“Whatever Works Best for You”- A New Method for a Priori and Progressive Multi-objective Optimisation

Rui Wang[1,2], Robin C. Purshouse[1], and Peter J. Fleming[1]

[1] Department of Automatic Control & Systems Engineering, University of Sheffield
{cop10rw,r.purshouse,p.fleming}@sheffield.ac.uk
http://www.shef.ac.uk/acse

[2] Department of Systems Engineering, College of Information Systems and Management, National University of Defence Technology, ChangSha, China, 410073

Abstract. Various multi-objective evolutionary algorithms (MOEAs) have been developed to help a decision maker (DM) search for his/her preferred solutions to multi-objective problems. However, none of these approaches has catered simultaneously for the two fundamental ways that DM can specify his/her preferences: weights and aspiration levels. In this paper, we propose an approach named iPICEA-g that allows the DM to specify his preference in either format. iPICEA-g is based on the preference-inspired co-evolutionary algorithm (PICEA-g). Solutions are guided toward regions of interest (ROIs) to the DM by co-evolving sets of goal vectors exclusively generated in the ROIs. Moreover, a friendly decision making technique is developed for interaction with the optimization process: the DM specifies his preferences easily by interactively brushing his preferred regions in the objective space. No direct elicitation of numbers is required, reducing the cognitive burden on DM. The performance of iPICEA-g is tested on a set of benchmark problems and is shown to be good.

Keywords: Preferences, interactive, decision making, co-evolution.

1 Introduction

Multi-objective optimization problems (MOPs) arise in many real-world applications, where multiple conflicting objectives must be simultaneously satisfied. Over the last two decades, multi-objective evolutionary algorithms (MOEAs) have become increasingly popular for solving MOPs since: (1) their population based nature is particularly useful for approximating trade-off surfaces in a single run; and (2) they tend to be robust to underlying cost function characteristics [1].

The fundamental goal of solving MOPs is to help a DM to consider the multiple objectives simultaneously and to identify one final Pareto optimal solution that pleases him/her the most [2]. Most of the proposed MOEAs aim to obtain a good approximation of the whole Pareto optimal front and subsequently let the DM choose a preferred one, i.e. a posteriori decision making [2]. Such a

R.C. Purshouse et al. (Eds.): EMO 2013, LNCS 7811, pp. 337–351, 2013.

process is effective on small-scale problems. However, it has difficulties on large-size problems (e.g. MOPs with many objectives), because approximation of the whole Pareto optimal front is computationally difficult and DM is usually only interested in some regions of the Pareto front.

To facilitate the process of decision making, a better choice is to consider DM preferences in an a priori (preferences are specified before the start of the search) or an interactive (preferences are articulated during the search) way [2]. In these cases, the search can be guided by the preferences toward the ROIs of the Pareto front and away from exploring non-interesting solutions. Since the final decision making process is based on a set of preferred solutions the burden to DM can be reduced significantly.

In the multi-criteria decision making (MCDM) community, various ways have been proposed to represent the preferences of a DM e.g. aspiration levels (goals), weights (search directions), local trade-off, utility function, outranking, fuzzy logic, etc. [2]. the The most frequently used ways are weights (or search direction) and aspiration levels (or goals) [3]. By using weights [4,5] it is easy to articulate DM's bias toward some objectives yet difficult to obtain a precise ROI. By using aspiration levels [6] it is easy to obtain a precise ROI yet difficult to incorporate the DM's bias. Moreover, in some cases it is easier for the DM to express the preferences by weights and in some cases by aspiration levels. The most flexible approach would be to develop a unified approach which enables the DM to articulate both types of the preferences.

In this paper, we describe such a unified approach. Three parameters: reference point (R), weight (W) and search range (θ) are introduced. Then a new interactive evolutionary multi-bjective optimization and decision-making algorithm, iPICEA-g, is proposed that incorporates the unified approach within the existing algorithm PICEA-g [7,8]. Similar to PICEA-g, in iPICEA-g candidate solutions are co-evolved with goal vectors and so guided toward the Pareto front. However, in iPICEA-g the co-evolved goal vectors are exclusively generated in the ROIs that are defined by the three parameters. Moreover, a very friendly interactive technique is developed with which the DM need not use any numeric values to specify his preferences; rather he describes his preferences by interactively brushing his preferred regions in objective space. iPICEA-g automatically configures the required parameters according to the brushed regions and therefore guides the solutions toward the ROIs.

The reminder of the paper is organized as follows: in Section 2 a brief review of preference based MOEAs is presented. This is followed, in Section 3, by an elaboration of the proposed approach iPICEA-g. Section 4 introduces the simulation results of using iPICEA-g to solve different problems in an a priori way or an interactive way. Section 5 concludes and discusses the future research.

2 Review of a Selection of Preference Based MOEAs

A variety of MOEAs that have integrated MCDM methods for preference articulation have been proposed in literature. In this section, we briefly review some representative preference based MOEAs. Two comprehensive survey papers can be found in [9,3].

MOGA [10,6] proposed by Fonscea and Fleming includes probably the earliest attempt to incorporate DM preferences. In their studies, preferences are expressed with goals and priorities. The incorporation of the preferences can be in either a priori or interactive manner. Candidate solutions are ranked based on the Pareto dominance relation together with the specified preferences and therefore the search space of interest gradually becomes smaller during the evolution. MOGA has been successfully used in a variety of applications, including the low-pressure spool speed governor of a Pegasus gas turbine engine [11,12]. The main disadvantage of this approach is that it cannot explore multiple ROIs at the same time. However, exploring multiple ROIs simultaneously is useful when the DM cannot decide which particular region to explore to be explored at the beginning, also for group decision making (different DMs can search for their preferred solutions and select the final solution at the end).

Molina et al. [13] suggested a dominance relation called g-dominance. Solutions satisfying all aspiration levels and solutions fulfilling none of the aspiration levels are preferred over solutions satisfying some aspiration levels. In [13] an approach that couples g-dominance and NSGA-II is proposed to search for ROIs. This algorithm works regardless of whether the specified goal vector is feasible or infeasible and also it is also easy to extended in an interactive manner. However, the g-dominance relation does not preserve a Pareto based ordering. Also, the performance of the algorithm is degraded as the number of objectives increases [14].

Branke et al [15] proposed a guided MOEA (G-MOEA). In the algorithm, considering DM preferences are expressed by modifying the definition of dominance using specified trade-offs between objectives: that is, how much improvement in one or more objective(s) is comparable to a unit degradation in another objective. G-MOEA works well for two objectives; however, providing all pair-wise information in a problem with many objectives is cognitively intensive.

In addition to the above Pareto related approaches, a large body of works are based on the use of reference point, reference direction and light beam search [2]. Two representative reference point based MOEAs are R-NSGA-II [16] and PBEA [17]. R-NSGA-II hybridized reference point with NSGA-II. Reference point is not applied in a classical way, i.e. together with an achievement scalarizing function [18], but rather to establish a biased crowding scheme. Specifically, solutions near reference points are emphasized by the selection mechanisms. The extent and the distribution of the solutions is maintained by an additional parameter ϵ. PBEA is hybridization of reference point method and the indicator based evolutionary algorithm (IBEA [19]). The preference is incorporated by a binary quality indicator (the ϵ-indicator) which is also Pareto dominance preserving. However, since the spread range of the obtained solutions are controled by an additional fitness scaling factor, it is difficult to control the range of the obtained solutions.

Deb et al. [4] combined the reference direction method with NSGA-II. Preferences are modelled by the reference direction (weights) encoded by a staring point and a reference point. This approach is able to find Pareto optimal solutions corresponding to reference points along the reference direction. Multiple ROIs can be obtained by using multiple reference directions. Deb et al. [5] also hybridized NSGA-II with the light beam search method, which enables searching part(s) of the Pareto optimal regions illuminated by the light beam emanating from the starting point to the reference point with a span controlled by a threshold.

Researchers from the MCDM community also developed some interactive MCDM approaches based on MOEAs. For example, Kaliszewski et al [20] proposed to incorporate the DM preference (expressed by search directions) with a Chebyshef scalarizing function and to execute the optimization search from both below (lower bounds) and above (upper bounds). The bounds are approximated based on the objective values of the solutions which are of interest to the DM.

All the above approaches have merit and are able to find Pareto optimal solutions in a ROI. However, none of the above approaches can simultaneously deal with preferences in the form of weights or in the form of aspiration levels. Moreover, among these approaches, some cannot explore multiple ROIs, e.g. MOGA; some do not perform well on many-objective problems [21,22,14], e.g. g-dominance based MOEA; some cannot search for a precise ROI, e.g. R-NSGA-II and PBEA.

3 A Unified New Approach for Articulating Decision Maker's Preference

In this section we introduce in detail the iPICEA-g algorithm. Since the iPICEA-g is based on PICEA-g [7,8], we firstly give a short introduction to PICEA-g.

3.1 Preference-Inspired Co-evolutionary Algorithms Using Goal Vectors

Preference-inspired co-evolutionary algorithms (PICEAs) represent a new class of MOEAs that were proposed by Purshouse et al. [7]. In PICEAs, incorporating concepts from Lohn et al [23], a population of candidate solutions are co-evolved with a set of preferences during the optimization process. Note that the co-evolved preference are not the real decision-maker preferences but are used as a means of comparing solutions for the purposes of a posteriori decision making.

Co-evolution of goal vectors (PICEA-g) is one realization of a PICEA [8]. In PICEA-g, a family of goal vectors and a population of candidate solutions are co-evolved as the search progresses. Candidate solutions gain fitness by meeting (weakly dominating [1]) a particular set of goal vectors in objective-space, but the fitness contribution is shared between other solutions that also satisfy those goals. Goal vectors only gain fitness by being satisfied by a candidate solution, but the fitness is reduced the more times the goals are met by other solutions in the population. The overall aim is for the goal vectors to adaptively guide the candidate solutions towards the Pareto optimal front. That is, the candidate solution population and the goal vectors co-evolve towards the Pareto optimal front. For more details readers are referred to [7,8].

3.2 Interactive Preference-Inspired Co-evolutionary Algorithms Using Goal Vectors

As argued earlier, in some cases it is easier for the DM to specify his preferences in the form of weights (reference/search direction) while other times it is more convenient for the DM to specify an aspiration level (goal). To meet the needs of both types of DM, a unified approach is proposed in this section.

The Unified Approach. Three parameters are defined for the unified approach: a reference point in objective space (R), a search direction (W) and a search range (θ). R is to describe the aspiration levels; W is to introduce the DM's bias toward some objectives where $\sum_{i=1}^{M} w_i = 1, \forall i, w_i \geq 0$. M is the number of objectives; θ is to control the range of the ROI. An example in the bi-objective case is shown in Figure 1. Note that R can also be unattainable; this will be described later.

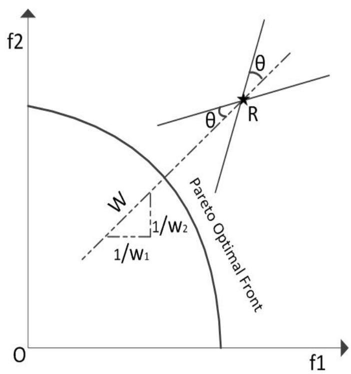


Fig. 1. Illustration of the parameters R, W and θ

Using the three parameters, DM preferences can be expressed either by weights or aspiration levels. If the DM specifies weights then R is set to the *ideal* point (or the coordinate origin, O), W represents specified weights and θ could be any value within the range $[0, \frac{\pi}{2}]$ radians. If the DM specifies aspirations then R is set as the aspiration levels, $w_i = 1/M, i = 1, \cdots, M$, $\theta = \arccos(\frac{\sqrt{M-1}}{\sqrt{M}})$, e.g., when $M = 2$, $w_1 = w_2 = 0.5$ and $\theta = \frac{\pi}{4}$.

The Proposed Algorithm: iPICEA-g. Using the concepts from PICEA-g, it is easy to imagine that if the goal vectors are exclusively generated in a region then candidate solutions inside this region will be encouraged in the evolution. The reason is that these candidate solutions can meet (weakly dominate) more goal vectors and so result in higher fitness, while candidate solutions outside this region can only meet(weakly dominate) few goal vectors and so have a lower fitness. Therefore, over the generations more and more candidate solutions will be guided toward the specified region. For example, in Figure 2, goal vectors are generated in regions $G1$ and $G2$. The objective vector $f(s_1)$ of solution s_1 is inside the region $G1$ while $f(s_1)$ of s_2 is outside the $G1$. Compared to $f(s_2)$, $f(s_1)$ can meet more goal vectors. That is, $f(s_1)$ would obtain a higher fitness than $f(s_2)$, thereby, $f(s_1)$ is more likely to be retained in the search process while $f(s_2)$ is likely to be disregarded.

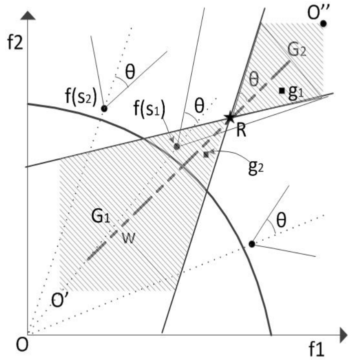


Fig. 2. Illustration of iPICEA-g

Inspired by this thinking, in iPICEA-g goal vectors are not generated in the whole objective space but somewhere which is related to the given ROIs (see the shaded regions in Figure 2). By co-evolving candidate solutions with these specially generated goal vectors, candidate solutions would be guided toward the ROIs. In details, goal vectors are generated in both the shaded regions ($G1$ and $G2$) that are determined by R, W, θ, O' and O''. The region extends both toward and away from the coordinate origin in order to handle the case where the supplied R is unattainable. O' and O'' are the lower and upper bounds of the regions that are to generate goal vectors. O and O' are estimated based on $f(S^*)$ (where S^* represents the current non-dominated solutions) and the specified reference point, R: see equation 1:

$$\begin{aligned} O' =& \alpha \times \min\left(R_i \uplus f_i(S^*)\right), i = 1, 2, \cdots, M, 0 < \beta < 1 \\ O'' =& \beta \times \max\left(R_i \uplus f_i(S^*)\right), i = 1, 2, \cdots, M, \beta > 1 \end{aligned} \tag{1}$$

where α and β are two scaling parameters, here, we use $\alpha = 0.5$ and $\beta = 1.5$. Note O' and O'' can be set equal to *ideal* and *nadir* point, if they are known.

A modified Pareto dominance relation named Pareto cone-dominance is applied in iPICEA-g. A goal vector, g_i is said to be satisfied (Pareto cone-dominated) by a candidate solution, $f(s_i)$ if and only if the angle between the vector $\overrightarrow{f(s_i)g_i}$ and the vector $\overrightarrow{Of(s_i)}$ is not larger than the specified search range, θ. For example, in Figure 2, g_1 is satisfied (Pareto cone-dominated) by candidate solution $f(s_1)$ while g_2 is not.

Apart from the benefit that iPICEA-g can handle the DM preference either as weights or aspiration levels, another major benefit of iPICEA-g is that multiple ROIs can be explored by simultaneously generating goal vectors for all the ROIs. Besides, we anticipate that iPICEA-g performs well on many-objective problems because PICEA-g has a good performance on MOPs with many objectives [7,8].

4 Experiments

In this section, we illustrate the performance of iPICEA-g on different benchmarks from the ZDT [24] and DTLZ [25] test suites. In all the experiments the population size of candidate solutions and goal vectors of iPICEA-g are set as $N = 100$ and $Ngoal = 100$, respectively. Simulated binary crossover (SBX, $p_c = 1$,$\eta_c = 15$) and polynomial mutation (PM, $p_m = \frac{1}{nvar}$ per decision variable and $\eta_m = 20$, where $nvar$ is the number of decision variables) [26] are applied as genetic variation operators. Firstly, we show the effects of R, W and θ. Secondly, we show the performance of iPICEA-g on searching for ROIs in an a priori and progressive way. Note that all the results are illustrative rather than statistically robust.

4.1 Demonstrations of the Effects of R, W and θ

The bi-objective 20-variable DTLZ2, which has a concave Pareto optimal front is selected as test problem to study the effect of the three parameters.

The Effect of R. Assuming the DM would like to have solutions around a point then we set R as the specified point. For example, the DM specify (1) one infeasible (0.6,0.6) reference point; (2) one feasible (0.8,0.8) reference point; and (3) two reference points (0.7,0.9) and (0.8,0.3). Figure 3 shows the obtained results after performing iPICEA-g for 200 generations. During the simulation the search direction is set as $W = [0.5, 0.5]$ which means there is no bias for any objective and the search range $\theta = \frac{\pi}{4}$ radians shows a range that is close to 50:50 emphasize. From Figure 3, we observe that in all cases iPICEA-g can find a set of well converged solutions. It illustrates that iPICEA-g is able to handle both the feasible and infeasible aspiration level, moreover, it can explore multiple ROIs simultaneously.

The Effect of W. Assuming that DM would like to specify a preference for one objective over another we use W. For example, the DM specifies that (1)

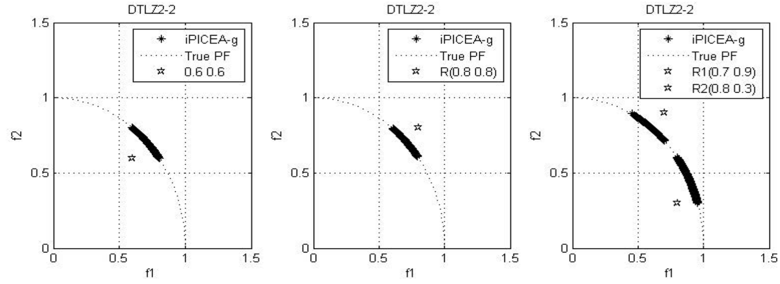


Fig. 3. The solutions obtained by iPICEA-g with different reference points

both the objectives are equally important then $W = [0.5, 0.5]$ or (2) objective f_1 is twice as important as f_2 then $W = [0.67, 0.33]$ or (3) objective f_1 is half as important as f_2 then $W = [0.33, 0.67]$. Figure 4 shows the obtained results after performing iPICEA-g for 200 generations with $W = [0.5, 0.5]$, $W = [0.33, 0.67]$ and $W = [0.67, 0.33]$, respectively. During the simulation, $R = (0.5, 0.5)$ and $\theta = \frac{\pi}{6}$ radians. From the Figure, we observe that the obtained solutions are along the given search direction, W. In other words, the obtained solutions are biased with different W. For example, in the case of $W = [0.67, 0.33]$ f_1 is more optimized.

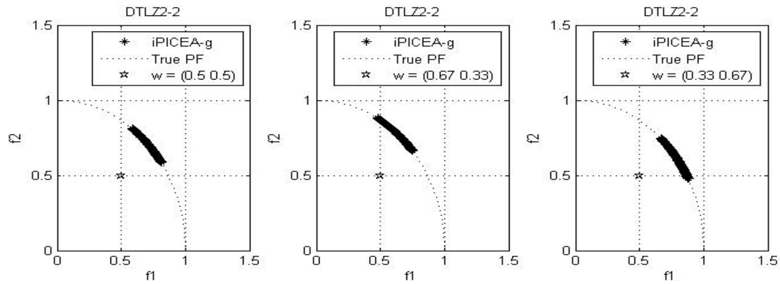


Fig. 4. Solutions obtained by iPICEA-g with different search directions

The Effect of $\boldsymbol{\theta}$. If the DM would like to obtain a large spread range of solutions then θ could be a large value e.g. having $\theta = \frac{\pi}{2}$ radians, the whole Pareto front can be obtained. If the DM would like to obtain some solutions that are exactly along the specified W then θ is set to $\frac{\pi}{180}$ radians. Figure 5 show the obtained results after performing iPICEA-g for 200 generations with $\theta = \frac{\pi}{2}, \frac{\pi}{4}$ and $\frac{\pi}{180}$, radians, respectively. During the simulation, $R = (0.3, 0.3)$ and $W = [0.5, 0.5]$. Clearly, the range of the obtained solutions decreases as θ decreases.

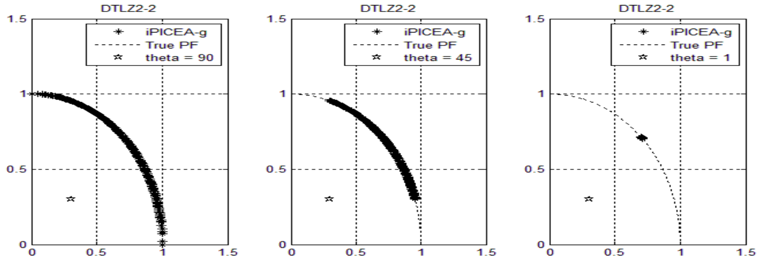


Fig. 5. The distribution of solutions obtained by iPICEA-g with different θ values

4.2 Results for a Priori Preference Expression

Search With Weights. Here we consider the case where the DM states the relative importance of each objective. Bi-objective ZDT2 and 4-objective DTLZ2 are used in the simulation and. iPICEA-g is run for 200 generations on each problem.

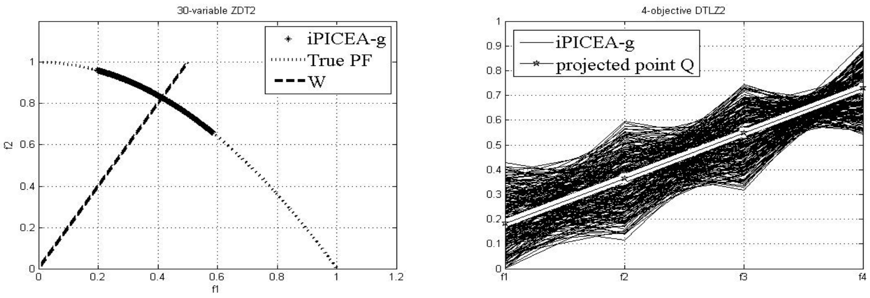
For ZDT2 assuming that DM specifies f_1 is twice as important as f_2 then $W = [\frac{2}{3}, \frac{1}{3}]$; correspondingly, θ is set as $\frac{\pi}{4}$ radians in order to obtain a moderate range of solutions; R is set as the O'. From Figure 6.a we can clearly observe the obtained solutions are biased to objective f_1. Also, the solutions are near the true Pareto front.

For DTLZ2 we assume the DM specifies that f_1 is four times as important as f_4, f_2 is three times as important as f_4, f_3 is twice as important as f_4 then $W = [0.4, 0.3, 0.2, 0.1]$; correspondingly, θ is set as $\frac{\pi}{12}$ radians so as to obtain a close range of solutions; R is set to the O'. Observed from Figure 6.b (parallel coordinates plots [12]), a set of solutions are obtained, which are located around the projected point Q shown as $-\star-$. Q is the projection of the coordinate origin to the Pareto optimal front along the direction $[0.4, 0.3, 0.2, 0.1]$. The true Pareto front of DTLZ2 is the surface of hyper-sphere with radius 1 ($\sum_{i=1}^{M} f_i^2 = 1$) in the first quarter [25]. Having computed $\sum_{i=1}^{4} f_i^2$ for all the obtained solutions, we find all values lies within the range [1.0391,1.0903] which confirm that the obtained solutions have almost converged to the true Pareto front.

Search With Aspiration Levels. Here we consider the case where the DM specifies preferences as aspiration levels. Again, the bi-objective ZDT1 and 4-objective DTLZ2 problems are used in the simulation.

For ZDT1 we assume that DM specifies his aspiration level as [0.7,0.7] and so R = (0.7,0.7); correspondingly, W is set as [0.5,0.5] and θ is set as $\frac{\pi}{4}$ radians. After running iPICEA-g for 200 generations a set of satisfied solutions are obtained shown in Figure 7.a. We can see that visually all the obtained solutions are very close to the true Pareto front.

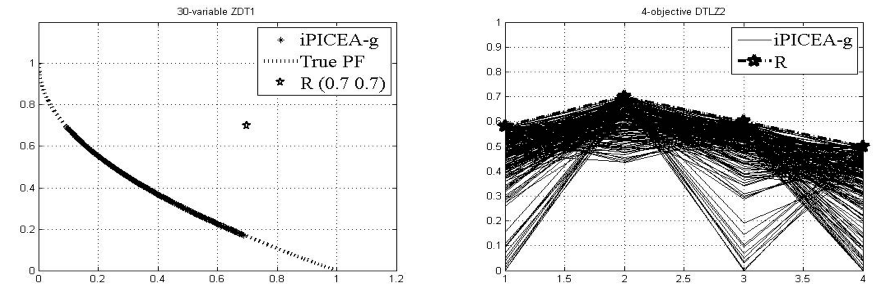
For DTLZ2, we assume the DM specifies that f_1, f_2, f_3 and f_4 should be better (smaller) than 0.58, 0.7, 0.6 and 0.5, respectively. Therefore, we set



(a) (b)

Fig. 6. Illustration of searching with weights

R = (0.58,0.7,0.6,0.5). Correspondingly, W is configured as [0.25, 0.25, 0.25, 0.25] and θ is set as $\arccos(\frac{\sqrt{M-1}}{\sqrt{M}}) = \frac{\pi}{6}$ radians. After running iPICEA-g for 200 generations a set of solutions is found as shown in Figure 7.b. All the solutions have met the aspiration level. After computing $\sum_{i=1}^{4} f_i^2$ for all obtained solutions, the values lie within the range [1.0141,1.0528], therefore indicating that all solutions have converged close to the true Pareto front.



(a) (b)

Fig. 7. Illustration of searching with aspirations

4.3 Results for a Progressive Preference Expression

Cognitively, DM may find it easier to specify preferences visually by drawing rather than using numbers. iPICEA-g allows DM to brush existing solutions or regions of the objective space that are of interest. These preferences would be automatically converted into R, W and θ parameters. Consider a 2-objective minimization example, see Figure 8. The brushed region is labelled as A.

Firstly, we find the extreme points of A, i.e. P_{f1} and P_{f2}. P'_{f1} and P'_{f2} are the normalized vector of P_{f1} and P_{f2}, respectively, i.e. $P_{fi} = \frac{P_{fi}}{L_i}, i = 1, 2$, where L_i is the Euclidean distance from O' to P_{fi}. $|O'P'_{f1}| = |O'P'_{f2}| = 1$. Then the search direction W is determined by vector $\boldsymbol{O'P}$, where P is the center of P'_{f1} and P'_{f2}. θ is then calculated by $\arccos(\overrightarrow{O'P} \cdot \overrightarrow{O'P'_{f1}})$. R is set as the O' (which can be obtained by equation 1). The co-evolved goal vectors are then generated in the shaded region closed by points R, P_{f1}, P_{f2} and O''.

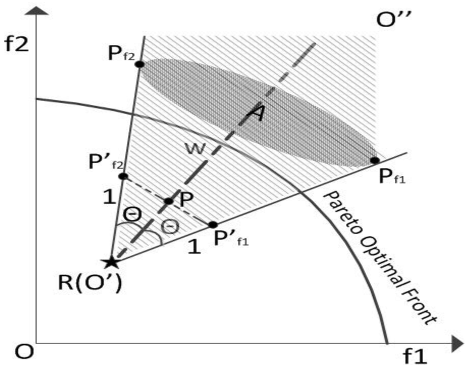


Fig. 8. Illustration of parameter calculation

To describe the working process, we solve the bi-objective ZDT1 and 4-objective DTLZ4 problems by simulating an interactive search process.

Bi-objective ZDT1. Firstly, iPICEA-g is run for 10 generations without incorporating any preferences. The aim is to roughly know the range of the objectives so as to give better preferences. The obtained solutions are shown in Figure 9.a.

Secondly, the DM brushes his preferred regions, i.e. the shaded regions in Figure 9.a. The related parameter settings of iPICEA-g are then calculated based on the brushed region, which are $W = [0.25, 0.75]$, $\theta = \frac{5\pi}{36}$ radians and $W = [0.75, 0.25]$, $\theta = \frac{5\pi}{36}$ radians for region A and B, respectively. After running iPICEA-g for 50 more generations, two sets of improved solutions are found. See Figure 9.b.

Thirdly, we assume that the DM is not satisfied with either of the two sets of solutions. However, he/she is interested in exploring a nearby region, C. The related parameter settings are $W = [0.6, 0.4]$, $\theta = \frac{\pi}{12}$ radians. By running iPICEA-g for another 50 generations, a set of solutions are found in C shown in Figure 9.c.

Fourthly, the DM is still dissatisfied. He/She would like to exploit these solutions. The preferred solutions are then brushed (See Figure 9.c) and iPICEA-g is run for 50 more generations. The related parameters are configured as

$W = [0.5, 0.5]$, $\theta = \frac{\pi}{18}$ radians. A set of better solutions are found. The DM is now happy to choose a single solution from this set. The solution D is selected; see Figure 9.d.

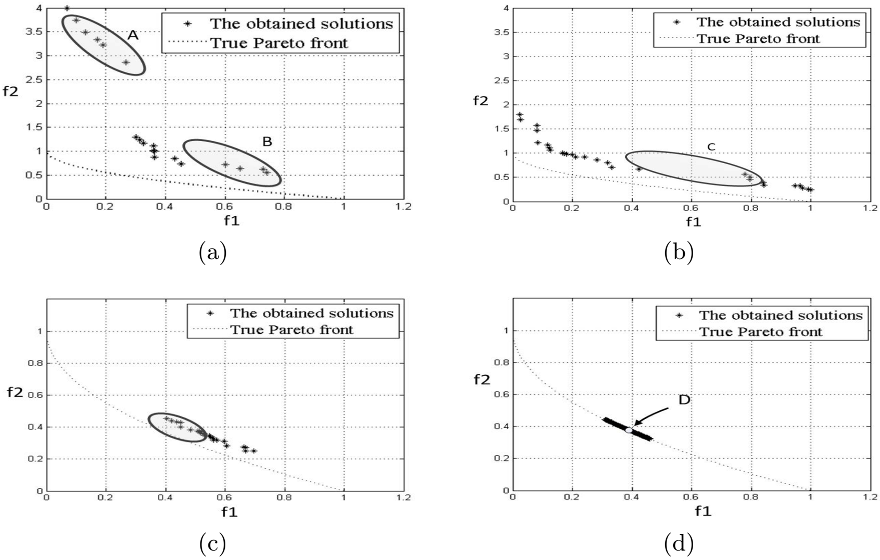


Fig. 9. Interactive scenario on 30-variable ZDT1

4-objective DTLZ2. Similarly, iPICEA-g is run without introducing any preference for 10 generations. A set of solutions are found as shown in Figure 10.a.

Secondly, DM brushes the preferred solutions for each objective (see Figure 10.a). Parameters W and θ are then calculated as $[0.25, 0.25, 0.25, 0.25]$ and $\frac{\pi}{6}$ radians. iPICEA-g is run for 50 more generations. An improved set of solutions are obtained (see Figure 10.b).

Thirdly, assuming DM is dissatisfied with the obtained solutions. He/She brushes some solutions that are of interest. Based on the brushed solutions, two ROIs are identified. The related parameters are configured by $W = [0.3986, 0.3500, 0.1105, 0.1409], \theta = \frac{\pi}{12}$ radians and $W = [0.1124, 0.2249, 0.3498, 0.3128], \theta = \frac{7\pi}{90}$ radians. The brushed solutions are shown in Figure 10.c. After running iPICEA-g for another 50 generations, more solutions are found. See Figure 10.d.

Fourthly, the DM is still not satisfied with the obtained solutions. He/she decides to explore one set of the obtained solutions. Again, he/she brushes his preferred solutions which are shown in Figure 10.e and run iPICEA-g for 50 more generations. W is set as $[0.3691, 0.2773, 0.1383, 0.2153]$, θ is set as $\frac{\pi}{36}$ radians. Seen from Figure 10.f, a set of refined solutions are found in this preferred region. We compute $\sum_{i=1}^{4} f_i^2$ for all the obtained solutions. The value lies within the range of [1.0190,1.041] which means the obtained solutions have well converged to the true Pareto front. The DM is now happy to choose a single solution from this set. The solution shown as the white dash line is selected; see Figure 10.d.

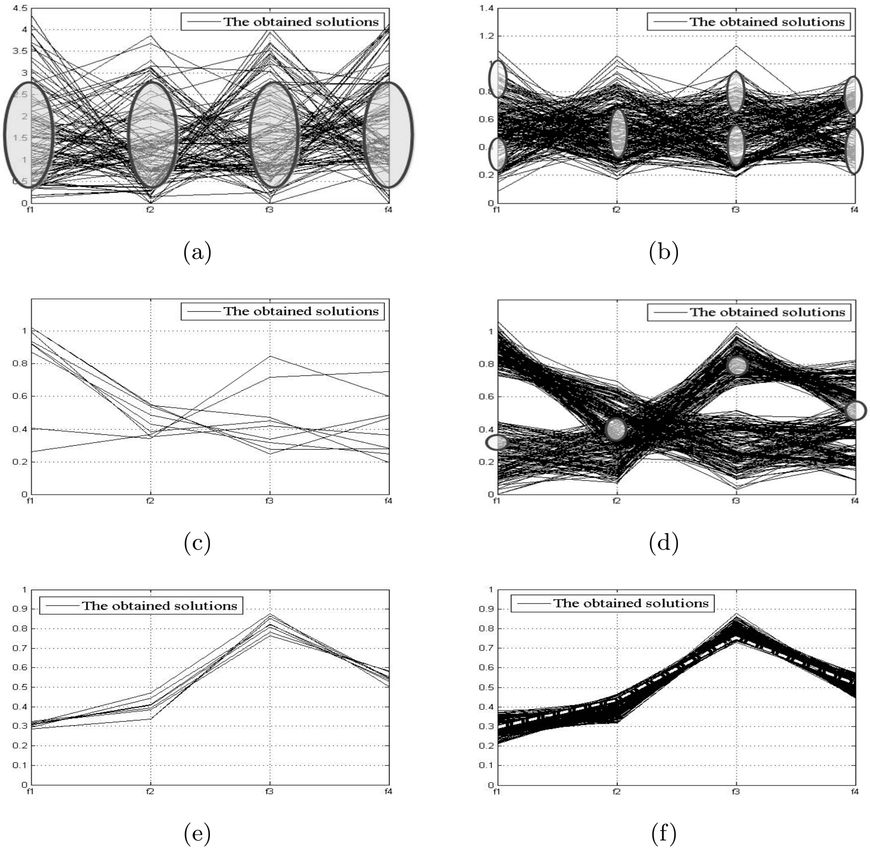


Fig. 10. Interactive scenario on 4-objective DTLZ2

5 Conclusions

Incorporation of DM preference is an important part of a real-world decision support system. However, current methods for preference-based multi-objective optimisation are unable to handle, comprehensively, the range of ways in which a DM likes to articulate his/her preferences. In this paper, we have presented, to the best of our knowledge, the first method that is simultaneously able to handle preferences expressed as weights or as aspirations and that is also able to support multiple regions of interest. We also enhance the DM-friendliness by allowing preferences to be expressed either numerically or by interactively drawing on cartesian coordinate plots or parallel coordinates plots. Simulation results have shown the effectiveness of the method.

There are three core directions for future research, firstly, since decision-making is often a group rather than individual activity, it would be useful to develop the method in order to support group decision making. Secondly, since the DM's preference is often expressed in fuzzy linguistic terms [27], it is important to study how to handle fuzzy preferences. Thirdly, the method should be trialled in a real decision making problem.

Acknowledgments. The work of Rui Wang was supported by the National Science Foundation of China (No. 70971132).

References

1. Deb, K.: Multi-objective optimization using evolutionary algorithms, vol. 16. Wiley (2001)
2. Miettinen, K.: Nonlinear multiobjective optimization, vol. 12. Springer (1999)
3. Rachmawati, L., Srinivasan, D.: Preference Incorporation in Multi-objective Evolutionary Algorithms: A Survey. In: The 2006 IEEE Congress on Evolutionary Computation, pp. 962–968. IEEE (2006)
4. Deb, K., Kumar, A.: Interactive evolutionary multi-objective optimization and decision-making using reference direction method. In: GECCO 2007: Proceedings of the Genetic and Evolutionary Computation Conference, vol. 1. ACM, London (2007)
5. Deb, K., Kumar, A.: Light beam search based multi-objective optimization using evolutionary algorithms. In: The 2007 IEEE Congress on Evolutionary Computation, pp. 2125–2132. IEEE (2007)
6. Fonseca, C., Fleming, P.J.: Multiobjective optimization and multiple constraint handling with evolutionary algorithms. I. A unified formulation. IEEE Transactions on Systems, Man and Cybernetics, Part A: Systems and Humans 28(1), 26–37 (1998)
7. Purshouse, R.C., Jalbă, C., Fleming, P.J.: Preference-Driven Co-evolutionary Algorithms Show Promise for Many-Objective Optimisation. In: Takahashi, R.H.C., Deb, K., Wanner, E.F., Greco, S. (eds.) EMO 2011. LNCS, vol. 6576, pp. 136–150. Springer, Heidelberg (2011)
8. Wang, R., Purshouse, R.C., Fleming, P.J.: Preference-inspired Co-evolutionary Algorithms for Many-objective Optimisation. IEEE Transactions on Evolutionary Computation (to appear) (accepted)
9. Coello, C.: Handling preferences in evolutionary multiobjective optimization: A survey. In: The 2000 IEEE Congress on Evolutionary Computation, vol. 1, pp. 30–37. IEEE (2000)
10. Fonseca, C., Fleming, P.J.: Genetic algorithms for multiobjective optimization: Formulation discussion and generalization. In: Proceedings of the 5th International Conference on Genetic Algorithms, pp. 416–423. Morgan Kaufmann Publishers Inc. (1993)
11. Fonseca, C., Fleming, P.J.: Multiobjective optimization and multiple constraint handling with evolutionary algorithms. II. Application example. IEEE Transactions on Systems, Man and Cybernetics, Part A: Systems and Humans 28(1), 38–47 (1998)

12. Fleming, P.J., Purshouse, R.C., Lygoe, R.J.: Many-Objective Optimization: An Engineering Design Perspective. In: Coello Coello, C.A., Hernández Aguirre, A., Zitzler, E. (eds.) EMO 2005. LNCS, vol. 3410, pp. 14–32. Springer, Heidelberg (2005)
13. Molina, J., Santana, L.V., Hernández-Díaz, A.G., Coello Coello, C.A., Caballero, R.: g-dominance: Reference point based dominance for multiobjective metaheuristics. European Journal of Operational Research 197(2), 685–692 (2009)
14. Said, L.B., Bechikh, S.: The r-dominance: a new dominance relation for interactive evolutionary multicriteria decision making. IEEE Transactions on Evolutionary Computation 14(5), 801–818 (2010)
15. Branke, J.: Guidance in evolutionary multi-objective optimization. Advances in Engineering Software 32(6), 499–507 (2001)
16. Deb, K., Sundar, J.: Reference point based multi-objective optimization using evolutionary algorithms. In: GECCO 2006: Proceedings of the Genetic and Evolutionary Computation Conference, pp. 635–642. ACM, New York (2006)
17. A preference-based evolutionary algorithm for multi-objective optimization. Evolutionary Computation 17(3), 411–436 (2009)
18. Wierzbicki, A.: The use of reference objectives in multiobjective optimization-theoretical implications and practical experiences. In: Proceedings of the Third Conference on Multiple Criteria Decision Making: Theory and Application, vol. 1979, p. 468. Springer, Hagen Konigswinter (1979)
19. Zitzler, E., Künzli, S.: Indicator-Based Selection in Multiobjective Search. In: Yao, X., Burke, E.K., Lozano, J.A., Smith, J., Merelo-Guervós, J.J., Bullinaria, J.A., Rowe, J.E., Tiňo, P., Kabán, A., Schwefel, H.-P. (eds.) PPSN VIII. LNCS, vol. 3242, pp. 832–842. Springer, Heidelberg (2004)
20. Kaliszewski, I., Miroforidis, J., Podkopaev, D.: Interactive multiple criteria decision making based on preference driven evolutionary multiobjective optimization with controllable accuracy. European Journal of Operational Research 216(1), 188–199 (2012)
21. Purshouse, R., Fleming, P.: Evolutionary many-objective optimisation: an exploratory analysis. In: The 2003 IEEE Congress on Evolutionary Computation, vol. 3, pp. 2066–2073. IEEE (2003)
22. Purshouse, R.C., Fleming, P.J.: On the Evolutionary Optimization of Many Conflicting Objectives. IEEE Transactions on Evolutionary Computation 11(6), 770–784 (2007)
23. Lohn, J., Kraus, W., Haith, G.: Comparing a coevolutionary genetic algorithm for multiobjective optimization. In: The 2002 IEEE Congress on Evolutionary Computation, vol. 2, pp. 1157–1162. IEEE (2002)
24. Zitzler, E., Thiele, L.: Multiobjective evolutionary algorithms: A comparative case study and the strength pareto approach. IEEE Transactions on Evolutionary Computation 3(4), 257–271 (1999)
25. Deb, K., Thiele, L., Laumanns, M., Zitzler, E.: Scalable multi-objective optimization test problems. In: The 2002 IEEE Congress on Evolutionary Computation, pp. 825–830. IEEE (2002)
26. Deb, K., Pratap, A., Agarwal, S., Meyarivan, T.: A fast and elitist multiobjective genetic algorithm: NSGA-II. IEEE Transactions on Evolutionary Computation 6(2), 182–197 (2002)
27. Rachmawati, L., Srinivasan, D.: Incorporation of imprecise goal vectors into evolutionary multi-objective optimization. In: The 2010 IEEE Congress on Evolutionary Computation, pp. 1–8. IEEE (2010)