Manipulation of Weighted Voting Games via Annexation and Merging

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Abstract. We conduct an experimental study of the effects of *manipulations* (i.e., dishonest behaviors) including those of manipulation by *annexation* and *merging* in weighted voting games. These manipulations involve an agent or agents misrepresenting their identities in anticipation of gaining more power at the expense of other agents in a game. Using the well-known Shapley-Shubik and Banzhaf power indices, we first show that manipulators need to do only a polynomial amount of work to find a much improved power gain, and then present two enumeration-based pseudopolynomial algorithms that manipulators can use. Furthermore, we provide a careful investigation of heuristics for annexation which provide huge savings in computational efforts over the enumeration-based method. The benefits achievable by manipulating agents using these heuristics also compare with those of the enumeration-based method which serves as upper bound.

Keywords: Agents, Weighted voting games, Annexation, Merging, Power indices.

1 Introduction

False-name manipulation in *weighted voting games* (WVGs), which involves an agent or some agents misrepresenting their identities in anticipation of power increase, has been identified as a problem. The menace can take different forms. With manipulation by *annexation*, an agent, termed, an *annexer*, takes over the voting weights of some agents in a game. Power is not shared with the annexed agents. Forming an *alliance* or manipulation by *merging* involves voluntary merging of weights by two or more agents to form a single bloc [2,13,18]. Merged agents expect to be compensated with their share of the power gained by the bloc. The agents whose voting weights are taken over or merged into a bloc are referred to as *assimilated* agents. The only difference between merging and annexation is that, in merging, the assimilated agents must be compensated for their participation by sharing of the power, while in annexation, only the annexer is compensated for the participation of the group. Annexed agents are assumed to either voluntary forfeit their weight or be compensated on a one-time basis that is not related to power. Thus, power increase is much easier to achieve with annexation.

WVGs are classic cooperative games which provide compact representation for coalition formation models in human societies and multiagent systems. Each agent in a WVG has an associated weight. A subset of agents whose total weight is at least the value of

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a specified *quota* is called a *winning coalition*. The weights of agents in a game correspond to resources or skills available to the agents, while the quota is the amount of resources or skills required for a task to be accomplished. For example, in *search and rescue*, robotic agents put their resources (i.e., weights) together in large natural disaster environments to reach the necessary levels (i.e., quota) to save life and property.

The relative power of each agent reflects its significance in the elicitation of a winning coalition. A widely accepted method for measuring such relative power in WVGs uses *power indices*. Two prominent power indices for measuring power are the *Shapley-Shubik* [23] and the *Banzhaf* [6] power indices. WVGs can be viewed as a form of competition among agents to share the available *fixed* power whose total value is always assumed to be 1. Agents may thus resort to manipulation by annexation or merging to improve their influence in anticipation of gaining more power. With the possibility of manipulation, it becomes difficult to establish or maintain trust, and more importantly, it becomes difficult to assure fairness in such games.

This paper continues the work of [2,13,18,19] on annexation and merging in WVGs. We first extend the framework of [18] on susceptibility of power indices to manipulation by annexation and merging in WVGs to consider a *much improved* power gain for manipulators. Then, we propose and evaluate heuristics that manipulating agents may employ to engage in such manipulations using the two power indices. Consider a WVG of n agents. The simulation of [18] is based on a *random* approach where some agents, say $k < n$, in the game are randomly selected to be assimilated in annexation or to form a voluntary bloc of manipulators in merging. This simple random approach shows that, on average, annexation can be effective for manipulators using the two power indices to compute agents' power. These results also show that merging only has a minor effect on the power gained for manipulators using the Shapley-Shubik index, while it is typically non-beneficial (i.e., no power is gained) for manipulators using the Banzhaf index.

Restricting the number of agents in the blocs of assimilation or merging as employed in the simple random simulation of [18], we show that manipulators need to do only a *polynomial* amount of work to find a much improved power gain over the random approach during manipulation. Given that the problem of computing the Shapley-Shubik and Banzhaf indices of agents is already NP-hard [21,22], *pseudopolynomial* or approximation algorithms [7,9,21] are available to compute agents' power. We then present two pseudopolynomial-time enumeration algorithms that manipulators may use to find a much improved power gain. We empirically evaluate our enumeration approach for manipulation by annexation and merging in WVGs. Our method is shown to achieve significant improvement in benefits over previous work for manipulating agents in several numerical experiments. Thus, unlike the simple random simulation of [18] where merging has little or no benefits for manipulators using the two power indices, results from our experiments suggest that manipulation via merging can be *highly* effective.

The remainder of the paper is organized as follows. Section 2 provides some preliminaries. In Section 3, we provide visual illustrations of manipulation in WVGs to give some insights into why it is difficult to predict how to merge. We present our pseudopolynomial-time manipulation algorithms for annexation and merging in Section 4. Section 5 presents results of evaluation of the manipulation algorithms. In Sections

6, we present heuristics for annexation in WVGs. Section 7 discusses related work and we conclude in Section 8.

2 Preliminaries

2.1 Weighted Voting Games

Let $I = \{1, \cdots, n\}$ be a set of n agents and the corresponding positive weights of the agents be $\mathbf{w} = \{w_1, \cdots, w_n\}$. Let a coalition $S \subseteq I$ be a non-empty subset of agents. A WVG G with *quota* q involving agents I is represented as $G = [w_1, \cdots, w_n; q]$. We assume that $w_1 \leq w_2 \leq \ldots \leq w_n$. Denote by $w(S)$, the weight of a coalition, S, derived as the summation of the weights of agents in S, i.e., $w(S) = \sum_{j \in S} w_j$. A coalition, S, wins in game G if $w(S) \geq q$, otherwise it loses. WVGs belong to the class of *simple voting games*. In simple voting games, each coalition, S, has an associated function $v : S \rightarrow \{0, 1\}$. The value 1 implies a win for S and 0 implies a loss. So, $v(S) = 1$ if $w(S) \geq q$ and 0 otherwise.

2.2 Power Indices

We provide brief descriptions of the two power indices we use in computing agents' power in WVGs. For further discussion, we refer the reader to [12,17].

Shapley-Shubik Power Index

The Shapley-Shubik index quantifies the marginal contribution of an agent to the *grand coalition* (i.e., a coalition of all the agents). Each permutation (or ordering) of the agents is considered. We term an agent *pivotal* in a permutation if the agents preceding it do not form a winning coalition, but by including this agent, a winning coalition is formed. Shapley-Shubik index assigns power to each agent based on the proportion of times it is pivotal in all permutations. We specify the computation of the index using notation of [2]. Denote by π, a permutation of the agents, so $\pi : \{1, \ldots, n\} \rightarrow \{1, \ldots, n\}$, and by Π the set of all possible permutations. Denote by $S_\pi(i)$ the predecessors of agent i in π, i.e., $S_\pi(i) = \{j : \pi(j) < \pi(i)\}$. The Shapley-Shubik index, $\varphi_i(G)$, for each agent i in a WVG G is

$$\varphi_i(G) = \frac{1}{n!} \sum_{\pi \in \Pi} [v(S_\pi(i) \cup \{i\}) - v(S_\pi(i))]. \quad (1)$$

Banzhaf Power Index

An agent $i \in S$ is referred to as being *critical* in a winning coalition, S, if $w(S) \geq q$ and $w(S \backslash \{i\}) < q$. The Banzhaf power index computation for an agent i is the proportion of times i is critical compared to the total number of times any agent in the game is critical. The Banzhaf index, $\beta_i(G)$, for each agent i in a WVG G is given by

$$\beta_i(G) = \frac{\eta_i(G)}{\sum_{j \in I} \eta_j(G)} \quad (2)$$

where $\eta_i(G)$ is the number of coalitions for which agent i is critical in G.

2.3 Annexation and Merging in Weighted Voting Games

Felsenthal and Machover [13] consider a real life example of annexation where a shareholder buys the voting shares of some other shareholders in a firm in order to use them for her own interest. Clearly, this action allows the manipulator to possess more shares and makes it easier for her to affect the outcomes of decisions in the firm. Yokoo et al., [25] have also considered false-name manipulation in open anonymous environments which they refer to as *collusion*. Like in merging, collusion involves many agents acting as a single agent. They have shown that the manipulation can be difficult to detect in such environments. Thus, the increase use of online systems (such as trading systems and peer-to-peer networks, where WVGs are also applicable) means that annexation or merging remains an important challenge that calls for attention. We now provide a formal definition of manipulation by annexation and merging in WVGs.

Let G be a WVG. Let Φ be either of Shapley-Shubik or Banzhaf power index. We denote the power of an agent i in G by $\Phi_i(G)$. Also, consider a coalition $C \subseteq I$, we denote by $\&C$ a bloc of assimilated voters formed by agents in C. We say that a power index Φ is *susceptible* to manipulation whenever a WVG G is *altered* by an agent i (in the case of annexation or by some agents in the case of merging) and such that there exists a new game G' where $\Phi_i(G') > \Phi_i(G)$. In other words, Φ is susceptible to manipulation when the power of the agent in the altered game is more than its power in the original game.

Definition 1. *Manipulation by Annexation.*

Let agent i alter game G by annexing a coalition C ($i \notin C$ assimilates the agents in C to form a bloc $\&(C \cup \{i\})$). We say that Φ is susceptible to manipulation via annexation if there exists a new game G' such that $\Phi_{\&(C\cup\{i\})}(G') > \Phi_i(G)$; the annexation is termed *advantageous*. The *factor of increment* by which the annexer gains is given by $\frac{\Phi_{\&(C\cup\{i\})}(G')}{\Phi_i(G)}$. If $\Phi_{\&(C\cup\{i\})}(G') < \Phi_i(G)$, then the annexation is *disadvantageous*.

We provide an example to illustrate annexation in WVG. The annexer and assimilated agents are all shown in bold.

Example 1. Annexation in Weighted Voting Game.

Let $G = [\mathbf{12}, 16, \mathbf{18}, \mathbf{19}, 23, 26, 43, 46, 50; 195]$ be a WVG. The Banzhaf power index of agent 1 with weight 12 is $\beta_1(G) = 0.026$. Suppose the agent annexes agents 3 and 4 with weights 18 and 19 respectively. An assimilated bloc of weight 49 is formed in the new game $G' = [16, 23, 26, 43, 46, \mathbf{49}, 50; 195]$. The new Banzhaf power index of the annexer $\beta_6(G') = 0.177 > \beta_1(G)$. The agent gains from the annexation and increases its power index by a factor of $\frac{0.177}{0.026} = 6.81$.

Definition 2. *Manipulation by Merging.*

Let a manipulators' coalition, S, alter G by merging into a bloc $\&S$. We say that Φ is susceptible to manipulation via merging if there exists a new game G' such that $\Phi_{\&S}(G') > \sum_{j\in S} \Phi_j(G)$; the merging is termed *advantageous*. The factor of increment by which the manipulators gain is given by $\frac{\Phi_{\&S}(G')}{\sum_{j\in S}\Phi_j(G)}$. If $\Phi_{\&S}(G') < \sum_{j\in S} \Phi_j(G)$, then the merging is *disadvantageous*. The agents in a bloc formed by merging are assumed to be working cooperatively and have transferable utility. For the sake of simplicity in our analysis, we also refer to the factor of increment as power gain or benefit.

Example 2 illustrates manipulation by merging in a weighted voting game.

Example 2. Merging in Weighted Voting Game.

Let $G = [12, 16, 18, 19, 23, \mathbf{26}, \mathbf{33}, \mathbf{40}, \mathbf{45}; 155]$ be a WVG. The last four agents in the game are designated as would-be manipulators. The Banzhaf indices of these agents are: $\beta_6(G) = 0.116$, $\beta_7(G) = 0.142$, $\beta_8(G) = 0.174$, and $\beta_9(G) = 0.200$. So, $\sum_{j=6}^{9} \beta_j(G) = 0.632$. Suppose the agents decide to merge their weights. A bloc of weight 144 is formed in the new game $G' = [12, 16, 18, 19, 23, \mathbf{144}; 155]$. The Banzhaf power index of the bloc $\beta_6(G') = 0.861 > 0.632$. The manipulators gain from the merging and increase their power indices by a factor of $\frac{0.861}{0.632} = 1.36$.

3 Visual Description of Manipulation by Annexation and Merging in Weighted Voting Games

We provide visual description of manipulation by annexation and merging in WVGs to further explain the intricases of what goes on during manipulation. We use the Shapley-Shubik power index for this illustration. Consider a WVG of three agents denoted by the following patterns: Agent 1 (■), Agent 2 (□), and Agent 3 (▤). The weight of each agent in the game is indicated by the associated length of the pattern. A box in the pattern corresponds to a unit weight. Suppose all permutations (or ordering) of the three agents are given as shown in Figure 1 where we vary the values of the quota of the game from $q = 1$ to $q = 6$. The Shapley-Shubik indices of the three agents are also computed from the figure and shown in the associated table of the figure. These power indices for the agents in the game correspond to using various values of the quota for the same weights of the agents in the game.

Consider the case of manipulation by merging where Agent 1 and Agent 3 merge their weights to form a new agent, say Agent X. In this situation, Agent 1 and Agent 3 cease to exist since they have been assimilated by Agent X. Thus, we have only two agents (Agent X and Agent 2) in the altered WVG. Figure 2 shows the results of the merging between Agent 1 and Agent 3. Consider the cases when the quota of the game is 1 or 6, the power of the assimilated agents for Agent X from Figure 1 shows that Agent 1 and Agent 3 each has a power of $\frac{1}{3}$ for a total power of $\frac{2}{3}$. Whereas, the power of Agent X which assimilates these two agents in the two cases is each $\frac{1}{2} < \frac{2}{3}$. On the other hand, the power of the manipulators stays the same for the cases where the quota is either 2 or 5. Specifically, the sum of the powers of Agent 1 and Agent 3 is $\frac{1}{2}$ for these cases. This is also true of Agent X for these cases. Finally, for the cases where the quota of the game is either 3 of 4, the power of Agent X is 1 which is greater than $\frac{5}{6}$, the sum of the powers of Agent 1 and Agent 3 in the original game. Note the difficulty of predicting what will happen when manipulators engage in merging.

Now, suppose Agent 1 annexes Agent 3 instead of the merging between the two agents as described earlier, then Agent 3 ceases to exist as shown in Figure 3. In this case, Agent 1's power does not decrease. Felsenthal and Machover [12] have already

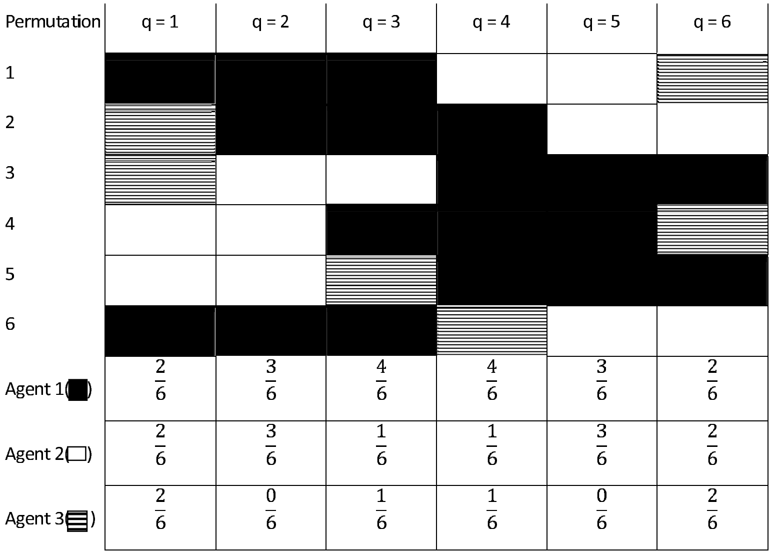


Fig. 1. Six permutations of three agents and corresponding power indices of the agents for various values of quota from $q = 1$ to $q = 6$

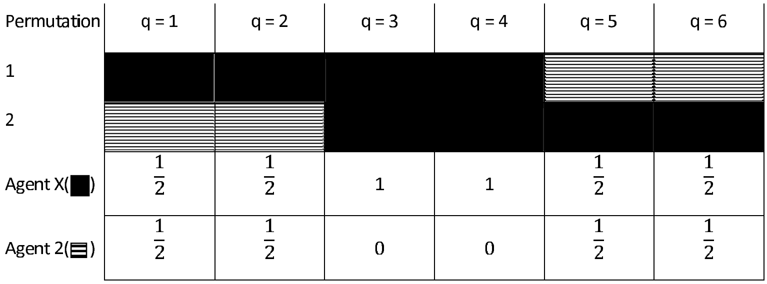


Fig. 2. Manipulation by merging between Agent 1 and Agent 3 (from Figure 1) to form a new Agent X. The power indices of Agent X and Agent 2 as computed by Shapley-Shubik index after the merging for various values of the quota is also shown.

shown that annexation is never disadvantageous for an annexer using the Shapley-Shubik power index. However, that is not the case when the annexer uses the Banzhaf power index. This they have dubbed the bloc paradox.

4 Manipulation Algorithms for Annexation and Merging

4.1 Overview

We propose an enumeration approach for manipulation by annexation and merging in WVGs using the two power indices to compute agents' power. To begin with, we recall that the problem of calculating the Shapley-Shubik and Banzhaf indices is NP-hard and admit pseudopolynomial algorithms using generating functions or dynamic

Permutation	q = 1	q = 2	q = 3	q = 4	q = 5	q = 6
1	■	■	■	■	□	□
2	□	□	■	■	■	■
Agent X(■)	$\frac{1}{2}$	$\frac{1}{2}$	1	1	$\frac{1}{2}$	$\frac{1}{2}$
Agent 2(□)	$\frac{1}{2}$	$\frac{1}{2}$	0	0	$\frac{1}{2}$	$\frac{1}{2}$

Fig. 3. Manipulation by annexation where Agent 1 assimilates Agent 3 (from Figure 1). The new power indices of Agent 1 as computed by Shapley-Shubik index after the annexation for various values of the quota is also shown.

programming [7,9,21]. Given that the problem of computing the two indices is already NP-hard, and only pseudopolynomial or approximation algorithms are available to compute agents' power, it is reasonable that the manipulation algorithms we propose are also pseudopolynomial since we necessarily need to use these indices in computing agents' benefits during manipulation.

4.2 Manipulation Algorithm for Merging

The brute force approach to determine a coalition that yields the most improved benefit in merging in a WVG is to simply enumerate all the possible coalitions of agents in the game and then compute the benefit for each of these coalitions. We can then output the coalition with the highest value. Unfortunately, enumerating all the possible coalitions is exponential in the number of agents. Also, computing the power indices naively from their definitions means that we have two exponential time problems to solve. We provide an alternative approach.

Let procedure *PowerIndex*(G, i) be a pseudopolynomial algorithm for computing the power index of an agent i in a WVG G of n agents for Shapley-Shubik or Banzhaf index according to any of [7,9,21]. We first use *PowerIndex*(G, i) as a subroutine in the construction of a procedure, *GetMergeBenefit*(G, S). Procedure *GetMergeBenefit*(G, S) accepts a WVG G and a would-be manipulators' coalition, S. It first computes the sum of the individual power index of the assimilated agents in S using *PowerIndex*(G, i). Then, it alters G by replacing the sum of the weights of the assimilated agents in G with a single weight in a new game G' before computing the power of the bloc $\&S$ in G'. Finally, *GetMergeBenefit*(G, S) returns the factor of increment of the merged bloc $\&S$. Let $A(G)$ be the pseudopolynomial running time of *PowerIndex*(G, i). Now, since $|S| \leq |I| = n$, procedure *GetMergeBenefit*(G, S) takes at most $O(n \cdot A(G))$ time which is pseudopolynomial.

We now use *GetMergeBenefit*(G, S) to construct an algorithm that manipulators can use to determine a coalition that yields a good benefit in merging. We first argue that manipulators tend to prefer coalitions which are small in size because they are easier to form and less likely to be detected. Also, intra-coalition coordination, communication, and other overheads increase with coalition size. Thus, we suggest a limit on the size of the manipulators' coalitions since it is unrealistic and impractical that *all* agents in a WVG will belong to the manipulators' coalition. This reasoning is consistent with the assumptions of the previous work on annexation and merging [2,18] as well as coalition formation [24]. We note, however, that limiting the manipulators coalitions' size this way does not change the complexity class of the problem as finding the coalition that yields the most improved benefit remains NP-hard.

Consider a WVG of n agents. Suppose the manipulators' coalitions, S, have a limit, $k < n$, on the size of the members of the coalitions, i.e., S, are bounded as $2 \leq |S| \leq k$. In this case, the number of coalitions that the manipulators need to examine is at most $O(n^k)$ which is polynomial in n. Specifically, the total number of these coalitions is:

$$\binom{n}{2} + \binom{n}{3} + \cdots + \binom{n}{k} = \sum_{j=2}^{k} \binom{n}{j}. \tag{3}$$

So, we have

$$\begin{aligned}
\sum_{j=2}^{k} \binom{n}{j} &= \sum_{j=2}^{k} \frac{n(n-1)\cdots(n-j+1)}{j!} \\
&\leq \sum_{j=2}^{k} \frac{n^j}{j!} \\
&\leq \sum_{j=2}^{k} \frac{n^j}{2^{j-1}} \\
&= \frac{n^2}{2^1} + \frac{n^3}{2^2} + \cdots + \frac{n^k}{2^{k-1}} = O(n^k).
\end{aligned}$$

Running *GetMergeBenefit*(G, S) while updating the most[1] improved benefit found so far from each of these coalitions requires a total running time of $O(n^k \cdot A(G))$ which is pseudopolynomial, and thus becomes reasonable to compute.

4.3 Manipulation Algorithm for Annexation

Our pseudopolynomial manipulation algorithm for annexation provides a modification of the merging algorithm. We first replace *GetMergeBenefit*(G, S) with another procedure, *GetAnnexationBenefit*(G, i, S). *GetAnnexationBenefit*(G, i, S) accepts a WVG G, an annexer, i, and a coalition S to be assimilated by i. The procedure then returns the factor of increment of the assimilated bloc $\&(S \cup \{i\})$.

We use *GetAnnexationBenefit*(G, i, S) to construct an algorithm that the annexer can use to determine the coalition that yields the most improved benefit in annexation. The

[1] We refer to the most improved benefit among the $O(n^k)$ polynomial coalitions and not from the original 2^n coalitions since we have restricted each manipulators' coalition size to a constant $k < n$.

method of construction of the algorithm is the same as that of the manipulation algorithm for merging with the exception that we add the weight of an annexer i to the weight of each coalition S and compare the power index $\Phi_{\&(S\cup\{i\})}(G')$ of the assimilated bloc in a new game G' to the power index $\Phi_i(G)$ of the annexer in the original game G. The annexer examines a polynomial number of coalitions of the agents assuming a limit $k < n$ on the size of each coalition. Since the annexer belongs to a coalition it annexes, the total number of coalitions examined by the annexer is:

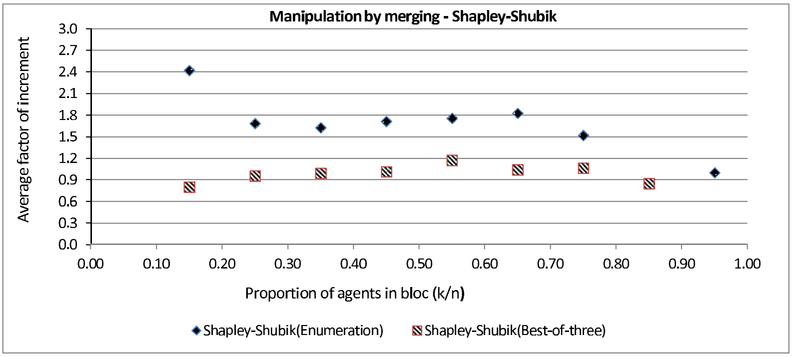
$$\binom{n-1}{1} + \cdots + \binom{n-1}{k-1} = \sum_{j=1}^{k-1} \binom{n-1}{j}. \tag{4}$$

Bounding this equation using similar approach as in Equation 3 shows that Equation 4 is $O(n^k)$. Thus, as before, the manipulation algorithm for annexation also runs in pseudopolynomial time, with a total running time of $O(n^k \cdot A(G))$.

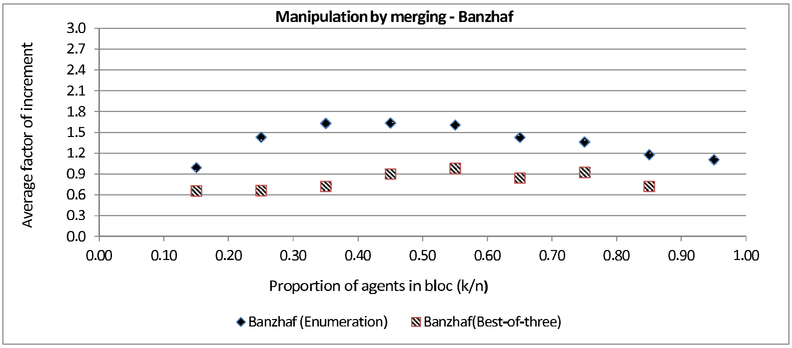
5 Evaluation of the Manipulation Algorithms

We first argue that the simple random simulation of [18] for manipulation by annexation and merging is unintelligent. Thus, it is impractical that strategic agents would employ such method to engage in manipulation [19]. This is because simply guessing a particular coalition from among all the exponential possible coalitions provides a rare chance for strategic agents to benefit in both manipulation by annexation and merging. We note also that this chance of success by the manipulators decreases as the number of agents in a game becomes large. Now, since it is impractical to exhaustively consider all the exponential possible coalitions, we have presented two pseudopolynomial manipulation algorithms where we have restricted the sizes of coalitions to be considered by the manipulators to a constant k which is less than the number n of the agents in the game. Our idea is for the manipulators to sacrifice optimality for good merging or annexation. By doing so, the manipulation algorithms potentially bypass a lot of search. Although, the manipulation algorithms are incomplete, nonetheless, they consider more search space than the simple random approach, and hence, are guaranteed to find a much improved factor of increment than the simple random simulation of [18].

We perform experiments to confirm the above hypothesis. First, we make a simple modification to the random simulation of [18] which provides manipulators with higher average factor of increment. The modification involves the selection of the best factor of increment from three random choices (which we refer to as the *best-of-three* method). We compare results of our enumeration-based method with those of the best-of-three method. We randomly generate WVGs. The weights of agents in each game are chosen such that all weights are integers and drawn from a normal distribution, $N(\mu, \sigma^2)$, where μ and σ^2 are the mean and variance. We have used $\mu = 50$ and values of standard deviation σ from the set $\{5, 10, \ldots, 40\}$. When creating a new game, the quota, q, of the game is randomly generated such that $\frac{1}{2}w(I) < q \leq w(I)$, where $w(I)$ is the sum of the weights of all agents in the game. The number of agents, n, in each of the original WVGs is chosen uniformly at random from the set $\{10, 11, \ldots, 20\}$ while the number of assimilated agents k is uniformly chosen at random from the set $\{5, 6, \ldots, 10\}$.



(a)

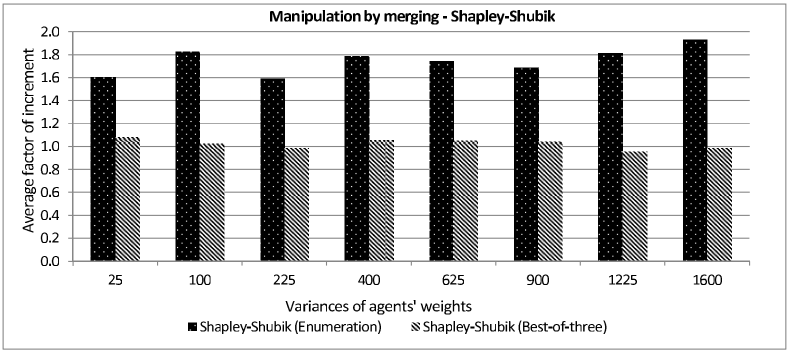


(b)

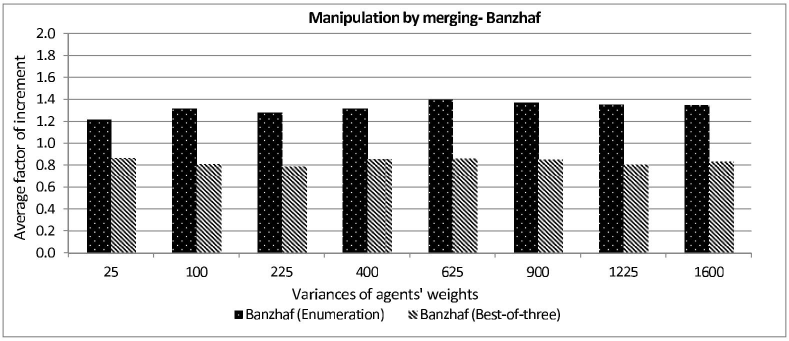
Fig. 4. The average factor of increment for merging for the enumeration and best-of-three methods using various values of size of bloc divided by the number of agents in the games

We have used a total of 500 original WVGs for each experiment and compute the average factor of increments over the entire set of games for the two power indices. The evaluation is carried out for the proportion of agents in manipulators' blocs and the variance of the weights of agents in the WVGs. We have implemented efficient exact polynomial methods of computing the Shapley-Shubik and Banzhaf indices for integer weights using generating functions [7,9] which can be exponential-time in complexity in the worst case.

We first present the results of the case of manipulation by merging and then provide discussion on the results of manipulation by annexation. Figure 4 shows the benefits from merging for both the best-of-three and enumeration-based methods using the two indices. The x-axes indicates the proportion of agents in the manipulators' bloc (i.e. $\frac{|S|}{|I|} = \frac{k}{n}$) whose factor of increment were reported while the y-axes are the average factor of increment achieved by manipulating agents in those coalitions. For the Shapley-Shubik power index, Figure $4(a)$ shows that manipulating agents achieved improved power using the enumeration approach than using the best-of-three method. There are cases where the manipulators achieved more than 2.4 times as much as the original power for the enumeration method, and in general the average factor of increment



(a)



(b)

Fig. 5. The average factor of increment for merging for the enumeration and best-of-three methods using different variances of agents' weights

is between 1.5 and 2.7 times the original power of the manipulators. Whereas, the best-of-three method has only minor effects for the manipulators as the average factors of increment in these tests are below 1.3. Similar trends between the enumeration and best-of-three methods are observed for the case of Banzhaf index too (See Figure $4(b)$). However, the average factor of increment is lower for the two methods using the Banzhaf index. On average, merging does not appear to significantly improve power using the best-of-three method for either the Shapley-Shubik or Banzhaf index, and in most cases is harmful for the agents. We conclude that since improvement in power over the best-of-three method can be achieved with only a polynomial amount of work, then, manipulators are more likely to seek a much improved power gain in merging using the enumeration-based approach.

Figure 5 provides further comparison of the enumeration and best-of-three methods for the two indices using different variances of agents' weights. It is clear from both Figures $5(a)$ and $5(b)$ that irrespective of the variances in the weights of agents in the games, the enumeration method is better in all cases. As before, the figures also show that on average, the best-of-three method does not appear to improve average factor of increment. Finally, for the case of manipulation by annexation and for both the Shapley-Shubik and Banzhaf indices, the impovement in power achieved by manipulating agents using the enumeration method is by far more than those achieved using the best-of-three method.

6 Heuristics for Annexation and Merging in WVGs

Unlike the manipulation algorithms for annexation and merging of Section 4 where we have n and k as the number of agents and the number of assimilated agents in a WVG, manipulating agents may not be interested in achieving the most improved power gain among the $O(n^k)$ polynomial coalitions described before. This is because the number of these coalitions may be large even for small values of n and k. Thus, we propose heuristics that agents may use for annexation in WVGs. Considering the basic requirements for a good heuristic as its ease of computation and to be as informative as possible [14], the heuristics we propose are designed to first avoid the enumeration approach of the manipulation algorithms in Section 4. Second, we avoid the unintelligent simple random approach of [18] to ensure that our heuristics provide good information for manipulating agents to make decisions on how to annex in WVGs.

6.1 Annexation Heuristics

We recall the definition of annexation in Section 2 and from [2,13], the power of the assimilated bloc in an altered WVG is compared to the power of the annexer in the original game. By this definition, intuition suggests that annexation should always be advantageous. This intuition is indeed true using the Shapley-Shubik index to compute agents' power. However, there exist situations where annexation is disadvantageous for the annexer using the Banzhaf index. See [2,3,13] for examples of WVGs where annexation is disadvantageous for the annexer using the Banzhaf index. This case where annexation results in power decrease for the annexer is refer to as the *bloc paradox* [13].

Again, recall from Equation 4 that an annexer needs to examine a polynomial number of assimilated coalitions of size at most $k-1$ to find the most improved power gain among these coalitions whose sizes we have restricted to k. We note also that the assimilated coalitions with maximal weights among these coalitions are those of sizes $k-1$. Thus, it is enough for a particular annexer to check only the assimilated coalitions of size exactly $k-1$ in order for the annexer to find the coalition with the most improved benefit using the two power indices. This indeed is the case for the Shapley-Shubik index. For the case of Banzhaf index, we conduct a test to check the highest factor of increment among all the coalitions of $2,000$ different WVGs. We found that the coalitions with maximal weights (i.e., those having sizes of $k-1$) yield the highest possible factor of increment in over 82% of the games. The remaining highest factor of increment are archieved by manipulators' blocs with lower number of agents in them. In none of these two situations do we experience the bloc paradox. This suggests that the bloc paradox for the Banzhaf power index may be a rare phenomenon in practice.

There are only $\binom{n-1}{k-1}$ such assimilated coalitions to be considered by this annexer. As seen, the amount of work carried out by the annexer is still polynomial, however, this heuristic requires smaller computational effort compared to that of the enumeration-based method and thus makes it more useful in practice. We can even do better using the idea in the following heuristic:

> **MaximalWeights Heuristic:**
> *Given*: A WVG of n agents with a distinguished annexer i.
> *Procedure*: Since agents' weights are given in ascending order, let agent i annex the last $k-1$ agents from the remaining $n-1$ agents in the game.

As before, let $A(G)$ be the pseudopolynomial running time of Shapley-Shubik or Banzhaf power index of an agent according to [7,9]. The running time of the *MaximalWeights* heuristic is $O(n + A(G))$. Here is a brief analysis. We just need to sum the weights of the last $k-1$ agents from the remaining $n-1$ agents to that of the annexer at a cost of $O(k)$. In the worst case, it takes $O(n)$ to sum the weights when $k = n$, i.e., the annexer is able to annex all the remaining $n-1$ agents. Computing the power of the annexer in each of the original and altered games takes $O(A(G))$. Therefore, the total running time of the heuristic is $O(n + A(G))$. Now, apart from the fact that this heuristic appears to find the most improved gain, its running time is by far less than the $O(n^k \cdot A(G))$ running times of the manipulation algorithm for annexation of Subsection 4.3 and the above heuristic especially when k is large. Thus, the *MaximalWeights* heuristic is more useful in practice for the annexer.

7 Related Work

Weighted voting games and power indices are widely studied [8,12,17]. Prominent real-life situations where WVGs have found applications include the United Nations Security Council, the International Monetary Fund [1,20], the Council of Ministers, and the European Community [12]. The need to compensate agents from jointly derived payoff in WVGs has also necessitated the assignment of power to players. A widely accepted method for measuring power of agents in WVGs uses power indices. *Fairness* in the assignment of power to players in a game is also a concern of most of the power indices. The two most prominent and widely used power indices are Shapley-Shubik [23] and Banzhaf [6] power indices. Other power indices found in the literature include Deegan-Packel [10], Johnsoton [16], and Holler-Packel [15] power indices. Computing the Shapley-Shubik and Banzhaf power indices of players in WVGs is NP-hard [22]. The power indices of voters using any of Shapley-Shubik and Banzhaf power indices can be computed in pseudo-polynomial time using dynamic programming [21]. Efficient exact algorithms using generating functions [7,9] also exist for both the Shapley-Shubik and Banzhaf power indices for WVGs where the weights of all agents are restricted to integers. There are also approximation algorithms [5,11] for computing the Shapley-Shubik and Banzhaf power indices in WVGs.

Felsenthal and Machover [12,13] originally studied annexation and alliance (or merging) in WVGs. They consider when the blocs formed by annexation or merging are advantageous or disadvantageous. They show that using the Shapley-Shubik index, it is always advantageous for a player to annex some other players in the game. However, this is not true for Banzhaf power index. Furthermore, they show that merging can be advantageous or disadvantageous for the two power indices. In contrast to our work, they do not consider the extent to which the agents involved in annexation or merging may gain, which we study in this paper. Aziz et al. [2], have also considered the computational aspects of the problem of annexation and merging in WVGs. They show that determining if there exists a beneficial merge in a WVG is NP-hard using both the Shapley-Shubik and Banzhaf indices. The same is also true for determining the existence of beneficial annexation using the Banzhaf index.

8 Conclusions

We investigate the effects of manipulations (i.e., dishonest behaviors) by annexation and merging in weighted voting games. These manipulations involve an agent or agents misrepresenting their identities in anticipation of gaining more power in a game. We evaluate the effects of these manipulations using two prominent power indices, *Shapley-Shubik* and *Banzhaf* indices, to compute agents' power. We first provide visual illustrations of manipulation in weighted voting games to give some insights into why it is difficult to predict how to merge. We then show that manipulators need to do only a polynomial amount of work to find a much improved power gain, and present two enumeration-based pseudopolynomial algorithms that manipulators can use. Furthermore, we provide a careful investigation of heuristics for annexation which provide huge savings in computational efforts over the enumeration-based method. The benefits achievable by manipulating agents using these heuristics also compare with those of the enumeration-based method which serves as upper bound.

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