

Chapter 10

Ionospheric Grid Modeling of Regional Satellite Navigation System with Spherical Harmonics

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Abstract As an important component of differential navigation service, ionospheric grid can greatly improve the accuracy of single frequency users. Wide-Area Augmentation System (WAAS) broadcasts a set of ionospheric grids data for authorized single frequency GPS users every 300 s. WAAS uses inverse distance weighting (IDW) algorithm to calculate the vertical total electron content (VTEC) at those ionosphere grid points (IGP). IDW algorithm has good correction accuracy, but has low availability in the area with few ionosphere pierce points (IPP). The availability and accuracy of ionospheric grid by IDW algorithm depend on the distribution of monitoring stations and the constellation of navigation system. As the constellation of regional satellite navigation system consists mainly of geostationary earth orbits (GEO) satellites and inclined geosynchronous orbits (IGSO) satellites, the IPPs have a concentrating distribution. Considering that the distribution of monitoring stations are all in China, using IDW algorithm to calculate ionospheric grid will not have good availability. This paper proposes a practical algorithm to calculate the regional ionospheric grid based on spherical harmonics (SH) model and International Reference Ionosphere (IRI-2012) model. Compared with IDW algorithm, the simulation data analysis indicates that the SH algorithm can greatly improve the availability from 82.62 to 99.83 % for mainland China and from 34.66 to 94.25 % for all grids, but the average correction RMS of ionospheric grid increases from 2.60TECU to 3.33TECU.

Keywords Regional satellite navigation system · Ionospheric grid · Spherical harmonics · Inverse distance weighting

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10.1 Introduction

As one of the error sources for navigation calculation, ionospheric delay seriously affects the positioning accuracy of the users. Dual-frequency users can remove ionospheric delay by linear ionosphere-free combination ignoring the higher-order term. Single-frequency users can calculate the ionospheric delay of IPP by the broadcasted parameters of ionosphere model. GPS broadcasts 8 parameters Klobuchar model on geomagnetic coordinate, and the correction accuracy is about 60 % [1, 2]; Galileo uses Nequick model, the designed correction accuracy is about 70 %. The error of ionosphere model becomes the most important factor determining single-frequency users' positioning accuracy.

To meet the needs of advanced users, especially the aviation users, institutions in different countries begin to build augmentation system, such as WAAS of America, EGNOS of European and the GAGAN of Indian. Each system has a similar principle to calculate the ionospheric grid: several monitoring stations and a center station are distributed in service area. Each monitoring station equipped with dual-frequency receivers collects the observation of all visible satellites, and calculates the IPP's coordinates and the ionospheric delays. Data mentioned above transmits to the center station every second through the network. Then ionospheric grid delay can be solved by the center station, and broadcasted by GEO satellites for the authorized single-frequency users.

WAAS broadcasts $5^\circ \times 5^\circ$ ionospheric grid corrections by GEO satellites and correction accuracy is about 80 % [3, 4]. Regional satellite navigation system provides differential services for authorized users, the service area covers from 7.5° to 55° north latitude, 70° to 145° east longitude, and using its GEOs to broadcast the ionospheric grid correction.

As regional satellite navigation system constellation consists mainly of high orbit satellites, the distribution of IPPs has a relative concentration. The availability of ionospheric grid by IDW algorithm greatly depends on the distribution of monitoring stations. IGP has low availability in case that there are few IPPs nearby.

To solve this problem, this paper will propose a spherical harmonics (SH) algorithm combined with regional satellite navigation system observation data and the IRI-2012 model to calculate the ionospheric grid. Real data and simulation analysis shows that this algorithm has a great increase in availability of grid points, and loss little accuracy compared with the ionospheric grid by the IDW algorithm.

10.2 IDW Algorithm

The IDW algorithm is widely used to calculate the ionospheric grid. It relies on the IPPs near the IGP. The algorithm is simple but has good accuracy at the available IGP. However, as the algorithm seriously relies on the distribution of IPPs,

it shows bad accuracy at IGP with not enough IPPs nearby. IDW algorithm is introduced as below.

According to global navigation satellite system (GNSS) observation equation, the dual-frequency linear geometry-free combination P_4 is:

$$P_4 = P_1 - P_2 \quad (10.1)$$

where P_1, P_2 are the P-code observations on B1, B2 frequency. Linear geometry-free combination eliminates all errors related to geometry, and the remaining factors are ionosphere delay, differential code bias (DCB) of satellite and receiver, multipath error and ranging noise. Ignoring the multi-path and measuring error,

$$P_4 = -\left(\frac{f_1^2 - f_2^2}{f_2^2}\right)F(z) \cdot I + c \cdot \Delta b^s + c \cdot \Delta b_r \quad (10.2)$$

where f_1, f_2 are the frequencies of B1, B2. Δb^s is the TGD of satellite s ; Δb_r is the IFB of receiver r ; c is the speed of light; I is the ionosphere delay from receiver r to the satellite s :

$$I = \frac{40.28}{f_1^2} \cdot VTEC \quad (10.3)$$

$F(\cdot)$ is the ionospheric mapping function depending on the zenith distance z .

$$F(z) = \frac{1}{\cos(z)}, \sin(z) = \frac{R}{R+H} \sin\left(z' + \frac{\pi}{2}\right) \quad (10.4)$$

where z' is the zenith distance at the IPP; $R = 6371$ km is the mean earth radius; $H = 375$ km is the ionospheric single layer height. As Δb^s and Δb_r are already known, VTEC of every epoch at every IPP can be got by Eq. (10.2).

For every IGP, IDW algorithm uses the VTEC of IPPs in the four adjacent grids to calculate the VTEC of IGP.

$$I_{IGP_i} = \sum_{j=1}^n I_{IPP_j} P_{ij} / \sum_{j=1}^n P_{ij} \quad (10.5)$$

where I_{IGP_i} is the VTEC of i th IGP; n is the number of IPPs in the four adjacent grids; I_{IPP_j} is VTEC of IPPs; P_{ij} is the weight of j th IPP to the IGP. Here we take P_{ij} as the inverse distance from j th IPP to i th IGP.

10.3 Solution Method of Ionospheric Grid Based on Spherical Harmonics

The SH algorithm uses all VTEC of IPPs to fit the ionosphere of whole area, but this algorithm is relatively complex.

Assume there are N IPPs during some time, $IPP_i (i = 1, \dots, N)$ is the IPP at the position of (lat_i, lon_i) at t_i , and its VTEC is $VTEC_i$. Using the SH to fit VTEC.

$$VTEC = \sum_{n=0}^{n_{max}} \sum_{m=0}^n P_{nm}(\sin \beta)(C_{nm} \cos(ms) + S_{nm} \sin(ms)) \quad (10.6)$$

where β is the latitude of IPP; $s = sod + lon/15$ is the local time of IPP; n_{max} is the maximum degree of SH expansion; P_{nm} is the normalized Legendre function; C_{nm} , S_{nm} are the unknown SH coefficients. According to the SH function above, we can solve C_{nm} and S_{nm} using the least squares. Then we can calculate the ionospheric grid VTEC by the SH coefficients.

As regional satellite navigation system has less monitoring stations in western and northern China, there are less IPPs in those areas. However, the SH algorithm relies on the distribution of IPPs, better results are presented in IPPs covered areas, and SH function diverges rapidly out of boundary. The greater degree of SH function, the more seriously the SH function diverges. In order to decrease the divergency of SH function in area out of China with no IPP, we use the IRI-2012 model to forecast the exact boundary of ionospheric grid service area.

IRI-2012 model is an experimental model based on observation data of satellites and receivers, aiming to describe the ionosphere in a peaceful geomagnetic condition. Yanyuevich showed that given the geomagnetic planetary index Kp under 7, IRI-2007 model has an accuracy of 30 % relative to measurement data from satellite altimeters, and the amplitude of the absolute error is about 11TECU in the abnormal areas [5].

Figure 10.1a is the VTEC map only using IPPs of navigation system by 6° SH functions for day 300 of 2012 at 21:00 (UT), and we use pseudo-color to present the VTEC value. It is assumed that the reasonable ionosphere delay is between 0TECU and 150TECU, we make the VTEC value equals 0TECU when it is out of the interval above. From the figure, it is obvious that SH algorithm has a normal condition in China, but diverges rapidly at high latitudes and low latitudes. Figure 10.1b is the VTEC map using both IPPs of navigation system and boundary IPPs of IRI-2012 model at the same time. In the same circumstance, SH algorithm covers all area at high latitudes and most area at low latitudes.

Figure 10.2 is the VTEC map using the SH algorithm at the daily peak of ionosphere. The SH function diverges at low latitudes. It is probably because IRI-2012 model has low accuracy at low latitudes, then the additional IPPs of ionospheric grid boundary lack of enough data supporting. So the SH algorithm has a bad performance in those areas.

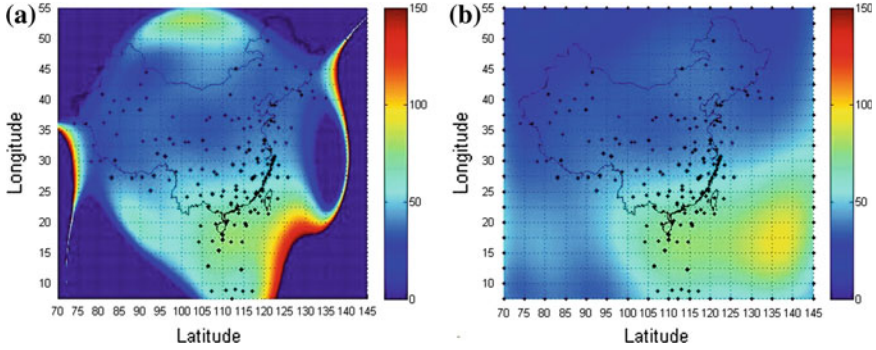
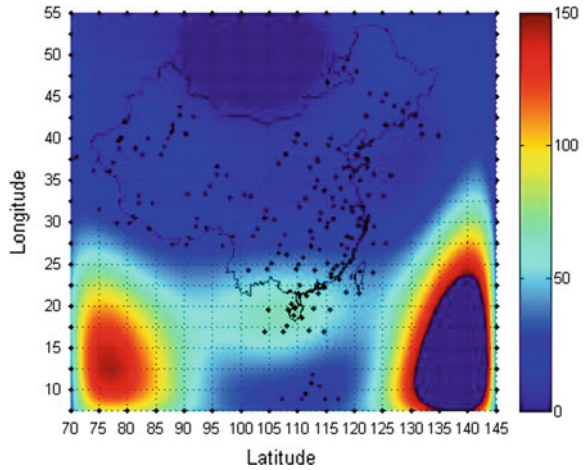


Fig. 10.1 Example of VTEC map using different IPPs, for day 300 of 2012, at 21:00 UT, unit TECU. **a** Using IPPs of navigation system receivers. **b** Using IPPs of navigation system receivers and boundary IPPs from IRI-2012

Fig. 10.2 Example of VTEC map using IPPs of navigation system receivers and boundary IPPs from IRI-2012, for day 300 of 2012, at 7:00 UT, unit TECU



10.4 Interpolation Algorithm of IGPs

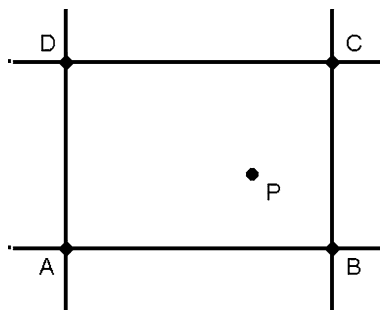
As the user receives the VTEC data of IGPs, interpolation algorithm is needed to calculate the VTEC at an IPP. There are many interpolation algorithms. Here we introduce a simple algorithm for the accuracy analysis of different ionospheric grid algorithms in the next section.

As shown in Fig. 10.3, the IPP P is located in the grid ABCD. We can calculate the VTEC of IPP with weighted VTEC of four IGPs. The weight of each IGP takes the way of inverse distance weighted.

$$Q_A = \omega_x \omega_y \tag{10.7}$$

$$Q_B = (1 - \omega_x) \omega_y \tag{10.8}$$

Fig. 10.3 Example of IPP and IGPs



$$Q_C = (1 - \omega_x)(1 - \omega_y) \quad (10.9)$$

$$Q_D = \omega_x(1 - \omega_y) \quad (10.10)$$

where

$$\omega_x = \frac{x_P - x_A}{\delta_x} \quad (10.11)$$

$$\omega_y = \frac{y_P - y_A}{\delta_y} \quad (10.12)$$

δ_x is the grid interval of longitude; δ_y is the grid interval of latitude. The VTEC of IPP P is:

$$I_P = \frac{Q_A I_A + Q_B I_B + Q_C I_C + Q_D I_D}{Q_A + Q_B + Q_C + Q_D} \quad (10.13)$$

10.5 Numerical Experiments

In three sections above, we introduce the IDW algorithm, SH algorithm and interpolation algorithm. This section will evaluate the performance of two ionospheric grid algorithms in both availability and accuracy.

We assume that 23 cities distributed in China include Nansha and Xisha islands have monitoring stations. Coordinates of IPPs at every epoch can be achieved by the coordinates of monitoring stations and broadcast ephemeris of navigation satellites. VTEC of every IPP can be got by global ionosphere map of IGS. Then we can use the VTEC of IPP to do simulation experiment and numerical analysis of IDW algorithm and SH algorithm.

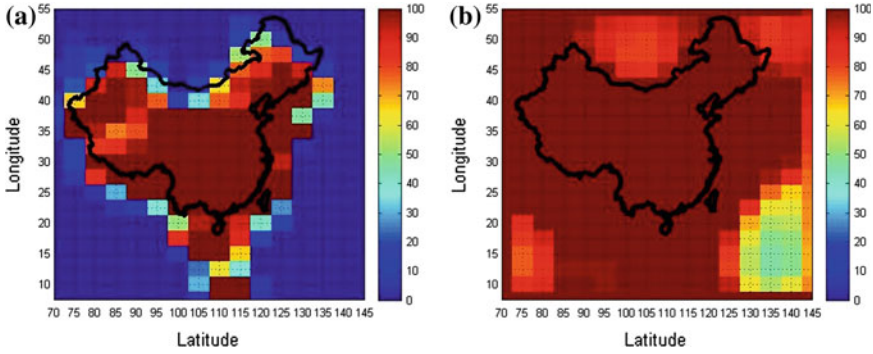


Fig. 10.4 Availability of two algorithms ionospheric grid, for day 300 of 2012, unit %. **a** Availability of IDW grid. **b** Availability of SH grid

10.5.1 Availability Analysis

If VTEC of a IGP can't be calculated, we set this IGP unavailable. The number and distribution of unavailable IGPs have a bad influence on differential service. Define the daily availability of a IGP ρ :

$$\rho = \frac{t}{86400} \tag{10.14}$$

where t is the time span in which this IGP is available for one day, units second. We define the areal availability as the average of daily availabilities of all IGPs in the area.

Figure 10.4a shows the availability of ionospheric grid by IDW algorithm for day 300 of 2012. Pseudo colors represent the availability. Dark red stands for the availability of 100 %. It is obvious that most area in China has good availability, however, the availability is quite low in northwestern China such as Inner Mongolia, Xinjiang and Tibet. The area out of China is almost unavailable in all daytime.

Figure 10.4b shows the availability of ionospheric grid by SH algorithm for day 300 of 2012. It is 100 % available in most China. The availability out of China is improved by a large margin, and there is no IGP unavailable.

Table 10.1 Availability of two algorithms ionospheric grid in different areas

	IDW algorithm (%)	SH algorithm (%)
Low latitudes	16.85	88.86
Mid-latitudes	60.04	97.23
High latitudes	18.62	95.65
Mainland China	82.62	99.83
All grids	34.66	94.25

Table 10.1 shows the availability in different areas. IDW algorithm has low availability (less than 20 %) at low latitudes and high latitudes, and the availability is 60.04 % at mid-latitudes. The availability of SH algorithm improved greatly, all latitudes are more than 88 %. The availability increased from 82.62 to 99.83 % in mainland China, and from 34.66 to 94.25 % in all service area. SH algorithm can not only improve the availability of mainland China, but also can greatly improve the availability of area out of China.

10.5.2 Accuracy Analysis

To comprehensively compare the effectiveness of IDW algorithm and SH algorithm, this part analyses the accuracy of two algorithms. Since GEO satellite has geostationary characteristics, it can be a good method to monitor ionosphere delay at a specific point. In mainland China, we take dual-frequency COMPASS receivers' real observations to calculate the VTEC of IPP. We take this VTEC as the reference. Ionospheric grid of IDW algorithm and SH algorithm are used to calculate the VTEC of IPP by the interpolating algorithm in Sect. 10.4, compared to the VTEC of dual-frequency observation can evaluate the accuracy of two algorithms.

The top graph in Fig. 10.5 describes VTECs calculated by three methods above, the bottom graph is the difference between VTECs of two grid algorithms and VTECs of dual-frequency observations. It is shown that SH algorithm and IDW

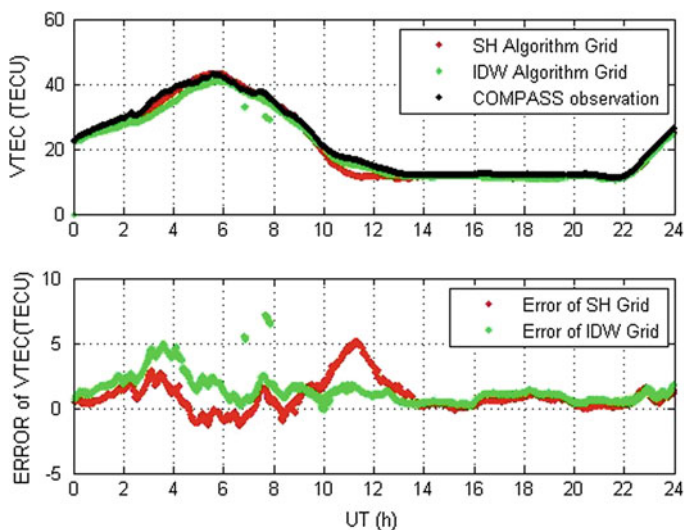


Fig. 10.5 VTEC of three methods, and the differences between two ionospheric grids and COMPASS observation, for day 300 of 2012

Table 10.2 Two kinds of ionospheric grid RMS in different regions, for day 300 of 2012, unit TECU

	IDW algorithm	SH algorithm
San Ya	4.81	6.74
Cheng Du	2.89	4.43
Ka Shi	2.75	2.06
Zheng Zhou	1.33	0.74
La Sa	1.96	3.97
Nan Jing	1.84	2.04

Table 10.3 RMS and correction accuracy of three grids solved by SH algorithm compared with IGS map, for day 300 of 2012, unit TECU

IGP	RMS	Correction accuracy (%)
(20°, 90°)	14.92	71.65
(35°, 135°)	11.21	76.86
(50°, 80°)	8.18	65.95

algorithm have almost same correction accuracy, and their RMS are 1.25TECU and 1.14TECU.

Table 10.2 is RMS of ionospheric grid by two algorithms in different areas. Chengdu and Sanya are ionosphere abnormal areas, but RMS has little difference by two algorithms. The difference of RMS in all areas is less than 2TECU (0.33 m of B1).

As there is no COMPASS receiver, global ionosphere map of IGS is used to evaluate the accuracy of IGP's out of China. We take three IGP's (20°, 90°), (35°, 135°) and (50°, 80°) as an example, which are all unavailable by IDW algorithm.

Table 10.3 shows RMS and correction accuracy of three IGP's by SH algorithm. RMS is small at high latitudes for 8.18TECU, with over 10TECU RMS at low latitudes. As large ionosphere delay at low latitudes, it has a big RMS but good correction accuracy. However, since ionosphere delay is small at high latitudes, it has a small RMS but bad correction accuracy.

10.6 Conclusions

The theoretical analysis and numerical analysis above shows that the availability in mainland China of IDW algorithm is 82.62 %, and almost unavailable out of China, but has good accuracy at available IGP's. Mean of daily RMS in different areas is 2.60TECU. The availability of SH algorithm improves greatly: all IGP's are 100 % available in mainland China, the availability is 95.65 % at high latitudes, and the availability is 88.86 % at low latitudes. SH algorithm and IDW algorithm have almost same correction accuracy in mainland China. The difference is less than 2TECU. Average RMS of SH algorithm in different areas is 3.33TECU. In conclusion, two algorithms both have advantages and

disadvantages. Compared with IDW algorithm, SH algorithm shows better availability and slightly lower accuracy. As for differential single-frequency users, availability is an important factor to ensure their navigation and positioning service. Under the circumstance of sacrificing accuracy in small range, it is very meaningful that SH algorithm can greatly improve the availability.

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