Iterative Phase Retrieval and the Important Role Played by Initial Conditions

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1 Introduction

Recovering the complex amplitude of a coherent wave field has many important applications in modern optics from practical metrology problems to basic diffraction research. Although several techniques exist, in this manuscript, we will examine the iterative Phase Retrieval (PR) [1-6] exclusively. To recover the phase information with this approach several intensity distributions, diffracted from the object of interest, are recorded at different optical planes. A significant advantage of the PR is the relative simplicity of the optical setup, compared with an interferometric approach. With this approach, noise sources such as an imperfect reference wave can be avoided. PR techniques are an ill-posed inverse problem and hence initial conditions play an important and relatively poorly understood role.

To begin the PR algorithmic process an estimate of the wave field's phase distribution is required. It is assumed that the errors in this initial guess are continuously improved through an iterative algorithm so that the final estimate approaches that of the actual wave field phase distribution. In practice, due to the ill-posed nature of the problem, this is not always the case. Additionally, the only means we have of estimating the accuracy of our final solution is the similarity of the actual measured distribution, to that, recovered from the PR process. Ideally if we have the correct phase information at a particular plane, we can use this to propagate the complex amplitude to other planes where we have recorded the actual intensity distribution. Differences between the measured and recovered intensity distributions are attributed to an error in the phase estimate.

In this manuscript we introduce a means to assess the accuracy of the final solution. By capturing an extra "reference plane" intensity distribution that is withheld from the PR process, we can examine some of the convergence properties of the PR algorithm. We use one data set of captured intensity distributions and different initial conditions to examine their role in PR. The initial guess phase distribution and the number of intensities are the initial conditions we examine in this study.

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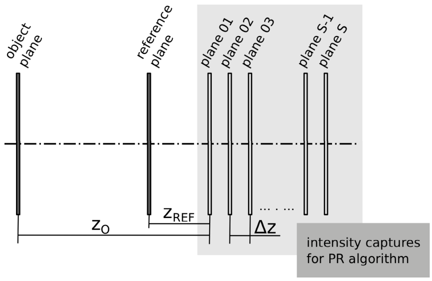

Fig. 1 Geometry of the PR e xperiment, intensity captures in plane 01…plane S are input intensities for the PR experiment, the reference plane is for evaluating the retrieved distributions

2 PR Experiment

In Fig. 1 we depict a schematic of the experiment that was carried out. The object is illuminated with a HeNe laser beam, wavelength 633 nm, and a series of intensity distributions (pixel value M×N, 2048×2048) are captured with a CCD sensor (Kodak KAI-4022, pixel pitch Δx = 7.4 μm) in axially displaced planes. The capture planes are denoted 01 to S, in Fig. 1 and are axially displaced from each other by Δz = 2 mm. Here we use the Forward Backward (FB) algorithm, presented in Ref. [2]. The square root of the captured intensity in plane 01 and a chosen initial phase form the initial guess of the complex amplitude. This artificial distribution is propagated from plane to plane until the last plane, plane S, is reached and then step-by-step back propagated to the first plane, plane 01. A complete run through the data set of captured intensities in forward and backward direction is one iteration step. The convergence of the FB is monitored using the RMS error

$$RMS = \sqrt{\frac{1}{MN}\sum_{xy}\left(\sqrt{I_1} - A_{1,n}\right)^2} \tag{1}$$

between the iterated and actual measured amplitude (square root of the captured intensity) in plane 01.

We assume the paraxial approximation is valid for our experiment and use the spectral method to implement a fast numerical calculation of the Fresnel transform, thereby describing the light propagation process [4].

In our PR experiment a ground glass diffuser serves as our object. The first intensity distribution is recorded at a distance of z_0 = 185 mm, from the diffuser plane. The size of the illumination spot, the propagation distance (here z_0) and the wavelength of the laser determine the lateral speckle size in plane 01. The speckle size of the measured intensity in plane 01 is (7.73·Δx), where Δx is the pixel pitch of the employed camera. Here we define the lateral (horizontal) speckle size, by performing an autocorrelation of the intensity captures, and measuring the full width of this peak at *one tenth of maximum* (FWTM).

Care has been taken to ensure that the axially displaced intensities, see Fig. 1, are sufficiently different from each other so as to provide a stable numerical solution. The iterative PR approach requires that there is sufficient diversity between each of the measured intensity distributions in order for the algorithm to converge properly [2]. So, when choosing Δz the lateral and longitudinal speckle size have to take in account. The bigger the lateral speckle size is, the longer the speckles are and so a larger axial distance Δz is required to fulfill these diversity criteria.

As noted earlier we also record, an intensity distribution in the reference plane, see Fig. 1. This reference intensity is kept out the PR process and used only for evaluating the result of the PR. If we have correctly estimated the phase distribution we should also be able to accurately reconstruct this reference plane intensity.

3 Results

We now investigate how the initial conditions influence the result of a PR. We refer to the initial guess phase and the number of intensity captures as the initial conditions.

The PR algorithm runs with a flat (constant) phase as initial guess and 2, 5, 10 and 20 intensity captures. The algorithm is terminated when the RMS error converges to a minimum. Sixty iterations are used for a PR with 5 intensities and thirty with 10 or 20 intensities. For a PR with two captures the algorithm is terminated after 280 iterations. Plots of the converging RMS error with increasing iterations are presented in [6].

Since we use intensity captures of a speckle field, we can determine the lateral speckle size in the measured and iterated intensities and compare these values. The speckle size is used as a second metric to evaluate the accuracy of the retrieved distribution. The speckle size of the measured intensity is (7.73·Δx) in plane 01, and that of the iterated it is (6.53·Δx) for a PR with 20 planes, and (7.20·Δx) when 10 planes are used. The RMS error of the iterated and measured amplitude for a PR with 20 planes and 10 planes is 0.089 and 0.057, resp. The results of the PR with 10 planes seem to be a bit better than these of a PR with 20 planes because the RMS error is lower, and the speckle size, is closer to that of the measured speckle size. When we use a FB algorithm with only two planes, the speckle size in the iterated intensity is (7.71·Δx) and the RMS error is 0.042 after 280 iterations.

These results would appear to indicate that a PR process where less intensity captures are used and hence less information, can nevertheless provide better results. This impression is misleading however which becomes apparent once we compare the actual intensity distribution in the reference plane to that reconstructed from the PR algorithm.

Table 1 RMS error and mean lateral speckle size. Δx_S of the captured $\sqrt{I_{REF, CAP}}$ and reconstructed amplitude $A_{REF, REC}$ in the reference plane for the FB algorithm with different numbers of planes and two different initial guess phase, Δx_S ($\sqrt{I_{REF, CAP}}$) - speckle size of the captured image, the speckle size is in pixel units.

initial phase	2 planes		5 planes		10 planes		20 planes		Δx_S ($\sqrt{I_{REF, CAP}}$)
	RMS	Δx_S	RMS	Δx_S	RSM	Δx_S	RMS	Δx_S	
plane	0.161	2.43	0.160	1.98	0.137	6.88	0.138	6.47	7.38
random	0.161	2.28	0.160	1.95	0.136	6.93	0.138	6.49	7.38

The complex amplitude distributions retrieved from the PR process are back propagated from plane 01 to the reference plane for z_{REF} = 10 mm, see Fig. 1. The back propagated distributions are referred as the "reconstructed" fields. To evaluate, whether a good retrieval has been achieved, the reconstructed and the captured reference images are compared.

Examining the RMS error and the speckle size of the iterated intensity in plane 01, the best results are obtained employing the FB algorithm with only two intensity captures. Propagating the retrieved complex amplitude back to the reference plane evaluable results are not achieved. But the more intensity captures are used in the FB algorithm the better the quality of the reconstructions become, see Tab. 1.

To evaluate the reconstructed intensity and hence the retrieved distribution one can use the RMS error, the speckle size or the correlation coefficient. Here however we plot cross sections of the captured and reconstructed intensity distributions. All images have a pixel number of 2048 × 2048. For the intensity profile a pixel row in the center of each image is read out, the start and end point in pixel coordinates is (925,1024) and (1124,1024), resp. Fig. 2 depicts the profiles of the reconstructed and captured intensities for the PR experiment and different numbers of planes. It is obvious, the more planes are used in the FB algorithm the reconstructed intensity becomes more similar to the captured one.

The left graphs in Fig. 2 show the results when as initial guess a plane phase is used. Can we improve the results when another initial phase is used? If a "good" initial guess is used, will be the role of the number of intensity captures reduced? Since in our PR experiment intensity captures of a speckle field are employed, we decided to use as initial guess a pattern with random phase values. For this phase pattern random numbers are generated, which are distributed uniformly. To get a finite correlation width the phase pattern is filtered in the Fourier domain and the correlation width is determined by the FWTM. Three different phase patterns are employed with a correlation width around the speckle size of the desired wave field in plane 01, a second with a smaller and a third with a bigger correlation width than the speckle size. The best results are achieved when a phase pattern is used with a correlation width around the speckle size. We determine this initial phase pattern as a "good" guess and employ it in a PR experiment with different

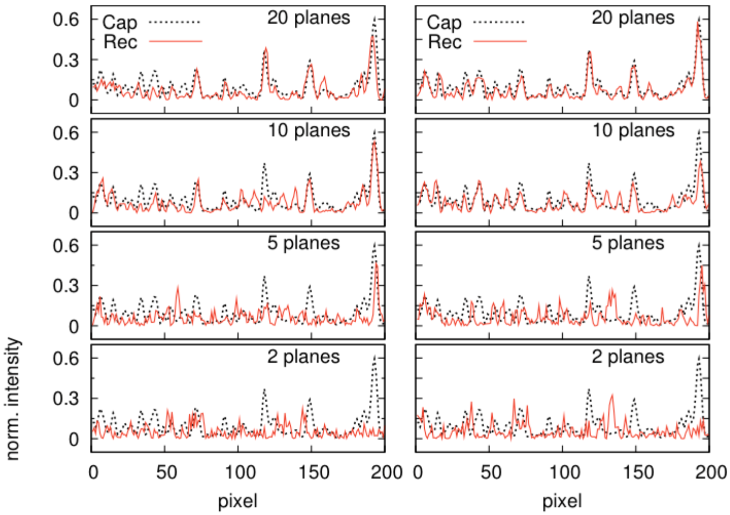

Fig. 2 Intensity profile of the captured and reconstructed distribution in the reference plane (1D depiction in x-direction), for retrieving the complex amplitude the FB algorithm is used with 2, 5 10, 20 intensity captures and as initial guess a plane phase (left plots) and a random phase pattern with a correlation width of (7.63·Δx) (right plots)

numbers of planes: 2, 5, 10 and 20. With this particular phase distribution and 20 captures are the best reconstructions achieved within all the presented PR experiments. Using less intensity captures the quality of the reconstructions gets worse.

4 Conclusions

We used one data set of intensity captures in a speckle field and examined the influence of the initial conditions on the retrieved distribution. The number of intensities and the initial guess phase we determine as initial conditions. The FB algorithm is terminated when the RMS error of the iterated and measured amplitude converges toward a minimum. To evaluate the quality of the phase retrieval process the calculated and captured intensity in a reference plane are compared. The captured reference intensity is not used in the PR algorithm and the retrieved complex amplitude must be propagated to the reference plane.

Based on our results it can be stated: The more intensity captures we use, the better the quality of the retrievals become and employing a “good” initial guess phase a further improvement is achievable. Besides the here used conditions, initial phase and number of planes, the axial distance between the captures also has an influence on the retrievals.

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