Fuzzy Decision Making Based on Hesitant Fuzzy Linguistic Term Sets

Li-Wei Lee[1] and Shyi-Ming Chen[2]

[1] Department of Computer and Communication Engineering, De Lin Institute of Technology, New Taipei City, Taiwan, R.O.C.
[2] Department of Computer Science and Information Engineering, National Taiwan University of Science and Technology, Taipei, Taiwan, R.O.C.

Abstract. This paper presents a new fuzzy decision making method based on likelihood-based comparison relations of hesitant fuzzy linguistic term sets. We also present a similarity measure between hesitant fuzzy linguistic term sets. The proposed method is more simple for fuzzy decision making than the method presented in [18]. It provides us with a useful way for decision making in a fuzzy environment.

Keywords: Hesitant fuzzy sets, hesitant fuzzy linguistic term sets, fuzzy decision making, likelihood-based comparison relations, similarity measures.

1 Introduction

The fuzzy linguistic approach has successfully been applied to deal with decision making problems [3]-[19], [21]-[23]. In a fuzzy decision making environment, experts maybe hesitate to choose appropriate linguistic terms to assess alternatives in some situations for reaching a final agreement. In order to deal with such situations, Torra [20] presented the concept of hesitant fuzzy sets, which is a generalization of fuzzy sets [26]. He also presented different generalizations and extensions of fuzzy sets and discussed the relationships among hesitant fuzzy sets and the other generalizations of fuzzy sets, such as intuitionistic fuzzy sets [1], [2], type 2 fuzzy sets [8], [15], type n fuzzy sets [8] and fuzzy multisets [16]. Based on the concept of hesitant fuzzy sets presented in [20], some researchers [18], [25], [27] have studied related issues of hesitant fuzzy sets. In [18], Rodriguez et al. presented the concept of hesitant fuzzy linguistic term sets based on the fuzzy linguistic approach [26] and hesitant fuzzy sets [20]. They pointed out that the fuzzy linguistic approach is very limited due to the fact that it assesses a linguistic variable by using a single linguistic term, whereas the hesitant fuzzy linguistic term sets approach assesses a linguistic variable by using several linguistic terms for decision making. They presented two symbolic aggregation operators to obtain a linguistic interval associated with each alternative and presented an exploitation process to get a preference order for decision making based on the nondominance choice degree of a preference relation obtained from linguistic intervals. However, the drawback of the method presented in [18] is that it is too complicated for dealing with fuzzy decision making problems. Therefore, we

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must develop a new fuzzy decision making method based on hesitant fuzzy linguistic term sets to overcome the drawback of the method presented in [18].

In this paper, we present a new fuzzy decision making method based on likelihood-based comparison relations of hesitant fuzzy linguistic term sets. We also present a similarity measure between hesitant fuzzy linguistic term sets. The proposed fuzzy decision making method is more simple for fuzzy decision making than the method presented in [18].

The rest of this paper is organized as follows. In Section 2, we briefly review the concept of hesitant fuzzy linguistic term sets [18]. In Section 3, we present the concept of likelihood-based comparison relations and present a similarity measure of hesitant fuzzy linguistic term sets. In Section 4, we present a fuzzy decision making method based on likelihood-based comparison relations of hesitant fuzzy linguistic term sets. The conclusions are discussed in Section 5.

2 A Review of Rodriguez et al.'s Decision Making Method Based on Hesitant Fuzzy Linguistic Term Sets

In [18], Rodriguez et al. presented the concept of hesitant fuzzy linguistic term sets for decision making. The basic concepts and operations of hesitant fuzzy linguistic term sets are reviewed from [18] as follows.

***Definition 2.1* [18]:** Let $S = \{s_0, s_1, \ldots, s_g\}$ be a linguistic term set. A hesitant fuzzy linguistic term set H_S is an ordered finite subset of consecutive linguistic terms of the linguistic term set S.

***Definition 2.2* [18]:** Let $G_H = (V_N, V_T, I, P)$ be a context-free grammar and let $S = \{s_0, s_1, \ldots, s_g\}$ be a linguistic term set, where V_N denotes a set of nonterminal symbols, V_T denotes a set of terminal symbols, I denotes the starting symbol and P denotes the production rules, shown as follows:

$$V_N = \{\langle\text{primary term}\rangle, \langle\text{composite term}\rangle, \langle\text{unary relation}\rangle, \langle\text{binary relation}\rangle, \langle\text{conjunction}\rangle\},$$

$$V_T = \{\text{lower than, greater than, between}, s_0, s_1, \ldots, \text{and } s_g\},$$

$$I \in V_N,$$

$$\begin{aligned} P = \{I &::= \langle\text{primary term}\rangle | \langle\text{composite term}\rangle, \\ \langle\text{composite term}\rangle &::= \langle\text{unary relation}\rangle\langle\text{primary term}\rangle | \langle\text{binary relation}\rangle \\ &\quad\ \langle\text{primary term}\rangle\langle\text{conjunction}\rangle\langle\text{primary term}\rangle, \\ \langle\text{primary term}\rangle &::= s_0 | s_1 | \cdots | s_g, \\ \langle\text{unary relation}\rangle &::= \text{lower than} | \text{greater than}, \\ \langle\text{binary relation}\rangle &::= \text{between}, \\ \langle\text{conjunction}\rangle &::= \text{and}\}. \end{aligned}$$

***Definition 2.3* [18]:** Let E_{G_H} be a function that transforms the linguistic expressions le obtained by the context-free grammar G_H into a hesitant fuzzy linguistic term set H_S of the linguistic term set S, shown as follows:

$$E_{G_H}: le \rightarrow H_S.$$

The linguistic expressions generated by production rules can be transformed into a hesitant fuzzy linguistic term set in different ways according to their meaning:

1) $E_{G_H}(s_i) = \{s_i | s_i \in S\}$,
2) $E_{G_H}(\text{less than} s_i) = \{s_j | s_j \in S \text{ and } s_j \leq s_i\}$,
3) $E_{G_H}(\text{greater than} s_i) = \{s_j | s_j \in S \text{ and } s_j \geq s_i\}$,
4) $E_{G_H}(\text{between } s_i \text{and } s_j) = \{s_k | s_k \in S \text{ and } s_i \leq s_k \leq s_j\}$.

***Definition 2.4* [18]:** Let $X = \{x_1, x_2, \ldots, x_n\}$ be a set of alternatives, let $C = \{c_1, c_2, \ldots, c_m\}$ be a set of criteria, let $S = \{s_0, s_1, \ldots, s_g\}$ be a linguistic term set and let $H_S^j(x_i)$ be a hesitant fuzzy linguistic term set associated with alternative x_i with respect to criterion c_j, where $1 \leq i \leq n$ and $1 \leq j \leq m$. The min_upper operator $H_{S_{min}^+}(x_i)$ and the max_lower operator $H_{S_{max}^-}(x_i)$ of alternative x_i are defined as follows:

$$H_{S_{min}^+}(x_i) = \min\{H_{S^+}^j(x_i) | 1 \leq i \leq n \text{ and } 1 \leq j \leq m\}, \quad (1)$$

$$H_{S_{max}^-}(x_i) = \max\{H_{S^-}^j(x_i) | 1 \leq i \leq n \text{ and } 1 \leq j \leq m\}, \quad (2)$$

where $H_{S^+}^j(x_i)$ and $H_{S^-}^j(x_i)$ are the upper bound of hesitant fuzzy linguistic term set $H_S^j(x_i)$ and the lower bound of hesitant fuzzy linguistic term set $H_S^j(x_i)$ associated with alternative x_i with respect to criterion c_j, respectively.

Based on the min_upper operator $H_{S_{min}^+}(x_i)$ and the max_lower operator $H_{S_{max}^-}(x_i)$, the linguistic interval $H'(x_i)$ for each alternative x_i can be obtained, shown as follows [18]:

$$H'(x_i) = \left[min\left\{H_{S_{min}^+}(x_i), H_{S_{max}^-}(x_i)\right\}, max\left\{H_{S_{min}^+}(x_i), H_{S_{max}^-}(x_i)\right\}\right]. \quad (3)$$

***Definition 2.5* [18]:** Let P be a preference relation defined over a set X of alternatives. For alternative x_i, its nondominance choice degree NDD_i is obtained as follows:

$$NDD_i = \min\{1 - p_{ji}^S, j \neq i\}, \quad (4)$$

where $p_{ij} = p(x_i \geq x_j) = \frac{\max(0, x_{iR} - x_{jL}) - \max(0, x_{iL} - x_{jR})}{(x_{iR} - x_{iL}) + (x_{jR} - x_{jL})}$ denotes the degree of the alternative x_i over x_j, $1 \leq i \leq n$, $1 \leq j \leq n$, $i \neq j$, $x_i = [x_{iL}, x_{iR}]$, $x_j = [x_{jL}, x_{jR}]$, and $p_{ji}^S = \max\{p_{ji} - p_{ij}, 0\}$ represents the degree in which x_i is strictly dominated by x_j. The larger the value of NDD_i, the better the preference order of alternative x_i, where $1 \leq i \leq n$.

3 Likelihood-Based Comparison Relations and Similarity Measures of Hesitant Fuzzy Linguistic Term Sets

In this section, we propose the concept of likelihood-based comparison relations of hesitant fuzzy linguistic term sets and propose a similarity measure between hesitant fuzzy linguistic term sets. Assume that there is a linguistic term set $S = \{s_0, s_1, \ldots, s_g\}$, where the membership functions of the linguistic terms in linguistic term set S are shown in Fig. 1.

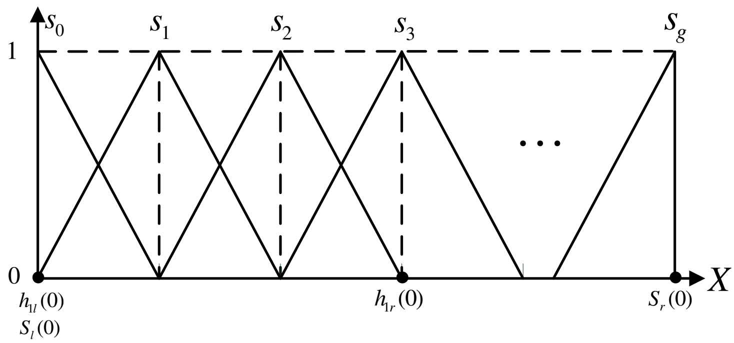

Fig. 1. The 0-cut $h_1(0)$ of the hesitant fuzzy linguistic term set $h_1 = \{s_0, s_1, s_2\}$ and the 0-cut $S(0)$ of the linguistic term set $S = \{s_0, s_1, \ldots, s_g\}$

Assume that there is a hesitant fuzzy linguistic term set $h_1 = \{s_0, s_1, s_2\}$, then the 0-cut $h_1(0)$ of the hesitant fuzzy linguistic term set h_1 is defined as follows:

$$h_1(0) = [h_{1l}(0), h_{1r}(0)],$$

where the 0-cut $h_1(0)$ of h_1 and the 0-cut $S(0)$ of S are shown in Fig. 1, respectively, where $S(0) = [S_l(0), S_r(0)]$. From Fig. 1, we can see that $h_{1l}(0)$ has the largest membership degree in the membership function of the linguistic term s_0 and $h_{1r}(0)$ has the largest membership degree in the membership function of the linguistic term s_3. Therefore, we can get $h_1(0) = [s_{0,}s_3]$. From Fig. 1, we also can see that $S_l(0)$ has the largest membership degree in the membership function of the linguistic term s_0 and $S_r(0)$ has the largest membership degree in the membership function of the linguistic term s_g. Therefore, we can get $S(0) = [s_{0,}s_g]$.

Definition 3.1: Assume that there is a linguistic term set $S = \{s_0, s_1, \ldots, s_g\}$. Based on the concept of likelihood-based comparison relations between intervals [24], the likelihood-based comparison relation $p(h_1 \geq h_2)$ between two hesitant fuzzy linguistic term sets h_1 and h_2 is defined as follows:

$$p(h_1 \geq h_2) = \max\left(1 - \max\left(\frac{\text{Ind}(h_{2r}(0)) - \text{Ind}(h_{1l}(0))}{L(h_1(0)) + L(h_2(0))}, 0\right), 0\right), \tag{5}$$

where $h_1(0) = [h_{1l}(0), h_{1r}(0)]$ is the 0-cut of the hesitant fuzzy linguistic term set h_1, $h_2(0) = [h_{2l}(0), h_{2r}(0)]$ is the 0-cut of the hesitant fuzzy linguistic term set h_2, $\text{Ind}(s_i) = i$ denotes the index associated with the linguistic term s_i, $L(h_1(0)) = \text{Ind}(h_{1r}(0)) - \text{Ind}(h_{1l}(0))$ and $L(h_2(0)) = \text{Ind}(h_{2r}(0)) - \text{Ind}(h_{2l}(0))$.

The likelihood-based comparison relation $p(h_1 \geq h_2)$ between two hesitant fuzzy linguistic term sets h_1 and h_2 has the following properties:

1) $0 \leq p(h_1 \geq h_2) \leq 1$.
2) $p(h_1 \geq h_2) + p(h_2 \geq h_1) = 1$.
3) If $\text{Ind}\big(h_{1r}(0)\big) \leq \text{Ind}\big(h_{2l}(0)\big)$, then $p(h_1 \geq h_2) = 0$.

4) If $\mathrm{Ind}(h_{1l}(0)) \geq \mathrm{Ind}(h_{2r}(0))$, then $p(h_1 \geq h_2) = 1$.
5) $p(h_1 \geq h_1) = 0.5$.

If $\mathrm{Ind}(h_{1r}(0)) = \mathrm{Ind}(h_{1l}(0))$ and $\mathrm{Ind}(h_{2r}(0)) = \mathrm{Ind}(h_{2l}(0))$, then the likelihood-based comparison relation $p(h_1 \geq h_2)$between two hesitant fuzzy linguistic term sets h_1 and h_2 is defined as follows:

$$p(h_1 \geq h_2) = \begin{cases} 1, \text{if} \mathrm{Ind}(h_{1l}(0)) > \mathrm{Ind}(h_{2l}(0)) \\ \frac{1}{2}, \text{if} \mathrm{Ind}(h_{1l}(0)) = \mathrm{Ind}(h_{2l}(0)) \\ 0, \text{if} \mathrm{Ind}(h_{1l}(0)) < \mathrm{Ind}(h_{2l}(0)) \end{cases}$$

Definition 3.2: Assume that there is a linguistic term set $S = \{s_0, s_1, \ldots, s_g\}$. The degree of similarity $s(h_1, h_2)$ between two hesitant fuzzy linguistic term sets h_1 and h_2 is defined as follows:

$$s(h_1, h_2) = 1 - |p(h_1 \geq S) - p(h_2 \geq S)|. \quad (6)$$

4 The Proposed Fuzzy Decision Making Method Based on Likelihood-Based Comparison Relations of Hesitant Fuzzy Linguistic Term Sets

In this section, we present a fuzzy decision making method based on likelihood-based comparison relations of hesitant fuzzy linguistic term sets. Assume that there is a set X of alternatives, $X = \{x_1, x_2, \ldots, x_n\}$, assume that there is a set C of criteria, $C = \{c_1, c_2, \ldots, c_m\}$, and assume that there is a set W of weights, $W = \{\omega_1, \omega_2, \ldots, \omega_m\}$, where ω_j denotes the weight of criterion c_j and $1 \leq j \leq m$. Assume that there is a linguistic term set $S = \{s_0, s_1, \ldots, s_g\}$ and assume that there is a context-free grammar G_H which produces the linguistic expressions of alternatives with respect to different criteria, where the linguistic expressions are transformed into hesitant fuzzy linguistic term sets by means of the transformation function E_{G_H}. Based on the proposed likelihood-based comparison relations of hesitant fuzzy linguistic term sets, the proposed fuzzy decision making method is now presented as follows:

Step 1: Construct the decision matrix Y, shown as follows:

$$Y = \begin{array}{c} \\ x_1 \\ x_2 \\ \vdots \\ x_n \end{array} \begin{array}{c} \begin{array}{cccc} c_1 & c_2 & \cdots & c_m \end{array} \\ \begin{bmatrix} y_{11} & y_{12} & \cdots & y_{1m} \\ y_{21} & y_{22} & \cdots & y_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ y_{n1} & y_{n2} & \cdots & y_{nm} \end{bmatrix} \end{array},$$

where y_{ij} is a hesitant fuzzy linguistic term set of alternative x_i with respect to criterion c_j, $1 \leq i \leq n$ and $1 \leq j \leq m$.

Step 2: Based on Eq. (5), construct the likelihood-based comparison relation P, shown as follows:

$$P = \begin{array}{c} \\ x_1 \\ x_2 \\ \vdots \\ x_n \end{array} \begin{array}{c} \begin{array}{cccc} c_1 & c_2 & \cdots & c_m \end{array} \\ \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1m} \\ p_{21} & p_{22} & \cdots & p_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ p_{n1} & p_{n2} & \cdots & p_{nm} \end{bmatrix} \end{array},$$

$$p_{ij} = p(y_{ij} \geq S) = \max\left(1 - \max\left(\frac{\mathrm{Ind}(S_r(0)) - \mathrm{Ind}(y_{ijl}(0))}{L(y_{ij}(0)) + L(S(0))}, 0\right), 0\right), \quad (7)$$

where $y_{ij}(0) = [y_{ijl}(0), y_{ijr}(0)]$ is the 0-cut of the hesitant fuzzy linguistic term set y_{ij}, $S(0) = [S_l(0), S_r(0)]$ is the 0-cut of the linguistic term set S, $1 \leq i \leq n$ and $1 \leq j \leq m$.

Step 3: Let

$$R(x_i) = \sum_{j=1}^{m} \omega_j p_{ij}, \quad (8)$$

where ω_j denotes the weight of criterion c_j, $1 \leq i \leq n$ and $1 \leq j \leq m$. The larger the value of $R(X_i)$, the better the preference order of alternative x_i, where $1 \leq i \leq n$.

In the following, we use an example to illustrate the process of the proposed fuzzy decision making method.

Example 4.1: Assume that there are three alternatives x_1, x_2, x_3, assume that there are three criteria c_1, c_2, c_3, and assume that the weights of the criteria c_1, c_2 and c_3 are 1/3, 1/3 and 1/3, respectively. Assume that there is a linguistic term set S = {s_0: nothing (*n*), s_1: very low (*vl*), s_2: low (*l*), s_3: medium (*m*), s_4: high (*h*), s_5: very high (*vh*), s_6: perfect (*p*)}. Thelinguistic expressions of the alternatives with respect to different criteria are shown in Table 1. Based on the transformation function E_{G_H} shown in *Definition 2.3*, Table 1 can be transformed into Table 2.

Table 1. Linguistic expressions of the alternatives with respect to different criteria [18]

Criteria / Alternatives	c_1	c_2	c_3
x_1	between *vl* and *m*	between *h* and *vh*	*h*
x_2	between *l* and *m*	*m*	lower than *l*
x_3	greater than *h*	between *vl* and *l*	greater than *h*

Table 2. Transformation of Table 1 into hesitant fuzzy linguistic term sets [18]

Criteria / Alternatives	c_1	c_2	c_3
x_1	{*vl*, *l*, *m*}	{*h*, *vh*}	{*h*}
x_2	{*l*, *m*}	{*m*}	{*n*, *vl*, *l*}
x_3	{*h*, *vh*, *p*}	{*vl*, *l*}	{*h*, *vh*, *p*}

The fuzzy decision making process based on the proposed method is shown as follows:

[Step 1]: We can get the decision matrix *Y*, shown as follows:

$$Y = \begin{matrix} & c_1 & c_2 & c_3 \\ x_1 & \{vl, l, m\} & \{h, vh\} & \{h\} \\ x_2 & \{l, m\} & \{m\} & \{n, vl, l\} \\ x_3 & \{h, vh, p\} & \{vl, l\} & \{h, vh, p\} \end{matrix}.$$

[Step 2]: Based on Eq. (7), we can get the likelihood-based comparison relation *P*, shown as follows:

$S(0) = [s_0, s_6], y_{11}(0) = [s_0, s_4], y_{12}(0) = [s_3, s_6], y_{13}(0) = [s_3, s_5], y_{21}(0) = [s_1, s_4], y_{22}(0) = [s_2, s_4], y_{23}(0) = [s_0, s_3], y_{31}(0) = [s_3, s_6], y_{32}(0) = [s_0, s_3], y_{33}(0) = [s_3, s_6]$,

$$\begin{aligned} p_{11} &= p(y_{11} \geq S) = \max\left(1 - \max\left(\frac{\text{Ind}(S_r(0)) - \text{Ind}(y_{11l}(0))}{L(y_{11}(0)) + L(S(0))}, 0\right), 0\right), \\ &= \max\left(1 - \max\left(\frac{\text{Ind}(s_6) - \text{Ind}(s_0)}{\text{Ind}(s_4) - \text{Ind}(s_0) + \text{Ind}(s_6) - \text{Ind}(s_0)}, 0\right), 0\right) \\ &= 0.4000, \end{aligned}$$

$$\begin{aligned} p_{12} &= p(y_{12} \geq S) = \max\left(1 - \max\left(\frac{\text{Ind}(S_r(0)) - \text{Ind}(y_{12l}(0))}{L(y_{12}(0)) + L(S(0))}, 0\right), 0\right), \\ &= \max\left(1 - \max\left(\frac{\text{Ind}(s_6) - \text{Ind}(s_3)}{\text{Ind}(s_6) - \text{Ind}(s_3) + \text{Ind}(s_6) - \text{Ind}(s_0)}, 0\right), 0\right) \\ &= 0.6667, \end{aligned}$$

$$\begin{aligned} p_{13} &= p(y_{13} \geq S) = \max\left(1 - \max\left(\frac{\text{Ind}(S_r(0)) - \text{Ind}(y_{13l}(0))}{L(y_{13}(0)) + L(S(0))}, 0\right), 0\right), \\ &= \max\left(1 - \max\left(\frac{\text{Ind}(s_6) - \text{Ind}(s_3)}{\text{Ind}(s_5) - \text{Ind}(s_3) + \text{Ind}(s_6) - \text{Ind}(s_0)}, 0\right), 0\right) \\ &= 0.6250, \end{aligned}$$

$$\begin{aligned} p_{21} &= p(y_{21} \geq S) = \max\left(1 - \max\left(\frac{\text{Ind}(S_r(0)) - \text{Ind}(y_{21l}(0))}{L(y_{21}(0)) + L(S(0))}, 0\right), 0\right), \\ &= \max\left(1 - \max\left(\frac{\text{Ind}(s_6) - \text{Ind}(s_1)}{\text{Ind}(s_4) - \text{Ind}(s_1) + \text{Ind}(s_6) - \text{Ind}(s_0)}, 0\right), 0\right) \\ &= 0.4444, \end{aligned}$$

$$\begin{aligned} p_{22} &= p(y_{22} \geq S) = \max\left(1 - \max\left(\frac{\text{Ind}(S_r(0)) - \text{Ind}(y_{22l}(0))}{L(y_{22}(0)) + L(S(0))}, 0\right), 0\right), \\ &= \max\left(1 - \max\left(\frac{\text{Ind}(s_6) - \text{Ind}(s_2)}{\text{Ind}(s_4) - \text{Ind}(s_2) + \text{Ind}(s_6) - \text{Ind}(s_0)}, 0\right), 0\right) \\ &= 0.5000, \end{aligned}$$

$$\begin{aligned} p_{23} &= p(y_{23} \geq S) = \max\left(1 - \max\left(\frac{\text{Ind}(S_r(0)) - \text{Ind}(y_{23l}(0))}{L(y_{23}(0)) + L(S(0))}, 0\right), 0\right), \\ &= \max\left(1 - \max\left(\frac{\text{Ind}(s_6) - \text{Ind}(s_0)}{\text{Ind}(s_3) - \text{Ind}(s_0) + \text{Ind}(s_6) - \text{Ind}(s_0)}, 0\right), 0\right) \\ &= 0.3333, \end{aligned}$$

$$p_{31} = p(y_{31} \geq S) = \max\left(1 - \max\left(\frac{\text{Ind}(S_r(0)) - \text{Ind}(y_{31l}(0))}{L(y_{31}(0)) + L(S(0))}, 0\right), 0\right),$$
$$= \max\left(1 - \max\left(\frac{\text{Ind}(s_6) - \text{Ind}(s_3)}{\text{Ind}(s_6) - \text{Ind}(s_3) + \text{Ind}(s_6) - \text{Ind}(s_0)}, 0\right), 0\right)$$
$$= 0.6667,$$
$$p_{32} = p(y_{32} \geq S) = \max\left(1 - \max\left(\frac{\text{Ind}(S_r(0)) - \text{Ind}(y_{32l}(0))}{L(y_{32}(0)) + L(S(0))}, 0\right), 0\right),$$
$$= \max\left(1 - \max\left(\frac{\text{Ind}(s_6) - \text{Ind}(s_0)}{\text{Ind}(s_3) - \text{Ind}(s_0) + \text{Ind}(s_6) - \text{Ind}(s_0)}, 0\right), 0\right)$$
$$= 0.3333,$$
$$p_{33} = p(y_{33} \geq S) = \max\left(1 - \max\left(\frac{\text{Ind}(S_r(0)) - \text{Ind}(y_{33l}(0))}{L(y_{33}(0)) + L(S(0))}, 0\right), 0\right),$$
$$= \max\left(1 - \max\left(\frac{\text{Ind}(s_6) - \text{Ind}(s_3)}{\text{Ind}(s_6) - \text{Ind}(s_3) + \text{Ind}(s_6) - \text{Ind}(s_0)}, 0\right), 0\right)$$
$$= 0.6667,$$

$$P = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{ccc} c_1 & c_2 & c_3 \end{array} \\ \begin{bmatrix} 0.4000 & 0.6667 & 0.6250 \\ 0.4444 & 0.5000 & 0.3333 \\ 0.6667 & 0.3333 & 0.6667 \end{bmatrix} \end{array}.$$

[Step 3]: Because the weights $\omega_1, \omega_2, \omega_3$ of the criterion c_1, c_2, c_3 are 1/3, 1/3 and 1/3, respectively, based on Eq. (8), we can get

$$R(x_1) = \sum_{j=1}^{3} \omega_j p_{1j} = \frac{1}{3} \times 0.4000 + \frac{1}{3} \times 0.6667 + \frac{1}{3} \times 0.6250 = 0.5639,$$

$$R(x_2) = \sum_{j=1}^{3} \omega_j p_{2j} = \frac{1}{3} \times 0.4444 + \frac{1}{3} \times 0.5000 + \frac{1}{3} \times 0.3333 = 0.4259,$$

$$R(x_3) = \sum_{j=1}^{3} \omega_j p_{3j} = \frac{1}{3} \times 0.6667 + \frac{1}{3} \times 0.3333 + \frac{1}{3} \times 0.6667 = 0.5556.$$

Because $R(x_1) > R(x_3) > R(x_2)$, the preference order of the alternatives x_1, x_2 and x_3 is: $x_1 > x_3 > x_2$. This result coincides with the one presented in [18].

5 Conclusions

We have presented the concept of likelihood-based comparison relations of hesitant fuzzy linguistic term sets. We also have presented a similarity measure between hesitant fuzzy linguistic term sets. Based on likelihood-based comparison relations of hesitant fuzzy linguistic term sets, we have presented a new method for fuzzy decision making. The proposed method is more simple for fuzzy decision making than the method presented in [18].

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