Approximation Algorithms for Wireless Spectrum Allocation with Power Control

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Radio spectrum is one of the most valuable resources necessary to run wireless networks. Unfortunately, somewhat wasteful allocation has made it very scarce nowadays. Therefore, for future wireless evolution, we need smart algorithms that manage wireless spectrum access inside and between networks.

Often algorithmic studies dealing with optimization in wireless networks have modeled interference essentially by a graph. This simplified view of interference neglects a number of important aspects. Therefore, attention has recently shifted to more realistic models, in which interference constraints are stated based on the signal-to-interference-plus-noise ratio (SINR). These models allow to take power control into account: By adjusting transmit powers individually, one can significantly reduce the effects of interference.

1 Capacity-Maximization Problem

The most fundamental combinatorial optimization problem with power control is the following *capacity-maximization problem*: Given n pairs of senders and receivers (links), the task is to select a subset of these links and for each selected link a transmit power such that the number of successful transmissions is maximized.

One of the easiest ways to tackle this problem is to ignore the possibility of power control by simply set all transmit powers to a common, uniform value, and apply a suitable selection algorithm. Maybe surprisingly, this already yields approximation factors as good as $O(\log \Delta)$ [1], where Δ is the ratio between the maximum and the minimum distance between a sender and its corresponding receiver. Improved performances of $O(\log \log \Delta)$ can be reached by setting transmit powers proportional to the square-root of the distance between the senders and its receiver and applying a suitable section algorithm [3,4,2].

However, both mentioned bounds are tight: For both uniform and square-root power assignments, there are instances with ($\Delta = 2^n$ resp. $\Delta = 2^{2^n}$) in which one cannot get better approximation factors than $\Omega(n)$. Asymptotically, this is not even better than the trivial algorithm selecting only a single link, which has approximation factor n.

In [7], we show that using a different approach, a constant-factor approximation can be reached. In contrast to the algorithms mentioned above, it first selects the subset of links and assigns the transmit powers afterwards. Extensions of this

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algorithm deal with limited transmit powers [9] rather than unlimited ones and with flexible data rates [8], in which the interference model

2 Secondary Spectrum Auctions

The insights from the considerations of the capacity-maximization problem can be reused in a more general context. In [6,5], we study a setting motivated by auctions for secondary spectrum markets. In these markets licenses allowing secondary-usage of currently unused parts of the spectrum are being sold. Licenses are valid for short periods of time and in local areas. Thus, they have to take interference into account, which we model by an edge-weighted graph. We extend the notion of independent sets to edge-weighted conflict graphs by requiring that the sum of incoming weights from all neighbors have to be less than 1 for all vertices in the set. By suitable choices of edge weights, the independent sets correspond the link sets for which there is a power assignment making all SINR constraints fulfilled.

Interestingly, the conflict graphs derived from the SINR model but also from a number of simpler interference models share a very important property. The *inductive independence number* ρ [10] is bounded by a constant or by a slowly growing function.

This property enables us to bypass the $\Omega(n^{1-\varepsilon})$ lower bound on the approximability of *maximum independent set* in general conflict graphs and get $O(\rho \cdot \log n)$-approximations for weighted maximum independent set edge-weighted conflict graphs. Also for the case of multiple channels, we devise approximation algorithms whose guarantees are almost optimal under standard complexity-theory assumptions. Furthermore, all approximation algorithms can be turned into truthful-in-expectation mechanisms ensuring that no bidder can benefit from lying about his true valuation.

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