Validation of Second-Generation GOCE Gravity Field Models by Astrogeodetic Vertical Deflections in Germany

C. Voigt and H. Denker

Abstract

High-precision astrogeodetic vertical deflections in Germany are utilized to validate recent satellite-only global gravity field models with emphasis on the first- and second-generation GOCE models. In order to account for the different spectral characteristics of the data sets involved, the comparisons are performed with a multistage filtering procedure. The comparisons demonstrate that the second generation of GOCE models is significantly improved (by roughly 30 %) compared to the first release, and that the GOCE models contain considerably more gravity field information than pure GRACE models.

Keywords

Astrogeodetic validation • GOCE gravity field models • Vertical deflections

1 Introduction

The second generation of global gravity field models (GGMs) from the GOCE mission have become available recently. The completed GOCE mission aims at providing the gravity anomalies and geoid heights with a precision of 1 mgal and 1–2 cm, respectively, both at a resolution of 100 km, corresponding to a spherical harmonic expansion up to degree and order (d/o) 200 (e.g., Pail et al. [2011\)](#page-5-0). With regard to the vertical deflections, no target precision is specified, but it can be derived from the predicted gravity anomaly error degree variances (cf. ESA [1999\)](#page-5-1) as about 0.15 ["] for a single vertical deflection component, again at a resolution of 100 km.

In order to reach the final mission goals, various internal and external calibration and validation techniques are applied. The validation of the GOCE products is one of the main objectives of the German REAL GOCE project, where the work package "GOCE Cal/Val, Quasigeoid and Height System" deals with the validation of the global GOCE gravity field models by various high-precision terrestrial data sets in Germany. As well as about 300 astrogeodetic vertical deflection stations, observed with a precision of about 0.1 ["] along two 500 km long profiles (Fig. [1\)](#page-1-0), an extensive terrestrial gravity data set of more than 260,000 points (precision about 0.1–1.0 mgal), and about 900 GPS/levelling stations (precision about 1–3 cm) exist (cf. Ihde et al. [2010\)](#page-5-2).

In this contribution, astrogeodetic vertical deflections in Germany are utilized for the external validation of GGMs in the spectral range up to d/o 180–240, corresponding to a resolution (half wavelength) of 110–80 km, respectively. Apart from the precision of about 0.1 ", the astrogeodetic data set fulfills the GGM validation requirements on area size (profile lengths about 500 km) and resolution (2.5–5 km station spacing, 4 km on average), which corresponds to the spherical harmonic d/o range 80–10,000. A comprehensive report on the preparation of the astrogeodetic profile data sets is given in Ihde et al. [\(2010\)](#page-5-2), while further details on the digital transportable zenith camera system TZK2-D can be found in Hirt [\(2004\)](#page-5-3).

Since the astrogeodetic vertical deflections are completely independent of any other gravity field data set, they are a useful validation tool at regional scale and can serve two purposes. First, they allow a cross-validation of the

of Geodesy Symposia 139, DOI 10.1007/978-3-642-37222-3_38,

C. Voigt $(\boxtimes) \cdot H$. Denker

Institut für Erdmessung, Leibniz Universität Hannover, Schneiderberg 50, 30167 Hannover, Germany e-mail: voigt@ife.uni-hannover.de

C. Rizos and P. Willis (eds.), *Earth on the Edge: Science for a Sustainable Planet*, International Association

[©] Springer-Verlag Berlin Heidelberg 2014

Fig. 1 Astrogeodetic vertical deflection stations along a North–south and a West–east profile in Germany

GPS/levelling data at identical points based on the method of astronomic levelling (cf. Voigt et al. [2009;](#page-5-4) Ihde et al. [2010\)](#page-5-2). Secondly, they may be employed for a regional validation of the GGMs. In addition, the astrogeodetic technique is complementary to other external validation procedures, as it provides precise information about the horizontal components of the gravity anomaly vector with other spectral characteristics. In this context, the different spectral behavior of the astrogeodetic and GGM data as well as the mainly 1D information of the astrogeodetic profile data has to be considered.

The first generation of GOCE GGMs was evaluated by Gruber et al. [\(2011\)](#page-5-5) based on GPS/levelling data sets in Germany, Europe, North America, Australia, and Japan. Additional evaluations were done with terrestrial gravity data and astrogeodetic vertical deflections in Switzerland and Australia (Hirt et al. [2011\)](#page-5-6). Further analyses of the firstgeneration GOCE GGMs were carried out by Voigt et al. [\(2010\)](#page-5-7) using terrestrial gravity data, gravimetric quasigeoid models and astrogeodetic vertical deflections in Germany and Europe. Additional results with vertical deflections in Germany and the first- and some selected second-generation GOCE GGMs are reported in Voigt and Denker [\(2011\)](#page-5-8). Numerous studies on the external validation of other GGMs, in particular the ultra-high degree model EGM2008, can be found in Newton's Bulletin [\(2009\)](#page-5-9), and comparisons between EGM2008 with astrogeodetic vertical deflection data were performed along local profiles or at single stations

in the German and Swiss Alps (Hirt [2010\)](#page-5-10) as well as in other parts of Europe (Hirt et al. [2010\)](#page-5-11).

In the following, the evaluation of the full set of secondgeneration GOCE models by two astrogeodetic vertical deflection profiles in Germany is outlined. Section [2](#page-1-1) describes the employed GGMs; Sect. [3](#page-2-0) explains the validation method with emphasis on the different spectral content of the data sets involved, while Sect. [4](#page-3-0) discusses the comparisons of various GGMs with the astrogeodetic data.

2 Global Gravity Field Models

The first generation of GOCE GGMs, published in mid-2010, utilized an observation period of approximately 2 months; the available models within the GOCE High-level Processing Facility (HPF) project are based on the time-wise approach model (TIM1; d/o 224), the direct approach (DIR1; d/o 240), and the space-wise approach (SPW1; d/o 210), as outlined in Pail et al. [\(2011\)](#page-5-0). On the other hand, the second generation of GOCE GGMs used an observation period of approximately 6 months and was published in early 2011; the corresponding models are TIM2 (d/o 250), DIR2 (d/o 240), and SPW2 (d/o 240). In addition, the GOCO01S (d/o 224) and the GOCO02S models (d/o 250) combine the respective 2 and 6 months of GOCE data with 7 years of GRACE as well as CHAMP and SLR data. The focus here is on the analyses of the performance of the second-generation GOCE GGMs compared to the first-generation models, as well as the combined models from GRACE, altimeter and terrestrial data, i.e. EGM2008 (d/o 2,160) and EIGEN-5C (d/o 360). The GGMs are available, e.g., from the International Centre for Global Earth Models (ICGEM), where the corresponding references are specified.

 $\sqrt{\xi^2 + \eta^2}$, where ξ and η are the North–south and East–
west components respectively the error degree variance of Considering the total vertical deflection defined as $\theta =$ west components, respectively, the error degree variance of the total vertical deflection (at degree *n*) can be derived in spherical approximation $(r = a = R)$ from the supplied standard deviations of the fully normalized spherical harmonic coefficients as (Rummel and van Gelderen [1995\)](#page-5-12)

$$
\sigma_n^2(\varepsilon_\theta) = n (n+1) \sum_{m=0}^n (\sigma_{\Delta \bar{C}_{nm}}^2 + \sigma_{\Delta \bar{S}_{nm}}^2).
$$
 (1)

Accordingly, the error degree variance of a single vertical deflection component is

$$
\sigma_n^2\left(\varepsilon_{\xi,\eta}\right) = \tfrac{1}{2}\sigma_n^2\left(\varepsilon_\theta\right). \tag{2}
$$

Furthermore, the cumulative error degree variance of a single vertical deflection component is

Fig. 2 Vertical deflection standard deviations (*t*) per degree (*top*) and cumulative (*bottom*) for selected GGMs

$$
\bar{\sigma}_n^2 \left(\varepsilon_{\xi,\eta} \right) = \sum_{k=2}^n \sigma_k^2 \left(\varepsilon_{\xi,\eta} \right). \tag{3}
$$

Figure [2](#page-2-1) shows the standard deviations (positive square root of the above error degree variances) for single vertical deflection components per degree and accumulated for each GGM tested here. In the spectral range up to d/o 100, the GOCE GGMs show higher standard deviations compared to the combined models including GRACE data, while above d/o 100 the impact of the GOCE gradiometer observations becomes visible. At about d/o 200, the GOCE GGMs are at the same precision level as the combined GGMs, while above d/o 200 the GOCE models deteriorate significantly.

Regarding the aspired GOCE resolution of d/o 200, the (accumulated) formal errors for the TIM1 and TIM2 models are 0.42 ⁿ and 0.28 ⁿ, respectively, which is an improvement by about 30 %, reflecting the increased volume of observations. The corresponding formal errors of the other HPF models (not all shown in Fig. [2\)](#page-2-1) reduce from 0.22 ["] to 0.17 " for DIR1 and DIR2, and from $0.49''$ to $0.42''$ for SPW1 and SPW2. In terms of geoid heights and gravity anomalies, e.g., the corresponding TIM2 formal errors up to d/o 200 are 7 cm and 2 mgal, respectively, which is still about a factor of two to three above the GOCE targets.

Some remarks about the error degree variance approach are appropriate. Firstly, the supplied errors of the spherical harmonic coefficients are formal for the GOCE models and calibrated for the combined GGMs. Secondly, the error degree variances do not take into account the correlations among the coefficients and may thus give too optimistic error estimates. Furthermore, the handling of the polar gap problem (Sneeuw and van Gelderen [1997\)](#page-5-13) in the GOCE processing, affecting the zonal and near-zonal coefficients, may have also an impact on the supplied potential coefficient standard deviations.

3 Validation Method

Regarding the comparison between the astrogeodetic vertical deflections and the GGMs, the different spectral characteristics of both data sets must be considered. The astrogeodetic observations represent the complete gravity field spectrum (all frequencies), while the GGM data are spectrally limited through the maximum d/o of the spherical harmonic expansion, which is due to (1) the strong attenuation of the gravity field signals at satellite altitudes for the satellite-only GGMs, or (2) the limited resolution of the terrestrial data sets associated with the combined GGMs. The signal beyond the maximum d/o of a given GGM is called the omission error.

Exemplarily, the spectral characteristics of vertical deflections and height anomalies are shown in Table [1](#page-3-1) based on the degree variance model of Tscherning and Rapp [\(1974\)](#page-5-14). The spectral range from d/o 81–200 is of special interest, as it is related to the lengths of the astrogeodetic profiles (500 km) and the GOCE target resolution. Furthermore, this is also the spectral window where the most significant improvements are expected from the GOCE gradiometer observations compared to the GRACE mission. The signal portion within the spectral band from d/o 81–200 is about 15 % for the vertical deflections, but only 0.1 % for the height anomalies. Nevertheless, in terms of absolute values, both the vertical deflection and height anomaly contributions within this spectral band are notable with about $3^{''}$ and 1 m, respectively, which is significantly above the GOCE measurement noise level (by roughly a factor 30). Hence, both functionals should be suitable for assessing the relevant spectrum (higher degrees) of the GOCE GGMs (cf. Jekeli [1999\)](#page-5-15).

Another important issue is to consider the high-frequency signals, which are not included in the GGMs, but represent a significant portion of the entire spectrum. In particular, the vertical deflection contribution beyond about d/o 200, largely lacking in the GOCE GGMs, amounts to about 40 % of the entire signal (cf. Table [1\)](#page-3-1) and thus has to be taken into account appropriately within the validation process. One option to overcome this problem is to filter out the

Table 1 Spectral characteristics of vertical deflections and height anomalies based on the anomaly degree variance model of Tscherning and Rapp [\(1974\)](#page-5-14)

		$\left[\sum_{n=n_i}^{n_j} \sigma_n(\xi,\eta)\right]^{\frac{1}{2}} \left[\sum_{n=n_i}^{n_j} \sigma_n(\zeta)\right]$		
Degree $n_i - n_i$ (km)	(11)		(m)	$(\%)$
$250 - 100$			0.966	0.10
$100 - 9$			0.424	0.02
$9 - 0$			0.023	0.00
$10,000 - 0$				
	Resolution		$(\%)$ 10,000-250 4.41 43.10 2.65 15.59 3.99 35.25 1.65 6.06 6.72 100	30.376 99.88 30.395 100

high-frequency components from the (terrestrial) evaluation data sets, e.g., by a Gaussian filter of 100 km width. This procedure works quite well in connection with gravity and height anomaly grids, covering larger areas such as Germany or Europe (Voigt et al. [2010\)](#page-5-7). However, this method is not well suited for the astrogeodetic vertical deflections because these are only available along slightly curved profiles (i.e., mainly 1D information) and edge effects may also play a role.

Therefore, the different spectral content of the relevant data sets is handled here by a stepwise procedure. First, the GGM to be evaluated is truncated at some maximum d/o n_{max} (in steps from 180 to 240), then the omitted harmonics are modelled by EGM2008 and detailed topographic information, and finally an additional low-pass filter is applied to the differences between the such derived quantities and the given terrestrial data. EGM2008 is utilized from d/o $n_{max} + 1$ up to 2160, corresponding to a spatial resolution of $5'$ or about 9 km, while the higher frequencies beyond d/o 2,160 are taken into account by the residual terrain model (RTM) approach (Forsberg and Tscherning [1981\)](#page-5-16) based on a $1'' \times 1''$
termin model and a reference topography with a resolution of terrain model and a reference topography with a resolution of $5' \times 5'$. The latter RTM contributions are strongly correlated
with the local topography and may be quite notable, easily with the local topography and may be quite notable, easily reaching a few seconds of arc for vertical deflections (cf. Table [1\)](#page-3-1).

The augmentation of a given GGM with EGM2008 may introduce some spectral break, which could be attenuated by a smooth transition from one data set to the other within a certain degree range. However, this was not attempted here as the use of other high-degree GGMs instead of EGM2008 (e.g., EIGEN-6C up to d/o 1,420) merely showed a degradation in the overall performance. Furthermore, the approach of complementing a given GGM by EGM2008 and RTM effects was also successfully applied by Gruber et al. [\(2011\)](#page-5-5) and Hirt et al. [\(2011\)](#page-5-6).

Another problem is that the RTM modelling does not work very well in areas with local density anomalies in the subsurface, e.g., related to salt domes in Germany. Therefore, in order to filter out such undesired high-frequency

noise, an additional spatial low-pass filter was applied to the astrogeodetic profiles, which is feasible due to the small spacing of the astrogeodetic sites (about 2.5–5 km, 4 km on average). A Gaussian filter with a width of 9 km was chosen, corresponding to d/o 2,160.

Consequently, the following residuals between the observed astrogeodetic vertical deflections and the corresponding GGM values, augmented by EGM2008 and RTM effects, are analysed:

$$
(d\xi, d\eta)_{\text{raw}} = (\xi, \eta)_{\text{astro}} - (\xi, \eta)_{\text{GGM}} \tag{4}
$$

with

$$
(\xi, \eta)_{\text{GGM}} = (\xi, \eta)_{2, n_{\text{max}}}^{\text{GGM}} + (\xi, \eta)_{n_{\text{max}}+1, 2160}^{\text{EGM2008}} + (\xi, \eta)_{\text{RTM}}.
$$
\n(5)

Furthermore, the low-pass filtered differences are given by

$$
(d\xi, d\eta)_{\text{filt}} = \text{Gauss}\left\{ (\xi, \eta)_{\text{astro}} - (\xi, \eta)_{\text{GGM}} \right\}. \tag{6}
$$

In addition, systematic differences between the astrogeodetic and GGM vertical deflections have to be considered in the comparisons (cf. Jekeli [1999\)](#page-5-15). The only relevant effect within this analysis is the curvature of the normal plumb line in North–south direction (see Heiskanen and Moritz [1967,](#page-5-17) p 196) with an RMS of 0.08". All other effects, including spherical approximations in the GGM computations, inconsistencies due to different reference systems and epochs of the astronomic and ellipsoidal coordinates, as well as tidal effects, especially the permanent parts, do not exceed 0.01 ["] (RMS) and are therefore not considered in this study.

4 Validation Results

As a typical example, the results from the comparisons of the astrogeodetic vertical deflections along the two German profiles with the TIM2 model to d/o 200, augmented by EGM2008 and RTM, are documented in Table [2,](#page-4-0) including the full statistics of the individual components involved in the processing, as well as the raw and filtered differences. Within the analyses, the second decimal place of the statistics between astrogeodetic and GGM values should be handled with care due to observation errors and approximations. The largest signal contribution is related to the spectral band up to d /o 200 $(3-5$ ["] RMS), while the EGM2008 and RTM parts have an RMS of about $2-3$ ["] and $0.5-0.6$ ", respectively, which is in reasonable agreement with the figures reported in Table [1.](#page-3-1) The raw differences are about $0.6-0.7$ ["] RMS, which reduce by more than 10 $%$ to about 0.55 $''$ RMS after Gaussian filtering. Furthermore, Table [3](#page-4-1) provides the RMS of the differences $\Delta \xi$, $\Delta \eta$ between the astrogeodetic and

Quantity Min Mean RMS Min RMS Max Mean $n_i - n_i$ 6.00 -3.78 4.25 21.67 1.97 3.35 -6.85 Astro $2-\infty$ TIM2 $2 - 200$ 5.17 -1.78 4.29 11.46 2.05 2.68 -3.64			
	Max		
	10.05		
	5.25		
EGM2008 $201 - 2,160$ 2.73 -5.83 12.17 -6.38 -0.07 -0.27 2.14	5.90		
RTM -0.02 -4.35 >2.160 0.61 0.47 -1.81 3.89 0.05	3.99		
Raw differences 0.68 0.04 -4.24 0.63 2.54 -1.25 0.14 $2-\infty$	3.55		
Filtered differences 0.57 0.55 0.04 -3.99 1.65 0.15 -0.97 $2-\infty$	3.06		

Table 2 Statistics (") related to the comparison of astrogeodetic vertical deflections with the TIM2 model up to d/o 200

Table 3 RMS differences $\Delta \xi$, $\Delta \eta$ ('') between astrogeodetic vertical deflections and corresponding GGM values after filtering

Truncation n_{max}	RMS $\Delta \xi$	RMS $\Delta \eta$
180	1.07	0.79
180	0.57	0.58
180	0.55	0.48
180	0.57	0.50
180	0.57	0.50
180	0.56	0.62
180	0.55	0.51
180	0.57	0.55
180	0.55	0.48
180	0.55	0.46
180	0.54	0.48
200	0.79	0.77
200	0.57	0.55
200	0.62	0.55
200	0.58	0.55
200	0.78	0.86
200	0.59	0.55
200	0.76	0.75
200	0.57	0.56
200	0.55	0.46
200	0.56	0.47
240	0.87	1.02
240	1.00	1.46
240	0.83	1.15
240	0.86	0.97
240	0.55	0.46
240	0.58	0.53

various GGM vertical deflections after Gaussian filtering, considering different maximum d/o in the range 180–240. The RMS differences between the astrogeodetic and the EGM2008 vertical deflections stay at a constant level due to the modelling of the high-frequency spectrum by EGM2008 itself; they are between 0.46 ["] and 0.55 ["] for the individual components and reflect commission errors of EGM2008 and the astrogeodetic observations (about 0.1 ["]) as well as unmodelled high-frequency effects. As the commission error of EGM2008 up to d /o 2,160 is 0.63^{$\prime\prime$} for a single vertical deflection component; this indicates that the (global) EGM2008 error estimate may be slightly pessimistic for the (local) investigation area.

Up to d/o 180, the RMS differences between the astrogeodetic data and all GOCE models are quite similar, where virtually all second-generation models perform slightly better than the corresponding first-generation models, the improvements arising mainly in the η component (up to about 18 %). In addition, the combined models EGM2008 and EIGEN-5C based on GRACE and terrestrial data show similar RMS differences to the second-generation GOCE models. On the other hand, the pure GRACE model ITG-Grace2010s shows a significantly worse performance (especially in the degree range 150–180), which is due to the restricted spatial resolution of the GRACE observations; hence, already at this point the impact of the GOCE gradiometer observations is obvious (see also Gruber et al. [2011\)](#page-5-5).

Regarding d/o 200, the improvement of the secondgeneration versus the first-generation GOCE models becomes more pronounced (cf. Table [3\)](#page-4-1); the RMS differences reduce from about $0.80''$ to $0.55''$ for TIM1 to TIM2, SPW1 to SPW2, as well as GOCO01S to GOCO02S. Moreover, all second-generation GOCE models perform similar to the combined models EGM2008 and EIGEN-5C. Finally, for d/o 240, the combined models stay at about the same level (about $0.55''$ RMS difference), while the GOCE models steadily deteriorate with GOCO02S giving the smallest RMS difference and DIR2 showing the largest value.

Furthermore, the RMS differences between the astrogeodetic observations in Germany and the GOCE models are within the formal error estimates of all relevant data sets involved in the comparison; for instance, up to d/o 200, the RMS differences of about 0.55 ["] between the astrogeodetic data and the second-generation GOCE models correspond to an astrogeodetic observation error of 0.1 ["] as well as the GOCE commission error $(0.17''$ for DIR2 to $0.42''$ for SPW2 up to d /o 200), the EGM2008 commission error (0.59) ["] for degrees 201–2,160), errors in the RTM contribution, and unmodelled effects. Hence, the RMS differences are even smaller than what could be expected from the error estimates associated with all relevant data sets involved in the comparisons. Finally, regarding again d/o 200, the RMS differences between the astrogeodetic and GOCE data improve by about 15–30 % from the first- to the secondgeneration models, in accordance with the formal error estimates of the GOCE models.

5 Summary and Conclusions

Astrogeodetic vertical deflections along two 500 km long profiles in Germany were utilized for an independent spot check of various GGMs with emphasis on the first- and second-generation GOCE models. The comparisons clearly show that the GOCE models contain significantly more gravity field information above d/o 150 than pure GRACE models. The improvement of the second- versus the firstgeneration GOCE models becomes most pronounced up to about d/o 200, where the RMS differences between the astrogeodetic and GOCE data improve by roughly 30 % (from about $0.80''$ to $0.55'$) in most cases, accompanied by a similar reduction of the formal GOCE error estimates. Moreover, the RMS differences are within the error estimates associated with all data sets involved in the comparisons. Finally, in the study area, the second-generation GOCE models perform similar to the combined models based on GRACE and terrestrial data such as EGM2008, while the pure GOCE models steadily deteriorate above d/o 200.

Acknowledgements The project REAL GOCE is part of the R&D-Programme GEOTECHNOLOGIEN, funded by the German Ministry of Education and Research (BMBF) and the German Research Foundation (DFG), Grant (03G0726C) of project REAL GOCE.

References

- ESA (1999) Gravity field and steady-state ocean circulation mission. SP-1233(1).
- Forsberg R, Tscherning CC (1981) The use of height data in gravity field approximation by collocation. J Geophys Res 86:7843–7854
- Gruber T, Visser PNAM, Ackermann C, Hosse M (2011) Validation of GOCE gravity field models by means of orbit residuals and geoid comparisons. J Geod 85:845–860
- Heiskanen WA, Moritz H (1967) Physical geodesy. W.H. Freeman and Company, San Francisco
- Hirt C (2004) Entwicklung und Erprobung eines digitalen Zenitkamerasystems für die hochpräzise Lotabweichungsbestimmung. Wissenschaftliche Arbeiten der Fachrichtung Vermessungswesen der Universität Hannover Nr. 253 (in German)
- Hirt C (2010) Prediction of vertical deflections from high-degree spherical harmonic synthesis and residual terrain model data. J Geod 84:179–190
- Hirt C, Marti U, Bürki B, Featherstone WE (2010) Assessment of EGM2008 in Europe using accurate astrogeodetic vertical deflections and omission error estimates from SRTM/DTM2006.0 residual terrain model data. J Geophys Res 115:B10404
- Hirt C, Gruber T, Featherstone WE (2011) Evaluation of the first GOCE static gravity field models using terrestrial gravity, vertical deflections and EGM2008 quasigeoid heights. J Geod 85:723–740
- Ihde J, Wilmes H, Müller J, Denker H, Voigt C, Hosse M (2010) Validation of satellite gravity field models by regional terrestrial data sets. In: System Earth via geodetic-geophysical space techniques (Advanced Technologies in Earth Sciences) pp 277–296
- Jekeli C (1999) An analysis of vertical deflections derived from highdegree spherical harmonic models. J Geod 73:10–22
- Newton's Bulletin (2009) External quality evaluation reports of EGM08 (Issue No. 4). BGI and IGeS Joint Bulletin ISSN 1810-8555, Publication of the International Association of Geodesy and the International Gravity Field Service
- Pail R, Bruinsma S, Migliaccio F et al (2011) First GOCE gravity field models derived by three different approaches. J Geod 85:845–860
- Rummel R, van Gelderen M (1995) Meissl scheme – spectral characteristics of physical geodesy. Manuscr Geodaetica 20:379–385
- Sneeuw N, van Gelderen M (1997) The polar gap. Geodetic boundary value problems in view of the one centimeter geoid. Lecture notes in Earth science, vol 65, pp 559–568
- Tscherning CC, Rapp RH (1974) Closed covariance expressions for gravity anomalies, geoid undulations, and deflections of the vertical implied by anomaly degree variance models. Report No. 208, Department of Geodetic Science and Surveying, The Ohio State University, Columbus
- Voigt C, Denker H, Hirt C (2009) Regional astrogeodetic validation of GPS and levelling data and quasigeoid models. In: IAG symposium, vol 133, pp 413–420
- Voigt C, Rülke A, Denker H, Ihde J, Liebsch G (2010) Validation of GOCE products by terrestrial data sets in Germany. Geotechnologien Science Report No. 17, pp 106–111
- Voigt C, Denker H (2011) Validation of GOCE gravity field models by astrogeodetic vertical deflections in Germany. In: Proceedings of the 4th international GOCE user workshop, SP-696, CD-ROM