Chapter 10
Design of Sliding Mode Controller with Actuator Saturation

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Abstract. This chapter discusses two methods of designing a sliding surface in the face of an actuator saturation constraint for a class of nonlinear uncertain systems. The first approach uses an ARE based approach to design the sliding surface and the second approach uses the parametric Lyapunov equation to design the surface. These methods are based on the low gain approach proposed by Lin et al. The design methods give a surface matrix as a function of the designed parameter. This parameter can be modulated to reduce the control amplitude which ensures that the control limits are respected in a region of the state space. This region can be made sufficiently large by choosing appropriate values of the design parameter.

10.1 Introduction

Beginning in the late 1970s and continuing today, sliding mode control has received plenty of attention due to its insensitivity to disturbances and parameter variations. The well known *sliding mode control* is a particular type of Variable Structure Control System (VSCS). Recently many successful practical applications of sliding mode control (SMC) have established the importance of sliding mode theory which has mainly been developed in the last three decades. This fact is also witnessed by many special issues of leading journals focusing on sliding mode control ⟦2,4⟧. The research in this field was initiated by Emel'yanov and his colleagues ⟦6, 7⟧, and the design paradigm now forms a mature and an established approach for robust control and estimation. The idea of sliding mode control (SMC) was not known to the control community at large until an article published by Utkin ⟦16⟧ and a book by Itkis ⟦11⟧.

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SMC is an established method to deal with uncertainty- inevitable in most practical systems. However, for any practical systems, the input is always limited in magnitude. Therefore it is necessary to consider this limitation *a priori* while designing the SMC. Design of a first order sliding mode is done in two steps viz. design of a stable sliding surface and a control law which produces a sliding mode in finite time. To ensure that the actuator does not saturate for a given set of initial conditions, the sliding surface (switching function) design should incorporate this limitation. Some authors have contributed in this regard, Corradini and Orlando ⟦5⟧ proposed a nonlinear surface to handle actuator saturation. Bartoszewicz and Nowacka ⟦3⟧ proposed an optimal sliding surface to handle input constraints. Ferrara and Rubagotti ⟦8⟧ proposed an effective algorithm to handle saturation in the higher order sliding mode framework.

In this chapter, we present two methods to design a sliding surface by which the control magnitude can be made arbitrarily small by choosing an appropriate surface matrix. Our method is based on the low gain approach proposed in ⟦12–14, 18⟧. To enforce sliding motion, the required control input has two components, one linear and the other, discontinuous (for first order sliding mode). The discontinuous component is decided by the maximum amplitude of uncertainty therefore it does not provide any flexibility to reduce the control input to avoid actuator saturation. The linear component (linearly) depends on the sliding function matrix; flexibility to design the sliding surface matrix can be explored to avoid actuator saturation. We present two methods, based on the low gain approach as mentioned earlier, to design the surface. In the first method, the sliding surface matrix is parameterized by the parameter ε and in the second method, it is parameterized by γ. These parameters can be altered to reduce the control amplitude. The rest of the chapter is organized as follows. The work presented in this chapter is based on our work in ⟦9⟧

Section 10.2 discusses the system description and problem statement. The surface design is discussed in Section 10.3. The effect of actuator saturation is discussed in Section 10.4. Section 10.5 discusses another method to design the sliding surface based on a parameterized Lyapunov equation. To verify the design methodology a numerical example is simulated in Section 10.6 followed by concluding Section 10.7.

10.2 System Description and Problem Statement

Consider the following class of nonlinear uncertain systems

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ &\vdots \\ \dot{x}_n &= f(x,t) + b_2 sat(u(t)) \end{aligned} \tag{10.1}$$

The function $f(x,t)$ satisfies the classical condition for the existence and uniqueness. $sat(u(t))$ is a saturation function and is defined as follows

$$sat(u(t)) = sign(u(t)) \times min(u_{max}, |u|)$$

In the above equation, u_{max} is the maximum value of $u(t)$. We make the following assumptions for the above system:

Assumption 1. b_2 is a non-zero scalar.
Assumption 2. Uncertain nonlinear function $f(x,t)$ satisfies
$\|f(x,t)\| \leq R_1\|x\| + R_2 \;\; \forall x \times t \; \in R^n \times R$ here R_1 and R_2 are positive constants.
Assumption 3. In a region Σ in the state space, there exists a constant Q such that $\forall x \in \Sigma, \quad Q \geq R_1\|x\| + R_2 + \beta$ where $\beta > 0$ is a small positive constant which satisfies

$$\forall x \in \Sigma, \quad Q \leq \delta u_{max} \tag{10.2}$$

where $0 < \delta < 1$.

Assumption 3 ensures that the maximum amplitude of disturbance/uncertainty is smaller than the available control amplitude $\forall x \in \Sigma$. This assumption is necessary to enforce sliding mode.

Let the switching function for the above system be

$$s := c^T(\varepsilon)x(t) \tag{10.3}$$

Here ε is a design parameter which will be discussed later. Control input to ensure sliding mode ($s = 0$) in finite time can be defined as

$$u(t) := -(c(\varepsilon)^T B)^{-1}\{c^T(\varepsilon)Ax + Qsgn(s)\} \tag{10.4}$$

It should be noted that the region Σ in the state space is the region where stability with saturated actuator is ensured. Furthermore, we define the following matrices:

$$B := \begin{bmatrix} 0 \\ 0 \\ 0 \\ b_2 \end{bmatrix}, \quad A := \begin{bmatrix} 0 & 1 & 0 & \cdot\cdot & 0 \\ 0 & 0 & 1 & \cdot\cdot & 0 \\ : & : & : & \cdot\cdot & : \\ 0 & 0 & 0 & & 0 \end{bmatrix}$$

The objective is to design a sliding surface matrix $c(\varepsilon)^T$ such that $\forall x \in \Sigma$, the control law (10.4) respects the saturation limit and resulting closed loop system remains stable.

10.3 Design of Switching Function

Consider the following representation of the system defined in (10.1)

$$\dot{z}_1 = A_{11}z_1 + A_{12}z_2 \tag{10.5a}$$

$$\dot{z}_2 = A_{21}z_1 + A_{22}z_2 + f(x,t) + b_2u(t) \tag{10.5b}$$

Here

$$z_1 := [x_1 \; x_2 \; \ldots \; x_{n-1}]^T$$

$$z_2 := x_n$$

$$A_{11} = \begin{bmatrix} 0 & I_{n-2} \\ 0 & 0 \end{bmatrix}, \quad A_{12} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}, \quad A_{21} = [0, \cdots, 0], \quad A_{22} = 0.$$

It is convenient to design the switching function in $z-$ coordinates. The switching function in $z-$ coordinates is defined as

$$s := c^T(\varepsilon)x := \begin{bmatrix} c_1(\varepsilon) & 1 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \tag{10.6}$$

Here $c_1(\varepsilon) \in R^{1\times(n-1)}$ is to be designed.

Design of $c_1(\varepsilon)$
The control law (10.4) ensures that sliding mode $s = 0$ occurs in finite time. This leads to

$$c_1(\varepsilon)z_1 + z_2 = 0$$

$$\Rightarrow \; z_2 = -c_1(\varepsilon)z_1 \tag{10.7}$$

Using (10.5a) and (10.7)

$$\dot{z}_1 = (A_{11} - A_{12}c_1(\varepsilon))z_1 \tag{10.8}$$

$c_1(\varepsilon)$ should be designed such that the above closed loop system remains stable. $c_1(\varepsilon)$ is defined as follows

$$c_1(\varepsilon) := A_{12}^T P_1(\varepsilon) \tag{10.9}$$

$P_1(\varepsilon) > 0$ is a symmetric matrix and obtained by solving the ARE

$$A_{11}^T P_1(\varepsilon) + P_1(\varepsilon)A_{11} - P_1(\varepsilon)A_{12}A_{12}^T P_1(\varepsilon) + Q_1(\varepsilon) = 0 \tag{10.10}$$

$Q_1(\varepsilon)$ can be chosen as $Q_1(\varepsilon) = \varepsilon I$ as proposed in low gain design approach [[13, 14]].

Theorem 10.1. *For some $\varepsilon \in (0,1]$ there exists a $P_1(\varepsilon)$ which solves ARE in* (10.10) *and it satisfies*

$$\lim_{\varepsilon\to 0} c_1(\varepsilon) = 0.$$

Proof. The ARE in (10.10) is the result of minimization of the following cost function

$$J(x,u) = \frac{1}{2}\int_0^\infty [\varepsilon z_1(t)^T z_1(t) + z_2^T(t)z_2(t)]dt \tag{10.11}$$

This is a standard LQR problem and the existence of a unique positive definite $P_1(\varepsilon)$ for $\forall\varepsilon > 0$ was proved by Willems (1971) [[17]] for LTI system of the form

$$\dot{x}(t) = Ax(t) + Bu(t)$$

The pair (A,B) should be stabilizable. Replacing A by A_{11} and B by A_{12}, the existence of $P_1(\varepsilon)$ can be proved in a similar way as it is proved in Willems (1971) [[17]]. The continuity of solution i.e. $P_1(\varepsilon) \to 0$ as $\varepsilon \to 0$ was proved in [[15]]. Using (10.9) it is straightforward to infer $\lim_{\varepsilon\to 0} c_1(\varepsilon) = 0$.

Remark 10.1. *With this condition ($\lim_{\varepsilon\to 0} c_1(\varepsilon) = 0$), we can find some ε to ensure arbitrarily small norm of $c_1(\varepsilon)$.*
We need to find a region Σ such that $\forall x \in \Sigma$ implies $| u | \leq u_{max}$. Define

$$c_1(\varepsilon) := A_{12}^T P_1(\varepsilon) := \left[\, \bar{c}_1(\varepsilon)\ \bar{c}_2(\varepsilon)\ \cdot\cdot\ \bar{c}_{n-1}(\varepsilon) \,\right] \tag{10.12}$$

Here $\bar{c}_i(\varepsilon), \quad i = 1\cdots(n-1)$ are constants which depend on ε, moreover, $\lim_{\varepsilon\to 0} c_i(\varepsilon)$ $= 0$.

Remark 10.2. *It should be noted that all eigenvalues of A_{11} are at origin. Any nonzero $\varepsilon \in (0,1]$ ensures that closed loop system* (10.8) *is stable which also ensures stability of sliding surface.*

10.4 Effect of Actuator Saturation

For most of the physically realizable systems, the actuator capacity (amplitude) is limited. This requires that for a given set of initial conditions, the control should respects its boundaries. Consider the control law (10.4)

$$\begin{aligned}
u(t) &= -(b_2)^{-1}\{c(\varepsilon)^T Ax + Qsgn(s)\} \\
&= -(b_2)^{-1}\left[\, \bar{c}_1(\varepsilon)\ \bar{c}_2(\varepsilon)\ \cdot\cdot\ \bar{c}_{n-1}(\varepsilon)\ 1 \,\right] \times \\
&\qquad \begin{bmatrix} 0 & 1 & 0 & \cdot\cdot & 0 \\ 0 & 0 & 1 & \cdot\cdot & 0 \\ \vdots & \vdots & \vdots & \cdot\cdot & \vdots \\ 0 & 0 & 0 & & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} - (b_2)^{-1} Qsgn(s) \\
&= -(b_2)^{-1}\left[\, \bar{c}_1(\varepsilon)\ \bar{c}_2(\varepsilon)\ \cdot\cdot\ \bar{c}_{n-1}(\varepsilon) \,\right] \begin{bmatrix} x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix} \\
&\qquad -(b_2)^{-1} Qsgn(s) \\
&= -(b_2)^{-1} c_1(\varepsilon)\bar{x} - (b_2)^{-1} Qsgn(s)
\end{aligned} \tag{10.13}$$

Where $\bar{x} = \begin{bmatrix} x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix}$

The control law in (10.13) has two parts linear and nonlinear. The nonlinear component of the control law is decided by the maximum value of uncertainty. However, norm of linear component $c_1(\varepsilon)$ can be made arbitrarily small by choosing a small value of ε which allows to reduce the contribution of linear component to the required proportion. Considering the equation (10.2), the linear part of the control law (10.13) is limited by

$$|(b_2)^{-1}c(\varepsilon)^T\bar{x}| \le (1-\delta)u_{max} \tag{10.14}$$

Suppose Σ is the region in state space, such that $\bar{x} \in \Sigma$ implies $|\, u \,| \le (1-\delta)u_{max}$. This region can be obtained as follows [[10]].

Find

$$g := \text{Max } \bar{x}^T P_1(\varepsilon)\bar{x} \Rightarrow |(b_2)^{-1}c^T(\varepsilon)\bar{x}| \le (1-\delta)u_{max} \tag{10.15}$$

This actually has an analytic solution which can be obtained using [[10]].

$$\begin{aligned} g &= \frac{(1-\delta)^2 u_{max}^2}{(b_2)^{-1}c(\varepsilon)^T P_1(\varepsilon)^{-1}c(\varepsilon)(b_2)^{-1}} \\ &= \frac{b_2^2(1-\delta)^2 u_{max}^2}{A_{12}^T P_1(\varepsilon)A_{12}} \end{aligned} \tag{10.16}$$

Remark 10.3. *We need only initial condition of $\bar{x} \in \Sigma$ to respect saturation limits. Recall that $\bar{x} = [x_2 \; x_3 \; \cdots \; x_n]^T$, this implies that the state x_1 can take any value without affecting the control input. Therefore initial condition of state x_1 does not influence control much. This is verified through simulation example.*

Remark 10.4. *It is desirable that the region Σ should include all possible initial conditions. This region can be made arbitrarily large by choosing sufficiently small value of ε. Consider* (10.16)*, as it is discussed earlier, $\lim_{\varepsilon\to 0} P_1(\varepsilon) = 0$ therefore a small value of ε results in a large value of parameter 'g' and thus region Σ also becomes large.*

Remark 10.5. *Using control law* (10.13)*, existence of sliding mode can be easily proved using* (10.2) *in the region Σ.*

10.5 Parametric Lyapunov Based Approach to Design Switching Function

In this section, we will study another method to design switching function. This method is based on low gain approach proposed in [[18]]. Consider the minimization of cost function for the system defined in (10.5)

$$J_1(z(t)) = \int_0^\infty e^{\gamma t}(z_1^T M z_1 + z_2^T N z_2)dt \tag{10.17}$$

where $M = E^T E \geq 0$, $N > 0$ and γ is a positive scalar. Using [[18]], we have the following proposition

Proposition 10.1. *Consider the system equation* (10.5a) *and the cost function* (10.17). *Assuming pair (A_{11}, E) is detectable and the pair (A_{11}, A_{12}) stabilizable. Stabilizability of pair* (A,B) *ensures the stabilizability of pair (A_{11}, A_{12}). With this assumptions, the value of z_2 which minimizes the cost function $J_1(z(t))$ in* (10.17)

$$z_2^* = -N^{-1}A_{12}^T P_2(\gamma) z_1(t) \tag{10.18}$$

$P_2(\gamma)$ is the unique positive-definite solution of the following ARE

$$(A_{11} + \frac{\gamma}{2}I)^T P_2(\gamma) + P_2(\gamma)(A_{11} + \frac{\gamma}{2}I) - P_2(\gamma)A_{12}N^{-1}A_{12}^T P_2(\gamma) = -M \tag{10.19}$$

Closed loop system (10.5) *and* (10.18) *is exponentially stable with convergence rate faster than $e^{-\gamma/2\, t}$.*
With $M = 0$, (10.19) *becomes*

$$A_{11}^T P_2(\gamma) + P_2(\gamma)A_{11} - P_2(\gamma)A_{12}N^{-1}A_{12}^T P_2(\gamma) = -\gamma P_2(\gamma) \tag{10.20}$$

and corresponding cost function becomes

$$J_1(z_2) = \int_0^\infty e^{\gamma t} z_2^T N z_2 dt \tag{10.21}$$

It should be noted that during the sliding mode, the state z_2 behaves as an 'input' to the system and with the above cost function we are not penalizing z_1. However, the convergence rate is controlled by the parameter γ.

The ARE (10.20) has a unique positive definite solution

$$P_2(\gamma) = W_2^{-1}(\gamma) \tag{10.22}$$

$W_2 > 0$ is obtained by solving the following Lyapunov equation

$$W_2(A_{11} + \frac{\gamma}{2}I)^T + (A_{11} + \frac{\gamma}{2}I)W_2 = A_{12}N^{-1}A_{12}^T \tag{10.23}$$

the above equation can be easily obtained by rearranging (10.20).
z_2 which minimizes (10.21)

$$z_2 = -N^{-1}A_{12}^T P_2(\gamma) z_1(t) \tag{10.24}$$

By using (10.24), sliding surface matrix can be obtained as

$$c^T(\gamma) = \left[N^{-1}A_{12}^T P_2(\gamma) \ 1 \right] \tag{10.25}$$

During the sliding mode $s = 0$, and the resulting closed loop system becomes

$$\dot{z}_1 = (A_{11} - A_{12}N^{-1}A_{12}^T P_2(\gamma))z_1 \tag{10.26}$$

To prove stability of sliding surface, we need to prove stability of the above closed loop system for $\gamma > 0$.

Theorem 10.2. *All the eigenvalues of the closed loop system in* (10.26) *have negative real parts, thus the system is stable.*

Proof. Consider (10.20)

$$\begin{aligned} A_{11}^T P_2(\gamma) + P_2(\gamma)A_{11} - P_2(\gamma)A_{12}N^{-1}A_{12}^T P_2(\gamma) &= -\gamma P_2(\gamma) \\ \Rightarrow P_2^{-1}(\gamma)A_{11}^T P_2(\gamma) + A_{11} - A_{12}N^{-1}A_{12}^T P_2(\gamma) &= -\gamma I \\ \Rightarrow A_{11} - A_{12}N^{-1}A_{12}^T P_2(\gamma) &= P_2^{-1}(\gamma)(-A_{11}^T - \gamma I)P_2(\gamma) \end{aligned}$$

It is clear from the above equation that the matrices $A_{11} - A_{12}N^{-1}A_{12}^T P_2(\gamma)$ and $-A_{11}^T - \gamma I$ are similar matrices and so have the same eigenvalues. It should be noted that all $n-1$ eigenvalues of A_{11} are located at the origin. Any nonzero positive value of γ ensures that eigenvalues of $-A_{11}^T - \gamma I$ have negative real part and thus stability of sliding surface is proved.

Remark 10.6. *When eigenvalues of A_{11} are located anywhere in the s-plane, the scalar γ should be selected such that* [1, 18]

$$\gamma > -2min\{Re(\lambda(A_{11}))\} \tag{10.27}$$

where $Re(\lambda(A_{11}))$ denotes real part of eigenvalue of A_{11}.

Remark 10.7. *The matrix $P_2(\gamma)$ is differentiable and monotonically increasing with respect to γ,* [1, 18]

$$\frac{dP_2(\gamma)}{d\gamma} > 0 \tag{10.28}$$

By choosing appropriate value of γ, norm of matrix $P_2(\gamma)$ can be chosen sufficiently small and, as we discussed in the previous section, we can limit linear part of the control. This property is needed to prove existence of sliding mode with saturated actuator in a region of state space.

With this method we can use parameter γ in a similar way as we used the parameter ε with the ARE based method. However, it should be noted that $\varepsilon \in (0,1]$ while γ can be any positive value for the given system.

10.6 Simulation Studies

In this section, we will simulate a fourth order nonlinear uncertain system. We will design surface only by one method i.e. ARE based method. The following parame-

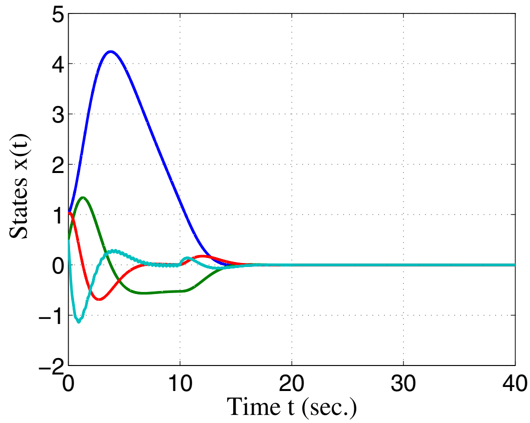

Fig. 10.1 Plot of states for Case-I

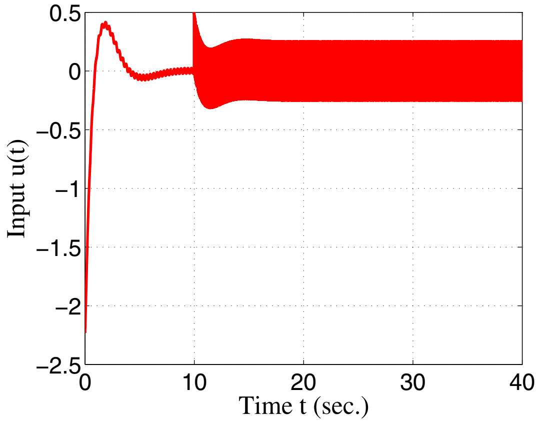

Fig. 10.2 Input for Case-I

ters are taken for the system in (10.1) $f(x,t) = 0.4sint(t)$, $b_2 = 2$, $u_{max} = 2.5$. The maximum value of $f(x,t)$ is 0.4, therefore parameter Q in control law is chosen as $Q = 0.5$ which satisfies (10.2). Linear part of the control depends on sliding surface matrix $c(\varepsilon)^T$. Linear part of the control can be reduced by choosing appropriate value of ε. The maximum possible value of linear part becomes 2. We design and simulate the system with two different values of ε parameter.

Case I. $\varepsilon = 0.9$
Solving ARE in (10.10) gives

$$P_1 = \begin{bmatrix} 2.1969 & 2.2312 & 0.9487 \\ 2.2312 & 4.4975 & 2.3157 \\ 0.9487 & 2.3157 & 2.3519 \end{bmatrix} \tag{10.29}$$

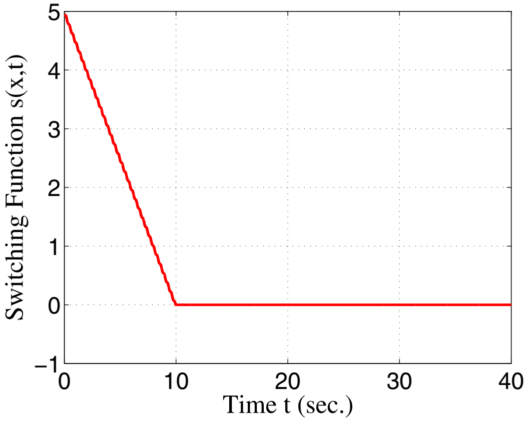

Fig. 10.3 Plot of switching function for Case-I

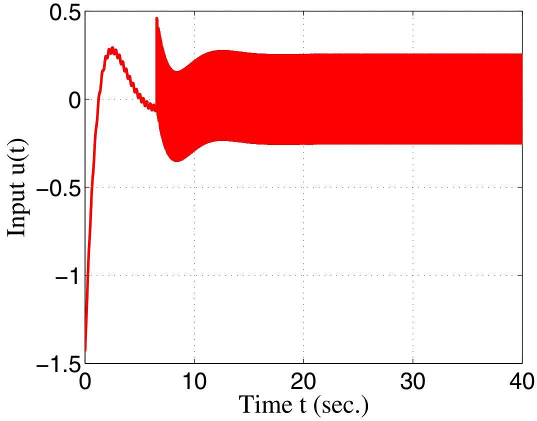

Fig. 10.4 Input for Case-II

Thus surface matrix becomes

$$c^T(\varepsilon) = \begin{bmatrix} 0.9487 \ 2.3157 \ 2.3519 \ 1.0000 \end{bmatrix} \tag{10.30}$$

With initial condition $x(0) = \begin{bmatrix} 1 \ 0.5 \ 1 \ 0.5 \end{bmatrix}^T$ the system is simulated with Runge-Kutta 4th order algorithm with maximum sampling time 0.001 sec. Fig. 10.2 shows input. Fig 10.1 shows evolution of states and Fig. 10.3 shows evolution of switching function with time. It is evident that system is stable and sliding mode establishes at $t = 10sec.$ and thereafter system remains in sliding mode.

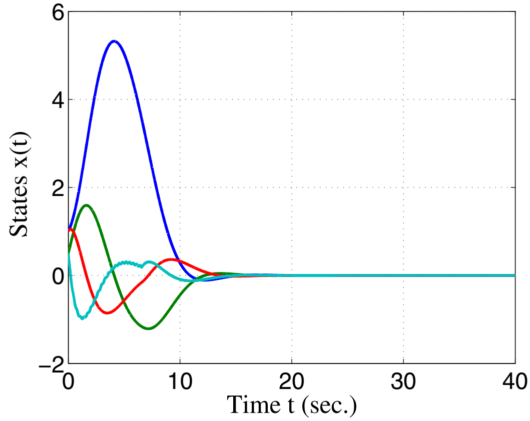

Fig. 10.5 Plot of states for Case-II

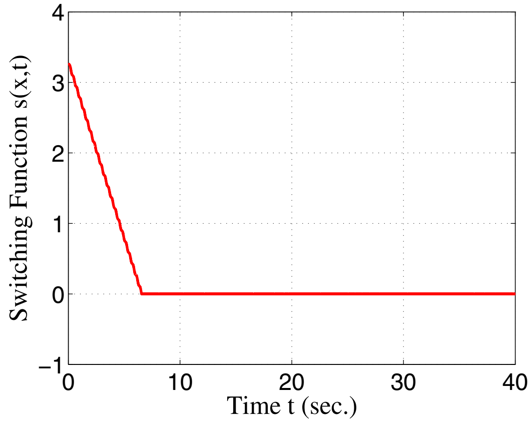

Fig. 10.6 Plot of switching function for Case-II

Case II. $\varepsilon = 0.2$

By proceeding in a similar way as we did in Case I, we obtain

$$P_1 = \begin{bmatrix} 0.5828 & 0.7492 & 0.4472 \\ 0.7492 & 1.7360 & 1.3032 \\ 0.4472 & 1.3032 & 1.6752 \end{bmatrix} \tag{10.31}$$

Thus surface matrix becomes

$$c^T(\varepsilon) = \begin{bmatrix} 0.4472 & 1.3032 & 1.6752 & 1.0000 \end{bmatrix} \tag{10.32}$$

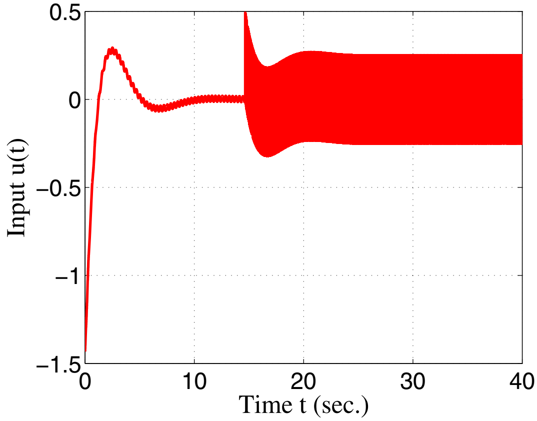

Fig. 10.7 Input for Case-III

The system is simulated using the same numerical algorithm as in Case I. Fig. 10.4 shows control input with $\varepsilon = 0.2$ and comparing it with Fig. 10.2 verifies that control input reduces as ε is reduced. Figs. 10.5, 10.6 show the states and the evolution of the switching function respectively.

Case III
The value of initial condition of x_1 state is increased by 10 fold and by keeping the same value of $\varepsilon = 0.2$. This case is simulated with initial condition $x(0) = \begin{bmatrix} 10 & 0.5 & 1 & 0.5 \end{bmatrix}^T$ Fig. 10.7 shows the control input plot and comparing it with Fig. 10.4 it is evident that influence of x_1 on thecontrol input is negligible which agrees with the discussion in Remark 2.

10.7 Conclusion

The sliding surface design to handle actuator saturation has been presented for a class of nonlinear system. The control amplitude can be controlled by a parameter ε. It has been observed that the control amplitude is not affected significantly by the value of the state x_1. Simulation studies verify the theoretical claims.

References

1. Amin, M.H.: Optimal pole shifting for continuous multivariable linear systems. International Journal of Control 41(3), 701–707 (1985)
2. Bartoszewicz, A., Kaynak, O., Utkin, V.I.: Special section on sliding mode control in industrial applications. IEEE Transactions on Industrial Electronics 55(11), 3806–4074 (2008)

3. Bartoszewicz, A., Nowacka-Leverton, A.: Itae optimal sliding modes for third-order systems with input signal and state constraints. IEEE Transactions on Automatic Control 55(8), 1928–1932 (2010)
4. Bartoszewicz, A., Patton, R.J.: Sliding mode control: Special issue. International Journal of Adaptive Control and Signal Processing 21, 635–822 (2007)
5. Corradini, M.L., Orlando, G.: Linear unstable plants with saturating actuators: Robust stabilization by a time varying sliding surface. Automatica 43, 88–94 (2007)
6. Emel'yanov, S.V.: Method of designing complex control algorithm using an error and its first time derivative only. Automation and Remote Control 18(10) (1957) (in Russian)
7. Emel'yanov, S.V., Burovoi, I.A., et al.: Mathematical models of process in technology and development of variable structure control system. Metallurgy (Moscow) 18(07) (1964) (in Russian)
8. Ferrara, A., Rubagotti, M.: A sub-optimal second order sliding mode controller for systems with saturating actuators. IEEE Transactions on Automatic Control 54(5), 1082–1087 (2009)
9. Fulwani, D., Bandyopadhyay, B.: Sliding surface design with saturated actuator. In: 2012 12th International Workshop on Variable Structure Systems (VSS), pp. 485–490 (2012)
10. Henrion, D., Garcia, G., Tarbouriech, S.: Piecewise linear robust control of systems with input saturation. European Journal of Control 5, 157–166 (1999)
11. Itkis, U.: Control Systems of Variable Structure. Wiley, New York (1976)
12. Lin, Z., Saberi, A.: Semi-global exponential stabilization of linear discrete-time systems subject to input saturation via linear feedbacks. Systems and Control Letters 24(2), 125–132 (1995)
13. Lin, Z., Stoorvogels, A.A., Saberi, A.: Output regulation for linear systems subject to input saturation. Automatica 32(1), 29–47 (1996)
14. Teel, A.: Semi-global stabilizability of linear null controllable systems with input nonlinearities. IEEE Transactions on Automatic Control 40(1), 96–100 (1995)
15. Trentelman, H.: Families of linear-quadratic problems: Continuity properties. IEEE Transactions on Automatic Control 32(4), 323–329 (1987)
16. Utkin, V.I.: Variable structure system with sliding mode. IEEE Transaction on Automatic Control 22(2), 212–221 (1977)
17. Willems, J.: Least squares stationary optimal control and the algebraic riccati equation. IEEE Transactions on Automatic Control 16(6), 621–634 (1971)
18. Zhou, B., Duan, G., Lin, Z.: A parametric lyapunov equation approach to the design of low gain feedback. IEEE Transaction on Automatic Control 53, 1548–1554 (2008)