Alan Turing and John von Neumann - Their Brains and Their Computers

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"There exists today a very elaborate system of formal logic, and specifically, of logic as applied to mathematics. This is a discipline with many good sides, but also with certain serious weaknesses. ... Everybody who has worked in formal logic will confirm that it is one of the technically most refractory parts of mathematics. The reason for this is that it deals with rigid, all-or-none concepts, and has very little contact with the continuous concept of the real or of complex number, that is, with mathematical analysis. Yet analysis is the technically most successful and best-elaborated part of mathematics. Thus formal logic is, by the nature of its approach, cut off from the best cultivated portions of mathematics, and forced onto the most difficult part of mathematical terrain, into combinatorics."

- John von Neumann

1 The Duo

Were it not for two decades of the intertwined intellectual lives of Alan Turing and John von Neumann, the disciplines of mathematics and computer science would not be what they are today.

Their shared intellectual path began in 1933, when college student Turing wrote to his mother, Sarah, that his prize book was von Neumann's Mathematical Foundations of Quantum Mechanics, which he described as being "very interesting, and not at all difficult reading, although the applied mathematicians seem to find it rather strong."

Shortly after, in 1935, von Neumann finds his way into the first line of the first sentence in Turing's first paper: "In his [1934] paper Almost periodic functions in a group, J. v. Neumann has used independently the ideas of left and right periodicity. I shall now show that these are equivalent." Such a demonstration of Turing's power of proof surely must have caught von Neumann's attention, for in 1937, he wrote a letter in support of a Princeton fellowship for Turing, and in 1938 offered Turing a position as his assistant which, although it paid $1,500 a year, Turing declined as the shadows of war lengthened in Europe.

The admiration was mutual. In a letter written home from Princeton, von Neumann's is the first name on a list of Princeton luminaries that included "Weyl, Courant, Hardy, Einstein, Lefschetz, as well as hosts of smaller fry."

Though Turing returned to his native England, the two continued to correspond and collaborate for the rest of their all-too-short lives. In 1939, after hearing of a continuous group problem from von Neumann, Turing proved the general negative solution and sent it to von Neumann for Annals of Mathematics (see von Neumann letter to Turing and Stan Ulam letter). A 1949 letter from von Neumann to Turing acknowledged receipt of Turing's submission of a paper for Annals of Mathematics for which von Neumann served as an editor. "Exceedingly glad to get your paper" and "agree with your assessment of the paper character ... our machine-project is moving along quite satisfactory but we are not at the point you are" [16]. (It may be interesting to note that von Neumann would be assigning Turing's famous paper on computable numbers as required reading for his collaborators in the EDVAC project of constructing his computers.)

Even in critical discourse, Turing and von Neumann are intertwined. "The fathers of the field had been pretty confusing," E. W. Dijkstra wrote. "John von Neumann speculated about computers and the human brain in analogies sufficiently wild to be worthy of a medieval thinker and Alan M. Turing thought about criteria to settle the question of whether Machines Can Think, which we now know is about as relevant as the question of whether Submarines Can Swim."

Although Turing was 10 years younger than von Neumann, they acknowledged one another's intellectual seniority, with von Neumann serving as an elder in mathematics to Turing and Turing the elder in computer science to von Neumann. Turing papers on almost periodicity, Lie groups, numerical matrix analysis and word problem for compact groups follow from two relatively deep theorems - one due to Tarski and the other to von Neumann. In a letter to Max Newman, Turing talks about Gödel and von Neumann: "Gödel's paper has reached me at last. I am very suspicious of it now but will have to swot up the Zermelo-v. Neumann system a bit before I can put objections down in black and white. The present report gives a fairly complete account of the proposed calculator. It is recommended however that it be read in conjunction with J. von Neumann's 'Report on the EDVAC' [Proposal for the Development of an Automatic Computing Engine]. Most of the most hopeful scheme, for economy combined with speed, seems to be the 'storage tube' or 'iconoscope' (in J. v. Neumann's terminology)."

Their age difference is irrelevant in another respect: We could consider Turing the grandfather of computer science and von Neumann its father, because the Turing machine was invented in the 1930s, while von Neumann's basic work in the field belongs to the 1940s and 1950s.

We find similarities on many fronts: Turing and von Neumann were essentially involved in the creative intellectual effort required by their governments during the Second World War against Nazism and Fascism, and each was considered a war hero by his country, with von Neumann receiving the Presidential Medal of Freedom and Turing the OBE; both showed interest for biology (although von Neumann's interest in this respect was much longer and deeper); they both

were struck by Gödel's incompleteness theorem and both contributed to a better understanding of its meaning and significance; they both were strongly related in some periods of their lives to Princeton University; they both were attracted by game aspects of computing and of life; and they both left some important unpublished manuscripts. Did they meet? No sign exists in this respect in the available writings. Both lived lives that were too short: Just 41 when he died, Turing lived two years longer than Bernard Riemann; von Neumann died at 53, four years younger than Henri Poincaré was at the time of his death.

Von Neumann was a high achiever from a young age. At 15, he began to study advanced calculus. At 19, he published two major mathematical papers, the second of which gave the modern definition of ordinal numbers. He was 21 when he published *An axiomatization of set theory*, 22 when he began his work on *Mathematical Foundations of Quantum Mechanics* (finished when he was 25) and 24 when he published his minimax theorem. By 26, he was one of the first four people (among them Einstein and Gödel) Princeton University selected for the faculty of its Institute for Advanced Study. He was the first to capture the meaning and significance of Gödel's incompleteness theorem, realizing that "if a system of mathematics does not lead into contradiction, then this fact cannot be demonstrated with the procedures of that system" [12].

In examining the totality of von Neumann's work, it is difficult to find names equal in class. If we refer to those historically near to him, maybe Poincaré and David Hilbert before him and A.N. Kolmogorov, after him. But even with respect to these great names, it is important to observe that von Neumann's impact spans the whole landscape of sciences, be they more or less exact, natural sciences or social sciences, science or engineering (like in his work related to nuclear weapons). From axiomatic foundations of set theory to the foundation of continuous geometry, from measure theory to ergodic theory, from operator theory to its use to build the foundations of quantum mechanics, from probability theory to lattice theory, from quantum logic to game theory, from mathematical economics to linear programming, mathematical statistics and nuclear weapons, computer science, fluid dynamics, weather systems, politics and social affairs, everywhere he shined new light upon the very essential roots of the respective problems. Mediocrity was not his neighbor.

Turing's achievements as a young man are no less remarkable than von Neumann's. On the strength of his fellowship dissertation, *On the Gaussian Error Function*, completed and submitted in November 1934, the 22-year-old Turing was elected a Fellow of King's College four months later, on March 16, 1935. Economist John Maynard Keynes was among the committee members electing him. The paper contained a proof of the Central Limit Theorem, one of the most fundamental in probability theory. In 1937 at age 25 he published his seminal paper *On Computable Numbers, with an Application to the Entscheidungsproblem*, solving one of the most famous problems in mathematics proposed by Hilbert. This paper, with negative and positive results of greatest depth, defining the Turing machine, and inspiring the designers of electronic computers in England and United States – von Neumann, in particular, in such a decisive way – is

without question the most important and influential paper in computer science, one offering proof positive that the new field had emerged.

2 From Leibniz, Boole, Bohr and Turing to Shannon, McCullogh-Pitts and von Neumann - The Emergence of the Information Paradigm

The middle of the past century has been very hot, characterized by the appearance of the new fields defining the move from the domination of the energy paradigm, characterizing the second half of the 19th century and the first half of the 20th century, to the domination of the information-communication-computation paradigms, appearing at the crossroad of the first and the second halves of the 20th century. So, John von Neumann's reflection, by which he became a pioneer of the new era, developed in the context of concomitant emergence in the fifth and the sixth decades of the 20th century of theory of algorithms (A.A. Markov), simply typed lambda calculus (Alonzo Church), game theory (von Neumann and Oskar Morgenstern), computer science (Turing and von Neumann), cybernetics (Norbert Wiener), information theory (Claude Shannon), molecular genetics (Francis Crick, James Watson, Maurice Wilkins, among others), coding theory (R. W. Hamming), system theory (L. von Bertalanffy), control theory, complexity theory, and generative grammars (Noam Chomsky). Many of these lines of development were no longer available to von Neumann and we are in the situation to question the consequences of this fact.

Von Neumann was impressed by Warren McCullogh and Walter Pitts's result connecting logic, language and neural networks, [10]. In von Neumann's formulation, this result shows "that anything that can be exhaustively and unambiguously described, anything that can be completely and unambiguously put into words, is ipso facto realizable by a suitable finite neural network. Three things deserve to be brought into attention in this respect: a) In the 19th century, George Boole's project to unify logic, language, thought and algebra (continuing Leibniz's dream in this respect) was only partially realized (An investigation in the laws of thought, on which are founded the mathematical theories of logic and probabilities, 1854) and it prepared the way for similar projects in the 20th century; b) Claude Shannon, in his master's thesis (A symbolic analysis of relay and switching circuits) submitted in 1937, only one year after Turing published his famous Non-computable..., proved the isomorphism between logic and electrical circuits; c) Niels Bohr, in his philosophical writings, developed the idea according to which the sphere of competence of the human language is limited to the macroscopic universe; see, in this respect, [5]. Putting together all these facts, we get an image of the strong limitations that our sensations, our intuitions, our logic and our language have to obey. We can put all these things in a more complete statement: The following restrictions are mutually equivalent: to be macroscopic; to be Euclidean (i.e. to adopt the parallel axiom in the way we represent space and spatial relations); to be Galileo-Newtonian in the way we represent motion, time and energy; to capture the surrounding and to act

according to our sensorial-intuitive perception of reality; to use and to represent language, in both its natural and artificial variants (moreover, to use human semiosis in all its manifestations).

So a natural sequence emerges, having Leibniz, Boole, Bohr, Turing, Shannon, McCullogh-Pitts and von Neumann as successive steps. It tells us the idea of the unity of human knowledge, the unifying trend bringing in the same framework logic, language, thought and algebra. But we have here only the discrete aspects, while von Neumann wanted much more.

3   John von Neumann's Brain - von Neumann's Unification: Formal Logic + Mathematical Analysis + Thermodynamic Error

It was only too fitting for von Neumann to study the most inspiring automaton of all: the brain. "Our thoughts ... mostly focused on the subject of neurology, and more specifically on the human nervous system, and there primarily on the central nervous system. Thus in trying to understand the function of the automata and the general principles governing them, we selected for prompt action the most complicated object under the sun - literally."

His theory of building reliable organs from unreliable components and the associated probabilistic logics was focused on modeling system errors in biological cells, central nervous systems cells in particular. His research program aimed boldly at the unification of the "most refractory" and "rigid" formal logic (discrete math) with the "best cultivated" mathematical analysis (continuous math) proposals via a concept of thermodynamic error. "It is the author's conviction, voiced over many years, that error should be treated by thermodynamical methods, and be the subject of a thermodynamical theory, as information has been, by the work of L. Szilard and C. E. Shannon." Turing also uses thermodynamics arguments in dealing with errors in computing machines. For von Neumann this was at the core of a theory of information processing for the biological cell, the nervous system and the brain. The error model was given the latitude to approximate and therefore was not an explicit model of "the more complicated aspects of neuron functioning: threshold, temporal summation, relative inhibition, changes of the threshold by aftereffects of stimulation beyond synaptic delay, etc." He proposed two models of error. One, concrete - ala Weiner and Shannon "error is noise" where in every operation the organ will fail to function correctly in a statistically independent way with respect with the state of the network, i.e. with "the (precise) probability epsilon" and another one, more realistic assuming an unspecified dependence of the errors on the network and among them. For detailing the dependence to the general state of the network, more needed to be known about the biological "microscopic" mechanism, about which von Neumann was growing increasingly frustrated since technology had not yet advanced to the point necessary. Indeed, it is here where molecular biology developments since von Neumann's time could bring the next well-defined concepts of errors that would satisfy his axioms. He managed in the paper to

prove a constructive version of biological channel "capacity" that Shannon could only prove nonconstructively.

In a 1946 letter to Norbert Wiener, von Neumann expresses his unhappiness with the results of "Turing-cum-Pitts-and-McCulloch:"

"What seems worth emphasizing to me is, however, that after the great positive contribution of Turing-cum-Pitts-and-McCulloch is assimilated, the situation is rather worse than before. Indeed, these authors have demonstrated in absolute and hopeless generality that anything and everything Browerian can be done by an appropriate mechanism, and specifically by a neural mechanism - and that even one, definite mechanism can be 'universal.' Inverting the argument: Nothing that we may know or learn about the functioning of the organism can give, without 'microscopic,' cytological work any clues regarding the further details of neural mechanism ... I think you will feel with me the type of frustration that I am trying to express."

He expresses skepticism that neurological methods would help in understanding the brain as much as experimenting with a fire hose on a computing machine. "Besides the system is not even purely digital (i.e. neural): It is intimately connected to a very complex analogy (i.e. humoral or hormonal) system, and almost every feedback loop goes through both sectors, if not through the 'outside' world (i.e. the world outside the epidermis or within the digestive system) as well. And it contains, even in its digital part, a million times more units than the ENIAC."

Another basic idea in von Neumann's writings is related to the analog-digital distinction and to the fact that the noise level is strongly inferior in digital machines than in the analog ones. However, in living organisms both analog and digital aspects are essential, and von Neumann indicates the contrast between the digital nature of the central nervous system and the analog aspect of the humoral system. To capture the novelty of these considerations, we have to point out several aspects. The analog-digital distinction is a particular form of the more general distinction between discreteness and continuity. In mathematics, the use of expressions such as discrete mathematics and continuous mathematics became frequent only in the second half of the 20th century, in contrast with other fields, such as biology, psychology and linguistics, where the discrete-continuous distinction appeared earlier. The famous mind-body problem considered by Leibniz is just the expression of the dual discrete-continuous nature of the human being. Leibniz is announcing both the computing era, by his digital codification, and the theory of dynamical systems as a framework of the mathematical model of the human body.

4   "You Would Certainly Say That Watson and Crick Depended on von Neumann"

Nobel laureate Sydney Brenner talks about von Neumann as one of his heroes in his memoir, [2]. Brenner was a close collaborator with Francis Crick. These reflections and story are possibly the greatest mathematical insight of all times for biology. That qualifies von Neumann as a prophet.

Freeman Dyson noted that what today's high school students learn about DNA is what von Neumann discovered purely by mathematics.

Brenner recalls a symposium titled "The Hixton Symposium on Cerebral Mechanism in Behaviour," held in Pasadena, California, in 1948. "The symposium was published in 1951, and in this book was a very famous paper by John von Neumann, which few people have read. The brilliant part of his paper in the Hixton Symposium is his description of what it takes to make a self-reproducing machine. Von Neumann shows that you have to have a mechanism for not only copying the machine, but copying the information that specifies the machine. So he divided - the automaton as he called it - into three components: the functional part of the automaton; a decoding section which actually takes a tape, reads the instructions and builds the automaton; and a device that takes a copy of this tape and inserts it into the new automaton.

"Now this was published in 1951, and I read it a year later in 1952. But we know from later work that these ideas were first put forward by him in the late 1940s. ... It is one of the ironies of the entire field that were you to write a history of ideas of the whole DNA, simply from the documented information as it exists in the literature - that is, a kind of Hegelian history of ideas - you would certainly say that Watson and Crick depended on von Neumann, because von Neumann essentially tells you how it's done. But of course no one knew anything about the other. It's a great paradox to me that in fact this connection was not seen."

He claims that von Neumann made him see "what I have come to call this 'Schrödinger's fundamental error' in his famous book *What is Life?* When asked who are his scientific heroes he lists three names. 'There are many people whom I admire, both people I've known and whom I've read about. Von Neumann is a great hero to me, because he seemed to have something special. Of course it may be envy rather than admiration, but it's good to envy someone like von Neumann.' " The other two names in his heroes list: Francis Crick and Leo Szilard.

5   Turing's Brain and the Most Important Paper in Computer Science

"The exactness of mathematics is well illustrated by proofs of impossibility. When asserting that doubling the cube ... is impossible, the statement does not merely refer to a temporary limitation of human ability to perform this feat. It goes far beyond this, for it proclaims that never, no matter what, will anybody ever be able to [double the cube]. No other science, or for that matter no other discipline of human endeavor, can even contemplate anything of such finality."

- Mark Kac and Stan Ulam, 1968

Turing's seminal paper solved Hilbert's Entscheidungsproblem (decision problem) in the negative. After Gödel's first hit to Hilbert's program to find a mechanical process for deciding whether a theorem is true or false in a given

axiomatic system, Turing provided the second hit, effectively terminating Hilbert's program.

Papers with negative results as such Turing's are the most impressive and deep in mathematics. To understand the magnitude of Turing's challenge to prove mathematically "such finality," one has to rule out "everything," and this needed a definition of what a most general "mechanical process" is, i.e., a machine that could compute "everything" that is computable. In turn, the Turing machine was one of the most positive and powerful results in mathematics. The computer era, with Turing and von Neumann as founding fathers, had this paper with negative-and-positive results of greatest depth possible as its foundation. For both von Neumann and Turing, mathematical proof was a philosophy of how truth is won. In discovering it, they possessed a power almost unequaled by mathematicians of any era.

6   From Universal Turing Machine to Universal Grammar

Universality is an important concept in mathematics, in computer science, in linguistics, in philosophy. There are universal sets in set theory and topology, universal functions in mathematical analysis, universal recursive functions in logic, universal grammars in linguistics.

According to a long tradition that originated with Roger Bacon and endures still, awareness of an idea of a universal grammar came from multiple directions – Joseph Greenberg [6] and Noam Chomsky [4] sought universals of natural languages; Richard Montague [11] for universals of all human languages, be they natural or artificial. In the theory of formal languages and grammars, results outline in what conditions universality is possible in the field of context free languages, of context sensitive languages, of recursively enumerable languages (Takumi Kasai [8], Sheila Greibach [7], Grzegorz Rozenberg [15]).

Each of these types of universal grammars can be used to obtain a specific cognitive model of the brain activity; it concerns not only language, but any learning process. The potential connection between universal Turing machines and the nervous system is approached just towards the end of CB, at the moment when von Neumann had to stop his work, defeated by his cancer. We are pushed to imagine possible continuations, but we cannot help but consider ideas, results, theories which did no yet exist at the moment of his death. A joint paper with Cristian Calude and Gheorghe Păun [3] adopted the assumption according to which any type of human or social competence is based on our linguistic generative competence. This assumption was motivated in a previous paper, see [9] The generative linguistic nature of most human competences may be interpreted as a hypothesis about the way our brain works. But it is more than this, because nature and society seem to be based on similar generative devices.

It seems to be more realistic to look for a metaphorical brain (see [1]), giving an a posteriori explanation of various creative processes. But, for Arbib, the metaphorical brain is just the computer. Other authors speak of artificial brains; see [14].

Our aim is to explain how so many human competences, i.e. so many grammars, find a place in our brain, how we successfully identify the grammar we need and, after this, how we return it to its previous place for use again when necessary. An adequate alternation of actualizations and potentialisations needs a hyper-grammar. For instance, if we know several languages, at each moment only one of them does, it is actualized, all the other are only, they remain only in a potential stage. We are looking for a hyper-competence, i.e. a universal competence, a competence of the second order, whose role is just to manage, to activate at each moment the right individual competence. This is *the universal grammar as a hypothetical brain*, appearing in the title of our joint paper.

Behind this strategy is the philosophy according to which any human action is the result of the activity of a generative machine, defining a specific human competence, while the particular result of this process is the corresponding performance. Chomsky used the slogan "linguistics is a branch of cognitive psychology." Learning processes are the result of the interaction among the innate and the acquired factors, in contrast with the traditional view, seeing these processes only as the interaction among stimuli and responses to them. The historical debate organized in 1979 between Chomsky and Piaget aimed just to make the point in this respect (see [13]). With respect to the claim formulated by von Neumann on page 82 in his final book - "The logics and mathematics in the central nervous system, when viewed as languages, must structurally be essentially different from those languages to which our common experience refers" – it seems that the prevalent view today, at least in the field of linguistics, is to replace the strong requirement asking for the grammar of the brain by the weak requirement asking for a grammar whose result is similar to that of the brain. In the first case, the form of the generative rules should be iconic images of the operations taking place in the brain; in the second case, this strong requirement, for which there is little evidence in the existing experiments, is replaced with the less demanding requirement that the result of the grammar is similar to the result of the brain activity. Chomsky never claimed that the regular, the context free and the context sensitive rules have their correspondent in the brain's activity, despite the fact that he imagined the architecture of his grammars having as term of reference the grammatical needs of natural languages. No such claims were formulated with respect to other generative devices used in logic or in computer science.

An idea emerging frequently in his writings is clearly expressed in GLTA (p. 526-527): "Natural organisms are, as a rule, much more complicated and subtle, and therefore much less understood in detail, than artificial automata." The highest complexity is realized by the human central nervous system. We can approach it by decomposing it in various parts and by analyzing each part (component) on its own. Physics, chemistry and, in a near future, quantum mechanics are involved here, believed von Neumann. But for the mathematician and the logician, the data of the first step can be organized in a system of axioms, adopting for each component the representation as a black-box metaphor used in Norbert Wiener's cybernetics. Then, in a second step, we try to understand

how these different components interact as a whole and how the functioning of the whole is obtained by the right interaction of the components. While the first step is just here, logic and mathematics are at home.

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