Deriving Weights from Group Fuzzy Pairwise Comparison Judgement Matrices

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Abstract. Several Multi-Criteria Decision Making (MCDM) methods involve pairwise comparisons to obtain the preferences of decision makers (DMs). This paper proposes a fuzzy group prioritization method for deriving group priorities/weights from fuzzy pairwise comparison matrices. The proposed method considers the different importance weights of multiple DMs by extending the Fuzzy Preferences Programming Method (FPP). The elements of the group pairwise comparison matrices are presented as fuzzy numbers rather than exact numerical values in order to model the uncertainty and imprecision in the DMs' judgments. Unlike the known fuzzy prioritization techniques, the proposed method is able to derive crisp weights from incomplete and fuzzy set of comparison judgments and doesn't require additional aggregation procedures. A prototype of a decision tool is developed to assist DMs to use the proposed method for solving fuzzy group prioritization problems. A detailed numerical example is used to illustrate the proposed approach.

Keywords: Fuzzy Non-linear Programming. Fuzzy Preferences Programming Method. Multiple Criteria Decision-Making. Triangular Fuzzy Number.

1 Introduction

There are various techniques for deriving priorities/weights for decision elements (e.g. attributes/criteria), see [1] and [2] for a review. These techniques are based on either direct weighting or on pairwise comparison methods.

In direct weighting, the decision maker (DM) is directly asked to give values between 0 and 1 to each decision element to assign their importance. Some methods for deriving attributes/criteria weights by direct assigning techniques are: the Simple Multi-Attribute Rating Technique (SMART) [3], SWING weighting methods [4], and SMART Exploiting Ranks (SMARTER) [5].

When the DM or a group of DMs is unable to directly assign decision elements' weights, the Pairwise Comparison (PC) method proposed in [6] can be used.

Psychological experiments have shown that weight derivation from PC is much more accurate than direct weighting [8]. Therefore, the PC methods are often used as an intermediate step in many MCDM methods, as Analytic Hierarchy Process (AHP)

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[7], Analytic Network Process (ANP) [8], PROMETHEE [9], and Evidential Reasoning (ER) [10].

The PC methods require construction of Pairwise Comparisons Judgment Matrices (PCJMs). In order to construct a PCJM, the DM is asked to compare pairwisely any two decision elements and provide a numerical / linguistic judgment for their relative importance. Thus, the DM gives a set of ratio judgments to indicate the strength of his/her preferences, which are structured in a reciprocal PCJM. Then, the weights or priority vectors of the decision elements can be derived from the PCJM by applying some prioritization method.

There are numerous Pairwise Comparisons Prioritization Methods (PCPMs), as the Eigenvector Method [7], the Direct Least Squares Method [11], the rank-ordering method [7] and the Logarithmic Least Square Method [12]. Choo and Wedley [1] summarized and analyzed 18 PCPMs for deriving a priority vector from PCJMs. They discussed that no method performs best in all situations and no method dominates the other methods.

However, in many practical cases, in the process of prioritization the DMs are unable to provide crisp values for comparison ratios. A natural way to deal with the uncertainty and imprecision in the DMs' judgments is to apply the fuzzy set theory [14] and to represent the uncertain DMs' judgments as fuzzy numbers. Thus, Fuzzy PCJMs can be constructed and used to derive the priority vectors by applying some Fuzzy PCPMs. Such methods are proposed by Laarhoven and Pedrycz's [15], Buckley [14], Chang [16]; Mikhailov [17], and applied for group decision making.

The existing fuzzy PCPMs have some drawbacks. They require an additional defuzzification procedure to convert fuzzy weights into crisp (non-fuzzy) weights. However, different defuzzification procedures will often give different solutions.

The linear and non-linear variants of the Fuzzy Preference Programming (FPP) method [17] do not require such defuzzification procedures, but their group modifications assume that all the DMs have the same weight of importance. However, in the real group decision making problems, sometimes some experts are more experienced than others. Therefore the final results should be influenced by the degree of importance of each expert.

In order to overcome some of the limitation of the group FPP method, a new group version of the FPP method is proposed by introducing importance weights of DMs in order to derive weights for decision elements in group decision problems. The proposed method has some attractive features. It does not require any aggregation procedures. Moreover, it does not require a defuzzification procedure and derives crisp priorities/weights from an incomplete set of fuzzy judgments and incomplete fuzzy PCJMs.

For applying the proposed method and solving prioritization problems, a Non-Linear FPP Solver is developed based on the MATLAB Optimisation Toolbox. This decision tool is used for solving a specific numerical example.

The remainder of this paper is organized as follows. In Section 2, representation of the fuzzy group prioritization problem is briefly explained. Then, the proposed method is presented in Section 3 and illustrated by a numerical example in section 4. The developed Non-Linear FPP Solver is presented in section 5, followed by conclusions.

2 Representation of the Fuzzy Group Prioritization Problem

Consider a group of K DMs ($DM_k, k = 1,2,..., K$) that evaluates n elements $E_1,.. E_n$ (in MCDM, these elements could be clusters, criteria, sub-criteria or alternatives). With respect to some fixed preference scale, each DM assesses the relative importance of any two elements (E_i, E_j) $(i, j = 1,2,.., n)$ by providing a ratio judgment a_{ijk}, specifying by how much E_i is preferred/not preferred to E_j.

In a fuzzy environment, suppose that each DM provides a set of y fuzzy comparison judgements $A^k = \{\tilde{a}_{ijk}\}$, $y \le n(n-1)/2$, where $i = 1,2,.., n-1$, , $j \succ i$, $j = 2,3,.....n$, $k = 1,2,.., K$ and those judgments are represented as Triangular Fuzzy Numbers (TFNs) $\tilde{a}_{ijk} = (l_{ijk}, m_{ijk}, u_{ijk})$, where l_{ijk}, m_{ijk} and u_{ijk} are the lower bound, the mode and the upper bound, respectively.

The set A^k can be used to form a Fuzzy PCJM of the form (1):

$$A^k = \begin{bmatrix} (1,1,1) & (l_{12k}, m_{12k}, u_{12k}) & .... & (l_{1jk}, m_{1jk}, u_{1jk}) \\ (l_{21k}, m_{21k}, u_{21k}) & (1,1,1) & .... & (l_{2jk}, m_{2jk}, u_{2jk}) \\ ... & ... & .... & ... \\ (l_{i1k}, m_{i1k}, u_{i1k}) & (l_{i2k}, m_{i2k}, u_{i2k}) & .... & (1,1,1) \end{bmatrix} \tag{1}$$

Then, the fuzzy group prioritization problem is to determine a crisp priority vector (crisp weights) $w = (w_1, w_2,..., w_n)^T$ from all A^k, $k = 1,2,..., K$, which represents the relative importance of the n elements.

3 Group Fuzzy Preference Programming Method

The non-linear FPP method [17] derives a priority vector $w = (w_1, w_2,..., w_n)^T$, which satisfies:

$$l_{ij} \tilde{\le} w_i / w_j \tilde{\le} u_{ij} \tag{2}$$

where $\tilde{\le}$ denotes 'fuzzy less or equal to'. If M is the overall number of fuzzy group comparison judgments, then $2M$ fuzzy constraints of the type (3) are obtained.

$$\begin{aligned} -w_i + w_j l_{ij} &\tilde{\le} 0 \\ w_i - w_j u_{ij} &\tilde{\le} 0 \end{aligned} \tag{3}$$

For each fuzzy judgement, a membership function, which represents the DMs' satisfaction with different crisp solution ratios, is introduced:

$$\mu_{ij}(w_i/w_j) = \begin{cases} \dfrac{(w_i/w_j) - l_{ij}}{m_{ij} - l_{ij}}, & w_i/w_j \le m_{ij} \\ \dfrac{u_{ij} - (w_i/w_j)}{u_{ij} - m_{ij}}, & w_i/w_j \ge m_{ij} \end{cases} \tag{4}$$

The solution to the prioritization problem by the FPP method is based on two assumptions. The first on requires the existence of a *non-empty fuzzy feasible area* $\tilde{P}$ on the $(n-1)$ dimensional simplex Q^{n-1},

$$Q^{n-1} = \{(w_1, w_2, ..., w_n), w_i \succ 0, \sum_{i=1}^{n} w_i = 1\} \tag{5}$$

The fuzzy feasible area $\tilde{P}$ is defined as an intersection of the membership functions (4). The membership function of the fuzzy feasible area $\tilde{P}$ is given by:

$$\mu_{\tilde{P}}(w) = [Min\{\mu_1(w), \mu_2(w), ...\mu_{2M}(w)\} \setminus \sum_{i=1}^{n} w_i = 1] \tag{6}$$

The second assumption identifies a selection rule, which determines a priority vector, having the highest degree of membership in the aggregated membership function (6). Thus, there is *a maximizing solution* w^* (a crisp priority vector) that has a maximum degree of membership λ^* in $\tilde{P}$, such that :

$$\lambda^* = \mu_{\tilde{P}}(w^*) = Max[Min\{\mu_1(w), ..., \mu_{2M}(w)\} \setminus \sum_{i=1}^{n} w_i = 1] \tag{7}$$

A new decision variable λ is introduced which measures the maximum degree of membership in the fuzzy feasible area $\tilde{P}$. Then, the optimization problem (7) is represented as

$$\begin{aligned} & Max \quad \lambda \\ & s.t. \\ & \lambda \le \mu_{ij}(w) \\ & \sum_{i=1}^{n} w_i = 1, \quad w_i \succ 0, \quad i = 1,2,...,n, \quad j = 1,2,..n, \quad j \succ i \end{aligned} \tag{8}$$

The above max-min optimization problem (8) is transformed into the following non-linear optimization problem:

$$\begin{aligned} & Max \quad \lambda \\ & s.t. \\ & (m_{ij} - l_{ij})\lambda w_j - w_i + l_{ij} w_j \le 0 \\ & (u_{ij} - m_{ij})\lambda w_j + w_i - u_{ij} w_j \le 0 \\ & i = 1,2,...n-1; \quad j = 2,3,..n \quad ; \quad j \succ i \ ; \\ & \textstyle\sum_{i=1}^{n} w_i = 1 \ ; \quad w_i \succ 0; \quad i = 1,2,...,n \end{aligned} \tag{9}$$

The non-linear FPP method can be extended for solving group prioritization problems. Mikhailov *el. at.* [20] propose a Weighted FPP method to fuzzy group prioritization problem by introducing the importance weights of DMs. However, Weighted FPP method requires an additional aggregation technique to obtain the priority vector at different α - threshold. Consequently, this process is time consuming due to several computation steps needed for applying the α - threshold concept. Therefore, this paper modifies the non-linear FPP method [17], which can derive crisp weights without using α - threshold and by introducing the DMs' importance weights.

When we have a group of K DMs, the problem is to derive a crisp priority vector, such that priority ratios w_i / w_j are approximately within the scope of the initial fuzzy judgments a_{ijk} provided by those DMs, i.e.

$$l_{ijk} \mathrel{\tilde{\leq}} w_i / w_j \mathrel{\tilde{\leq}} u_{ijk} \tag{10}$$

The ratios w_i / w_j can also express the satisfaction of the decision makers, because ratios explain how similar the crisp solutions are close to the initial judgments from the DMs.

The inequality (10) can be represented as two single-side fuzzy constraints of the type (3):

$$R_q^{\ k} W \mathrel{\tilde{\leq}} 0, \quad k = 1,.., K \;\; q = 1,2,..2M_k \tag{11}$$

The degree of the DMs' satisfaction can be measured by a membership function with respect to the unknown ratio w_i / w_j :

$$\mu_q^{\ k}(R_q^{\ k} W) = \begin{cases} \dfrac{\left(w_i^k / w_j^k\right) - l_{ijk}}{m_{ijk} - l_{ijk}} & , w_i^k / w_j^k \leq m_{ijk} \\ \dfrac{u_{ijk} - \left(w_i^k / w_j^k\right)}{u_{ijk} - m_{ijk}} & , w_i^k / w_j^k \geq m_{ijk} \end{cases} \tag{12}$$

We can define K fuzzy feasible areas, $\tilde{P}_k$ as intersection of the membership functions (12), corresponding to the k -th DMs' fuzzy judgments and define the group fuzzy feasible area $\tilde{P} = \bigcap \tilde{P}_k$.

By introducing a new decision variable λ_k , which measures the maximum degree of membership of a given priority vector in the fuzzy feasible area $\tilde{P}_k$, we can formulate a max-min optimization problem of the type (8), which can be represented into:

$$\begin{aligned} & Max \quad \lambda_k \\ & s.t. \\ & \lambda_k \leq \mu_q^{\ k}(R_q^{\ k} W) \\ & \sum_{i=1}^{n} w_i = 1, \quad w_i \succ 0, \quad i = 1,2,...,n, \quad k = 1,.., K \;\; q = 1,2,..2M_k \end{aligned} \tag{13}$$

For introducing the DMs' importance weights, let us define I_k as the importance weight of the $DM_k; k = 1,2,..., K$. For aggregating all individual models of type (13) into a single group model a weighted additive goal-programming (WAGP) model [18] is applied.

The WAGP model transforms the multi-objective decision-making problem to a single objective problem. Therefore, it can be used to combine all individual models (13) into a new single model by taking into account the DMs' importance weights.

The WAGP model considers the different importance weights of goals and constraints and is formulated as:

$$\begin{aligned} &\mu_D(x) = \sum_{s=1}^{p} \alpha_s \mu_{z_S}(x) + \sum_{r=1}^{h} \beta_r \mu_{g_r}(x) \\ &\sum_{s=1}^{p} \alpha_s + \sum_{r=1}^{h} \beta_r = 1 \end{aligned} \tag{14}$$

Where:

μ_{z_S} are membership functions for the p–th fuzzy goal $z_S, s = 1,2,...p$;

μ_{g_r} are membership functions of the h -th fuzzy constraints $g_r, r = 1,2,...h$;

x is the vector of decision variables;

α_s are weighting coefficients that show the relative important of the fuzzy goals;

β_r are weighting coefficients that show the relative important of the fuzzy constraints.

A single objective model in WAMP is the maximization of the weighted sum of the membership functions μ_{z_S} and μ_{g_r} . By introducing new decision variables λ_s and γ_r, the model (14) can be transformed into a crisp single objective model, as follows:

$$\begin{aligned} &Max \ \ \sum_{s=1}^{p} \alpha_s \lambda_s + \sum_{r=1}^{h} \beta_r \gamma_r \\ &s.t. \\ &\lambda_s \le \mu_{z_s}(x), \qquad s = 1,2,...p \\ &\gamma_r \le \mu_{g_r}(x), \qquad r = 1,2,...h \\ &\sum_{s=1}^{p} \alpha_s + \sum_{r=1}^{h} \beta_r = 1 \\ &\lambda_s, \gamma_r \in [0,1]; \qquad \alpha_s, \beta_r \ge 0 \end{aligned} \tag{15}$$

In order to derive a group model, where the DMs have different importance weights, we exploit the similarity between the models (13) and (15). However, the non-linear FPP model (13) does not deal with fuzzy goals; it just represents the non-linear fuzzy constraints. Thus, by taking into the account the specific form of $R_q^k W \tilde{\le} 0$, and introducing the important weights of the DMs, the problem can be further presented into a non-linear program by utilizing WAGP model as:

$$
\begin{aligned}
& Max \quad Z = \sum_{k=1}^{K} I_k \lambda_k \\
& s.t. \\
& (m_{ijk} - l_{ijk})\lambda_k w_j - w_i + l_{ijk} w_j \le 0 \\
& (u_{ijk} - m_{ijk})\lambda_k w_j + w_i - u_{ijk} w_j \le 0 \\
& i = 1,2,\dots n-1; \quad j = 2,3,..n \quad ; \quad j \succ i \;;\; k = 1,2,\dots K \\
& \sum_{i=1}^{n} w_i = 1 \;; \quad w_i \succ 0; \qquad i = 1,2,\dots, n
\end{aligned}
\tag{16}
$$

Where the decision variable λ_k measures the degree of the DM's satisfaction with the final priority vector $w = (w_1, w_2, \dots\dots, w_n)^T$, I_k denotes the importance weight of the k-th DM, $k = 1,2,\dots K$.

In (16), the value of Z can be considered as a consistency index, as it measures the overall consistency of the initial set of fuzzy judgments. When the set of fuzzy judgments is consistent, the optimal value of Z is greater or equal to one. For the inconsistent fuzzy judgments, the maximum value of Z takes a value less than one.

For solving the non-linear optimization problem (16), an appropriate numerical method should be employed. In this paper, the solution is obtained by using MATLAB Optimization Toolbox, and a Non-linear FPP solver is developed to solve the prioritization problem.

4 An Illustrative Example

This example is given to illustrate the proposed method and also the solution by using the Non-linear FPP Solver. Moreover, this example demonstrates how the importance weights of DMs influence the final group ranking.

We consider the example in [20], where three DMs ($K = 3$) assess three elements ($n = 3$), and the importance weights of DMs are given: $I_1 = 0.3$, $I_2 = 0.2$, $I_3 = 0.5$.

The DMs provide an incomplete set of five fuzzy judgments, presented as TFNs:

DM 1: $a_{121} = (1,2,3)$; $a_{131} = (2,3,4)$.

DM 2: $a_{122} = (1.5,2.5,3.5)$; $a_{132} = (3,4,5)$.

DM 3: $a_{123} = (2,3,4)$.

The group fuzzy prioritization problem is to derive a crisp priority vector $w = (w_1, w_2, w_3)^T$ that approximately satisfies the following fuzzy constraints:

For DM 1: $1 \tilde{\le} w_1/w_2 \tilde{\le} 3$; $2 \tilde{\le} w_1/w_3 \tilde{\le} 4$.

For DM 2: $1.5 \tilde{\le} w_1/w_2 \tilde{\le} 3.5$; $3 \tilde{\le} w_1/w_3 \tilde{\le} 5$.

For DM 3: $2 \tilde{\le} w_1/w_2 \tilde{\le} 4$.

The weights obtained by applying the method proposed in the previous section are $w_1 = 0.621$, $w_2 = 0.212, w_3 = 0.167$.

This solution can be compared with the crisp results from the example in [20] as shown in Table 1. We may observe that we have the same final ranking $w_1 \succ w_2 \succ w_3$, from applying the two different prioritization methods. However, the Weighted FPP method [20] applies an aggregation procedure for obtaining the crisp vector from different values of priorities at different α - threshold, while, the proposed non-linear group FPP method does not require an additional aggregation procedure.

Table 1. Results from the two prioritization methods

Methods	w_1	w_2	w_3
Weighted FPP method [a]	0.615	0.205	0.179
Non-linear FPP method [b]	0.623	0.216	0.161

[a] The method proposed in [16] with applying α - threshold.
[b] The method proposed in this paper without applying α - threshold.

If the third DM, who has the highest important weight provides a new fuzzy comparison judgment $a_{323} = (1,2,3)$, which means that the third element is about two times more important than the second element, the weights obtained by using the proposed Non-Linear FFP method are: $w_1 = 0.538$, $w_2 = 0.170$, $w_3 = 0.292$ and the final ranking is $w_1 \succ w_3 \succ w_2$. Consequently, it can be observed that the third DM's judgments strongly influence the final ranking. However, if the importance weight of the third DM is lower to the first two DMs' weights, then the new fuzzy comparison judgment does not change the final ranking. Thus, we can notice the significance of introducing importance weights of the DMs to the fuzzy group prioritization problem.

According to the computation time for solving the fuzzy group prioritization problem, the proposed method does not need an additional procedure to aggregate the priorities at the different α -levels. Therefore, the proposed method in this paper demands less computation time than the Weighted FPP method [20].

The computation time of the proposed method has been investigated by using the Non-Linear FFP Solver. It was found that the group non-linear FFP method performs significantly faster compared to the Weighted FPP [20] with different α - threshold ($\alpha = 0, 0.2 , 0.5 , 0.8 , 1$), as seen in Fig. 1.

We can conclude that the average of computation time (Minutes) for the Weighted FPP method highly increases as the number of decision elements n increases, comparing with the proposed method. Hence, these results show that the method proposed in this paper is more efficient with respect to the computation time.

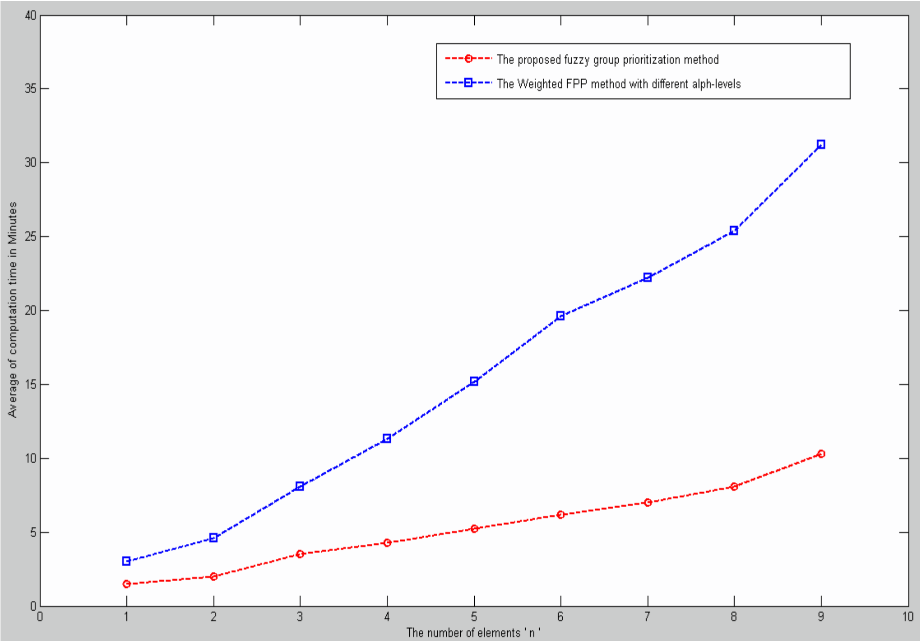

Fig. 1. Average Computation Time (Minutes)

5 Software Implementation Using MATLAB

We use the Optimization Toolbox in MATLAB functions and the functions MATLAB Graphical User Interface (GUI) to implement the proposed group non-linear FPP method. Essentially, there are three steps for programming and developing of the Non-Linear FFP solver:

Step 1: Coding the model into the system. A number of functions are available in MATLAB to solve the non-linear programming problem. In our prototype, the optimization problem is solved using sequential quadratic programming procedure [19].

Step 2: Creating a basic user interface. In this step the interface is designed which can run in the MATLAB command window. The aim of this user interface is to obtain the input from the DMs. The input information which should be acquired includes the total number of decision elements, the name of these elements, the total number of DMs, the importance weights of the DMs, and the fuzzy judgments.

The main feature in the developed interface is that the user can input the fuzzy judgments into the system directly and easily. According to the example from the previous section, the fuzzy judgments for the DM 1 are illustrated in Fig.2. However, if the fuzzy judgments between two elements are missing, the user can click the **'Missing Data'** button then the system will temporarily put -1 for the comparison, the negative value is not a true judgment in the real world, it just indicates that those elements should not be included in the further calculations. For instance, in the given example, the judgment a_{231} is missing for the DM1 and can be sorted as $(-1,-1,-1)$, Fig. 2.

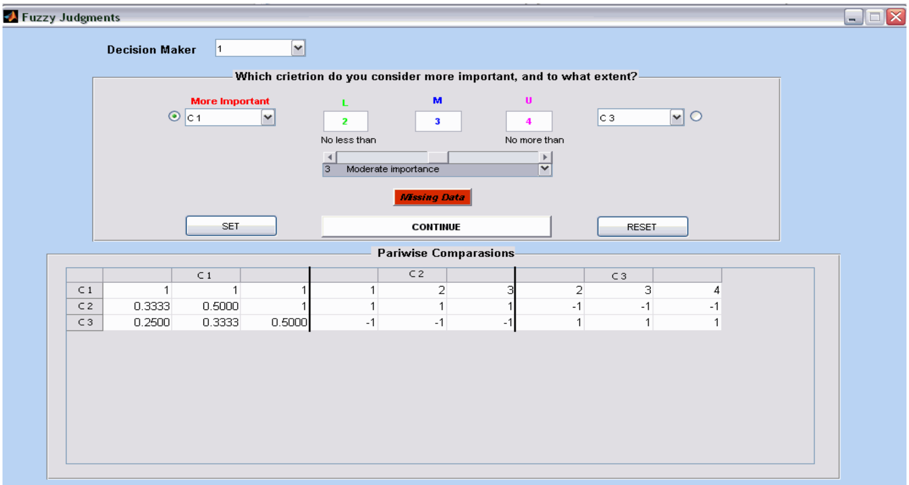

Fig. 2. The fuzzy comparison judgments window for the DM 1

Step 3: Developing the system based on the GUI functions. In this step, the MATLAB Graphical User Interface (GUI) functions are employed to develop a more user-friendly system.

6 Conclusions

This paper proposes a new method for solving fuzzy group prioritisation problems. The non-linear FPP is modified for group decision-making by introducing DMs' importance weights. The proposed method derives crisp priorities/weights from a set of fuzzy judgements and it does not require defuzzification procedures. Moreover, the proposed method is capable to derive crisp priorities from an incomplete set of DMs' fuzzy pairwise comparison judgments. The method is very efficient from a computational point of view, and a promising alternative to existing fuzzy group prioritization methods.

A Non-Linear FPP Solver is developed for solving group prioritization problems, which provide a user-friendly and efficient way to obtain the group priorities.

References

1. Choo, E., Wedley, W.: A common framework for deriving preference values from pairwise comparison matrices. Computers & Operations Research 31, 893–908 (2004)
2. Ittersum, K.V., Pennings, J.M., Wansink, B., van Trijp, H.C.: The validity of attribute-importance measurement: A review. Journa of Business Research 60, 1177–1190 (2007)

3. Edwards, W.: How to use multiattribute utility measurement for social decision making. IEEE Transactions on Systems, Man, and Cybernetics 7, 326–340 (1977)
4. Von Winterfeldt, D., Edwards, W.: Decision analysis and behavioural research. Cambridge University Press, Cambridge (1986)
5. Edwards, W., Barron, F.H.: SMARTS and SMARTER: Improved simple methods for multiattribute utility measurement. Organizational Behavior and Human Decision Processes 60, 306–325 (1994)
6. Thurstone, L.: A law of comparative judgment. Psychological Review 34, 273–286 (1927)
7. Saaty, T.L.: The analytic hierarchy process: Planning, Priority setting, resource allocation. McGraw-Hill, NY (1980)
8. Saaty, T.L.: Decision making with dependence and feedback: the Analytic Network Process. RWA Publications, PA (1996)
9. Brans, J., Mareschal, B.: Promethee methods: In multiple criteria decision analysis: state of the art surveys. International Series in Operations Research & Management Science 78, 163–186 (2005)
10. Xu, D.L., Yang, J.B., Lama, J.: Group-based ER–AHP system for product project screening. Expert Systems with Applications 35, 1909–1929 (2008)
11. Chu, A., Kalaba, R., Springam, K.: A comparison of two methods for determining the weights of belonging to fuzzy sets. Journal of Optimization Theory and Application 27, 531–541 (1979)
12. Saaty, T.L., Vargas, L.G.: Comparison of Eigenvalue, Logarithmic Least Squares and Least Squares Methods in Estimating Rations. Mathematical Modelling 5, 309–324 (1984)
13. Mikhailov, L.: A Fuzzy Programming Method for Deriving Prioritises in Analytic Hierarchy Process. The Journal of Operational Research Society 51, 341–349 (2000)
14. Buckley, J.J.: Fuzzy Hierarchical Analysis. Fuzzy Sets and System 17, 233–247 (1984)
15. Laarhoven, P.J.M., Pedrycz, W.: A fuzzy extension of Saaty's priority theory. Fuzzy Sets and System 11, 229–241 (1983)
16. Chang, D.: Applications of the extent analysis method on fuzzy AHP. European Journal of Operational Research 95, 649–655 (1996)
17. Mikhailov, L.: Deriving priorities from fuzzy pair-wise comparison judgments. Fuzzy Sets and System 134, 365–385 (2003)
18. Tiwari, R.N., Dharmar, S., Rao, J.R.: Fuzzy goal programming-an additive model. Fuzzy Sets and System 24, 27–34 (1987)
19. Higham, D.J., Higham, N.J.: MATLAB Guide, 2nd edn. SIAM (2005)
20. Mikhailov, L., Didehkhani, H., Sadi-Nezhad, S.: Weighted Prioritization Models in the Fuzzy Analytic Hierarchy Process. International Journal of Information Technology & Decision Making 10, 681–694 (2011)