An Economic Production Quantity Problem with Fuzzy Backorder and Fuzzy Demand

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Abstract. Optimization models based on fuzzy set theory are relevant to the process industry, where there are many uncertainties that are inherently fuzzy. In this paper, we incorporate backorders (i.e the inventory to go below zero) and cycle time in a fuzzy Economic Production Quantity (EPQ) model. The uncertainties in the backorders and in the demand for different products are modeled using triangular possibility distributions. We illustrate the model with an example that describes a typical decision making problem in the paper industry.

Keywords: Economic Production Quantity, Fuzzy Sets, Signed Distance, Production optimization, Supply Chain Management.

1 Introduction

The contemporary business environment has increased the strategic importance of Decision Support Systems (DSS) in improving the efficiency of a company. In the optimal situation, the decision making steps and procedures and the supporting systems should fit the specific needs of the industry under consideration to identify the "optimal" (most efficient) decisions in different (business) processes instead of using some very generic models. In this paper, we will specifically focus on the specific features and needs of production-inventory problems. According to the APICS dictionary [1], process industries are businesses that add value to materials by mixing, separating, forming, or triggering chemical reactions.

Process industries account for a significant portion of the Gross Domestic Product (GDP) in most of the countries. For example in Finland, manufacturing (17.3 % of the GDP in 2011 [2]) is the second most important sector of the economy after services and the key sector considering foreign trades. The key branches [3] include pulp and paper industry (9.3 %), electronic products (9.8 %), and chemicals and chemical products (5.6 %).

Process industry in the Nordic countries is facing severe challenges. To meet up with the added logistics costs, the production efficiency needs to be extremely high. But not even that is enough and therefore, new ways of looking at the supply chain is needed. Production and distribution planning constitute very important task in the supply chain. Traditional methods, like the EOQ (Economic Order Quantity) model

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first presented by [4] and its extensions still play an important role in the decision making processes in present days. On the other hand, many schemes, like the Bullwhip counter-acting models, cannot be introduced unaltered in the process industries [5,6].

Finding the answer to the "when" and "how much" questions in different industries is always a difficult task when the uncertainty present in the processes is significant. Uncertainties often stem from different aspects in both production and markets. These uncertainties can sometimes be captured with probabilistic measures, but quite often they derive from expert opinions in the production planning or marketing departments. Possibilistic measures and fuzzy EOQ-models provide an appropriate tool to handle these uncertainties in many applications [7].

The uncertainties in the production-inventory management decisions are today taken into consideration as a standard procedure. In the EOQ literature already [8] started to use probabilistic measures for EOQ models with backorders. However, the context often calls out for a possibilistic way of dealing with the uncertainties [9,10]. Chang et al. [11] were the first to solve a fuzzy EOQ model with backorders. They solved the model numerically, however. [9] and [12] solved the same model analytically using the signed distance defuzzification method [13]. Yao et al. [14] introduced an EOQ-model, without backorders, but for two replaceable merchandizes. [15] used the signed distance method for a fuzzy demand EOQ-model without backorders. In the references above, none account for a model, where the production rate is finite (i.e. models for producing entities). However [16] and [17] solved a fuzzy EPQ problem, where the entire cycle times are assumed to be fuzzy.

However, none of the above has allowed for backorders. [18] considered a EOQ problem with one item, backorders and fuzzy parameters. One of the first applications of fuzzy theory in EPQ models is [19], where the authors solve the EPQ model with fuzzy demand and production quantity. Bag et al. [20] introduce a model with inventory model with flexibility and reliability (of production process) consideration; the demand is modeled as a fuzzy random variable. The authors in [21] consider the EPQ problem with backorder with the setup cost, the holding cost and the backorder cost as fuzzy variables.

This paper presents a multi-item (one machine) EPQ (Economic Production Quantity) model with fuzzy backorders and demand values. It is an extension of the model in [17]. As analytical solution to the problem cannot be obtained, the optimal values are determined numerically. The rest of the paper is structured as follows. We describe the crisp and fuzzy models and check the convexity of the defuzzified version in Section 2. Section 3 provides an illustrative example using a typical decision making problem from paper industry. Finally, conclusions and future research directions are discussed in Section 4.

2 The Model

In this section, we are first going to present the crisp model, and the fuzzy model along with some basic assumptions and a suitable defuzzification strategy. Finally, we are going to solve the defuzzified optimization problem to global. The parameters and variables (can be assumed strictly greater than zero) in the classical multi-item EPQ model with shared cycle time and backorders are the following (where the index $i \in I = \{1,2,...,|I|\}$ denotes the products):

- Q_i is the production batch size (variable)
- K_i is the fixed cost per production batch (parameter)
- D_i is the annual demand of the product (parameter)
- B_i is maximum shortage (just after a production run starts, variable)
- b_i is the unit shortage penalty cost per year (parameter)
- h_i is the unit holding cost per year (parameter)
- T is the cycle time (variable)
- P_i is the annual production rate (parameter)

The total cost function (TCU), including production setup costs and inventory holding costs for all products, is given by

$$TCU(Q_1,...,Q_n,B_1,...,B_n) = \sum_{i=1}^{n}\frac{K_iD_i}{Q_i} + \sum_{i=1}^{n}\frac{B_i^2b_i}{2Q_i\rho_i} + \sum_{i=1}^{n}\frac{h_i(Q_i\rho_i - B_i)^2}{2Q_i\rho_i} \tag{1}$$

where $\rho_i = 1 - D_i/P_i$. The production batch size Q_i can also be replaced with the cycle time T according the formula Q_i=TD_i . The insertion of this formula into Eq. (1) yields the total cost function to minimize

$$TCU(T,B_1,...,B_n) = \sum_{i=1}^{n}\frac{K_i}{T} + \sum_{i=1}^{n}\frac{B_i^2(b_i+h_i)}{2TD_i\rho_i} + \sum_{i=1}^{n}\frac{Th_iD_i\rho_i}{2} - \sum_{i=1}^{n}B_ih_i \tag{2}$$

Eq. (2) is one version of the crisp (classical) multi-item EOQ-model with shared production capacity, cycle time and backorders. This problem can be solved using the derivatives, since all the terms in Eq. (2) are convex.

In order to present the fuzzy model, we will start by assuming that the cycle time is uncertain but it is possible to describe it with a triangular fuzzy number (symmetric).

Definition 1. Consider the fuzzy set $\tilde{A} = (a,b,c)$ where $a < b < c$ and defined on R, which is called a triangular fuzzy number, if the membership function of $\tilde{A}$ is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x-a}{b-a} & a \le x \le b \\ \dfrac{c-x}{c-b} & b \le x \le c \\ 0 & otherwise \end{cases}$$

The fuzzy shortage $\tilde{B}_i$ and fuzzy demand $\tilde{D}_i$ will then be $\tilde{B}_i = (B_i - \Delta_i, B_i, B_i + \Delta_i)$ and $\tilde{D}_i = (D_i - \Lambda_i, D_i, D_i + \Lambda_i)$. In order to find non-fuzzy values for the model, we need to use some distance measures, and as in [11] we will use the signed distance [13].

Before the definition of this distance, we need to introduce the concept of α-cut of a fuzzy set.

Definition 2. Let $\tilde{B}$ be a fuzzy set on R and $0 \le \alpha \le 1$. The α-cut of $\tilde{B}$ is the set of all the points x such that $\mu_{\tilde{B}}(x) \ge \alpha$, i.e. $\tilde{B}(\alpha) = \{x | \mu_{\tilde{B}}(x) \ge \alpha\}$.

Let Ω be the family of all fuzzy sets $\tilde{B}$ defined on R for which the α-cut $\tilde{B}(\alpha) = \left[\tilde{B}_l(\alpha), \tilde{B}_u(\alpha)\right]$ exists for every $0 \le \alpha \le 1$, and both $\tilde{B}_l(\alpha)$ and $\tilde{B}_u(\alpha)$ are continuous functions on $\alpha \in [0,1]$.

Definition 3. For $\tilde{B} \in \Omega$ define the signed distance of $\tilde{B}$ to $\tilde{0}$ as

$$d(\tilde{B}, \tilde{0}) = \frac{1}{2}\int_0^1 \left[\tilde{B}_l(\alpha) + \tilde{B}_u(\alpha)\right] d\alpha$$

The Total Annual Cost in the fuzzy sense will be

$$TCU(T, \tilde{B}_1, ..., \tilde{B}_n) = \sum_{i=1}^{n} \frac{K_i}{T} + \sum_{i=1}^{n} \frac{\tilde{B}_i^2 (b_i + h_i)}{2T\tilde{D}_i \rho_i} + \sum_{i=1}^{n} \frac{Th_i \tilde{D}_i \rho_i}{2} - \sum_{i=1}^{n} \tilde{B}_i h_i \tag{3}$$

The signed distance between TCU and $\tilde{0}$ is given by

$$TCU(T, \tilde{B}_1, ..., \tilde{B}_n) = \sum_{i=1}^{n} \frac{K_i}{T} + \sum_{i=1}^{n} \frac{(b_i + h_i)}{2T\rho_i} d(\tilde{B}_i^2 / \tilde{D}_i, \tilde{0}) + \sum_{i=1}^{n} \frac{h_i T \rho_i}{2} d(\tilde{D}_i, \tilde{0}) - \sum_{i=1}^{n} h_i d(\tilde{B}_i, \tilde{0}) \tag{4}$$

If we calculate the signed distances, we obtain that

$$d(\tilde{B}_i, \tilde{0}) = \frac{1}{2}\int_0^1 \left[(\tilde{B}_i)_l(\alpha) + (\tilde{B}_i)_u(\alpha)\right] d\alpha = \frac{1}{2}\int_0^1 [(B_i - \Delta_i + \Delta_i \alpha) + (B_i + \Delta_i - \Delta_i \alpha)] d\alpha = B_i \tag{5}$$

$$d(\tilde{D}_i, \tilde{0}) = \frac{1}{2}\int_0^1 \left[(\tilde{D}_i)_l(\alpha) + (\tilde{D}_i)_u(\alpha)\right] d\alpha = \frac{1}{2}\int_0^1 [(D_i - \Lambda_i + \Lambda_i \alpha) + (D_i + \Lambda_i - \Lambda_i \alpha)] d\alpha = D_i \tag{6}$$

$$\begin{aligned} d(\tilde{B}_i^2 / \tilde{D}_i, \tilde{0}) &= \frac{1}{2}\int_0^1 \left[(\tilde{B}_i^2 / \tilde{D}_i)_l(\alpha) + (\tilde{B}_i^2 / \tilde{D}_i)_u(\alpha)\right] d\alpha = \frac{1}{2}\int_0^1 \left[\frac{(B_i - \Delta_i + \Delta_i \alpha)^2}{(D_i - \Lambda_i + \Lambda_i \alpha)} + \frac{(B_i + \Delta_i - \Delta_i \alpha)^2}{(D_i + \Lambda_i - \Lambda_i \alpha)}\right] d\alpha = \\ &-\frac{2B_i \Delta_i}{\Lambda_i} - \frac{D_i \Delta_i^2}{\Lambda_i^2} + \ln\left(\frac{D_i + \Lambda_i}{D_i - \Lambda_i}\right)\left[\frac{B_i^2 \Delta_i}{\Lambda_i} + \frac{B_i D_i \Delta_i}{\Lambda_i^2} + \frac{D_i^2 \Delta_i}{\Lambda_i^3}\right] \end{aligned} \tag{7}$$

The defuzzified total cost function is

$$\begin{aligned} TCU &= \sum_{i=1}^{n} \frac{K_i}{T} + \sum_{i=1}^{n} \frac{Th_i D_i \rho_i}{2} - \sum_{i=1}^{n} B_i h_i - \sum_{i=1}^{n} \frac{B_i \Delta_i (b_i + h_i)}{T\rho_i \Lambda_i} - \sum_{i=1}^{n} \frac{D_i \Delta_i^2 (b_i + h_i)}{2T\Lambda_i^2 \rho_i} \\ &+ \sum_{i=1}^{n} \frac{(b_i + h_i)}{2T\rho_i} \ln\left(\frac{D_i + \Lambda_i}{D_i - \Lambda_i}\right)\left[\frac{B_i^2 \Delta_i}{\Lambda_i} + \frac{B_i D_i \Delta_i}{\Lambda_i^2} + \frac{D_i^2 \Delta_i}{\Lambda_i^3}\right] \end{aligned} \tag{8}$$

To find the optimal solutions of this problem, first we have to examine the convexity of the defuzzified cost function. For this the Hessian matrix of the second derivatives needs to be computed. We will calculate the derivatives for a fixed *i*, to check under which conditions will be the terms in Eq. (8) convex. The Hessian matrix of the following function needs to be calculated:

$$
\begin{aligned}
f(T,B_i) &= \frac{K_i}{T}+\frac{Th_iD_i\rho_i}{2}-B_ih_i-\frac{B_i\Delta_i(b_i+h_i)}{T\rho_i\Lambda_i}-\frac{D_i\Delta_i^2(b_i+h_i)}{2T\rho_i\Lambda_i^2}\\
&+\frac{(b_i+h_i)}{2T\rho_i}\ln\left(\frac{D_i+\Lambda_i}{D_i-\Lambda_i}\right)\left[\frac{B_i^2\Delta_i}{\Lambda_i}+\frac{B_iD_i\Delta_i}{\Lambda_i^2}+\frac{D_i^2\Delta_i}{\Lambda_i^3}\right]
\end{aligned} \tag{9}
$$

The partial derivatives are the following:

$$
\begin{aligned}
\frac{\partial f}{\partial T} &= -\frac{K_i}{T^2}+\frac{h_iD_i\rho_i}{2}+\frac{B_i\Delta_i(b_i+h_i)}{T^2\rho_i\Lambda_i}+\frac{D_i\Delta_i^2(b_i+h_i)}{2T^2\rho_i\Lambda_i^2}\\
&-\frac{(b_i+h_i)}{2T^2\rho_i}\ln\left(\frac{D_i+\Lambda_i}{D_i-\Lambda_i}\right)\left[\frac{B_i^2\Delta_i}{\Lambda_i}+\frac{B_iD_i\Delta_i}{\Lambda_i^2}+\frac{D_i^2\Delta_i}{\Lambda_i^3}\right]
\end{aligned} \tag{10}
$$

$$
\frac{\partial f}{\partial B_i} = -h_i-\frac{\Delta_i(b_i+h_i)}{T\rho_i\Lambda_i}+\frac{(b_i+h_i)}{2T\rho_i}\ln\left(\frac{D_i+\Lambda_i}{D_i-\Lambda_i}\right)\left[\frac{2B_i\Delta_i}{\Lambda_i}+\frac{D_i\Delta_i}{\Lambda_i^2}\right] \tag{11}
$$

$$
\begin{aligned}
\frac{\partial^2 f}{\partial T^2} &= \frac{2K_i}{T^3}-\frac{2B_i\Delta_i(b_i+h_i)}{T^3\rho_i\Lambda_i}-\frac{D_i\Delta_i^2(b_i+h_i)}{T^3\rho_i\Lambda_i^2}\\
&+\frac{(b_i+h_i)}{T^3\rho_i}\ln\left(\frac{D_i+\Lambda_i}{D_i-\Lambda_i}\right)\left[\frac{B_i^2\Delta_i}{\Lambda_i}+\frac{B_iD_i\Delta_i}{\Lambda_i^2}+\frac{D_i^2\Delta_i}{\Lambda_i^3}\right]
\end{aligned} \tag{12}
$$

$$
\frac{\partial^2 f}{\partial B_i^2} = \ln\left(\frac{D_i+\Lambda_i}{D_i-\Lambda_i}\right)\left[\frac{\Delta_i(b_i+h_i)}{T\rho_i\Lambda_i}\right] \tag{13}
$$

$$
\frac{\partial^2 f}{\partial B_i\partial T} = \frac{\Delta_i(b_i+h_i)}{T^2\rho_i\Lambda_i}-\frac{B_i(b_i+h_i)}{T^2\rho_i\Lambda_i}\ln\left(\frac{D_i+\Lambda_i}{D_i-\Lambda_i}\right)-\frac{D_i\Delta_i(b_i+h_i)}{2T^2\rho_i\Lambda_i^2}\ln\left(\frac{D_i+\Lambda_i}{D_i-\Lambda_i}\right) \tag{14}
$$

In Eq. (13) all the terms are non-negative which implies that the first principal minor of the matrix is non-negative. To check the second determinant, we need to calculate

$$
\frac{\partial^2 f}{\partial B_i^2}\frac{\partial^2 f}{\partial T^2}-\left(\frac{\partial^2 f}{\partial B_i\partial T}\right)^2
$$

and check when this expression is non-negative:

$$\frac{\partial^2 f}{\partial B_i^2}\frac{\partial^2 f}{\partial T^2}=\left[\frac{\Delta_i^2(b_i+h_i)^2}{T^4\rho_i^2\Lambda_i^2}\right]\Bigg(\frac{2K_i\rho_i\Lambda_i}{\Delta_i(b_i+h_i)}\ln\left(\frac{D_i+\Lambda_i}{D_i-\Lambda_i}\right)-2B_i\ln\left(\frac{D_i+\Lambda_i}{D_i-\Lambda_i}\right)$$
$$-\frac{D_i\Delta_i}{\Lambda_i}\ln\left(\frac{D_i+\Lambda_i}{D_i-\Lambda_i}\right)+B_i^2\left(\ln\left(\frac{D_i+\Lambda_i}{D_i-\Lambda_i}\right)\right)^2+\frac{D_iB_i}{\Lambda_i}\left(\ln\left(\frac{D_i+\Lambda_i}{D_i-\Lambda_i}\right)\right)^2+\frac{D_i^2\Delta_i}{\Lambda_i^2}\left(\ln\left(\frac{D_i+\Lambda_i}{D_i-\Lambda_i}\right)\right)^2\Bigg) \tag{15}$$

and

$$\left(\frac{\partial^2 f}{\partial B_i\partial T}\right)^2=\left[\frac{\Delta_i^2(b_i+h_i)^2}{T^4\rho_i^2\Lambda_i^2}\right]\Bigg(1-2B_i\ln\left(\frac{D_i+\Lambda_i}{D_i-\Lambda_i}\right)-\frac{D_i}{\Lambda_i}\ln\left(\frac{D_i+\Lambda_i}{D_i-\Lambda_i}\right)$$
$$+B_i^2\left(\ln\left(\frac{D_i+\Lambda_i}{D_i-\Lambda_i}\right)\right)^2+\frac{D_iB_i}{\Lambda_i}\left(\ln\left(\frac{D_i+\Lambda_i}{D_i-\Lambda_i}\right)\right)^2+\frac{D_i^2}{4\Lambda_i^2}\left(\ln\left(\frac{D_i+\Lambda_i}{D_i-\Lambda_i}\right)\right)^2\Bigg) \tag{16}$$

After some calculations one can obtain that the second determinant is non-negative if the following inequality holds:

$$\frac{2K_i\rho_i\Lambda_i}{\Delta_i(b_i+h_i)}\ln\left(\frac{D_i+\Lambda_i}{D_i-\Lambda_i}\right)+\frac{D_i(1-\Delta_i)}{\Lambda_i}\ln\left(\frac{D_i+\Lambda_i}{D_i-\Lambda_i}\right)+\frac{D_i^2(\Delta_i-0.25)}{\Lambda_i^2}\left(\ln\left(\frac{D_i+\Lambda_i}{D_i-\Lambda_i}\right)\right)^2\geq 1 \tag{17}$$

Since this formula depends only on the parameters, before applying the model, one needs to check whether Eq. (17) is satisfied for every $i=1,\ldots,n$. Therefore, to obtain the minimum of Eq. (9), the system of *(n+1)* equations to be solved is the following:

$$\frac{\partial TCU}{\partial T}=0,\qquad \frac{\partial TCU}{\partial B_i}=0,\forall i=1,\ldots,n.$$

If we look at the derivatives, we can formulate the equations for T:

$$\frac{\partial TCU}{\partial T}=-\sum_{i=1}^{n}\frac{K_i}{T^2}+\sum_{i=1}^{n}\frac{h_iD_i\rho_i}{2}+\sum_{i=1}^{n}\frac{B_i\Delta_i(b_i+h_i)}{T^2\rho_i\Lambda_i}+\sum_{i=1}^{n}\frac{D_i\Delta_i^2(b_i+h_i)}{2T^2\rho_i\Lambda_i^2}$$
$$-\sum_{i=1}^{n}\frac{(b_i+h_i)}{2T^2\rho_i}\ln\left(\frac{D_i+\Lambda_i}{D_i-\Lambda_i}\right)\left[\frac{B_i^2\Delta_i}{\Lambda_i}+\frac{B_iD_i\Delta_i}{\Lambda_i^2}+\frac{D_i^2\Delta_i}{\Lambda_i^3}\right] \tag{18}$$

and for $B_i,\forall i=1,\ldots,n$:

$$\frac{\partial TCU}{\partial B_i}=-h_i-\frac{\Delta_i(b_i+h_i)}{T\rho_i\Lambda_i}+\frac{(b_i+h_i)}{2T\rho_i}\ln\left(\frac{D_i+\Lambda_i}{D_i-\Lambda_i}\right)\left[\frac{2B_i\Delta_i}{\Lambda_i}+\frac{D_i\Delta_i}{\Lambda_i^2}\right] \tag{19}$$

Hence the (numerical) solution of this system of equations will give us the optimal value of T and $B_i,\forall i=1,\ldots,n$:. In the next section, we will calculate the solution for a numerical example.

3 Numerical Example

In this section we illustrate the model with a numerical example. This problem is a fictive one, even if the numbers are in the likely range of a real Finnish paper producer. The problem consists of 8 products, where the an annual demand (D_i) of products are 1200 tons, 1100 tons, 1600 tons, 1100 tons, 1000 tons, 1500 tons, 1200 tons and 1700 tons respectively. The production rates (P_i) are 2900, 2700, 2400, 2800, 3000, 2500, 2100, 2500 tons / year. There is a fixed cost incurring each time a product starts to be produced (setup costs, K_i): 1200, 1100, 800, 1500, 700, 1200, 900, and 1300 euro respectively. The holding costs are 0.8 euro per kg and annum for each product and the unit shortage costs are 1.5 euro per kg and annum for each product. The Δ_i-parameters in the fuzzy case are assumed to be 5% for every product and the Λ_i parameters representing the uncertainty in the demand are specified in this example as 10\% of the demand values. All the requirements for the problem to be convex according to Eq. (17) are satisfied. The optimal solutions for the crisp and fuzzy case are given in Table 1 (the optimal values are obtained using Excel Solver optimization package).

Table 1. Results for the test example

	Crisp	Fuzzy
T	**2,6138**	**2,2093**
B_1	**639,51**	**389,43**
B_2	**592,64**	**382,33**
B_3	**484,89**	**364,93**
B_4	**607,20**	**384,56**
B_5	**606,12**	**384,39**
B_6	**545,43**	**374,94**
B_7	**467,59**	**361,94**
B_8	**494,57**	**366,58**
TCU	**6656,95**	**7500,40**

From Table 1, it can be seen that the cycle time decreases from 2,61 to 2,21 days if the possibility distribution in the cycle time is correctly accounted for. The optimal total cost is increased from 6656,95 to 7500,40 (12,7% increase). Each of the backorders decreased significantly. The increase in the total cost comes mainly from the smaller batch sizes and the uncertainty incorporated in the demand and the backorders.

To analyze the results through sensitivity analysis, we have to examine the optimal solutions and the value of the total cost function for different initial values of the parameters. In this example two important parameters were investigated:

- the b_i values (the unit shortage penalty cost),
- the h_i values (the unit holding cost).

As for the unit shortage penalty costs, the results are listed in Table 2. The results show that the total cost value increases as we increase the unit shortage penalty cost as it is expected, but the difference in the values is not significant. On the other hand we can observe that the optimal cycle time decreases with the penalty cost: if the penalty cost increases, the optimal decision is to avoid shortages as efficiently as possible; this can be achieved by changing the products in the machines more frequently.

The results of the simple sensitivity analysis concerning the unit holding cost can be found in Table 3. As it is clear from the comparison of the crisp and the fuzzy optimal solution and total cost function, if we increase the holding cost in the Economic Production Quantity model, the total cost value is increased significantly more than for the shortage cost. The results in Table 3 reflect this observation: the total cost is an increasing function of the imprecision. On the other hand, the cycle time is much shorter: because of the high holding cost, it is very expensive to produce extra products.

Table 2. The results for different values b_i

	b_i	T	TCU
1	3	4,9848	4965,42
2	2,5	4,6808	4820,49
3	2	4,4157	4696,92
4	1	4,0339	4523,96
5	0,5	3,9324	4479,04

Table 3. The results for different values of h_i

	h_i	T	TCU
1	1,20	4,6960	4801,23
2	1,22	4,6897	4809,09
3	1,25	4,6808	4820,49
4	1,28	4,6722	4831,42
5	1,30	4,6667	4838,47

4 Summary and Future Research Directions

In Supply Chain Management (SCM), track of research that tries to increase collaboration in these supply chains (in order to reduce the Bullwhip effect, for instance), there is a track to create fundamental and generic models for each part of the supply chain. For this track of research, the uncertainties involved should not be neglected. Some of these uncertainties are captured through possibilistic measures based on expert opinions. A promising research direction in this context has been the fuzzy EPQ (Economic Production Quantity) development.

This paper contributes to the track of fuzzy EPQ-theory with a model that takes fuzzy backorders and demands together with cycle times into consideration along with multi-item EPQ (Economic Production Quantity) features. The uncertainty in backorders and demands originates from the imprecise information about different decision process components. The fuzzy model was defuzzified using the signed distance method. The system of equations needed to be solved numerically. The problem was illustrated with a small test example along with some simple sensitivity analysis. This analysis showed that results from the uncertainties in the demand have significant impact on the overall annual cost function value. Future research directions will introduce different types of fuzzy numbers and defuzzification methods in the model.

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