A Heuristic Approach Based on Shape Similarity for 2D Irregular Cutting Stock Problem

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Abstract. Cutting stock problem is an important problem that arises in a variety of industrial applications. This research constructs an irregular-shaped nesting approach for two-dimensional cutting stock problem. The techniques of shape similarity are utilized, drawn from computer vision and artificial intelligence. These techniques enable the approach to find potential matches of the unplaced pieces within the void regions of the sheet, and thus the packing density and the performance of solutions are highly improved. The proposed approach is able to deal with complex shapes in industrial application and achieve high-quality solution with shorter computational time. We evaluate the proposed method using 15 established benchmark problems available from the EURO Special Interest Group on Cutting and Packing. The results demonstrate the effectiveness and high efficiency of the proposed approach.

Keywords: Cutting Stock Problem, Grid Approximation, Shape Similarity, Fourier Descriptor.

1 Introduction

Cutting and Packing Problem are a large family of problems arising in a wide variety of industrial applications, including the cutting of standardized stock units in the wood, steel and glass industries, packing on shelves or truck beds in transportation and warehousing, and the paging of articles in newspapers. There are many classic cutting and packing problems, including knapsack problem, bin packing and cutting stock problem etc. In this paper, we focus on the cutting stock problem (CSP). In CSP, a number of two-dimensional pieces must be cut from a couple of same stocks. The objective is to minimize the number of stocks. Using the typology of Wäscher, this is a Two-Dimensional Single Stock-Size Cutting Stock Problems (2DCSP) [1]. The 2DCSP has been proved to be NP-hard [2].

The 2DCSP can be defined as follows: Given a set L = (a1, a2, ..., an) of regular and irregular pieces to be cut (size of each piece $s(a_i) \in (0, A_o]$) from a set of rectangular stock-cutting sheets (objects) of size A_o (with fixed Length L and Width W), the CSP is to find cutting patterns to minimize the number of objects used. Typical assumptions are summarized as:

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1. All pieces must be within the object (closeness);
2. Pieces must not overlap with each other (disjoint);
3. In this research, rotation of piece is allowed. And each piece has eight orientations: 0°, 45°, 90°, 135°, 180°, 225°, 270° and 315°.

This paper is organized as follows. A brief review of previous work in the field is presented in Section 2. The outline of our method is introduced and a novel heuristic placement strategy based on shape similarity is presents in detail in section 3. Section 4 gives experimental results on benchmark problems from the literatures that demonstrate the capabilities of the proposed approach. In Section 5 the research is concluded and possible issues for future work are suggested.

2 Literature Review

The Cutting and Packing problem involving irregular shape is also called nesting problem. A number of approaches have been proposed to tackle different nesting problems. We only give a brief discussion on some of the most interesting approaches previously presented in the literature.

The most visible attribute of nesting problems and the first obstacle researchers come up against is the geometry. As a result, developing a set of tools to assimilate the geometry is a non-trivial task and potentially a barrier that stifles academic research in this area. There exist a couple of solutions to this problem such as Rectangle Enclosure, Orbital Sliding of No Fit Polygon, Minkowski Sum of No Fit Polygon, and Phi Function [3-7].

In terms of solution methods a number of approaches are proposed depending on the type and the size of the problem. There have been many different strategies for producing solutions to the irregular cutting stock problem. These include exact algorithm (e.g. linear programming, dynamic programming, column generation etc.), heuristic placement strategy, meta-heuristic search techniques, and other novel approaches. For less constrained and simpler packing tasks, exact algorithms are developed along with problem-specific heuristic procedures [8]. For more complex packing tasks, heuristic search methods have been applied successfully for their solution. Heuristic placement strategy such as bottom-left (BL), bottom-left-fill is proposed to supply a rule for pieces to be placed on sheet [9-11]. Furthermore, this research utilizes techniques of shape similarity drawn from computer vision and artificial intelligence, and achieves high-quality solutions with shorter computational times [12].

The 2DCSP is NP hard due to the combinatorial explosion encountered as the size of problem increases. As a result, a number of published solution approaches focus on heuristic and meta-heuristics methodologies. Meta-heuristics are general frameworks for heuristics in solving combinatorial optimization problems. These meta-heuristic approaches include simulated annealing, tabu search, neural networks and genetic algorithms [13-16].

Although numerous approaches based on computational geometric description have obtained good performance, computational complexity for large and complex data sets is yet a huge difficulty. We adopt another method to represent shapes called grid approximation, in which pieces are represented by two-dimensional matrices. With use of grid approximation, it's not necessary to introduce additional routines to identify enclosed areas and geometric tool to detect overlap. The grid approximation is used in several literatures [17, 18].

3 Methodology

In 2DCSP, two kinds of heuristics are generally used, namely the selection heuristic for selecting pieces and objects, and the placement heuristic for placing pieces and objects. Many heuristics have been studied in the literatures and have their own superiority in some instances. In this research, Best Fit and Bottom Left irregular are respectively chosen to be the selection heuristic and placement heuristic. Firstly, it is decided by heuristic Best Fit that which stock sheet is chosen for the allocating piece. Then a hybrid approach combining heuristic Bottom Left irregular and other placement strategy is proposed to place the piece on this stock.

The proposed approach is composed of three steps. Firstly, the grid approximation is used to represent irregular-shaped pieces in two-dimensional matrices. The geometry of the stocks and pieces are converted into discrete form in order to make the nesting process faster and the actual geometry of the sheets and pieces independent. Secondly, a hybrid approach combining with heuristic Bottom Left irregular and a two-stage placement strategy is used to pack the ordered pieces. Finally, void regions are generated between the packed pieces and stock sheet after some pieces are placed. Then these void regions are matched with the unpacked pieces, according to their similarity of shape. Among the pieces that can be allocated in the void region, the most similar one will be allocated on the void region. A detailed description of every step is showed in following sections.

In order to evaluate the resulting solutions, we consider the ratio of the total area of all the allocated shapes to the area of the occupied stocks as a performance measure called packing density (PD). Its maximum value is 1, when there is no waste of resource material.

$$PD = \frac{\text{Area of Allocated Pieces}}{\text{Area of Occupied Stocks}}.$$

3.1 Pre-layout Phase

The operation of the method is divided into two phases: the pre-layout phase and the layout phase. In the pre-layout phase, the pieces are represented as a matrix by taking the grid approximation method and the initial sequence and orientation of the pieces based on their geometrical features are determines.

In this research, the grid approximation, a digitized representation technique, is used to represent multiple-shaped pieces including convex and concave. The matrix representation approach was proposed by Dagli 1990[19]. Each piece is represented by a matrix of size K by L which is the smallest rectangular enclosure of the irregular-shaped piece. The detailed technique applied in the paper is referred to W.K. Wong and Z.X. Guo 2009. By using this technique, each piece is enclosed by an imaginary rectangle for the sake of obtaining the reference points during the nesting process. Then, this particular rectangular area is divided into a uniform grid of 1mm×1mm size. In the case, the value of a pixel is ‘1’ when the material of the sheet is occupied, otherwise the value of the pixel is ‘0’. $P_L^{(k)}$ and $P_W^{(k)}$ denote the length and the width of an enclosing rectangle corresponding to the piece p_k. R=1 mm, denotes the square side of a piece. Fig.1 gives an example of grid approximation.

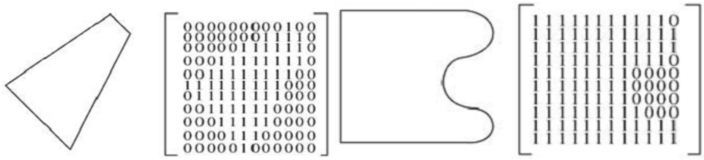

Fig. 1. Binary representations

Similar to the piece representation, the object is discretized as a finite number of equal-size pieces of size R^2. The object with a two-dimensional matrix of size $A_W^{(k)} \times A_L^{(k)}$ is represented as follows:

$$\mathrm{A}^{(k)} = \begin{pmatrix} a_{11}^{(k)} & a_{12}^{(k)} & \cdots & a_{1A_L^{(k)}}^{(k)} \\ a_{21}^{(k)} & a_{22}^{(k)} & \cdots & a_{2A_L^{(k)}}^{(k)} \\ \vdots & \vdots & \ddots & \vdots \\ a_{A_W^{(k)}1}^{(k)} & a_{A_W^{(k)}1}^{(k)} & \cdots & a_{A_W^{(k)}A_L^{(k)}}^{(k)} \end{pmatrix}, \text{ where } A_W^{(k)} = \frac{P_W^{(k)}}{R} \quad \mathit{and} \quad A_L^{(i)} = \frac{P_L^{(k)}}{R}$$

For each entry, $$a_{i,j}^{(k)} = \begin{cases} 1\ , \text{ if pixel } (i, j) \text{ is occupied} \\ 0\ , \text{ otherwise} \end{cases}.$$

The initial sequence of the pieces is determined according to their area. The initial orientation for each packing piece is confirmed by the MRE (Minimum Rectangular Enclosure) of each piece. It is worth mentioning that the method does not replace the piece shapes by their MREs. Rather, it uses MREs as additional information for orienting the pieces.

3.2 Layout Phase

The placement of the pieces follows a single-pass placement strategy and takes place in a sequential manner. The manner entails that the method considers only one layout. In layout phase, a hybrid approach combining with heuristic Bottom Left irregular and two-stage placement strategy is proposed to construct a packing pattern according to the sequence and orientation of the packing pieces. Furthermore, a packing approach based on shape similarity draw from computer vision and artificial intelligence is adopted to increase the packing density of the nesting piece at a particular stage by reusing the complements of the allocated pieces on the sheet.

3.2.1 Hybrid Approach Combining with Heuristic Bottom Left Irregular and Two-Stage Placement Strategy

The heuristic Bottom Left irregular is used to allocate the current pieces on the stock sheet without affecting any other allocated pieces. The purpose is to minimize the total required area and the objective value can be stated as:

$$\text{objective value of sliding part} = x * W + y$$

Where x and y is respectively the coordinate of the piece along X axis and Y axis of the matrix representation of the stock sheet; W is the width of the stock sheet. By using this approach, the first packing piece is placed at the lower left-hand corner of the empty stock sheet. The following pieces are allocated along the Y axis until there is no enough space. Then a new row of pieces along X axis is formed. The two-stage placement strategy is referred to W.K. Wong and X.X. Wang 2009. In this case, the enclosing rectangles of the packing pieces are first examined, and then the packing pieces are compacted directly. The differences are that the compaction routine is done when each enclosing rectangle is placed rather than implement it in a single step after all the enclosing rectangles are allocated. This compaction routine is able to obtain a tighter packing pattern and provide more space for the unpacked pieces. The hybrid approach improves the quality of packing pattern without more computational effort.

3.2.2 Packing Approach Based on Shape Similarity

After pieces are placed, a number of void regions are generated by previous allocations, namely the complements of the allocated pieces. These void regions may be reused to increase the packing density at a particular stage. The packing approach is based on shape similarity draw from computer vision and artificial intelligence. The characteristic is the utilization of a shape similarity criterion for matching void regions of the layout and remaining pieces. As the grid of the allocated pieces in sheet partially or wholly covered is assigned by '1', then the void region between these pieces is the region assigned by '0' among '1's. The void regions are extracted from the stock sheet by scanning each row of the matrix representation of the stock plate. The result of scanning the sheet is a pool of line segments as sections of void regions. These line segments are later combined and form void regions.

Whenever a new row along X axis of the stock sheet appears as described in 3.2.1, the rectangular area with the size of the width of the stock sheet at end of the new row is considered as the active portion of the stock sheet at each allocation stage. Then the scanning operation as described above is done in the active portion.

Having defined what constitutes a void region in the layout, we can now describe how the approach attempts to find effective placements of the remaining pieces. This approach is referred to Alexandros and Murray 2007, but this research employs another method to solve it. In order to determine whether it is appropriate to place a piece to a void region, the approach considers the ratio of their areas. Firstly, the packing piece must be able to be placed into the void region. Then the similarity of shapes between the piece and void region is under considered. Thus it's effective to decrease computational time. The following equation is employed as an evaluation function (EF) for each possible placement

$$\mathrm{EF}=\begin{cases}\mathrm{M}\ , & \text{if } 1\le \dfrac{\text{area of void region}}{\text{area of part}} \le 2.5 \\ 1000\ , & \text{otherwise}\end{cases}.$$

Where, M is a measure that expresses the similarity between the piece's shape and the region's shape. When the area ratio of the void region and the piece is not between the thresholds we have defined, the EF takes a great value in order to prefer other possible solutions. However, the evaluation function is just a quick and approximate measure and the actual quality of a potential placement still need to consider the performance measure to the layout by first placing the piece.

3.2.3 Fourier Descriptors Method for Shape Similarity

The problem of shape similarity has been well studied in the literature of computer vision, and a number of different methods have been proposed [20, 21]. In this paper, the Fourier descriptors method is adopted as it can achieve both good representation and easy normalization. The normalization can make the similarity measure of two shapes is invariant under translation, rotation, and change of scale.

For a given shape defined by a closed curve C. At every time t, there is a complex u(t), $0 \le t < T$. Since u(t) is periodic, $u(t + nT) = u(t)$ can be implied. Therefore, it is possible to expand u(t) into Fourier series.

The discrete Fourier transform is given by

$$a_n = \frac{1}{N}\sum_{t=0}^{N-1} u(t)\exp(-j2\pi nt/N) \qquad n = 0,1,\ldots \mathrm{N}\text{-}1$$

u(t) is often called a shape signature which is any one dimensional function representing shape boundary. It has been shown that FDs derived from centroid distance function outperforms FDs derived from other shape signatures. The centroid distance function r(t) is expressed by the distance of the boundary points from the centroid (x_c, y_c) of the shape and N is the number of boundary points.

$$\mathrm{r(t)} = ([x(t)-x_c]^2 + [y(t)-y_c]^2)^{1/2}, \text{ where } x_c = \frac{1}{N}\sum_{t=0}^{N-1} x(t),\ y_c = \frac{1}{N}\sum_{t=0}^{N-1} y(t)$$

Since shapes generated through rotation, translation and scaling of a same shape are similar shapes, shape descriptors should be invariant to these operations. Based on the above analysis of FD properties, it is possible to normalize FDs into shape invariants. Now considering the following expression

$$b_n = \frac{a_n}{a_0} = \frac{\exp(jn\tau)\cdot\exp(j\phi)\cdot s\cdot a_n^{(0)}}{\exp(j\tau)\cdot\exp(j\phi)\cdot s\cdot a_0^{(0)}} = \frac{a_n^{(0)}}{a_0^{(0)}} = b_n^{(0)}\exp[j(n-1)\tau)]$$

Where b_n and $b_n^{(0)}$ are normalized Fourier coefficients of the derived shape and the original shape respectively. The set of magnitudes of the normalized Fourier coefficients of the shape $\{|b_n|, 0<n<N\}$ can now be used as shape descriptors, denoted

as $\{FD_n, 0<n<N\}$. The similarity between the query shape Q and the target shape T is given by the Euclidean distance d between their FDs.

$$d = (\sum_{i=1}^{N} \left| FD_i^Q - FD_i^T \right|^2)^{\frac{1}{2}}.$$

4 Performance Evaluation

All algorithms are implemented in Visual C++ and we also use the library CGAL to do some geometrical operations. The tests are performed on a computer with processor Intel Pentium Dual 1.8 GHz, 2 GB of RAM, and Windows XP operating system.

Few work for packing problems with pieces of irregular shape are done in the literature, and especially we can hardly find any related work for the 2DCSP and 2DBPP when pieces have irregular shape. As a result, we adapt some other known instances for packing problems with one open dimension to test our algorithms. The generated instances are adapted from the Two-Dimensional Irregular Strip Packing problem, and they can be found at the ES-ICUP website. In these instances, the following information is available: the quantity of pieces; the set of allowable rotations for these pieces; and, the size of the stock.

The instances from AM Del Valle .et .al 2012 are adapted and adopted in this research [22]. Table 1 presents solutions computed by our algorithm. We use the total area of the pieces divided by the area of one stock as a lower bound for the optimal solution. The rows in this table contain the following information: instance name (Name); solution value computed by our algorithm (Solution); the lower bound (LB); the difference (in percentage) on number of stocks computed by Solve 2DCSP and LB; the time spent in seconds (Time).

From Table 1 we can see that the value of the solutions returned by the algorithm is on average 38.487% larger than the lower bound (in the worst case), However, for instance DIGHE1, it is 84.375% larger. The results of the algorithm should be much closer to the optimal solutions and these differences are mainly due to the weakness of the lower bound. The instance SHIRTS took 1,232,468.32 s ($\approx$14 days) of CPU processing. The CPU time spent is on average 140896.16s. Since the complexity of the working with pieces of irregular shape is largely increased, the problem becomes harder than the working with rectangular pieces and some instances even take a long time to be solved.

It is worth to mention that all the instances are executed 30 times, except SHIRTS (2 time), SWIM (20 times) and TROUSERS (10 times) due to the high CPU time required by them. All the results presented on Table 1 are the average values of all executions. Such results show that the algorithm returns good solutions for the cutting stock problem with pieces of irregular shape. However, it requires high CPU time when solving instances with several pieces of completely irregular shape.

Table 1. Results obtained for the 2CS

Name	Solution	LB	Difference(%)	Time(s)
FU	72	56	22.222	208.82
JACKOBS1	48	38	26.315	6706.62
JACKOBS2	45	30	50.000	6877.99
SHAPES0	53	30	76.666	26681.03
SHAPES1	54	32	68.750	59581.59
SHAPES2	61	50	22.000	9393.39
DIGHE1	59	32	84.375	232.29
DIGHE2	44	29	51.724	25.07
ALBANO	82	65	26.153	6730.06
DAGLI	56	43	30.232	9284.62
MAO	47	33	42.424	7053.12
MARQUES	51	44	15.909	8278.96
SHIRTS	43	37	16.216	1232468.21
SWIM	58	34	70.588	466799.09
TROUSERS	51	42	21.428	273121.59

5 Conclusion

Cutting and Packing problems exist almost everywhere in real world situation. In this paper, a novel heuristic approach for two-dimensional irregular cutting stock problem is presented, based on grid approximation and shape similarity. The approach is mainly drawn on techniques from computer vision and artificial intelligence and has shown its capability of finding high-quality solutions. Specifically, the advantages include the following aspects: firstly, the placement approach based on grid approximation provides the system designers with an easier way to detect whether overlap occurs. Secondly, the two-stage placement strategy improves the quality of packing pattern without compromising the computational effort. Thirdly, the packing approach based on shape similarity gets higher occupancy rate and better performance compared with conventional methods. The proposed method is assessed on 15 established benchmark problems and performs very well.

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