# Chapter 44 Research and Implementation of Ambiguity Resolution for Combined GPS/GLONASS/COMPASS Positioning

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Abstract As GLONASS and COMPASS systems are approaching their full constellations, and more Galileo satellites are to be launched, the need of combined positioning is increasing quickly. For high precise applications, data preprocessing and ambiguity resolution are the most important parts. Because Code Division Multiple Access technology is applied by both GPS and COMPASS, so the current data processing methods are also suitable for combined GPS/COMPASS positioning, but for GLONASS the cycle slip detecting and ambiguity resolution will biased by satellite wavelength differences because the Frequency Division Multiple Access technology. To solve this problem, a single difference phase observable differenced in time is proposed in this paper. With a majority voting procedure using the observable residuals from all the satellites we can detect and mark satellites obviously suffering from a cycle slip and "clean" satellites, then fixed the cycle slip using the receiver clock term computed by "clean" satellite. Iterative search approach is applied in ambiguity resolution. One double differenced ambiguity is fixed to integer according to the specified criteria in each iteration until all double differenced ambiguities are fixed. The data experiment shows that even one cycle of slip can be detected and fixed and ambiguities can be resolved correctly.

Keywords GPS  $\cdot$  GLONASS  $\cdot$  COMPASS  $\cdot$  Combined positioning  $\cdot$  Difference in time  $\cdot$  Cycle slip  $\cdot$  Ambiguity

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## 44.1 Introduction

With the 16th COMPASS satellite launched on October 2012, the COMPASS system network in the Asia–Pacific region has been completed, at the same time GLONASS system reached full constellation, experimental Galileo satellite has been lifted off, it is widely known that the combination of multiple GNSS constellations for positioning will be a future trend. Compared to a single system, multi-system positioning has efficiency advantages in terms of continuity, availability, reliability, accuracy, can greatly improve the usability, precise integrity and reliability of the user by the advantage of wealthy navigation information [1].

Although the combination of multi-system positioning make satellite positioning and navigation applications more widely, it also faces a number of challenges: the COMPASS system with GPS system using code division multiple access technology, the GPS processing technology is equally applicable to the GPS/COMPASS combination positioning; but the GLONASS system uses frequency division multiple access system, each satellite has different carrier frequency, so original cycle slip detection methods (such as triple differential method) and ambiguity resolution would be effected by the wavelength difference between the satellites. Therefore, how to weaken or eliminate the impact of the wavelength differences between GLONASS satellites is the key point in the relative positioning with the combined GPS/GLONASS/COMPASS constellations. The data preprocessing especially ambiguity resolution of GPS/GLONASS/ COMPASS is studied and a data processing method suitable for multi-system applications is proposed in this paper, through the implementation of the new method into our commercial software. The corresponding results are obtained and presented here.

### 44.2 Cycle-Slip Detection and Repair

### 44.2.1 Triple Difference Cycle-Slip Detection

In GPS relative positioning, the triple difference carrier phase observations are commonly used to the detection and repair of cycle slip. Triple difference observations eliminate the initial ambiguity and avoid constant value for the integer cycles after slip occurs and almost eliminate the clock error and common error terms of two stations. The ionospheric delay, tropospheric delay and multipath error are also considerably weakened after differencing for shorter baseline. So the triple difference model can be used to obtain the initial position to fix the ambiguities and cycle slip detection. Through examine the triple difference observation residuals based on initial baseline solution after adjustment, we can use the triple difference observation residuals changes to detect and repair cycle slips.

Triple difference observation model can be expressed as:

$$\nabla \Delta \varphi_{ij}^{pq}(t_1, t_2) = \nabla \Delta \varphi_{ij}^{pq}(t_2) - \nabla \Delta \varphi_{ij}^{pq}(t_1)$$
  
=  $\nabla \Delta \varphi_{ij}^{pq}(t_1, t_2) + \lambda^p b^p - \lambda^q b^q$  (44.1)

with:

$$b^{p} = N_{ij}^{p}(t_{2}) - N_{ij}^{p}(t_{1})$$
  
$$b^{q} = N_{ij}^{q}(t_{2}) - N_{ij}^{q}(t_{1})$$

Residual can be expressed:

$$\delta \nabla \Delta r = \lambda^p b^p - \lambda^q b^q \tag{44.2}$$

For data without cycle slip,  $\delta \nabla \Delta r$  should be a small value, but if the observation exists cycle slips, residual will result in great changes. We can think observations exist cycle slips when this change large than a certain limit. But for same cycle slip occurs both on base station and rover station, this method also can't accurate detecting.

For GPS,  $\lambda^p = \lambda^q$ , the only requirement is the double difference phase observations without cycle slip effect, we do not need to determine which satellite suffers cycle slip; but for GLONASS as different satellite has different signal wavelength, so after the cycle slip detection, we can eliminate its influence only if the satellite can be indicated exactly, so triple difference method can't detect and repair GLONASS satellites cycle slips [2].

# 44.2.2 Single Difference Phase Observable Differenced in Time

We form a new type of difference starting from the single difference phase observable:

$$\Delta \varphi_{ij}^{p}(t_{1}, t_{2}) = \Delta \rho_{ij}^{p}(t_{1}, t_{2}) + c \cdot \Delta t_{ij}(t_{1}, t_{2})$$
(44.3)

with:

$$\begin{aligned} \Delta \varphi_{ij}^p(t_1, t_2) &= \Delta \varphi_{ij}^p(t_2) - \Delta \varphi_{ij}^p(t_1) \\ \Delta \rho_{ij}^p(t_1, t_2) &= \Delta \rho_{ij}^p(t_2) - \Delta \rho_{ij}^p(t_1) \\ \Delta t_{ij}(t_1, t_2) &= \Delta t_{ij}(t_2) - \Delta t_{ij}(t_1) \end{aligned}$$

called a single difference phase observable "differenced in time". The residuals derived from the observation type (44.3) may be interpreted as sum of a possible cycle slip and the change of the receiver clock in the time interval, neglecting other error sources. We may thus write:

$$\Delta \mathbf{r}_{ij}^{p}(t_1, t_2) = \lambda^{p} \cdot b^{p} + c \cdot \Delta t_{ij}(t_1, t_2)$$
(44.4)

This residual can be expressed in cycles of satellite i and shows the integer nature of the cycle slip, but it is biased by the receiver clock change. If the receiver clock change would be known to a few cm, Eq. (44.4) could be used directly to detect cycle slips. The receiver clock term derived from code measurements (single point positioning) shows an error of a few nanoseconds or some tens of cycles (a few 0.1  $\mu$ s or some hundreds of cycles) and is certainly not good enough.

### 44.2.3 Cycle Slip Detection Algorithm

We have seen in Sect. 44.2.1 that the triple difference cannot be used to detect all possible cycle slips and to correct them on the single difference level. However, the single difference residuals "differenced in time" (44.4) may be used to detect a cycle slip on the single difference level by computing:

$$b^{p} = \frac{\Delta r_{ij}^{p}(t_{1}, t_{2}) - c \cdot \Delta t_{ij}(t_{1}, t_{2})}{\lambda^{p}}$$
(44.5)

But it requires the receiver clock change is known exactly. In order to keep the receiver clock term smaller than 0.1 cycles, the receiver clock change has to be determined with a precision of  $6 \times 10^{-11}$  s (or a few cm in units of length). In order to achieve this purpose, we first calculated all satellite single difference phase observable "differenced in time" of each epoch, then use a majority vote procedure we detect and mark satellites obviously suffering from a cycle slip and "clean" satellites and then calculate the receiver clock change using clean satellite:

$$\Delta \bar{t}_{ij}(t_1, t_2) = \frac{\prod_{i=1}^{n} \Delta r_{ij}^p(t_1, t_2)}{n \cdot c}$$
(44.6)

New ambiguities for all satellites are introduced if the number n of "clean" satellites is lower than two. Using Eq. (44.5) for each satellite with the receiver clock estimate  $\Delta \bar{t}_{ij}(t_1, t_2)$  the cycle slip is:

$$b^{p} = \frac{\Delta r_{ij}^{p}(t_{1}, t_{2}) - c \cdot \Delta \bar{t}_{ij}(t_{1}, t_{2})}{\lambda^{p}}$$
(44.7)

The data measurement proved this method can detect and repair more than one cycle slip, and at the same time, this method can be used for GPS and COMPASS and GPS/GLONASS/COMPASS combination observations.

### 44.3 Ambiguity Resolution

### 44.3.1 Mathematical Model

Traditional double difference observation model can be expressed as:

$$\nabla \Delta \varphi_{ij}^{pq} = \nabla \Delta \rho_{ij}^{pq} + \lambda^p N_{ij}^p - \lambda^q N_{ij}^q$$
  
=  $\nabla \Delta \rho_{ii}^{pq} + \lambda^p N_{ii}^{pq} + (\lambda^q - \lambda^p) N_{ii}^q$  (44.8)

For GPS and COMPASS system, since the code division multiple access technology is adopt, satellite wavelength equal for every satellite, the last term in Eq. (44.8) can eliminate, and the site the initial position and ambiguity floating solution can be acquired using the least square principle. Finally, with a certain ambiguity resolution methods (such as FARA, LAMBDA, etc.) to get the integer ambiguity, and then accurate three-dimensional site coordinates can be resolved [3].

Frequency division multiple access technology make the GLONASS satellite signals emitted at a different wavelength, when forming double difference observation equation a new single differential bias term  $b_{SD} = (\lambda^q - \lambda^p)N_{ij}^q$  can't be eliminated. The single difference ambiguity and double difference ambiguity cannot be separated, the normal equation become singular. One solution is to use other information such as pseudorange to obtain single difference ambiguity:

$$N_{ij}^{q} = \frac{1}{\lambda_q} (R_{ij}^{q} - \lambda_q \varphi_{ij}^{q})$$
(44.9)

With  $R_{ij}^q$  is the single differential pseudorange observation value. If we want to make the double difference ambiguity well fixed, its precision must be less than 0.1 cycles, the requirements of single difference ambiguity for different GLONASS satellites combination is showed on Table 44.1.

It is easy to see from the previous discussion that properly estimation of the Ambiguity greatly depends on the precision pseudorange observations. In many practical situation, however, pseudorange may be seriously biased by multipath and hardware delay. For example, a 5 m error in pseudorange can lead to an error of 26 cycles in single difference ambiguity. These errors can be negligible for the smaller wavelength difference satellite combination, but for the satellites combination with large wavelength difference the pseudorange accuracy will greatly affect the ambiguity resolution.

 Table 44.1 GLONASS wavelength differences in cycles and maximum bias allowed for the single differences ambiguities

0	8			
Satellite pare	Wavelength difference (cycle)	Maximum bias allowed		
Min	0.000351	285		
Max	0.00810	12		

# 44.3.2 Ambiguity Resolution Algorithm

In view of that the single difference ambiguity has smaller effect on satellite combination with small wavelength difference, an iterative solution of the double difference ambiguity is adopted, one double ambiguity in each iteration step. A specific algorithm is as follows:

- 1. For n satellites n single difference ambiguities are set up as unknown parameters in the normal equation system, assuming that there are no breaks or problems in the data forcing us to set up additional ambiguities.
- 2. After introducing code observations to remove the singularity of normal equation system, single difference can be estimated as real values.
- 3. Using the estimated single difference ambiguities and their covariance matrix, all possible double difference ambiguities are computed with the corresponding formal errors.
- 4. After the computation of all possible double difference ambiguities and their formal errors, a first double difference ambiguity parameter with the smallest wavelength difference is fixed to an integer number, according to specified resolution criteria (such as FARA, LAMBDA, etc.) [4].
- 5. After fixing the first double difference ambiguity, one of the two single difference ambiguities involved in the double difference ambiguity may be eliminated from the normal equation system and go to the next iteration until n-1 double ambiguities is fixed to integer.
- 6. In the final solution the unresolved single difference ambiguities and the baseline components are estimated at the same time using the fixed double ambiguities.

Theories prove the above method is applicable to GLONASS and combined GPS/GLONASS/COMPASS solution applies to both the original and a combination of carrier phase observations [5].

# 44.4 Applications and Results

### 44.4.1 Software Implementation

According to the model, the author added and modified a number of modules on the basis of Guangzhou Hi-Target Survey Instruments Co. Ltd new version data processing software HGO (Hi-Target Geomatics Office), including cycle slip detection and repair, GPS/GLONASS/COMPASS ambiguity resolution, developed a software oriented multi-system data integration and processing. When processing the data, you can set a certain kind of system separately or set using a variety of systems integration to conduct relative positioning. The following testing and analysis of the measured data is based on new HGO software.

## 44.4.2 Results

As there is no multi-system baseline data, GPS/GLONASS and GPS/COMPASS experiments are implemented separately. The measured data is acquired by Hi-target Vnet6 and Vnet8 receiver. Vnet6 and VNet8 is CORS reference station receivers of GPS/GLONASS and GPS/COMPASS systems. GPS/GLONASS data collected in Hainan in November 13, 2012 and GPS/COMPASS data collected in Guangzhou in March 21, 2012 is adapted for example, the baseline length is 34.4 and 20.1 km, observation time span is two hours, the interval is 5 s, the total COMPASS satellites is 11 and average observed number is 8 during observing period. The baseline fixed solution reference value is the previous day's single-day solution results.

In order to fully evaluate the combination of relative positioning performance, two different programs is used based on the observation environment. The first program is the ideal observing environment, more than four satellites and geometry strength is good; the second program is the non-ideal observation environment, the number of observation satellites is few. In each program, three ways including independent positioning and combined positioning is used for data processing.

#### 44.4.2.1 Ideal Environment Results

In ideal environment, Hainan and Guangzhou baselines processing results shown in Tables 44.2 and 44.3.

From Tables 44.2 and 44.3, we can see:

- 1. Independent positioning accuracy of GLONASS system is poor due to defects in the design and cannot meet the demand of relative positioning.
- 2. Under ideal conditions, the GPS/GLONASS combined positioning ratio value is lower than GPS, but the positioning accuracy is better than the standalone GPS, which is due to an increase in the number of satellites, space geometric distribution conditions improved.
- 3. COMPASS system has met the need of independent relative positioning and positioning accuracy is comparable with GPS. Under ideal conditions, as separate system has meet the requirements, positioning accuracy of combination of GPS/COMPASS is not significant improved, the advantage is not obvious.

System	Satellite number	Precisi	on (mm)	Ratio	RMS (mm)	
		X	Y	Ζ		
GPS	9	2.0	15.9	6.6	39.7	7.4
GLONASS	6	19.4	-37.1	-9.8	2.5	9.9
GPS/GLONASS	15	5.0	1.7	4.0	2.1	11.5

Table 44.2 Comparison of relative positioning results in good environment in Hainan

System	Satellite number	Precision (mm)			Ratio	RMS (mm)
		X	Y	Ζ		
GPS	8	0.7	-2.5	-2.2	26.4	10.9
COMPASS	8	-1.4	2.0	2.6	56.6	8.9
GPS/COMPASS	16	0.8	-1.6	-3.0	22.7	9.8

Table 44.3 Comparison of relative positioning results in good environment in Guangzhou

#### 44.4.2.2 Non-ideal Environment Results

In many observations case, the user cannot guarantee continuously tracking many GNSS satellites, such as in urban areas with dense buildings or serious occlusion area, sometimes the number of satellites is too insufficient to positioning. In order to artificially simulate the harsh environment of observations, part of the GPS satellite and part of the COMPASS satellite as well as GLONASS satellite is disabled in HGO software respectively, only remaining four satellites to solve, and the results are shown in the following Tables. 44.4 and 44.5.

We can see that in the case of less simultaneous observing satellites the positioning accuracy of the single system descend sharply, especially GLONASS and COMPASS system. This may be because the satellite number is few, space geometric distribution is poor, wrong ambiguity resolution is likely to appear in this regard. In combined relative positioning the positioning accuracy is declined but still within the allowable range due to the large number of synchronous satellite, we can see the advantage of combination of GPS/GLONASS/COMPASS positioning performs well in non-ideal observing conditions.

System	Satellite number	Precision (mm)			Ratio	RMS (mm)
		X	Y	Ζ		
GPS	4	-4.7	20.1	11.6	23.6	7.6
GLONASS	4	-23.5	62.3	13.9	1.9	9.6
GPS/GLONASS	8	2.2	16.3	10.4	6.3	9.6

Table 44.4 Comparison of relative positioning results in bad environment in Hainan

Table 44.5 Comparison of relative positioning results in bad environment in Guangzhou

System	Satellite number	Precision (mm)			Ratio	RMS (mm)
		X	Y	Ζ		
GPS	4	10.1	-7.1	7.7	3.5	10.6
COMPASS	4	-13.8	20.4	10.1	1.6	8.3
GPS/COMPASS	8	-0.6	2.5	-3.5	26.3	9.9

# 44.5 Conclusions

The correctness and feasibility of the proposed relative positioning algorithm for combined GPS/GLONASS/COMPASS constellations have been demonstrated by our experimental results. The COMPASS system has been used to carry out relative positioning independently, but the latest COMPASS positioning results are not yet as good as the results from the GPS system because the low number of satellites and weak constellation distribution. In addition, independent GLONASS positioning is not an easy task. GPS/GLONASS and GPS/COMPASS combination for relative positioning have no obvious advantages compared to a single system under ideal observing conditions, but in non-ideal observing conditions the combined positioning approach can well enhance observing satellite geometry strength, thus improving reliability and accuracy significantly.

As the lack of multi-system baseline data, the multi-system data experiment didn't take in this article, so the effect of multi-system positioning accuracy still needs further discussion.

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