cost, i.e.

$$R_c^z = N_s - N_c \tag{36.17}$$

36.2.5 The Departure Frequency Optimization Model

In summary, the departure frequency optimization model is as following:

$$\max R = R_p^z - w_s R_C^Z \tag{L1}$$

Subject to

$$\left|\left(t_{j+1} - t_j\right) - \left(t_j - t_{j-1}\right)\right| \le \sigma \quad (j = 2, 3, \ldots, n-1)$$
$$h_{\min} \le t_j - t_{j-1} \le h_{\max} \quad (j = 2, 3, \ldots, n)$$
$$N_s \ge R_{\min}$$

36.3 Model Algorithm

Based on the Genetic Algorithm, the solution is expressed using One-dimensional string structure data. The repeatedly genetic manipulations bring the constantly evolutionary processes; finally it would converge to the best solution.

Step 1: The alleles are constituted by {0, 1}; we set the length of the encoded string as the scheduling period represented as letter T, the accuracy of the scheduled time is 1 min, each of the encoded bits is corresponded to 1 min in the scheduled time. The value of the encoded bit represents the selection of docking site, '1' means pit stop, and '0' means pass by [7].
Step 2: Find and note the position of the '1' in the encoded string in proper order, and then note the number of the '1'.
Step 3: First decoding the individual the code string getting the individual phenotype. Than calculate the objective function value. At last, according to the conversion calculate the individual fitness.
Step 4: Using penalty function to express the constraints.
Step 5: Using heuristic methods to generate the initial population of each individual gene.
Step 6: The length of the individual l is T, the size of the population is 100, the crossover probability is 0.6, the mutation probability is 0.005, the terminate algebra corpse is 500.
Step 7: Iterative to P(500), the algorithm terminates; if 10 consecutive generations keeps the same, the algorithm terminates.

36.4 Example

The parameter value is shown in Table 36.1.

Through the matlab simulation, the most suitable departure interval is 1 min in peak and 10 min in Non-peak.

Table 36.1 The parameter value

Parameter	Value	Parameter	Value
Operation time	6:00–22:00	Fare	2 yuan
Peak:	7:00–9:00	$h_{\min}$	1
Second peak	9:00–10:00, 16:00–17:00, 19:00–20:00	$h_{\max}$	10
Waiting time (peak)	3 min	σ	3
waiting time (Non-peak)	6 min	$R_{\min}$	120 yuan

36.5 Conclusion

The model describe the process of transit operators accurately, such as the use of the passenger arriving function, the proportion of the passengers to alight function and the remaining passengers in the station.

But in the specific use, it is pending further investigation and analysis whether the comfort of the membership function can reflect the actual situation.

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