

A Multi-objective Optimization for Multi-period Planning in Multi-item Cooperative Manufacturing Supply Chain

Wafa Ben Yahia^{1,*}, Naoufel Cheikhrouhou², Omar Ayadi¹, and Faouzi Masmoudi¹

¹ Mechanics, Modelling and Production Research Unit,
Ecole Nationale d'Ingénieurs de Sfax (ENIS), Université de Sfax, Tunisia
Route de Sokra B.P.1173 - 3038, Sfax, Tunisia
wafa.benyahia@hotmail.fr, omar.ayadi@yahoo.fr,
faouzi.masmoudi@enis.rnu.tn

² Laboratory for Production Management and Processes,
Ecole Polytechnique Fédérale de Lausanne (EPFL), Switzerland
EPFL STI IGM LGPP, Station 9, CH-1015 Lausanne, Switzerland
naoufel.cheikhrouhou@epfl.ch

Abstract. Consumer goods are mainly manufactured in multiple steps often done by separate, independent production nodes, related to each others to form manufacturing supply chains (MSC). Mostly, each member of a supply chain optimizes his own local objective and accordingly, plans his operations (e.g., production, inventory, capacity planning). The purpose of this work is to improve the efficiency of production networks as a whole by developing a multi-objective optimization model for cooperative planning which aims at minimizing simultaneously the total production cost and the average inventory levels in a multi-period, multi-item environment. To solve this problem, we adopt an elitist non-dominated Sorting Genetic Algorithm (NSGA-II) to find optimal solutions. Several tests are developed to show the performance of the model.

Keywords: Multi-objective optimization model, cooperative planning, manufacturing supply chain, elitist genetic algorithm, NSGA-II.

1 Introduction

Planning operations across a supply chain (SC) is considered in the literature as a major component of supply chain management (SCM). Christopher (1998) defined the SC as “*the network of organizations that are involved, through upstream and downstream linkages, in the different processes and activities that produce value in the form of products and services in the eyes of the ultimate consumer*”. In other words, a supply chain is composed of two organizations or more that are

* Corresponding author.

connected by materials, information and financial flows, in order to fulfil a customer request /demand. According to Stadtler (2009), planning is regarded as the activity to choose sequences and evaluate future activities for a specific decision making unit (e.g. a company).

Coordination in manufacturing supply chain (MSC) depends on the decision-making nature, which can be either centralized or decentralized. According to Holland (1995), developing a cooperative relationship is an effort to make the SC as a whole more competitive. Schneeweiss and Zimmer (2004) used hierarchical planning to coordinate between producers and suppliers in order to minimize the total cost for both partners and improve the performance of the supply chain. Dudek and Stadtler (2005, 2007), Seifert (2003), Chan and Zhang (2011) and Lyu et al. (2010) addressed the collaborative planning effectiveness in improving SC performances, by minimizing the total cost, in the case of a single objective. Kelle and Akbulut (2005) provided quantitative models that showed potential advantages of cooperation between enterprises in a SC context. They showed that cooperation minimizes the total SC cost. According to Rudberg (2004), centralized management offers better cost-effectiveness due to better coordination, possibilities of higher utilisation, and avoidance of duplication of activities. SCs coordinated on a centralized basis lead to better results, in regard to overall costs, than a SC coordinated on a decentralized basis (Axsater and Rosling, 1993; Lee and Billington, 1993; Haehling von Lanzener and Pilz-Glombik, 2002). Timpe and Kallrath (2000), Berning et al. (2002), and Kerschbaum et al. (2010) presented different models and algorithms for centralized master planning in chemical industry supply chains that show potential advantages of cooperation between partners in SCs. All these proposed models aimed at minimizing a total cost function which is composed of production costs, shipping costs and holding costs.

Very few works address cooperative SC from the mathematical programming perspective. Moreover, when they exist, the models are generally based on mono-objective optimization formulation. The purpose of this paper is to develop and to optimize a multi-objective model for optimal cooperative planning in MSCs in order to improve SC performances. The idea is to provide the external demand to the MSC partners in order to cooperate and generate a global optimal production plan to achieve a global goal. So the whole system is considered as one entity. To solve the multi-objective model, we adapt an elitist genetic algorithm based on the non-dominated Sorting Genetic Algorithm -II (NSGA-II).

The paper is organized as follows. In section 2 the cooperative scheme is described and formulated. The resolution methodology is presented in section 3, followed by the computational results in section 4. Finally, a conclusion and discussion of future research directions close the paper.

2 Problem Statement

This paper aims to develop an optimisation model that provides an optimal production plan for a multi-echelon manufacturing supply chain within a fixed time horizon with a finite capacity of personnel and machines. The complexity of the problem can be viewed not only from the multi-objective perspective but also in

the multi-level product complexity structure, where products are related to each others by successor and predecessor items according to the bill of materials and the sequences of operations. The demand for every finished product or semi-finished product is assumed to be known. The deadline to satisfy the customer's demand corresponds to the end of the planning horizon.

The following assumptions must be satisfied by the multi-objective optimisation problem:

- Several resources, with limited availabilities, can process several items.
- Raw materials are always available.
- Inventories at the initial period are void.
- Items can be only produced if all their predecessor components are available.
- Periodic external demand of each item is known.
- Overtime is allowed to extend fixed capacity availabilities.
- Backlogging is not allowed.
- The sequence of operations required to produce an item is fixed, and any alternative routing is prohibited.
- Inventory is calculated at the end of a time period.
- Setup time is neglected.
- External demand has to be fully met in time and quantity.

We consider a cooperative MSC, where different production sections or manufacturing plants cooperate together with the intention of generating a global optimal production plan. The manufacturing plant, which produces the final product requested by the customer, receives orders from its customers and transmits it to the other plants. Besides the inherent nature of MSC actors, these manufacturing plants share different information with each others, such as the production capacity and the production costs. Each manufacturing plant has eight working hours per day, but has different capacities and operations times. Products are transferred from a plant to the next one until reaching the last plant, where the finished products are stored to be delivered to the customer.

2.1 Planning Model

In this work, production plans are generated in order to simultaneously minimize the total production cost of the MSC and minimize the average inventory levels.

Consider the following notations:

Indexes

- t planning period, $t = 1, \dots, T$.
 j operation, $j = 1, \dots, J$.
 r resource, $r = 1, \dots, R$.

Set Indexes

- T set of planning periods
 J set of operations
 R set of resources
 S_j set of direct successors of operation j

Parameters

cv_j	unit cost of operation j
cf_i	setup cost of operation j
co_r	unit cost of overtime (capacity expansion) for resource r
$D_{j,t}$	(external) demand for operation j in period t
$C_{r,t}$	Capacity of resource r in period t
$L_{j,t}$	Large constant
$a_{r,j}$	Unit requirement of resource r by operation j
$r_{j,k}$	Unit requirement of operation j by successor operation k (Depends of the manufacturing process)

Variables

C	total production cost
I_{moy}	average of inventory level for all operations
$x_{j,t}$	output level of operation j in period t (lot size)
$i_{j,t}$	inventory level of operation j at the end of period t
$y_{j,t}$	setup variable of operation j in period t ($y_{j,t}=1$ if product j is set up in period t ; $y_{j,t}=0$ otherwise)
$o_{r,t}$	overtime of resource r in period t

Formulation

$$\text{Min } (C, I_{moy}) \quad (1)$$

$$\text{S.t } C = \sum_{t=1}^T \sum_{j \in J} [(cv_j x_{j,t}) + (cf_j y_{j,t})] + \sum_{t=1}^T \sum_{r \in R} co_r o_{r,t} \quad (2)$$

$$I_{moy} = \frac{1}{T} \sum_{t=1}^T \sum_{j \in J} i_{j,t} \quad (3)$$

$$i_{j,t-1} + x_{j,t} = D_{j,t} + \sum_{k \in S_j} r_{j,k} x_{k,t} + i_{j,t} \quad \forall j \in J, \forall t \quad (4)$$

$$\sum_j a_{r,j} x_{j,t} \leq C_{r,t} + o_{r,t} \quad \forall r \in R, \forall t \quad (5)$$

$$x_{j,t} \leq L_{j,t} y_{j,t} \quad \forall j \in J, \forall t \quad (6)$$

$$x_{j,t} \geq 0, i_{j,t} \geq 0 \quad \forall j \in J, \forall t \quad (7)$$

$$o_{r,t} \geq 0 \quad \forall r \in R, \forall t \quad (8)$$

$$y_{j,t} \in \{0, 1\} \quad \forall j \in J, \forall t \quad (9)$$

Equations (2) and (3) present the objective functions. The first criterion considered is the total production cost, which is the sum of the costs of operations, setup and overtime. The second objective considered is the average level of inventory with respect to the number of planning periods. The model output consists of the operations levels ($x_{j,t}$) (units of item j to be produced at period t), the inventory levels ($i_{j,t}$) (units of item j in the inventory at period t) for all operations considered, and the expansions of resource capacities through overtime ($o_{r,t}$) (overtime needed for resource r during the period t). The Equation (4) provides the constraints capturing the flow balance between output, inventory and consumption by external demand

or successor operations. In fact, demand has to be fulfilled at any stage and any time using the items either produced at that period or stored in previous periods. The constraints (5) represent the capacity restrictions in using the resources to produce the different items. This limitation in capacities is a representation of real-life MSC situation, where overtime can be used as a means to extend the capacity of a manufacturing plant at each period of time. Lot-sizing relationships and the choice of the items to be produced at each time period and at each plant location are expressed in (6). The constraints (7), (8) and (9) specify the domains of the different variables.

In this model, there are $J \cdot T$ equality constraints and $(2 \cdot R \cdot T + 4 \cdot J \cdot T)$ inequality constraints to satisfy.

3 Solving the Multi-objective Minimization Problem

3.1 Choice of a Resolution Method

There has been a growing interest in using genetic algorithms (GAs) to solve a variety of single as well as multi-objective problems in production and operations management. The main reason of using GAs is the high complexity of these problems that are combinatorial and NP hard (Gen and Cheng, 2000; Dimopoulos and Zalzal, 2000; Aytug et al., 2003; Altiparmak et al., 2006). Moreover, GAs show good performances in finding near-optimal solutions for multi-level lot sizing, which is the basic problem in the considered work (Dellaert and Jeunet 2000, Dellaert et al. 2000, Xie and Dong 2002, Jung et al. 2006). We adapt here a genetic algorithm to solve our problem which is NSGA-II, initially developed by Deb (2002). This algorithm is chosen for the many reasons, first the use of elitism, which shows its importance in the comparison made by Zitzler (2002) on a set of test problems. Second, according to Deb (2002), NSGA-II has a computational complexity equal to $O(MN^2)$ (M is the number of objectives and N is the population size), compared to other Multi-objective Evolutionary Algorithms (MOEAs), where the computational complexity is equal to $O(MN^3)$. In addition, NSGA-II is one of the contemporary MOEAs that demonstrated high performances and was successfully applied in various problems (Bekele and Nicklow, 2007; Hnaien et al., 2010). Finally, Deb (2002) shows the ability of NSGA-II to maintain a better spread of solutions and to converge better than two other elitist MOEAs: Pareto Archived Evolution Strategy (PAES) and Strength Pareto Evolutionary Algorithm (SPEA).

3.2 NSGA-II Description

Like any genetic algorithm, NSGA-II deals simultaneously with a set of possible solutions (called population), which allows finding Pareto fronts (set of optimal solutions with equal performances).

Initially, a random parent population P_0 is created. The population is sorted in order to provide different fronts composed of feasible solutions having the same

rank. In fact, individuals are ranked based on the concept of domination: an individual x_1 dominates another individual x_2 if the following two conditions are verified: first, all the objective functions of x_1 are not worse than x_2 , second, at least x_1 is strictly better than x_2 in one objective function. In addition, we define the parameter *crowding distance*, calculated for each individual. This parameter is calculated to estimate the density of solutions surrounding a particular individual in the population. The solution located in a lesser crowded region is selected. Selection is made using tournament between two individuals. From N parents, N new individuals (offspring) are generated in every generation by the use of the Simulated Binary Crossover (SBX) and Polynomial mutation (Deb et al., 2002 and 2001). Since elitism is introduced by comparing current population with previously found best non-dominated solutions.

4 Experimental Results

4.1 Tests Description

We consider a MSC constituted of two production plants. The demand for products is given and has to be fulfilled while facing finite capacities of personnel and machines (resources). Three types of items are produced: product 1 made from one unit of operation 1, product 2 made from one unit of operations 1 and 2, and product 3 made from one unit of operations 1, 2 and 3. The planning horizon has duration of two periods; the time period considered here is equal to one week with six working days and eight hours per day.

The genetic parameters shown in table 1 are selected after a sensitivity analysis.

Table 1 Genetic parameters

NSGA-II (parameters)	N, Population size	G, generation number	P_c , crossover probability	P_m , mutation probability	η_c , Crossover Index	η_m , Mutation Index	r, controlled elitism
Parameter values	150	1000	0.99	$1/n$ (n=number of variables)	50	100	0.123

Two examples are studied, where the difference is the demand trend. As shown in table 2, in the first example, the external demand has an increasing pattern and in the second example it has a decreasing pattern.

Table 2 Customer demand features

	Example 1		Example 2	
Demand of	1st period	2nd period	1st period	2nd period
Product 1	20	140	90	15
Product 2	15	70	40	5
Product 3	10	70	50	5

4.2 Test Results

The development of NSGA-II and its implementation are done in C-language. The execution time does not exceed 15 minutes for all tests. The results of tests are shown in Table 3. At convergence, only one optimal solution is found and not a complete Pareto front as expected. This is due to the complexity of the problem and particularly to the flow equality constraint between MSC tiers.

For the first example the proposed solution is to produce in advance the needed products and to store them in the first period. As a consequence, there is no need to use overtime to meet the demand, since overtime is more expensive compared to nominal capacity. Besides, at the end of the planning period, the inventory level is null.

In the second example, the optimal solution provided by the algorithm is to produce the exact quantities needed to satisfy the demand of the first period. So it uses only the needed overtime in that period. Hence, there is no storage in that period. In the second period, the production quantities are higher than the demand. This explains the needs for storage of the second and the third operation at the end of the planning period, which can be considered as safety storage.

Table 3 Outputs of the developed test

	$y_{j,t}$	$x_{1,1}$	$x_{2,1}$	$x_{3,1}$	$x_{1,2}$	$x_{2,2}$	$x_{3,2}$	$o_{1,1}$	$o_{2,1}$	$o_{1,2}$	$o_{2,2}$	$i_{1,1}$	$i_{2,1}$	$i_{3,1}$	$i_{1,2}$	$i_{2,2}$	$i_{3,2}$	C	I_{moy}
Exp1	1	113	85	58	212	80	22	0	0	0	0	8	12	48	0	0	0	2669	34
Exp2	1	180	90	50	91	76	36	60	0	0	0	0	0	0	0	35	31	3130	33

5 Conclusion and Future Work

This paper proposes a cooperative planning framework for multi-tier and multi-item linear MSC. The developed bi-objective multi-period optimization model aims at minimizing the total production cost and the average inventory level, taking into account capacities and demand constraints. The proposed model shows different advantages over those discussed in the literature. In fact, compared to mono-objective models, this model considers not only costs but also inventory levels as a performance measure. Moreover, it does not consider the inventory as a cost, where an artificial extrapolation is needed, but as a performance to be minimized. The model is solved using NSGA-II coded with C language. In order to evaluate the proposed model, it can be compared to the mono-objective model developed by Dudek (2004).

For future work, we can add a constraint that forces the use of the full main capacity before using of overtime. Besides, a constraint on the storage capacity can be added to the model.

References

- Altıparmak, F., Gen, M., Lin, L., Paksoy, T.: A genetic algorithm approach for multi-objective optimization of supply chain networks. *Computers & Industrial Engineering* 51(1), 196–215 (2006)
- Axsater, S., Rosling, K.: Notes: installation vs. echelon stock policies for multilevel inventory control. *Management Science* 39(10), 1274–1280 (1993)
- Aytug, H., Khouja, M., Vergara, F.E.: Use of genetic algorithms to solve production and operations management: a review. *International Journal of Production Research* 41(17), 3955–4009 (2003)
- Bekele, E.G., Nicklow, J.W.: Multiobjective automatic calibration of SWAT using NSGA-II. *Journal of Hydrology* 341(3-4), 165–176 (2007)
- Berning, G., Brandenburger, M., Gürsoy, K., Mehta, V., Tölle, F.J.: An integrated system solution for supply chain optimization in the chemical process industry. *OR Spectrum* 24(4), 371–402 (2002)
- Chan, F.T.S., Chung, S.H.: A multi-criterion genetic algorithm for order distribution in a demand driven supply chain. *International Journal of Computer Integrated Manufacturing* 17(4), 339–351 (2004)
- Chan, F.T.S., Zhang, T.: The impact of Collaborative Transportation Management on supply chain performance: A simulation approach. *Expert Systems with Applications* 38(3), 2319–2329 (2011)
- Christopher, M.: *Logistics and Supply Chain Management –Strategies for Reducing Cost and Improving Service*, 2nd edn. Financial Times Pitman, London (1998)
- Deb, K., Pratap, A., Agarwal, S., Meyarivan, T.: A Fast and Elitist Multiobjective Genetic Algorithm: NSGA-II. *IEEE Transactions on Evolutionary Computation* 6(2), 182–197 (2002)
- Deb, K.: *Multi-objective optimization using Evolutionary Algorithms*. John Wiley & Sons Ltd., Singapore (2001)
- Dellaert, N., Jeunet, J.: Solving large unconstrained multi-level lot-sizing problems using a hybrid genetic algorithm. *International Journal of Production Research* 38(5), 1083–1099 (2000)
- Dellaert, N., Jeunet, J., Jonard, N.: A genetic algorithm to solve the general multi-level lot-sizing problems with time varying costs. *International Journal of Production Research* 68(5), 241–257 (2000)
- Dimopoulos, C., Zalzala, A.M.S.: Recent developments in evolutionary computation for manufacturing optimization: problems, solutions and comparisons. *IEEE Transactions on Evolutionary Computation* 4(2), 93–113 (2000)
- Dudek, G.: *Collaborative planning in supply chains: a negotiation-based approach*. Springer (2004)
- Dudek, G., Stadler, H.: Negotiations-based collaborative planning between supply chain partners. *European Journal of Operational Research* 163(3), 668–687 (2005)
- Dudek, G., Stadler, H.: Negotiation-based collaborative planning in divergent two-tier supply. *International Journal of Production Research* 45(2), 465–484 (2007)
- Ertogral, K., Wu, S.D.: Auction-theoretic coordination of production planning in the supply chain. *IIE Transactions* 32(10), 931–940 (2000)
- Haehling von Lanzener, C., Pils-Glombik, K.: Coordinating supply chain decisions: an optimization model. *OR Spectrum* 24(1), 59–78 (2002)

- Hnaien, F., Delorme, X., Dolgui, A.: Multi-objective optimization for inventory control in two-level assembly systems under uncertainty of lead times. *Computers & Operations Research* 37(11), 1835–1843 (2010)
- Hollond, C.: Cooperative supply chain management: the impact of interorganizational information systems. *The Journal of Strategic Information Systems* 4(2), 117–133 (1995)
- Jung, H., Song, I., Jeong, B.: Genetic algorithm-based integrated production planning considering manufacturing partners. *The International Journal of Advanced Manufacturing Technology* 32(5-6), 547–556 (2006)
- Kelle, P., Akbulut, A.: The role of ERP tools in supply chain information sharing, cooperation, and cost optimization. *International Journal of Production Economics* 93-94, 41–52 (2005)
- Lee, H.L., Billington, C.: Material management in decentralized supply chains. *Operations Research* 41(5), 835–847 (1993)
- Lyu, J., Ding, J., Chen, P.S.: Coordinating replenishment mechanisms in supply chain: From the collaborative supplier and store-level retailer perspective. *International Journal of Production Economics* 123(1), 221–234 (2010)
- Rudberg, M.: Linking competitive priorities and manufacturing networks: a manufacturing strategy perspective. *Journal of Manufacturing Technology Management* 6(1/2), 55–80 (2004)
- Schneeweiss, C., Zimmer, K.: Hierarchical coordination mechanisms within the supply chain. *European Journal of Operational Research* 153(3), 687–703 (2004)
- Seifert, D.: Collaborative planning, Forecasting, and Replenishment: how to create a supply chain advantage, 1st edn. AMACOM, New York (2003)
- Stadtler, H.: A framework for collaborative planning and state-of-the-art. *OR Spectrum* 31(1), 5–30 (2009)
- Timpe, C.H., Kallrath, J.: Optimal planning in large multisite production networks. *European Journal of Operational Research* 126(2), 422–435 (2000)
- Xie, J., Dong, J.: Heuristic genetic algorithms for general capacitated lot sizing problems. *Computers Mathematics with Applications* 44(1-2), 263–276 (2002)
- Zitzler, E., Laumanns, M., Thiele, L.: SPEA2: Improving the strength pareto evolutionary algorithm for multiobjective optimization. *Computer Engineering* 3242(103), 1–21 (2001)