

Computing Nature – A Network of Networks of Concurrent Information Processes

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1 Introduction

The articles in the volume *Computing Nature* present a selection of works from the Symposium on Natural/Unconventional Computing at AISB/IACAP (British Society for the Study of Artificial Intelligence and the Simulation of Behaviour and The International Association for Computing and Philosophy) World Congress 2012, held at the University of Birmingham, on the occasion of the centenary of Alan Turing's birth.

This book is about nature considered as the totality of physical existence, the universe. By physical we mean all phenomena - objects and processes - that are possible to detect either directly by our senses or via instruments. Historically, there have been many ways of describing the universe (the cosmic egg, the cosmic tree, the theistic universe, the mechanistic universe) while a particularly prominent contemporary model is the computational universe.

One of the most important pioneers of computing, Turing, seen by Hodges [1] as natural philosopher, can be identified as a forerunner and founder of the notion of computing nature and natural computing through his ideas about morphological computing, "unorganized" (neural-network type) machines and "oracle" machines. Turing's impact on the development of computing can be seen as two-fold: laying down the foundations of the theory of computing by his Turing Machine model he provided such powerful paradigm that soon led to the belief that it is all we can do when it comes to computing. But, "There are assumptions underlying the paradigm which constrain our thinking about the realities of computing", as Cooper in this volume rightly observed. On the other hand, his work on natural computing points towards the development in different directions. It is obvious from Turing's own research that he did not consider Turing Machine model the only possible way of computation.

After many decades of development, present day computers are distinctly different from the early stand-alone calculating machines that Turing helped construct, that were designed to mechanize computation of mathematical functions. Computers today are networked and largely used for world-wide communication and variety of information processing and knowledge management. They are cognitive tools of extended mind (in the sense of Clark and Chalmers) used in social interactions and they provide ever growing repositories of information. Moreover, computers play an

important role in the control of physical processes and thus connect directly to the physical world in automation, traffic control, robotics and more. Apart from classical engineering and hard-scientific domains, computing has in recent decades pervaded new fields such as biology and social sciences, humanities and arts – all previously considered as typical soft, non-mechanical and unautomatable domains.

Computational processes running in networks of networks (such as the internet) can be modeled as distributed, reactive, agent-based and concurrent computation. The main criterion of success of this computation is not its termination, but its behavior - response to changes, its speed, generality and flexibility, adaptability, and tolerance to noise, error, faults, and damage. Internet, as well as operating systems and many database management systems are designed to operate indefinitely and termination for them would be an error. We will return to the topic of concurrent computing and its relationship with Turing machine model of computation in more detail later on.

One of the aims of this book is to show the state of the art of developments in the field of natural/unconventional computation which can be seen as generalization and enrichment of the repertoire of classical computation models. As a generalization of the traditional algorithmic Turing Machine model of computation, in which the computer was an isolated box provided with a suitable algorithm and an input, left alone to compute until the algorithm terminated, natural computation models *interaction* i.e. communication of computing processes with each other and with the environment. In natural systems, computation is information processing that can proceed on both symbolic and sub-symbolic (signal-processing) level. For human cognitive processes it means that not only the execution of an algorithm can be seen as computation, but also learning, reasoning, processing of information from sense organs, etc.

Hewitt [2] characterizes the Turing machine model as an *internal (individual)* framework and the Actor model of concurrent computation as an *external (sociological)* model of computing. This tension between an (isolated) individual *one* and (interacting) social *many* resonates with two articles from this volume: Cottam et al. who distinguish "conceptual umbrella of *entity* and *its ecosystem*" and Schroeder's view that "Information can be defined in terms of the categorical opposition of one and many, leading to two manifestations of information, selective and structural. These manifestations of information are dual in the sense that one always is associated with the other." Here information is directly related with computation defined as information processing. [3]

The frequent objection against the computational view of the universe, elaborated by Zenil in this volume, is that "it is hard to see how any physical system would not be computational." The next frequently mentioned issue is: if the universe computes, what are the input and the output of its computation? This presupposes that a computing system must have an input from the outside and that it must deliver some output to the outside world. But actor system [2] for example needs no input. Within pan-computationalist framework, the whole universe computes its own next state from its current state [4]. As all of physics is based on quantum mechanical layer of information processing, zero-point (vacuum) oscillations can be seen as constant input for the computational network of the universe. What causes different processes in the universe is the interaction or exchange of information between its parts. The universe is

a result of evolution from the moment of big-bang or some other primordial state, through the complexification of the relationships between its actors by computation as a process of changes of its informational structure. Physical forces are established through particle exchanges (message exchanges) which necessarily connect particles into a web of physical interactions which are manifestation of natural laws. The whole of the universe is in the state of permanent flow, far from steady state, which results in forming increasingly complex structures, [5]. So much on the input-output objection.

As to the objection that not all of the universe can be computational, as it is a too powerful a metaphor, [6] it is essential to keep in mind the layered architecture of the computing nature, as not all of computation is the same – computation is proceeding on many scales, on many levels of hierarchical organization. Moreover, in tandem with computation, universe is described by information, representing its structures. *Given that computation follows physical laws, or represents/implements physical laws*, generative model of the universe can be devised such that some initial network of informational processes develops in time into increasingly complex (fractal, according to Kurakin, [5]) information structures.

The parallel could be drawn between natural computationalism and atomic theory of matter which is another general theory which implies that all of matter is made of atoms (and void). We may also say that all of physics (structures and processes) can be derived from elementary particles (and void that is an ocean of virtual particles which for short time, obeying Heisenberg uncertainty relations, pop into existence and quickly thereafter disappear). This does not make the world a soup of elementary particles where no differences can be made, and nothing new can emerge. Those basic elements can be imagined as neodymium ball magnets from which countless structures can be constructed (in space and time, through interactions).

Unified theories are common and valued in physics and other sciences, and natural computationalism is such a unified framework. It is therefore not unexpected that physicists are found among the leading advocates of the new unified theory of informational and computational universe – from Wheeler, via Feynman, to our contemporaries such as Fredkin, Lloyd, Wolfram, Goyal and Chiribella. For the articles of latter two physicists on the topic of informational universe, see the special issue of the journal *Information* titled *Information and Energy/Matter* [7] and the special issue of the journal *Entropy* titled *Selected Papers from Symposium on Natural/ Unconventional Computing and its Philosophical Significance* [8].

Conceptualizing the physical world as a network of information networks evolving through processes of natural computation helps us to make more compact and coherent models of nature, connecting non-living and living worlds. It presents a suitable basis for incorporating current developments in understanding of biological, cognitive and social systems as generated by complexification of physicochemical processes see Deacon [9] through self-organization of molecules into dynamic adaptive complex systems that can be understood as morphogenesis, adaptation and learning—all of which can be understood as computation (information processing), [50].

2 Re-Conceptualizing of Nature as Hierarchically Organized Network of Networks Computational Architecture

2.1 Natural Hierarchy

“If computation is understood as a physical process, if nature computes with physical bodies as objects (informational structures) and physical laws govern process of computation, then the computation necessarily appears on many different levels of organization. Natural sciences provide such a layered view of nature. One sort of computation process is found on the quantum-mechanical level of elementary particles, atoms and molecules; yet another on the level of classical physical objects. In the sphere of biology, different processes (computations = information processing) are going on in biological cells, tissues, organs, organisms, and eco-systems. Social interactions are governed by still another kind of communicative/interactive process. If we compare this to physics where specific “force carriers” are exchanged between elementary particles, here the carriers can be complex chunks of information such as molecules or sentences and the nodes (agents) might be organisms or groups—that shows the width of a difference.” [3]

Searching for a framework for natural computation and looking at nature from variety of perspectives and levels of organization, Cottam et al. in this book address the general question of hierarchy in nature and point to Salthe who “*restricts the term hierarchy to two forms: the scale (or compositional) hierarchy and the specification (or subsumption) hierarchy.*” However, they find that for the description of natural systems a third form they name the representation or model hierarchy is most suitable. The central tenet is the birational ecosystemic principle: “*Nature seen through sciences brings all of Science under a generalized umbrella of entity and its ecosystem, and then characterizes different types of entity by their relationships with their relevant ecosystems.*” (emphasis added)

In spite of suggested tree-structure with representation on top, followed by model hierarchy with subsequent compositional and subsumption hierarchy, the authors emphasize the movement between the bottom and the top. Parts define the whole, which once established, affect its parts. As a case in point, they provide an example of a (Natural) model hierarchy for a tree represented at different scales: “{a tree described in terms of atoms}, {a tree described in terms of molecules}, {a tree described in terms of cells}... up to {a tree described in terms of branches}, {a tree as itself – a tree}”. Here inter-scale interfacing and consequently digital-analog interfaces are discussed and it is pointed out that naturally-hierarchical multi-scale organisms function qualitatively differently from a digital computer. The article concludes with the hope that this birational ecosystemic hierarchical framework will be capable of providing a new definition of computation, closer to physical processes in nature.

2.2 Cognitive Level of Information Processing

In a hierarchy of organizational levels in nature the most complex level of information processing is cognitive level and it subsumes all lower levels that successively emerge from their antecedent lower levels. Lindley in this volume addresses the problems

encountered in the development of engineered autonomous and intelligent systems caused by *exclusively linguistic models of intelligence*. The alternative proposed is “taking inspiration more directly from biological nervous systems”. This approach is argued to be able to go “far beyond twentieth century models of artificial neural networks (ANNs), which greatly oversimplified brain and neural functions”. This implies study of computation as information processing in neural and glial systems in order to *implement* “asynchronous, analog and self-* architectures that digital computers can only *simulate*.” (emphasis added) The difference between physical process as it appears in nature and its computational simulation is essential when we not only talk about them and represent them them, but also use them as computational resources in AI systems.

Continuing on the level of neural systems, Phillips’s paper addresses the important topic of coordination of concurrent probabilistic inference. Adaptively organized complexity of life builds on information processing and in cognitive agents with neural systems also on inference. The paper discusses the theory of Coherent Infomax in relation to the Theory of free energy reduction of probabilistic inference. Coherent Infomax shows how neural systems combine local reliability with context-sensitivity and here we recognize the leitmotif from several other papers: individual in relation to the social, or agent and its eco-system.

Basti and Dodig-Crnkovic recognize significant role Turing played as a pioneer of natural computing, especially in the field of morphological computing and neural networks (unorganized machines).

Bull et al. in this book address Turing’s unorganized machines as models of neural systems. Turing in his 1948 paper [10] made an essential insight about the *connection of social aspects of learning and intelligence*. From the contemporary perspective of natural computing we see networks as information processing mechanisms and their role in intelligence is fundamental. Suggesting that natural evolution may provide inspiration for search mechanisms to design machines, Bull et al investigate Turing’s dynamical representation for networks of vesicles (membrane-bound compartments with Belousov-Zhabotinsky mixture) used as liquid information processing system. Communication between vesicles via chemical signals - excitations propagating between vesicles, was seen as imitation or cultural information communication. The authors hope that this may provide “a useful representation scheme for unconventional computing substrates”.

Arriola-Rios, Demery, Wyatt, Sloman and Chappell contribute to this book with a study of object representation in animals (especially parrots) and robots from segregate information about physical objects. This work helps better understanding of the mechanisms of information processing on the cognitive level. Information in a cognizing agent forms internal representations dependent on the way of its use, or the way of the interaction of the agent with the environment. Information could be compressed and re-used for interpretations, and identification of causal relationships and functions. It is described “how a selection of key elements from the environment could be used as a basis for an object representation, and considered possible underlying exploration strategies for gathering information by observing natural behaviour.” Particular analysis is devoted reasoning about deformable objects through key frames.

Still on the cognitive level of an agent processing information through its physical structures, it is instructive to connect to Cooper's observation about Turing's model.

“We might have been told at some point that it was devised as a disembodied model of machine computation. Not so, of course. 2012 has made everyone aware of the very specific physicality of the computing situation that Turing was modeling, the predominantly women ‘computers’ following instructions. (...) The underlying physicality may be highly complex. But such things as the human computer's aches and pains, her feelings of hunger or boredom, are factored out of the process.”

Yet, on a more basic level, “her feelings of hunger or boredom” were part of her being alive which made it possible for her to compute. If we want to construct such self-sustained, intelligent, adaptive computers capable of not only following instructions but even creating new algorithms, we might need to take the boredom and joy and other human characteristics into our broader model of computing. Those are qualities that may fuel creativity, even though they act as disturbance when performing lengthy mechanical calculations. Nature uses both mechanical and creative computing in cognitive agents.

3 The Unreasonable Effectiveness of Mathematics in the Natural Sciences (Except for Biology). Mathematicians Bias and Computing Beyond the Turing Limit

Mathematician's contribution to the development of the idea of computing nature is central. Turing as an early proponent of natural computing put forward a machine model that is still in use. How far can we hope to go with Turing machine model of computation?

In the context of computing nature, living systems are of extraordinary importance as up to now science haven't been able to model and simulate the behavior of even the simplest living organisms. “The unreasonable effectiveness of mathematics” observed in physics (Wigner) is missing for complex phenomena like biology that today lack effective mathematical models (Gelfand), see Chaitin [11].

Not many people today would claim that human cognition (information processing going on in our body, including brain) can be adequately modeled as a result of computation of one Turing machine, however complex function it might compute. In the next attempt, one may imagine a complex architecture of Turing machines running in parallel as communicating sequential processes (CSPs) exchanging information. We know today that such a system of Turing machines cannot produce the most general kind of computation, as truly asynchronous concurrent information processing going on in our brains. [4]

However, one may object that IBM's super-computer Watson, the winner in man vs. machine "Jeopardy!" challenge, runs on contemporary (super)computer which is claimed to be implementation of the Turing machine. Yet, Watson is connected to the Internet. And Internet is not a Turing machine. It is not even a network of Turing

Machines. Information processing going on throughout the entire Internet includes signaling and communication based on complex asynchronous physical processes that cannot be sequentialized. (Hewitt, Sloman) As an illustration see Barabási et al. article [12] on parasitic computing that *implements computation on the communication infrastructure of the Internet, thus using communication for computation.*

Zenil in this volume examines the question: “What does it mean to claim that a physical or natural system computes?” He proposes a behavioural characterisation of computing in terms of a measure of programmability, which reflects a system’s ability to react to external stimuli. To that end Zenil investigates classical foundations for unconventional computation.

Cooper in his chapter “What Makes a Computation Unconventional?” investigates the relationships between method and matter, process and embodiment. He addresses the phenomenon of emergence, which is recurrent theme of this book, but in Coopers approach emergence is related to unconventional, higher type computation:

“Although Stephen Kleene provided formal content to the notion of higher type computation via a series of papers spanning over 30 years (1959 - 1991), the physical relevance of his take on the topic needs to be clarified. A forthcoming book on "Computability At Higher Types" by John Longley and Dag Normann is eagerly anticipated. The intuition is that computational unconventionality certainly entails higher type computation, with a correspondingly enhanced respect for embodied information."

Hernandez-Espinosa and Hernandez-Quiroz, starting from the old computationalism defined as the belief that the human mind can be modeled by Turing Machines, analyze Wolfram’s Principle of Computational Equivalence based on his studies of cellular automata – the claim that “any natural (and even human) phenomenon can be explained as the interaction of very simple rules.” The next step in cellular automata models may be to replace present basic simple rules of cellular automata with more elaborate ones. Instead of synchronous update of the whole system; they can be made asynchronous networks of agents, placed in layered architectures on different scales etc. Here we recognize the basic idea of generative science which is to generate apparently unanticipated and infinite behaviour based on deterministic and finite rules and parameters reproducing or resembling the behavior of natural and social phenomena. As an illustration see Epstein, [13].

If we want to generalize the idea of computation so to be able to encompass more complex operations than mechanical execution of an algorithm, simulating not only a person executing strictly mechanical procedure, but the one constructing a new theory, we must go back to underlying mathematics.

While Cooper in this volume asks “*To what extent can the explanatory power of the mathematics clarify key issues relating to emergence, basic physics, and the supervenience of mentality on its material host?*” Dodig-Crnkovic and Burgin investigate the explanatory power of mathematics. They analyze methodological and philosophical implications of algorithmic aspects of unconventional/natural computation that extends the closed classical universe of computation of the Turing machine type. The new extended model constitute an open world of algorithmic constellations,

allowing increased flexibility and expressive power, supporting constructivism and creativity in mathematical modeling and enabling richer understanding of computation, see [14].

3.1 Hypercomputation - Beyond the Turing Limit

Hypercomputation is the research field that formulated the first ideas about the possibility of computing beyond Turing machine model limits. The term hypercomputation was introduced by Copeland and Proudfoot [15]. The expression "super-Turing computation" was coined by Siegelman and usually implies that the model is physically realizable, while hypercomputation in general typically relies on thought experiments. Present volume offers two contributions that sort under hypercomputation, written by Franchette and Douglas.

Franchette studies the possibility of a physical device that hypercomputes by building an oracle hypermachine, which would be a device able to use external information from nature in order to go beyond Turing machines limits. The author also addresses an analysis of the verification problem for oracle hypermachines.

Douglas in his contribution presents a critical analysis of Siegelmann Networks.

3.2 Physical Computation "In Materio" - Beyond the Turing Limit

Several authors at the Symposium on Natural/Unconventional Computing at AISB/IACAP World Congress 2012 (Stepney, Cooper, Goyal, Basti, Dodig-Crnkovic) underlined the importance of physical computing, or as Stepney [16] termed it, "computation in materio". Along the same lines, Cooper in his article Turing's Titanic Machine? [17] diagnoses the limitations of the Turing machine model and identifies the ways of overcoming those limitations by introducing:

- Embodiment invalidating the 'machine as data' and universality paradigm.
- The organic linking of mechanics and emergent outcomes delivering a clearer model of supervenience of mentality on brain functionality, and a reconciliation of different levels of effectivity.
- A reaffirmation of experiment and evolving hardware, for both AI and extended computing generally.
- The validating of a route to creation of new information through interaction and emergence.

Related article by the same author, The Mathematician's Bias and the Return to Embodied Computation, in [18], analyses the role of physical computation vs. universal symbol manipulation.

The theme of embodied computation is addressed in this volume by Hernandez-Quiroz and Padilla who examine actual physical realizability of mathematical constructions of abstract entities - a controversial issue and important in the debate about the limits of the Turing model. The authors study a special case of physical realizability of the enumeration procedure for rational numbers via Cantor's diagonalization by an Ising system.

3.3 Higher Order Computability - Beyond the Turing Limit

One of the main steps towards the new paradigm of natural/unconventional computing is to make visible host of myths which are surrounding the old paradigm and helping it to survive. One of those myths is that our modern computers with all their programming languages are just diverse implementations of Turing machines. However, as Kanneganti and Cartwright already argued twenty years ago:

“Classic recursion theory asserts that all conventional programming languages are equally expressive because they can define all partial recursive functions over the natural numbers. This statement is misleading because programming languages support and enforce a more abstract view of data than bit strings. In particular, most real programming languages support some form of higher-order data such as potentially infinite streams (input and output), lazy trees, and functions.” Kanneganti and Cartwright [19]

Kleene was a pioneer of higher order computability as he “opened the frontiers of computability on higher type objects in a series of papers first on constructive ordinals and hierarchies of number-theoretical predicates and later on computability in higher types.” Soare [20]

Also Cooper [21] underlines the importance of higher-order computational structures as characteristic of human thinking. This can be connected to higher-order functional programming, which means, among others, programming with functions whose input and/or output may consist of other functions.

“Kreisel [21] was one of the first to separate cooperative phenomena (not known to have Turing computable behaviour), from classical systems and proposed [22] (p 143, Note 2) a collision problem related to the 3-body problem as a possible source of incomputability, suggesting that this might result in “an analog computation of a non-recursive function (by repeating collision experiments sufficiently often)”. This was before the huge growth in the attention given to chaos theory, with its multitude of different examples of the generation of informational complexity via very simple rules, accompanied by the emergence of new regularities (see for example the two classic papers of Robert Shaw [33], [32]). We now have a much better understanding of the relationship between emergence and chaos, but this still does not provide the basis for a practically computable relationship.” Cooper [21] (emphasis added)

4 Concurrent Computing and Turing Machine Model

4.1 Bi-directional Model Development of Natural Computation

Turing machine (originally named “logical calculating machine”) model of computation was developed by Turing in order to describe a human (at that time called” a computer”) executing an algorithm:

“It is possible to produce the effect of a computing machine by writing down a set of rules of procedure and asking a man to carry them out. Such a combination of a man with written instructions will be called a ‘Paper Machine’. A man provided with paper, pencil, and rubber, and subject to strict discipline, is in effect a universal machine.” Turing [10]

The underlying logic of Turing’s “logical calculating machine” is fully consistent standard logic. Turing machine is assumed always to be in a well defined state. [2] In contemporary computing machinery, however, we face both states that are not well defined (in the process of transition) and states that contain inconsistency:

“Consider a computer which stores a large amount of information. While the computer stores the information, it is also used to operate on it, and, crucially, to infer from it. Now it is quite common for the computer to contain inconsistent information, because of mistakes by the data entry operators or because of multiple sourcing. This is certainly a problem for database operations with theorem-provers, and so has drawn much attention from computer scientists. Techniques for removing inconsistent information have been investigated. Yet all have limited applicability, and, in any case, are not guaranteed to produce consistency. (There is no algorithm for logical falsehood.) Hence, even if steps are taken to get rid of contradictions when they are found, an underlying paraconsistent logic is desirable if hidden contradictions are not to generate spurious answers to queries.” Priest and Tanaka [22]

Open, interactive and asynchronous systems have special requirements on logic. Goldin and Wegner [23], and Hewitt [2] argue e.g. that computational logic must be able to model interactive computation, and that classical logic must be robust towards inconsistencies i.e. must be paraconsistent due to the incompleteness of interaction.

As Sloman [24] argues, concurrent and synchronized machines are equivalent to sequential machines, but some concurrent machines are asynchronous, and thus not equivalent to Turing machines. If a machine is composed of asynchronous concurrently running subsystems, and their relative frequencies vary randomly, then such a machine cannot be adequately modeled by Turing machine, see also [4].

Turing machines are discrete but can in principle approximate machines with continuous changes, yet cannot implement them exactly. Continuous systems with non-linear feedback loops may be chaotic and impossible to approximate discretely, even over short time scales, see [25] and [2]. Clearly Turing machine model of computation is an abstraction and idealization. In general, instead of idealized, symbol-manipulating models, more and more physics-inspired modeling is taking place.

Theoretical model of concurrent (interactive) computing corresponding to Turing machine model of algorithmic computing is under development. (Abramsky, Hewitt, Wegner) From the experience with present day networked concurrent computation it becomes obvious that Turing machine model can be seen as a special case of a more general computation. During the process of learning from nature how to compute, we both develop computing and at the same time improve understanding of natural phenomena.

“In particular, the quantum informatic endeavor is not just a matter of feeding physical theory into the general field of natural computation, but also one of using high-level methods developed in Computer Science to improve on the quantum physical formalism itself, and the understanding thereof. We highlight a seemingly contradictory phenomenon: passing to an abstract, categorical quantum informatics formalism leads directly to a simple and elegant graphical formulation of quantum theory itself, which for example makes the design of some important quantum informatic protocols completely transparent. It turns out that essentially all of the quantum informatic machinery can be recovered from this graphical calculus. But in turn, this graphical formalism provides a bridge between methods of logic and computer science, and some of the most exciting developments in the mathematics of the past two decades“ Abramsky and Coecke [25]

The similar two-way process of learning is visible in biocomputing, see Rozenberg and Kari [26]. As we already mentioned “the unreasonable effectiveness of mathematics in the natural sciences” does not (yet) apply to biology, as modeling of biological systems attempted up to now was too crude. Living systems are essentially open and in constant communication with the environment. New computational models must be interactive, concurrent, and asynchronous in order to be applicable to biological and social phenomena and to approach richness of their information processing repertoire.

Present account of models of computation highlights several topics of importance for the development of new understanding of computing and its role: natural computation and the relationship between the model and the physical implementation, interactivity as fundamental for computational modeling of concurrent information processing systems (such as living organisms and their networks), and new developments in logic needed to support this generalized framework. Computing understood as information processing is closely related to natural sciences; it helps us recognize connections between sciences, and provides a unified approach for modeling and simulating of both living and non-living systems. [4]

4.2 Concurrency and Actor Networks in Nature All the Way Down

In his article: What is computation? Concurrency versus Turing's Model, Hewitt [2] makes the following very apt analysis of the relationship between Turing machines and concurrent computing processes:

“Concurrency is of crucial importance to the science and engineering of computation in part because of the rise of the Internet and many-core architectures. However, concurrency extends computation beyond the conceptual framework of Church, Gandy [1980], Gödel, Herbrand, Kleene [1987], Post, Rosser, Sieg [2008], Turing, etc. because there are effective computations that cannot be performed by Turing Machines. In the Actor model [Hewitt, Bishop and Steiger 1973; Hewitt 2010], computation is conceived as distributed in space where computational devices communicate asynchronously and the entire computation is not in any well-defined state. (An Actor can have information about other Actors that

it has received in a message about what it was like when the message was sent.) Turing's Model is a special case of the Actor Model." Hewitt [2] (emphasis added)

According to natural computationalism/pancomputationalism [4] every physical system is computational, but there are many different sorts of computations going on in nature seen as a network of agents/actors exchanging "messages". The simplest agents communicate with simplest messages such as elementary particles (with 12 kinds of matter and 12 kinds of anti-matter particles) exchanging 12 kinds of force-communicating particles. Example from physics that we can recast into actor model is Yukawa's theory of strong nuclear force modeled as exchange of mesons (as messages), which explained the interaction between nucleons. Complex agents/actors like humans communicate through languages which use very complex messages for communication. Also, exchange of information causes change of actors. Those changes are simple in simple actors such as elementary particle that can change its state (quantum numbers) and in complex agents with memory, communication results in substantial changes in agents' way of response.

Natural computational systems as networks of agents exchanging messages are in general asynchronous concurrent systems. Conceptually, agent-based models and actor models are closely related, and as mentioned, understanding of interactions between agents in interaction networks fits well in those frameworks.

Physical Computing - New Computationalism.

Non-symbolic vs. Symbolic Computation

It is often argued that computationalism is the opposite of connectionism and that connectionist networks and dynamic systems do not compute. This implied that human mind as a processes powered by human brain as a network of neurons cannot be adequately modeled in computational terms. However, if we define computation in a more general sense of natural computation, instead of high level symbol manipulation of Turing machine, it is obvious that *connectionist networks and dynamical systems do compute*. Computational modeling of cognitive processes requires computing tools that contain not only Turing Machine model but also connectionist network models. That is also the claim made by Scheutz in the Epilogue of the book Computationalism: New Directions [27], where he notices that:

"Today it seems clear, for example, that classical notions of computation alone cannot serve as foundations for a viable theory of the mind, especially in light of the real-world, real-time, embedded, embodied, situated, and interactive nature of minds, although they may well be adequate for a limited subset of mental processes (e.g., processes that participate in solving mathematical problems). Reservations about the classical conception of computation, however, do not automatically transfer and apply to real-world computing systems. This fact is often ignored by opponents of computationalism, who construe the underlying notion of computation as that of Turing-machine computation." Scheutz [27] p. 176

Classical computationalism was the view that classical theory of computation (Turing-machine-based, universal, and disembodied) might be enough to explain cognitive phenomena. New computationalism (natural computationalism) emphasizes that embodiment is essential and thus physical computation, hence natural computationalism.

The view of Scheutz is supported by O'Brien [28] who refers to Horgan and Tien-son [29] arguing that "cognitive processes, are not governed by exceptionless, representation-level rules; they are instead the work of defeasible cognitive tendencies subserved by the non-linear dynamics of the brains neural networks."

Dynamical characterization of the brain is consistent with the analog interpretation of connectionism. But dynamical systems theory is often not considered to be a computational framework. O'Brien [28] notices that "*In this sense, dynamical systems theory dissolves the distinction between intelligent and unintelligent behaviour, and hence is quite incapable, without supplementation, of explaining cognition. In order for dynamical engines to be capable of driving intelligent behaviour they must do some computational work: they must learn to behave as if they were semantic engines.*"

O'Brien and Opie [30] thus search for an answer to the question how connectionist networks compute, and come with the following characterization:

"Connectionism was first considered as the opposed to the classical computational theory of mind. Yet, it is still considered by many that a satisfactory account of how connectionist networks compute is lacking. In recent years networks were much in focus and agent models as well so the number of those who cannot imagine computational networks has rapidly decreased. Doubt about computational nature of connectionism frequently takes the following two forms.

1. (W)hile connectionists typically interpret the states and activity of connectionist networks in representational terms, closer scrutiny reveals that these putative representations fail to do any explanatory work, and since there is "no computation without representation" (Pylyshyn 1984, p. 62), the connectionist framework is better interpreted non-computationally.
2. "the connectionist networks are better characterized as dynamical systems rather than computational devices."

In the above denial of computational nature of connectionist models the following confusions are evident.

1. Even though it is correct that there is "no computation without representation", representation in this context can be any state of activation in a cognizing agent that causes the agent to "recognize" the information. It can be a dynamical state induced in the agents' brain as a consequence of perception and that dynamical state, even though it has no apparent resemblance of the source of information, is causally connected to it.
2. Dynamical systems compute and their computation in general is natural computation. One of the central questions in this context is the distinction between symbolic and non-symbolic computing. Trenholme [31] describes the relationship of analog vs. symbolic simulation:

“Symbolic simulation is thus a two-stage affair: first the mapping of inference structure of the theory onto hardware states which defines symbolic computation; second, the mapping of inference structure of the theory onto hardware states which (under appropriate conditions) qualifies the processing as a symbolic simulation. Analog simulation, in contrast, is defined by a single mapping from causal relations among elements of the simulation to causal relations among elements of the simulated phenomenon.” Trenholme [31] p.119. (emphasis added)

Both symbolic and sub-symbolic (analog) simulations depend on causal/analog/physical and symbolic type of computation on some level but *in the case of symbolic computation it is the symbolic level where information processing is observed*. Similarly, even though in the analog model symbolic representation exists at some high level of abstraction, it is the physical agency of the substrate and its causal structure that define computation (simulation).

Basti in this volume suggests how to “integrate in one only formalism the physical (“natural”) realm, with the *logical-mathematical* (“computation”), studying their relationships. That is, the passage from the realm of the *causal* necessity (“natural”) of the physical processes, to the realm of the *logical* necessity (“computational”), and eventually representing them either in a sub-symbolic, or in a symbolic form. This foundational task can be performed, by the newborn discipline of *theoretical formal ontology*.” Proposed formal ontology is based on the information-theoretic approach in quantum physics and cosmology, the information-theoretic approach of dissipative QFT (Quantum Field Theory) and the theoretical cognitive science.

Freeman offers an accurate characterization of the relationship between physical/sub-symbolic and logical/symbolic level in the following passage:

“Human brains intentionally direct the body to make symbols, and they use the symbols to represent internal states. The symbols are outside the brain. Inside the brains, the construction is effected by spatiotemporal patterns of neural activity that are operators, not symbols. The operations include formation of sequences of neural activity patterns that we observe by their electrical signs. The process is by neurodynamics, not by logical rule-driven symbol manipulation. The aim of simulating human natural computing should be to simulate the operators. In its simplest form natural computing serves for communication of meaning. Neural operators implement non-symbolic communication of internal states by all mammals, including humans, through intentional actions. (...) I propose that symbol-making operators evolved from neural mechanisms of intentional action by modification of non-symbolic operators.” [32] (emphasis added)

Consequently, our brains use non-symbolic computing internally in order to manipulate relevant external symbols/objects!

4.3 Physical Computation/Natural Computation vs. Turing Machine Model

So in what way is physical computation/natural computation important vis-à-vis Turing machine model? One of the central questions within computing, cognitive science, AI and other related fields is about computational modeling (and simulating) of intelligent

behaviour. What can be computed and how? It has become obvious that we must have richer models of computation, beyond Turing machine, if we are to adequately model and simulate biological systems. What exactly can we learn from nature and especially from intelligent organisms?

It has taken more than sixty years from the first proposal of the test Turing called the "Imitation Game", as described in Turing [33] p. 442, to the Watson machine winning Jeopardy. That is just the beginning of what Turing believed one day will be possible - a construction of computational machines capable of generally intelligent behavior as well as the accurate computational modeling of the natural world. So there are several classes of problems that deserve our attention when talking about computing nature.

To "*compute*" nature by any kind of computational means, is to model and/or simulate the behaviors of natural systems by computational means. Watson is a good example. We know that we do not function like Watson or like chess playing programs that take advantage of brute force algorithms to search the space of possible states. We use our "gut feeling" and "fingertip-feeling"/ "fingerspitzengefühl" and they can be understood as embodied, physical, sub-symbolic information processing mechanisms we acquire by experience and use when necessary as automatized *hardware-based, automatic* recognition tools.

To *compute nature* means to interpret natural processes, structures and objects as a result of natural computation which is in general defined as information processing. This implies understanding and modeling of physical agents, starting from the fundamental level of quantum computing via several emergent levels of chemistry, biology, cognition and extended cognition (social, and augmented by computational/ information processing machinery).

At the moment we have bits and pieces of the picture – *computing* nature, that is computational modeling of nature and *computing nature*, that is nature understood in itself as a computational network of networks.

5 The Relationship between Human Representation, Animal Representation and Machine Representation

We would like to highlight the relevance of the relationship between human representation and machine representation to show the main issues concerning "functionalism" and "connectionism". We propose to discuss the notion of "representation" because an important challenge for AI is to simulate not only the "phonemic" and "syntactic" aspects of mental representation but also the "semantic" aspect. Traditionally, philosophers use the notion of "intentionality" to describe the representational nature of mental states namely intentional states are those that "represent" something, because mind is directed toward objects. We think that it is important to consider the relevance of "embodied cognition" for contentful mental states (see, for instance, the classical thought experiment of the "Chinese room" introduced by Searle to criticize the important results of the Turing test, [34]).

The challenge for AI is therefore to approximate to human representations i.e. to the semantic content of human mental states. There are two competing interpretations of mental representations relevant for AI. The first focuses on the discreteness of mental representations and the second focuses on their inter-relation [35]. The first corresponds to the symbolic paradigm in AI, according to which mental representations are symbols. Proponents of the symbolic representation point on a semantic that rests on the relation between tokens of the symbol and objects of representation. The intentional mechanism functions in a way that the content of a symbol does not depend on the content of other symbols. In this sense, each symbol is discretely conferred with its intentional content. The second corresponds to connectionism in AI, according to which mental representations are distributed patterns. Proponents of this view intend the way in which a mental representation is conferred with its intentional content as mediated by relations with other representations. The virtue of connectionism as presented in the neural networks resides in the fact that the categories represented admit borderline cases of membership. As regards the composition of mental representations, it reveals itself to be the complex, contextually modulated interaction of patterns of activation in a highly interconnected network. We aim to describe the main aspects of the two approaches to make clear: the mechanisms characterizing the different way by which representations are conferred with their intentional content; the nature and structure of the categories represented and the ways in which mental representations interact.

The task to consider the similarity between human and artificial representation could involve the risk of skepticism about the possibility of “computing” this mental capacity. If we consider computationalism as defined in purely abstract syntactic terms then we are tempted to abandon it because human representation involves “real world constrains”. But, a new view of computationalism could be introduced that takes into consideration the limits of the classical notion and aims at providing a concrete, embodied, interactive and intentional foundation for a more realistic theory of mind [27]. We would like to highlight also an important and recent debate on “digital representation” [36] that focus on the nature of representations in the computational theory of mind (or computationalism). The starting point is the nature of mental representations, and, particularly, if they are “material”. There are authors such as Clark who maintain that mental representation are material [37] while others like Speaks think that thought processes use conventional linguistic symbols [38]. The question of digital representation involves the “problem of physical computation” [39] as well as the necessity of the notion of representation [40] so that we only have the problem of how to intend the very notion of representation [41, 42]. But, there is also the possibility of understanding computation as a purely physical procedure where physical objects are symbols processed by physical laws on different levels of organization that include “every natural process” in a “computing universe” [43]. In this context, we need a plausible relation between computation and information. Info-computational naturalism describes the informational structure of the nature i.e. a succession of level of organization of information. Morphology is the central idea in the understanding of the connection between computation and information. It proceeds by abstracting the principles via information self-structuring and sensory-motor coordination. The sensory-motor coordination provides an “embodied” interaction with the environment:

information structure is induced in the sensory data, thus facilitating perception, learning and categorization.

Among the possibilities to compute human representational processes, Basti in this volume proposes a natural account from the field of formal ontology. In particular, he implements the so-called “causal theory of reference” in dynamic systems.

We think it is necessary to find a plausible philosophical strategy to consider the capacities that are common to human and machine representation (Giovagnoli in this volume). Analytic Pragmatism that is represented by the American philosopher Brandom [44] suggests relevant ideas to describe human, animal and artificial capacities for representing the external world. It is easier to start with the human case and so to describe discursive practices and to introduce norms for deploying an autonomous vocabulary, namely a vocabulary of a social practice (science, religion etc.). These norms are logical and are at the basis of an “inferential” notion of representation. But, inference in this sense, recalling Frege, is material. Brandom refuses the explanation of representation in terms of syntactical operations as presented by “functionalism” in “strong” artificial intelligence (AI or GOFAI). He does not even accept weak AI (Searle), rather he aims to present a “logical functionalism” characterizing his analytic pragmatism. According to Brandom, we are not only creatures who possess abilities such as to respond to environmental stimuli we share with thermostats and parrots but also “conceptual creatures” i.e. we are logical creatures in a peculiar way and we need a plausible view to approach human capacities.

Very interesting results are offered by Arriola-Rios and Demery et al. who discuss in this book how salient features of objects can be used to generate compact representations in animals and robots, later allowing for relatively accurate reconstructions and reasoning. They would like to propose that when exploration of objects occurs for forming representations, it is not always random, but also structured, selected and sensitive to particular features and salient categorical stimuli of the environment. They introduce how studies into artificial agents and into natural agents are complementary by emphasizing some findings from each field.

Along this line, Bull, Holley, De Lacy Costello and Adamatzky present initial results from consideration of using Turing’s dynamical representation within unconventional substrate – networks of Belousov-Zhabotinsky vehicles – designed by an imitation based i.e. cultural approach. Over sixty years ago, Alan Turing presented a simple representation scheme for machine intelligence namely a discrete dynamical system network of two-input NAND gates. Since then only a few other explorations of these unorganized machines are known. As the authors underscore in their paper, it has long been argued that dynamic representations provide numerous useful features, such as an inherent robustness to faults and memory capabilities by exploiting the structure of their basins of attraction:

“For example, unique attractors can be assigned to individual system states/outputs and the map of internal states to those attractors can be constructed such that multiple paths of similar states lead to the same attractor. In this way, some variance in the actual path taken through states can be varied, e.g., due to errors, with the system still responding appropriately. Turing appears to have been thinking along these lines also”.

6 Conclusions, Open Problems and Future Work

“It turns out to be better to use the world as its own model.” Brooks [45]

As already argued, we enjoy and appreciate what Wigner named *“the unreasonable efficiency of mathematics in natural sciences”* [46] – except for biology. Time is right to address biology at last and try to find out how best to use computation to model and simulate behavior of biological systems. In this context it can never be overemphasized that: *“nothing in biology makes sense except in the light of evolution”* – an insight made by the evolutionary biologist Dobzhansky [47]. In order to model (simulate) evolution we need generative models. As demonstrated by e.g. Epstein [13] and Wolfram [48], such models are capable of producing complex behaviors starting from simple structures and processes (rules).

Of all biological phenomena, cognition (the ability of living organisms to process information beyond simple reactivity) seems to be the most puzzling one, as in more complex organisms it is related to phenomena such as mind, intelligence and mental (thought) processes. Cognition in highly developed organisms indeed looks like a miracle if one does not take into account that it took several billions of years in nature to develop through the process of evolution from simplest forms to increasingly complex ones. Through the reverse engineering of evolution we are learning how organisms function through computational models such as *Human Brain/Blue Brain project*. At the same time we learn to compute in novel and more powerful ways, such as developed in the IBM’s project on *Cognitive Computing*.

For the future work on computing nature it remains to reconstruct the process of evolution of life in terms of information and computation (as information processing), starting from the process of abiogenesis i.e. the transition from amino acids to first living organisms. Of special interest are the evolution of nervous systems and brains in animals and thus the development of complex cognitive capacities, such as intelligence. This understanding of the evolution and the development in terms of information and computation will lead to improved understanding of underlying mechanisms of morphological computing as information self-structuring [48].

Here follows the list of some important questions to answer in the framework of natural computation (information processing in physical systems).

- *Generative modeling of the evolution and development of physical structures of the universe*, starting with minimum assumptions about primordial universe in terms of information and computation, based on actor (agent) networks exchanging information (messages/particles).
- *Generative modeling of hierarchical structure of emergent layers of organization in physical systems in terms of natural computing*. Modeling of the process in which the whole constraints its parts and showing how its (higher level) properties emerge.
- *Understanding and describing of the evolution and development of living organisms on earth within the framework of natural computation* (morphological computation, self-organization of informational structures through computational processes – concurrent computational processes, modeled as above. [49])

- *Understanding intelligence and consciousness*, in terms of information and computation. Explaining how representations (symbolic level) emerge from sub symbolic information processes. Understanding how exactly our brains process information, learn and act in terms of information and natural computation on different levels of organization. Working out the connections between connectionist networks/dynamic systems and symbol manipulation, sub-symbolic and symbolic information processing.
- *Explaining how physics connects to life* and how the fact that we evolved from physical matter defines the ways we interact with the universe and form our concepts and actions (observer problem in epistemology), continuation of the project started by Deacon [9].
- *Uncovering details of info-computational mechanisms* involved in DNA control of cellular processes.
- *Application of natural computation to program nano-devices* and universally programmable intelligent matter. [50]
- *Answering questions for which natural computationalism is especially suitable framework*, such as: why is the genetic difference between humans and other animals much smaller than we imagined before genome sequencing? How does the evident difference between humans and apes developed, given our social communication system as computational infrastructure that acts as a basis of human social intelligence established by natural computing?

From all above proposed research a richer notion of computation will emerge, which in its turn will help in the next step to better address natural phenomena as computations on informational structures. As Penrose in the foreword to [18] states:

“(S)ome would prefer to define “computation” in terms of what a physical object can (in principle?) achieve (Deutsch, Teuscher, Bauer and Cooper). To me, however, this begs the question, and this same question certainly remains, whichever may be our preference concerning the use of the term “computation”. If we prefer to use this “physical” definition, then all physical systems “compute” by definition, and in that case we would simply need a different word for the (original Church-Turing) mathematical concept of computation, so that the profound question raised, concerning the perhaps computable nature of the laws governing the operation of the universe can be studied, and indeed questioned.”

With this new idea of natural computation generalizing current Turing model of computation, nature indeed can be seen as a network of networks of computational processes and what we are trying is to compute the way nature does, learning its tricks of the trade. So the focus would *not* be *computability* but *computational modeling*. How good computational models of nature are we able to produce and what does it mean for a physical system to perform computation, where computation is implementation of physical laws.

It is evident that natural computing/ computing nature presents a new natural philosophy of generality and scope that largely exceed natural philosophy of Newton’s era, presented in his *Philosophiae Naturalis Principia Mathematica*. Natural computation

brings us to the verge of a true paradigm shift in modeling, simulation and controlling the physical world, and it remains to see how it will change our understanding of nature and especially living nature including humans, their societies and ecologies.

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