Abduction and Model Based Reasoning in Plato's *Meno*

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Abstract In the *elenchus* of *Meno*, Socrates employs simultaneous algorithmic and abductive visual model-based reasoning. Even though the algorithmic method would quickly provide the answer, Socrates' purpose is to make the slave boy recollect the Form of Diagonal. Recollection itself is abductive discovery and hypothesis generation. Contrary to standard interpretation true opinion rather than knowledge is recollected. For knowledge, a tether, an account or justification is required that cannot be recollected. Rather it involves abduction–deduction–induction chains of reasoning. The algorithm method is also deficient because whereas the squaring algorithm is easily grasped and employed by the slave boy, the inverse square rooting algorithm is not available to him and would be extremely difficult for him to grasp for he has not been educated in mathematics. The visual abductive model which involves counting as well as seeing is hence essential for the boy to acquire knowledge of a simple geometric proposition.

> HAIL to thee blithe abduction!
>
> Deduction or Algorithm thou never wert—
> That from discovery or recollection
> True opinions from your soul
> In profuse generation of visual model-based reasoning art[1]
> Adapted from Percy Bysshe Shelley, 'To a Skylark' (1820)

[1] Some of the words of Shelley's poem 'To a Skylark' [26] have been changed, the spirit is maintained.

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1 Introduction

Plato (c.427–c.347 BCE) is recognized as the master of deduction. His use of induction and abduction is most often overlooked but it is methodologically embedded in most of his deductions. There is an abundance of induction and abduction in Plato including analogical reasoning and model-based abduction. Most of Plato's compact reasoning provides paradigm examples of Magnani's 'abduction–deduction–induction cycle' [12, p. 77]. Olsen states: 'In general Plato presents puzzles, problems, and incomplete analysis, from which the reader may infer (abduct) the solutions (or adequate hypotheses)' [19, p. 86].

Plato's *elenchus* in *Meno* is an abduction–algorithm–induction cycle. The purpose of this part of the dialogue is to conclude that knowledge is recollection, and recollection is achieved through the dialectic. Dialectic is considered to be abduction by Olsen [19, p. 88].

2 Stages of the Socratic *Elenchus* with the Slave Boy

The *elenchus* is from *82b* to *85b* [21]:

I. Socrates draws a square of length two and asks the boy what the area is.

  – The boy understands that the area is four.

II. Socrates asks the boy what is the length of the square with double this area of four?

  – The boy responds that it would be the double of two, that is, four.

III. Socrates demonstrates by sketching a square with side four, and drawing boxes inside of one square unit each, what the area of the square of length four would be.

  – The boy understands that this area would be 16, four times the area of the original square.

IV. With the help of Socrates' questioning and prodding, the boy comes to realize that the length of the side of the square with area eight will be less than four but greater than two.

V. Socrates asks again, what will the length of this side be?

  – The boy responds quickly again: "three".

VI. Socrates demonstrates to the boy and the boy understands that the area of the square with side of length three will be nine.

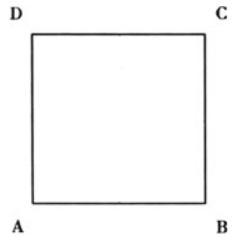
VII. The boy now admits that he is in a state of confusion.

VIII. Now, Socrates draws a diagonal of the square and constructs a square on the diagonal.

– So the boy is able to understand that the square built on the diagonal of the original square of length two will have the area double of the area of the original square, i.e. eight.

I. **Abductive (Visual)–Algorithmic Model**

Socrates draws a square of side length two and asks: 'you know that a square is a figure like this?' [21, *82b*][2]



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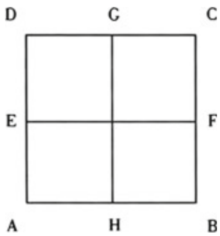
Even though Socrates draws a nominal square of side length two, he asks whether the boy knows that a square is a figure of this type. Hence, Socrates appeals to the boy to recollect the Form of Square of which this particular square is an instantiation and simultaneously wants the boy to generalize square from this particular visual square so that the properties of the abstract square can be applied to this concrete instantiation. The visual square then represents an abductive visual model described by Magnani: '[…] as Peirce noted, abduction plays a role even in relatively simple phenomena. *Visual abduction* […] occurs when hypotheses are instantly derived from a stored series of previous experiences' [12, p. 42]. The purpose is the acquisition of knowledge by recollection, and once the Form of Square is recollected it is applied to know that what is seen is a square because it is an instantiation of the Form Square. Knowledge of Forms is knowledge of universal definitions so that when the boy grasps the definition of 'square' then he can easily answer Socrates' next question: 'It has all four sides equal?' Since having four sides equal is a necessary condition in the *definiens* of the definition of 'square', the boy responds 'Yes' [21, *82c1–2*]. 'It' refers simultaneously to the visual square that has been drawn with side length equal to two as well as to the abstract square of any side length. This is neither universal instantiation nor universal generalization. It is not a deductive model, but a combination of a visual abduction, an abductive recollection and an algorithmic calculation.

[2] This diagram is obviously drawn by Socrates but is not shown in the dialogue, but only the square at the next stage with the bisectors is shown.

Socrates immediately draws the bisectors and makes the boy admit that EF = GH.



[21, *82c*]

(Square accessed from http://kireetjoshiarchives.com/teachers_training/good_teacher/learning_recollection.php, p. 8, accessed on 11 December, 2012.)

Why does Socrates introduce the bisectors?

SOCRATES: Now if this side is two feet long, and this side the same, how many feet will the whole be? Put it this way, if it were two feet in this direction and only one in that, must not the area be two feet taken once?
BOY: Yes.
SOCRATES: But since it is two feet this way also, does it not become twice two feet?
BOY: Yes.
SOCRATES: And how many feet is twice two? Work it out and tell me.
BOY: Four [21, *82c–d*].

When Socrates says: 'if this side is two feet long, and this side the same, how many feet will the whole be?', the boy could take this to mean 'when a side of the square is two feet long, then what will the area of the square be?' which the boy could answer by simple calculation 'two times two equals four'. And this the boy can easily do at this point since Socrates has already made him recollect the Form and hence the definition of 'square'. But the recollection itself is abductive. Socrates wants the boy to be able to see that the bisectors create four equal one by one squares, so that he can simply count the number of squares as four, the total area of the square. This is an effective abductive visual model. Hence, the visual and algorithmic models are simultaneously used. The visual model is abductive. One model provides a reason for the other, as Moriarti states: '[…] abduction provides a logical explanation for visual interpretation […]. Abduction begins with observation—and observations are usually visual' [15, p. 181]. The connection between the algorithmic and the visual model is abductive as the algorithm provides the algebraic answer for the visual counting. The visual (counting) and the algorithmic (calculating) hence support each other.

II. **Failure of Finding Square Root Algorithm Leading to Error**

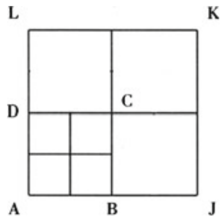
> SOCRATES: Now could one draw another figure double the size of this, but similar, that is, with all its sides equal like this one?
> BOY: Yes.
> SOCRATES: How many feet will it be?
> BOY: Eight.
> SOCRATES: Now then, try to tell me how long each of its sides will be. The present figure has a side of two feet. What will be the side of the double-sized one?
> BOY: It will be double, Socrates, obviously.
> SOCRATES: You see, Meno, that I am not teaching him anything, only asking. Now he thinks he knows the length of the side of the eight-foot square?
> MENO: Yes.
> SOCRATES: But does he?
> MENO: No [21, *82d–e*].

Socrates continues to employ the simple algorithm in getting the answer eight for the square double the area of the square with area four. The boy need look at no figure to get this answer, but simply needs to understand what 'double of' means. Now, Socrates throws a monkey's wrench, asking in stride, 'how long each side of the square double the area of the original square would be?' The slave boy, deceived in thinking that the answer will be just as automatic as the previous answer, says 'double'. If asked why the area of the square double of the given square would be eight, the boy would immediately respond because the double of four is eight. But here he cannot immediately give an algorithm. In fact if he understood the squaring algorithm he would not have given the wrong answer. In other words, the boy can employ the squaring algorithm without understanding what the algorithm is. And even if he understood it, he surely would not understand the inverse algorithm of the square root, nor would he understand that the length of the side of a square is the square root of the area. He may have understood this if Socrates had begun with drawing a square and given its area as four without telling the boy what the side of the square was, and then asked the length of the side, but then Socrates could not have accomplished the purpose of running the algorithmic method and the visual model simultaneously.

III. **Visual Model to Realize the Error**

Socrates now clearly demonstrates to the boy that the area of the square with length four, double of the side of the original square, will give us a square with area 16 and not the required eight[3]:

[3] Like the very fist square this square is not drawn by itself by Socrates in the dialogue but the completed square with the square of side three also displayed in it drawn as a composite.



[21, *83a-b*]

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Let us look at how Socrates comes to '16':

SOCRATES: How big is it then? Won't it be four times as big?
BOY: Of course. {visual model}
SOCRATES: And is four times the same as twice?
BOY: Of course not. {algorithmic}
SOCRATES: So doubling the side has given us not a double but a fourfold square?
BOY: True. {visual–algorithmic}
SOCRATES: And four times four are sixteen, are they not?
BOY: Yes [21, *83b–c*]. {algorithmic}

After making the boy see his error visually as the area we get is four times not double, Socrates immediately turns to purely algorithmic reasoning without bringing in the vision of the square at all. This is to emphasize the importance of the algorithmic method. It seems like that the boy will ultimately have to understand the notion of and the algorithm of square root as well.

IV. **Deductive Inference to Length being Between Two and Four**

> SOCRATES: Then how big is the side of the eight-foot square? This one has given us four times the original square, hasn't it?
> BOY: Yes [21, *83c*].

Socrates asks two questions, but the boy answers only the second question and does not even attempt to answer the first. Why? The boy is now realizing that the answer will not come easy but he is sure of the answers found so far, so he confidently responds 'yes' to the second question. Neither does Socrates expect the boy to answer the first question that is why he does not persist with it in the immediately following sequence.

Socrates makes the boy realize that the length of the side of the square with area eight, double of the area of the original square with side length two will be greater than two but less than four:

SOCRATES: Good. And isn't a square of eight feet double this and one half that?
BOY: Yes.

SOCRATES: Will it not have a side greater than this one but less than that?
BOY: I think it will.
SOCRATES: Then the side of the eight-foot figure must be longer than two feet but shorter than four?
BOY: It must [21, *83c–d*].

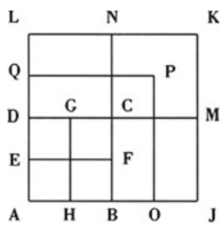
V. **Abductive Guessing**

SOCRATES: Try to say how long you think it is?
BOY: Three feet [21, *83e*].

Understanding this topological point the boy now ventures another guess, that of three. But Socrates had already anticipated this. The progression of guesses, from positing four first to positing three now after realizing that the length is less than four but greater than two, is abductive reasoning. Guessing is to be taken as a meaningful step by the slave boy towards the acquisition of knowledge, as Peirce says: '[…] every step in the development of primitive notions into modern science was in the first instance mere guess-work, or at least mere conjecture. But the stimulus to guessing, the hint of the conjecture, was derived from experience' [20, CP 2.755].[4] The experience here is the visual models being drawn by Socrates combined with algorithmic thinking. As Magnani states one of the purposes here is: 'to illustrate the relevance of the activity of guessing hypotheses, dominant in *abductive reasoning*, […]' [12, p. 2].

VI. **Visual Model Again to Demonstrate New Error**

Socrates quickly makes a three by three square in the same figure by adding the segments BO and DQ to AB and AD respectively:



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SOCRATES: If it is three feet this way and three feet that, will the whole area be three times three feet?
BOY: It looks like it. {visual}

[4] Accessed from [16, p. 218].

SOCRATES: And that is how many?
BOY: Nine. {algorithm} [21, *83e*]

This is a repeat process of when the guess of the length was four. However, this time we quickly get to nine as the area of the square with side length three, but again with both the visual model and the algorithmic model working hand in hand. Even though the boy says 'it looks like it' he is not using the visual model to calculate by counting squares but the algorithm. Since the diagram was altered by extending EF and HG to new vertices P and Q, Socrates could easily have created the nine one by one squares inside the three by three square. And then the boy could have answered visually, perhaps he sees it anyway. But Socrates wants the boy to be thinking in the stream of the algorithm. What follows about the correct length of the side of the square with area eight and the figure that displays that cannot be confirmed by counting one by one squares, as in the case with perfect squares, that is those of areas four, 16 and nine.

VII. **Reaching the State of Confusion before the Final Step to Recollection**

SOCRATES: Whereas the square double our first square had to be how many?
BOY: Eight.
SOCRATES: But we haven't got the square of eight feet even from the three-foot side?
BOY: No.
{The emphasis on 'three' here is Socrates' hint or rather the obvious inference that the length of the side of the desired square will not be a natural number and this leaves the boy perplexed}
SOCRATES: Then what length will give it? Try to tell us exactly. If you don't want to count it up, just show us on the diagram.
BOY: It's no use Socrates, I just don't know. [21, *83e–84a*] {The state of confusion}

The choice that Socrates gives the boy is between counting and seeing it on the diagram. Both are part of the same process as the dialogue has proceeded so far. The counting is to be done on the diagram not independent of it and is hence part of the visual model. All through Socrates has employed simultaneous visual model and algorithmic reasoning. 'Show us' however challenges the boy to use the visual model to demonstrate the length of the side of the square we want, but the boy is unable to do it at the moment. Algorithm has been used exactly three times in 'two times two is four', 'four times four is sixteen', and 'three times three is nine'. Then why not direct the boy towards the algorithmic 'x times x is eight'? Because the square root of eight is an incommensurable number and neither Socrates nor anyone in his time understood incommensurable numbers so how could Socrates expect the boy to understand this. Nor does the boy at this point understand the converse algorithm that the square root of four is two, the square root of sixteen is four and the square root of nine is three.

SOCRATES: Observe Meno, the stage he has reached on the path of recollection. At the beginning he did not know the side of the square of eight feet. Nor indeed does he know it now, but then he thought he knew it and answered boldly, as was appropriate—he felt no perplexity. Now however he does feel perplexed. Not only does he not know the answer; he doesn't even think he knows.
MENO: Quite True.
SOCRATES: Isn't he in a better position now in relation to what he didn't know?
MENO: I admit that too.
SOCRATES: So in perplexing him and numbing him like the sting ray, have we done him any harm?
MENO: I think not.
SOCRATES: In fact we have helped him to some extent toward finding out the right answer, for now not only is he ignorant of it but he will be quite glad to look for it. Up to now, he thought he could speak well and fluently, on many occasions and before large audiences, on the subject of a square double the size of a given square, maintaining that it must have a side of double the length.
MENO: No doubt.
SOCRATES: Do you suppose then that he would have attempted to look for, or learn, what he thought he knew, though he did not, before he was thrown into perplexity, became aware of his ignorance, and felt a desire to know?
MENO: No.
SOCRATES: Then the numbing process was good for him?
MENO: I agree [21, *84a–c*].

Stage VII is the paramount feature of the Socratic *elenchus*. The student, or answerer, the boy, has reached an authentic state of confusion, which was exactly the aim of the interrogator or teacher, Socrates. This is the stage where due to the rigorous progression of the earlier stages the answerer comes to know that he does not know what he earlier claimed to know. In *Euthyphro*, Euthyphro who thinks he knows what piety is reaches the state of confusion by the end of the dialogue and instead of persisting with acquiring authentic knowledge of what piety is he simply abandons the project. In *Crito*, Crito makes a series of knowledge claims which are one by one dismissed by Socrates' interrogation leaving Crito in a state of confusion and he does not pursue any of these further as his pragmatic aim is to convince Socrates to escape from prison.

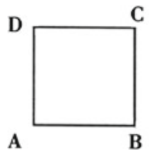
In *Meno* the boy reaches the state of confusion because now he knows that he does not know what the length of the side of the square with area eight square feet is. However, in the process of the *elenchus* he has come to know that it is neither four nor three and further that it is greater than two and less than three, so he has made remarkable progress towards authentic knowledge.

VIII. **Recollection as Discovery and Abduction**

SOCRATES: Now notice what, starting from this state of perplexity, he will discover by seeking the truth in company with me, though I simply ask him questions without teaching him. (*Socrates here rubs out the previous figures and starts again*) [21, *84c–d*].

The final stage has begun where the boy will come to have knowledge of what is the length of the square with the area of eight square feet.

SOCRATES: Tell me boy, is not this our square of four feet? [ABCD.] You understand?



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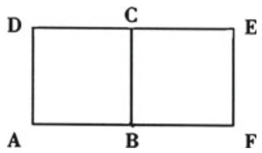
BOY: Yes.
SOCRATES: Now we can add another equal to it like this? [BCEF.]



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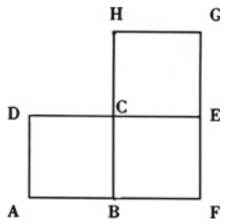
BOY: Yes.
SOCRATES: And a third here, equal to each of the others? [CEGH.]



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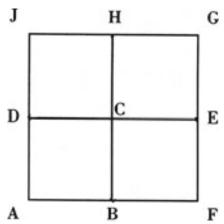
BOY: Yes.
SOCRATES: And then we can fill in this one in the corner? [DCHJ.]



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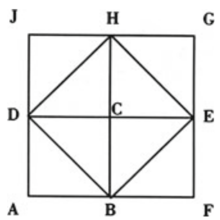
BOY: Yes.
SOCRATES: Then here we have four equal squares?
BOY: Yes.
SOCRATES: Then how many times the size of the first square is the whole?
BOY: Four Times.
SOCRATES: And we want one double the size. You remember?
BOY: Yes.
SOCRATES: Now does this line going from corner to corner cut each of these squares in half?
BOY: Yes [21, *84d–85a*].



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Even though there are four lines Socrates uses the singular 'line' in order for the boy to abstract the concept of a diagonal.

SOCRATES: And these are four equal lines enclosing this area? [BEHD.]
BOY: They are.

Now, the visual model is coming on strong.

SOCRATES: Now think. How big is this area?
BOY: I don't understand.

Of course the boy does not understand as so far Socrates has dissuaded the boy from counting but now he wants him to count.

SOCRATES: Here are four squares. Has not each line cut off the inner half of each of them?
BOY: Yes.

Again the emphasis is on the visual model.

SOCRATES: And how many such halves are there in this figure? [BEHD.]
BOY: Four.

The answer here can only be gotten through seeing and counting.

SOCRATES: And how many in this one? [ABCD.]
BOY: Two.

The answer here can only be gotten through seeing and counting.

SOCRATES: And what is the relation of four to two?
BOY: Double [21, *85a*].

The answer to this question does not depend on the visual model but on a simple algorithm.

SOCRATES: How big is this figure then?
BOY: Eight feet [21, *85a–b*].

Again this answer is based on a simple algorithm as the double of four, the area of the original square, is eight. Also, both the doubles refer to areas. The four equal areas seen are double of the two equal areas seen and it is also seen that each of these two equal areas is equal to each of the four equal areas.

SOCRATES: On what base?
BOY: This one.

The visual model is now used as the boy points out to the base, the diagonal BD.

SOCRATES: The line which goes from corner to corner of the square of four feet?
BOY: Yes.

The diagonal is pointed to by Socrates while emphasizing its diagonal nature.

SOCRATES: The technical name for it is diagonal; […] it is your personal opinion that the square on the diagonal of the original square is double of its area.
BOY: That is so Socrates [21, *85b*].

This is the end of Stage VIII.

3 Recollection as Abduction

Is this recollection? It is a whimper rather than a bang. It does not seem convincing at all that the slave boy has recollected anything leave alone first the Form of Diagonal and then the knowledge that the area of the square double the area of the original square will be the square with the diagonal of the original square as its side. The boy has answered in the same expression and has not said 'eureka!' or expressed any excitement or given any indication of having made a discovery.

Has Plato cheated us? Not really! As Plato has reminded us through his rigorous toils in every dialogue, recollection does not come easy, whether it is the Form of Piety, which is never recalled by Euthyphro at the end of *Euthyphro*, or the Form of Knowledge, which Theaetetus comes close to recollecting at the end of *Theaetetus*, but falls short of it as the definition he has found so far, true belief with an account is not yet knowledge: 'So, my friend, there is such a thing as right belief together with an account, which is not yet entitled to be called knowledge' [22, *208b*]. In the *Republic* it is Plato, through Socrates, who finally provides the definition of 'justice'. Yet, one can read the dialogue many times but fail to see at what point Socrates recollects the Form of Justice. But what we find in the *Republic* in the construction of the Platonic definition of 'justice' is the finest complex of craftsmanship in philosophy. Plato did not use the examples of craft persons like carpenters, cobblers and weavers just for entertainment. He wanted to convey that philosophy as systematic thinking and theorizing requires the finest craftsmanship. Socrates as a teacher was a craftsman of unmatched skills and perseverance.

In Stage VIII we have seen a display of this craftsmanship. The dialectic has picked up pace from the earlier part where it moved rather slowly and meticulously, it is like a symphony reaching a crescendo. Socrates begins this section with a brick by brick construction of the visual square which should finally convince the boy that the area of the square with the side double of the original side will be four times the original square not two times. Not only does the boy see this for this particular square which is drawn in front of him, he is able to grasp the generalization perhaps through an abduction–deduction–induction cycle.

Next, Socrates constructs the diagonals for each of the four squares and visually demonstrates, or rather has the slave boy visually realize by looking and simple calculation that the area of the square with the diagonal as the side is the desired eight, as it is four halves of the square with area four. At this point one would hope that the boy has a flash and has discovered a geometrical truth about the area of the square of the diagonal of a square to be double of the area of the original square. Perhaps, if the boy has not actually reached the moment of discovery, the same procedure could be repeated with squares of different lengths and sooner or later the discovery will come.

Isn't this the daily procedure of so many human processes from pottery to music to tennis to mathematics? Each of us must have so many childhood memories when we had difficulty with understanding some concept such as why the square root of a positive number could be either positive or negative, and finally some flash came and we understood it and have understood it ever since. Socrates is well aware that the slave boy even if he has had a flash cannot at the moment describe it:

SOCRATES: What do you think, Meno? Has he answered with any opinions that were not his own?
MENO: No, they were all his.
SOCRATES: Yet he did not know, as we agreed a few moments ago.
MENO: True.
SOCRATES: But these opinions were somewhere in him, were they not?
MENO: Yes.
SOCRATES: So a man who does not know has in himself true opinions on a subject without having knowledge.
MENO: It would appear so.
SOCRATES: At present these opinions, being newly aroused, have a dreamlike quality. But if the same questions are put to him on many occasions and in different ways, you can see that in the end he will have a knowledge on the subject as accurate as anybody's.
MENO: Probably.
SOCRATES: This knowledge will not come from teaching but from questioning. He will recover it for himself.
MENO: Yes.
SOCRATES: And the spontaneous recovery of knowledge that is in him is recollection, isn't it?
MENO: Yes [21, *85b–d*].

Even though Socrates uses the term 'spontaneous recovery' we can term it as *discovery* in the Peircean sense since it is at least not consciously available to us before the dialectic process but only at the end of a long and arduous dialectic process. Recollection then as being 'spontaneous' or in a flash cannot be either deduced or induced but only abduced. Though the dialectic is the path to recollection, it is not a deductive or inductive inference to recollection, in fact recollection may or may not happen at the end of a dialectic process. How does recollection actually happen? How does the moment of 'eureka' actually happen? The inference for recollection is as follows:

The surprising flash of knowledge occurs.
But if there were recollection, flashes of knowledge would be a normal occurrence.
Hence, there is a reason to suspect that recollection happens.

This is an instance of abductive reasoning as described by Jaime Nubiola:

The surprising fact, C, is observed.
But if A were true, C would be a matter of course.
Hence, there is a reason to suspect that A is true [18, p. 126].

Almost all Plato scholars agree that recollection involves discovery. Scott states: 'Everyone would agree that the theory of recollection is intended to explain how philosophical and mathematical discoveries are made' [24, p. 7]. The explanation of how discoveries are made is the domain of abduction. As Polanyi states: '[…] scientific discovery cannot be achieved by explicit inference, nor can its true claims be explicitly stated. Discovery may be arrived at by the tacit powers of the mind, and its content, so far as it is indeterminate, can be only tacitly known' [23, p. 158].[5] We could not have a better modern account of Plato's knowledge as recollection. If we can reconcile the notion of discovery with recollection in the form of tacit knowing, then we can bring in Peirce's statement: 'Abduction is the process of forming an explanatory hypothesis. It is the only logical operation that introduces any new idea; for induction does nothing but determine a value, and deduction merely evolves the necessary consequences of a hypothesis' [20, CP 5.171].[6] Peirce may not have approved of recollection and might have taken discovery to be more authentic than Plato or Polanyi, but Mullins finds a nice synthesis: 'Examining tacit knowing in conjunction with Peirce's ideas about abduction provides a new and rich context within which to appreciate Polanyi's claims for tacit knowing' [16, p. 199]. And if Polanyi's tacit knowledge is a revival of Plato's knowledge as recollection, then Plato's recollection is a type of Peircean abduction.

Magnani expresses some doubt whether recollection can be considered as abduction: '[…] in order to solve a problem one must in some sense already know the answer, there is no real generation of hypotheses, only recollection of them' [12, p. 1]. I am in slight disagreement here as the notion of tacit knowledge in Plato has to be taken within the context of Platonic epistemology. The claim that knowledge is propositional, that the object of the verb 'know' is a proposition is firmly established in *Theaetetus*, where knowledge is also tentatively defined as true opinion with an account. Plato does have some notion of this in the *Meno* as well, as later Socrates clearly states: 'true opinion is as good a guide as knowledge for the purposes of acting rightly. […] So right opinion is something no less useful than knowledge' [21, *97b–c*]. Meno immediately throws a doubt as surely, he thinks, knowledge must be superior to true opinion. Socrates gives the reason for Meno's intuition: 'True opinions […] run away […] they are not worth much until you tether them by working out the reason. That process […] is recollection […]. Once they are tied down, they become knowledge, […] [21, *97e–98a*].' Hence, there is a closure within *Meno*. True opinion is good enough for practical knowledge, knowledge required for action; however, for theoretical knowledge, true opinion needs to be tied down as knowledge by providing an account.

This failure to be clear about what exactly is being recollected, true opinion or knowledge, leads to misconceptions of recollection being the one and all of Plato's epistemology such as that by Stefanson: 'Knowledge (*epistēmē*), on Platonic

[5] Accessed from [16, p. 207].

[6] Accessed from [16, p. 199].

understanding, is an entirely separate activity and a distinct possession to belief or opinion (*doxa*). […] Understanding is only possible when a man allows his *psychē* to contemplate the metaphysical realm of knowledge' [27, pp. 102–103]. However, I contend that what is recollected through what Stefanson calls 'to contemplate the metaphysical realm' is true opinion, whereas understanding has to do with giving an account.

Even famous Plato scholars like F. M. Cornford do not clearly see that what is being recollected in *Meno* is not knowledge but true opinion and though true opinion may be practical knowledge it is not theoretical knowledge: 'The *Meno* had already announced the theory of *Anamnesis*: that knowledge is acquired […] by recollection […] truths seen and known by the soul […] for seeking and learning is nothing but recollection. […]' [3, p. 2]. Is the use of 'knowledge' here ambiguous, referring either to 'true opinion' or to 'true opinion with an account' or true belief with understanding? What follows makes it clear that Cornford is not making a conflation: '[…] and if he were questioned again and again in various ways, he would end up having knowledge in place of true belief—knowledge which he would have recovered from his own soul. This knowledge must have been acquired before birth' [3, p. 3]. So, Cornford clearly believes that what is being recollected by the slave boy is not true opinion but knowledge. This is rather unfortunate as my exegesis of the passage shows just the opposite, namely, that as Plato takes pain to demonstrate later in *Meno*, knowledge requires a tether that is not there in true opinion and the tether cannot be recalled or recollected but must in a way be constructed as an account or understanding.

Norman Gulley also believes that what is recollected is 'knowledge': 'Thus the knowledge sought by Socrates […] is described by Plato as aknowledge of 'forms', […]' [7, p. 3]. Gulley goes on to make this assessment about: '[…] passage of the *Meno* […] that these ideas embody a rudimentary theory of recollected knowledge' [7, p. 9]. Gulley then discusses the distinction between true belief and knowledge. I believe he is in error when he describes the second stage of recollection as '[…] recognition that certain propositions are true, but not as yet *why* they are true. […] The level of apprehension now reached is described as "true belief" (85c, 86a)' [7, p. 13]. This has taken place as recollection of a true opinion as clearly stated:

SOCRATES: The technical name for it is diagonal; […] it is your personal opinion that the square on the diagonal of the original square is double of its area.
BOY: That is so Socrates [21, *85b*].

This is the point of recollection. And in the immediately following discussion with Meno Socrates emphasizes that what is recollected is not knowledge but true opinion: 'So a man who does not know has in himself true opinions on a subject without having knowledge' [21, *85c*]. So unless at the second stage Gulley means recollection by 'recognition' and 'apprehension' he is mistaken. However, Gulley, unlike Cornford does state that for Plato at this point the tether is required to make true opinion into knowledge: 'To become knowledge it must be "tied down" by a "chain of causal reasoning" […] the method of analysis […] to find the antecedent conditions […] a method practiced by geometers' [7, pp. 14–15]. Incidentally the geometric method of analysis seems to be abductive reasoning as it is reasoning to antecedents or hypotheses.

Magnani also sees that what is recollected is true opinion, not knowledge: 'The true opinion is given by recollection and science is the system of true opinions when related by the activity of reasoning and thereby made permanent and definitive' [12, p. 7]. Science then turns true opinion into knowledge by what Plato calls the 'tether', the account that gives proper understanding.

Plato himself is responsible for an equivocation in the use of the words 'knowledge' and 'know'. In the passage just quoted above, first Socrates says that true opinions, not knowledge, were in the boy: 'So a man who does not know has in himself true opinions without having knowledge'. 'True opinion' here is ambiguous between the tacit true opinion and the conscious true opinion after recollection. What is recollected is true opinion not knowledge: 'At present, these opinions, being newly aroused have a dreamlike quality'. Knowledge then is not recollected, true opinions are, and they can then be turned into knowledge: 'if the same questions are put to him […] in the end he will have a knowledge on the subject […]'. All of this seems to be clear but then Socrates says 'But this knowledge does not come from teaching but from questioning. He will recover it for himself.' The last sentence indicates that knowledge after all is recollection. So, if my earlier claim is correct that the boy has not at this point in the dialogue recollected the Form of Diagonal, but with repeated questioning he will eventually recollect and thereby have knowledge; then Magnani's line of 'tacit knowledge' is correct. This is further supported by the immediately following text: 'Either then he has at some point acquired the knowledge he now has, or he has always possessed it' [21, *85d*].

However, I insist that we take 'knowledge' in the larger context of Plato's epistemology, especially in the context of the definition of propositional knowledge established later in *Theaetetus*. In recollecting Forms one does not recollect propositions but abstract objects. Then one puts these abstract objects together with concrete particulars to form beliefs or opinions. Hypotheses after all are not objects but propositions. So it is not even true opinion that is recollected but a true opinion is formed immediately at the time of recollection.

Whether he recollects Forms or true opinions, he does not recollect an account or adequate justification of the opinion. An account is surely not something that can be recovered, but something that must be constructed from understanding. Hence, if the concept of 'knowledge' includes understanding then providing an account becomes essential. Hence, recollection, at the end of the process of the dialectic is necessary for knowledge but it is not sufficient as the additional condition of account is required which may or may not be available at the time of recollection.

This is a dynamic example. Let us suppose that not only the boy, but Socrates, Meno and everyone else present at the time, and for that matter anyone reading the dialogue for hundreds of years after that, including the greatest philosophers and mathematicians, all recollected the Form of Diagonal and reached the true opinion that the square which has double the area of the original square must be the square constructed on the diagonal of the original square; but none of them have knowledge, because whenever a square has a length that is a discrete number like 1, 2, 3, 4, 5, and so on, then the diagonal of the square will be an incommensurable number like $\sqrt{2}$, $\sqrt{8}$, $\sqrt{18}$, $\sqrt{32}$, $\sqrt{50}$, and so on. At the time of Socrates and Plato no one properly understood incommensurable numbers, so at least a complete justification was lacking, hence if complete justification is a necessary condition for knowledge,[7] then no one could have had knowledge of the hypothesis that is being generated through the abductive process of recollection here. So, we should not be surprised at all that despite recollection the boy is falling short of knowledge because he lacks an account and understanding.

Induction and abduction are intricately linked in the *elenchus* in *Meno* so that we cannot really say which comes first, as Nguifo states: '[…] in the ML systems, abduction cannot be used without induction, and induction needs abduction' [17, p. 49].

4 The Parallel Geometric Visual Model and the Algorithm

Even near the end of the dialectic part of the *elenchus* an opinion or what we today call a 'belief' is reached not as a whim but as one with a reasoning process backing it, but it is not yet knowledge. At the end of the long dialectic process one reaches a belief and one still has a long way to go in making this belief into knowledge.

Socrates, if he were interested only with the right answer, would have worked only with the algorithmic model. Reasoning as follows:

$$2^2 = 4$$
$$\sqrt{4} = 2$$
$$\sqrt{8} = 2\sqrt{2}$$

In this case none of the figures drawn would be required. But the whole point that Socrates wants to make about recollection as a progressive dialectical process of reaching true opinion would have been lost. Furthermore, the simultaneity of the visual model and the algorithm is required to come to realize that the square root of eight is an incommensurable number.

Elaine Landry claims that in *Meno* Plato has two supplemental methods running parallel, namely the *elenchus* and the hypothetical, or mathematical [10, pp. 1–2]. I would rather claim that the hypothetical method is embedded in the *elenchus* as the *elenchus* is the process. Landry's hypothetical method requires the parallel geometric visual model and an algorithm.

[7] Keith Lehrer and Thomas Paxson Jr. take 'S is completely justified in believing that *h*' to be one of the necessary conditions of knowing [11, p. 225].

The arithmetical solution requires neither drawing figures nor the calculation of the length of the diagonal by the Pythagorean Theorem. Algorithms are generally considered to be deductive, but are they? Once an algorithm is constructed then it can be used to deductively churn out a result by plugging in values. Compact deductive arguments, operating on a simple algorithm, sometimes may not lead to knowledge even if one immediately grasps the soundness of the argument.

The implicit induction here is that a method of counting works in determining the areas of squares. The use of geometric visual models by Plato is abductive model-based reasoning. The squares with side two, three and four are presented as geometric models. This brings upon the realization that the side of the square with area eight is an incommensurable number, the length of which can nonetheless be determined by the Pythagorean theorem so deduction comes into aid model-based reasoning. Furthermore the boy uses abductive reasoning of making closer guesses or conjectures as he moves from four to three to realizing that the answer is between two and three, closer to three. As Peirce states: 'The order in the march of succession in retroduction is from experience to hypothesis' [20, 2.755].

Meno presents a classic case of abduction as 'reasoning which starts from data and moves towards hypothesis' as Fann claims [5, p. 5]. When Socrates asks the initial question, the boy goes into search for a hypothesis but he does not quite know how to arrive at it. Through the dialectic process he finally arrives at the hypothesis that the length of a side of a square double the area of another square is the length of the diagonal of the original square as he recollects the Form of Diagonal. Recollection is discovery of sorts and this makes it abduction.

If the boy knew the diagonal hypothesis from the beginning he would have answered the question right away. But he does not have this hypothesis available to him but only arrives at it through the dialectic within *elenchus*. Socrates does not give the boy the option of pursuing the much easier algorithmic method because through the algorithmic method he would understand it arithmetically but would not understand it geometrically, which is the more complete knowledge that Plato was seeking. Once the first diagram is drawn, the boy could easily internally visualize with the aid of the simple algorithmic method. However, the advantage of the continual use of the visual model by Socrates is best explained by Mary Hegarty: 'viewing an external visualization […] can be a substitute for internally visualizing […] the availability of external visualizations relieves us of the necessity of internally visualizing […]' [8, p. 5].

5 Socrates–Slave Boy *Elenchus* as a Hybrid Algorithm—Abductive (Visual) Model

The *Meno* hence provides us with a paradigm hybrid method as Magnani says 'visual and algorithmic may be intertwined, and so, hybrid so to say' [14]. In modern algebraic geometry by understanding the algorithmic method one may well understand the geometrical hypothesis as well. Another reason why Socrates does not end matters with the algorithmic method is because this would not lead to the realization that the square root of 8 is an incommensurable number, which may be the ultimate aim of the inquiry, not so much for the slave boy but for the rest of us, at the time of Plato of course. Gabbay and Woods state: ‘abduction is the finding and engaging of a hypothesis (H) that, when combined with what one already knows (K), enables one to presumptively attain a cognitive target (T) that one could not attain via K (or a ready expansion of K) alone’ [6, p. 290]. If K is the knowledge available at the point before recollection of the diagonal including the knowledge of the extended algorithm, it would not be sufficient to realize that the length of the diagonal is an incommensurable number, yet its square is a commensurable number.

The use of perceptual models by Socrates of drawing squares on the ground and explaining to the slave boy through these visual models is what Peirce would call ‘abduction’: ‘[…] our first premises, the perceptual judgments are to be regarded as an extreme case of abductive inference […]’ [28, p. 393].

Both the algorithm and the diagrammatic model are alternative computational methods where each ‘computational model is the theory not a simple instantiation of a theory’ [9, p. 511]. The reasoning presented in the *Meno* may best be described as what Magnani calls ‘manipulative abduction’: ‘the exploitation of external logical and computational abductive–but also inductive—systems/agents to form hybrid and multimodal representations and ways of inferring in organic agents’ [13, p. 396].

6 The Surprise: No Deduction in the *Elenchus* Between Socrates and the Slave Boy

There is a great surprise after this long trek through the Socrates–slave boy *elenchus*. I started the introduction with the sentence ‘Plato is recognized as a master of deduction’. In this *elenchus* and dialectic, one of the most famous in Plato, where is deduction? There are some very simple straightforward deductive arguments in some of the eight stages, but these too are more implicit than explicit, like in deducing that the length of the side of the square we are looking for is between two and four since it is greater than two and less than four. But the spots of deduction that are present are hardly anything to write home about. If algorithm is not deduction as I have argued above, then the *elenchus* has the parallel abductive visual and guessing models and an algorithm embedded in it. Within the abduction visual and guessing as two species of abduction are embedded; and the sub-species of counting is further embedded in the visual. The counting is interwoven with induction. Within the guessing there is some minimal deduction woven in as the slave boy has realized that the length of the side must be between two and four because he knows at that point that it is greater than two and less than four; so he makes the guess of three. All of this part of the *elenchus* leads to the dialectic and recollection as a species of abduction is embedded in the dialectic. This surprise of a prime piece of Platonic reasoning that is not centered on deduction is itself a hypothesis inferred through abduction.

7 The *Meno Elenchus* in the Context of the Narrative of the Whole Dialogue

The Narrative Sequence of the *Meno*

Meno asks: Is virtue acquired or natural? If acquired, can it be taught or is it acquired through practice?
↓
Socrates answers that since he does not know what virtue is he cannot answer any of Meno's questions.
↓
Meno claims to know that virtue is relative to each person and it is what each person desires and obtains.
↓
Socrates points out the circularity of first defining virtue in terms of part of virtue, justice, but we do not know what virtue as a whole is. This is an example of claiming to know what one does not know.
↓
Meno accuses Socrates of being a trickster who has numbed him in the process of questioning him. {Socrates has equivocated knowledge of what virtue is with the ability to define what 'virtue' is}
↓
Socrates softens his claim: Meno may have known what virtue is but presently he seems not to know
↓
Meno raises a paradox: How can one begin an inquiry in search of either knowing or not knowing any concept without some knowledge of that concept? Because if one does not know it then how will one know that one does not know it and if one knows that one knows it then he must already have known it.
↓
Socrates' solution to the paradox is the theory of recollection: The soul has already acquired knowledge of the Forms in a previous state of her existence and this is innate knowledge, the conscious process of knowing then is the process of recollection.
↓
The *elenchus* with the slave boy recollecting the concept of diagonal is an example of knowledge by recollection.
↓
The eight stages of the Socratic *elenchus* between Socrates and the boy (discussed above)

↓
The Boy acquired knowledge by recollection of the Form of a diagonal through the *elenchus.* (stage VIII)
↓
Neither true opinion nor knowledge can be taught but they are acquired through questioning, the *elenchus*.
↓
If virtue were a form it would be known through the *elenchus*, but knowledge of virtue is not that.
↓
If virtue is knowledge it is teachable.
↓
Virtue is not knowledge but wisdom, that is, it is the ability to know and not knowledge itself.
↓
If virtue were teachable who would be the teachers of virtue? Would it be the Sophists?
↓
Anytus who now walks into the conversation protests that the Sophists cannot be teachers.
↓
Neither the Sophists nor those who possess and display virtue are teachers of virtue.
↓
Who then can teach virtue? No one. So, there are neither teachers nor students of virtue.
↓
True opinion and knowledge are not natural. So they are acquired.
↓
Since virtue is not teachable it is not knowledge. Yet virtue is not natural but acquired.
↓
Virtue, like true opinion, is acquired and can be practiced without being taught or known

Taking the whole narrative into account the starting point is the question: is virtue teachable? The conclusion is that virtue is not teachable and if knowledge is teachable then virtue is not knowledge. This is essentially a deductive argument. Embedded in this main argument is the *elenchus* of Socrates and the boy, which is essentially an abductive–algorithmic argument. Hence the *elenchus* in the context of the entire dialogue is embedded in an overarching deductive argument for why virtue is not teachable.

8 Conclusion

One just does not want to leave *Meno* as there is so much there. We have seen varieties of abductive reasoning in *Meno elenchus* with the slave boy, including diagrammatic, visual, guessing, and discovery (which is recollection, the heart of the narrative). If we consider model-based reasoning to be 'the consideration and manipulation of various kinds of representations' [12, p. 45], then the algorithm and the geometric visual model are simultaneous model-based reasoning processes embedded in the *elenchus*. Plato was actually extremely complex, extremely sophisticated and way ahead of his times as any attempt to interpret him, including mine probably does not come close to what he really had in mind. Nonetheless, it would only be in Platonic spirit to entertain all interpretations but at the same time try to establish the viability of each interpretation.

Gerald Boter argues that the lines to be drawn in the beginning inside the square are the diagonals and not the transversals so that everyone who follows the literal reading of the instructions given by Socrates is mistaken. Boter's reason is that Socrates wants the boy to determine the area of the square by calculating and not by counting, in which the transversals help by dividing the square up into four one by one squares; and in the end the answer to the desired question is the diagonal so Socrates wants to draw it first [2; 25, p. 220]. This would be a viable interpretation only if Plato meant to use only one method here or thought that there was only one method. I have on the contrary shown that Socrates requires both the algorithmic method as well as the visual geometric method in order to make the full use of the dialectic towards recollection.

Boter's conjecture then is motivated by the myopic view of Plato that he used only deduction as a method, whereas what we have seen is that the dialectic is not possible without abduction as the final step of recollection itself is a stage of discovery and thereby abductive and not deductive. In fact deduction plays a marginal role or no role at all in the entire *elenchus* of *Meno*. The reason why Socrates chose the slave boy instead of a regular school boy with instructions in geometry; is that Socrates was neither trying to show off his knowledge in geometry as there were better geometers than him as the audience of the dialogue, nor was Socrates after any kind of theorem or proof. And a slave boy with no traditional schooling was not likely to either think of or be interested in a deductive proof, nor would he have understood it.

The purpose of the early point of the dialectic is for the slave boy to realize that he made an error and also to understand why he made the error. The visual method of counting is an effective pedagogical device to realize the error and then with the algorithmic 'and four times four is sixteen', and so on, helps the boy understand his error. First, in the visual square the boy sees that the square with the side four would have four squares at the bottom and hence four squares at the top, but once he simultaneously calculates this algorithmically he does not actually need to count all sixteen. This is why the two methods are purposefully used simultaneously.

The three stages of the dialectic are best captured by Bluck:

> First, Socrates dispels the slave's false supposition that a square twice the size of the original will have sides twice the length […] and his next incorrect answer, 'three feet', is treated in the same way. This elenchic or refutative procedure has a positive aspect: […] and aid towards the recollection of the correct answer […] a very important stage in the recollection process. […] The second stage is the 'stirring up' of latent true opinions, and the third is the conversion of these into knowledge [1, p. 15].

Bluck makes it clear that in order to accomplish the first stage effectively Plato requires the geometrical visual model and the algorithmic method alone would not have produced this:

> […] recollection can be aided by careful questioning and perhaps also by sense-experience, and that it is a process not a sudden jump […] recognition that neither a two- nor three- nor four-foot line will give a square of eight square feet […] lead(s) to a discovery of the correct solution […] the method of 'stirring up' true opinions on the question at issue is not radically different from the method of eliminating false opinions […] [1, pp. 16–17].

It is gratifying to find support in a scholar like Bluck of almost everything I have maintained in my discussion of Plato. The most important gratification is that what is being recollected are true opinions and not knowledge, as knowledge according to Bluck is the third stage, the tether stage. Bluck also makes an important point earlier that since the *Meno* is before the *Phaedo* and the theory of Forms does not begin to be formed until the *Phaedo*, it would not be appropriate to claim that the boy or Meno, in the dialogue, would recollect Forms [1, p. 6], and if knowledge is to be of Forms then they cannot recollect knowledge; however they could well recollect true opinions.

Finally, *86d-87c Meno* is more crucial in resolving the paradox of virtue and knowledge and this also involves the geometrical method. Mark Faller claims that the main purpose of Plato's analogy, in this passage, of the geometric problem of inscribing a triangle in a given circle and in determining whether virtue is knowledge, is not simply to show that virtue cannot be knowledge but to point out the logical form of analogical reasoning. Plato takes care to demonstrate the clear formal isomorphism between the geometric problem and the *Meno* paradox of virtue and knowledge that is required for the analogical argument to go through [4]. So, we can pin down Plato's *Meno* as the origins of a formal account of analogical reasoning.

> Teach me half the ampliativeness
> That thy powers must inferentially grow
> Such surprise and hypotheses generation madness
> From my reasonings would flow
> The world will abductively know then, as I am abductively knowing now[8]
> Adapted from Percy Bysshe Shelley, 'To a Skylark' (1820)

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[8] Again, the words from the last verse of Shelley's 'To a Skylark' have been changed. [26].

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