Computational Social Choice (with a Special Emphasis on the Use of Logic)

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Abstract. This is a summary of a tutorial on computational social choice, delivered at the *9th Tbilisi Symposium on Language, Logic and Computation* in Kutaisi, Georgia, in September 2011. The tutorial specifically focussed on the use of logic in the field.

1 Introduction

Social choice theory deals with questions regarding the design and analysis of methods for collective decision making [1]. In recent years there has been a growing interest in the computational aspects of collective decision making, giving rise to the field of *computational social choice* [5,4]. This tutorial provided an introduction to this field, highlighting in particular the role of logic. The first lecture was devoted to an exposition of the axiomatic method in social choice theory; the second lecture was about social choice in combinatorial domains; and the third lecture was an introduction to judgment aggregation.

Below we briefly introduce each of these three topics and provide references for further reading. Full details are available elsewhere [7].

2 The Axiomatic Method in Social Choice Theory

Much of the classical work in social choice theory is concerned with the formalisation of normative intuitions about the proper way of aggregating preferences by stating so-called *axioms* and with the investigation of the consequences of those axioms. The best-known example is Arrow's Theorem [2].

In Arrow's model, n individuals each express a preference over a set of alternatives $\mathcal{X}$ (where a preference is a linear order $\succ$ over $\mathcal{X}$). To aggregate this into a collective preference order that accurately reflects the views of the group, we are looking for a function F from n-tuples of linear orders on $\mathcal{X}$ to a single linear order on $\mathcal{X}$. Arrow argued that F should satisfy the following axioms:

- The *Pareto condition:* in case every individual agrees that $x \succ y$, then also the collective preference order should stipulate $x \succ y$.
- *Independence of irrelevant alternatives:* whether $x \succ y$ holds for the collective preference should only depend on the relative ranking of x and y in the individual orders (and not on, say, how many individuals agree with $x \succ z$).

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– *Absence of dictators:* it should not be the case that there exists one fixed individual such that the collective preference order is always identical to that individual's preference order, independently of what the others say.

Arrow's deeply surprising and thought-provoking theorem states that it is *impossible* to simultaneously satisfy these requirements—at least in case there are more than just two alternatives to rank [2]. A number of different proofs of Arrow's Theorem are known. What might be of particular interest to logicians is that some proofs make use of concepts from the theory of ultrafilters, and also that there have been several attempts to automate the process of proving this and similar results. One proof technique that is particularly helpful in understanding the issues raised by Arrow's result is based on the notion of a *decisive coalition*—it amounts to investigating the structure of the family of coalitions (sets of individuals) who together can determine the relative ranking of two given alternatives, independently of the preferences of the other individuals [7].

3 Social Choice in Combinatorial Domains

For many applications the set of alternatives the individuals are asked to express preferences over will have a combinatorial structure. For instance, if we are asked to elect a committee of size k by choosing from a field of n candidates, then there are $\binom{n}{k}$ possible committees; and in a referendum in which voters are asked to accept or reject n different propositions there are 2^n possible outcomes.

Because of this combinatorial explosion, we need to carefully choose the *language* used to represent the preferences of individuals. This is why work on social choice in combinatorial domains often borrows ideas from knowledge representation and reasoning [6]. In particular, several logic-based languages for preference modelling are used in the field [11]. Three approaches are particularly promising:

– *Distance-based voting:* The basic idea is to ask each individual for her most preferred combination and then make a collective choice that minimises the aggregated distance to the combinations—for a suitable notion of distance and, in some cases, restricting attention to a particular set of admissible outcomes. Examples include the *minimax rule* [3] and the *average-voter rule* [8].
– *Sequential voting:* Here the idea is to vote on each issue in turn, rather than to vote on all of them at the same time, which reduces the complexity of the problem as well as the potential for paradoxical outcomes [10,12].
– *Combinatorial vote:* The central idea in this approach is that individuals encode their preferences using a compact preference representation language and transmit an expression in the language of choice rather than an explicit preference order [11]. The main challenge is to develop efficient algorithms that are able to aggregate the compactly represented preferences.

4 Judgment Aggregation

Preferences are not the only structures we may wish to aggregate. *Judgment aggregation* studies the aggregation of judgments on mutually dependent propositions [13,9]. These dependencies are often modelled using propositional logic.

For instance, suppose Alice judges both p and q to be true, Bob judges p to be true and q to be false, and Carlo judges p to be false and q to be true. How should we aggregate this information? Going with the obvious choice of majority voting is problematic: there is a majority for p as well as a majority for q, but at the same time there is a majority *against* $p \wedge q$.

List and Pettit [13] were the first to treat this problem formally. They showed that it is in fact *impossible* to devise a method of aggregation that would treat all individuals as well as all propositions symmetrically (anonymity and neutrality), for which the collective judgment on a proposition solely depends on the individual judgments on that same proposition (independence), and for which the outcome is always a consistent set of formaulas including, for every formula φ under consideration, either φ or $\neg\varphi$. Subsequent work has investigated topics such as the connections to preference aggregation and Arrow's Theorem, ways of circumventing the impossibility, precise conditions on the set of formulas to be judged for impossibilities to arise, specific aggregation rules, and the computational complexity of working in the framework of judgment aggregation [9].

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