An Introduction to Multicriteria Decision Aid: The PROMETHEE and GAIA Methods

Yves De Smet and Karim Lidouh

Computer and Decision Engineering Departement CoDE-SMG, Ecole polytechnique de Bruxelles, Université libre de Bruxelles, Belgium
yves.de.smet@ulb.ac.be
http://code.ulb.ac.be/∼yvdesmet

Abstract. Most strategic decision problems involve the evaluation of potential solutions according to multiple conflicting criteria. The aim of this chapter is to introduce some basic concepts of Multicriteria Decision Aid (MCDA) with a special emphasis on the PROMETHEE and GAIA methods. First, we will introduce the specific vocabulary of this research area as well as traditional modelling issues. The main part of the presentation will be dedicated to explain in detail the PROMETHEE and GAIA methods. Finally, an illustrative example will be analyzed with the D-Sight software. This will highlight the added value of using interactive and visual tools in complex decision processes.

Keywords: multicriteria decision aid, outranking methods, PROMETHEE, GAIA.

1 Introduction

Multicriteria Decision Aid (MCDA) has been an active field of research for more than 40 years. Summarizing it in a few pages is, of course, impossible. Consequently, the only ambition of this chapter is to constitute a rough introduction to the subject. Additionally, we have decided to detail a given methodology, namely the PROMETHEE and GAIA methods, rather than to present an over-simplified summary of different methods. As a consequence, the reader should keep in mind that plenty of other approaches do exist and deserve attention (for instance AHP [39], MAUT [20], ELECTRE [24], MACBETH [3], . . .).

As shown hereafter, a brief analysis of the terms *"multicriteria decision aid"* already allows the novice to understand the underlying motivations of this research area [18]. It is, first of all, a decision **aid** activity (versus decision **making**) that has its root in the **multicriteria** paradigm. These statements will be further commented in the next two subsections. We refer the interested reader to [2,8,18,36,37,40,43,44] for detailed discussions.

1.1 What Is Decision Aid ?

Selecting an investment project, appointing a new employee, choosing a site to establish a garbage dump, diagnosing a disease, etc. All these examples show

M.-A. Aufaure and E. Zimányi (Eds.): eBISS 2012, LNBIP 138, pp. 150–176, 2013.

that *deciding* is a complex activity that, in many cases, can have important consequences.

A decision is, first of all, the result of a more or less time consuming process that is made of partial decisions, negotiations and learning phases, search for (additional) information, etc. During this process, new potential solutions can appear while others become not feasible anymore. The context of the problem can be such that the evaluation of the potential solutions has to be made according to several conflicting points of views (possibly integrating subjective elements). The data are often imprecise, uncertain or simply not available. Social, economic and political constraints further increase the complexity of the decision process. Finally, most decisions involve several actors with different interests and goals.

Facing the complexity of this activity, one may try to *build* a model i.e. an abstraction of the reality that will be used, during the decision process, as a support for investigation and communication. The limited, approximate, and imperfect nature of this model has to remind us of its modesty. This observation has led Bernard Roy [37] to define decision aid as follows:

Definition 1. *Decision aiding is the activity of the person who, through the use of explicit but not necessary completely formalized models, helps obtain elements of responses to the questions posed by a stakeholder*[1] *in a decision process. These elements work towards clarifying the decision and usually towards recommending, or simply favoring, a behavior that will increase the consistency between the evolution of the process and this stakeholder's objectives and value system.*

1.2 What Is Multicriteria Decision Aid ?

In the fifties, the pioneers of Operational Research (O.R.) were convinced of the natural and promising applicability of their models. Twenty years later, the reality was somewhat different: some problems had been successfully treated by using classic operational research tools while, in other cases, their application had disappointed [35].

As noted by Schärlig [40], the success stories were essentially related to situations where the decision problem could have been *isolated* from its context: the search for optimal mixtures, an optimal traveling salesman problem, an optimal stock management, etc. In the other cases, the underlying assumptions of classic OR models appeared to be too restrictive to constitute an adequate model of the reality.

Indeed, most of unicriterion optimisation approaches rely on the following (implicit) assumptions [40]:

– stable set of actions: the set of alternatives is assumed to be known prior to the analysis and to remain unchanged during the decision process. On contrary, in most decision problems, new alternatives can appear during the analysis while others become not topical anymore.

[1] *Here, the term stakeholder refers to any individual or entity that may intervene in the decision making process.*

– exclusive actions: every alternative is assumed to perfectly reflect all the facets of the problem.
– transitivity: the preferences of the Decision Maker (DM) are assumed to be transitive. As a consequence it is possible to rank the alternatives from the worst to the best one and thus to find a so-called *optimal* solution.

Among the critics listed above, the one related to the non-transitivity of preferences is definitively the most crucial one. Indeed, in unicriterion optimisation models, one assumes that the decision maker is able to determine admissible alternatives i.e. those satisfying a given set of constraints. Then, these admissible alternatives are ranked according to the unique criterion (that is assumed to perfectly represent the preferences of the decision maker). Therefore it is possible to rank the alternatives from the worst to the best and to find a so-called "optimal" solution[2]. As a consequence, in unicriterion optimisation models, the apparent universality of the *optimal solution* concept leads the analyst[3] to search for *a hidden truth* [18,37,40,45].

"Where are you going on holidays next year?" This question has nothing to do with a complex optimisation or strategical decision problem. Yet it allows to illustrate the problem induced by multicriteria evaluations. Table 1 summarizes a fictitious problem. If you only consider the price, you shoud go hiking in the montains. If you only consider to party, Ibiza is the best alternative. Obviously, there is no *objective* best ranking and therefore, no *objective* optimal solution (due to the conflicting nature of the criteria). If one agrees with these evaluations, the only objective information that could be stressed is that visiting the Pyramids in Egypts is a better choice than selecting a cruise in the Bahamas (since it is at least as good for all the criteria and strictly better for the price and culture). Then, you cannot compare the three other alternatives without adding subjective judgments such that *the criterion party is more important than culture, etc.*

As stressed in the previous example, the notion of *optimal* solution no longer exists in multicriteria contexts; researchers will rather look for *compromise solutions* i.e. solutions that are *"globally good"* according to the different criteria (without necessarily being the *best* for a given criterion) and that are not too bad on any given criterion.

We end this section by giving a few examples inspired by real applications. These will serve us through the chapter to illustrate the different concepts and methods.

Example 1. **The Portfolio Management Problem (PMP)**. Let us consider a set of n equities and an investment capital K. The portfolio management

[2] In most cases, an alternative that is optimal for a specific criterion will not be optimal for another criterion (on the contrary, it is likely to be a bad solution according to this second point of view). In fact, most of people interpret the term *optimal* solution in an erroneous way because they assign it to a global meaning. On the contrary, in practice, one should ask the question *"optimal with respect to which criteria?"*

[3] i.e. the person that helps the decision maker during the decision process.

Table 1. The holidays problem

Type	Price	Exostism	Culture	Sports	Party
Cruise in the Bahamas	Very expensive	Very good	Low	Low	Low
Pyramids in Egypt	Medium	Very good	Very good	Low	Low
Hiking in the mountains	Very low	Very low	Very low	High	Low
Party in Ibiza	Low	Medium	Very low	Good	Outstanding

problem can be stated as follows: *"How much do we have to invest in the different equities in order to maximize the expected return and to minimize the risk?"* This famous question was first addressed by Markowitz [34]. Of course, there is not a unique portfolio that could be objectively considered as the best one (since the two criteria are in conflict: increasing the expected return will also increase the risk). In this problem, a crucial step is thus to compute the so-called Pareto-optimal frontier i.e. the set of portfolios such that the expected return cannot be increased without also increasing the risk (see Fig. 1; dots represent potential portfolios - the continuous curve represent the Pareto-optimal frontier). Once this set has been identified, the decision maker will have to select a given combination that best fit his risk aversion (or in other words his preferences). Let us stress that this bi-objective optimisation problem can easily be extended to the optimisation of other criteria such as liquidity, robustness, etc.

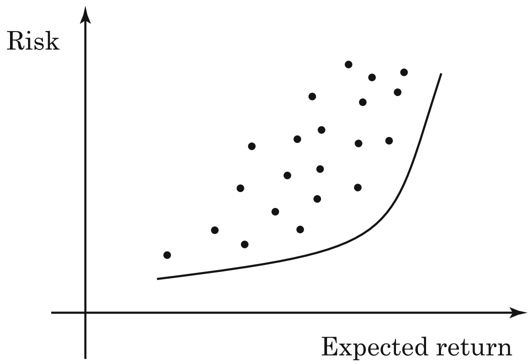

Fig. 1. Bi-objective portfolio problem

Example 2. **The Criminality Assessment Problem** (CAP). The Belgian police records statistics about criminal activities. These facts belong to predefined crime types such as robbery, road accidents, murders, burglary, sex offenses, prostitution, fraud, vandalism, etc. The severity of each crime type can be assessed according to different points of views: number of deaths, number of victims, financial impacts, social impacts, evolution over the last 5 years, type of organizations, etc. Every year, a ranking of these crime types is performed. This allows to allocate human, financial and material resources to efficiently fight the most severe activities. Of course, modelling the severity of these crime types is not easy. If we consider the number of deaths, the worst crime type is related to road accidents. On the other hand, this is not related to criminal organizations and other related illegal activities such as in the case of prostitution. Vandalism is not characterized by a high number of victims or deaths and is not directly related to important criminal organizations however it has a high social impact, etc.

Example 3. **The Diagnosis of Alzheimer's Disease (DAD)**. Being able to detect Alzheimer's disease is naturally of the uttermost importance. Any patient can be characterized by a set of criteria that are related to it. These encompass the age, heredity, etc. but also results to well-established memory tests. Given the evaluation of a patient, the problem consists of verifying if he or she suffers from this disease and assessing its severity. Once it is detected, one may consider three different grades: mild, medium, or severe.

Example 4. **The Academic Ranking of World Universities (ARWU)**. Assessing the academic quality of world universities has become a topical issue over the last years. Based on criteria such as the number of articles published in top quality scientific journals, the number of awards received by alumni, etc. one establishes a ranking of institutions. If the so-called *Shanghai ranking* was initially developed to quantify the gap between Chinese and international universities, it is nowadays considered as a reference ranking that has been integrated in public life (see also section 3.3). Let us note that these rankings remain subject to criticism and has recently initiated a lot of debates [7].

2 Main Concepts and Terminology

Facing the complexity of a decision problem, the decision maker (DM) tries to rationalize it. Therefore, he has to identify the key elements that will intervene in the decision process i.e. the object of the decision, the set of potential solutions, a way to evaluate and compare them, the factors that can influence the decision(s), etc. This structuring phase is at the core of any multicriteria decision aid activity.

In this section, we will introduce the basic terminology that is used within the MCDA community and, consequently, increase the reader's awareness of the MCDA problem's formulation.

2.1 The Alternatives

At first, let us introduce the notion of an **action**. Intuitively, actions are the set of objects, alternatives, items, candidates, projects, potential decisions, etc. on which the decision is based. More formally,

Definition 2. *[38] An action is a generic term used to designate that which constitutes the object of the decision, or that which decision aiding is directed towards.*

In what follows, we will denote the set of actions $\mathcal{A} = \{a_1, \ldots, a_n\}$. As stressed by Vincke [45], $\mathcal{A}$ can be:

- *stable*: if $\mathcal{A}$ can be defined a priori and is not likely to change during the decision process;
- *evolutive*: if, on the contrary, $\mathcal{A}$ is likely to change during the decision process. Indeed, the decision process being an dynamic activity, intermediary results and/or the potential evolution of the decision context can lead to consider new actions while others are not topical anymore.

Furthermore, let us stress that $\mathcal{A}$ is said to be **globalized**, if each element of $\mathcal{A}$ excludes any others, and **fragmented** if it is not the case i.e. if combinations of elements from $\mathcal{A}$ constitute possible outcomes of the decision process. Finally, one generally distinguishes contexts where $\mathcal{A}$ can be defined by extension (the cardinality of $\mathcal{A}$ is finite and relatively small. As a consequence, its elements can be enumerated) and situations where it is defined by comprehension (the cardinality of $\mathcal{A}$ is infinite or relatively high. The elements of $\mathcal{A}$ are identified as those satisfying a set of specific constraints).

In the Criminality Assessment Problem, the alternatives are the different predefined crime types. The set of alternatives is defined by extension (since its elements can be easily enumerated) and stable (unless a new form of crime type appears). In the Portfolio Management Problem, the set of alternatives is defined by comprehension i.e. all the investment options that do not exceed the capital limit.

2.2 The Criteria

Once the set $\mathcal{A}$ has been determined, one has to characterize the actions (according to different points of views). This is formalized by the notion of criterion.

Definition 3. *[45] A criterion is a function f, defined on $\mathcal{A}$ taking its value in a totally ordered set and representing the decision maker's preferences according to some point of views.*

$$f : \mathcal{A} \rightarrow E \text{ , where } E \text{ is a totally ordered set}$$

Without loss of generality, we will assume that all criteria have to be minimized[4]. Let $f_j(a_i)$ denote the evaluation of action a_i according to the criterion f_j. Let us assume that q distinct criteria are involved in the decision problem and let $F = \{f_1, \ldots, f_q\}$ be the set of all criteria.

In the previous definition, we see that the only restriction about E is the fact that it is a totally ordered set. In other words, given two elements $e, e' \in E$ it is always possible to state if $e \succ e'$, $e = e'$ or $e' \succ e$. The poorest scale that respects this condition is the ordinal one. Of course, richer scales can be

[4] We assume that any totally ordered set E can be represented by real numbers. If a given criterion has to be maximized, taking the symmetric values of the evaluations allows to consider it as a criterion to be minimized.

considered such interval or ratio scales (see [10]). These differ with respect to the kind of mathematical operations that are allowed. In the Diagnosis of Alzheimer's Disease, one may consider the judgment of a physican about the severity of the disease. Five values could be considered: very bad, bad, medium, good and very good. Even if these values are coded using respectively the numbers 0,1,2,3,4, one may only state that 1 is worst than 2 (bad is worst than medium). Saying that medium is two times better than bad is not correct (since this depends on the arbitrary nature of the coding). When modelling a multicriteria problem, the decision maker should always keep in mind the nature of the scale characterizing the different criteria since this will restrict the kind of mathematical operations that are allowed.

At this point of the analysis, the only objective information that can be extracted from the decision problem is based on the Pareto dominance relation:

Definition 4. *Let D denote the Pareto dominance relation i.e.* $aDb \Leftrightarrow f_j(a) \leq f_j(b)\, \forall j \in \{1, \ldots, q\}$ *and* $\exists k \in \{1, \ldots, q\} | f_k(a) < f_k(b)$.

This relation leads to distinguish efficient and dominated actions from $\mathcal{A}$.

Definition 5. *An action a is said to be efficient if* $\nexists\, b \in \mathcal{A} : bDa$

If the purpose of the Decision Maker is to select a single action, he would be tempted to first remove all dominated actions from $\mathcal{A}$. Unfortunately, the number of remaining efficient actions will still remain important (since generally there is no action that is simultaneously the best for all the criteria). On the other hand, one can explicitly build a *virtual* action, called observed ideal point, that satisfies the aforementioned condition:

Definition 6. *The observed ideal point, $i(\mathcal{A})$, associated to $\mathcal{A}$, is the point the coordinates of which are* $(i(\mathcal{A})^1, \ldots, i(\mathcal{A})^q)$ *where:*

$$i(\mathcal{A})^j = \min_{a \in \mathcal{A}} f_j(a)$$

Since the ideal point (or assimilated actions i.e. that are the best for all criteria) does not usually belong to $\mathcal{A}$, the notion of optimal solution is not adapted to multicriteria problems. On the contrary, in most cases, the presence of conflicting criteria will lead the decision maker to rather focus on *compromise* solutions among efficient alternatives. As a consequence, he will be forced to express subjective judgements in order to make trade-offs between the different criteria, to interpret the evaluation scales, etc. Naturally, this leads to the question of formally modelling his preferences.

2.3 Preference Modelling

As already stressed, most multicriteria decision aid problems cannot be solved if we simply rely on the dominance relation (since the cardinality of the efficient set is too high). Therefore, additional information has to be asked to the decision maker. This leads to the parametrization of a particular mathematical model to represent in the best possible way the choice of given decision maker. As a consequence, it is crucial to be able to represent his preferences in a formal way. This section will introduce the basics of preference modelling.

When modelling the decision maker's preferences, one usually distinguishes the three following binary relations[5]: Preference (P), Indifference (I) and Incomparability (J), which result from the comparison between two actions a_i and $a_j \in \mathcal{A}$

$$\begin{cases} a_i P a_j & \text{if } a_i \text{ is preferred to } a_j \\ a_i I a_j & \text{if } a_i \text{ is indifferent to } a_j \\ a_i J a_j & \text{if } a_i \text{ is incomparable to } a_j \end{cases}$$

Indeed, these relations translate situations of preference, indifference and incomparability and it can be assumed that they satisfy the following requirements:

$$\forall a_i, a_j \in \mathcal{A} \begin{cases} a_i P a_j \Longrightarrow a_i \neg P a_j & : P \text{ is asymmetric} \\ a_i I a_i & : I \text{ is reflexive} \\ a_i I a_j \Longrightarrow \quad a_j I a_i & : I \text{ is symmetric} \\ a_i \neg J a_i & : J \text{ is irreflexive} \\ a_i J a_j \Longrightarrow \quad a_j J a_i & : J \text{ is symmetric} \end{cases}$$

Definition 7. *[45] The three relations $\{P, I, J\}$ make up a preference structure on $\mathcal{A}$ if they satisfy the above conditions and if, given any two elements a_i, a_j of $\mathcal{A}$, one and only one of the following properties is true: $a_i P a_j, a_j P a_i, a_i I a_j, a_i J a_j$.*

Intuitively [37]:

- *aPb* corresponds to the existence of clear and positive reasons that justify significant preference in favor of a;
- *aIb* corresponds to the existence of clear and positive reasons that justify equivalence between the two actions;
- *aJb* corresponds to an absence of clear and positive reasons that justify any of the two preceding relations.

In the classic *unicriterion* optimisation models, the pairwise comparisons of actions can only lead to two situations: preference or indifference. In the same way, many multicriteria methods, such as multi-attribute utility functions for instance, aggregate all the criteria into a unique (artificial) value and, therefore, transform the multicriteria problem into a unicriterion optimisation problem. In this context, both the indifference and preference relations are assumed to be transitive. These assumptions have, nevertheless, been criticized by several authors. For example, Luce [30] illustrates the non-transitivity of the indifference relation with the following example: let us consider 401 cups of coffee, noted $C_0, C_1, \ldots, C_{400}$. One assumes that the cup C_i contains exactly $(1 + \frac{i}{100})$ grams of sugar. In this context, any normal person is unable to differentiate two successive cups. Therefore, we have: $C_0 I C_1, C_1 I C_2, C_2 I C_3, \ldots, C_{399} I C_{400}$. However, it is obvious that nobody will state $C_0 I C_{400}$.

[5] R is a binary relation on $\mathcal{A} \Leftrightarrow R \subseteq \{(a_i, a_j) | a_i, a_j \in \mathcal{A}\}$

Let us note that some authors [37] further enrich the previous structure by a relation Q which stands for a *weak preference* relation (versus the *strict preference* relation P). In other words, if $a_i Q a_j$, the decision maker knows that $a_j \neg P a_i$ but cannot clearly choose between $a_i I a_j$ or $a_i P a_j$. However this specific relation will not be considered in the present work.

The potential presence of incomparability is a distinctive feature of the so-called French school of multicriteria decision aid. As already stressed, aJb is stated when the decision maker cannot clearly choose among the three possibilities: aPb, bPa or aIb. This can happen, for instance, due to a lack of information, to uncertainty or conflicting preferences (see [37] for illustrative examples).

Finally, let us define the relation $\mathcal{S} = (P \cup I)$. Thus, $a_i \mathcal{S} a_j$ will stand for a_i *is at least as good as* a_j. A direct consequence of this definition is:

$$\forall a_i, a_j \in A \begin{cases} a_i P a_j \Leftrightarrow a_i \mathcal{S} a_j, a_j \neg \mathcal{S} a_i \\ a_i I a_j \Leftrightarrow a_i \mathcal{S} a_j, a_j \mathcal{S} a_i \\ a_i J a_j \Leftrightarrow a_i \neg \mathcal{S} a_j, a_j \neg \mathcal{S} a_i \end{cases}$$

We refer the interested reader to [9] for a detailed introduction to binary relations and preference modelling.

Until now, we have restricted ourselves to binary relations for preference modelling. Let us note that another important trend relies on valued relations. This will be illustrated in section 3.1 which presents the PROMETHEE methods.

2.4 Consistent Family of Criteria

A fundamental difficulty in multicriteria decision aid is to represent the decision maker's preferences on the basis of the evaluations of the actions according to the different criteria. The selection of these criteria is thus a crucial first step of the modelling activity. One way to formalize this selection is to require certain properties such as exhaustivity, cohesion and non redundancy. Intuitively:

- **exhaustivity**: if a_i and a_j are two actions that are identical with respect to all criteria, then one cannot have $a_i P a_j$, $a_j P a_i$ or $a_i J a_j$. Should one of these relations hold, then at least one other differentiating criterion would have been forgotten and would thus ought to be added to the set of considered criteria.
- **cohesion**: let us assume that a_i and a_j are indifferent ($a_i I a_j$). Weakening a_i and reinforcing a_j on one criterion (different or the same) lead to $a_i (P \cup I) a_j$. This condition ensures some coherence between the criteria and the global preferences.
- **non redundancy**: the family of criteria $F = \{f_1, f_2, \ldots, f_q\}$ does not contain any redundant criteria in the sense that the family obtained by removing any single criterion f_j from F would violate at least one of the two previous conditions.

These three properties put together allows to define a **consistent family of criteria**. We refer the interested reader to [5,26,29,37] for formal definitions.

2.5 The Different Multicriteria Problematics and Methods

Now that the basic multicriteria terminology and notions have been introduced, we are ready to define a **multicriteria decision problem** is.

Definition 8. *[45] A multicriteria decision problem can be defined as a situation where given a set of actions $\mathcal{A}$ and a consistent family of criteria F over $\mathcal{A}$, we want to solve one of the following problems:*

- *determine a subset of actions considered as the best considering F (choice problem),*
- *partition $\mathcal{A}$ in subsets with respect to pre-established norms (sorting problem), or*
- *rank order the set of actions $\mathcal{A}$ from best to worst (ranking problem).*

Of course, many real problems involve a mixture of these three main issues. Moreover, additional considerations may be cited:

- **The description problem**: helps to describe actions and their consequences in a formalized and systematic manner to develop a cognitive procedure [37].
- **Choosing k among m actions** [2]: this problematic can be viewed as a mixed of choice and ranking problematics.
- **The design problem**: to search for, identify or create new decision alternatives to meet the goals and aspirations revealed through the MCDA process [6].
- **The porfolio problem**: to choose a subset of alternatives from a larger set of possibilities, taking into account not only the characteristics of the individual alternatives, but also the manner in which they interact (their positive and negatives synergies [6]).
- **The clustering problem**: to define homogeneous groups of alternatives with respect to the preferences of the decision maker. These can be ordered (see for instance [21]) or nominal (see for instance [19]).

Of course, the different problematics allow to clearly identify the final goal of the decision. Obviously, in the Portfolio Management Problem, we are facing a choice problematic since we are looking to select a given portfolio from the Pareto Optimal frontier (α problematic). By definition, in the Criminality Assessment Problem, we are trying to rank the different crime types (β problematic). In the Diagnosis of Alzheimer's Disease, we are sorting a given patient into one of the following categories: healthy, mild, medium, severe (γ problematic).

Different methods have been developed in order to address these problematics. Without being exhaustive, we can distinguish three main families [45]:

- **Interactive methods:** these techniques are based on strong interactions with the Decision Maker. After a first computation step, an initial solution is proposed. If the current solution is not satisfying, the DM reacts by providing additional information about his preferences (for instance; *"I would*

like to improve the value of the current solution on a specific criterion and, therefore, I do accept to deteriorate it on other criteria"). This information is integrated in the optimization model and a new solution is computed. The process is repeated until it converges towards a satisying solution (see [28]);
- **Multiple attribute utility theory:** these methods rely on the assumption that all criteria can be aggregated into a single function that has to be optimized. Therefore, the multicriteria problem is transformed into a single optimization problem (see for instance UTA [41], AHP [39], MACBETH [3], etc.);
- **Outranking methods:** these approaches are based on the construction and the exploitation of an outranking relation [45]:

  Definition 9. *An outranking relation is a binary relation S defined in $\mathcal{A}$ such that aSb if, given what is known about the decision-maker's preferences and given the quality of the evaluation of the actions and the nature of the problem, there are enough arguments to decide that a is at least as good as b, while there is no essential reason to refute the statement (Bernard Roy).*

  Major families of outranking methods are ELECTRE [24] and PROMETHEE [17].

3 The PROMETHEE and GAIA Methods

3.1 PROMETHEE

PROMETHEE [6] I (partial ranking) and PROMETHEE II (complete ranking) were developed by J.P. Brans and presented for the first time in 1982 at a conference organized by R. Nadeau and M. Landry at the Université Laval, Québec, Canada (L'Ingénierie de la Décision. Elaboration d'Instruments d'Aide à la Décision). Since this seminal work, a lot of developments [11,13,16,17] have been proposed including visual representations [33], tools for robustness and sensitivity analysis [14,32], an extension to address the portfolio problematic, called PROMETHEE V, etc. More recently, a literature review [4] listed more than 200 PROMETHEE-based papers published in 100 different journals. The application fields cover finance, health care, logistics and transportation, hydrology and water management, manufacturing and assembly, etc.

The PROMETHEE methods are based on pairwise comparisons. When comparing two actions a_i and a_j on criterion f_k, the difference of evaluations between these two actions should be taken into account. Assuming that criterion f_k has to be minimized, this difference can be stated as follows,

$$d_k(a_i, a_j) = f_k(a_j) - f_k(a_i)$$

[6] PROMETHEE is the acronym of Preference Ranking Organisation METHod for Enrichment Evaluations.

When the difference $d_k(a_i, a_j)$ is small and the DM can neglect it, there is no reason to say that a_i is preferred to a_j and consequently the actions are indifferent (for the specific criterion f_k). The higher the value of d_k, the larger the preference $P_k(a_i, a_j)$ in favor of a_i over a_j, on criterion f_k. This preference can be defined through a *preference function* in the following way,

$$P_k(a_i, a_j) = H_k(d_k(a_i, a_j)), \ \forall \, a_i, a_j \ \in \ \mathcal{A}$$

and we can assume that $P_k(a_i, a_j) \in [0, 1]$ (if $P_k(a_i, a_j) > 0$, then $P_k(a_j, a_i) = 0$).

The pair $(f_k, P_k(a_i, a_j))$ is called a *generalized criterion* associated with criterion f_k, for all $k \in \{1, \ldots, q\}$. Generally, 6 types of generalized criteria are considered. Generalized criterion of type 5 requires the definition of both q_k and p_k (see Fig. 2). The value p_k is called the preference threshold and is defined as the smallest difference on criterion f_k between actions for which the decision maker can say without a doubt that he prefers the better one. Similarly, q_k is called the indifference threshold and is defined as the biggest difference on criterion f_k for which the decision maker can say without a doubt that he is indifferent between the two actions.

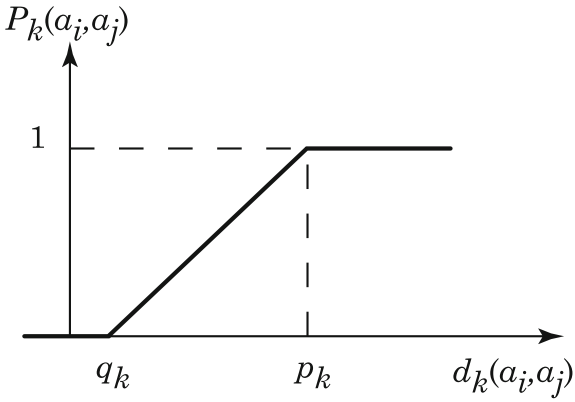

Fig. 2. Generalized criterion of type 5

Once the preference degrees between two actions a_i and a_j have been computed for every criterion, one needs to aggregate this marginal contribution to obtain $P(a_i, a_j)$ i.e. a global measure of the preference of a_i over a_j. This is performed using a classical weighted sum (ω_k is assumed to be the weight associated to criterion f_k):

$$P(a_i, a_j) = \sum_{k=1}^{q} \omega_k \cdot P_k(a_i, a_j)$$

$P(a_i, a_j)$ represents the valued preference of a_i over a_j (and not a binary preference as introduced in Section 2.3). Obviously, we have

$$P(a_i, a_j) \geq 0$$

and

$$P(a_i, a_j) + P(a_j, a_i) \leq 1.$$

The fundamental idea underlying the PROMETHEE methods is the quantification of how an action a *outranks* all the remaining $(n-1)$ actions and how a is *outranked* by the other $(n-1)$ actions. This idea leads to the definition of the *positive* $\phi^+(a)$ and *negative* $\phi^-(a)$ *outranking flows*. More formally:

$$\phi^+(a_i) = \frac{1}{n-1} \sum_{a_j \in \mathcal{A}, i \neq j} P(a_i, a_j)$$

$$\phi^-(a_i) = \frac{1}{n-1} \sum_{a_j \in \mathcal{A}, i \neq j} P(a_j, a_i)$$

Given these two measures, two total pre-orders[7] of $\mathcal{A}$ can be obtained (one associated to the values of ϕ^+ and another associated to the values of ϕ^-). The intersection of these two pre-orders leads to a partial pre-order called the PROMETHEE I ranking. In this context, two actions a_i and a_j will be judged to be incomparable if $\phi^+(a_i) > \phi^+(a_j)$ and $\phi^-(a_i) > \phi^-(a_j)$ or if $\phi^+(a_i) < \phi^+(a_j)$ and $\phi^-(a_i) < \phi^-(a_j)$.

On the other hand, the complete pre-order obtained with the PROMETHEE II method is based on the net flow $\phi(a_i)$ assigned to each action $a_i \in \mathcal{A}$.

$$\phi(a_i) = \phi^+(a_i) - \phi^-(a_j)$$

Let us note that,

$$\phi(a_i) = \frac{1}{n-1} \sum_{k=1}^{q} \sum_{a_j \in \mathcal{A}} \{P_k(a_i, a_j) - P_k(a_j, a_i)\} \cdot \omega_k = \sum_{k=1}^{q} \phi_k(a_i) \cdot \omega_k$$

where $\phi_k(a_i)$ is called the k^{th} unicriterion net flow assigned to action a_i. Intuitively, these values allow to better position action a_i, according to criterion f_k, with respect to all the other actions in $\mathcal{A}$.

In addition to these rankings, Mareschal and Brans [33] have proposed a geometrical tool that helps the decision maker both to interactively explore and structure the decision problem, and to better understand the results provided by the PROMETHEE rankings. This is referred to as the GAIA plane. The underlying idea of this approach is to perform a principal components analysis on the unicriterion net flows assigned to each action [16]. This will be further developed in the next section.

---

[7] A pre-order is a binary relation that is both transitive and reflexive.

3.2 GAIA and Its Interpretation

When we take our set of alternatives into account, it is often difficult to get a visual representation of it because of the numerous criteria that we try to keep in mind. Indeed, if we think of a multidimensional space defined by taking each of those criteria into account, the alternatives could be represented as points that each have specific coordinates depending on their evaluations. Such a space is represented in Fig. 3 with a set of actions positioned with respect to five criteria. Of course, it is impossible to represent a five dimensional space on paper and the representation in Fig. 3 is merely a projection of the actual space on a two dimensional plane. Furthermore, if the view point of such a projection is poorly chosen, the representation will rarely teach us anything useful.

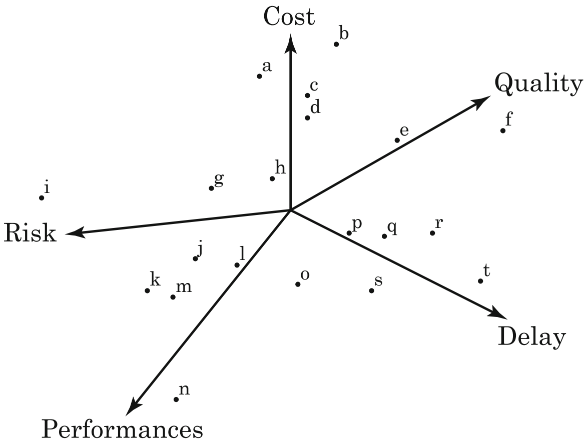

Fig. 3. Criteria space

The aim of GAIA[8] will be to find the best view point for this multidimensional space in order to extract as much information as possible from the representation. In order to do that we are going to resort to a Principal Component Analysis (PCA) applied on the unicriterion net flows $\phi_k(a_i)$ computed by PROMETHEE. This will ensure that the actions we represent will be defined as they are seen by the decision maker. Indeed, the unicriterion net flows contain information on how the decision maker perceives the different criteria and compares the actions pair by pair.

Let us consider a matrix Φ that contains all the unicriterion net flows for our problem. We have:

$$\Phi = (\phi_k(a_i)) \qquad \forall a_i \in \mathcal{A}; k \in \{1, 2, ...q\}$$

We begin by calculating the variance-covariance matrix C of our problem.

$$nC = \Phi'\Phi$$

[8] GAIA is the acronym of Graphical Analysis for Interactive Assistance.

We then compute the eigenvectors and eigenvalues of matrix C. All of these eigenvectors are orthogonal because of the properties of matrix C and they each indicate a direction towards which we have a certain dispersion of the alternatives' positioning. That dispersion is given to us by the respective eigenvalues of each vector.

Finally, we select the two eigenvectors u and v with the highest associated eigenvalues λ_1 and λ_2 and use those to define a two-dimensional plane in the criteria space. This plane will be the canvas that will be used to represent the decision problem and all of its defined elements (i.e. the actions, the criteria, the direction of the best compromise).

Since we have selected the vectors with the highest eigenvalues to define our plane, it means the plane will capture the maximum dispersion of the alternatives in two dimensions. It is also possible to evaluate the amount of information kept that way. That amount is called the delta value of the plane and is denoted δ:

$$\delta = \frac{\lambda_1 + \lambda_2}{\sum_{k=1}^{q} \lambda_k}$$

Once the plane for the projection has been defined, we project the actions defined by their coordinates (i.e. the unicriterion net flows) on it. The actions' coordinates in the criteria space can be written as:

$$\alpha_i : (\phi_1(a_i), \phi_2(a_i), ..., \phi_k(a_i), ..., \phi_q(a_i)), \forall a_i \in \mathcal{A}$$

The projection of the actions can thus be found as follows:

$$\begin{cases} |Op_i| = \alpha_i' u \\ |Oq_i| = \alpha_i' v. \end{cases}$$

We then add the projection of the axes e_k representing each criterion on the plane:

$$e_k : (0, 0, \ldots, 1, 0, \ldots, 0) \qquad k \in \{1, 2, ..., q\}.$$

Those projections are denoted c_k.

Finally, to give an idea of which actions are closest to the best compromise, we add the projection of the weights vector.

$$w : (w_1, w_2, \ldots, w_k, \ldots, w_q).$$

That projection is often referred to as the decision stick and is computed as follows:

$$\pi = \sum_{k=1}^{q} c_k \cdot \overline{w_k},$$

where $\overline{w_k}$ is the k-th coordinate of the normalised vector corresponding to w i.e. $\overline{w} = w/|w|$.

When all of the components have been added, we are able to display a projection similar to the one on Fig. 4. All of these elements and their relative positions can now be interpreted.

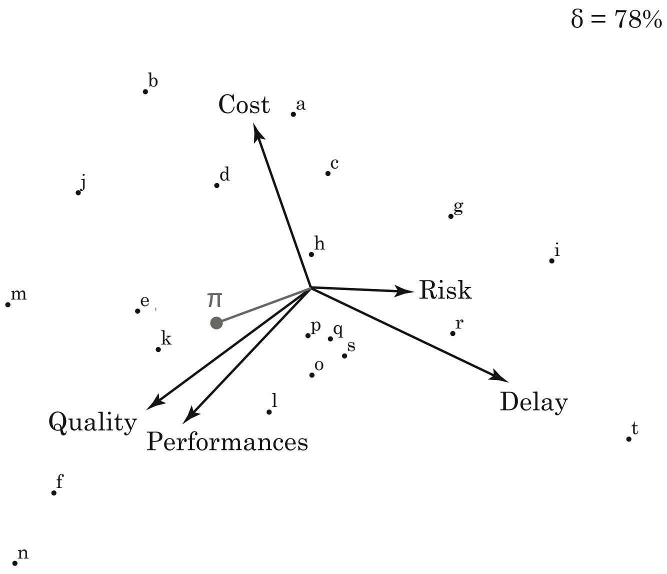

Fig. 4. GAIA plane

– **Positions of the criteria:** The orientation of the axes will indicate which criteria are compatible and which ones are in conflict. We can see for example that in the case of Fig. 4 the axes for "Quality" and "Performances" are very close to each other and therefore compatible. That means that we can easily find alternatives that excel in both quality and performance simultaneously, or, on the contrary, that some alternatives have bad evaluations on both of these criteria. Also, we can notice that "Cost" and "Delay" are in conflict because they are pointing towards very different directions. The same can be said for "Quality" and "Risk". This means that it is very difficult to find an action that presents good scores on both criteria for each pair. Usually, when an action is good on one of those criteria, it is bad on the other.

  Furthermore, the size of the criteria axes will point out the discriminant criteria within the problem. Indeed, since the plane has been chosen to capture the maximum variation of the actions, the criteria that do not present a high enough variation of the evaluations will likely end up being orthogonal to the plane. In this last case we can see that all the criteria axes have a relatively good size with the exception of "Risk". We can therefore say that the risk measured on the actions for this problem does not differentiate them as well as the other criteria do.
– **Position of the decision stick π:** In this multivariate view, the indication of an objective is of high importance. It will indicate us the importance that the decision maker has given to each criterion. In the representation on Fig. 4, the decision stick points slightly more towards "Quality" and

"Performances" than the other criteria. The weight associated to those two criteria must therefore be bigger than for criteria such as "Risk" or "Delay". Of course, changing the weights i.e. the relative importances of the criteria, will change the direction of the decision stick and make it point in a different direction.

- **Relative positions of the alternatives:** Groups of alternatives on the plane will represent solutions with similar profiles. Actions o, p, q, and s seem to have the same characteristics. But also, alternatives b, n, and t have very different profiles that ultimately give them projections very far from each other.
- **Positions of the alternatives (according to the criteria):** The location of an alternative on the plane will give us an indication on the type of profile it has. It will point out the strongest and weakest features of a solution. By taking a look at the actions in the direction of each criterion, we can see that actions n and f have strong evaluation in quality and performances but low ones on risk. Action t has a good evaluation on delay and a fairly good one regarding risk. Actions a, b, c, and d are oriented towards costs and behave poorly in terms of delay. Actions m and j are good on cost, quality, and performance, but bad on delay and risk.
- **Positions of the alternatives (according to the decision stick):** When projected on the decision stick, the alternatives take their positions from the PROMETHEE II ranking. Even though the ranking inferred from a projection could present differences due to loss of data, it still is an interesting use of the tool when more precise information is not available. In our example, the inferred ranking we obtain would be, from best to worst: n, f, m, e, k, j, b, l, d, o, p, q, s, h, a, c, r, g, i, t.
- **Delta value δ:** These results would not be complete without an indication on their reliability. The delta value i.e. the amount of information preserved by the plane, will give us a confidence level for the results and will have to be indicated alongside them. In most software implementations, the delta value is therefore given in one of the corners of the plane as a percentage. In the given representation a value of 78% means that 22% of the variation of the actions is lost and not represented on the plane. We can thus say that the two dimensions that were chosen for this projection due to their associated eigenvalues successfully represent 78% of the information from the five criteria in this problem.

The results we extract from the GAIA plane are, of course, an approximation of the reality. Because of the loss of data due to the projection on the plane, some of the actions might not be well represented in two dimensions. For example, alternatives that seem close on the plane, might actually be apart from each other but have projections that are close. Every time we use the GAIA plane to draw conclusions, we will need to pay attention to the delta value and compare the inferred ranking to the complete ranking from PROMETHEE II.

3.3 A Pedagogical Example with D-Sight

During the recent years, we have witnessed the development of indexes allowing to evaluate countries, cities, universities, companies, etc. Among them, we may cite the Human Development Index (HDI), the Environmental Performance Index (EPI), the Global Peace Index (GPI), the Academic Ranking of World Universities (ARWU), the European Cities Monitor, etc. These evaluations are, of course, typical multicriteria decision aid problems that are, most of the time, solved by using a classical weighted sum (after a first normalization step). In the end, all the alternatives are characterized by a global score allowing to rank them from the best to the worst one. These results are often easily available on the web.

In what follows, we do not claim that computing a net flow score (like in the PROMETHEE II ranking) instead of a weighted value is more appropriate. We let this methodological question to further investigations. However, we assert that *"solving"* a multicriteria decision aid problem cannot be limited to the computation of a global score in order to rank the alternatives. In what follows, we will illustrate different steps that could lead to a better understanding of the problem and to more robust conclusions. In order to illustrate this, we will consider the ten first ranked universities of the ARWU in the field of computer sciences (see table 2): Stanford University (SU), Massachusetts Institute of Technology (MIT), University of California Berkeley (UCB), Princeton University (PU), Carnegie Mellon University (CMU), Cornell University (CU), University of Southern California (USC), The University of Texas at Austin (UTA), Harvard University (HU) and University of Toronto (UT). The universities are evaluated according to 5 criteria [1] (their relative importance is given between the parentheses):

- Alumni (10%): number of alumni from the institution winning Turing Awards in Computer Science since 1951;
- Awards(15%): staff of an institution winning Turing Awards in Computer Science since 1961;
- HiCi (25%): highly cited researchers in Computer Science category;
- PUB (25%): papers Indexes in Science Citation Index-Expanded in Computer Science;
- TOP (25%): percentage of papers published in the top 20% journals on the field of Computer Science compared to the papers published in all journals of that subject field;

We refer the interested reader to [1] for a detailed description of these criteria and their computation. Of course, we are aware of the fact that such kind of rankings are subject to criticisms. However, this debate exceeds the illustrative purpose of this section.

We propose to use the D-Sight software [25], that implements the PROMETHEE and GAIA methods, in order to analyze the problem. For the sake of simplicity, we have decided to use linear preference functions (with an indifference threshold equal to 0 and a preference threshold equal to 100) for all the

Table 2. Evaluations of the 10 first Universities listed in the ARWU in Computer Sciences for 2010

Name	ARWU score	Alumni	Awards	HiCi	PUB	TOP
SU	100	90,7	86,6	100	80,9	97,9
MIT	94,8	54,2	100	89,2	87,8	89,3
UCB	82,7	100	96,8	42,9	76,7	86,1
PU	78,7	68,6	71,8	60,6	63	94,7
CMU	76,4	42	79,1	55,3	85,4	75,4
CU	67,9	42	57,3	55,3	57,3	85,5
USC	66,6	0	39,5	65,5	68,4	86,8
UTA	66,3	42	39,5	55,3	70,4	77,2
HU	65,6	97	0	42,9	65,5	93,7
UT	65,5	24,3	53	49,5	71,1	78,3

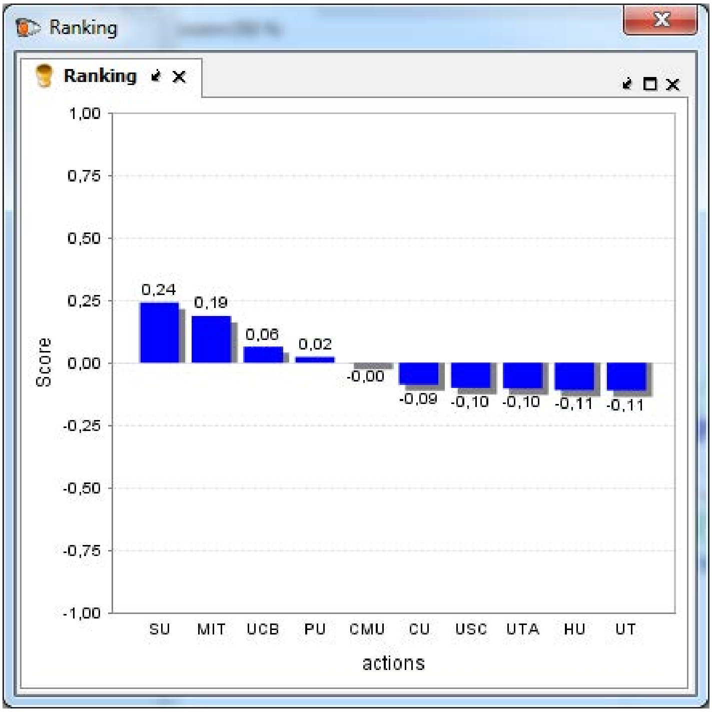

Fig. 5. Promethee II ranking

criteria (one more time, the aim of this section is to demonstrate the usefulness of visual and interactive tools in MCDA rather than to justify modelling choices). Additionally, this parametrization leads to the same ranking as the one induced by the ARWU score (see Fig. 5). As already stressed, in most cases, the analysis is stopped at this level i.e. the ranking of the alternatives according to

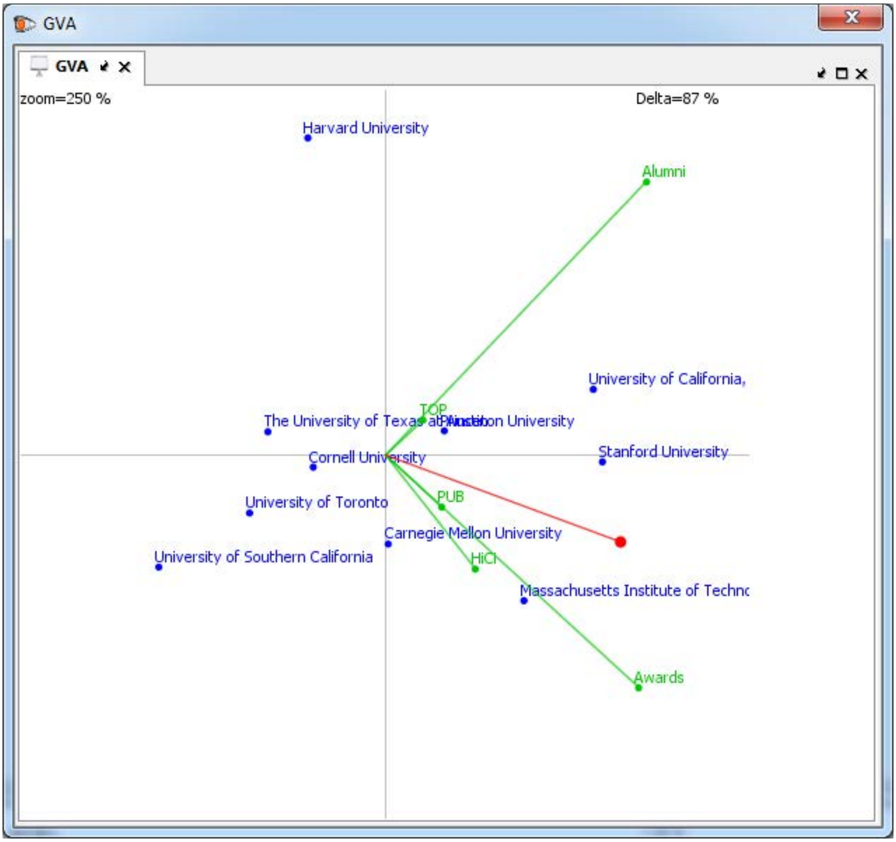

Fig. 6. GAIA plane

their scores. Let us now investigate how a software like D-Sight can help us to deepen our understanding of the problem.

A look on the GAIA plane (see Fig. 6) already helps us drawing some interesting conclusions:

- **Delta value:** the delta value is equal to 87%, which is a rather important value; the information loss due to the projections seems to be limited;
- **Relative positions of the criteria:** two groups of criteria can be identified: $\{PUB, HiCI, Awards\}$ and $\{Alumni, TOP\}$. These two sets seem to be independent from each other. In other words, there are no strong conflicts between the criteria;
- **Relative postions of the alternatives:** clearly the Harvard University is distinguishing itself from the cloud of other universities. Additionally, one may observe a similar effect for the group constituted by the University of California and Stanford University: these two institutions seem to

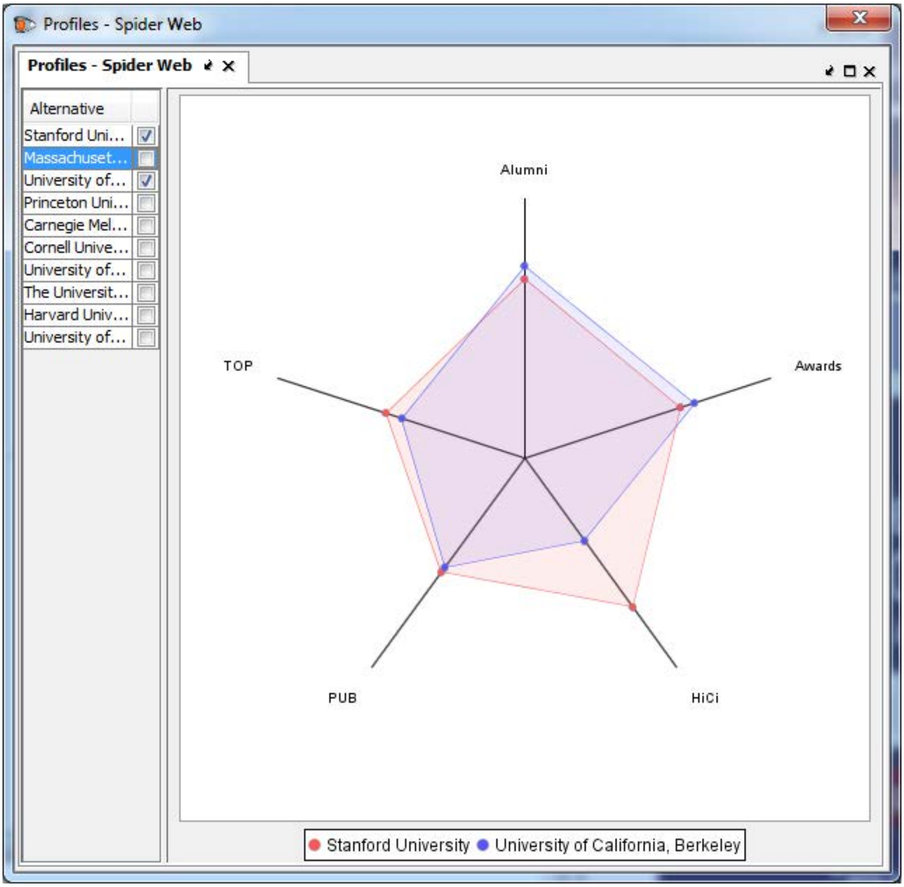

Fig. 7. Spider chart between SU and UCB

have close profiles. This is indeed confirmed by the spider chart shown on Fig. 7;

- **Relative position of the alternatives with respect to the criteria:** the Harvard University has a very particular evaluation; it is very good regarding Alumni and TOP and bad or average for the other criteria. Clearly, the University of California and the Stanford University have average good scores for both families of criteria. The Massachusetts Institute of Technology, which is ranked at the second position in the PROMETHEE II ranking, is very good on all criteria but has an average score on the Alumni criterion;
- **Decision Stick:** the projection of the different alternatives on the decision stick allows to find the total ranking (especially for the first ranked alternatives, see Fig. 8).

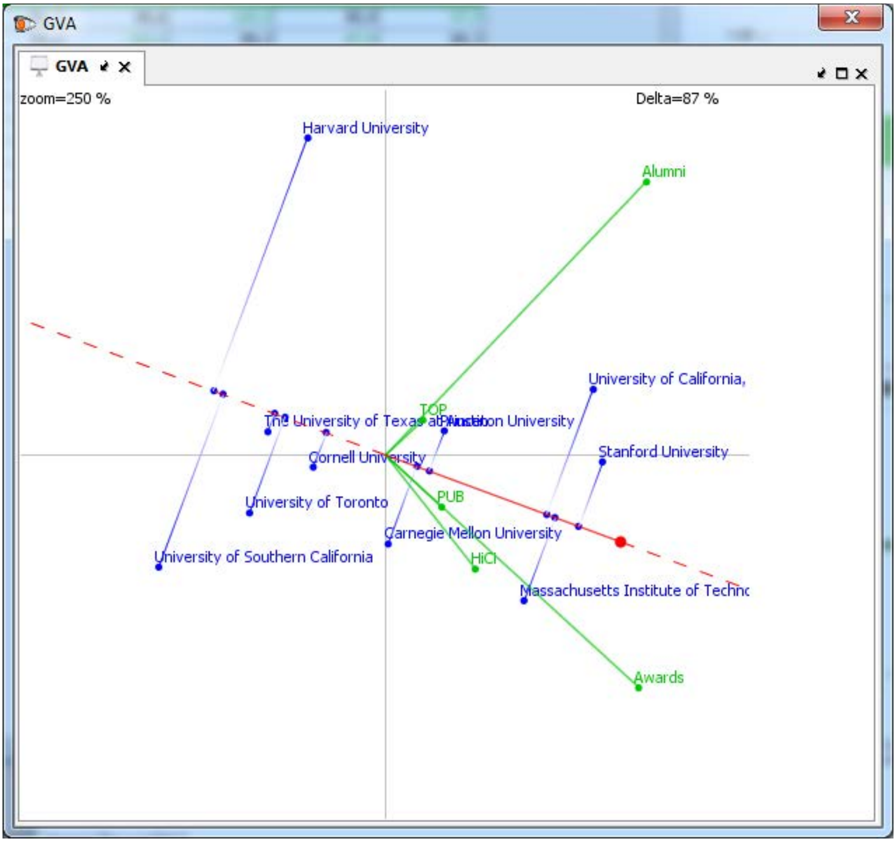

Fig. 8. Projection on the Decision Stick

As already stressed, a number of authors have criticized the legitimacy of such kind of rankings. Among others, the weight values can be discussed. If we slightly change the relative importance of a given criterion, would we get a totally different ranking ? An interactive tool called *walking weights* allows the decision maker to perform a sensitivity analysis directly on the results while changing the weight values. For instance, multiplying the relative importance of the Alumni criterion by three (while the relative importance of other criteria remains the same) does not have an impact on the first ranked alternative (see Fig. 9). Finally, one could also address this question in a different way: *For every criterion, what are the interval values that will not affect the first or the two first ranked alternatives (under the assumption that the relative importance of other criteria remain the same)?* Fig. 10 shows these values when we want to hold the top ranking constituted by the two first alternatives. Clearly, we may observe that the interval values are rather large. More particularly, we may notice that even important modifications of the weight values of HiCi, PUB and TOP will not affect the top of the ranking. This proves that the selection of the two first alternatives seems to be rather robust.

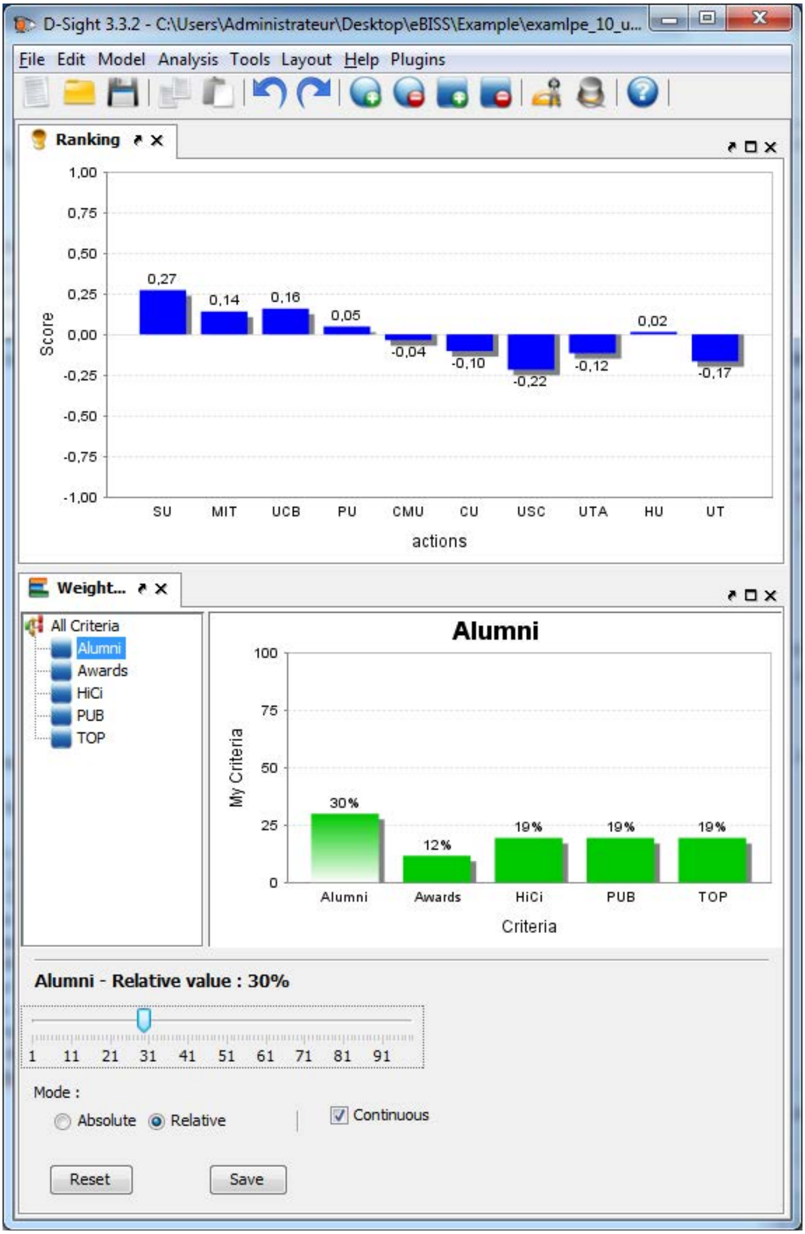

Fig. 9. Walking Weights

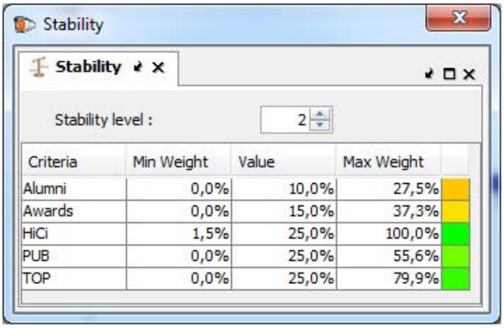

Fig. 10. Stability Intervals

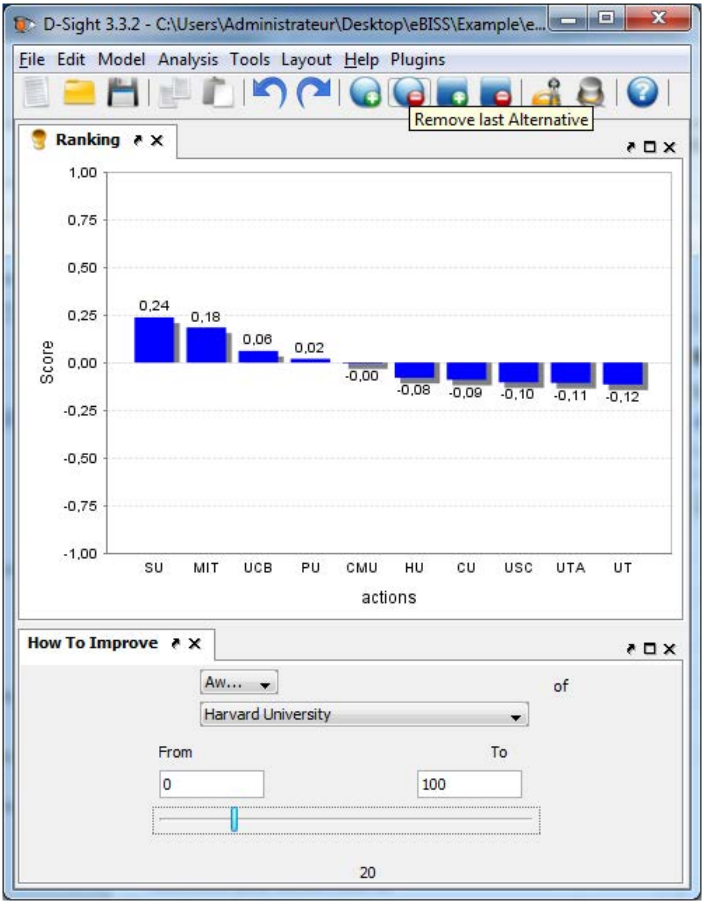

Fig. 11. How to improve?

Finally, a last strategical question could be: *How does the Harvard University need to improve itself on a given criterion in order to gain some positions?* Fig. 11 shows that increasing the score of the criterion Awards to 20 (which remains a relatively low value with respect to the evaluation table) allows Harvard to pass from the 9^{th} position to the 6^{th}.

This section has shown that the success of a multicriteria analysis heavily depends on the availability of user-friendly software. D-Sight is the third generation of PROMETHEE-based software (following PROMCALC [13] and Decision Lab 2000). We may not conclude this section without citing other multicriteria software such as Expert Choice (for the AHP method), Electre IS (for a generalization of the ELECTRE I method), M-Macbeth (as expected for the Macbeth method), etc. Another interesting initiative that has to be mentioned is the Decision Deck project, which is an open source source software that is collaboratively developed and which implements various methods. The reader is also referred to [46] for a review on MCDA software.

4 Conclusion

Multicriteria decision aid is an exciting research field. The only ambition of this chapter was to introduce the basics of this domain. We refer the interested reader to [23] for a recent and complete state of the art of MCDA. Additionally, interesting resources can be found on the websites of the EURO working group on MCDA or of the MCDM international society.

As a demonstration of the growing interest in MCDA, we may point out that it has been applied to other research areas such as Artificial Intelligence [22], Geographic Information Systems [31], Classification and Pattern Recognition [19,21], System Dynamics [15], Group Decision and Negotiation [27], Scheduling [42], etc.

References

1. Academic Ranking of World Universities, `http://www.arwu.org`
2. Bana e Costa, C.A.: Convictions et Aide à la Décision. Newsletter of the Euro Working Group Multicriteria Decision Aiding 2 (1993)
3. Bana e Costa, C.A., De Corte, J.M., Vansnick, J.C.: On the Mathematical Foundations of MACBETH. In: Figueira, J., Greco, S., Ehrgott, M. (eds.) Multiple Criteria Decision Analysis: State of the Art Surveys, pp. 133–162. Springer, Boston (2005)
4. Behzadian, M., Kazemzadeh, R.K., Albadvi, A., Aghdasi, M.: PROMETHEE: A comprehensive literature review on methodologies and applications. European Journal of Operational Research 100(1), 198–215 (2010)
5. Belton, V., Ackerman, F., Shepherd, I.: Integrated support from problem structuring through to alternative evaluation using COPE and VISA. Journal of Multi-Criteria Decision Analysis 7, 115–130 (1997)
6. Belton, V., Steward, T.: Multiple Criteria Decision Analysis: An Integrated Approach. Kluwer Academic Publisher, Boston (2002)

7. Billaut, J.C., Bouyssou, D., Vincke, P.: Should you believe in the Shanghai ranking? Scientometrics 84(1), 237–263 (2010)
8. Bouyssou, D.: Décision Multicritère ou Aide Multicritère? Newsletter of the Euro Working Group Multicriteria Decision Aiding 3 (1993)
9. Bouyssou, D., Vincke, P.: Relations binaires et modélisation des préférences. Technical report SMG, IS-MG 2003/02 (2003)
10. Bouyssou, D., Marchant, T., Pirlot, M., Tsoukias, A., Vincke, V.: Evaluation and decision models with multiple criteria: stepping stones for the analyst. International Series in Operations Research and Management Science, vol. 86. Springer, Boston (2006)
11. Brans, J.P., Vincke, P.: A preference ranking organization method. Management Science 31(6), 647–656 (1985)
12. Brans, J.P., Mareschal, B.: PROMETHEE V: MCDM Problems with Segmentation Constraints. INFOR 30(2), 85–96 (1992)
13. Brans, J.P., Mareschal, B.: PROMCALC and GAIA: A new decision support system for Multicriteria Decision Aid. Decision Support Systems 12, 297–310 (1994)
14. Brans, J.P.: The space of freedom of the decision maker modelling the human brain. European Journal of Operational Research 92(3), 593–602 (1996)
15. Brans, J.P., Macharis, C., Kunsch, P.L., Chevalier, A., Schwaninger, M.: Combining multicriteria decision aid and system dynamics for the control of socio-economic processes. An iterative real-time procedure. European Journal of Operational Research 109(2), 428–441 (1998)
16. Brans, J.P., Mareschal, B.: Prométhée - GAIA: une méthode d'aide à la décision en présence de critères multiples. Statistiques et Mathématiques Appliquées. Ellipses, Editions de l'Université Libre de Bruxelles, Paris (2002)
17. Brans, J.P., Mareschal, B.: PROMETHEE Methods. In: Figueira, J., Greco, S., Ehrgott, M. (eds.) Multiple Criteria Decision Analysis: State of the Art Surveys, pp. 163–196. Springer, Boston (2005)
18. De Smet, Y., Vincke, P.: L'aide à la décision, IT-Scan 7 (2002)
19. De Smet, Y., Montano Guzman, L.: Towards multicriteria clustering: An extension of the k-means algorithm. European Journal of Operational Research 158(2), 390–398 (2004)
20. Dyer, J.S.: MAUT - Multiattribute Utility and Value Theories. In: Figueira, J., Greco, S., Ehrgott, M. (eds.) Multiple Criteria Decision Analysis: State of the Art Surveys, pp. 133–162. Springer, Boston (2005)
21. De Smet, Y., Nemery, P., Selvaraj, R.: An exact algorithm for the multicriteria ordered clustering problem. Omega 40(6), 861–896 (2006)
22. Doumpos, M., Grigoroudis, E. (eds.): Multicriteria decision aid and artificial intelligence: Theory and applications. John Wiley and Sons (to appear)
23. Figueira, J., Greco, S., Ehrgott, M.: Multiple Criteria Decision Analysis: State of the Art Survey. Springer, Boston (2005)
24. Figueira, J., Mousseau, V., Roy, B.: ELECTRE methods. In: Figueira, J., Greco, S., Ehrgott, M. (eds.) Multiple Criteria Decision Analysis: State of the Art Surveys, pp. 133–162. Springer, Boston (2005)
25. Hayez, Q., De Smet, Y., Bonney, J.: D-SIGHT: a new decision making software to address multi-criteria problems. International Journal of Decision Support Systems Technologies 4(4) (2012)
26. Keeney, R.L.: Value-Focused Thinking. Harvard University Press, Cambridge (1992)

27. Kilgour, D.M., Chen, Y., Hipel, K.W.: Multiple Criteria Approaches to Group Decision and Negotiation. In: Ehrgott, M., Figueira, J., Greco, S. (eds.) Trends in Multiple Criteria Decision Analysis. International Series in Operations Research and Management Science, vol. 142, pp. 317–338. Springer (2010)
28. Korhonen, P.: Interactive methods. In: Figueira, J., Greco, S., Ehrgott, M. (eds.) Multiple Criteria Decision Analysis: State of the Art Surveys, pp. 133–162. Springer, Boston (2005)
29. Neves, L.P., Dias, L.C., Antunes, C.H., Martins, A.G.: Structuring a MCDA model using SSM: A case study in Energy Efficiency. European Journal of Operational Research 199(3), 834–845 (2009)
30. Luce, R.D.: Semiorders and a theory of utility discrimination. Econometrica 24, 178–191 (1956)
31. Malczewski, J., Rinner, C.: Multicriteria Decision Analysis in Geographic Information Science. Springer (to appear)
32. Mareschal, B.: Weight stability intervals in multicriteria decision aid. European Journal of Operational Research 33(1), 54–64 (1988)
33. Mareschal, B., Brans, J.P.: Geometrical representations for MCDA: the GAIA module. European Journal of Operational Research 34, 69–77 (1988)
34. Markowitz, H.M.: Portfolio Selection. The Journal of Finance 7(1), 77–91 (1952)
35. Roy, B.: Il faut désoptimiser la Recherche Opérationelle. Bulletin de l'AFIRO 7 (1968)
36. Roy, B.: Decision-aid and decision-making. European Journal of Operational Research 45, 324–331 (1990)
37. Roy, B.: Multicriteria Methodology for Decision Aiding. Kluwer Academic Publishers, Dordrecht (1996)
38. Roy, B.: A French-English Decision Aiding glossary. Newsletter of the Euro Working Group Multicriteria Decision Aiding 3 (2000)
39. Saaty, T.L.: The Analytic Hierarchy and Analytic Network Processes for the Measurement of Intangible Criteria and for Decision-Making. In: Figueira, J., Greco, S., Ehrgott, M. (eds.) Multiple Criteria Decision Analysis: State of the Art Surveys, pp. 133–162. Springer, Boston (2005)
40. Schärlig, A.: Décider sur plusieurs cirtères: Panorama de l'aide à la décision multicritère. Presses Polytechniques Universitaires Romandes, Lausanne (1985)
41. Siskos, Y., Grigoroudis, E., Matsatsinis, N.: UTA methods. In: Figueira, J., Greco, S., Ehrgott, M. (eds.) Multiple Criteria Decision Analysis: State of the Art Surveys, pp. 133–162. Springer, Boston (2005)
42. T'kindt, V., Billaut, C.: Multicriteria scheduling: Theory, Models and Algorithms. Springer (2006)
43. Vansnick, J.C.: L'aide multicritère à la Décision: une Activité Profondément Ancrée Dans Son Temps. Newsletter of the Euro Working Group Multicriteria Decision Aiding 6 (1995)
44. Vincke, P.: L'aide multicritère à la décision, Statistiques et Mathématiques Appliquées, Ellipses, Editions de l'université Libre de Bruxelles, Paris (1989)
45. Vincke, P.: Multicriteria Decision-Aid. John Wiley and Sons, New York (1992)
46. Weistroffer, H.R., Smith, C.H., Narula, S.C.: Multiple Criteria Decision Support Software. In: Figueira, J., Greco, S., Ehrgott, M. (eds.) Multiple Criteria Decision Analysis: State of the Art Surveys, pp. 133–162. Springer, Boston (2005)