Chapter 11
Sliding Mode Control for Nonlinear Discrete Time Systems with Matching Perturbations

Yang Li, Quanmin Zhu, Xueli Wu and Jianhua Zhang

Abstract This chapter considers sliding mode control of nonlinear discrete time systems with matching perturbations. The nonlinear sliding mode controller, whose parameters assure the closed-loop system stable, is designed in order to drive the state trajectories toward to a small bounded region. The controller is approximated by a polynomial equation in current control term $u(k)$ according to Taylor series expansion. The algebraic solutions can be obtained by resolving a polynomial equation in the latest control term $u(k)$. The integrated procedure provides a straightforward methodology to apply sliding mode control design technique for nonlinear systems. The simulation results are provided to illustrate the effectiveness of the proposed scheme.

Keywords Nonlinear discrete time system · Nonlinear controller · Sliding mode control · Matching perturbations

11.1 Introduction

The discrete time sliding mode control algorithm is very important when it is implemented by digital controller [1]. However, the discrete time sliding mode controller is not easily designed from the continuous counterpart through simple equivalence. So it is necessary to study the discrete time sliding mode control algorithm.

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Reference [2] proposed the necessary reaching condition of the discrete time sliding mode control system. Reference [3] proposed the sufficient and necessary reaching condition of the discrete time sliding mode control system. Reference [4] proposed the discrete reaching condition based on Lyapunov function by using equivalent form in Ref. [3]. Reference [1] proposed the "reaching law approach" which is the equivalence of the reaching condition in inequality form and defined notions of reaching condition. References [4] and [5] developed the idea of an equivalent control and the sector of sliding mode. Reference [6] proposed discrete-time equivalent controller in the prescribed boundary layer for Sampled-Data Systems. Reference [7] proposed a simple methodology for designing sliding mode control that can eliminate chattering for discrete time systems with matching perturbations. However, these methods cannot assign the desired closed-loop eigenvalues directly. Reference [8] proposed the output feedback sliding mode controller for a sampled data linear systems with matching disturbances which can assign the desired closed-loop eigenvalues directly. The applications of the sliding mode control method have been extensively studied, Refs. [9, 10] considered the application of the sliding mode control for discrete time systems.

On the other hand, a number of reports for discrete time nonlinear controller have been presented from application demand. However, it is not easy for the description of the controller nonlinearities because the lack of general modeling framework for a wide range of nonlinearities. Some researches into the nonlinear controllers have been presented, such as Ref. [11] proposed a control-oriented model to represent a wide range of nonlinear discrete-time dynamic plants, a pole placement controller is designed for providing a straightforward methodology when designing systems with nonlinear controller. Reference [12] proposed state-dependent parameter (SDP) models to deal with the nonlinearities of system states and system controllers. In all of the above methods with sliding mode control, there is no direct method to handle the nonlinearities of the discrete time system. This is the motivation to propose the new study in which a direct nonlinear sliding mode controller is proposed to handle the nonlinearities in such control system design.

With reference to some previous results, a simple design technique of nonlinear sliding mode controller for discrete time nonlinear system with matching perturbations is discussed in this chapter. The controller is designed by using the sliding mode control concept, but the controller proposed in this chapter is more general than Ref. [8]. The nonlinear controller is approximated by a polynomial equation in current control term $u(k)$ according to Taylor series expansion [13]. The algebraic solutions can be obtained by resolving a polynomial equation in the latest control term $u(k)$. It provides a straightforward method to deal with the sliding mode nonlinear controller for the discrete time nonlinear system.

The organization of this chapter is: in Sect. 11.2, the problem formulation is proposed. In Sect. 11.3, the structure of the general controller is designed to drive the state trajectories into a small region with respect to the bound of perturbations. In Sect. 11.4, the parameters of the general controller are designed according to a set of preassigned eigenvalues. In Sect. 11.5, the direct sliding mode controller is obtained by using Newton–Raphson algorithm to resolve the general controller polynomial equation. In Sect. 11.6, two examples are given to illustrate the effectiveness of the proposed procedure.

11.2 Preliminaries

The nonlinear discrete time system can be described as follows:

$$x(k+1) = Ax(k) + B(\varphi(u(k)) + v(k, x, u)), \tag{11.1}$$

where $x \in R^n$ is the state vector, $\varphi(u(k)) \in R^m$ is the nonlinear controller, $u(k)$ is the control input, where $m \leq n$, $v(k, x, u)$ is bounded matching perturbations, A and B are constant matrixes with appropriate dimensions.

Lemma 1 [8]**.** The sufficient discrete time sliding condition $|\overline{\sigma}_i(k+1)| < |\overline{\sigma}_i(k)|$ is held if the following inequality satisfies:

$$\sigma(k)\,\Delta\sigma(k+1) < -\frac{1}{2}(\sigma(k+1))^2, \quad \sigma(k) \neq 0, \tag{11.2}$$

where $\Delta\sigma(k+1) = \sigma(k+1) - \sigma(k)$.

Assumption 1. The norm of the matching perturbation is bounded as follows:

$$\|v(k, x, u)\|_p \leq \delta, \tag{11.3}$$

where $p = 1, 2, \infty$.

The sliding coefficient matrix $S \in R^{m\times n}$ of the sliding function is chosen such that SB is nonsingular matrix. The sliding function is designed as

$$\sigma(k) = Sx(k). \tag{11.4}$$

11.3 Design of Controller Structure

The controller structure is designed in this section. The sliding mode control idea is inspired by Ref. [8], but the nonlinear sliding mode controller is introduced in this section.

Theorem 1. Given the system equation described in (11.1), and the sliding function described in (11.4). Let $\overline{\sigma} = (SB)^{-1}\sigma = \left[\overline{\sigma}_1\ \overline{\sigma}_2\ \cdots\ \overline{\sigma}_m\right]^T$. If the sliding mode controller is designed as

$$\varphi\left(u\left(k\right)\right)=\varphi\left(u_e\left(k\right)\right)+\varphi\left(u_\sigma\left(k\right)\right), \tag{11.5}$$

where

$$\varphi\left(u_e\left(k\right)\right)=\left(SB\right)^{-1}\left(\beta S-SA\right)x\left(k\right). \tag{11.6}$$

$$\varphi\left(u_\sigma\left(k\right)\right)=\left[\varphi\left(u_{\sigma 1}\right)\ \varphi\left(u_{\sigma 2}\right)\ \cdots\ \varphi\left(u_{\sigma m}\right)\right]^T=-K_0\overline{\sigma}\left(k\right), \tag{11.7}$$

where β is constant, and $K_0=\text{diag}\left[k_{01}\ k_{02}\ \cdots\ k_{0m}\right]$.

Then the following cases can be obtained:
(A) The state trajectories of the controlled system whose controller is described by (11.5) is driven into the following region:

$$\Re_A=\left\{\begin{array}{l} z\left(k\right)\in R^n:\left\|z\left(k\right)\right\|_p \\ \le\frac{\left\|P^{-1}B\right\|_p\delta}{1-\left\|P^{-1}\left(A+B(SB)^{-1}(\beta S-SA)-BK_0(SB)^{-1}S\right)P\right\|_p}\end{array}\right\}, \tag{11.8}$$

where $z\left(k\right)=P^{-1}x\left(k\right)$, P is diagonal transformation matrix.
(B) The discrete sliding condition $\left|\overline{\sigma}_i\left(k+1\right)\right|<\left|\overline{\sigma}_i\left(k\right)\right|$ will be satisfied outside the following region:

$$\Re_B=\left\{z\left(k\right)\in R^n:\left|\sum_{j=1}^{n}l_{ij}z_j\left(k\right)\right|\le\max\left[\frac{\rho\left(k\right)}{k_{0i}},\frac{\rho\left(k\right)}{2-k_{0i}}\right]\right\}, \tag{11.9}$$

where $0<k_{0i}<2$, $L=\left[l_{ij}\right]=\left(SB\right)^{-1}SP$, and $\left\|v\left(k\right)+\left(\beta-1\right)Lz\left(k\right)\right\|_p\le\rho\left(k\right)$.

Proof. (A) The system equation with nonlinear controller described by (11.10) can be obtained by substituting (11.5) into (11.1).

$$\begin{aligned}x\left(k+1\right)=&\left(A+B\left(SB\right)^{-1}\left(\beta S-SA\right)-BK_0\left(SB\right)^{-1}S\right)x\left(k\right)\\&+Bv\left(k\right).\end{aligned} \tag{11.10}$$

Equation (11.10) can be transformed into the following equation by using the transformation $z\left(k\right)=P^{-1}x\left(k\right)$:

$$\begin{aligned}z\left(k+1\right)=&P^{-1}\left(A+B\left(SB\right)^{-1}\left(\beta S-SA\right)-BK_0\left(SB\right)^{-1}S\right)Pz\left(k\right)\\&+P^{-1}Bv\left(k\right),\end{aligned} \tag{11.11}$$

where P is the transformation matrix.

If the state trajectories satisfy:

$$\|z\|_p > \frac{\left\|P^{-1}B\right\|_p \delta}{1 - \left\|P^{-1}\left(A + B\,(SB)^{-1}\,(\beta S - SA) - BK_0\,(SB)^{-1}\,S\right)P\right\|_p}. \tag{11.12}$$

Then

$$\begin{aligned}
&\|z\,(k+1)\|_p \\
&= \left\|P^{-1}\left(A + B\,(SB)^{-1}\,(\beta S - SA) - BK_0\,(SB)^{-1}\,S\right)Pz\,(k) + P^{-1}Bv\,(k)\right\|_p \\
&\le \left(\left\|P^{-1}\left(A + B\,(SB)^{-1}\,(\beta S - SA) - BK_0\,(SB)^{-1}\,S\right)P\right\|_p\right. \\
&\quad \left.+\frac{\left\|P^{-1}B\right\|_p \delta}{\|z\|_p}\right)\|z\|_p \\
&< (\left\|P^{-1}\left(A + B\,(SB)^{-1}\,(\beta S - SA) - BK_0\,(SB)^{-1}\,S\right)P\right\|_p \\
&\quad + 1 - \left\|P^{-1}\left(A + B\,(SB)^{-1}\,(\beta S - SA) - BK_0\,(SB)^{-1}\,S\right)P\right\|_p)\,\|z\|_p \\
&\le \|z\,(k)\|_p\,.
\end{aligned} \tag{11.13}$$

Therefore, it can indicate that the system state trajectories are driven into the region as follows:

$$\Re_A = \left\{\begin{array}{c} z\,(k) \in R^n : \|z\,(k)\|_p \le \\ \frac{\left\|P^{-1}B\right\|_p \delta}{1-\left\|P^{-1}\left(A+B(SB)^{-1}(\beta S - SA) - BK_0(SB)^{-1}S\right)P\right\|_p}\end{array}\right\}. \tag{11.14}$$

Which means the system state trajectories can be driven into a small closed and bounded region by the controller. The bound of the region is determined by the system parameters and the magnitude of perturbation.
(B) The following equation can be obtained by substituting (11.3) into (11.10):

$$\begin{aligned}
\Delta\sigma\,(k+1) &= \sigma\,(k+1) - \sigma\,(k) \\
&= SB\varphi\,(u_\sigma\,(k)) + SBv\,(k) + (\beta - 1)\,Sx\,(k)\,.
\end{aligned} \tag{11.15}$$

Equation (11.15) indicates that

$$\Delta\overline{\sigma} = (SB)^{-1}\Delta\sigma = \varphi(u_\sigma) + v + (\beta - 1)Lz, \tag{11.16}$$

where $\overline{\sigma} = (SB)^{-1}\sigma = \left[\overline{\sigma}_1\ \overline{\sigma}_2 \cdots \overline{\sigma}_m\right]^T$ and $L = [l_{ij}] = (SB)^{-1}\,SP$.

If the system (11.11) is stabilized, then $z\,(k)$ is bounded. Because $v(k)$ is bounded, so $\bar{v}\,(k) = v\,(k) + (\beta - 1)\,Lz\,(k)$ is bounded, too. Therefore, it gives $\|\bar{v}\,(k)\|_p \le$

$\rho(k)$. Equation (11.16) implies that

$$\Delta\overline{\sigma}_i = \varphi(u_{\sigma i}) + \bar{v}_i = -k_{oi}\overline{\sigma}_i + \bar{v}_i, \quad i = 1, 2, \ldots, m. \tag{11.17}$$

Now, suppose that the following equation is satisfied with

$$|\overline{\sigma}_i(k)| > \max\left\{\frac{\rho(k)}{k_{0i}}, \frac{\rho(k)}{2 - k_{0i}}\right\} = \frac{\rho(k)}{2 - k_{0i}}, \tag{11.18}$$

where $0 < k_{0i} < 2$.
Equation (11.18) can lead to the following two cases:

(a) $\overline{\sigma}_i > 0$. Equation (11.18) implies that $2\overline{\sigma}_i(k) - \overline{\sigma}_i(k)k_{0i} > \rho(k)$, which means that $1 > \frac{\rho(k)+\overline{\sigma}_i(k)k_{0i}}{2\overline{\sigma}_i(k)} > \frac{\bar{v}_i+\overline{\sigma}_i(k)k_{0i}}{2\overline{\sigma}_i(k)}$, that means $-1 < \frac{-k_{oi}\overline{\sigma}_i+\bar{v}_i}{2\overline{\sigma}_i}$.

(b) $\overline{\sigma}_i < 0$. Equation (11.18) implies that $-2\overline{\sigma}_i(k) + \overline{\sigma}_i(k)k_{0i} > \rho(k)$, which means that $1 > \frac{\rho(k)-\overline{\sigma}_i(k)k_{0i}}{-2\overline{\sigma}_i(k)} > \frac{\bar{v}_i-\overline{\sigma}_i(k)k_{0i}}{-2\overline{\sigma}_i(k)}$, that means $-1 < \frac{-k_{oi}\overline{\sigma}_i+\bar{v}_i}{2\overline{\sigma}_i}$.

On the other hand, Eq. (11.18) also implies $|\overline{\sigma}_i| > \frac{\rho}{k_{0i}}$, which indicates that

$$-k_{oi} + \frac{\bar{v}_i}{\overline{\sigma}_i} \le -k_{oi} + \frac{|\bar{v}_i|}{|\overline{\sigma}_i|} \le -k_{oi} + \frac{\rho}{|\overline{\sigma}_i|} < 0. \tag{11.19}$$

According to case (a), case (b), and (11.19), it gives

$$-1 < (-k_{oi}\overline{\sigma}_i + \bar{v}_i)/2\overline{\sigma}_i < 0. \tag{11.20}$$

According to (11.17) it gives

$$-\infty < \frac{2\overline{\sigma}_i}{\Delta\overline{\sigma}_i} < -1, \tag{11.21}$$

which also implies $\Delta\overline{\sigma}_i\overline{\sigma}_i < -\frac{1}{2}(\Delta\overline{\sigma}_i)^2$.
where $\overline{\sigma} = (SB)^{-1}Sx = Lz$. Therefore, from Lemma 1, it can be obtained that $|\overline{\sigma}_i(k+1)| < |\overline{\sigma}_i(k)|$ which also means that the sliding function $\overline{\sigma}_i(k)$ is decreasing outside $\Re_B$ when the nonlinear sliding mode controller is used.

Remark 1. It should be noted that the obtained controller in Theorem 1 is not strict sliding mode controller due to the nonlinearities. We can use the U model approach to solve it in Sect. 11.5.

Remark 2. There are two advantages of the controller. First, the upper bound of the perturbations need not be known before the controller implement. Second, the chattering phenomenon will never happen because there is no switching action in the controller. A multiple robotic manipulators system (MRMS) is composed of n robotic manipulators. An MRMS containing four robotic manipulators is shown in Fig. 11.1.

11.4 Design of Controller Parameters

The controller parameters are determined in this section. The design method has been proved by Ref. [14] for single-input systems, and has been proved by Ref. [8] for multi-input systems.

(A) Determination of the controller parameter β

If the sliding coefficient matrix S is full rank matrix and $K_0 = I$, then $n - m$ eigenvalues of the system described in Eq. (11.10) is determined by the following reduced order system:

$$x(k+1) = \left(A - B(SB)^{-1}SA\right)x(k) = \bar{A}x(k) \tag{11.22}$$

$$\sigma(k) = Sx(k) = 0. \tag{11.23}$$

And the rest m eigenvalues are $\beta - 1$.

(B) Determination of the controller parameter S

First, the eigenvector matrix W is determined. This method has been proved by Ref. [14]. Consider the following system:

$$\begin{aligned}\dot{x} &= Ax + Bu \\ \sigma &= Sx,\end{aligned} \tag{11.24}$$

where $A \in R$, $u \in R$, $y \in R$, and SB is nonsingular, the feedback system eigenvalue assignment question is that

$$(A + BK)W = WJ, \tag{11.25}$$

where K is an $m \times n$ feedback matrix and it is chosen to yield the desired closed-loop poles specified by the eigenvalues of J.

The problem of arbitrary eigenvector assignment has been tackled by Ref. [15], where it has been shown that, in general, it is possible to specify all components of any one eigenvector arbitrarily using state feedback method. In matrix form, (11.25) is equivalent to

$$AW - WJ = BL, \tag{11.26}$$

where L is an arbitrary $m \times (n - m)$ matrix and it is chosen to provide linear combinations of the columns of matrix B.

Second, the eigenvector matrix S is determined.

Let the matrix S satisfy

$$SB = X, \tag{11.27}$$

where X is an arbitrary $m \times m$ nonsingular matrix and

$$SW = 0. \tag{11.28}$$

A solution to (11.28) always exists, since B is full rank, giving the particular solution

$$S = XB^{-1}. \tag{11.29}$$

This solution also satisfies (11.28), since it is required from $B^{-1}W = 0$. A systematic method of finding B^{-1} which will always satisfy $B^{-1}W = 0$ is by constructing $\begin{bmatrix} B & W \end{bmatrix}^{-1}$. The first m rows of this inverse matrix is B^{-1} that satisfies $B^{-1}W = 0$.

11.5 Solution of the Nonlinear Controller

The nonlinear controller is solved in this section. The method has been proved by Ref. [11]. According to Taylor series (1721), analytic functions of the Taylor series at a given point are finite order functions of its Taylor's series, which completely determines the function in some neighborhood of the point. So the following polynomial equation in the current control term $u(k)$, was proposed to approximate the nonlinear controller $\varphi\ (u(k))$ that is described by (11.5):

$$\varphi(u(k)) = \sum_{j=0}^{M} \alpha_j(k)\, u^j(k-1) \tag{11.30}$$

The control input $u(k)$ can be obtained by Newton–Raphson algorithm.

Remark 4. As far as the authors know, there is almost no straightforward approach for nonlinear system control [16–22], the method of Ref. [11] provides the straightforward approach.

11.6 Algorithm for Implement

In this section, a step-by-step procedure is listed to implement the control scheme.

Step 1. Calculate the controller parameter β and sliding matrix S.

Step 2. Assign initial values of the state $x(1)$, the ideal state of the system and assign $i = 1$.

Step 3. Calculate the controller $\varphi(u(i))$ based on Eqs. (11.5) to (11.7) and initial values of the state $x(i)$.

Step 4. Calculate the controller $u(i)$ of (11.30) based on the Newton–Raphson algorithm.

Step 5. $i = i + 1$ go to Step 3.

This is an online algorithm for sliding model control of discrete time nonlinear dynamic system.

11.7 Simulation

Example 1. The selected discrete time nonlinear system is described as follows:

$$x(k+1) = \begin{bmatrix} 1 & 0.01 & 0 \\ 0 & 1 & 0.01 \\ -0.01 & 0.02 & 0.99 \end{bmatrix} x(k) + \begin{bmatrix} 0.01 & -0.01 \\ 0 & 0.01 \\ 0.01 & 0 \end{bmatrix} \left(v(k) + \begin{bmatrix} \varphi(u_1(k)) \\ \varphi(u_2(k)) \end{bmatrix} \right). \tag{11.31}$$

The nonlinear controller outputs are expressed in terms of (11.30):

$$\begin{bmatrix} \varphi(u_1(k)) \\ \varphi(u_2(k)) \end{bmatrix} = \begin{bmatrix} u_1^3(k) + u_1^2(k) + u_1(k) + 1 \\ u_2^3(k) + u_2^2(k) + u_2(k) + 1 \end{bmatrix}. \tag{11.32}$$

The matching perturbations of the system can be described as follows:

$$v(k) = \begin{bmatrix} x_1 x_2 - 0.2x_3 + 0.1x_1 u_1 - x_2 \cos(k) + 0.2 \sin(k) \\ x_2^2 - x_3 - 2x_3^2 + 0.1x_1 + 0.01x_2 u_2 + 0.5 \cos(k) \end{bmatrix}. \tag{11.33}$$

The eigenvalues of the discrete time nonlinear system (11.31) are assigned to {0.6,0.6,0.8}, and $\beta = 1.6$, and x_d is the ideal value of the state, and y_d are the desired output value of the system.

The sliding coefficient matrix S is obtained according to Ref. [13].

$$S = \begin{bmatrix} 2111.8 & 2111.8 & -2011.8 \\ 2117.6 & 2217.6 & -2117.6 \end{bmatrix}. \tag{11.34}$$

The equivalent control law is designed as

$$\varphi(u_e) = \begin{bmatrix} 1247 & 1286.2 & -1248.3 \\ 1249.4 & 1331.7 & -1313.9 \end{bmatrix} x. \tag{11.35}$$

And,

$$\varphi(u_\sigma) = -\overline{\sigma}(k) = (SB)^{-1}\sigma = -\begin{bmatrix} 2111.8 & 2111.8 & -2011.8 \\ 2117.6 & 2.2176 & -2117.6 \end{bmatrix} x, \tag{11.36}$$

where $x_1(1) = 1, x_2(1) = -0.5, x_3(1) = 0.5$.

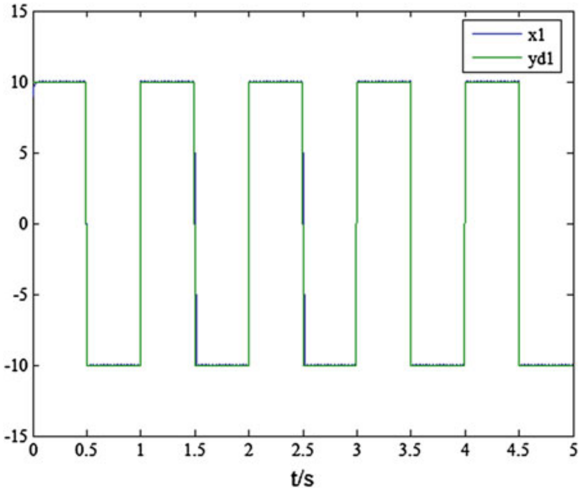

Fig. 11.1 State variable x_1 and desired output y_{d1}

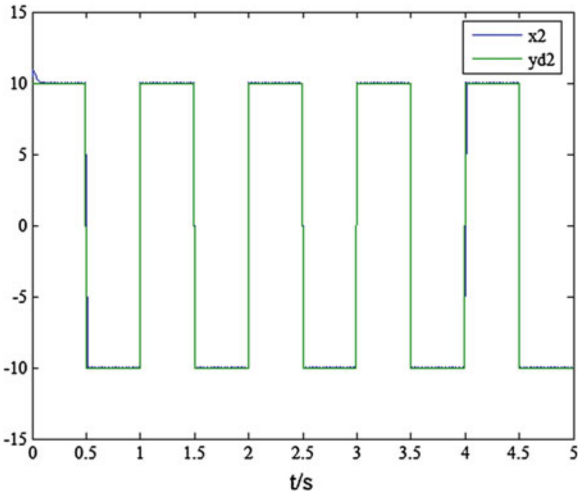

Fig. 11.2 State variable x_2 and the desired output y_{d2}

Figures 11.1, 11.2, and 11.3 show that the state trajectories are all tracked to the desired output y_d rapidly. Figures 11.4 and 11.5 show the sliding function σ also into small bounded region rapidly. Figures 11.6 and 11.7 show that the controller output trajectories obtained by Newton–Raphson algorithm.

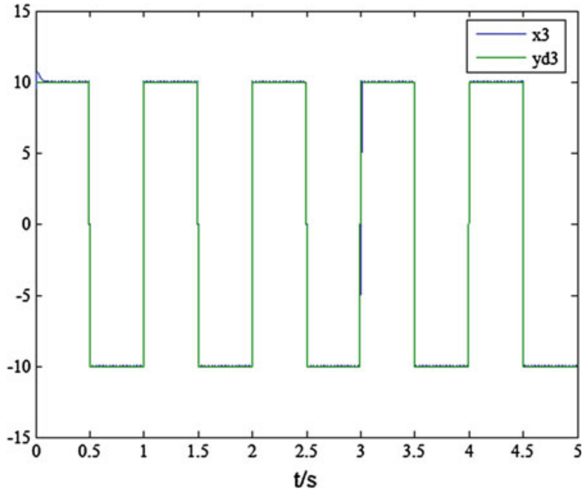

Fig. 11.3 State variable x_3 and desired output y_{d3}

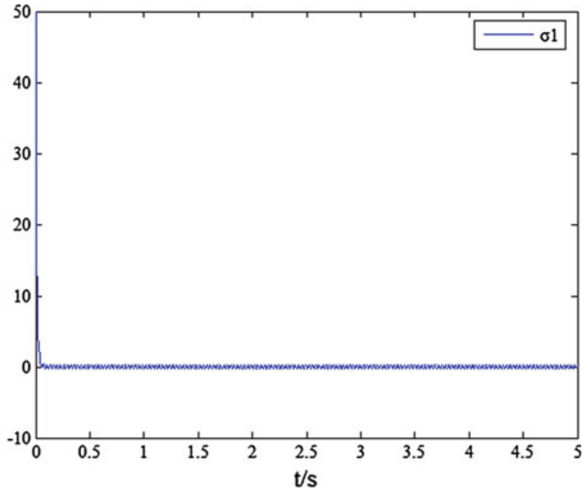

Fig. 11.4 Sliding surface σ_1

Note that the controllers proposed in Ref. [7] can be used for discrete time linear system with matching perturbations, but it does not contain controller nonlinearities. Therefore, the controller proposed in Ref. [8] cannot be used directly for those system with controller nonlinearities.

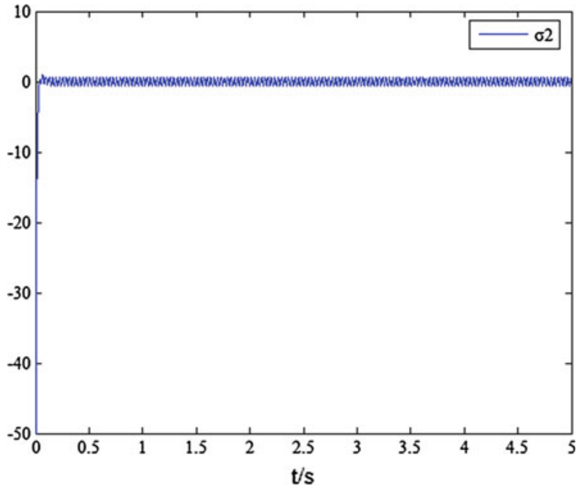

Fig. 11.5 Sliding surface σ_2

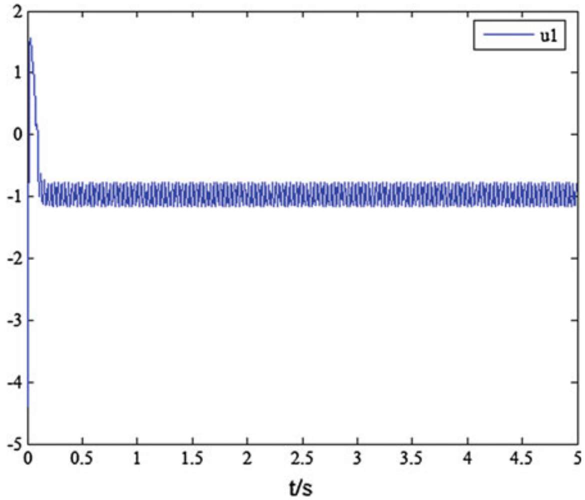

Fig. 11.6 controller output u_1

Example 2. Comparison with the PID method.

Figures 11.8, 11.9, and 11.10 show that the state trajectories are all tracked to the desired output y_d by using the PID method. The plots show very clearly that the peak overshoot and the settling time can be minimized and the system performance improves significantly by using the sliding mode controller proposed in this chapter.

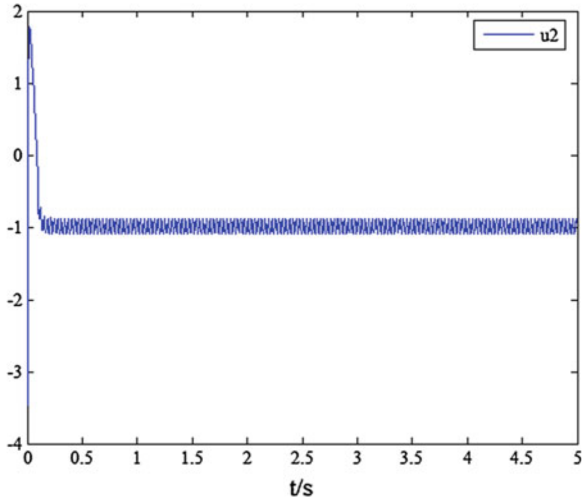

Fig. 11.7 controller output u_2

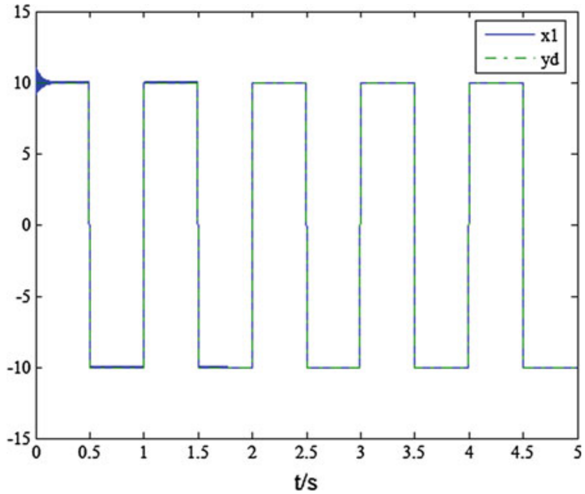

Fig. 11.8 State variable x_1 and desired output y_{d1}

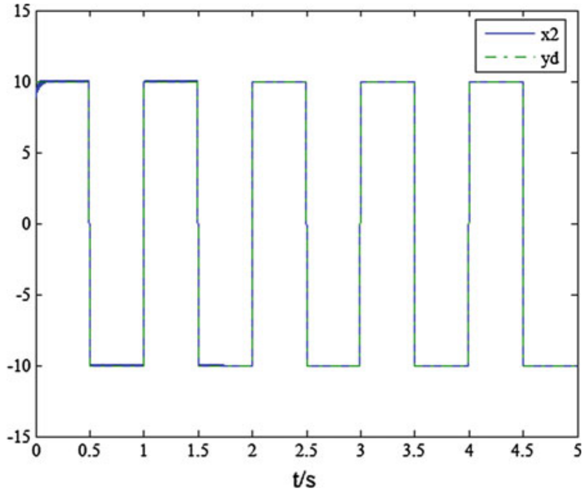

Fig. 11.9 State variable x_2 and the desired output y_{d2}

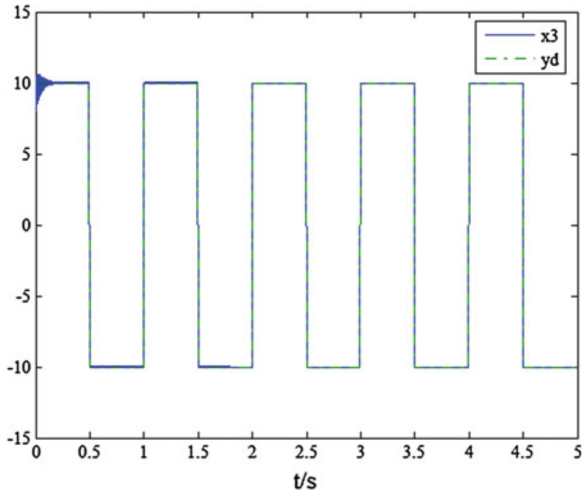

Fig. 11.10 State variable x_3 and desired output y_{d3}

11.8 Conclusions

The nonlinear sliding mode controller is designed for the discrete time nonlinear systems. The controlled system state trajectories are driven into small bound region, and the system is stable by the determination of the nonlinear controller parameters. The nonlinear controllers are represented by a polynomial equation, and the algebraic solutions can be obtained by Newton–Raphson algorithm. The method proposed in this chapter provides a straightward methodology to use sliding mode control design techniques when nonlinearities embedded in the controller. Here, only the sliding mode control of nonlinear systems is investigated, but it is strongly believed that the idea of this chapter is effective for most other classes of discrete time control systems. Further studies on the developed methodology, such as discrete time switched systems, discrete time neutral systems, time delay systems, will be conducted to provide a comprehensive framework in designing discrete time nonlinear control systems.

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