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# Geocenter Coordinates from GNSS and Combined GNSS-SLR Solutions Using Satellite Co-locations

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## Abstract

Satellite Laser Ranging (SLR) data to LAGEOS, ETALON and to Global Navigation Satellite Systems (GNSS) were combined with GNSS microwave data for 5 years. Including SLR data to GNSS satellites and estimating common orbit parameters allows it to connect both space-geodetic techniques using satellite instead of station co-location. We show that only SLR data to the spherical satellites can improve the geocenter estimates, whereas SLR data to the GNSS satellites suffer from the same GNSS orbit modelling deficiencies as in the analysis of microwave data.

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## Keywords

Inter-technique combination • Satellite co-locations • GNSS • SLR • Geocenter

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## 1 Introduction

The strengths of SLR and GNSS solutions regarding the terrestrial reference frame are quite different. SLR has a clear advantage for the definition of the datum parameters scale and origin. The strength of the SLR observation technique lies in the absolute distance information, which is contaminated by only few and small error sources, which can be calibrated rather well (e.g., range biases). SLR data therefore provide the scale. The situation is quite different for GNSS microwave observations, where the modelling problems related to the uncalibrated satellite antenna phase centers are the major error source for the scale. More details on this issue may be found in, e.g., [Ge et al. \(2005\)](#). [Thaller et al. \(2011\)](#) showed that a combined solution with SLR data provides valuable information about the scale allowing for

the estimation of GNSS antenna phase center offsets without constraining the scale of the GNSS ground network.

The strength of SLR and the weakness of GNSS for the determination of the scale is visible in the computation of the International Terrestrial Reference Frame (ITRF) as well: the scale of the latest realization, i.e., ITRF2008 ([Altamimi et al. 2011](#)) is based solely on SLR and on VLBI. The agreement between the SLR- and VLBI-derived scale was shown to be at the level of 1 ppb at the reference epoch of ITRF2008. [Rothacher et al. \(2011\)](#) showed that the agreement of the scale can be even improved if the a priori models are carefully homogenized when generating the individual technique solutions.

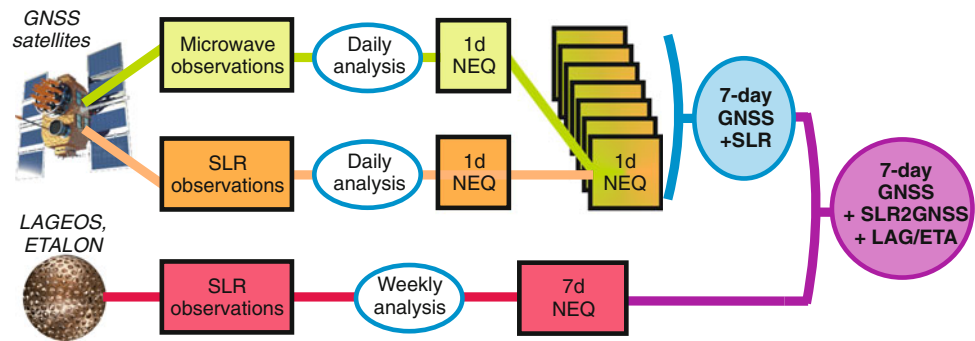
Whereas the strength of SLR to determine the scale is independent of the particular satellite tracked, the situation is different for the geocenter, because geocenter estimates are correlated with the once-per-revolution terms for the solar radiation pressure acting on the satellites ([Meindl et al. 2013](#)). The SLR solutions used in the ITRF are based solely on geodetic satellites LAGEOS and ETALON, where modelling is rather simple. The opposite is true for the GNSS satellites: modelling solar radiation pressure is difficult and, thus, it is the main error source for the geocenter coordinates. For more information we refer to [Springer \(2000\)](#). As a consequence of this satellite-dependency of radiation

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**Fig. 1** Processing strategy for GNSS, SLR and combined solutions



pressure models, SLR observations to GNSS satellites cannot provide better geocenter information than microwave observations (what we will demonstrate later on). SLR observations to LAGEOS and ETALON are therefore indispensable in a combination for stabilizing the geocenter. But the SLR observations to GNSS satellites are important as well, as they provide the link between the two observation techniques at the satellites. The link at the stations is not necessarily needed then, which is why the problems related to local ties can be avoided.

We use the GPS and GLONASS satellites tracked by the SLR stations as satellite co-locations. SLR observations to LAGEOS and ETALON are included in addition in our combination in order to stabilize the SLR station coordinate and the geocenter estimates. Different combinations were analyzed to study the behavior of the geocenter. The main focus is on the correlation between the geocenter coordinates and the once-per-revolution empirical orbit parameters.

Section 2 describes our analysis of the GNSS and SLR observations and the solution setup. The geocenter coordinates resulting from GNSS and combined GNSS-SLR solutions are presented in Sect. 3.

## 2 Analysis and Solution Description

The *Bernese GPS Software* (Dach et al. 2007) is used for processing GNSS microwave (MW) as well as SLR data. The *Bernese GPS Software* is a well established software package for GNSS data analysis, and has recently been extended to a fully operational SLR analysis software. Processing both observation types with the same software guarantees the highest possible consistency level regarding a priori models and parameterization. The analysis consists of three parts (Fig. 1):

- GNSS microwave observations to GPS and GLONASS satellites,
- SLR observations to GPS and GLONASS satellites (“SLR2GNSS”),
- SLR observations to LAGEOS and ETALON satellites.

Normal equation systems (NEQs) result from each step. The NEQs contain all relevant parameters including all

parameters common with the other parts (satellite orbits, EOPs, geocenter and station coordinates). This is essential for a combination on the NEQ level in order to be as close as possible to a combination at the observation level. Since the three groups of observations are independent, this combination at the NEQ level is equivalent to the combination at the observation level.

The years 2006–2010 have been considered for the studies.

### 2.1 Processing of GNSS Microwave and SLR Data

For the first step MW observations to GPS and GLONASS were processed with the current strategy of the CODE Analysis Center of the IGS (Dach et al. 2009), with the exception that only 1-day orbital arcs were generated instead of 3-day arcs. The GNSS orbits are parameterized according to Beutler et al. (1994) with six osculating elements, nine empirical parameters and one stochastic pulse per component in the middle of the arc. The empirical parameters comprise a constant and a once-per-revolution (OPR) acceleration for each of the three orthogonal axes of the Sun-oriented coordinate system at the satellite. The empirical parameters should account for modelling deficiencies, mainly related to solar radiation pressure, which is modelled a priori by the CODE model (Springer 2000). The phase center model for the ground and satellite antennas were taken from the *igs05.atx* file (Schmid et al. 2007), but satellite antenna offsets are set up as parameters. This allows it to either estimate corrections or fix the parameters to their a priori values. Dach et al. (2011) showed that the *igs05* values might be wrong for the GLONASS satellites, so that an estimation of corrections is justified.

SLR observations to GPS and GLONASS satellites are analyzed in the second processing step. Data screening, outlier detection and solution setup is done as described by Thaller et al. (2011). In order to be as close as possible to a combination on the observation level, the GNSS orbits are parameterized in the same way as in the analysis of

microwave data described above. As the orbit parameters refer to the center of mass of the satellite, whereas SLR measurements refer to the Laser Reflector Array (LRA), the offsets of the LRA provided by the ILRS<sup>1</sup> have to be applied. Offsets for the LRA are also set up as parameters in analogy to the microwave processing, allowing that either corrections to the official values can be estimated or the parameters can be fixed to the official values. A range bias parameter is set up for each station and each satellite, as well.

SLR data to the LAGEOS and ETALON satellites are analyzed as well. Due to the small amount of data it is not possible to generate 1-day orbital arcs, but longer arcs are needed. This is why 7-day NEQs are the outcome of this part of the analysis obeying the standards of the ILRS Analysis Working Group.

## 2.2 Combined Solutions

The combination is performed in two steps (see Fig. 1): first, only observations to GNSS satellites are combined, i.e., the NEQs from the GNSS-MW analysis and the SLR2GNSS analysis are stacked on a daily basis and then accumulated to weekly NEQs (but keeping 1-day arcs). Then, the NEQs of the LAGEOS-ETALON analysis are stacked with the weekly NEQs of the first combination step.

The orbit parameters for the GNSS satellites, the EOPs and the geocenter coordinates are common parameters to the two observation techniques for all combinations. The local ties are not applied in order to avoid problems related to inconsistencies of station coordinate estimates based on space geodetic data and local surveys. Thaller et al. (2011) showed that a combination using only the GNSS satellite co-locations is feasible: the SLR station coordinates resulting from combined solutions based on satellite co-locations only agreed with the SLRF2005 reference frame at the level of 2 cm. The agreement between the SLR station coordinates and the GNSS station coordinates corrected by the local tie values was shown to be at the level of 1–2 cm, which is on a similar level as it can be reached by independently generated multi-year SLR and GNSS solutions. Taking into account that only 1 year of data and only SLR observations to GNSS satellites were used in the aforementioned publication, this level of agreement is remarkable. And—as mentioned in this paper—the extension of the time span and especially the inclusion of LAGEOS data should stabilize the estimation of the SLR station coordinates, thus, we can expect an even better performance of the combined solutions presented here.

Microwave data and SLR data to LAGEOS have the same weight in the combination.

The relation  $\sigma_{LAGEOS}/\sigma_{ETALON}$  is set to 1/3, and  $\sigma_{LAGEOS}/\sigma_{SLR2GNSS}$  is 1/10.

The factor of three between LAGEOS and ETALON data is the standard value used within the ILRS Analysis Working Group for generating the official ILRS products. It is mainly based on the quality measures of the normal point data: the average bin RMS of LAGEOS and ETALON normal points (as given in the normal point data files) is about 1 and 3 cm, respectively.

The factor of ten between LAGEOS and SLR-GNSS data was derived in a less empirical way. If we would just look at the average bin RMS of SLR normal points to GNSS satellites (which is about 1.5 cm) we would get a relationship of 1:1.5 for LAGEOS w.r.t. SLR2GNSS. On the other hand, we see RMS values of about 1–5 cm in the residual statistics for SLR observations to GNSS satellites (depending on the station and on the satellite), suggesting a bigger down-weighting of the SLR-GNSS data. This led us to the decision to down-weight the SLR-GNSS observations significantly w.r.t. the LAGEOS data, although we have to admit that the factor of ten is rather an arbitrary value than a calculated value based on dedicated numbers. Detailed studies to determine the optimal weighting for the SLR-GNSS data have to be done in near future, of course.

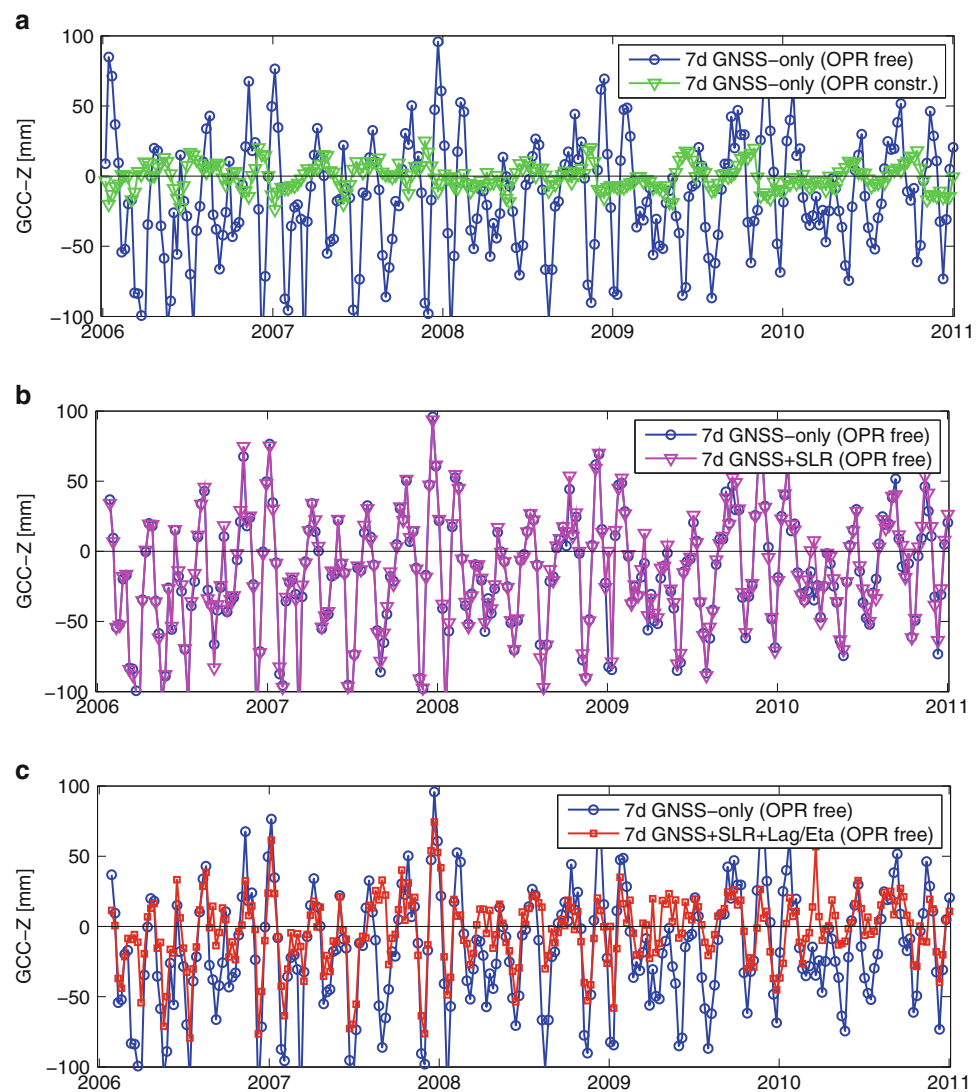
## 3 Geocenter Coordinates

Orbit modelling deficiencies for GNSS satellites may show up as periodical signal in time series of geocenter coordinates with the main period equal to the draconitic year of the satellites, i.e., about 352 days for GPS (Meindl et al. 2013). Empirical orbit parameters normally account for these modelling deficiencies, but due to correlations with geocenter coordinates, the OPR terms for two of the three axis have to be constrained in GNSS-only solutions (we estimate the x-component only). Figure 2 (top) illustrates, for the z-component of the geocenter, what would happen if OPR terms for all three axes would be estimated together with the geocenter coordinates: the scatter in the geocenter time series grows dramatically (i.e., from 9.2 to 43.2 mm). Additionally, a spectral analysis shows that the amplitude for a 358.2 day period increases from 5 to 17 mm (see Fig. 3). Due to the length of the time series, an annual and the draconitic period cannot be separated. But the increase of the amplitude is only due to the handling of the OPR parameters, this is why the 358.2 day period in the spectra must be associated with the draconitic GNSS year rather than with the annual period of 365 days.

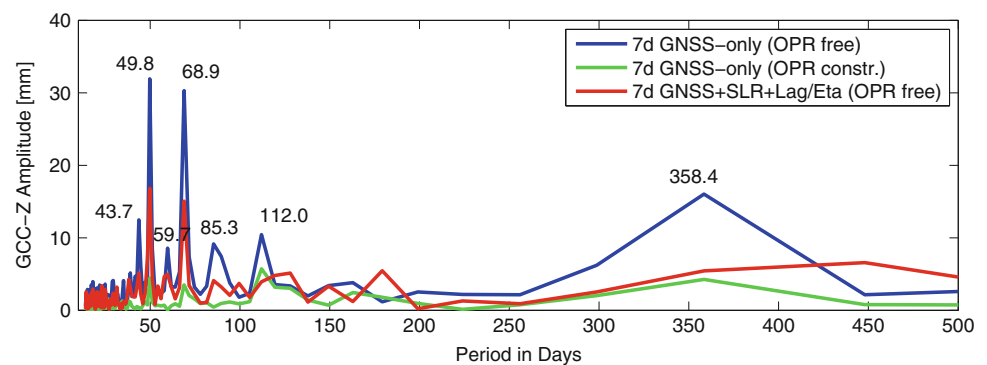
If the SLR observations to GNSS satellites are included, the time series of geocenter coordinates do not change significantly (see Fig. 2 (middle)). This was expected because the draconitic periods visible in the geocenter time series are

<sup>1</sup>[http://ilrs.gsfc.nasa.gov/satellite\\_missions/](http://ilrs.gsfc.nasa.gov/satellite_missions/)

**Fig. 2** Geocenter coordinates from different solution types. *Top:* Impact of constraining OPR terms in GNSS-MW solutions. *Middle:* Impact of SLR observations to GNSS satellites. *Bottom:* Impact of LAGEOS and ETALON observations if OPR parameters for GNSS satellites are unconstrained



**Fig. 3** Spectra of geocenter coordinates estimated from GNSS-only and combined GNSS-SLR solutions. The periods of the major peaks are indicated

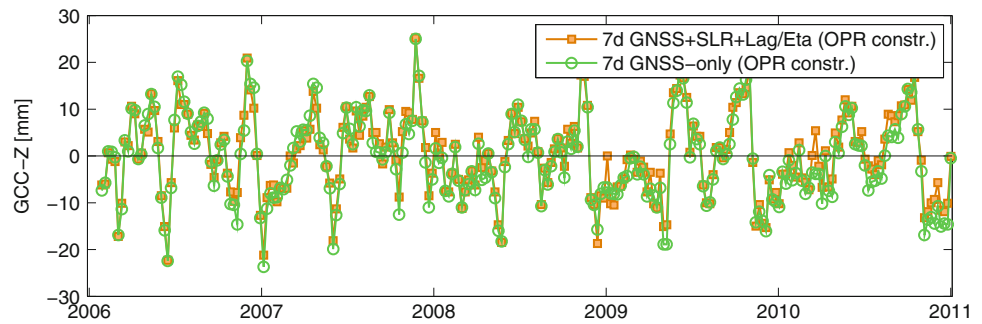


due to orbit modelling deficiencies for the GNSS satellite, and not due to the observation technique.

As soon as SLR observations to the spherical satellites are included, as well, the scatter in the resulting geocenter time series is clearly reduced: from 43.2 to 25.0 mm for the z-component (see Fig. 2 (bottom)).

A spectral analysis shows that the amplitude of the draconitic GNSS year is significantly reduced and reaches a similar level as a solution with constrained OPR orbit parameters (see Fig. 3). The higher-order harmonics of the draconitic year (mainly 1/5 and 1/7) are still larger than for a solution with constrained OPR orbit parameters, but the

**Fig. 4** Geocenter coordinates from different solution types with fixed OPR parameters: impact of LAGEOS and ETALON observations



amplitudes are only half the size of a GNSS-only solution with freely estimated OPR orbit parameters.

If the OPR orbit parameters are constrained, the inclusion of LAGEOS data only marginally decreases the RMS, i.e., from 9.2 to 8.5 mm (Fig. 4).

For all analysis, the x- and y-components show a similar behavior, although the effects are much smaller.

## 4 Summary and Conclusions

We performed a combined SLR-GNSS analysis including SLR observations to LAGEOS, ETALON and GNSS satellites. The latter allows the connection of SLR and GNSS using the satellite instead of the station co-location, implying that no local ties are needed.

The focus of our work is on the potential of different GNSS and combined SLR-GNSS solutions to determine reliable time series of geocenter coordinates. GNSS-only solutions for geocenter coordinates suffer from model deficiencies associated with solar radiation pressure. Especially the OPR parameters are critical: on the one hand, they should account for the modelling deficiencies, but on the other hand we showed that the geocenter estimates are heavily degraded if OPR parameters for all three dimensions are estimated, i.e., a factor of about four in terms of RMS, and a factor of about three for the amplitude associated with the annual period and the draconitic GNSS year.

We showed that SLR observations to GNSS satellites cannot improve the geocenter estimates because the modelling problems for the satellites are the same as in solutions using MW observations.

The inclusion of LAGEOS and ETALON observations, on the other hand, clearly improves the geocenter estimates. We demonstrated that constraints on empirical orbit parameters—which have to be applied in GNSS-only solutions—are not necessarily needed in such combined solutions. The combined geocenter series is mainly driven by the LAGEOS-ETALON contribution. This behavior is good to see because the GNSS contribution clearly suffers from artifacts. And in view of consistently combined space-geodetic solutions it is important that the strength of the SLR

contribution regarding geocenter is not destroyed by these artifacts. In this context it is as well of special importance that we did not use local ties because discrepancies between the local tie values and the coordinate differences derived from the space-geodetic techniques could have a negative impact on the geocenter estimates if the local ties were introduced in the combined solution.

The benefit for other (GNSS-related) parameters due to the improved geocenter estimates in the combined solution has not been studied in this paper.

Studies focussing on the best relative weighting between all types of observations are not yet performed, but will be done as a next step. The results presented, however, show that the relative weighting chosen for the combination is reasonable.

An extension of the time series is highly desired, especially in view of a clear distinction between the signals with an annual period and those related to the draconitic GNSS year.

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