Mathematical Reconstructions Out, Textual Studies In: 30 Years in the Historiography of Greek Mathematics [1998]

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History of Greek Mathematics Before and After 1970

Thirty years ago, at the end of the sixties, the history of Greek mathematics was considered an almost closed subject, just like physics was at the turn of the twentieth century. People felt that they had constructed a definitive picture of the essence of Greek mathematics, even though some details remained unclear due to irrecoverable document losses. Critical editions had been established, mainly by Heiberg, while two of the great scholars of the history of Greek mathematics, Tannery and Zeuthen, had built on this material. Then, the standard book [Heath 1921] brought much of this material together. Through his discoveries in Mesopotamian mathematics, Neugebauer was led to think that he had given substance to legends about the Oriental origin of Greek mathematics. Originally published in Dutch in 1950, the book [van der Waerden 1954] reflected scholars' self-confidence in this period.

One may well compare what happened after 1970 in the historiography of Greek mathematics to the developments of physics in the first decades of this century. In some sense the change in the history of Greek mathematics was even more dramatic, because no new important material was discovered since 1906, at which time the *Method* was brought to light by Heiberg. This great interpretative change was mainly due to a shift in scholars' attitudes. | 132

In the following, the historiography of Greek mathematics before 1970 will be briefly contrasted, with no pretense of being exhaustive, with that which followed.[1]

The Origins: Who Was the First Mathematician?

A tradition reaching as far back as Eudemus (late 4th century BC), via citations found in Proclus' *Commentary on the First Book of Euclid's Elements*, considers Thales (ca. 585 BC) to have been the founder of the Greek mathematical sciences. However, if by the phrase *"the origins of Greek mathematics"* we mean an embryo of the rigorous deductive structure found in the *Elements*, Thales had little to do with it. Eudemus may well have constructed a story of a mathematician from fragmentary sources at his disposal, which described the practical knowledge of a wise man (see [Dicks 1959], and also [Vitrac 1996]).

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[1] Accounts given below are a personal view, even if I tried to be as impartial as possible in the bibliography. I restricted myself to studies of mathematics in the classical period, that is, mathematics before Apollonius, although developments of research in late antiquity, including the rediscovery of part of the lost books of Diophantus' *Arithmetica* in Arabic, should not be underestimated.

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Dismantling the myth of origins became the subject of hot debates centered around the figure of Pythagoras (ca. 572–ca. 494 BC), who enjoys no less enthusiastic advocates today than in the ancient world. However, considerable if not decisive, is the damage done to *"Pythagoras the mathematician"* by the blow of the epoch-making study [Burkert 1972].[2] Now we see Pythagoras as the founder of a prevalently (but not exclusively) religious community, established on doctrines of reincarnation and metempsychosis. To be a Pythagorean meant choosing a certain way of life based on these doctrines, without being necessarily involved in philosophical or scientific inquiries.

Thus we are rather concerned, now, with the role of the Pythagoreans in the development of Greek mathematics after the middle of the fifth century. The picture once prevailed that the discovery of
133 incommensurability was a scandal for the Pythagoreans and provoked a *crisis*. This | belief was deduced from 1) the alleged Pythagorean monopoly on the mathematical sciences in the fifth century; 2) their central dogma "all is number;" and 3) Iamblichus' testimony. However, 1) has no good evidence to support it; 2) is very likely an Aristotelian summary deduced from Philolaus' (ca. 470–ca. 390 BC) book; and 3) is so confused that it is hardly reliable (which means that we have no authentic document to credit the Pythagoreans with the discovery of incommensurability). The scandal, or foundations-crisis thesis has thus turned out to be scarcely plausible (see [Freudenthal 1966], [Knorr 1975, 21–61], [Fowler 1987, 294–308]). More recently [Fowler 1994] has even suggested that this discovery itself may have been no more than an incidental event. After all, the above thesis may have been a retroprojection of early twentieth-century interests in the foundations of mathematics.

Therefore, the roles traditionally ascribed to Pythagoreans are also to be reconsidered and greatly modified, a point to which we shall later return. For the moment let us examine modern studies devoted to the theory of proportions.

Mathematical Reconstructions

If a foundation-crisis theory was soon dismissed, the assumption that incommensurability constituted a turning point in Greek mathematics enjoyed better support. In fact, it seems natural to us, today, to suppose that the discovery of incommensurability called for a new definition of proportions (sameness of ratios) applicable to incommensurable magnitudes. This assumption gave birth to the most influential historical approach in this century: mathematical reconstruction.

[Becker 1933] pointed out that a passage of Aristotle's *Topics* can be construed as evidence for the existence of a definition of proportions based on *anthyphairesis* (Euclidean algorithm), which can be dated to a period between the discovery of incommensurability (probably second half of the fifth century) and Eudoxus' time (ca. 390–ca. 337). This paper not only called attention to the technique of *anthyphairesis*, but also encouraged scholars to use mathematical reconstructions in order to venture new conjectures and hypotheses. One eminent example of this technique is [Fritz 1945], which proposed, with no direct textual evidence, that incommensurability had first been found in a study of the relation between the
134 side and diameter of regular pentagons by the method of | *anthyphairesis*. Even [Knorr 1975], the most critical and thoroughgoing study of the development of incommensurability theory to date, remained highly speculative, and in a sense, this book marked a culmination in the tradition of the reconstruction approach opened by Becker.[3]

Although the significance of this kind of study cannot be denied, its danger also is obvious: one has no general criterion to judge whether the reconstructed argument ever existed in antiquity. Moreover, while most reconstructions deal with the period around and before 400 BC, sources come from later

[2] Reasonable doubts over whether Pythagoras actually was the first mathematician and philosopher go back at least to [Vogt 1908/09], and perhaps even to [Zeller 1844–1852]. Burkert's central thesis, as well as scholarly developments after 1972, are very skillfully, and with extraordinary clarity and concision, described in [Centrone 1996]. For opinions sympathetic to the view of Pythagoras and the Pythagoreans as scientists, see [van der Waerden 1979] and [Zhmud 1997].

[3] I exclude from this tradition the book [Fowler 1987] whose reconstructions undoubtedly are more sophisticated: see below.

periods.[4]

From Mathematical Reconstructions to Textual Studies

However, since any significant interpretation of ancient mathematics is bound to involve some kind of conscious or unconscious reconstruction, one may well ask whether it is actually possible to distinguish recent research from previous reconstructive approaches. Let me try to justify this distinction, granted that, here, my account inevitably is more personal than other parts of this paper.

Previous scholars (say, from Tannery and Zeuthen to van der Waerden) were, I believe, confident in the power of something like universal reason, and took it for granted that a careful mathematico-logical reasoning was able to restore the essence of ancient mathematics. Today scholars are more skeptical: the type of reasoning that once played an essential role tends to be regarded as a mere rationalizing conjecture. They are even convinced that the modern mind will always err when it tries, without the guide of ancient texts, to think as the ancients did (I personally think | that this opinion can be attributed 135
to an indirect influence of Thomas Kuhn).

Thus, texts are read in a different manner by recent historians of mathematicians, as well as by historians of science in general. For example, apparently redundant or roundabout passages call for more attention, because these might reveal some of the ancients' particular thoughts of which modem minds are unaware.[5] This is one of the attitudes typical of what I call "textual studies" in a broad sense (I do not restrict them to textual criticism), an attitude based on reasonable doubts as to the validity of logical conjectures.

Happily for the French-reading public, the spirit and results of this new textual approach are best embodied in the French translation of the *Elements* now in progress (see [Euclide 1990-2001]),[*] but a review of other studies will also help us understand the new historiography. Renewed interest in text led to careful examinations of the extant mathematical documents and their logical structure. Since most of these documents are series of propositions, the logical interdependence of propositions, or the "deductive structure," is one of the important subjects of recent studies. Most of this work has been limited to Euclid's *Elements*: see [Beckmann 1967], [Neuenschwander 1972], [Neuenschwander 1973], and the comprehensive and influential book [Mueller 1981].[6] Lately, [Netz 1999][**] has proposed brand-new, insightful approaches to texts.[7]

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[4] Anomalies and idiosyncrasies in the logical structure of the *Elements* have been used by many scholars (including myself) in order to reconstruct the earlier phase of Greek mathematics. For example, the first four books of the *Elements* contain several demonstrations more easily proved with the theory of proportions. These demonstrations have been either located in the period when no adequate theory of proportions was available or attributed to some mathematician who compiled earlier versions of the *Elements*. [Artmann 1985] and [Artmann 1991] are a remarkable outcome of this approach. However, this approach relies on the assumption that Euclid's editorial intervention was minimal and the extent to which this assumption can be justified is unknown to us. With a bit of irony, Vitrac called this kind of approach an *"enquête archéologique"* [Vitrac 1993, xi]. See also [Gardies 1998], which developed very specific reconstructions based on logical analyses, and [Caveing 1994–1998].

[5] Here, one cannot but recall the attractive work of Árpád Szabó (I am thinking of [Szabó 1969] and less known [Szabó 1964]), who, using philosophical arguments, was the first seriously to criticize the trend of mathematico-logical reconstructions. His approach predated the present research trend. He was however concerned with finding traces of the earliest developments of Greek mathematics, and his arguments inevitably remain no less speculative than the theses he challenges.

[*] Editors: The original paper reads "[Euclide 1990–]." This project has been completed and we have updated the references.

[6] Concerning the proposition used by later authors, indices devoted to Pappus, Apollonius and Archimedes are available on my web page, where one will find how propositions of the *Elements* were used (or not used) in other mathematicians' works. My paper [Saito 1994] indicates the reasons that prompted me to assemble such indices.

[**] Editors: The original paper reads "[Netz (Forthcoming)]," however, this book has been published and we have updated the references.

[7] In this book, Netz examines the form and style of Greek mathematical texts, which, like Homer's epics, largely depended on "formulae" — fixed expressions regularly used to denote certain mathematical objects or relations. He illustrates the

136 The shift from reconstructions to textual studies can also be illustrated | by the works of Wilbur Knorr (1945–1997), the greatest historian of Greek mathematics in the second half of the twentieth century (and the restriction "second half" may be unnecessary). After the book [Knorr 1975], based on his Ph.D. thesis, he proposed a reconstruction of another theory of proportion in [Knorr 1978], an outcome of a thorough reading of Archimedes, attesting Knorr's shift toward a more substantial study of documents. Then, after producing [Knorr 1986], which is important for its emphasis on autonomous developments of problem-solving techniques independent of alleged philosophical interests, he arrived at textual studies in his monumental work [Knorr 1989].

A shift to textual studies entails a change in the subject of investigations. Even if one does not always embark on studies of Arabic and medieval Latin documents as Knorr did,[8] the weight necessarily moves from the fifth century (where the scarcity of documents provides ample room for conjectures and reconstructions) to the fourth century (where Plato and Aristotle are contemporary witnesses and more documents available: for example see [Mueller 1991]) and to the third century (where Archimedes' and Apollonius' works are at our disposal). When we calmly consider the status of extant documents, we can surely observe that it is extremely hazardous to speak of the history of Greek mathematics before 399 BC, the dramatic date of Plato's *Theaetetus*.

But reconstructions have not been dismissed. Rather, this approach also benefited from the same skepticism concerning rationalising conjectures. For example, [Fowler 1987], an outstanding work among recent attempts, is characterized by thorough investigations of extant documents. Its basic attitudes have more in common with recent research trends than with the anthyphairetic reconstruction tradition.

Algebraic vs. Geometrical Interpretation

A word is in order concerning the decline of the algebraic interpretation that served to combine such ingredients as Babylonian mathematics, Pythagorean interests, incommensurable magnitudes, and the
137 so-called geometric algebra. Following the bitter conflict caused by the provocative | paper [Unguru 1975], which strenuously argued against the prevailing algebraic interpretation, and the reactions to it (for a survey of this polemic, see [Berggren 1984]), scholars were reluctant to discuss this problem for some time. With the changes in research approach described above, the belief that algebraic interpretation could provide a sufficient description of the treatment of magnitudes in Greek geometry has become less convincing, and scholars can now speak as calmly about this as in [Grattan-Guinness 1996].

Pythagoreans, Are They Out?

Now, let us return to Pythagoreans, and try to replace them in the history of Greek mathematics. Two outstanding figures have drawn scholars' attention: Philolaus of Croton (ca. 470–ca. 390) and Archytas of Tarentum (fl. ca. 400–350).

In the study of these figures, the difficulty consists, above all, in the evaluation of documents. Since it already was very common among Plato's disciples in the Academy to attribute their own ideas to Pythagoreans, any such attribution is in itself suspicious. Of course, this hardly means that all

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way in which mathematical deduction is constructed and how formulae work. He moreover analyses the relation between text and diagram, and elucidates their interdependence, showing the indispensable role played by diagrams.

[8] [Knorr 1989] thoroughly investigated Arabic and Latin traditions of Archimedes' *Dimensions of the Circle*. [Knorr 1996] convincingly showed that an Arabo-Latin tradition of Book XII of Euclid's *Elements* preserved a text preferable to Heiberg's Greek edition.

attributions are wrong, and the first task of scholars is to distinguish genuine fragments from spurious ones.

As for Philolaus, a contemporary of Socrates, [Huffman 1993] marked a great step ahead with a thoroughgoing analysis of extant fragments and the attempts to identify genuine ones. He described Philolaus as a natural philosopher facing epistemological problems with the help of the model given by the rigorous mathematical sciences. What is striking here is that Philolaus himself does not seem to have made any original contributions to mathematics, yet it was mathematics that provided him with a model for science. Philolaus' place is not yet settled and needs further research.

Archytas, Plato's contemporary and friend, was no doubt the most brilliant mathematician among the Pythagoreans. He was the first to solve the problem of finding two mean proportionals; and if his fragment n°1 is genuine,[9] as recent scholars are inclined to believe, his importance in music theory would be well established, although music theory was not exclusively a Pythagorean interest (see [Bowen 1982] and [Bowen 1991]). | 138

The important question, in my own opinion, is how much of Archytas' achievements owed to the Pythagorean tradition. In other words, can we assume that Archytas was a great mathematician *because* he was a Pythagorean? No document proves this directly and incontestably. And if the answer turned out to be negative, what sense would it make to speak of Pythagorean mathematics?

Let me give an example. In the *Elements* (Book VII to IX), arithmetic consists of two levels of propositions. Proposition VII-20 serves as a breakthrough to the higher level, and to several propositions concerning the non-existence of mean proportional numbers (for example, that no integer x satisfies $n : x :: x : 2n$ can easily be deduced from *Elements* VIII-8). These propositions are not likely to have been proved by pebble (*psephoi*) arithmetic ascribed to the Pythagoreans [Burkert 1972, 433–436]. Moreover, they are, in a sense, extensions of the recognition that the side and diagonal of squares are incommensurable (although we do not know whether the former were *historical* extensions of the latter). Our attention therefore is focused on whether or not these "propositions of a higher level" are Pythagorean achievements.

Boethius credits Archytas with the proof of the non-existence of mean proportional numbers for an epimoric ratio,[10] a special case of a group of propositions to be found in Book VIII of the *Elements*. If the Pythagoreans had actually discovered incommensurability (and it must have been discovered sometime before Archytas), and if some continuous Pythagorean tradition had enabled Archytas to prove his theorem, then we would have every right to speak of "Pythagorean mathematics." If, on the contrary, Archytas simply applied a theorem already known outside of the Pythagorean community to his interests in harmonics (perhaps even because of the Pythagorean tradition), then Pythagorean mathematics would look somewhat faded. Many other assumptions are possible between these two extremes, and we should try to determine how "Pythagorean" Archytas' achievements were. In this sense, Pythagoreans are by no means out, and they will continue to be the subject of studies and discussions.[11] | 139

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[9] Once doubted by [Burkert 1972, 379–380 n.46], the authenticity of this fragment (DK 47B1) is effectively defended by [Bowen 1982, 83–85] and [Huffman 1985]. However, [Centrone 1996, 69–70, n. 21] expressed some reservations.

[10] The same proposition is included in Euclid's *Sectio Canonis* (prop. 3). The epimoric ratio is the ratio of two magnitudes whose greater term's excess over the lesser is a part (divisor) of both: it is therefore expressed in the form $(n + 1) : n$.

[11] A warning concerning astronomy seems appropriate here. Even though Archytas may have said that music and astronomy are sister sciences (DK 47B1), his astronomy was at best the very beginning of the Greek mathematical astronomy tradition culminating in Ptolemy. Note also that the word *sphairikas* in this fragment is found only in some of Nichomachus' manuscripts, not in the parallel passage in Porphyry, and [Bowen 1982, 80] omits this word. Greek geometrical astronomy began with Archytas' contemporary Euxodus (see [Berggren and Thomas 1996, 6 ff.]). Then, from the third century onwards, it developed more and more elaborate spherical models, and incorporated the purely arithmetical Babylonian data into this geometric framework (see [Jones 1991] and [Jones 1996]).

Conclusion

With some oversimplifications, I would sum up this discussion as such: Pythagoras out, Pythagoreans in (but without attributing to them a monopoly over the mathematical sciences). Fifth century out, fourth and third centuries in. Mathematical reconstructions out, textual studies in. What the Greeks could and should have done out, but what they actually did in. Today the history of Greek mathematics (and probably the history of mathematics in general) has become a branch of the history of ideas more than a branch of mathematics, as it used to be.

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