On the Kinematics of Spherical Parallel Manipulators for Real Time Applications

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Abstract. This paper deals with the computation of the forward and inverse kinematic model of a 3-RRR spherical parallel mechanism (SPM) for a teleoperation solution. The context of real time application is aimed; thus, the determination of the desired solution out of several possibilities, is crucial to guarantee motion continuity. The SPM structure kinematics is then detailed and a strategy, based on Newton Raphson method, is proposed to solve these models. Experimental results are presented to validate the proposed approach.

Keywords: Spherical parallel manipulator, kinematics, real time,teleoperation system, Newton raphson.

1 Introduction

The spherical parallel architecture represents an interesting alternative for applications with a fixed center of motion. The platform of the mechanism is moving over a spherical surface which center coincides with the base reference origin; Different varieties of this mechanism have been studied before. These studies covered a wide range of characteristics such as workspace (Bulca et al., 1999), kinematic analysis (Gosselin and Lavoie, 1993), design parameters optimization (Chaker et al., 2012), singularity (Wang and Gosselin, 2004) and dexterity (Merlet, 2006). However, being a parallel structure, the forward displacement, which calls for the position and orientation of the platform when the actives joints are given, is a difficult problem for which no general procedure has been determined yet. Bai et al. (2009) presented a strategy based on the input-output equation of spherical four bar linkages which yields the eight solutions of SPM model. This method makes possible the identification of all the solution when the mechanism is in a fixed position. In the same context, Bombin et al. (2001) took advantage of the subdivision and convex hull properties of polynomials in Bernstein form to propose a procedure to solve the forward kinematics of the SPM. But this method is not reliable since it depends on the intuition of the researcher when simplifying equations. The presented strategies do

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not consider real time constraints such as the resolution time and the determination of the exact one of the eight solutions in order to guarantee the continuity of the platform movement. In fact, such constraints are very important when passing from theoritical study to practical realisation. In this paper, the problem of the SPM is revisited with the aim of finding a robust method that takes in consideration real time and teleoperation applications constraints . The method is applicable for both forward and inverse displacement of the mechanism. The solution is based on the Newton Raphson method (NRM) that solves the model with consideration of the initial position of the platform. Thus we can guarantee a continuous solution when platform is moving and a reduced time of computation.

2 Teleoperation Context

The context of this work is a teleoperation system for minimally invasive surgery. The expert site is compose by an experimented operator and a haptic device based on a spherical parallel architecture that controls a slave robots operating on the patient. Fig. 1 describes the principle of teleportation. The main constraint of this system is the real time exchange of information related to the position and orientation of every mechanism. Thus, computing these parameters have to be very accurate and respects time conditions.

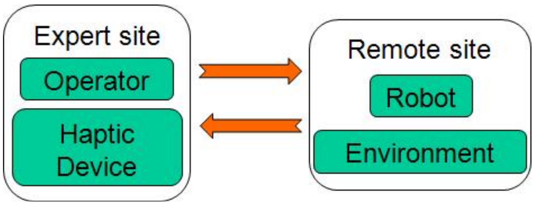


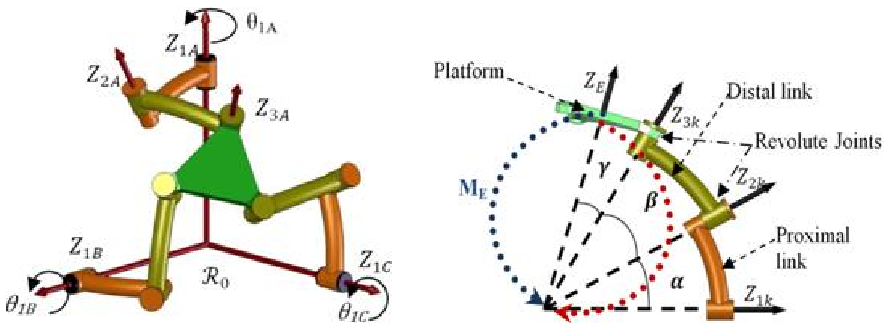
Fig. 1 Teleoperation system

The parallel structure of the haptic device (Fig. 6) presents a complicated kinematic models. In the case of the forward one, one has to solve a nonlinear system. The respect of a predifined time period of 10ms is imposed by the real time constraint. The inverse kinematics presents also the risk of losing the continuity of motion when the problem admits multiple solutions.

3 Kinematics of the SPM

Figure 2.a presents the 3-RRR architecture of the proposed SPM. Three identical legs A, B and C relate the base to the platform. Each leg of the SPM is made out of two links and three revolute joints.The three actuated revolute joints with the base have orthogonal axes $\mathbf{Z}_{1k}$ *(k= A,B and C)*. All the axes of the joints are intersecting in point O, the center of motion of the platform $\mathbf{Z}_E$.

Each link is characterized by a constant angle between the axes of its two joints that represents its dimension. Figure 2.b shows the geometric parameters of one leg. The angles α,β, γ are respectively between the first two joint axes, the second and the third one , and between the third joint axis and platform axis .



(a) SPM architecture (b) One leg parameters

Fig. 2 Architecture and parameters of the SPM

The three legs of the SPM are identical and the actuated joint axes are located along the base frame axes **X**, **Y**and **Z**, respectively. The workspace of the platform is then the intersection of the workspaces of three legs considered each as a spherical serial kinematic chain.

The motion of the SPM is generated by only revolute joints. The kinematics of the mechanism can be described by the following relation:

$$\mathbf{Z}_{2k} \cdot \mathbf{Z}_{3k} = cos(\beta) \quad (1)$$

Where $\mathbf{Z}_{2k}$ and $\mathbf{Z}_{3k}$ are respectively the axes of the second and the third joint of each leg and detailed as:

$$\mathbf{Z}_{2k} = Rot(\mathbf{Z}_{1k}, \theta_{1k}).Rot(\mathbf{X}_{2k}, \alpha).\mathbf{Z}_{1k} \quad (2)$$

$$\mathbf{Z}_{3k} = Rot(\mathbf{Z}_{1k}, \psi).Rot(\mathbf{X}, \theta).Rot(\mathbf{Z}_E, \varphi).Rot^{-1}(\mathbf{X}_{3k}, -\gamma).\mathbf{Z}_{1k} \quad (3)$$

The $\mathbf{Z}_E$ platform axis is described by the three ZXZ-Euler angles, ψ, θ and φ. θ_{1k} and θ_{2k} are, respectively, the joint variables of the revolute joint and the cylindrical intermediate joint of leg k (k= A,B and C) .The axes $\mathbf{X}_{2k}$ and $\mathbf{X}_{3k}$ are given respectively by $\mathbf{X}_{2k} = \mathbf{Z}_{1k} \times \mathbf{Z}_{2k}$ and $\mathbf{X}_{3k} = \mathbf{Z}_{2k} \times \mathbf{Z}_{3k}$.

The system of three equations **F** resulting from applying equation (1) for the three legs of the mechanism can be exploited for both forward and inverse displacement.

3.1 Forward Kinematics

The forward displacement determines the posture of the platform defined by the Euler angles when knowing the active joints parameters θ_{1k}. The operational vector of parameters $V = [\psi, \theta, \phi]^T$, are the three ZXZ-Euler angles of the platform, representing the orientation of the platform with respect to the base.

Applying the equation 1, the forward kinematics consists on solving the following system:

$$\mathbf{V} = f(\mathbf{q}) -> \begin{cases} \psi = f_1(\theta_{1A}, \theta_{1B}, \theta_{1C}) \\ \theta = f_2(\theta_{1A}, \theta_{1B}, \theta_{1C}) \\ \varphi = f_3(\theta_{1A}, \theta_{1B}, \theta_{1C}) \end{cases} \tag{4}$$

As for all parallel mechanisms,this forward kinematics is a non linar system combining polynomial trigonometric parameters. This make very difficult the resolution of the system and the determination of the platform position. Usual methods are not efficient in this case even with a powerful computing capacity.

3.2 Inverse Kinematics

The inverse model of the SPM is easier to obtain, it yields the actuators angles on the base $[\theta_{1A}, \theta_{1B}, \theta_{1C}]$ corresponding to a given platform position.

$$\begin{cases} A_1 cos(\theta_{1A}) + B_1 sin(\theta_{1A}) + C_1 = 0 \\ A_2 cos(\theta_{1B}) + B_2 sin(\theta_{1B}) + C_2 = 0 \\ A_3 cos(\theta_{1C}) + B_3 sin(\theta_{1C}) + C_3 = 0 \end{cases} \tag{5}$$

The inverse kinematics seems to be easier to solve than the forward one. Equations (6) can also be written by using the trigonometric tan-half identities. Which yields a polynomial problem that can be computed easily.

$$\begin{cases} A'_1 T_1^2 + B'_1 T_1^2 + C'_1 = 0 \\ A'_2 T_2^2 + B'_2 T_3^2 + C'_2 = 0 \\ A'_3 T_3^2 + B'_3 T_3^2 + C'_3 = 0 \end{cases} \tag{6}$$

where T_i are respectively $tan(\frac{\theta_{1k}}{2}), i = (A, B, C)$

However, this system admits eight possible solutions. The difficulty is in identifying the correct configuration that ensures no change in the aspect and the continuity of movement.

3.3 Jacobian Matrix

The jacobian matrix can be obtained by differentiating Eq. (1) with respect to time. The obtained equation can be written as:

$$\dot{\mathbf{Z}}_{2k}\mathbf{Z}_{3k} + \mathbf{Z}_{2k}\dot{\mathbf{Z}}_{3k} = 0 \tag{7}$$

with $\dot{\mathbf{Z}}_{2k} = \theta_{1k}\mathbf{Z}_{1k}\times\mathbf{Z}_{2k}$ et $\dot{\mathbf{Z}}_{\mathbf{3k}} = \omega\times\mathbf{Z}_{3k}$

ω is the angular velocity of the end effector..

For the whole manipulator and in a matrix form, we can write:

$$\omega = \mathbf{J}\dot{\mathbf{q}} = \mathbf{A}^{\mathbf{-1}}\mathbf{B}\dot{\mathbf{q}} \tag{8}$$

where $\mathbf{J}$ is the jacobian matrix and $\mathbf{B}$ a 3×3 diagonal matrix:

$$\mathbf{B} = Diag\,[\mathbf{Z}_{1A}\times\mathbf{Z}_{2A}\cdot\mathbf{Z}_{3A}, \mathbf{Z}_{1B}\times\mathbf{Z}_{2B}\cdot\mathbf{Z}_{3A}, \mathbf{Z}_{1C}\times\mathbf{Z}_{2C}\cdot\mathbf{Z}_{3C}] \tag{9}$$

And

$$\dot{\mathbf{q}} = \left[\dot{\theta}_{1A}, \dot{\theta}_{1B}, \dot{\theta}_{1C}\right]^T \tag{10}$$

$$\mathbf{A} = \left[(\mathbf{Z}_{3k}\times\mathbf{Z}_{2k})^T\right] \tag{11}$$

4 Proposed Algorithm

The proposed algorithm is based on NRM (Galantai, 2000) for solving equations numerically. It is based on the simple idea of linear approximation. he solution of the problem is sought in the neighborhood of an initial gess designed by the user. In the case of SPM kinematics and for our real time teleoperation, this strategy can ensure a rapid convergence to the desired solution especially for a nonlinear system of equations such as the ones obtained for forward displacement problem of the parallel structure. In fact, the forward displacement will be solve in a limited number of iterations since the initial guess meets with the last orientation (ψ, θ, ϕ) of the platform. In the case of the inverse kinematics, this strategy helps to avoid the multiple solution problem by choosing the nearest posture to the initial one. The figure 3 presents the flow chart of the algorithm. Depending on the treated problem (direct or inverse), the algorithm is initialized with the known parameters. The system F describing the kinematics is generated and the Jacobian matrix is computed. A loop calculation is then launched whith an initial gess vector $\mathbf{S_0}$ corresponding to

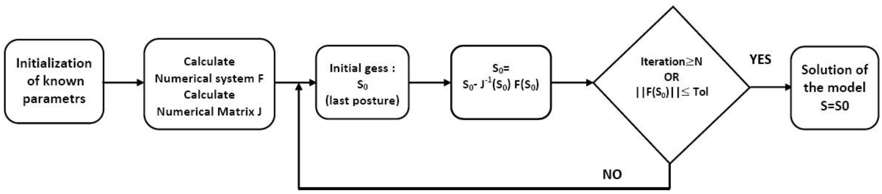
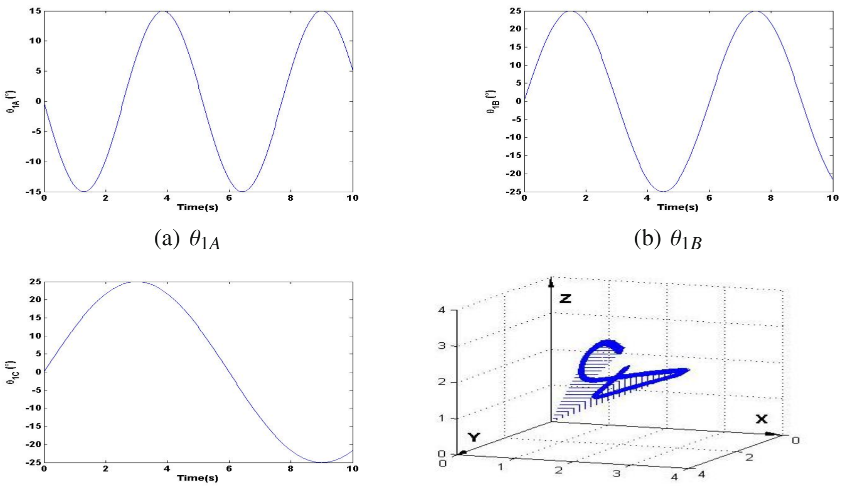


Fig. 3 Newton raphson flow chart

the last position of the mechanism. For every i^{th} loop, the initial gess is set to taking the value of the $i^{th}-1$ loop solution. The final solution **S** is retained if a maxima of iterations N is reached or it satisfies a predifined precision **Tol** fixed in our case to 10^{-4} which represents the quadratic error of the solution.

5 Results

The implementation of the NRM to solve both forward and inverse kinematics of the SPM, made possible the determine precisely and in a continuous way the motion of the SPM . as an example to validate the algorithm we have chosen an arbitrary trajectory for every active joint illustrated in the Fig. 4. The aim is to evaluate the ability of the algorithm to manipulate real time movements and its response time caracterized by the number of iterations needed touconverge to a solution.

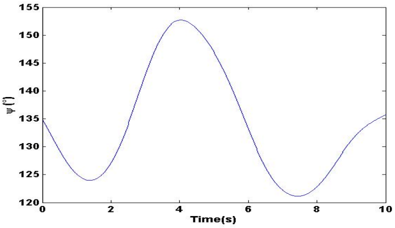


(a) θ_{1A} (b) θ_{1B}

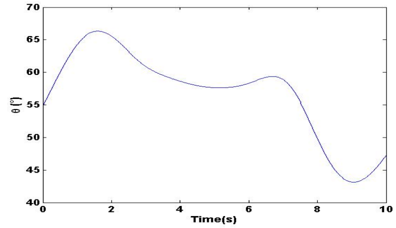
(c) θ_{1C} (d) Trajectory

Fig. 4 Actuator Joints angles injected to the algorithm (a,b,c) and the trajectory generated(d)

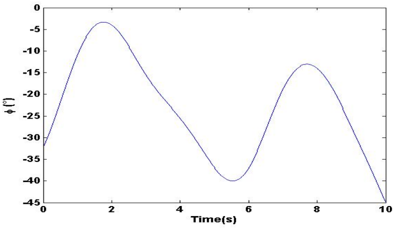
Fig. 5 represent the results of the Euler angles describing the platform orientation. The profile of each is continuous which confirms the performance of the strategy in preserving the aspect of the movement. Fig. 5.d shows that a maximum of two iterations was needed to solve the system. This means that the trajectory is followed instantly while keeping the imposed precision of 10^{-4} degree for every parameter.



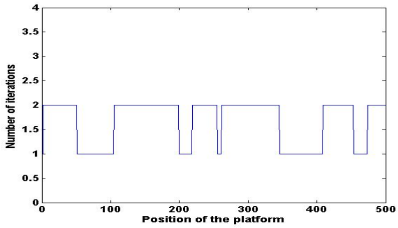
(a) ψ



(b) θ



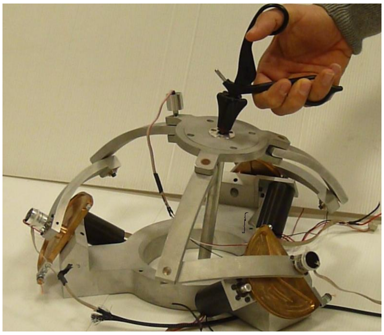
(c) φ



(d) Number of iterations

Fig. 5 Resulting Euler angles from the algorithm (a,b,c) and the number of iterations(d)

Fig. 6 First prototype of the SPM



6 Conclusion

An algorithm based on the NRM used to solve kinematics of an SPM is presented. The strategy is well suited for teleoperation context since it is applicable for both forward and inverse kinematics with similar performance. This makes intersesting its implementation for a telesurgery haptic device (Fig. 6. The kinematics of the SPM was then revisited and a formulation of the strategy was proposed. The results on an arbitrary trajectory of the manipulator confirmed its capacities to determine the exact solution over the eight possibilities. The respect of the real time constraint is garanteed. The performances of this method are also validated on the prototype of the haptic device realized on lab.

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