# Subjectivity and Objectivity of Trust

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**Abstract.** Trust plays an important role in the fields of Distributed Artificial Intelligence (DAI) and Multi-agent Systems (MAS), which provides a more effective way to reduce complexity in condition of increasing social complexity. Although a number of computational issues about trust have been studied, there has to date been little attempt to investigate the differences between Objective Trust (OT) and Subjective Trust (ST). In this paper, we will rectify this omission. Particularly, we study the relationship between OT and ST, and propose Transitive Trust (TT) based on ST. We show that, differing with OT, ST is related to preferences of agents. We propose three rules to form trust framework, and give an example to illustrate the process of trust formation. We finally characterize some useful properties of OT and ST.

**Keywords:** Multi-agent System, Subjective Trust, Objective Trust, Transitive Trust, Recommendation.

## 1 Introduction

Nowadays, both Agent Technology and Data Mining technologies have reached an acceptable level of maturity. A fruitful synergy of the two technologies has already been proposed, that would combine the benefits of both worlds and would offer computer scientists with new tools in their effort to build more sophisticated software systems [1,2,3]. In conditions of increasing social complexity, an agent can make much efforts by data mining to overcome the complexity of choices. Particularly, trust constructs a more effective form than utility theory of complexity reduction [4].

As Teacy and Khosravifar [5,6] defined, an agent's (trustor) trust in another (trustee) is defined as the measure of willingness that trustee will fulfill what he agrees to do and computed by considering personal interaction experiences and collecting suggested ratings from others. We believe that trust has three modes which are Objective Trust (OT), Subjective Trust (ST) and Transitive Trust (TT). We assume that trust only based on personal interaction experiences is OT, and ST means that an agent put some personal preferences on interactions, while trust relation formed without interactions is TT which is recommended from other agents.

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Most of existed researches are based on probability computational models of trust. It is not consistent with the definition of trust as Teacy and Khosravifar announced, as well as lack of strong persuasion. Particularly, ST is a common issue. Because ST is not a transitive relationship due to the subjectivity of trust, trust relay is not a reliable way in our society. Actually, inaccurate information provided by others is more or less due to the subjectivity of trust.

Unfortunately, little efforts were done for considering ST in previous work. Furthermore, no attempts has been done to bridge the gap of OT and ST. A trust network can not be built up because of non-transitivity of ST. We hope to investigate the subjectivity and objectivity of trust thoroughly. Some important properties of trust are proposed in the following sections. We also consider TT, the transitivity of ST, in our society to form trust network.

The first contribution of this paper is the proposition of subjectivity and objectivity of trust. Therefore, preferences and the transitivity of ST are considered systemically. The second contribution is that some useful properties are also given. It may help the future researches in some aspects.

## 2 Basic Notions

We assume that under a cooperative environment several agents, which have individual interaction states, form a society $Ag$. Some of agents interacted with each other inside this society. Therefore, based on objective interaction results and subjective preferences, trust relation will be formed.

We Assume that the environment where agents interacted is stable, and we assume that we discuss only a fixed attribute of agents in this paper and the attribute is invariable, i.e. all of agents interact in a fixed environment and on the same issue all the while.

First of all, let $OT$ and $ST$ translate from multiple value to binary value in this paper to discuss the following properties. Therefore, for $\forall a_i, a_j \in Ag$, either $\langle a_i, a_j \rangle \in OT$ (it means that $a_i$ trusts $a_j$)or $\langle a_i, a_j \rangle \notin OT$ (it means that $a_i$ does not trust $a_j$) is true. i.e. for $\forall a_i, a_j \in Ag$, $OT_i(j) = \{0, 1\} \wedge ST_i(j) = \{0, 1\}$.

In order to take advantage of the data of agent's interactions, we give informational states of individual agents [12].

**Definition 1.** Informational states of individual agents

$$V_i(j) = \langle v_i^1(j), v_i^2(j), \ldots, v_i^{s_i^j}(j) \rangle$$

where, $V_i(j)$ is a vector which saves interaction history between agent $a_i$ and agent $a_j$ ranked by time in a fixed situation.

$s_i^j$ denotes the quantity of interactions between agent $a_i$ and agent $a_j$ in a fixed situation. Clearly $s_j^i = s_i^j$.

$v_i^l(j)$ denotes the result of $l$th interaction between agent $a_i$ and $a_j$ in a fixed situation. $l = 1, 2, \ldots, s_i^j$. Under a cooperative environment, the result of interaction is same to the bilateral agents, i.e. if the interaction is successful, then the value of interaction is 1 for two agents. Otherwise, the value of interaction is 0 for two agents. We can derive that $v_i^l(j) = v_j^l(i)$ under a cooperative environment.

$$v_i^l(j) = \begin{cases} 1 & \text{If the interaction is successful} \\ 0 & \text{Otherwise} \end{cases} \tag{1}$$

where, successful interaction means that agent $a_i$ and agent $a_j$ have achieved their goals. On the other hand, unsuccessful interaction means that agent $a_i$ and agent $a_j$ have lost their goals.

**Definition 2.** Trust Model (TM)

$$TM = \langle Ag, \eta_i(j), OT_i(j), P_i(j), ST_i(j), TT_i(j) \rangle,$$

where, agents $a_1, a_2, ..., a_n$ form a set of $Ag$, $Ag = \{a_1, a_2, ..., a_n\}$.

The method for computing $\eta_i(j)$ is cited from Teacy [5] shown in the followings. $\eta_i(j) \in [0, 1]$ indicates probability of successful interactions between agent $i$ and agent $j$.

$$\eta_i(j) = \begin{cases} \frac{1+\sum_{l=1}^{s_i^j} v_i^l(j)}{2+s_i^j}, & i \neq j \\ 1, & i = j \end{cases} \tag{2}$$

We denote OT as $OT_i(j)$, which means that agent $a_i$ trusts $a_j$ on a fixed attribute in a fixed situation. If $\eta_i(j) \geq c$, then $OT_i(j) = 1$. Otherwise, $OT_i(j) = 0$. $c \in (0.5, 1]$ is a fixed constant which means the threshold of trust for all agents in $Ag$.

$$OT_i(j) = \begin{cases} 1 & \eta_i(j) \geq c \\ 0 & \text{Otherwise} \end{cases} \tag{3}$$

Due to $v_i^l(j) = v_j^l(i)$, $s_i^j = s_j^i$, and $c$ is a constant, so $\eta_i(j) = \eta_j(i)$, and $OT_i(j) = OT_j(i)$.

We denote the preference of agent as $P_i(j)$, which means the preference of agent $i$ on agent $j$. We consider $P_i(j)$ as the threshold of ST based on $\eta_i(j)$.

We denote ST as $ST_i(j)$, which means that agent $a_i$ trusts $a_j$ in a fixed situation.

$$ST_i(j) = \begin{cases} 1 & \eta_i(j) \geq P_i(j) \\ 0 & \text{Otherwise} \end{cases} \tag{4}$$

We denote TT as $TT_i(j)$, which means that agent $a_i$ has no interactions with agent $a_j$, but agent $a_i$ received a recommendation from agent $a_k$ which has interactions with agent $a_j$. We illustrate OT, ST and TT in Fig.1.

$$TT_i(j) = \begin{cases} 1 & \{\eta_k(j) \geq P_i(j)\} \wedge \{ST_i(k) = 1\} \\ 0 & \text{Otherwise} \end{cases} \tag{5}$$

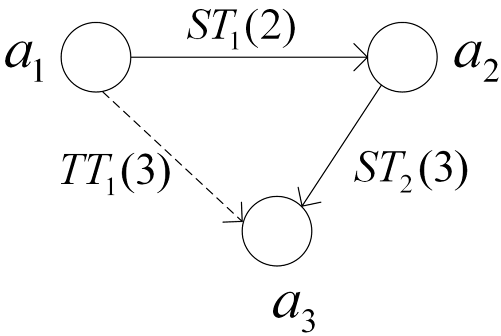


**Fig. 1.** ST and TT

## 3 Formation of OT, ST and TT

In this section we will discuss the process of trust formation and some examples are given to understand trust more thoroughly.

### 3.1 Formation of Trust

**Axiom 1.** $\forall a_i, a_j \in Ag$, If agent $a_i$ trusts $a_j$, then agent $a_i$ will delegate his authority to agent $a_j$. It means that all of interactions did by agent $a_j$ is regarded as agent $a_i$.

In order to discuss TT, we firstly give some rules of trust shown as follows.

**Rule 1. (OT determination rule)** $\forall a_i, a_j \in Ag$, If $s_i^j > 0$, and $\eta_i(j) \geq c$, then $OT_i(j) = OT_j(i) = 1$, otherwise $OT_i(j) = OT_j(i) = 0$.

**Rule 2. (ST determination rule)** $\forall a_i, a_j \in Ag$, If $s_i^j > 0$, and $\eta_i(j) \geq P_i(j)$, then $ST_i(j) = 1$, otherwise $ST_i(j) = 0$.

**Rule 3. (TT determination rule)** $\forall a_i, a_j, a_k \in Ag$, If agent $a_i$ trusts agent $a_j$, then interactions between agent $a_k$ and agent $a_j$ are regarded as interactions between agent $a_i$ and agent $a_k$. The formal description is as follows.

If $s_i^j > 0$, $s_j^k > 0$,$s_i^k = 0$, $ST_i(j) = 1$, and $\eta_j(k) \geq P_i(k)$, then $TT_i(k) = 1$, otherwise $TT_i(k) = 0$.

We consider that agents $a_1, a_2, a_3$ form trust relationship among them. If $\langle a_1, a_2 \rangle \in ST$, then $a_1$ regards that the data of $a_2$ is same to $a_1$. If agent $a_2$ recommends $a_3$ to $a_1$, then agent $\langle a_2, a_3 \rangle \in ST$, which means that agent $a_2$ does trust $a_3$, i.e. $\eta_2(3) \geq P_2(3)$.

However, it does not mean that agent $a_1$ will trust $a_3$. Because the data of $a_2$, integrated with the preference of agent $a_2$, is transferred to $a_1$. When $a_1$ received $ST_2(3)$, it should replace $P_2(3)$ by $P_1(3)$. Agent $a_1$ derives $TT_1(3)$ based on $P_1(3)$. Therefore, according to Rule 3, the key criterion for the determination of transitivity of ST in this example is whether $\eta_2(3) \geq P_1(3)$.

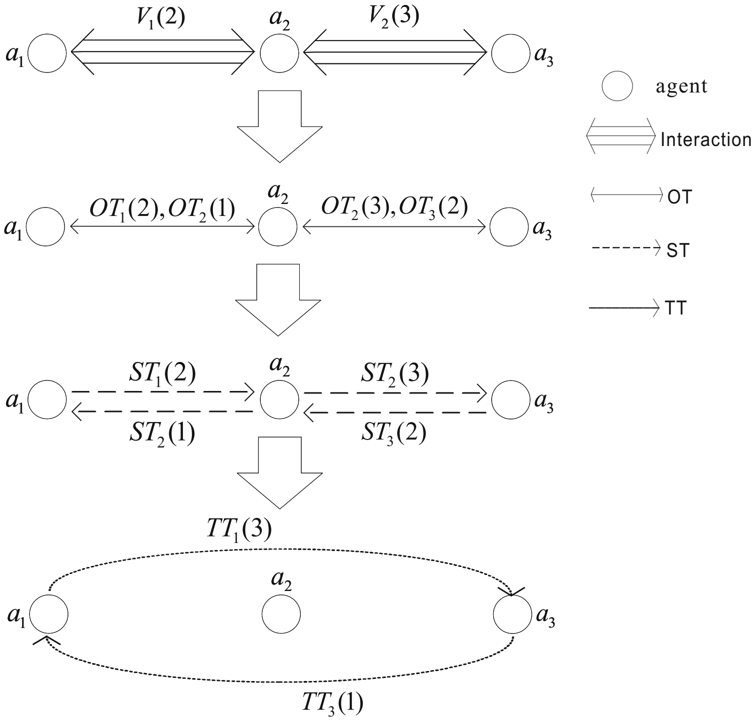


**Fig. 2.** Formation of OT and ST

Fig.2 describes the formation process of a trust relation. Agents firstly obtain OT based on interactions. Sequentially, agents consider preferences to form ST, and at the end agents should determine whether TT exists among them.

### 3.2 An Example

**Example 1.** We discuss trust among buyers and sellers in Amazon for deals of books, and consider that three agents $a_1, a_2, a_3$ interacted each other. For example, agent $a_1$ wants to buy a book from agent $a_2$ in Amazon, because agent $a_1$ bought lots of books from agent $a_2$ in Amazon before. But agent $a_2$ does not have this book, so he recommends another agent $a_3$ to $a_1$. Actually, agent $a_2$ interacted with $a_3$ before, and agent $a_1$ did not interact with $a_3$ in the past. The problem is whether agent $a_1$ should trust $a_3$.

We assume that $s_1^2 = s_2^1 = 28$, $\sum_{l=1}^{s_1^2} v_1^l(2) = 26$, $s_2^3 = s_3^2 = 18$, $\sum_{l=1}^{s_2^3} v_2^l(3) = 16$. We also assume that $c = 0.88$, $P_1(2) = 0.88$, $P_2(1) = 0.87$, $P_2(3) = 0.85$, $P_3(2) = 0.82$, $P_1(3) = 0.87$, $P_3(1) = 0.88$.

So according to formula 2, we can get $\eta_1(2) = (26+1) \div (28+2) = 0.9$, $\eta_2(3) = (16+1) \div (18+2) = 0.85$, $\eta_1(3) = (0+1) \div (0+2) = 0.5$.

Because $\eta_1(2) = 0.9 > c = 0.88$, $\eta_2(3) = 0.85 < c = 0.88$, then based on Rule 1, we get $OT_1(2) = 1$, $OT_2(1) = 1$, $OT_2(3) = 0$, $OT_3(2) = 0$.

Because $\eta_1(2) = 0.9 > P_1(2) = 0.88$, $\eta_2(1) = 0.9 > P_2(1) = 0.87$, $\eta_2(3) = 0.85 \geq P_2(3) = 0.85$, $\eta_3(2) = 0.85 \geq P_3(2) = 0.82$, then based on Rule 2, we get $ST_1(2) = 1$, $ST_2(1) = 1$, $ST_2(3) = 1$, $ST_3(2) = 1$.

Finally, because $ST_1(2) = ST_2(3) = 1$, and $\eta_2(3) = 0.85 < P_1(3) = 0.87$, then based on Rule 3, we can get $TT_1(3) = 0$. Moreover, because $ST_3(2) = ST_2(1) = 1$, and $\eta_2(1) = 0.9 > P_3(1) = 0.88$, then based on Rule 3, we can get $TT_3(1) = 1$. The process of trust formation and the result of Example 1 is illustrated in table 1.

**Table 1.** OT, ST and TT of agents ($c = 0.88$)

| Items | $\langle a_1, a_2\rangle$ | $\langle a_2, a_1\rangle$ | $\langle a_2, a_3\rangle$ | $\langle a_3, a_2\rangle$ | $\langle a_1, a_3\rangle$ | $\langle a_3, a_1\rangle$ |
|---|---|---|---|---|---|---|
| $s_i^j$ | 28 | 28 | 18 | 18 | 0 | 0 |
| $\sum_{l=1}^{s_i^j} v_i^l(j)$ | 26 | 26 | 16 | 16 | 0 | 0 |
| $\eta_i(j)$ | 0.9 | 0.9 | 0.85 | 0.85 | 0.5 | 0.5 |
| $P_i(j)$ | 0.88 | 0.87 | 0.85 | 0.82 | 0.87 | 0.88 |
| $OT_i(j)$ | 1 | 1 | 0 | 0 | 0 | 0 |
| $ST_i(j)$ | 1 | 1 | 1 | 1 | 0 | 0 |
| $TT_i(j)$ | - | - | - | - | 0 | 1 |

# 4 Properties of OT and ST

In this section, we will propose some useful properties of OT and ST which were not mentioned before. These properties are the base of following conclusions.

## 4.1 Properties of Trust

**Property 1.** In the set of $Ag$, Trust has a property of persistence.

Let $\forall a_i, a_j \in Ag$, if $\langle a_i, a_j\rangle \in OT$, then it is easier to strengthen this relation than destroy it in any situation. We say that this relation once formed, it is stable and becomes firmer and firmer in most situations. Because Once a choice (trust relation) has been made, the truster will tend to seek evidence in favour of this choice.

For a result of interaction, if it is successful, then it will strengthen trust. We also know that the probability of success is about $\eta_i(j)$. On the other hand, if it is not successful, then trust also has much more possibility to be 1 than 0. Because only a result of interaction is not enough to change $\eta_i(j)$ more, and $\eta_i(j) \geq P_i(j)$ is much more possible to be true all along.

**Property 2.** In the set of $Ag$, Trust has multi-dimensions.

Let $\forall a_i, a_j \in Ag$, if $\langle a_i, a_j\rangle \in OT$. It means that agent $a_i$ trust $a_j$ on a particular dimension. It does not mean that agent $a_i$ trust all of things of agent $a_j$. For example, I may trust my brother to drive me to the airport, I most certainly would not trust him to fly the plane!

In this paper we discuss trust on a fixed dimension. Multi-dimensions of trust will be investigated in our another paper.

### 4.2 Properties of OT

**Property 3.** In the set of $Ag$, $OT$ is a reflexive, symmetric, and transitive relation.

**Axiom 2.** Let $\forall a_i \in Ag$, then $\langle a_i, a_i \rangle \in OT$. In other words, any agent in $Ag$ trusts herself inherently.

Because the interactional statics is symmetric for all agents and the method of computation is same also, $\forall a_i, a_j \in Ag$, if $\langle a_i, a_j \rangle \in OT$, then $\langle a_j, a_i \rangle \in OT$.

**Proof:** $\forall a_i, a_j \in Ag$, if $\langle a_i, a_j \rangle \in OT$, it means that $\eta_i(j) \geq c$. Because $\eta_i(j) = \eta_j(i)$, so $\eta_j(i) = \eta_i(j) \geq c$. According to definition 2, then $\langle a_j, a_i \rangle \in OT$. **End.**

Suppose any information in $Ag$ is excise and complete, and any agent in $Ag$ is willing to provide her information to others. Thus, $\forall a_i, a_j, a_k \in Ag$, if $\langle a_i, a_j \rangle \in OT$, and $\langle a_j, a_k \rangle \in OT$, then $\langle a_i, a_k \rangle \in OT$.

**Proof:** $\forall a_i, a_j, a_k \in Ag$, if $\langle a_i, a_j \rangle \in OT$, and $\langle a_j, a_k \rangle \in OT$, it means that $\eta_i(j) \geq c$, and $\eta_j(k) \geq c$. According to Axiom 1, $\eta_j(k) = \eta_i(k) \geq c$, then $\langle a_i, a_k \rangle \in OT$. **End.**

**Corollary 4.** OT is an equivalence relation.

**Proof.** With respect to $OT$, it is a reflexive, symmetric, and transitive relation according to Property 3. So it is an equivalence relation therefore. **End.**

### 4.3 Properties of ST

**Property 5.** In the set of $Ag$, $ST$ is a reflexive, asymmetric, and nontransitive relation.

**Axiom 3.** Let $\forall a_i \in Ag$, then $\langle a_i, a_i \rangle \in ST$. In other words, any agent in $Ag$ trusts herself inherently.

Although the interactional statics is symmetric for all agents and the method of computation is same also, due to the different preferences of agents, so $\forall a_i, a_j \in Ag$, if $\langle a_i, a_j \rangle \in ST$, then $\langle a_j, a_i \rangle \in ST$ is not always true.

Suppose any information in $Ag$ is excise and complete, and any agent in $Ag$ is willing to provide her information to others. Because of the different preferences of agents. Thus, $\forall a_i, a_j, a_k \in Ag$, if $\langle a_i, a_j \rangle \in ST$, and $\langle a_j, a_k \rangle \in ST$, then $\langle a_i, a_k \rangle \in ST$ is not always true.

According to Rule 3, the key criterion for determination of transitivity of ST is whether $\eta_j(k) \geq P_i(k)$.

## 5 Background and Related Work

Paul Marsh [4] firstly studied trust by game theory and distributed artificial intelligence. He gave a computational model for trust and put forward some basis properties of trust. Trust is a complex subject relating to belief in honesty, truthfulness, competence, reliability etc. of the trusted person or service.

The mainstream of trust researches is to describe agent trust based on interaction history by probability function, which is OT in nature. In particular, it is well known that Josang and Ismail [7,8] firstly proposed the Beta Reputation System (BRS), which is based on the beta distribution of probability theory. Agents are required to collect interactions data among them, such as success, failure or others. Moreover, agents will give ratings to the performance of other users in the community. Here, ratings consist of a single value that is used to obtain positive and negative feedback values. However, BRS system is specifically designed for online communities and is centralized. Tong [15] took Fuzzy reasoning method to describe agent trust relationship and a long-term coalition system was proposed as a result.

BRS system is totally depended on objective data between agents who interacted with each other in the past. Even so, there are some unfair ratings, either unfairly positive or negative, towards a certain agent. Whitby et al. [9] extended BRS system and show how it can be used to filter unfair ratings. Yu described another method to filter inaccurate reputation [10]. Subsequent observations of trustee behavior are provided to the system as opinion sources. At this point, different methods are adopted to represent trust, ground trust in trustee observations, and implement reputation filtering. Teacy extended BRS and put forward TRAVOS model [5], which can treat with inaccurate information effectively. TRAVOS also provided the confidence of trust computation. If the value of confidence is under minimum predefined value, trust is substituted by reputation. An important advantage is that it can overcome noise and lying sources of reputation effectively. Furthermore, Tong [13] paid more attention to dynamic variety of agent trust for precise prediction and abnormal behavior detection of trust. CMAIT model was proposed based on derivative of trust.

Hang [18] synthesized operators for Propagating Trust in the social networks, such as concatenation, aggregation. A new operator, selection, was suggested to improve the computational system. JensWitkowski and Kastidou [16,17] studied on the honesty of trustees who offered the data of interactions.

Another method is to investigate structure model of trust, which will integrate trust computational model to produce a comprehensive assessment of another agent's performance. Huynh proposed an open multi-agent system, named FIRE model [11], which integrated trust and reputation model. It incorporated interaction trust, role-based trust, witness reputation, and certified reputation to provide a trust metric in most circumstances, where role-based trust is a rule reasoning trust, witness reputation is depended on external observation and certified reputation is computed through third party.

All of the above work are based on probability computational models of trust and never consider subjective component of trust. Particularly, inaccurate information offered by others is partly due to the subjectivity of trust. So an in-depth study is urgently necessary for the subjectivity and objectivity of trust. Tong

[12,14] has investigated ST integrated with OT from agent preferences. Based on probability theory method, a preference is considered to revise the trust.

## 6 Conclusions

In this paper, we clarified the differences between ST and OT firstly. Particularly, we investigated ST and OT and proposed some useful properties of trust. We discussed the formation of trust and gave an example to understand trust deeply.

For the future work on trust researches, we think that trust is indeed a kind of belief, so trust evaluation naturally can be treated as belief revision. In our another papers we will investigate trust as belief revision.

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