Mechanical Astronomy: A Route to the Ancient Discovery of Epicycles and Eccentrics

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Abstract The ancient Greek art of *sphairopoiïa* was devoted to the building of models of the universe such as celestial globes and armillary spheres. But it also included the construction of geared mechanisms that replicated the motions of the Sun, Moon, and planets, such as the famous orrery that Cicero attributed to Archimedes or the spectacular Antikythera mechanism, found in an ancient shipwreck of about 60 BC. Was *sphairopoiïa* merely an imitative art, in which the modelers followed the precepts of the theoretical astronomers? Or could theoretical astronomy also learn something from the art of mechanics? In this paper, we examine the relation of astronomy to mechanics in the ancient Greek world, and argue that we should imagine astronomy and mechanics in conversation with one another, rather than in a simple, one-way transmission of influence.

Introduction

The emergence of deferent and epicycle theory in Greek planetary astronomy is shrouded in mystery. The homocentric spherical models associated with Eudoxus, Callippus and Aristotle were abandoned in planetary theory sometime in the century after the death of Aristotle (322 BC), though the idea that the universe consists of nested spherical orbs continued to dominate cosmological thinking until the Renaissance. The next theoretical, geometrical tools known to us are epicycles and eccentrics, which probably appeared within a few decades one way or the other of 200 BC. We do not know what may have motivated them. One key development was Greek absorption and adaptation of Babylonian astronomy, which was already well under way in the third century BC, though of course a decisive episode was centered around the work of Hipparchus in the second. It is possible that Greek epicycle-and-eccentric theories arose, in part, as an effort to model Babylonian "phenomena" — i.e., the phenomena predicted by the Babylonian theories, which provided a much more convenient and comprehensive account of the planetary motions than mere observation ever could.

At least from the time of Archimedes, in the late third century, Greek astronomers and mechanics (*mechanikoi*) also constructed models to imitate the workings of the heavens. Such a model was called, in Greek, a *sphairopoiïa* (spherical construction), or often simply a *sphaira*. (The corresponding Latin term was *sphaera*.) *Sphairopoiïa* was also the name for the branch of mechanics devoted to this art.[1] The art of *sphairopoiïa* included the art of building simple teaching tools such as celestial globes and armillary spheres. But it also included the construction of more elaborate machines intended to replicate the motions of the Sun, Moon, and planets. We know from the remains of the Antikythera mechanism that the operative principle of these more elaborate planetarium-style *sphairopoiïai* was the concrete realization of astronomical period relations by means of gear trains. That gears emerged in Greek

[1] For an introduction to *sphairopoiïa*, see Evans and Berggren [2006, 47, 52–53, 246–249]. For a study stressing *sphairopoiïa* as a tool of discovery rather than merely of representation, see Aujac [1970].

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mechanics and epicycles appeared in Greek astronomy at roughly the same time (almost certainly within a century of one another), is certainly suggestive. And it is all the more remarkable that the gears are probably the older of the two. So the question arises: Were gearwork *sphairopoiïai* simply imitative models meant to illustrate the theories of the geometrical astronomers? This view, which has been almost universally held, we might describe as "theory first, mechanical model later." Or was theoretical astronomy also able to borrow inspiration from mechanics? In this paper we shall explore the possibility that mechanical invention played a role in the development of Greek theoretical astronomy. Gearwork mechanisms may have provided the insight that led to the invention of epicycles and eccentrics.

Chronology

The oldest mention known to us of something like gears appears in the pseudo-Aristotelian *Mechanics* (sometimes also called *Mechanical Problems*).[2] In later antiquity, it was generally believed to have been written by Aristotle himself. Diogenes Laertius [v, 26] mentions a *Mechanikon* in his list of books by Aristotle. And Athenaeus the Mechanic (perhaps first century BC) provides an earlier attestation, mentioning Aristotle in his list of authors on mechanics who can provide a reader with a theoretical introduction but not with instruction in anything practical [Whitehead and Blythe 2004, 44–45]. Since the middle of the nineteenth century, most (though not all) authorities have held that it is not really by Aristotle, but comes from the Peripatetic school of the late fourth or early third century BC. There is a voluminous literature on the question, spanning the whole period from the early nineteenth century to our own day. Summaries of the debate up to the recent past have been made by Berryman [2008, 107–109] and, in much greater detail, by Bottecchia Dehò [2000, 27–51]. Here we shall only mention what is essential for our argument.

The text attempts to derive the principle of the lever from that of the circle, and those of other simple powers from that of the lever. Because of its unsophisticated approach to the lever itself, it is reasonable to suppose that it was written some time before Archimedes' *On the Equilibrium of Planes*. Of course, by itself this argument would not be enough, as we have plenty of instances of "crude" treatises being written later than more sophisticated ones. But in any case, the text makes no use of Archimedes' results, nor of the concept of a center of gravity. Moreover, as Berryman [2008, 108] points out, in its enumeration of various mechanical powers (lever, wheel, pulley, wedge) with their many applications (to rudders, forceps, nutcrackers, rollers, etc.) the text makes no mention of the screw, which, again, suggests a pre-Archimedean date. Historians of mathematics place the composition in the Peripatetic school on the basis of the similarity of some of its demonstrations to demonstrations found in genuine works of Aristotle, such as *On the Heavens* and the *Physics* [Heath 1921, 1:344–346]. A strong argument for placing it rather early in the history of the school is based on its mathematical terminology — a case developed by Heiberg, based on a detailed examination of the technical vocabulary. Thus, according to Heiberg, the *Mechanical Problems* could have been written either before Euclid had made mathematical terminology more consistent and convenient, or perhaps a while after Euclid but in circles that were still dominated by the older, Aristotelian terminology [Heiberg 1904, 30–32]. An effort has occasionally been made to ascribe it to Strato of Lampsacus, who was head of the school c. 288–269 BC, based partly on the ascription of a work on mechanics to Strato [Diogenes Laertius v, 59], but this has not won wide support.[3] Marshall Clagett [1959, 4–9] revived interest in the *Mechanical Problems* when he argued for the significance of its dynamic approach to problems of statics (unlike the approach of the later Archimedean treatises), which was to become so important in the Middle Ages. Since then, the original ascription to Aristotle has been again defended, notably by Krafft [1970, 13–20].[4] Clearly, the authorship of the text remains an open question; but the authorship issue is not important for our

[2] Pseudo-Aristotle, *Mechanical Problems* 848a25–38 [Hett 1936, 334–337].

[3] The attribution to Strato has been most recently reassessed (unfavorably) by Bodnár [2011].

[4] Arguments against Krafft's conclusion were given by Knorr [1982, 100–101, n27], whose own view was for a date early in the third century.

purposes. In view of the history and present state of scholarship, we shall be adopting a conservative position if we take the *Mechanical Problems* as no later than the middle of the third century BC. Indeed, there seems to be no recent authority who would make it later than Strato.[5]

It is true that the text does not mention teeth, but only circles in contact, so some scholars have been reluctant to credit it with a discussion of real gears, but only "friction wheels."[6] Of course, the best way to improve a friction wheel is to eliminate the possibility of slipping by adding teeth. And, as far as we know, no one has pointed to an actual ancient artifact that involves a pair of interacting "friction wheels." By contrast, gears are not merely theoretical entities postulated by modern historians, but things that really did exist. Moreover, the writer says that the objects he is discussing are sometimes seen in temples, where they have been dedicated as offerings. And they are arranged so that, from one motion, many circles move at the same time. Although a *pair* of "friction wheels" may be workable, the reliable movement of many wheels by one driver seems plausible only for toothed wheels. The writer says that, using the principle of the circle, the craftsmen construct an instrument in which the first cause (τὴν ἀρχήν, perhaps referring to the first wheel in the system) is concealed, "so that only the wonder of the machine is apparent, while the cause is unseen" (ὅπως ᾖ τοῦ μηχανήματος φανερὸν μόνον τὸ θαυμαστόν, τὸ δ' αἴτιον ἄδηλον). It appears, then, that the writer is describing actual machines that he has seen. The use of θαυμαστόν also suggests that we are dealing with an early stage of the wonder-working art, in which the mere fact of multiple circular motions produced from one input would have been enough to amaze. Finally, it is noteworthy that Book I of the *Mechanica* of Hero of Alexandria begins (after the discussion of a winch that is almost certainly misplaced or interpolated) with a general discussion of the theory of gears and here the discussion makes no use or mention of teeth — simply ratios of circumferences, etc.[7] Teeth are introduced later on. So it seems that an introductory discussion of the mathematics of gear systems that makes no mention of the details of teeth would not be out of the ordinary.

Let us turn now to the other evidence for early gearing. The dates of the Alexandrian mechanic Ctesibius have been contested. But Drachmann places his *floruit* around 270 BC on the basis of an epigram by Hedylos, quoted by Athenaeus of Naucratis (11.497a–e), which tells of a musical cornucopia that he made for the statue of Arsinoë, the sister and wife of Ptolemy II Philadelphus (reigned 285–247 BC) [Drachmann 2008]. Now, Vitruvius discusses a water clock that he claims was made by Ctesibius, in which a rack engaged a toothed wheel.[8] So here is an argument (not a proof, certainly) for situating the first gears by the middle of the third century. Unfortunately, Athenaeus (4.174d) makes the situation a bit murky by saying elsewhere in the same work that Ctesibius, the inventor of a hydraulic organ, lived in the reign of "the second Euergetes" (Ptolemy VIII Euergetes, who reigned jointly with Ptolemy VI and Cleopatra II, in 170–164 BC, and on his own, 146–116 BC). So either Athenaeus has made a slip or there was a second Ctesibius. This issue, which has a long history, has been discussed in detail by Drachmann [1951], who argues that the second Ctsebius is unlikely. In any case, the placement of *a* Ctsebius in the reign of Ptolemy II, based as it is on the detail added by Hedylos's account of the statue of Arsinoë, seems reasonably secure.

Archimedes, who died in 212 BC, is said by Diodorus Siculus (twice) as well as by Athenaeus of Naucratis to have invented the water pump called the *cochlias*, now often known as the Archimedean screw.[9] According to Athenaeus, a water screw was used to pump out the bilge of the ship *Syracosia*, which Hieron II of Syracuse (reigned c. 271–216 BC) built and sent to Egypt as a gift for King Ptolemy. Athenaeus's source was a certain Moschion, who wrote a book in which the construction of this ship was treated in considerable detail. The mention of the water-screw bilge pump is embedded in the course of the longer description of the ship, which lends this detail more credibility. A water screw, of course, is not a gear; but its central element, the helical screw, is similar in form to one of the two

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[5] An outlying position is that of Winter [2007], who argues that the text is even older and was written by Archytas.

[6] Drachmann [1963, 13] opts for friction wheels. Berryman [2009, 113] comes down on the side of gears.

[7] Carra de Vaux [1894, 42–45]. The *Mechanica* of Hero survives only in Arabic.

[8] Vitruvius, *On Architecture* ix, 8.5.

[9] Diodorus Siculus, *Bibliotheca historica* i, 34.2 and v, 37.3–4 [Oldfather 1933, 1:112–115 and 3:199]. Athenaeus of Naucratis, *Deipnosophistae* v, 208f.

key elements of an endless screw. And the endless screw, in which a helical worm gear engages a plane gear, can be regarded as a natural development of a rack and pinion. These three technologies, then, are closely allied. It is, of course, irrelevant for our purposes whether Archimedes really invented the water screw or drew upon an already existing technology.[10]

For the endless screw or screw-windlass itself, the picture is less clear. Athenaeus of Naucratis [v207b], again, tells us that Archimedes was the discoverer of the screw-windlass (ἕλιξ), which he reportedly used to launch a ship. However, other writers say that he used pulleys for this task. The various testimonia have been collected and discussed by Drachmann [1958]. We will not indulge in legends of Archimedes' ship-launching, nor speculate on just which power he was thinking of when he boasted that if he had a place to stand he could move the Earth. That the rack and pinion, the water-screw, and the screw-windless should have emerged around the same time is inherently plausible and the key point is that we have attestations of all three for the third century BC.

Several Arabic manuscripts preserve a work on a water clock attributed to Archimedes. This device involves a crown gear engaging a lantern pinion, which are illustrated in diagrams. A key feature of the clock is its bird's head that disgorges a ball once each hour [Hill 1976]. Three nearly complete manuscripts exist in Paris, London, and New York and a fragment is preserved at Oxford. All four present Archimedes as the inventor. The work is also mentioned in the *Fihrist* of Ibn al-Nadīm, an Arabic bio-bibliographical work of the tenth century, in which it is also attributed to Archimedes [Dodge 1970, 636]. Some scholars have regarded the work as Byzantine or Muslim in origin (though drawing on Hellenistic ideas), Carra de Vaux going so far as to characterize the use of the name of Archimedes as most likely the "banal ruse of an author desirous of being read."[11] Moreover, the Oxford fragment is dedicated to one "Māristūn." Since Philo of Byzantium dedicated his works to "Ariston," Drachmann [1948, 38] remarks, "One might almost regard the dedication to Ariston the hall-mark of a work by Philon." And he concludes that the "whole thing is the work of a Moslem inventor, who has put together details from several sources, one of them doubtless Philon, another probably Heron...." The fullest discussion of its possible origins is given by Hill [1976, 6–9], who notes that the problem is complex. But, as Hill points out, the Arabic writers are unanimous in ascribing the first section (involving the water machinery and the release of the balls) to Archimedes. He concludes that the treatise is at least based on Hellenistic models and that it shows signs of having been translated into Arabic from Greek. Hill's own view is that the roots of the treatise are indeed Archimedean, but that it was reworked by Philo, and that the later sections with their Eastern motifs are later additions. For Philo, it is difficult to arrive at a secure date, but usually his *floruit* is placed in the latter part of the third century BC.[12]

Archimedes is also said to have devised a machine that represented the movements of the Sun, the Moon, and the planets. This instrument (along with a simpler celestial globe) was reportedly taken to Rome by the general Marcellus after the sack of Syracuse. Our chief source is Cicero, who describes Archimedes' device in a philosophical dialogue, the *Republic*, which was modeled on Plato's.[13] Now, Cicero wrote his *Republic* around 54 BC, but its dramatic date is set around 129. In the course of the dialogue one of the speakers recounts an episode in the life of Gaius Sulpicius Gallus, around 166, when Gallus saw and explained the *sphaera* that had been brought back to Rome in 212. Whether this device still survived in Cicero's day we have no way to know. Such a wonderful machine would certainly have been a "keeper" and its location in a wealthy household may possibly have helped to preserve it. However, Cicero does not say that he himself had seen it and perhaps he was supplying details on the

[10] Stephanie Dalley has argued that the water screw appeared at Nineveh in the 8th century BC; see Dalley and Oleson [2003]. A more commonly encountered proposal is that Archimedes drew upon an already-existing Egyptian technology, for a discussion (and refutation) of which see Oleson [1984, 291–294]. Oleson also discusses the iconographical evidence for the water screw, none of which pre-dates the Roman period. Terracotta reliefs in London and Cairo show a man or boy treading a water screw (Figure 71 and 86) and a wall painting in Pompeii shows the same (Figure 101).

[11] Carra de Vaux [1891, 296]. A German translation of the text with commentary was published by Wiedemann and Hauser in 1918, reprinted in Wiedemann [1970].

[12] Drachmann [2008b] has him flourishing c. 250 BC; Toomer, c. 200 BC in his entry "Philon of Byzantium" in the Oxford Classical Dictionary, 3rd ed.

[13] Cicero, *Republic* i, 21–22 [Keyes 1994, 40–43].

basis of what he had seen of the *sphaera* of Posidonius of Rhodes, his teacher and friend. But that Archimedes built some sort of device seems certain, and he is said by Pappus of Alexandria, on the authority of Carpos of Antioch, to have written a treatise on *sphairopoiïa* [Ver Eecke 1933, 2:813–814]. This may plausibly have included a description of his machine or of its principles. If we accept that Archimedes did build such a machine — even while admitting that we can say nothing with certainty about its features — it is difficult to see what its fundamental principle might have been if it was not the gear. And there must have been some line of development before anything as complex as a *sphairopoiïa* could have been built. (We may, for example, imagine the first gearwork mechanisms as simple displays of an amazing principle, such as the machine mentioned in the *Mechanical Problems*).

Here, then, are half a dozen strands of evidence placing gears in the third century. While no single episode in the early history of Greek mechanics is decisive, and any one of them may certainly be subject to reservations, together they allow us, with reasonable confidence, to situate the appearance of gears by about the middle of the third century BC.

As for epicycles and eccentrics, the tradition has been to place their origin around the time of Apollonius of Perge, if not with Apollonius himself, based on some remarks of Ptolemy.[14] Of course, it is possible that someone might have imagined epicycles and eccentrics before Apollonius. An epicycle for an inferior planet seems such an obvious way of explaining the planet's limited elongations from the Sun that it could conceivably have been invented more than once.[15] But Apollonius is the first figure for whom we have any evidence for an interest in epicycles or eccentrics as mathematical objects for which theorems can be proven. For this reason, it is necessary to say a little about how Apollonius's lifetime is best established.

Eutocius, in his *Commentary* (early sixth century AD) on the *Conics* of Apollonius, says that Apollonius lived (or, perhaps, was born) in the reign of Ptolemy III Euergetes (247–222 BC) [Heiberg 1891–93, 168]. The verb used is γέγονε, a perfect of γίγνομαι, so the basic meaning is usually (though not always) "was in existence" rather than "came into being."[16] Heiberg translated it by *vixit* ("lived") and Heath [1921, 2:126] by "flourished." Photius, the ninth-century patriarch of Constantinople, quoting a more doubtful source, the second-century grammarian Ptolemaeus Chennus of Alexandria, says that an Apollonius was famous for astronomy in the reign of Ptolemy IV Philopater (222–205 BC), and it is usually supposed that this must be Apollonius of Perge.[17] Heath, following Hultsch, discussed this evidence and concluded that Apollonius "was probably born about 262 BC, or 25 years after Archimedes," a conclusion that is still widely quoted.[18]

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[14] Ptolemy, *Almagest* xii, 1 [Toomer 1984, 555 & 558].

[15] However, it should be pointed out that the common attribution of circumsolar orbits for Venus and Mercury to Heraclides of Pontus has been thoroughly refuted. See Eastwood [1992] and Toomer [2008b].

[16] Rhode [1878] studied 129 instances of the use of γέγονε in the *Suda* (a Byzantine historical compilation of about the tenth century). He found that the meaning was

"certainly the time of flourishing in	88 cases
probably the time of flourishing in	17
certainly the time of birth in	6
perhaps the time of birth in	4
no obstacle to meaning ἤκμαζεν ["he was in his prime"] in	9
wholly undecidable in	5."

[17] Bekker [1824–1825, Codex 190, 1:151b18]. Henry and Schamp [1959–1991, 3:66]. This passage is the source of the well-known story that Apollonius was given the nickname ε, on account of the resemblance of the letter to the shape of the Moon, which he had investigated most thoroughly. It occurs in a short list of people who had letters of the alphabet as nicknames.

[18] Apollonius addressed the first three books of his *Conics* to Eudemus, but the fourth and following books to Attalus. Heath supposed that this is King Attalus I of Pergamum (reigned 241–197 BC), which supported his dating. But Toomer [2008a] has argued that Attalus was a common name among those of Macedonian descent and that "it is highly unlikely that Apollonius would have neglected current etiquette so grossly as to omit the title of 'King' (βασιλεύς) when addressing the monarch." We set the Attalus association aside as too insecure. Similarly, the mention by Pappus in Book 7 of the *Mathematical Collection* [Hultsch 1876–1878, 2: 678–679; Ver Eecke 1933, 507] that Apollonius studied with the pupils of Euclid is set aside, as it is not clear whether Pappus's source meant pupils that Euclid himself had taught, or the intellectual descendants in the "school."

But Apollonius himself provides some autobiographical detail that may contradict so early a dating. In the introduction to Book II of the *Conics*, Apollonius says that he is sending the work to Eudemus (of Pergamum) by way of his own son Apollonius and he requests that Eudemus should also give a copy of it to "Philonides the geometer, whom I introduced to you in Ephesus," if the latter ever be in the vicinity of Pergamum. Now an Epicurean philosopher named Philonides is known from an anonymous papyrus biography found at Herculaneum (*P. Herc.* 1044). The text was published in 1900 by Crönert, who pointed to its possible utility for dating Apollonius.[19] (This text was available when Heath published his *History of Greek Mathematics*, but he did not use it in his discussion of the date of Apollonius.)[20] Philonides is known also from inscriptions found at Athens and Delphi.[21] From the biography, we see that Philonides the Epicurean was well-connected at the Seleucid court of Antiochus IV Epiphanes (reigned 175–c. 164 BC) and his nephew Demetrius I Soter (162–150), which provides key evidence for dating him. Moreover, we learn that his first teacher was one Eudemus and that he then followed lectures by "Dionysodorus the son of Dionysodorus of Caunus" [Fragment 25; Gallo 1980, 82].[22] Philonides wrote an exegesis "of Book 8 of [Epicurus's] *On Nature* and many others of various kinds concerning his doctrines, many [of these exegeses] geometrical concerning the Minimum" (ἐλάχιστον).[23] *Elachiston* here perhaps refers to the Epicurean theoretical minimum magnitude. As Sedley [1976, 24] has suggested, resolving the apparent conflict between the existence of a minimum magnitude and the ordinary practices of Greek geometry (which assumed continuously divisible lines) could have been a significant issue for an Epicurean geometer. Of course, Epicurus's animosity towards geometry is well known, but Sedley [1976] and Mueller [1982, 95] argue that the later Epicureans were not all anti-geometers. Still, the papyrus life has little to say of Philonides' geometrical accomplishments — we have no actual geometrical discovery attributed to him. Rather the emphasis is on his role as an Epicurean philosopher, his conversion of Demetrius to Epicureanism, and his role in saving his home town of Laodicea-by-the-Sea (Syria) from destruction. Philonides also composed epitomes of the letters of Epicurus, Metrodorus, Polyaenus and Hermarchus that could be "useful to lazy young people." Thus Philonides seems, at least later in life, to have been more a philosopher than a geometer. On the other hand, the papyrus life gives us the names of his geometry teachers and also mentions a Zenodorus with whom Philonides was on friendly terms [Fragments 31, 34, Gallo 1980, 88–89], who is perhaps, though not certainly, the geometer mentioned by Pappus and Theon of Alexandria as the author of a treatise on isometric figures.[24] Finally, the papyrus seems also to associate Philonides with Ephesus at one stage of his life[25] — where, of course, we know that Apollonius introduced Eudemus and Philonides. In all, the identification of Apollonius's Philonides the geometer with Philonides the Epicurean seems reasonably secure, based as it is on the latter's demonstrated interest in geometry, his association with a certain Eudemus and his probable connection to Ephesus. But having Apollonius overlap with Philonides the Epicurean obviously requires taking Euctocius's γέγονε as indicating the birth of Apollonius and not his flourishing. This is somewhat unusual but not terribly rare.

The birth date of Philonides the Epicurean is often taken as about 200 BC, on the basis of his

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[19] Crönert [1900]. A new edition of the text, with Italian translation and copious notes, is provided by Gallo [1980 2:23–166]. Much of the evidence for Apollonius's date is discussed in Huxley [1963, 100–103] as well as Fraser [1972, 1: 415–418]. Fraser [1972, 2:600–604] prints most of the relevant Greek texts. Toomer [2008a] provides a concise and cogent discussion.

[20] Heath did discuss this papyrus in the section of his book dealing with the identity and dating of the geometer Dionysodorus [Heath 1921, 2:218].

[21] Köhler [1900]. The inscription from Delphi has been published in Plassart [1921]; Philonides and his brother appear at IV 78–80 (p. 24).

[22] A mathematician named Dionysodorus is said to have solved a cubic and to have studied the torus [Heath 1921, 2:218–219]. Vitruvius, *On Architecture* ix, 8, also mentions someone of this name as the inventor of a conical sundial. But there was more than one mathematician named Dionysodorus, as is clear from Strabo, *Geography* xii, 548c. See Knorr [1986, 263–276].

[23] Fragments 13 inf.–14. See Gallo [1980, 67–68], which corrects the text of Crönert [1900, 947] by removing the first line to a greater distance on the papyrus.

[24] See Heath [1921, 2:207], Knorr [1986, 233–234, 272–274] and Toomer [1972].

[25] Fragment 37 [Gallo 1982, 91]. The reading of Ephesus is not certain, however, due to damage to the papyrus.

association with Antiochus and Demetrius, as well as the evidence of the inscriptions.[26] However, Gera [1999] has argued that Philonides' association with the Seleucid court began, not in the reign of Antiochus Epiphanes, as is usually supposed, but in the reign of his predecessor on the throne, his brother Seleucus IV. If this is correct, we should probably back up Philonides' career by a decade or so. So Apollonius's introduction of the young Philonides to Eudemus could have happened as early as about 190, but not much earlier. Some years later, when Apollonius sent Book II to Eudemus, Apollonius had a grown son but was not yet very far along in the revision of his *Conics*; so he was then perhaps 40 years old. If he did his astronomy early in life, we might suppose that the work on epicycles and eccentrics was as early as about 210 BC, but not much earlier. Thus it is clear that Apollonius' mathematical work on planetary theory came well after (by one or two generations) the introduction of gears into Greek mechanics. Gears would pre-date Apollonius's astronomical investigations even with Heath's birth date for Apollonius; but the Philonides connection makes it all the more certain.[27]

Mechanical Geometry

In pure geometry, there was early discussion of mechanical contrivances for solving problems. A striking example involves the problem of finding the two mean proportionals in a continued proportion [Heath 1921, 1:255–258]. That is, given a and b, to find x and y such that $b/y = y/x = x/a$. This problem is closely related to the duplication of the cube. For, suppose that we are given a cube of side a and volume a^3 and we seek b, such that $b^3 = 2a^3$. If we can find two mean proportionals x and y, where $b/y = y/x = x/a = r$, say, we have

$$\left(\frac{b}{a}\right)^3 = \left(\frac{b}{y}\frac{y}{x}\frac{x}{a}\right)^3 = 2,$$

or $r^3 = 2$.

Thus the duplication of the cube is solved, for the required side length is $b = ra$. Hippocrates of Chios is said to have been the first to show that the duplication of the cube could be reduced to finding two mean proportionals[28] and from then on the problem was usually approached in the latter form.

As Pappus was later to say, geometrical problems could be divided into three categories. Plane problems (ἐπίπεδα προβλήματα) could be solved with straightedge and compass alone.[29] Solid problems (στερεά προβλήματα) required the use of conic sections. They are called solid, says Pappus, because they make use of solid figures for their construction. Finally, the most complex problems required the use of other special curves, such as the quadratrix, or spirals, or conchoids. These problems are called *grammika* (γραμμικά προβλήματα). There is no satisfactory English translation of *grammika* ("linear" won't do, but we might approximate it by "making use of curves").[30] Now, the duplication of the cube (and, equivalently, the finding of two mean proportionals) does not belong to the class of plane problems, but to the class of solid problems. Ancient geometers did not, therefore, abandon all hope when

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[26] Philippson [1941] gives 200–130 BC for Philonides' lifespan, on the basis that he was in his prime in the reign of Demetrius. Gallo [1980, 36] holds that Philonides' birth date is unlikely to have been after 200 BC, and rather likely to have been before this date, though not by much. Knorr [1986, 276] prefers a birth date of 220 BC. Fortunately, none of this fine-tuning is consequential for our argument.

[27] We know, e.g., from the inscription at Delphi that Philonides the Epicurean had a father who also was named Philonides; see Köhler [1900] and Plassart [1921, 24]. This elder Philonides was a man of some distinction, who played a role in facilitating diplomacy with the Seleucid court. On balance, it does seem probable that it is indeed the son, Philonides the Epicurean, who was Apollonius's "Philonides the geometer." But if it were the father who was Apollonius's Philonides, this would back Apollonius up by perhaps twenty years.

[28] Eutocius, in his Commentary on Archimedes' *On the Sphere and Cylinder II* [Mugler 1972, 64; Netz 2004, 294].

[29] Pappus, *Mathematical Collection* iii, 7 [Ver Eecke 1933, 1:38].

[30] Alexander Jones translated γραμμικά with "curvilinear" in his translation of Pappus's *Mathematical Collection* vii, 27 [Jones 1976, 2:112–113].

faced by a problem that could not be solved by ruler and compass alone, but had recourse to more complex methods.

Eutocius, in his commentary on Archimedes' *On the Sphere and Cylinder*, describes a mechanical solution to the problem of finding two mean proportionals, which he (probably wrongly) attributes to Plato, and which makes use of a special sliding instrument.[31] Eutocius attributes a second mechanical solution to Eratosthenes, and this involves an instrument called a *mesolabon* (a "mean-taker"). Pappus also discusses a number of solutions, including the *mesolabon*.[32]

According to Plutarch, Plato condemned such methods as the corruption of the good of geometry, since they involved a descent from the incorporeal things of pure thought to the realm of the perceptible.[33] (And this is a good reason for doubting Eutocius' attribution of a mechanical solution to Plato. A second reason is that Pappus does not mention a solution by Plato and he surely would have, had he known about it.) If Plutarch's story is not apocryphal, Plato must have been reacting to what he regarded as an unfortunate trend in geometry, so it is possible that geometers were engaged with mechanical solutions already in the early Academy. In any case, we have good evidence for mechanical approaches all through the Hellenistic period. The mechanical methods added to the quiver of available techniques.

And, of course, we have Archimedes' famous discussion, in the *Method*, of the use of mechanical methods as an *aid in discovery* of new theorems, which must then be proven in more conventional ways.[34] Archimedes finds it easier to divine the areas or volumes of figures if he imagines slicing them up and weighing them against slices of other figures by means of a balance. Here, then is a case in which mechanics helps guide speculation in pure geometry.

But it would be wrong to think of mechanics simply as an assistant to geometry. Rather, as Sidoli and Saito [2009, 605–607] have stressed, Greek geometry emerged in the context of instruments and this instrumental context helped shape the methods of geometry. Thus it is no accident that Euclid's rules of construction admitted of compass and straightedge only — it was by playing about with straightedge and compass that the early geometers imagined new problems and clarified their thinking about them. Sidoli and Saito point out that also in the *Spherics* of Theodosius, the great majority of constructions mentioned could actually be carried out on the surface of a real globe (leaving aside those that mandate a slicing of the globe): mechanical thinking — thinking about instruments — played a role in the development of the treatise. Perhaps most importantly, in the case of plane geometry, mechanical approaches served to broaden the scope of geometry, and to extend its range beyond the field of play imagined by the early geometers.

Mechanical Astronomy

If even pure geometry could benefit from mechanics, is it possible that astronomy benefitted from its relation to the art of *sphairopoiïa*? Ordinarily, we are disposed to think of *sphairopoiïa* as merely representational, as involving models meant to inspire contemplation or wonder, or tools that could be used in teaching. But a well-made celestial globe could also be used to solve problems of spherical trigonometry without tedious calculation — that is, a globe could serve as a specialized analogue computer. The use of a globe has often been claimed for Ptolemy's treatment of the heliacal risings and settings of the fixed stars (in his *Phaseis*) for climes outside of Alexandria, as well as for Hipparchus's sidereal phenomena in his *Commentary on the Phenomena of Aratus and Eudoxus* [Neugebauer 1975, 930–931].

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[31] The mechanical proofs are discussed in Heath [1921, 1:255–260]. Also relevant is the construction of a chonchoid curve by Nicomedes, using a special mechanical instrument [Heath 1921, 1:238–240].

[32] Pappus, *Mathematical Collection* iii, 7 [Ver Eecke 1933, 40–41].

[33] Plutarch, *Marcellus* xiv 5 [Perrin 1917, 471–473]. Plutarch, *Symposia* viii 2.1 (718 E–F) [Minar et al. 1961, 121–123].

[34] In this volume, see the chapter contributed by Ken Saito and Pier Daniele Napolitani, "Reading the Lost Folia of the Archimedean Palimpsest: The Last Proposition of the *Method*."

But we wish to suggest something well beyond the simple use of a globe as an instrument of calculation. For *sphairopoiïai* could also be used as tools of discovery. In the early period of Greek astronomy, when mathematicians were still mastering the theory of the celestial sphere, it is likely that propositions were sometimes discovered with the aid of a celestial globe or an armillary sphere before being proven geometrically. Many propositions of early spherics, of the kind that appear in Autolycus of Pitane's *On the Moving Sphere* and *On Risings and Settings* (c. 320 BC) may have been discovered and demonstrated on actual models, before being reduced to geometrical proof. For example, Autolycus writes that if two points of the celestial sphere rise at the same time, the one which is further north will set later.[35] This proposition can be seen to be true with a mere glance at a celestial globe; but the proof using methods available in Autolycus's day runs to two pages. This sort of mechanical astronomy — using an instrument to discover possible theorems and then subjecting them to geometrical proof — would be analogous to what Archimedes claims he did in the *Method*.

To take an example from much later, in the *Sphere* of Sacrobosco (13th century), widely used for teaching elementary astronomy in the medieval universities, we read that in far northern latitudes some of the zodiac signs may rise or set *prepostere*, that is, in the reverse of the usual order. "They rise backwards, as Taurus before Aries, Aries before Pisces, Pisces before Aquarius. Yet the signs opposite these rise in the right order. They set backwards, as Scorpio before Libra, Libra before Virgo. Yet the signs opposite these set in the direct order."[36] It seems that Sacrobosco or one of his sources has simply noticed this odd phenomenon on a celestial globe; in a non-mathematical introduction such as the *Sphere*, a detailed proof was not required.

It is not obvious whether these examples of the usefulness of mechanics to spherics are applicable to Greek planetary astronomy, cosmology, or philosophy of nature. For, a common metaphor for expressing the ancient Greek attitude toward the cosmos was to consider the universe, not as a machine, but as a living animal. In Plato's *Timaeus*, the demiurge creates not only a body, but also a soul for the world. In the first century AD, Pliny could refer to the Sun as the soul and mind of the world.[37] And in the second century, Ptolemy still ascribed souls to the planets. However, mechanical metaphors and models are not unknown in ancient natural philosophy, a point recently emphasized by Sylvia Berryman, who points to Aristotle's explanation of the action of limbs (in *On the Motion of Animals*) by analogy to that of a rudder [Berryman 2009, 67]. Berryman adduces a good deal of evidence to demonstrate that mechanical hypotheses sometimes supplied analogies for the functioning of organisms, and played a role in medical theory.

But let us turn to evidence for mechanical thinking in Greek astronomy. In Geminus's work (first century BC), we see "*sphairopoiïa*" used with a range of meanings. It can mean, of course, the branch of the mechanical art devoted to building models of the heavens.[38] But, in his *Introduction to the Phenomena*, Geminus sometimes uses the word to mean a theoretical picture of the world that can be said to be after or according to nature. For example, Geminus criticizes Krates the grammarian for readjusting Homer to make his verses appear to agree with contemporary astronomy. Homer mentions Aethiopians living near the rising of the Sun, and others living near the setting of the Sun, both equally burned by the Sun. Krates attempted to make sense of this by claiming Homer meant there must be Aethiopians living around the winter tropic as well as around the summer tropic. But Krates' interpretation is nonsense, says Geminus. For Homer and the other ancient poets believed that the Earth is flat and that it extends all the way to the sphere of the cosmos, with Ocean ranged all around, and that the risings are out of the Ocean and the settings are into the Ocean. Naturally enough, Aethiopians living at the extreme east and west could both be burned. This notion was consistent with their idea of the world, "but alien to the spherical construction (*sphairopoiïa*) in accord with nature," for, says Geminus, the Earth lies at the middle of the whole cosmos, and "the risings and settings of the Sun

[35] Autolycus of Pitane, *On the Moving Sphere*, proposition 9 [Aujac 1979, 60].

[36] Thorndike [1949, 138], slightly modified.

[37] Pliny, *Natural History* ii, 13 [Rackham 1947, 178–179].

[38] Geminus discussed the branches of mathematics, including *sphairopoiïa*, and their relations to one another in his *Philokalia*, which was cited at length by Proclus in his *Commentary on the First Book of Euclid's* Elements [Evans and Berggren 2006, 243–249].

are from the ether and into the ether, since the Sun is always equally distant from the Earth."[39]

Yet again, Geminus uses *sphairopoiïa* for a spherical arrangement that actually exists in nature. Geminus says, for example, that "there is a certain spherical construction (*sphairopoiïa*) proper for each [planet], in accordance with which they pass sometimes toward the following [signs], sometimes toward the preceding, and they sometimes stand still."[40] Or, again (at xvi 19), Geminus invokes the spherical construction (*sphairopoiïa*) to prove that there exists a second temperate zone in the southern hemisphere of the Earth. In these passages, there is no question of a "model" to save the phenomena; rather, Geminus is speaking of the *sphairopoiïa* of the world itself. If the world is considered in terms appropriate to a mechanical construction, then perhaps understanding can proceed in either direction — from world to model, or from model to world.

In Theon of Smyrna (early second century AD), we have a nice example of mechanical imagination leading from a gearwork machine to the world. Theon is in the course of discussing the nested spheres postulated by Eudoxus. He raises the question of how it can be that in the universe some spheres turn eastward and some turn westward, when it might be more natural to suppose that they all turn in one direction. Here he is referring to the fact that in Eudoxus's system, for each planet, the outermost sphere turns toward the west and is responsible for the daily revolution, while the next sphere interior to it turns eastward and is responsible for producing the planet's zodiacal motion. But, says Theon, maybe there are gears between these spheres, which could reverse the motion, just as in the case of a *mechanosphairopoiïa*.[41] Theon is pondering a machine and reasoning from the machine to answer to a question about the natural world. And he apparently coins a word, ***mechano****sphairopoiïa* (which seems not to be used by any other author), to make it clear that he means a man-made machine, and not the *sphairopoiïa* of the cosmos itself.

And when Ptolemy begins to describe his theory of latitudes for the planets, he makes a famous plea that no one should complain about the difficulty of his hypotheses. For it is not appropriate to compare human contrivances with the divine, nor to form beliefs about celestial things on the basis of very dissimilar analogies. And he goes on: "We see that in the models constructed on Earth the fitting together of these [elements] to represent the different motions is laborious, and difficult to achieve in such a way that the motions do not hinder each other, while in the heavens no obstruction whatever is caused by such combinations."[42] We should not, says Ptolemy, judge simplicity in celestial things from what might appear to be simple on the Earth. This is perhaps a sign that in Ptolemy's day there were people trying to argue about celestial reality on the basis of mechanical models and that this is what motivated his criticism.

The Case of the Antikythera Mechanism

This gearwork astronomical computing machine was discovered in an ancient shipwreck at the beginning of the twentieth century. The date of the shipwreck is most securely established from the coins found in association with the wreck, from which it appears that the ship sank in the decades just after 60 BC.[43] This is well supported by the dating of everyday objects (such as the crew's pottery dishes) that were carried on board [Davidson Weinberg et al. 1965, 4]. But the date for the mechanism itself is not as tightly constrained. Analysis of the letter forms in the Greek inscriptions has been said to imply a most likely date in the range 150–100 BC,[44] but some epigraphers believe that one should allow a century in either direction of 125 BC.[45]

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[39] Geminus, *Introduction to the Phenomena* xvi, 28–29. See also the discussion in Evans and Berggren [2006, 1–53].

[40] Geminus, *Introduction to the Phenomena* xii, 23.

[41] Theon of Smyrna iii, 30 [Dupuis 1892, 290].

[42] Ptolemy, *Almagest* xiii, 2 [Toomer 1984, 601].

[43] See Panogiotis Tselekas, "The Coins" [Kaltsas 2012, 216–219].

[44] Freeth et al. [2006, Supplementary Information, p. 7].

[45] We thank Alexander Jones for sharing this view.

Most of the moving parts of the mechanism were actuated by gear wheels driven by a single input. However, one part had to be moved by hand. This is the Egyptian calendar ring, which was divided into the 12 months (30 days each) and five additional days of the Egyptian year. Because the Egyptian calendar year was always 365 days long, with no leap days, the calendar ring had to be displaced "by hand" by one day every four years. Beneath the Egyptian calendar ring is a circle of closely spaced holes drilled into the underlying plate. There was probably a little post (or posts) on the back of the calendar ring. The ring could therefore be pulled off, turned to the appropriate orientation for the year under consideration, and then plugged back in.

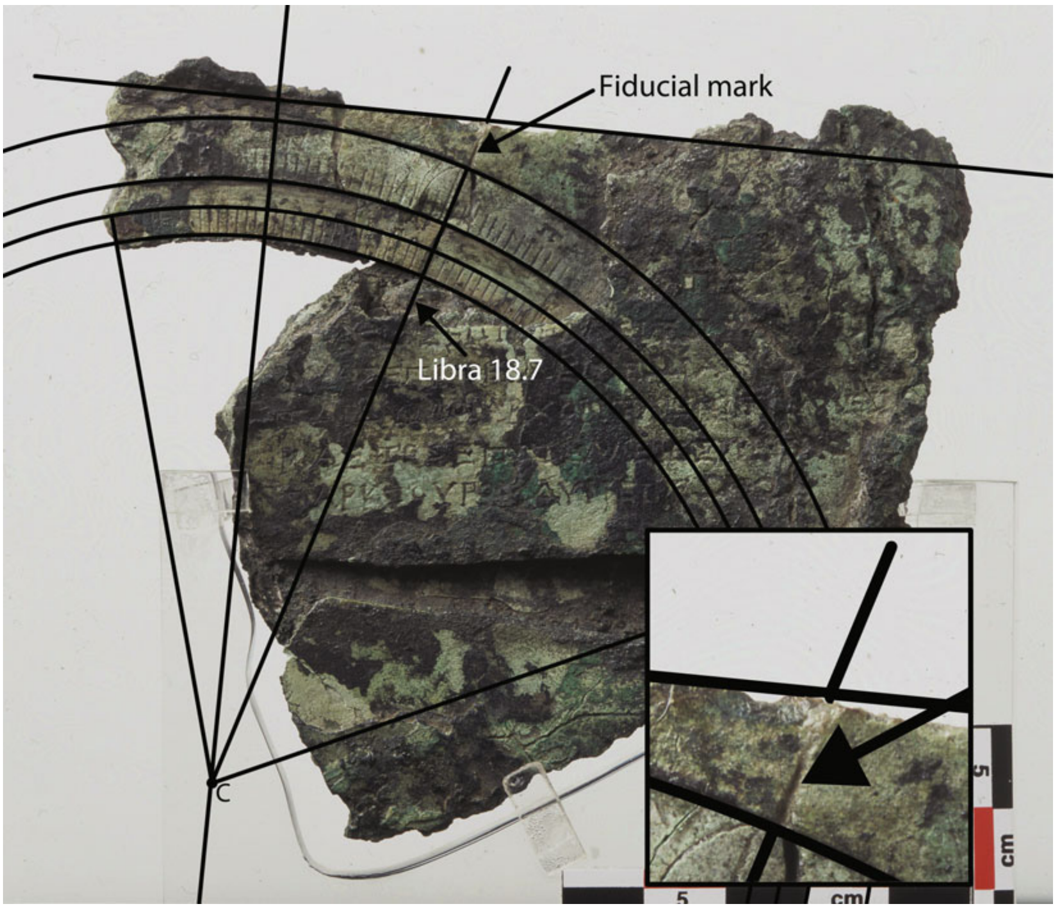

Figure 1: Fragment C of the Antikythera mechanism, carrying the remains of the zodiac and Egyptian calendar scales. National Archaeological Museum, Athens. © Hellenic Ministry of Education and Religious Affairs, Culture and Sports / Archaeological Receipts Fund. (Photograph by Kostas Xenikakis.)

On the plate just outside the Egyptian calendar scale, at about the beginning of the month of Payni, is a clear, uniformly made mark, shown in Figure 1. Price [1974, 19–20] drew attention to this and argued that it was a fiducial mark for setting the Egyptian calendar ring for some initial date. But in his analysis Price assumed that the calendar ring is still in its original position and, when this led to impossible dates, that it was set at the correct day of the month, but the wrong month of the year. However, as is known, the Egyptian calendar ring is out of its proper position by several months for the epoch of the Antikythera mechanism, so no inference can be drawn from the day of the year that now happens to lie against the fiducial mark. But, as we shall see, something interesting can be said about the zodiac degree corresponding to the mark, as the mark is inscribed on the same plate as the zodiac. To be sure, the x-ray CT (computed tomography) scans show that the region of the plate in the vicinity of the mark has cracks under the surface (and one crack is even visible in the surface images), so one could wonder whether this is a deliberately made mark or some sort of damage — a break in the plate, for example. However, to Price, who had the advantage of examining it directly, it seemed a deliberately made mark. The authors, separately on two different occasions, had the chance to view the fragment in its glass case at the National Archaeological Museum in Athens. To us, it also seems deliberately made, and all the more convincingly so when the mark is viewed in person (as opposed to in photographs or x-rays). But we must admit that this is not certain.[46]

We point out that the fiducial mark is nearly perfectly radial, that is, directed toward the center of the circular scales. In Figure 1, C is the geometrical center of the system of scales, found by simultaneous fitting of the four circles shown. The radial direction of the mark supports the view that it is indeed associated with the scales. Like Price, we ask whether it might have been intended as the "$t = 0$" setting mark for the Egyptian calendar ring. That is, we imagine that, someplace in the inscriptions, there would have been a line that read, "For such and such a year, set the first of Thoth (i.e., the first day of the Egyptian year) at the mark." (Alternatively, it would be conceivable to prescribe the setting of the calendar ring without the use of a separate fiducial mark, if, for example, there were an inscription that said: for such and such a year, place 1 Thoth against a certain degree of the zodiac.)

We are lucky in the portion of the zodiac that is preserved (approximately a quadrant). For the Sun's position on the first of Thoth fell in the extant portion of the zodiac between the years 425 BC and 72 BC. This encompasses practically the whole range of possible dates for the construction of the Antikythera mechanism, except perhaps for a very few years at the most recent end of the interval, immediately before the shipwreck. Thus, if there were a calibration mark for the first of Thoth, it would almost certainly have to fall in the preserved portion of the zodiac. But, there is one, and only one, such mark visible in the CT. As there is only one, it must in all likelihood be the setting mark for the calendar scale.

Year	1 Thoth	Sun
198	Oct 12	195.2
202	Oct 13	196.2
206	**Oct 14**	**197.2**
210	**Oct 15**	**198.2**
214	**Oct 16**	**199.1**
218	Oct 17	200.1
222	Oct 18	201.1

Table 1: Longitudes of the Sun (calculated from modern theory) at noon on the first day of Thoth, for geographical longitude 23° E.

Let us enquire for just which year the beginning of Thoth would be aligned with the fiducial mark. In Table 1, for each year in column 1, column 2 indicates the Julian calendar date corresponding to 1 Thoth [Bickerman 1980, 115–112]. Column 3 gives the longitude of the Sun calculated from modern theory for noon of 1 Thoth in the given year, at 23° east longitude (roughly the longitude of Antikythera itself, but, more importantly, in the middle of the Greek cultural zone from which the mechanism likely originated).[47]

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[46] Other scholars of the Antikythera mechanism whom we have consulted (each of whom has had ample opportunity to view it in person) are of divided opinion: two were of the opinion that the mark was most likely purposely made, one was convinced that it is due to accidental damage, and one was noncommittal.

[47] The National Renewable Energy Laboratory maintains a solar calculator at http://www.nrel.gov/midc/solpos/spa.html

The fiducial mark lies at about Libra 17.7°, i.e., longitude 197.7°, according to the modern convention, which assigns to the first mark of Libra the value 0°. However, it is probable that the ancient mechanic would have considered the long mark at the beginning of Libra to represent Libra 1°, which means the fiducial mark lies at 198.7°. To allow for either possibility, we look for years in which the Sun's noon longitude on the first of Thoth falls in the range from about 197.2° to 199.2° (thus allowing half a day one way or another about noon for either possibility). As can be seen, the result is the range 214–206 BC (shown in bold print in table). But we do not know just how the ancient mechanic would have calculated solar longitudes for this calendrical problem. Would he simply have used mean longitudes, for example? Moreover, how accurate was the equinox or solstice date that was used to tie the Sun to the calendar? If we allow a total of 2½° (roughly the size of the maximum solar equation) above 198.7° and below 197.7°, we look for years for which the Sun's longitude at local noon on the first of Thoth fell in the broader range 195.2°–201.2°. This gives us the more conservative estimate of 222–198 BC.[48] Of course, we have no way to know whether or not a single epoch date characterized the entire mechanism — the lunar and eclipse gear trains, for example, along with the Egyptian calendar. A single epoch is a plausible assumption, but no more than that.

Even if the mechanism should have an epoch date in the range 222–198 BC, it does not necessarily follow that it was built in this period. For example, (1) the extant machine could be a later copy or an elaboration of a mechanism built in this period. Or (2) perhaps a later designer drew upon a body of knowledge (a list of epoch positions of the Sun, Moon, planets, eclipse cycle, and calendar, etc.) from the late third century BC. Or (3) the designer, for some reason of convenience, could have adopted an epoch date that was substantially earlier than the date of construction. Ptolemy, in the planetary tables of the *Almagest* that he constructed in the second century AD adopted as his epoch the beginning of the reign of Nabonassar (747 BC), to ensure that neither he nor his readers would ever have to calculate a planetary position for a date before the epoch, which would have required a separate set of precepts. However, it is not so clear that the example of planetary tables applies very well to a gearwork mechanism: for a mechanical device, there would be no need for separate precepts for dates before "$t = 0$" — one would simply turn the input knob backwards. Also, while in the case of tables a far-distant epoch would pose no significant extra labor for practical calculation, the same cannot be said of a mechanical device. Here an epoch in the remote past would be inconvenient, as it would require lots of manual cranking to bring the machine up to the user's own date. Thus, if the fiducial mark is genuine, there are grounds to consider the possibility that it reflects a date not too remote from the date of construction (at least of a prototype or ancestor, if not of the extant machine).

The Lunar Anomaly in the Antikythera Mechanism

One of the most remarkable aspects of the Antikythera mechanism is its incorporation of a device to represent the lunar anomaly — the speeding up and slowing down of the Moon as it moves around the zodiac. The central idea is that one gear is mounted on another one of the same size and tooth count, but with the two axles slightly eccentric to one another. The driving wheel engages the follower by means of a pin that fits in a slot of the follower.[49] Because the wheels rotate about different centers, the uniform

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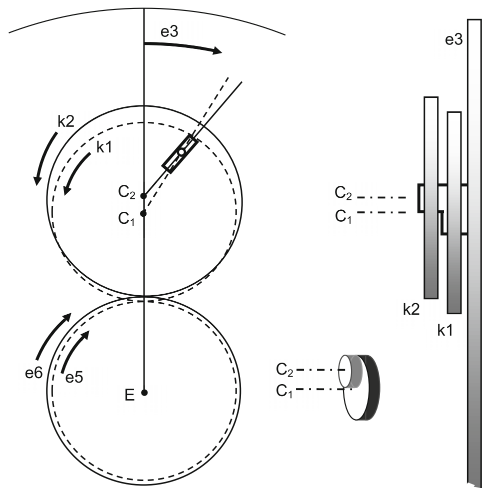
that was used for these calculations. However, this calculator does not accept values of ΔT greater than 8000s, so adaptations have to be made. For ΔT (the excess of atomically defined Terrestrial Time over Universal Time, which arises from the "clock error" of the Earth as its rotation decelerates), we used the value $3\frac{1}{2}^{h}$, which is appropriate for the years around −200 (201 BC). See Morrison and Stephenson [2004].

[48] In the course of our work, we discussed our conclusions with Alexander Jones, and found that he had independently arrived at a rather similar view: "(1) that if the mark is deliberately engraved, it must indicate the epoch position of Thoth 1 and thus an epoch date in the latish 3rd century BC, and (2) that the mark does indeed look deliberate though the coincident break makes certainty impossible." (Private communication.)

[49] Price [1975, 35] noticed this slot, but conjectured that it might be the result of an attempt to repair a broken gear. Wright [2002] described the pin-and-slot device and observed that it would suitable for modeling an anomaly, and perhaps a lunar anomaly. But because of problems with the tooth counts in the lunar gear trains, he could not settle on

motion of the first produces *nonuniform* motion of the second. Let us see how this pin-and-slot device works.



(A) (B)

Figure 2: The device for producing the non-uniform motion of the Moon in the Antikythera mechanism. Reproduced from Evans, Carman and Thorndike [2010].

The mechanism for the lunar inequality involves four gears of identical tooth number (50), called e5, e6, k1, and k2, illustrated in Figure 2A. This figure is based on the reconstruction by Freeth et al. [2006]. The input motion is from a hollow pipe at E (perpendicular to the plane of the diagram) that turns e5 at the rate of the Moon's mean sidereal frequency, ω_{si}. Concentric with e5, but turning freely from it, is a large wheel e3, which turns at the rate of the Moon's line of apsides. From a modern point of view, the orientation of the Moon's major axis does not stay invariable. Rather, the Moon's elliptical orbit itself turns in its own plane, so that the perigee advances in the same direction as the Moon moves, taking about 9 years to go all the way around the zodiac. The ancient astronomers were aware that the position of fastest speed in the Moon's orbit itself advances around the zodiac, the Greeks modeling the motion geometrically and the Babylonians by means of arithmetical period relations. In the Antikythera mechanism the advance of the Moon's perigee and apogee is modeled by letting e3 turn with a frequency we shall denote ω_A.

Riding on e3 at center C_1 is gear k1, which is driven by e5. A second gear, k2, turns about an axis, C_2, also attached to e3 but slightly offset from C_1. (In Figure 2, we have drawn wheels e5 and k1 slightly smaller than e6 and k2, in order to show the relationships among the wheels more clearly. But these wheels should all be thought of as having the same size.) The offset is achieved by using a stepped stud, with its larger diameter centered at C_1 and its smaller diameter centered at C_2, as shown in Figure 2B and in the perspective view. Wheel k1 has a small pin that engages a radial slot in k2. Thus k1, turning at a uniform angular speed, drives k2, producing a quasi-sinusoidal oscillation in the angular speed of k2. The motion of k2 is transferred to e6, rotating freely about axis E, which communicates the nonuniform motion of the Moon to the other parts of the mechanism. Uniform motion in (at e5) is transformed into non-uniform motion out (at e6) around the same axis.[50]

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this interpretation of the pin and slot and left it as an unexplained mystery. The first complete demonstration of the pin and slot as a device for the lunar anomaly is in Freeth et al. [2006].

[50] The output is by means of a central shaft attached to e6; and this shaft runs inside the hollow pipe to which e5 is attached.

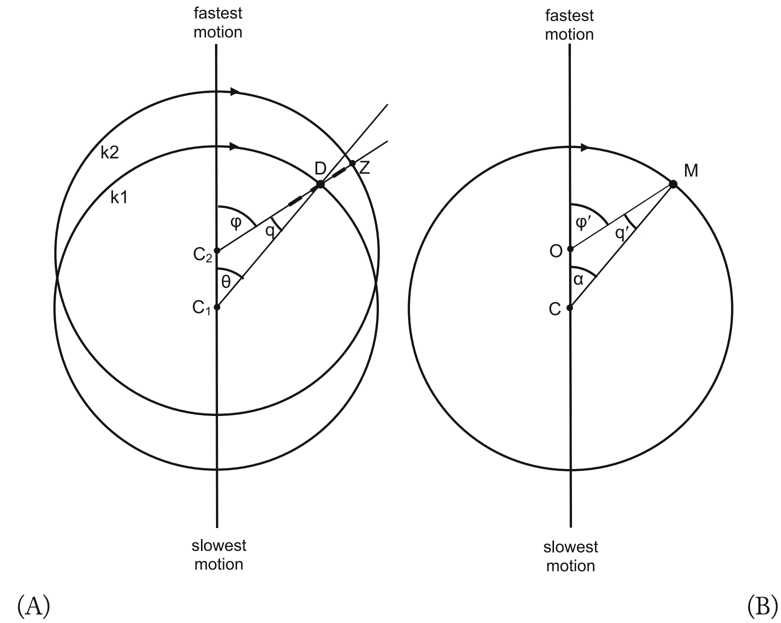


Figure 3: The pin-and-slot device of the Antikythera mechanism (A) compared with a standard eccentric-circle model (B).

In Figure 3A we see k1 and k2 in isolation. Wheel k1, carrying the pin D, turns uniformly about center C_1. On wheel k2, which rotates about center C_2, the radial slot is represented by the heavy dashed line.[51] Suppose k1 rotates uniformly, so that θ increases uniformly with time. Then a point Z on the perimeter of k2 will be seen from C_2 to rotate at a non-uniform angular speed. Angle φ increases more rapidly than θ when D is up in the diagram, and more slowly when D is down (though, of course, both wheels complete one period in the same time). At any instant, $\varphi = \theta + q$, where q functions as an equation of center. It is easy to show that

$$\sin q = \frac{e \sin \theta}{\sqrt{1 + e^2 - 2e \cos \theta}} \tag{1}$$

where $e = C_1C_2/C_1D$. This is the same equation of center as one gets with an ordinary eccentric circle.

To see the equivalence to an eccentric circle in terms of simple geometry (rather than trigonometry) let us examine Figure 3B, which represents the standard eccentric-circle model. (For the time being we suppose that the eccentric is fixed — i.e., we temporarily ignore the advance of the line of apsides. We will take up the question of a moving line of apsides below.) O is the Earth, and C is the center of the eccentric circle, around which the Moon M moves uniformly, so that angle α (the mean anomaly) increases uniformly with time. Then, as viewed from the Earth O, the angular position of the Moon at any time is $\varphi' = \alpha + q'$, where q' is the equation of center in the standard eccentric-circle model. It is immediately obvious that the pin-and-slot mechanism will reproduce the angular position of the Moon provided we put $\theta = \alpha$ (as we obviously need the mean motion to be the same in both models), and we require that $C_1C_2/C_1D = OC/CM$. Then the two equations of center will always be equal, i.e., $q = q'$. To put it another way, the direction C_2D (defined by φ) in the pin-and-slot mechanism is the same as the direction OM (defined by φ') of the Moon as viewed from the Earth in the eccentric-circle

[51] In Figure 3A, we have for the sake of simplicity shown k1 and k2 turning in the same direction as the Moon goes in Figure 3B. The key point is that the pin and slot reproduce the angular motion around an eccentric circle; the reversals of direction at the e5/k1 and k2/e6 interfaces need not concern us at this stage.

model. Now, the clever thing about the mechanism is that the non-uniform rotation of k2 is then transferred to e6, which rotates about the geometrical center of the zodiac scale. So a Moon marker driven by e6 will travel around a circle that is centered on E in Figure 2, but it will speed up and slow down on this circle.[52]

As far as is known, there is no extant ancient mention of the quasi-equivalence of the pin-and-slot mechanism to the eccentric circle model. This is a quasi-equivalence because the pin-and-slot mechanism produces the same motion in angle, but not the same physical motion in space as the eccentric-circle model. The output of the pin-and-slot device is a point moving at non-uniform speed on circle k2 — and, ultimately, after the motion is transferred back to e6, nonuniform motion on a circle concentric with the Earth. But the output of the eccentric-circle model is a point moving uniformly around a circle that is eccentric to the Earth, O.

An ancient Greek astronomer trained in the philosophical-geometrical tradition of Hipparchus and Ptolemy would not have regarded the pin-and-slot mechanism as a realistic representation of the lunar theory, for the pin-and-slot mechanism suppresses the motion in depth, though it does give a motion in longitude that agrees with what the eccentric theory prescribes. In the *Planetary Hypotheses*, Ptolemy criticized *sphairopoiïa* as traditionally practiced, saying that it "presents the phenomenon only, and not the underlying [reality], so that the craftsmanship, and not the models, becomes the exhibit."[53] It is possible that Ptolemy is merely complaining about a closed box, on the exterior of which the phenomena are displayed, but whose inner workings are kept sealed out of sight. But it seems to us likely that he is complaining just as much about the nature of the inner workings themselves. Suppose one took the lid off the box and saw inside, not epicycles and eccentrics, but pin-and-slot mechanisms, whose motions are a far cry from the real motions of the planets. *Quelle horreur*! For Ptolemy, the best *sphairopoiïa* would be one that offered a faithful display of the phenomena on the outside but that, when opened, revealed the true nature of planetary motion. Neither would Aristotle have approved of the pin and slot, as he maintains that each simple body (e.g., a celestial orb) should be animated by a single simple motion.[54] And here, the final output motion is the rotation of e6, which consists in a steady rotation with a superimposed oscillation. Did the ancient mechanic who designed the Antikythera mechanism realize the equivalence in angle of the pin-and-slot mechanism to the eccentric-circle theory? Or was this mechanism considered a rough-and-ready approximation to the behavior of the Moon — good for giving the final output angle, but not necessarily considered exact? In any case, no proof of the equivalence survives.

Perhaps the contrast between applied mechanics and accepted celestial physics should not surprise us, for there is a well-documented example of a similar contrast. Greek astronomers grounded in the philosophical-geometrical tradition (e.g., Theon of Smyrna, early second century AD) wrote treatises on deferent and epicycle theory while their contemporaries were busy mastering and adapting the non-geometrical planetary theory of the Babylonians [Jones 1999]. The philosophically-based astronomy of the high road explicitly endorsed uniform circular motion as the only motion proper to celestial bodies, while the numerically-minded astronomers (who needed quick and reasonably reliable results for astrology) made free and easy with nonuniformity of motion. In a similar way, it is possible that mechanical tricks of the trade such as the pin-and-slot mechanism were used in a craft tradition of model-building, quite apart from the practices of the "serious" (i.e., geometrically-minded) astronomers. On the other hand, Figure 3 shows that a proof of the equivalence in angle would have been well within the reach of Greek geometry. But the first historical accounts of ancient Greek astronomy were written by travelers of the high road (e.g., Ptolemy's historical remarks in the *Almagest* and Proclus's account in his *Sketch of Astronomical Hypotheses*). We should not be surprised that their accounts left no trace of the influence of mechanics on theoretical astronomy. Their silence on the issue should not be taken as evidence.

One last detail: We gave our equivalence proof above for a stationary line of apsides. Now we must

[52] The equivalence (in angular motion) of the pin-and-slot mechanism to a standard epicycle model was demonstrated by Freeth et al. [2006]. For a simpler demonstration of this equivalence, see Carman, Thorndike and Evans [2012].

[53] Ptolemy, *Planetary Hypotheses* i, 1 [Hamm 2011, 45].

[54] Aristotle, *On the Heavens* 268b28–269a2.

show that the pin-and-slot mechanism remains equivalent (in angular motion) to the eccentric-circle model, even when the pin-and-slot device is mounted on the turning wheel e3. The generalization is very simple. The trick is to view the motions in the frame of reference of wheel e3. This approach, involving the change of a frame of reference, was well within the scope of Greek mathematics. Ptolemy, for example, often subtracts angular velocities to effect a change of reference frame.

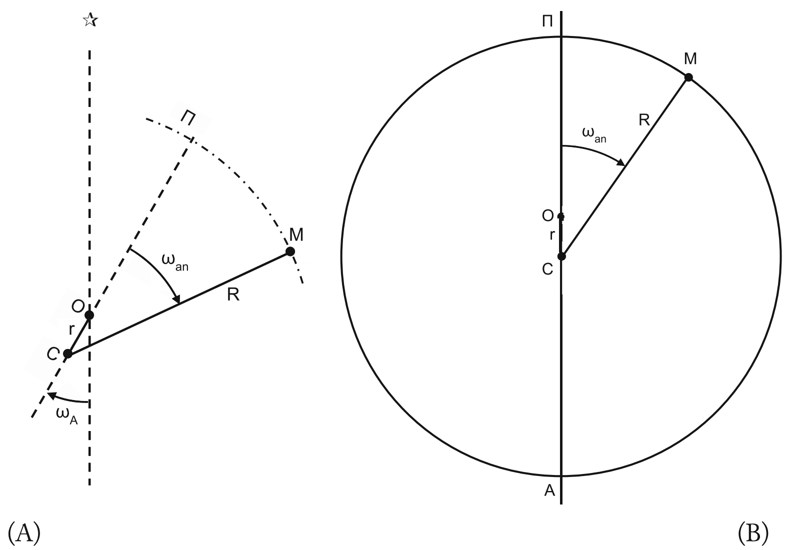
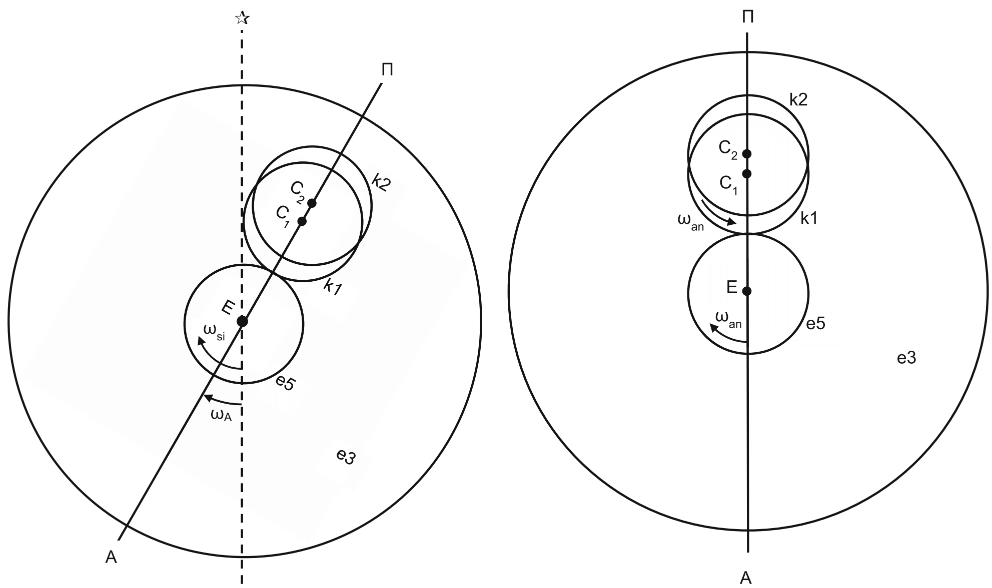


Figure 4: (A) The standard eccentric-circle model for the motion of the Moon, incorporating an advancing line of apsides. (B) The lunar theory of (A) as viewed in the frame of reference in which the line of aspides ACOΠ is stationary.

In Figure 4A, we see the standard eccentric-circle lunar theory, often associated with Hipparchus. The Earth O and the vertical reference line are fixed with respect to the stars. C is the center of the Moon's eccentric, and it moves in a circle of radius r around the Earth at a frequency ω_A. Thus the line of apsides CO slowly rotates. At the moment shown in the figure, the instantaneous position of the perigee is Π, and the instantaneous position of the Moon's circle (radius R) is shown by the dot-dash arc. Meanwhile, the Moon M moves in such a way that the mean anomaly increases uniformly with time, at the anomalistic frequency ω_{an}. (And $\omega_{an} = \omega_{si} - \omega_A$, where ω_{si} is the sidereal frequency.) In Figure 4B, we see the same lunar theory, but as viewed in a rotating frame of reference that is at rest with respect to the line of apsides ACOΠ.

Let us turn now to the Antikythera mechanism. In Figure 5A, we see the front face of the Antikythera mechanism as viewed in "absolute space." Wheel e5 turns at the Moon's mean sidereal frequency ω_{si} (corresponding to the sidereal month). Wheel e3, on which wheels k1 and k2 are mounted, turns at the frequency ω_A at which the line of apsides advances. Thus it is clear that the rate at which e5 turns with respect to the line of apsides is $\omega_{si} - \omega_A = \omega_{an}$, the anomalistic frequency. In Figure 5B, we see the same system, as viewed by an observer standing on e3. The line of apsides is fixed, and e5 rotates at angular speed ω_{an}. Thus it is clear that k1 also rotates at angular speed ω_{an} in this frame.

Comparing Figures 5B and 4B, we see that we have in each case a stationary line of apsides, and a mean angular speed of the Moon with respect to the line of aspides that is equal to ω_{an}. Thus the equivalence proof (in the rotating frame of reference) may proceed exactly as we outlined above for the case of a stationary line of apsides. And if the angular motion of k2 in Figure 5B is the same as the angular motion of the M about O in Figure 4B, as viewed in the rotating frame of reference, these two motions will remain equivalent to one another as viewed in any other frame of reference, including the space frame.



(A) (B)

Figure 5: (A) The lunar device of the Antiykthera mechanism, as viewed in the frame of reference at rest with respect to the box that contains the machine. (B) The mechanism of (A) as viewed in the frame of reference of wheel e3, such that the line of apsides AOCΠ is stationary.

Outer Planets

Recently, it has been shown that a pin-and-slot mechanism could also be used to represent the synodic cycle of a superior planet, including retrograde motion.[55] Again the pin-and-slot mechanism turns out to be exactly equivalent, in terms of angular motion (but ignores or suppresses the variation in distance), to the output of a simple concentric-deferent-plus-epicycle model. In the case of the lunar inequality, the pin-and-slot mechanism must be mounted on wheel e3 (which is fixed with respect to the lunar line of apsides). In the case of the outer planets, which retrograde when they are in opposition to the Sun, the main solar gear b1 plays the role of e3. That is, the pin-and-slot mechanisms for the outer planets must be mounted on the main solar wheel.

One simple way to accomplish this is shown in Figure 6. The input frequency of wheel u is the planet's sidereal frequency ω_{si}. (Thus we begin with a simple construction similar to that for the Moon shown in Figure 2: for the moment we do not worry about how the input frequency ω_{si} is obtained.)[56] The main solar wheel b1 turns at the Sun's sidereal frequency $\omega_{\odot}$. Thus, in the frame of reference of b1, u turns at frequency $\omega_{si} - \omega_{\odot}$. For a superior planet we must have $\omega_{\odot} = \omega_{si} + \omega_{an}$, where ω_{an} is the anomalistic (or synodic) frequency. Thus, in the frame of b1, the rotation frequency of wheel u is $\omega_{si} - \omega_{\odot} = -\omega_{an}$. Therefore, in the frame of b1, wheel u is turning counterclockwise at the correct angular speed to produce an anomaly with respect to the Sun. A pin-and-slot mechanism can be mounted on b1, as shown. In the frame of b1, the rotation frequency of x is ω_{an} and the mechanism introduces a nonuniformity into the rotary motion of wheel y in the usual way. The nonuniform motion of y would be transferred to a final wheel z (not shown), concentric with point E. Note that one need

[55] See Carman, Thorndike and Evans [2012] as well as Freeth and Jones [2012]. A film of a metal pin-and-slot mechanism producing retrograde motion may be seen at http://www2.ups.edu/faculty/jcevans/.

[56] As we shall see below, this is probably not the way the motions of the superior planets were modeled on the Antikythera mechanism. A more efficient design would allow us to produce the input frequency without using more gears. But the simple conceptual model shown in Figure 6 displays all that is essential for understanding the key points.

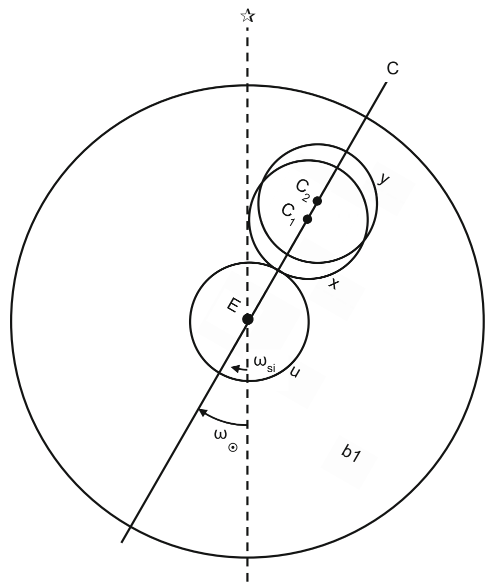


Figure 6: A pin-and-slot pair (wheels x and y) for a superior planet, riding on the main solar gear b1. b1 turns at the Sun's mean frequency $\omega_{\odot}$. In this hypothetical model the input wheel u turns at the planet's sidereal frequency ω_{si}, which is less than $\omega_{\odot}$.

not know anything about epicycles or eccentrics to arrive at such a mechanical solution. It is enough to know that the planet's retrograde motion is an anomaly with respect the Sun and that the pin and slot can be used to introduce a suitable wiggle into the steady motion.

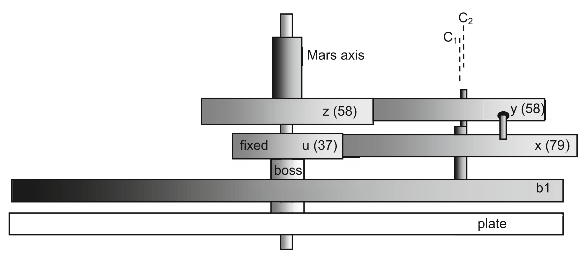


Figure 7: A more realistic pin-and-slot mechanism for Mars, as it might possibly have been realized on the Antikythera mechanism. The "input wheel" u is fixed, and x is driven from b1. From Carman et al. [2012].

Although the planetary gearing has not survived, most researchers believe that the Antikythera mechanism did also model the motions of the planets. The evidence for this comes from the inscriptions, which mention most of the planets by name [Freeth and Jones 2012, 8–10], as well as planetary phenomena, such as "stations" [Freeth at al. 2006, 587]. Additional evidence comes from the remnants of mounting hardware on b1: it looks as if *something* substantially complicated were originally mounted on b1, though it is now nearly all lost [Wright 2002; Freeth and Jones 2012]. While we have no way to know whether the pin-and-slot mechanism was used to model the motions of the planets, we do know that such a mechanism was used to represent the lunar anomaly. It seems plausible to suppose that a similar mechanism was used for the planets. The output of such a planetary device is motion in a circle concentric with the point representing the Earth, but motion that speeds up and slows down and occasionally reverses direction.

But the actual machinery for the superior planets built into the Antikythera mechanism probably more resembled Figure 7. Rather than supplying an initial drive of wheel u at frequency ω_{si}, one could make use of the freedom to vary the number of teeth on wheels u and x, and let wheel x be driven directly from b1. So, in Figure 7, drawn for Mars, wheel u is fixed to a stationary boss attached to the underlying plate. Wheel b1, carrying axle C_1, turns around once in a year. Because x engages u, x will be forced to rotate. If the gear ratio u/x is properly chosen, then x can be made to rotate at the anomalistic frequency, as seen in the frame of reference of b1. If x has 79 teeth and u has 37, for example, we will get for the rotation frequency of x with respect to b1 $(37/79)\omega_{\odot}$, which is a good match to ω_{an}, the anomalistic frequency of Mars. (79 years = 37 anomalistic periods is a preserved Babylonian period relation.) As far as the operation of the pin-and-slot mechanism goes, it makes no difference whether we use a solution like Figure 6 or one like Figure 7. The key thing is to mount the pin-and-slot device on the main solar wheel and to have x turn at the anomalistic frequency in the frame of reference of b1. Of course, economy of construction might well have an influence on just how we choose to get x turning at the right frequency.

Inner Planets

The inner planets may also be modeled with a pin and slot. Consider Figure 8, which shows the view in the frame of reference of "absolute space". The question is: At what frequency ω_* must the input wheel u turn? (The output wheel z, not shown, is concentric with u.) When we transform to the frame of reference of b1, by subtracting $\omega_{\odot}$, the motion of wheel u must be at the anomalistic frequency ω_{an} of the inferior planet. Thus it must be the case that $\omega_* = \omega_{an} + \omega_{\odot}$. In the frame of reference of b1, the frequency of wheel u is therefore just ω_{an}, as is appropriate for generating an anomaly with respect to the Sun.

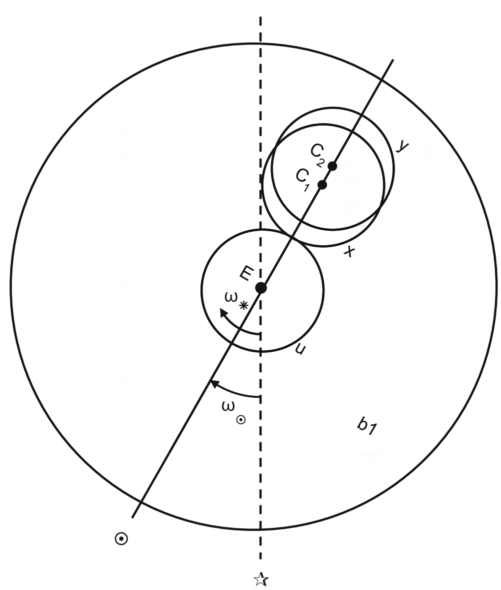


Figure 8: A hypothetical pin-and-slot representation of an inner planet, as viewed in the frame of reference of the box.

However, for an inferior planet, in frame b1 there can be no progressive motion of the planet — merely an oscillation, with no net forward progress. Thus the pin-and-slot mechanism must be modified, in conformity with Figure 9. The key thing is that the distance C_1C_2 between the centers of wheels x and y must now be *greater* than the distance C_1D of the pin from the center of wheel x. As before, the heavy dashed line shows the radial slot in wheel y. Wheel x turns at the constant anomalistic frequency, so θ increases uniformly with time. The resulting motion of wheel y is then a quasi-sinusoidal oscillation, with no net forward progress. It is easy to show that the equation of center q is also given by Equation 1, where $e = C_1D/C_1C_2$. Thus, a point Z on the perimeter of wheel y simply oscillates back and forth in the frame of b1. This motion is transferred to wheel z (concentric with u). Finally, when we return to the frame of reference of the box, the steady forward motion at frequency $\omega_\odot$ is added to the oscillation. The result is exactly what we need — a back and forth oscillation superimposed on a steady forward motion that keeps pace with the mean Sun.

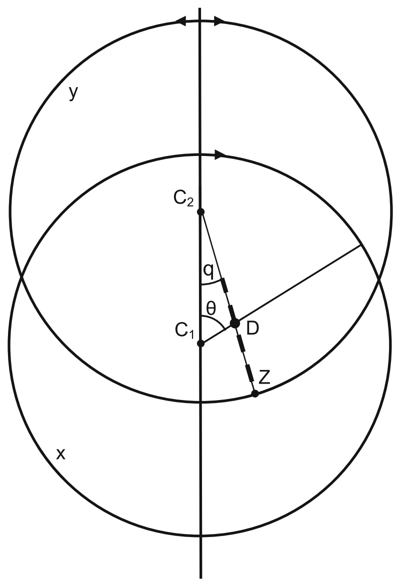


Figure 9: The arrangement of the pin-and-slot mechanism required for an inner planet: $C_1D<C_1C_2$. This effectively destroys the steady forward motion and results in a simple oscillation of wheel y in the frame of b1.

And here we may establish a connection with Ptolemy's *Planetary Hypotheses*. For the input frequency that we have denoted $\omega_*(= \omega_{an} + \omega_\odot)$ is precisely the angular speed of the planet on its epicycle, reckoned from the direction of the vernal equinox. (See Figure 10.) Now, in giving planetary parameters in forms that he felt would be helpful for those who wish to build models, Ptolemy formed his so-called compound or complex periods. For the inferior planets, the period in question is precisely that of the planet on its epicycle, but as reckoned from the vernal equinox.[57] It has never been quite clear why Ptolemy introduced this period. But if Ptolemy is thinking that builders are going to be using pins and slots, we can see why. This suggests a continuous tradition of working with pin-and-slot mechanisms at least from the time of the Antikythera mechanism (second century BC ?) to Ptolemy's own day (second century AD).

It should be noted that in seeking a pin-and-slot mechanism to use for the inner planets, we had no need of, and did not use, anything from epicycle or eccentric theory. It was enough to know that ω_* must necessarily be equal to $\omega_{an} + \omega_\odot$ and that, in the frame of b1, wheel y can make no net forward

[57] Ptolemy, *Planetary Hypotheses* i, 2 [Hamm 2011, 172–174]. These complex periods are discussed in Duke [2009].

progress.[58] Also, in the particular case of the Antikythera mechanism, if the motions of the inferior planets were represented kinematically by means of pins and slots, it is more likely that a construction like that in Figure 7 was used, since it allows one to obtain the correct rotation frequency of wheel x more economically.

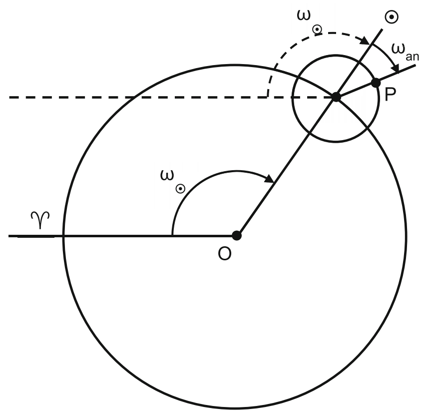


Figure 10: Standard epicycle model for an inner planet. $\omega_\odot + \omega_{an}$ is equal to the angular frequency of the planet P on its epicycle, as measured from a line parallel to the direction to the vernal equinox ♈.

Possible Origins

How might the pin-and-slot mechanism have originated? One possibility, of course, is that it is an after-the-fact adaptation of eccentrics or epicycles. In such a case, an astronomer steeped in geometrical planetary theory, or the mechanic with whom he was collaborating, dreamed up a clever way to separate out the angular part of the motion and display it alone. In this case, in Figure 3, the predominant influence went from right to left, that is, from pure theoretical astronomy to mechanical simulation. This is probably the majority view among historians of ancient astronomy. It is perfectly possible. And we do know of two mechanisms from times suitably later than the invention of epicycles and eccentrics. According to Cicero, Posidonius built such a model.[59] (Cicero invokes the *sphaera* of Posidonius in an early instance of the "watchmaker" version of the argument from design for the existence of god: if we came across this *sphaera* we would hardly doubt that it was built by a rational being; but what then of the cosmos that it imitates?) And Strabo mentions that when the Roman general Lucullus took Sinope (on the north coast of Asia Minor) in 70 BC, one of the objects carried off was the "sphere" (*sphaira*) of Billarus.[60] As there is no detail, we cannot be sure whether this was a planetarium-style "sphere" like those of Archimedes and Posidonius or a simple celestial globe; but the former seems somewhat more likely, as an ordinary globe would have been rather a commonplace by 70 BC. So here are one or two devices, known to have existed in the early first century BC, which comfortably post-date the invention of epicycles and eccentrics. The Antikythera mechanism could well be a third.

[58] We should note the existence of an alternative solution. Above, we argued that ω_* for an inferior planet must be chosen so that the rotation frequency of u in frame b1 is equal to ω_{an}. But it would also be possible to have the rotation frequency of u in frame b1 equal to $-\omega_{an}$. Then we would have $\omega_* = -\omega_{an} + \omega_\odot$. In epicycle theory, this would correspond to the planet revolving in the backwards direction on its epicycle.

[59] Cicero, *On the Nature of the Gods* ii, 88 [Rackham 1952, 206–208].

[60] Strabo, *Geography* xii, 3.11. For a proposal that the Antikythera mechanism may be the lost sphere of Billarus, see Mastrocinque [2009].

A second possibility is that the pin-and-slot mechanism owes its inspiration to the system of Eudoxus and Callippus. We don't mean to suggest that the pin and slot are a literal representation of Eudoxian planetary theory. But a mechanic, in an attempt to adapt the spherical models to a plane in building a machine for display, may have borrowed an idea. Here the connection is most easily seen in the case of planetary motion. Let us recall that for replicating retrograde motion, Eudoxus imagines two spheres that turn in opposite directions, with their axes offset at a small angle, and the axle of one sphere set into the surface of the other. The outermost sphere of this pair has its axis inserted into the "equator" of a sphere that rotates with the planet's zodiacal motion. If we imagine flattening the system out — taking say just the northern hemisphere this might readily suggest something close to Figure 3A. Against this is the fact that in Eudoxus's system, the two spheres we have mentioned turn in opposite directions, rather than in the same direction. But the general idea of an off-axis wobble might have been transferable. In addition, Eudoxus's idea of *homocentric* spheres might well be modeled by pin-and-slot devices, as they do effectively produce suitably non-uniform motion on a circle concentric with the Earth.

A third possibility is that the pin and slot arose from an effort to model Babylonian phenomena (and this need not be inconsistent with Eudoxian influence on the choice of mechanism). If this were the case, we need not suppose that the mechanic had a detailed understanding of the equivalence of the pin-and-slot mechanism to epicycles or eccentrics, or of epicycles and eccentrics to one another. It would be enough to provide a back-and-forth motion superimposed on a mean motion, without any worry about the geometrical details. Here the simplest case to consider is that of the Moon. Geminus, in his *Introduction to the Phenomena* (first century BC) describes for Greek readers the essential features of the Babylonian lunar theory now known as System B.[61] The Moon's daily motion changes from day to day according to a simple saw-tooth-pattern, with uniform daily changes of 0;18° between maximum and minimum daily displacements of 15;14,35° and 11;6,35°. (We use the Neugebauer notation, in which whole degrees stand to the left of the semi-colon, and successive sexagesimal parts stand to the right and are themselves separated by commas.) This leads to a lunar "equation of center" of quadratic form:

$$\begin{aligned} q &= DT\left[\frac{1}{2}\frac{t}{T} - \left(\frac{t}{T}\right)^2\right], \qquad \text{for any time } t \text{ between 0 and } T/2\text{, and} \\ q &= DT\left[-\frac{1}{2}\left(\frac{t}{T}-\frac{1}{2}\right) + \left(\frac{t}{T}-\frac{1}{2}\right)^2\right], \qquad \text{for } t \text{ between } T/2 \text{ and } T, \end{aligned} \tag{2}$$

where q is the difference between the Moon's longitude according to System B and the longitude it would have if it moved uniformly. The time t is reckoned from the moment of fastest motion, T is the length (in days) of the anomalistic month, and D is the difference between the greatest and least daily motion (in degrees per day). The associated length of the anomalistic month is T = 27;33,20 days. The curves for q are segments of parabolas (given by Eq. 2), alternately concave upward and concave downward. See Figure 11.

The graph in Figure 11 also shows the equation of center given by Equation 1 and using the dimensions of the pin-and-slot device in the Antikythera mechanism. Freeth [2006, 590] reported distance C_1C_2 (in Figure 3A) as 1.1 mm and distance C_1D as 9.6 mm. Clearly, the pin-and-slot device of the Antikythera mechanism does a fine job of modeling Babylonian lunar theory. The maximum equation of center is larger than one would expect for a Greek theory based on an epicycle or eccentric.[62] Of course, this could be an accidental result of construction — a few tenths of a millimeter in C_1C_2 would make a big difference. For example, if C_1C_2 were 0.8 mm (instead of the measured 1.1), the maximum equation would be in the 5° zone that Ptolemy adopted.

For comparison's sake, we also show the equation of center for Ptolemy's first lunar model (a simple epicycle model with epicycle radius 5;15 and deferent radius 60), as well as for the two versions of Hipparchus's lunar theory. According to Ptolemy, Hipparchus found two different values for the lunar

[61] Geminus, *Introduction to the Phenomena* xviii, 4-19. See Evans and Berggren [2006, 228–230 and 96–98].

[62] We are grateful to Dennis Duke for pointing this out.

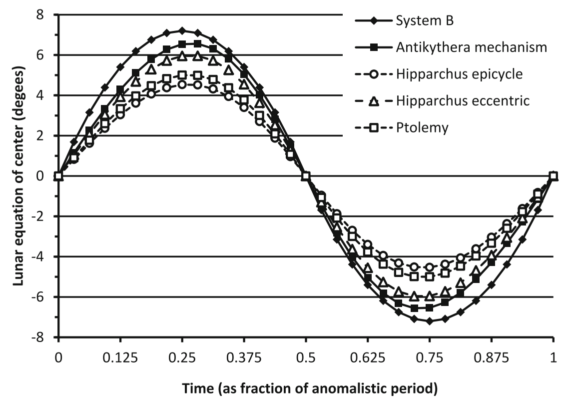


Figure 11: The equation of center of the Moon, according to various ancient models.

eccentricity.[63] Using the eccentric-circle model he found 327.67/3144 for the ratio OC/CM (Figure 3B). But using the epicycle model, he found the ratio 247.5/3122.5 for the ratio of the epicycle's radius to the deferent's radius. According to Ptolemy, Hipparchus used different data for the two determinations, but the results were also marred by faulty computation. But, Ptolemy says, some people have wrongly thought that the difference in the results must be due to some difference between the two hypotheses. This interesting remark shows us that, in the period between Hipparchus and Ptolemy, the equivalence of epicycle and eccentric may not have been thoroughly understood by all astronomers. Toomer [1967] presented a good argument that Hipparchus eventually adopted the smaller of his two results for the epicycle radius.

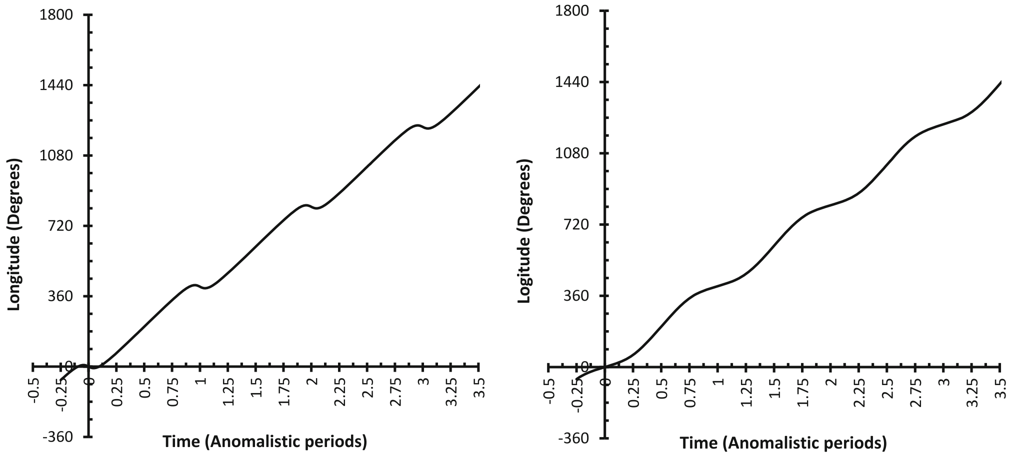
However, we would like to stress that we are not making an argument about whether it is Greek or Babylonian lunar theory that is represented on the Antikythera mechanism. We just wish to suggest that the Babylonian planetary theory could conceivably have served as model for a gearwork lunar mechanism that incorporated a moving line of apsides, that the Babylonian theory is a reasonably close match to the behavior of the later Greek epicycle theory, and that no knowledge of epicycles and deferents would have been required to incorporate the Babylonian theory into a gearwork mechanism.

In the lunar theory of System B, each parabolic segment is symmetric about its maximum or minimum. But the equation of center for an epicycle or an eccentric has a slight asymmetry, due to the form of the equation printed above. The pin-and-slot device, being equivalent in angle to either an eccentric or an epicycle, shows a similar asymmetry. However this assumes that the pin leads the slot — i.e., that the input motion is to the wheel carrying the pin (as is the case on the Antikythera mechanism). If the slot leads the pin, it turns out that the functional form of the equation of center is different [Carman, Thorndike and Evans 2012, 99], and one has instead:

$$\sin q = e \sin \theta. \tag{3}$$

In this case, the bumps of the equation of center curve are symmetric about their maxima and minima. At the level of precision of astronomy in the second century BC, there would be no way to choose empirically between these two forms. Is the fact that the pin leads the slot in the Antikythera mechanism a sign that the mechanic understood the equivalence to an eccentric-circle model? This could be the case, of course, but there are two reasons why the evidence is ambiguous. First, the mechanic would likely have regarded the pin as the active element, and the slot as passive, so it would have been intuitive to place the pin on the wheel with the input motion. And second, the lesson of the planets could have been determinative.

63 Ptolemy, *Almagest* iv, 11 [Toomer 1984, 211]. See the discussion by Neugebauer [1975, 317–319].



(A) (B)

Figure 12: (A) Motion in longitude produced by a pin-and-slot device scaled appropriately for Mars, with the pin leading the slot. The equation of center is given by Equation 1. (B) Motion in longitude produced by a pin-and-slot device scaled appropriately for Mars, but with the slot leading the pin. The equation of center is given by Equation 3.

For it happens that in the case of a planet with a large epicycle and with an anomalistic period longer than the sidereal period (such as Mars), the version of the mechanism with the slot leading the pin will not actually produce retrograde motion at all [Carman, Thorndike and Evans 2012, 113]. In Figure 12A we see a graph of longitude versus time for a pin and slot device with the synodic and sidereal frequencies and the eccentricity *e* chosen appropriately for Mars, and with the pin leading the slot: retrogradation appropriately occurs once every synodic cycle. But in Figure 12B we see the outcome with the same frequencies and the same *e*, but with the slot leading the pin. So, once the mechanism was applied to the planets, the "correct" relation between pin and slot would be settled on almost automatically. Our research group discovered this behavior of the pin and slot *empirically* — the fact that it makes an important difference whether the pin leads the slot or the slot leads the pin. Due to a miscommunication between the designer of our pin-and-slot device for Mars and the machinist who made it, our first model was built with the slot leading the pin, and it displayed no retrograde motion. This is an example of the way that mechanics and theoretical astronomy, in contact with one another, may have helped lead to refinements in planetary theory in its early, formative period.

If there were pins and slots before there were epicycles and eccentrics, the predominant direction of historical transmission in Figure 3 would be from left to right. Epicycles would have emerged through some mathematical astronomer steeped in natural philosophy taking seriously the possibility of one turning wheel riding on another. Did epicycles emerge, then, through an effort to model Babylonian phenomena with a mechanical invention? While we cannot know, there are some arguments that can be offered in support of this possibility. First, there is the fact that the gear trains in the Antikythera mechanism are based on Babylonian period relations. Moreover, the gearwork shows its designer's preference for directly relating synodic months to years and to anomalistic months, without using the day as a fundamental unit.[64] This is characteristic of Babylonian lunar theory.

Second, the representation of the solar anomaly on the Antikythera mechanism appears to be based simply on a nonuniform (but piecewise-uniform) division of the zodiac.[65] The whole of the extant portion of the zodiac happens to lie in the fast zone of the Babylonian solar theory of System A. In this

[64] We thank Alexander Jones for his insight on this point.

[65] Evans, Carman Thorndike [2010]. It is not quite possible to exclude an underlying geometrical theory (eccentric circle), but the statistics favor System A.

theory, the Sun travels at a uniform speed of 30° per synodic month in the fast zone (and at a speed of 28⅛° per synodic month in the slow zone). Simply by matching degrees on the zodiac against days on the Egyptian calendar scale, one can see that over the useable 69° of the scales the correspondence is indeed consistent with 30° per synodic month. This result does not depend on being able to find accurate centers of the extant portions of the scales; it comes from simply looking to see which degree mark is against which day mark. And it is hard to see how such a large effect could result from a sloppily performed division. One would imagine that in inscribing a zodiac, for example, the mechanic would begin by dividing the circle into quadrants. So it is difficult see how an effective equation of center could rise steadily from 0° to more than 2° over the course of 69° of longitude simply by chance error.[66]

And third, there are some complexities in the application of the pin-and-slot mechanism that would have to be mastered to go from epicycles and eccentrics to pins and slots. Notably, there is the fact that in standard theory, the Moon goes around backwards on its epicycle, but the planets go forward on theirs. It turns out, however, that it all works out fine with pins and slots. The key thing is that for the superior planets, the solar wheel turns faster than the sidereal frequency of the planets, but in the case of the Moon, wheel e3 turns more slowly than the sidereal frequency of the Moon. This leads to a very nice sort of geometrical reversal that allows both the outer planets and the Moon to be modeled in the same way.[67] And to transfer the design from the superior planets to the inferior, there is also the reversal in the relative sizes of C_2C_1 and C_1D (compare Figures 3 and 9) that was discussed above. Everything considered, we believe it would have been easier to arrive at a mechanical representation of Moon and inner and outer planets based on the pin-and-slot mechanism simply by starting from the phenomena than by starting from epicycle-and-deferent theory.

We know from the preface to Book I that Apollonius composed his *Conics* while living in Alexandria, which was the capital city of mechanical modelers. Apollonius himself may have been involved with mechanics and wonder-working, for several Arabic manuscripts preserve all or part of a description of an automaton flute-player attributed to a certain Apollonius.[68] This apparatus involves a water tank, valves, and gears. In the manuscripts the title reads: "Apollonius (a-b-l-n-y-w-s) the carpenter [and] the geometer. The art of the flute player." Now, "the Carpenter" is an epithet often attached to the name of Apollonius of Perge in medieval Arabic literature. For example, Ṣāʿid al-Andalusī in his *Book of the Categories of Nations* (11th century) wrote, "Among the Greek mathematicians, we have Ablūniūs the Carpenter, who wrote the book on *Makhrūṭāt* [Conics], which discusses bent lines that are neither straight lines nor arc segments ..." [Salem and Kumar 1991, 27].[69] The treatises in the Paris manuscript containing the "Apollonius" treatise are:[70] 1st, a "revision" or "improvement" of Theodosius on the sphere by Muḥyi al-Dīn al-Maghribī. 2nd (from fol. 29v), the treatise on a water-clock attributed to Archimedes, discussed above. 3rd (fol. 39v-42v), the treatise of Apollonius the Carpenter on the flute player. So, whether the compiler of this manuscript had the attribution right or not, it does seem likely, in view of the presence of Theodosius and Archimedes, that he had an ancient (and not a later) Apollonius in mind. Moreover, the "revision" of Theodosius does indeed begin with a selection of theorems from the first book Theodosius's *Spherics*; the second and third parts show more originality.[71]

[66] New work, still to be published, shows that the equation of center effect extends over the entire preserved 88° of the zodiac.

[67] This is discussed in detail in Carman et. al. [2012, 101-103].

[68] Bibliothèque Nationale, Paris, ms. Arabe 2468 (which may be viewed at http://gallica.bnf.fr/ark:/12148/btv1b52000453w/f90.image), British Library, Add. 23391, New York Public Library, Spencer, Indo-Persian ms. 2. For a description of this ms. see Schmitz [1992, 165–168]. An Arabic text based on all three manuscripts, with English translation and discussion, may be found in Shehadeh, Hill and Lorch [1994]. There is a German translation and discussion, based on the Paris and London mss., first published in 1914, in Wiedemann [1970, 2:50–56]. An additional manuscript at the Université St. Joseph, Beirut, was closely related to the London ms; this has disappeared, but photographs of it survive (see Shehadeh, Hill and Lorch [1994], who also mention a fragment of the treatise at Damascus).

[69] Perhaps, as Len Berggren has suggested (personal communication), an early Arabic writer thought that "carpenter" was a suitable thing to call a man who occupied himself with taking sections of cones.

[70] For details, see the on-line BN catalogue description of Arabe 2468 at http://archivesetmanuscrits.bnf.fr/ead.html?id=FRBNFEAD000030385.

[71] We have compared Carra de Vaux's [1891, 291-294] summary of the contents of the Arabic "revision" with Ver Eecke's

4th (from fol. 36), a hodgepodge on various topics, including regular polygons, the comparative weights of various minerals, and leveling. 5th and final (from fol. 52), a description of a "perfect compass," by Abū Sahl al-Kūhī and presented to the sultan Saladin, by means of which one can draw all the conic sections.[72] As Shehadeh, Hill and Lorch [1998, 355–356] remark, the authorship of the "Apollonius" flute-player treatise may never be known, and it could conceivably be of Hellenistic, Byzantine or even Islamic origin. However, Hill [1976, 9] previously suggested that the first part of the "Archimedes" treatise, including the arrangement of the gears, may possibly be the work of Archimedes. This, they argue, somewhat strengthens the case that the "Apollonius" treatise was composed by Apollonius of Perga. However, they note that the Arabic of the "Apollonius" treatise differs from that of the "Archimedes," which suggests different translators, so one has no way of knowing whether these two treatises were grouped together in Hellenistic times or long afterward. Finally, the authors point out that the mention by the "Apollonius" treatise of the vertical water wheel as a recent invention supports a Hellenistic date of composition.[73]

We do have the earlier example of Archimedes as a geometer who was also interested in mechanics and astronomy. And much later, according to the Suda, Ptolemy wrote a work on mechanics in three books.[74] That Apollonius might have had an interest in mechanics is not implausible. Certainly, as a mathematician living in Alexandria he could not have been unaware of gears and their uses. Much of what we have proposed in this paper must remain speculative. We have used the Antikythera mechanism frequently in our argument, as it offers us the only real insight we have into the design of early astronomical gearing. But our goal has been to explore a new approach to the early, formative period of Greek planetary theory, rather than the history of this particular machine, which could well turn out to be considerably later. We are left with a fascinating possibility to consider. Early Greek mechanics may have contributed in a significant way to the development of Greek theoretical astronomy. We should imagine planetary astronomy in conversation with mechanics, rather than a one-way transmission.

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[72] The text of this treatise has been published with a French translation by Woepcke [1874].

[73] On the other hand, Wilson [2008, 338] does not hesitate to ascribe the treatise to Apollonius of Perga.

[74] See *Suda On Line: Byzantine Lexicography* at http://www.stoa.org/sol/.

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