The “Second” Arabic Translation of Theodosius’ *Sphaerica*

Richard Lorch

Abstract From a short comparison of terminology it is suggested that the Arabic text of Theodosius’ *Sphaerica* carried by two manuscripts in Hebrew script is a translation different from the one translated by Gerard of Cremona into Latin. An edition of a lemma for Proposition III.11 *inter alia* supports the hypothesis that this translation is the basis of Moses b. Tibbon’s Hebrew version.

Like many other Greek mathematical and scientific works, Theodosius’ *Sphaerica* was translated into Arabic in the ninth century. Of the Arabic version that Gerard of Cremona translated into Latin in the twelfth century at least three manuscripts are known:[1]

A: Istanbul, Seray, Ahmet III 3464, ff. 20v–53v
N: Lahore, private library, M. Nabī Khān, pp. 185–281
H: Paris, Bibliothèque nationale de France, heb. 1101, ff. 1r–53r, 86r–87r.

It will be referred to as **ANH**.

There was at least one other Arabic translation of the work: that in manuscripts

F: Florence, Laur. Med. 124
C: Cambridge, University Library, add. 1220, ff. 1r–50r.

Both manuscripts are in Hebrew script — and so may be dated, perhaps, to the fourteenth century and assigned to the western area of the Arabic tradition. A preliminary comparison with the text translated by Gerard may be made by taking as examples four short enunciations specifying construction (ex. 1–4), I 19, I 20, I 21 and I 22.[2]

1. Τοῦ δοθέντος ἐν σφαίρᾳ κύκλου τὴν διάμετρον ἐκθέσθαι.

ANH كيف نجد خطاً مساوياً لقطر دائرة معلومة فى كرة.
FC نريد أن نجد قطر دائرة مفروضة على كرة.

2. Τῆς δοθείσης σφαίρας τὴν διάμετρον ἐκθέσθαι.

ANH كيف نخط خطاً مثل قطر كرة معلومة.
FC نريد أن نجد قطر كرة مفروضة.

It is a pleasure to thank Paul Kunitzsch for considerable help in writing this paper. He is not responsible for the opinions expressed in it.

[1] The text was edited by Kunitzsch and Lorch [2010].

[2] The numeration of the edition (see previous note) is used here and throughout the article. It is taken from manuscript **A**. The corresponding propositions in Czinczenheim’s Greek text are I 18, 19, 20 and 21 respectively [Czinczenheim 2000].

N. Sidoli and G. Van Brummelen (eds.), *From Alexandria, Through Baghdad: Surveys and Studies in the Ancient Greek and Medieval Islamic Mathematical Sciences in Honor of J.L. Berggren*, DOI 10.1007/978-3-642-36736-6_13,

3. Διὰ δοθέντων σημείων, ἅ ἐστιν ἐπὶ σφαιρικῆς ἐπιφανείας, μέγιστον κύκλον γράψαι.

ANH كيف نرسم دائرة عظيمة تمر بنقطتين معلومتين فى بسيط كرة.

FC نريد أن نخط دائرة عظيمة على نقطتين معلومتين على بسيط الكرة.

4. Τοῦ δοθέντος ἐν σφαίρᾳ κύκλου τὸν πόλον εὑρεῖν.

ANH كيف نجد قطب دائرة معلومة فى كرة.

FC نريد أن نجد قطب دائرة معلومة الذى على الكرة.

In all four examples the Greek for "to find" or "to determine" is represented in **ANH** by كيف + an imperfect and in **FC** by نريد أن + an imperfect. **FC** favours مفروض ("assumed") to translate δοθείς ("given"), which it has in several places where **ANH** consistently has معلوم ("known").

There is some inconsistency in the terminology. For instance, ἐκθέσθαι is represented by وجد ("to find") in both translations, but in ex. 2 by خط ("to draw") in **ANH** and by وجد in **FC**. Again, in the first proposition τέμνω generally becomes قطع ("to cut") in **ANH** and فصل ("to cut off") in **FC**, but قطع is to be found in **FC**, about half-way through the proof.

In the definitions at the beginning of the work, the most striking difference between **ANH** and **FC** is the translation of ἄξων as محور ("axis") in **ANH** and as قطر ("diameter") in **FC**. But there are plenty more differences, e.g. in the definition of "sphere" **FC** has مستوية for ἴσαι ἀλλήλαις εἰσίν, where **ANH** has مساوٍ بعضها لبعض, a more accurate rendering. Similarly, in the definition of the pole of a circle **ANH** again has مساوٍ بعضها لبعض and **FC** has this time متساوية. In this definition, **FC** has, simply, وقطب الدائرة ... على الكرة هى نقطة, which agrees with some Greek manuscripts; the reading chosen for the edited Greek text of Czinczenheim has additionally λέγεται; this is represented in the fuller version of **ANH** by الشيء الذى يقال له فى الكرة قطب دائرة, which in Gerard's Latin becomes "Res que in spera polus circuli dicitur."

On the whole, the great differences in terminology and style indicate two translations. They are too numerous and not consistent enough to be the work of a redactor.

As an extended specimen of **FC**'s style (and to show further its independence from **ANH**), we give its version of the lemma to Proposition III 11. It will be noted that it corresponds to none of the forms of the lemma presented in manuscripts **ANH**; even the name of the point that carries the right angle is different (*A* in **FC**, *B* in the proofs in **A** and **H**). But it corresponds very well to the proof in the Hebrew translation by Moses b. Tibbon, as may be seen by comparing **FC** with Knorr's translation of a manuscript of the Moses b. Tibbon version in the Jewish Theological Seminary in New York [Knorr 1986, 235–237].[3] The following is a "mathematical translation:"[4]

When triangle *ABG* is right and the right angle is point *A*, draw *BD* to base *AG*. *I say:* $GA : AD > \angle ADB : \angle DGB$

Proof: Let $DE \parallel GB$

$\therefore DE > AD$ and $< DB$

Construct a circle about centre *D* and with radius *DE*, going beyond *A* and cutting *DB* at *Z*

Produce *DA* to meet the circle at *H*

$\therefore$ sect. $DEH > \triangle DAE$; and sect. $DEZ < \triangle DBE$

$\therefore \triangle DAE : \triangle DBE <$ sect. $DEH :$ sect. DEZ

But $\triangle DAE : \triangle DBE = \overline{AE} : \overline{EB} = AD : DG$

and sect. $DEH :$ sect. $DEZ = \angle ADE : \angle EDZ$

$\therefore \angle ADE : \angle EDB > \overline{AD} : \overline{DG}$

Componendo $AG : GD < \angle ADB : [\angle]BDE$

---

[3] The text by Jacob b. Machir (1290) is apparently an adaptation of the Moses b. Tibbon translation [Knorr 1986, 235–237; and private communication from Knorr].

[4] This is not an exact translation. It is intended to reproduce the mathematical reasoning. It is followed by the full Arabic text.

Convertendo $[\angle]ADB : [\angle]ADE < AG : AD$
And $[\angle]ADE = [\angle]DGB$
$\therefore AG : AD > [\angle]ADB : [\angle]DGB$. Q.E.D.

The only difference from the Hebrew of any consequence is in the line

Componendo $AG : GD < \angle ADB : [\angle]BDE$.

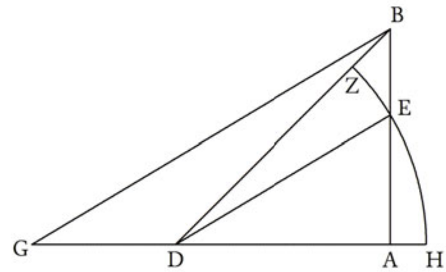

Figure 1: Lemma to Proposition III 11.

The term for *convertendo* in this Arabic text, إذا خلفنا,[5] that introduces the next line probably arose from a colloquial rendering of إذا قلبنا ("when we turn around"),[6] for قلب meant the conversion of a ratio $a : b$ into $a : a - b$.[7] It seems probable that the text is disturbed at this point. The Hebrew text translated by Knorr also has a deduction by *componendo*, but it is chosen so that it is the desired result (the argument forms the ratio $a + b : a$ from $a : b$, rather than **FC**'s $a + b : b$).

In conclusion, we may say that the text represented by **FC** was probably a translation, independent of **ANH**, and that it was the basis of Moses b. Tibbon's Hebrew.

إذا كان مثلث ا ب جـ قائم الزاوية وزاويته القائمة نقطة ا وخرج من نقطة ب إلى قاعدة ا جـ خط مستقيم كيف اتفق وهو خط ب د ، فأقول إن نسبة جـا إلى ا د أعظم من نسبة زاوية ا د ب إلى زاوية د جـ ب ، برهانه أنا نخرج من نقطة د خطاً موازياً لخط جـ ب وهو خط د ه فخط د ه أعظم من خط ا د وأصغر من خط د ب فلذلك إذا عملنا دائرة على مركز د ببعد د ه تجوز ا وتفصل خط د ب على نقطة ز ونخرج د ا إلى أن يلقى الدائرة على نقطة ح فقطع د ه ح أعظم من مثلث د ا ه وقطع د ه ز أصغر من مثلث د ب ه فنسبة مثلث د ا ه إلى مثلث د ب ه أصغر من نسبة قطع د ه ح إلى قطع د ه ز لكن نسبة مثلث د ا ه إلى مثلث د ه ب كنسبة خط ا ه إلى خط ه ب وهي كنسبة ا د إلى د جـ وقطع د ه ح إلى قطع د ه ز كزاوية ا د ه إلى زاوية ه د ز فإذاً نسبة زاوية ا د ه إلى زاوية ه د ب أعظم من نسبة خط ا د إلى خط د جـ وإذا ركبنا تكون نسبة ا جـ إلى جـ د أصغر من نسبة زاوية ا د ب إلى زاوية ب د ه وإذا خلفنا كانت نسبة ا د ب إلى ا د ه أصغر من نسبة ا جـ إلى ا د وزاوية ا د ه مساوية لزاوية د جـ ب فإذاً نسبة ا جـ إلى ا د أعظم من نسبة ا د ب إلى د جـ ب ، وذلك ما أردنا أن نبين .

[5] *sic* **C**, خالفنا **F**.

[6] This was suggested by Paul Kunitzsch (private communication).

[7] See the Euclid texts (Book V, definitions) in MSS Tehran, Malik 3586 (there is no visible foliation) and Leiden 399,1 [Besthorn and Heiberg 1932, 22] for the definition of قلب. The translation is by Isḥāq ibn Hunayn and revised by Thābit ibn Qurra.

1 قائم] עלי *add. et del.* C. 4 يلقى] ילאק F، ילקא C. 5 وقطع د ه ز... د ا ه] *marg.* F. 7 وقطع] נסבה *supra* C. كزاوية] נסבה *supra* C. 9 خلفنا] כאלפנא F. أصغر] אעגם C. 10 أعظم] אצגר F، אעטם *supra* F. أن نبين] *om.* F.

References

Ancient Authors

Euclid: Besthorn and Heiberg [1932].
Theodosius: Czinczenheim [2000], Kunitzsch and Lorch [2010].

Modern Scholarship

Besthorn, R.O., Heiberg, J.L., 1932. Codex Leidensis 399,1. Euclidis elementa elementa ex interpretatione al-Hadschdschadschii cum commentariis al-Narizii, 3 vols. Finished by G. Junge, J. Raeder, W. Thomson. Gyldendaliana, Copenhagen.

Czinczenheim, C., 2000. Edition, traduction et commentaire des Sphériques de Théodose. Thèse de docteur de l'Universite Paris IV. Atelier National de Reproduction des Thèses, Lille.

Knorr, W.R., 1986. The medieval tradition of a Greek mathematical lemma. Zeitschrift für Geschichte der arabisch-islamischen Wissenschaften 3, 230–261.

Kunitzsch, P., Lorch, R., 2010. Theodosius, Sphaerica: Arabic and Medieval Latin Translations. Franz Steiner Verlag, Stuttgart.

Lorch, R., 1996. The transmission of Theodosius' Sphaerica. In: Folkerts, M. (ed.), Mathematische Probleme im Mittelalter. Der lateinische und arabische Sprachbereich, Harrassowitz, Wiesbaden, pp. 159–183.