Asset Value Game and Its Extension: Taking Past Actions into Consideration

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Abstract. In 1997, a minority game (MG) was proposed as a non-cooperative iterated game with an odd population of agents who make bids whether to buy or sell. Since then, many variants of the MG have been proposed. However, the common disadvantage in their characteristics is to ignore the past actions beyond a constant memory. So it is difficult to simulate actual payoffs of agents if the past price behavior has a significant influence on the current decision. In this paper we present a new variant of the MG, called an asset value game (AG), and its extension, called an extended asset value game (ExAG). In the AG, since every agent aims to decrease the mean acquisition cost of his asset, he automatically takes the past actions into consideration. The AG, however, is too simple to reproduce the complete market dynamics, that is, there may be some time lag between the price and his action. So we further consider the ExAG by using probabilistic actions, and compare them by simulation.

Keywords: Multiagent, Minority game, Mean asset value, Asset value game, Contrarian, Trend-follower.

1 Introduction

Background. A minority game (MG) has been extensively studied since it was originally proposed [7]. It is considered as a model for financial markets or other applications in physics. It is a non-cooperative iterated game with an odd population size N of agents who make bids whether to buy or sell. Since each agent aims to choose the group of minority population, he is called a *contrarian*. Every agent makes a decision at each step based on the prediction of a strategy according to the sequence of the m most recent outcomes of winners, where m is said to be the memory size of the agents. Though MG is a very simple model, it captures some of the complex macroscopic behavior of the markets.

It is also known that the MG cannot capture large price drifts such as bubble/crash phenomena, but just can do the stationary state of the markets. This can be intuitively explained as follows. Suppose that a group of buyers can keep a majority for a long time. Then a group of sellers must continuously win in the bubble phenomenon. However, since every agent wants to win and thus joins the group of sellers one after another,

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it will gain a majority soon. That is, the group of buyers cannot keep a majority, a contradiction. Thus, it is difficult to simulate the bubble phenomenon by MG.

Related Work. Much work has been done for the purpose of adapting MG to a real financial market. For example, first, several authors investigated the majority game (MJ), consisting of *trend-followers*. Marsili [14] and Martino et al. [15] investigated a mixed majority-minority game by varying the fraction of trend-followers. Tedeschi et al. [17] considered agents who change themselves from contrarians to trend-followers, and vice versa, according to the price movements. Second, another way is to incorporate more realistic mechanism. A grand canonical minority game (GCMG) [5,10,11] is considered as one of the most successful models of a financial market. In the GCMG, a set of agents consist of two groups, called producers and speculators, and the speculators are allowed not to trade in addition to buy and sell. Third, it is also useful to improve the payoff function. Andersen and Sornette[1] proposed a different market payoff, called $-game, in which the timing of strategy evaluation is taken into consideration. Ferreira and Marsili[9] compared the behavior of the $-game with that of the MG/MJ. The difficulty of the $-game is to evaluate its payoff function because we have to know one step future result. Kiniwa et al. [12] proposed an improved $-game, in which the timing of evaluation is delayed until the future result is turned out. Fourth, there are some other kinds of improvement. Liu et al. [13] proposed a modified MG, where agents accumulate scores for their strategies from the recent several steps. Recent work by Challet [4] proposed a more sophisticated model using asynchronous holding positions which are driven by some patterns. Finally, two books [6,8] comprehensively described the history of minority games, mathematical analysis, and their variations. Beyond the framework of MG, efforts to reproduce the real market dynamics are continued [16,18].

Motivation. The purpose of this paper is also to improve MG by the thirdly mentioned above. Though the framework of MG and its variants seem to be reasonable, we have a basic question — "Do people always make decisions by using their strategies depending on the recent history ?" Some people may just take actions by considering losses and gains. For example, if one has a company's stock which has rapidly risen (resp. fallen), he will sell (resp. not sell) it soon without using his strategy as illustrated in Figure 1. Such a situation gives us the idea of an acquisition cost, or a mean asset value. In the conventional games, like the original MG, an agent forgets the past events and makes a decision by observing only the price up/down within the memory size [1]. In our game, however, each agent evaluates the strategies by whether or not the current price exceeds his mean asset value. Since the mean asset value contains all the past events in a sense, he can increase his net profit by reducing the mean asset value. We call the game an *asset value game*, denoted by AG.

However, there is still an unsolved problem in AG that stems from the framework of MG: the payoff function does not give an action, but just adds points to desirable strategies. Thus, if the adopted strategy is not desirable, the agent has to wait until the

[1] Recently, several studies [2,3] in this direction have been made from the viewpoint of evolutionary learning.

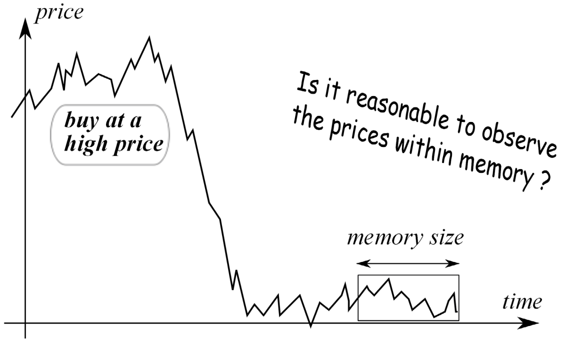

Fig. 1. Illustration of our idea

desirable one gains the highest score. So, there is a time lag between the rapid change of a price and the adjustment of an agent's behavior.

To improve the time lag, we allow each agent another action that satisfies the payoff function with some probability. If the price rises/falls rapidly and the difference between the price and agent i's mean asset value exceeds some threshold, the agent i may take the action according to the payoff function (regardless of the strategy). By tuning up the threshold, etc., we can reproduce the real market dynamics. We call the variant of AG an *extended asset value game*, denoted by ExAG.

Contributions. Our contributions in this paper are summarized as follows:

- We present a new variant of the MG, called an asset value game.
- To improve the problem of AG, we further consider an extended AG.
- We investigate the behavior of AG and ExAG in detail.

The rest of this paper is organized as follows. Section 2 states our model, which contains MG, MJ, AG and ExAG. Section 3 presents an analysis of AG. Section 4 describes a simulation model and shows some experimental results. Finally, Section 5 concludes the paper.

2 Models

In this section, we first describe MG and MJ in Section 2.1, then the difference between MG and AG in Section 2.2. Finally, we describe the difference between AG and ExAG in Section 2.3.

2.1 Previous Model — MG and MJ

At the beginning of the game, each agent $i \in \{1, \ldots, N\}$ is randomly given s *strategies* $\mathbf{R}^{i,a}$ for $a \in \{1, \ldots, s\}$. The number of agents, N, is assumed to be odd in order to break a tie. Any strategy $R_{i,a}(\mu) \in \mathbf{R}^{i,a}$ maps an m-length binary string μ into a decision -1 or 1, that is,

$$\mathbf{R}^{i,a} : \{-1, 1\}^m \longrightarrow \{-1, 1\}, \tag{1}$$

where m is the memory of agents. A *history* H, e.g., $[-1, 1, 1, \ldots]$, is a sequence of -1 and 1 representing a *winning decision* $h(t)$ for each time step $t \in T = (0, 1, 2, \ldots)$. The winning decision of MG (resp. MJ) is determined by the minority (resp. majority) group of -1 or 1. Each strategy $R_{i,a}(\mu) \in \mathbf{R}^{i,a}$ is given a *score* $U_{i,a}(t)$ so that the best strategy can make a winning decision. For the last m winning decisions, denoted by $\mu = h^m(t-1) \subseteq H$, agent i's strategy $R_{i,a}(\mu) \in \mathbf{R}^{i,a}$ determines -1 or 1 by (1). Among them, each agent i selects his highest scored strategy $R_i^*(\mu) \in \mathbf{R}^{i,a}$ and makes a decision $a_i(t) = R_i^*(\mu)$ at time $t \in T$. The highest scored strategy is represented by

$$R_i^*(\mu) = \arg \max_{a \in \{1, \ldots, s\}} U_{i,a}(t), \tag{2}$$

which is randomly selected if there are many ones. An aggregate value $A(t) = \sum_{i=1}^{N} a_i(t)$ is called an excess demand. If $A(t) > 0$, agents with $a_i(t) = -1$ win, and otherwise, agents with $a_i(t) = 1$ win in MG, and vice versa in MJ. Hence the payoffs g_i^{MG} and g_i^{MJ} of agent i are represented by

$$g_i^{MG}(t+1) = -a_i(t)A(t) \text{ and} \tag{3}$$
$$g_i^{MJ}(t+1) = a_i(t)A(t), \text{ respectively.} \tag{4}$$

The winning decision $h(t) = -1$ or 1 is added to the end of the history H, i.e., $h^{m+1}(t) = [h^m(t-1), h(t)]$, and then it will be reflected in the next step. After the winning decision has been turned out, every score is updated by

$$U_{i,a}(t+1) = U_{i,a}(t) \oplus R_{i,a}(\mu) \cdot sgn(A(t)), \tag{5}$$

where $\oplus$ means subtraction for MG (addition for MJ) and $sgn(x) = 1\ (x \geq 0),\ = -1\ (x < 0)$. In other words, the scores of winning strategies are increased by 1, while those of losing strategies are decreased by 1. We simply say that an agent increases selling (resp. buying) strategies if the scores of selling (resp. buying) strategies are increased by 1. Likewise the decrement of scores. Notice that the score is an accumulated value from an initial state in the original MG. In contrast, we define it as a value from the last H_p steps according to [13]. That is, we use

$$U_{i,a}(t+1) = U_{i,a}(t) \oplus R_{i,a}(\mu) \cdot sgn(A(t)) - U_{i,a}(t - H_p). \tag{6}$$

The constant H_p is not relevant to m, but is only used for selecting the highest score. Analogous to a financial market, the decision $a_i(t) = 1$ (respectively, -1) represents buying (respectively, selling) an asset. Usually, the price of an asset is defined as

$$p(t+1) = p(t) \cdot \exp \frac{A(t)}{N}. \tag{7}$$

2.2 Asset Value Game

The difference between MG and our asset value game is the payoff function. Let $v_i(t)$ be agent i's mean asset value at time t, and $u_i(t)$ the number of units of his asset. The payoff function in AG is defined as

$$g_i^{AG}(t+1) = -a_i(t)F_i(t), \tag{8}$$

where $F_i(t) = p(t) - v_i(t)$. The mean asset value $v_i(t)$ and the number of asset units $u_i(t)$ are updated by

$$v_i(t+1) = \frac{v_i(t)u_i(t) + p(t)a_i(t)}{u_i(t) + a_i(t)} \tag{9}$$

and

$$u_i(t+1) = u_i(t) + a_i(t), \tag{10}$$

respectively. That is, the payoff function (3) in MG is replaced by (8) in AG. Without loss of generality, we assume that $v_i(t), u_i(t) > 0$ for any $t \in T$.

The basic idea behind the payoff function is that each agent wants to decrease his acquisition cost in order to make his appraisal gain. Figure 2(a) shows the relationship between the price and the mean asset values of $N = 3$ agents, where the price is represented by the solid, heavy line. Notice that if the population size N is small, the price change becomes drastic.

The most important feature of the AG is to appreciate the past gains and losses. Even though an agent has bought a high-priced asset during the asset-inflated term (see Figure 1), the mean asset value of the agent reflects the fact and an appropriate action compared with the current price is recommended.

2.3 Extended Asset Value Game

Here we consider the drawbacks of AG, and present an extended AG, denoted by ExAG, to improve them. Though the AG captures a good feature of an agent's behavior, the payoff function indirectly appreciates desirable strategies. If the adopted strategy is not desirable, the agent has to wait until the desirable one gains the highest score. So, there is a time lag between the rapid change of a price and the adjustment of an agent's behavior.

More precisely, the movement of price is followed by the asset values (see arrows in Figure 2(a)). This behavior can be explained by the following reasons. If the price rapidly rises, it exceeds almost all the mean asset values. Then, $F_i(t) = p(t) - v_i(t)$ becomes plus and the $a_i(t) = -1$ (i.e., sell) action is recommended. So, some agents change from trend-followers to contrarians in a few steps. During the steps, such agents remain trend-followers, that is, buy assets at the high price. Thus, their mean asset values follow the movement of price.

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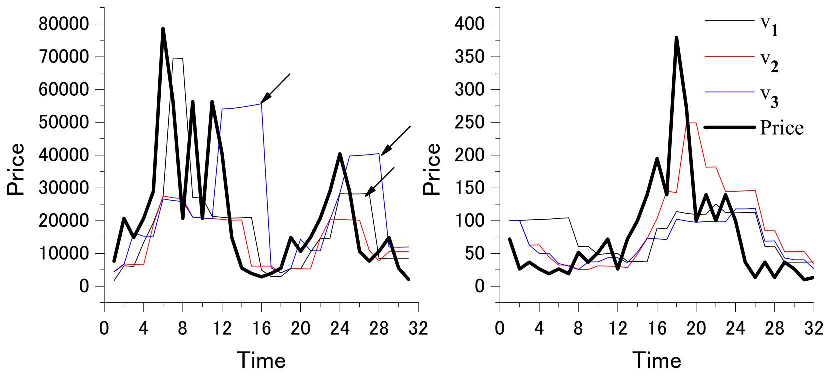
Our solution is to provide another option of the agent. That is, the agent who has much higher/lower asset value than the current price can directly act as the payoff function, called a *direct action*. However, if so, every agent may take the same action when the price go beyond every asset value. To avoid such an extreme situation, we give the direct action with some probability.

Let $K = K^+ \ (F_i \geq 0),\ K^- \ (F_i < 0)$ be the F_i's threshold over which the agent may take the direct action, and let λ be some constant. Each agent takes the same action as the payoff function (without using his strategy) with probability

$$p = \begin{cases} 1 - \exp\{-\lambda(|F_i| - K)\} & (K \leq |F_i|) \\ 0 & (|F_i| < K), \end{cases} \tag{11}$$



(a) Asset value game (b) Extended asset value game

Fig. 2. Price influence on mean asset values

where

$$K = \begin{cases} K^+ & 0 \leq F_i \\ K^- & F_i < 0 \end{cases} \quad \text{such that } K^- < K^+$$

and takes the action according to his strategy with probability $1 - p$. In short, in ExAG

- agent i takes an action $a_i(t)$ satisfying $g_i^{AG}(t+1) > 0$ with probability p, and
- an action $a_i(t) = R_i^*(\mu)$ with probability $1 - p$.

Figure 2(b) shows the behavior of the price and the mean asset values for $N = 3$ agents in our extended AG, where $K^+ = 300$, $K^- = 50$ and $\lambda = 0.001$. Notice that the change of price in Figure 2(b) is not so drastic as that in Figure 2(a). In addition, all the values do not follow the price movement.

3 Analysis of AG

In this section we briefly investigate the features of AG. Though we mainly discuss the bubble in the following, similar arguments hold for the crash. For convenience, we define a contrarian as follows. If $a_i(t) = -1$ (resp. $a_i(t) = 1$) for a history $h^m(t-1) = \{1\}^m$ (resp. $h^m(t-1) = \{-1\}^m$), agent i is a contrarian. Let t_r be the first time at which the winning decision is reversed after $t - m$. Let $C^{MJ}(t)$, $C^{AG}(t)$ and $C^{MG}(t)$ denote the set of contrarians in MJ, AG and MG, respectively. The next theorem means that the bubble phenomenon is likely to occur in the order of MJ, AG and MG.

Theorem 1. *Suppose that the same set of agents experience $h^m(t-1) = \{1\}^m$ starting from the same scores since $t - m$. Then, for any $t' \in T = (t, \ldots, t_r - 1)$ we have*

$$C^{MJ}(t') \subseteq C^{AG}(t') \subseteq C^{MG}(t').$$

Proof. First, we show that $C^{AG}(t) = C^{MG}(t)$ at time t. Consider an arbitrary agent i. Notice that agent i has the same score both in *AG* and in *MG*. Since $h^m(t-1) = \{1\}^m$, agent i takes the same action based on the same strategy both in *AG* and in *MG*. Thus, we have $C^{AG}(t) = C^{MG}(t)$ at time t.

Next, we show that $C^{AG}(t') \subseteq C^{MG}(t')$ at time $t' \in T$. Notice that all the agents in *MG* increase the selling strategies for $h^m(t-1) = \{1\}^m$. On the other hand, notice that the agents in *AG* that have smaller mean asset values $v(t)$ than the price $p(t)$ increase the selling strategies for $h^m(t-1) = \{1\}^m$. Since every contrarian refers to the same part (i.e., $\{1\}^m$) of the strategy, he does not change his decision during the interval T. If an agent increases the selling strategies in *AG*, it also increases the selling strategies in *MG*. Thus we have $C^{AG}(t') \subseteq C^{MG}(t')$ at time $t' \in T$.

The similar argument holds for $C^{MJ}(t') \subseteq C^{AG}(t')$. □

We call an agent a *bi-strategist* if he can take both buy and sell actions, that is, has strategies $\mathbf{R}^{i,a}$ containing both actions, for $h^m(t-1) = \{1\}^m$ or $h^m(t-1) = \{-1\}^m$. The following lemma states that there is a time lag between the price rising and the action of agent's payoff function.

Lemma 1. *In AG, suppose that a history H contains $h^m(t-1) = \{1\}^m$. Even if a bi-strategist keeps the opposite action of the payoff function for H_p steps, he takes the same action as the payoff function after the $H_p + 1$-st step.*

Proof. Suppose that a bi-strategist i has a strategy R_{i,a_1} (resp. and a strategy R_{i,a_2}) which takes the opposite action of (resp. the same action as) the payoff function. If i adopts the strategy R_{i,a_1} now, the score difference between R_{i,a_1} and R_{i,a_2} is at most $2H_p$. Since the difference decreases by 2 for a step, the scores of R_{i,a_1} and R_{i,a_2} becomes the same point at the H_p-th step. Then, after the $H_p + 1$-st step, he takes the strategy R_{i,a_2}. □

For simplicity, we assume that the size of H_p is greater than m enough.

Lemma 2. *In AG, suppose that a history H contains $h^m(t-1) = \{1\}^m$. For any time steps $t_1, t_2 \in T = (t, \ldots, t_r - 1)$, where $t_1 < t_2$, we have*

$$C^{AG}(t_1) \subseteq C^{AG}(t_2).$$

Proof. Suppose that agent i belongs to $C^{AG}(t_1)$. We show that once the rising price $p(t_1)$ overtakes the mean asset value $v_i(t_1)$ of agent i, $v_i(t_1)$ will not overtake $p(t_1)$ as long as $p(t_1)$ is rising. Since

$$v_i(t+1) - v_i(t) = \frac{a(p-v)}{u+a} > 0 \text{ and } 0 < \frac{a}{u+a} < 1,$$

$p > v$ holds as long as $p(t_1)$ is rising. Thus, agent i is contrarian at time $t_1 + 1$. We have $C^{AG}(t_1) \subseteq C^{AG}(t_1 + 1)$, and can inductively show $C^{AG}(t_1) \subseteq C^{AG}(t_2)$. □

We say that the bubble is *monotone* if $h^m(t-1) = \{1\}^m$ holds for any $t \in T = (t, \ldots, t_r - 1)$.

Lemma 3. *In AG, as long as more than half population are bi-strategists, the price in a monotone bubble will reach the upper bound.*

Proof. First, the mean asset values that has been overtaken by the price will not exceed the price again from the proof of Lemma 2.

Second, any bi-strategist i with $v_i(t) > p(t)$ will take a buying action in the $H_p + 1$ steps from Lemma 1. Since $v_i(t+1) - v_i(t) = a(p-1)/(u+a) < 0$, the mean asset value decreases. Thus, the rising price will eventually reach the greatest mean asset value in the set of contrarians.

Third, since all the bi-strategists increase the selling strategies, they will take selling actions in $H_p + 1$ steps. After that, $A/N < 0$ holds and the price falls down. □

From Lemma 3, the following theorem is straightforward.

Theorem 2. *In AG, as long as more than half population are bi-strategists, the monotone bubble will terminate.* □

4 Simulation

Here we present simulation results by using the basic constants in Table 1 [2].

Table 1. Basic constants

Symbol	Meaning	Value
N	Number of agents	501
S	Number of strategies	4
m	Memory size	4
H_p	Score memory	4
T	Number of steps	5000
—	Initial agent's money	10000
—	Initial agent's assets	100
r	Investment rate	0.01

Our first question with respect to ExAG is :

1. What values are suitable for the constant λ and the threshold K in ExAG ?

Our next question with respect to AG is :

2. How does the inequality of wealth distribution vary in AG ?

Then, our further questions with respect to several games are as follows.

3. How widely do the Pareto indices of games differ from practical data ?
4. How widely do the skewness / kurtosis of games differ from practical data ?
5. How widely do the volatilities differ in several games ?
6. How widely do the volatility autocorrelations differ from practical data ?

For the first issue, Figure 3 shows the patterns of price behavior for three kinds of λ values. From the definition of the direct action probability (see (11)), the smaller the λ

[2] We repeated the experiments up to 30 times and obtained averaged results.

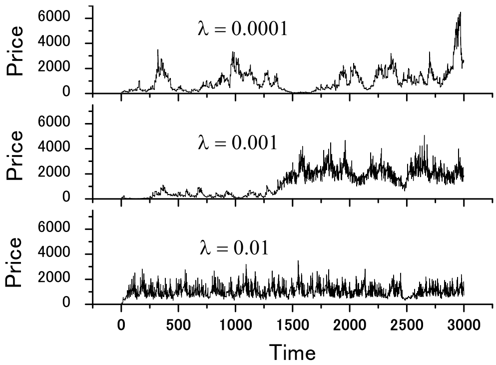


Fig. 3. Price behavior for varying λ in ExAG

becomes, the fewer the number of direct actions occur. Thus, the ratio of trend-followers is high for $\lambda = 0.0001$ and that of contrarians is high for $\lambda = 0.01$.

In addition, Figure 4 shows the skewness and the kurtosis for varying the constant λ, where the skewness (α_3) and the kurtosis (α_4) are defined as

$$\alpha_3 = \sum_{i=1}^{N} \frac{(x_i - \overline{x})^3}{N\sigma^3} \quad \text{and} \quad \alpha_4 = \sum_{i=1}^{N} \frac{(x_i - \overline{x})^4}{N\sigma^4},$$

respectively, for time series variable x_i and its average $\overline{x}$. If the skewness is negative (respectively, positive), the left (respectively, right) tail of a distribution is longer. A high kurtosis distribution has a sharper peak and longer, fatter tails, while a low kurtosis distribution has a more rounded peak and shorter, thinner tails. In other words, the more the patterns of price fluctuation occur, the smaller the kurtosis becomes. Thus, if λ is small and the reversal movements of contrarians are rare, the kurtosis becomes large. On the other hand, if we vary K^- with keeping $K^+ = 500$, the kurtosis is distributed as shown in Figure 5, where a regression curve is depicted.

From the observation above, we set $\lambda = 0.001$, $K^- = 50$ and $K^+ = 500$ in what follows.

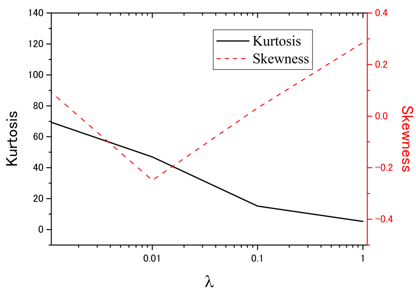


Fig. 4. Skewness / kurtosis vs λ in ExAG

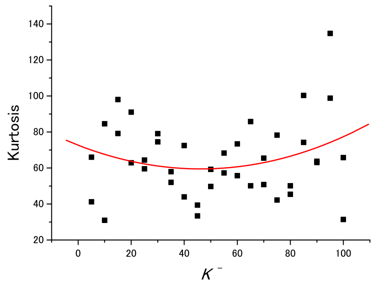
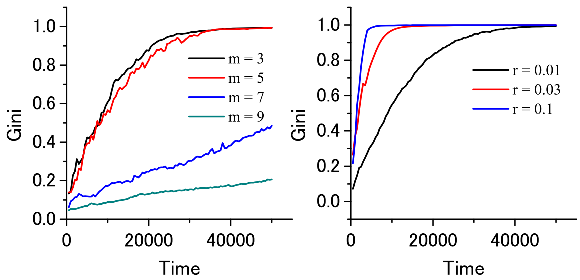


Fig. 5. Kurtosis vs K^- in ExAG



(a) Gini vs Memory Sizes (b) Gini vs Investment Rates

Fig. 6. Influence on Gini coefficient in AG

For the second issue, we present our results in Figure 6. The Gini coefficient is used as a measure of inequality of wealth distribution. Given a set of N agents' wealth $(X_1, X_2, \ldots, X_N)$, the Gini coefficient G is defined as

$$G = \frac{1}{2N^2\overline{X}} \sum_{i=1}^{N} \sum_{j=1}^{N} |X_i - X_j|,$$

where $\overline{X} = \sum_{i=1}^{N} X_i/N$. If $G = 0$, the wealth is completely even. If G is close to 1, an agent has a monopoly on the wealth.

Figure 6(a) shows that the influence of memory size on the Gini coefficient. It means that the smaller the memory size is, the wider the inequality of wealth becomes. If the memory size is small, some successful agents earn much money and the others not. So their mean asset values are widely distributed in the long run. Thus, the Gini coefficient tends to be large.

Figure 6(b) shows that the influence of investment rate on the Gini coefficient. It means that the larger the investment rate is, the wider the inequality of wealth becomes. If the investment rate is large, the successful agents earn much money and the others not. So their mean asset values are widely distributed in the long run. Thus, the Gini coefficient tends to be large.

For the third issue, Figure 7 shows the price decreasing change distribution for several games and NYSE, where NYSE is the Dow-Jones industrial average 20,545 data (1928 /10/1 — 2010/7/26) in New York Stock Exchange. That is, the normalized decreasing change of price $|R| = |\Delta\text{Price}/\sigma|$ and its distribution is compared. The straight lines represent the Pareto indices. At a glance, the curves of ExAG and AG resemble that of NYSE, which means their distributions are likewise. The Pareto index of ExAG is also not far from that of NYSE.

For the fourth issue, we obtained the following results. Both ExAG and AG have better values of skewness and kurtosis than MG does as shown in Tables 2 and 3, where "stdev." and "95% int." mean standard deviation and 95% confidence interval, respectively.

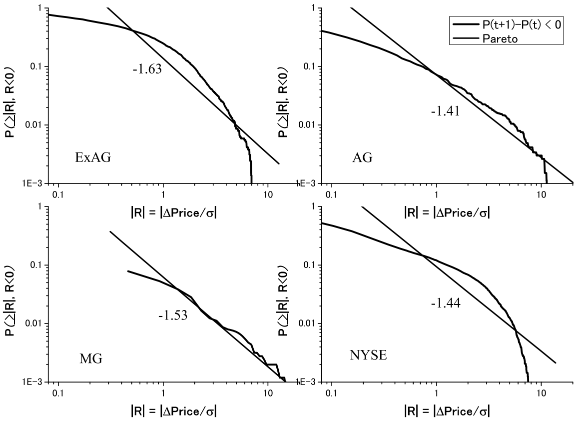


Fig. 7. Pareto indices for several games and NYSE

Table 2. Skewness

method	ExAG	AG	MG	NYSE
average	0.098	0.39	-0.32	3.725
stdev.	1.82	1.03	1.86	—
95% int.	[-0.58,0.77]	[0.007,0.77]	[-1.02,0.37]	—

Table 3. Kurtosis

method	ExAG	AG	MG	NYSE
average	42.3	72.9	148	18.92
stdev.	61.6	96.3	231	—
95% int.	[21.3,67.3]	[36.9,109]	[62.2,235]	—

For the fifth issue, we present our results in Figure 8. The volatility is defined as the standard deviation of the number of excess demand. The figure shows that the volatility of AG is lower than other games for every memory size. This means the memory size does not have a great impact on the price formation in AG.

For the sixth issue, the autocorrelation function $C(\tau)$ is defined as

$$C(\tau) = \frac{\langle A(t)A(t+\tau)\rangle}{\langle A(t)^2\rangle},$$

where τ is a time lag. The value of $C(\tau)$ becomes 1 (respectively, -1) if there is a positive (respectively, negative) correlation between $A(t)$ and $A(t+\tau)$. As shown in Figure 9, only MG has the alternating, strong positive/negative correlation for every time lag. Other games, AG and ExAG, have weak correlations which reduce as the time lag grows. The practical data, NYSE, has a negative correlation only when the time lag is $\tau = 1$. Since the excess demand in NYSE is unknown, we assume the number of

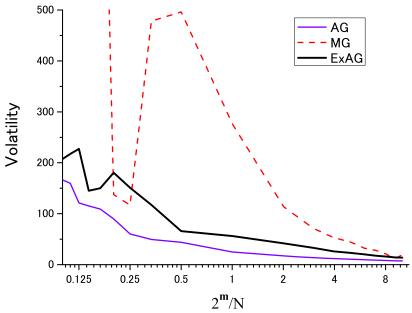


Fig. 8. Volatility ($N = 51 \sim 5119$, $S = 4$, $m = 9$)

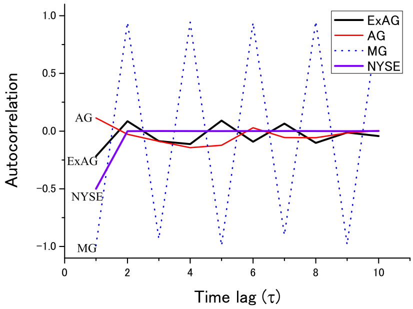


Fig. 9. Autocorrelation of volatility

agents is equal to $N = 501$ and estimate $A(t)$ from the equation (7). Notice that ExAG has the same (negative) correlation as NYSE when $\tau = 1$, while AG has the positive correlation.

5 Conclusions

In this paper, we proposed an asset value game and an extended asset value game. The AG is a simple variant of MG such that the only difference is their payoff functions. Though the AG captures a good feature of an agent's behavior, there is a time lag between the rapid change of a price and the adjustment of an agent's behavior. So we consider the ExAG, an improvement of AG, by using parameters which contain some probabilistic behavior. The ExAG has two parameters by which the balance of trend-followers and contrarians can be controlled. We examined several values for the parameters and then fixed to specified values. We obtained several experimental results which reveals some characteristics of ExAG. The advantages of ExAG are twofold. First, we can restrict a drastic movement of price in AG by tuning the parameters. Second, we can reduce the time lag generated by recovering score losses in AG.

Our future work includes investigating the influence of market intervention, an in-depth analysis of the AG, and other applications of the games.

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