A Tandem Queueing System with Batch Session Arrivals

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Abstract. We consider a tandem queueing system with session arrivals. Session means a group of customers which should be sequentially processed in the system. In contrast to the standard batch arrival when a whole group of customers arrives into the system at one epoch, we assume that the customers of an accepted session arrive one by one in exponentially distributed times. Generation of sessions at the first stage is described by a Batch Markov Arrival Process ($BMAP$). At the first stage of tandem, it is determined whether a session has the access to the second stage. After the first stage the session moves to the second stage or leaves the system. At the second stage having a finite buffer the customers from sessions are serviced. A session consists of a random number of customers. This number is geometrically distributed and is not known at a session arrival epoch. The number of sessions, which can be admitted into the second stage simultaneously, is subject to control. An accepted session can be lost, with a given probability, in the case of any customer from this session rejection.

Keywords: tandem system, batch Markovian arrival process, session admission control, performance modeling.

1 Introduction

Queueing theory is widely used for modelling and performance evaluation of modern telecommunications networks. Typically, an user of telecommunication system can generate not a single request but a group of requests. That is the reason why a Batch Markovian Arrival Process ($BMAP$) as an arrival process is assumed when the queueing system that modelles a real telecommunication system is considered. The $BMAP$ was introduced by D. Lucantoni in [1]. In [1], a single-server queueing system with the $BMAP$ arrival process, the general service time distribution and an infinite buffer is analyzed. In [2], a departure process of $BMAP/G/1$ is analysed. In [3, 4], $BMAP/G/1$ queue with controlled service intensity is investigated. $BMAP/G/1$ queue with generalized vacations is considered in [5] and $BMAP/G/1$ with disasters is considered in [6].

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$BMAP/G/1$ cyclic polling models are investigated in [7]. $BMAP/G/1$ retrial queueing system is considered in [8, 9].

Single-server queues with $BMAP$ arrivals, semi-Markovian service process and an infinite buffer are analysed, e.g., in [10–12].

If a queueing system with a finite buffer and $BMAP$ arrivals is considered, it is assumed that at a batch arrival epoch all requests of the batch arrive into the system simultaneously and decision whether or not the batch should be admitted into the system is based on comparison of the batch size and the available capacity of the system, see, e.g., [13], [14].

Very general model of the $BMAP/SM/1/N$ type with discipline of partial admission is investigated in [15], and the $BMAP/G/1/N$ type with disciplines of complete rejection and complete admission is investigated in [16]. Numerically stable algorithms, which are taking into account special structure of transition probability matrix and are suitable even if the buffer capacity N is equal to several thousands, are presented there.

Tandem queueing systems with $BMAP$ arrivals are considered, e.g., in [17–20].

The queues with $BMAP$ arrival process are well suited for modeling the real systems in which the requests can arrive simultaneously. However, in many nowadays communication networks, IP networks in particular, customers can arrive in groups, but the arrival of customers from a group is not simultaneous. To distinguish the standard batches from the group with non-simultaneous customers arrivals the latter ones are called sessions.

Session arrivals are typical for multiple access telecommunication system which resources are shared by a set of users. An user establishes a session (sends the first request) when it enters the system. If this user's request is admitted to the system, the session is considered as established. Once the user has established the session, he(she) can generate the sequence of requests. Belonging of the requests to established sessions is determined by means of IP address. Note that the number of requests at a session is random and unknown at the session arrival epoch. If the arrival request belongs to existed session, it is accepted to the system. If the request belongs to a new session (the first request of the session), the buffer and channel capacity is still available, and the number of session in the system is non critical, the session and request are admitted into the system and the session count is increased. Otherwise, the session and its first request are rejected. When the requests from admitted session do not arrive to the system during a certain time interval, the session is assumed to be finished and the session count is decreased by one.

Due to the requests from a session arrive to the system non-simultaneously and the number of requests in a session is unknown at the session arrival epoch it is impossible to make a decision to accept or not the arriving session to the system based on comparison the session size with the available capacity of the system. Under consideration queues with session arrivals, it is assumed that the number of sessions is restricted by means of so called tokens. The number of tokens, which defines the maximal number of flows that can be admitted into the system simultaneously, is very important control parameter.

In paper [21], a novel finite capacity queueing model of $M/M/N/R$ type with request arrivals in sessions is investigated. In paper [22], the $MAP/PH/1/N$ queueing system with session arrivals is investigated. It was assumed in [21] and [22], that the sessions arrival is regulated by means of tokens. The pool of tokens consists of K tokens and a new session is admitted to the system only if there is an available token and the buffer is not full at a session arrival epoch. Otherwise, the session leaves the system forever.

In paper [23], the mechanism of requests arrival within a session is significantly generalized comparing to the model considered in [22] by suggesting that the customers from the admitted session can arrive in groups. Session arrivals are directed by a MAP (Markovian Arrival Process) and customers' arrivals in session are directed by the $BMAP$ in [23].

In presented paper the tandem queueing system with $BMAP$ arrivals of session is investigated. At the first stage of the system it is determined whether an arriving session has the access to the system. After the first stage the session moves to the second one if it has the access or leaves the system. At the second stage admitted sessions are serviced.

The paper is organized as follows. In section 2, the mathematical model is described. The stationary distribution of system states is analyzed in section 3. The expressions for the main system performance measures are given in section 4. Section 5 concludes the paper.

2 Mathematical Model

The system consists of two stages. The first stage is a single server queueing system with a finite buffer of capacity $R, 1 \leq R < \infty$.

The customers arrive to the system in sessions. Groups of sessions arrive at the first stage according to the Batch Markov Arrival Process ($BMAP$). Sessions arrival in the $BMAP$ is directed by an irreducible continuous time Markov chain ν_t, $t \geq 0$, with the finite state space $\{0, 1, \ldots, W\}$. The sojourn time of the Markov chain ν_t in the state ν has an exponential distribution with the parameter λ_ν, $\nu = \overline{0, W}$. After this sojourn time expires, with probability $p_l(\nu, \nu')$ the process ν_t transits to the state ν', and l, $l \geq 0$, sessions arrive to the system.

The intensities of jumps from one state into another, which are accompanied by an arrival of l sessions, are combined into the square matrices $D_l, l \geq 0$, of size $\bar{W} = W + 1$. The matrix generating function of these matrices is $D(z) = \sum\limits_{l=0}^{\infty} D_l z^l, |z| \leq 1$.

The (ν, ν')th entry of the matrix D_l has form

$$(D_l)_{\nu,\nu'} = \lambda_\nu p_l(\nu, \nu'),\ \nu,\ \nu' = \overline{0, W},\ l \geq 1,$$

$$(D_0)_{\nu,\nu'} = \begin{cases} \lambda_\nu p_0(\nu, \nu'),\ \nu \neq \nu',\ \nu,\ \nu' = \overline{0, W}; \\ -\lambda_\nu, \qquad\qquad \nu = \nu',\ \nu = \overline{0, W}. \end{cases}$$

The matrix $D(1)$ is the infinitesimal generator of the process ν_t, $t \geq 0$. The stationary distribution vector $\boldsymbol{\chi}$ of this process satisfies the equations $\boldsymbol{\chi} D(1) = \mathbf{0}$, $\boldsymbol{\chi}\mathbf{e} = 1$. Here and in the sequel $\mathbf{0}$ is a zero row vector and $\mathbf{e}$ denotes unit column vector.

The average intensity λ (fundamental rate) of the sessions arrivals is defined as $\lambda = \boldsymbol{\chi} D'(z)|_{z=1}\mathbf{e}$. The intensity λ_b of group session arrivals is defined as $\lambda_b = \boldsymbol{\chi}(-D_0)\mathbf{e}$. The coefficient of variation c_{var} of intervals between group session arrivals is defined by $c_{var}^2 = 2\lambda_b \boldsymbol{\chi}(-D_0)^{-1}\mathbf{e} - 1$. The coefficient of correlation c_{cor} of the successive intervals between group session arrivals is given by $c_{cor} = (\lambda_b \boldsymbol{\chi}(-D_0)^{-1}(D(1) - D_0)(-D_0)^{-1}\mathbf{e} - 1)/c_{var}^2$.

The service time of a session at the first stage is exponentially distributed with the parameter η.

If at the arrival epoch of a batch of sessions the size of the batch does not exceed the number of available waiting places, the whole group is admitted to the system. Otherwise, the sessions, for which there is no available place in the buffer, leave the system forever. This means that we assume so called partial sessions admission discipline. Complete rejection and complete admission disciplines need separate treatment.

After service at the first stage a session leaves the system forever with probability q, $0 \leq q \leq 1$, or proceeds to the second stage with complementary probability.

The second stage consists of N identical independent servers and a finite buffer of capacity $M, 1 \leq M < \infty$.

We assume that admission of sessions (they are called also flows, connections, sessions, exchanges, windows, etc. in different real-life applications) to the second stage is restricted by means of tokens. The total number of available tokens is assumed to $K,\ K \geq 1$.

If there is no available token at a session arrival epoch at the second stage or the buffer at the second stage is full, the session is rejected, and leaves the system forever. If the number of available tokens at the session arrival epoch at the second stage is positive and the buffer is not full, this session is admitted into the second stage and the number of available tokens decreases by one. We assume that the first request of a session arrives at the session arrival epoch and if it meets a free server at the second stage, it occupies the server and is processed. If all servers are busy, the customer moves to a buffer and later it is picked up for the service according to the First Came - First Served discipline. After admission of the session at the second stage, the next customer of this session should arrive directly into the second stage in a random interval length which is exponentially distributed with the parameter γ.

If there is an available server at the second stage, the customer is admitted, otherwise, it is rejected and leaves the system forever. If the customer from admitted session is rejected, this session leaves the system forever with probability p, $0 \leq p \leq 1$, and releases the token. The rejection of customer does not affect on the future behavior of the session with complementary probability $1 - p$.

The number of customers in the session has geometrical distribution with parameter θ, i.e., probability that the flow consists of k customers is equal to $\theta^{k-1}(1-\theta),\ k \geq 1$. If the random time since arrival of the previous customer of a session expires and a new customer does not arrive, it means that the arrival of the session is finished. The token, which was obtained by this flow upon arrival, is returned into the pool of available tokens. The customers of this session, which stay in the buffer of the second stage at the epoch of returning the token, should be completely processed by the second stage.

The service time of a customer at the second stage is exponentially distributed with the parameter μ.

3 The Process of System States

Let $i_t,\ i_t = \overline{0, R+1}$, be the number of sessions at the first stage, $n_t,\ n_t = \overline{0, N+M}$, be the number of customers at the second stage, $k_t,\ k_t = \overline{0, K}$, be the number of sessions having token for admission to the system, $\nu_t,\ \nu_t = \overline{0, W}$, be the state of the directing process of the $BMAP$ arrival process at the epoch $t,\ t \geq 0$.

It is obvious that the four-dimensional process $\xi_t = \{i_t, n_t, k_t, \nu_t\},\ t \geq 0$, is the irreducible regular continuous time Markov chain.

Let us enumerate the states of this Markov chain in lexicographic order and refer to (i, n) as macro-state consisting of $\bar{K} = \bar{W}(K+1)$ states $(i, n, k, \nu),\ \ k = \overline{0, K}$, $\nu = \overline{0, W}$.

Introduce the following notation:

- I_m is an identity matrix of size m, O_m is a zero matrix of size $m \times m$;
- $\gamma^- = \gamma(1-\theta),\ \gamma^+ = \gamma\theta$;
- $\otimes$ and $\oplus$ are symbols of Kronecker's sum and product respectively, see, e.g., [24];
- $\tilde{C} = \text{diag}\{0, 1, \ldots, K\}$, $C = \tilde{C} \otimes I_{\bar{W}}$;
- E^- is the square matrix of size $K+1$ with all zero entries except entries $(E^-)_{i,i-1},\ i = \overline{1, K}$, which are equal to 1;
- E_l^+, $l = N+M, K$ is the square matrix of size $l+1$, with all zero entries except entries $(E_l^+)_{i,i+1},\ i = \overline{0, l-1}$, $(E_l^+)_{l,l} = 1$, which are equal to 1;
- $A = (-\gamma\tilde{C} + \gamma^-\tilde{C}E^-) \otimes I_{\bar{W}}$;
- $\delta_{i,j}$ is Kronecker delta, $\delta_{i,j}$ is equal to 1 if $i = j$ and equal to 0 otherwise.

Let Q be the generator of the Markov chain $\xi_t,\ t \geq 0$, with blocks $Q_{i,j}$ consisting of intensities $(Q_{i,j})_{n,n'}$ of this chain transitions from the macro-state (i, n) into the macro-state $(j, n'),\ n, n' = \overline{0, N+M}$. The diagonal entries of the matrix $Q_{i,i}$ are negative and the modulus of the diagonal entry of $(Q_{i,i})_{n,n}$ defines the total intensity of leaving the corresponding state (i, n, k, ν) of the Markov chain. The block $Q_{i,j},\ i, j = \overline{0, R+1}$, has dimention $\bar{M} \times \bar{M}$, where $\bar{M} = \bar{K}(N+M+1)$.

Lemma 1. *The generator Q of the Markov chain ξ_t, $t \geq 0$, has the following block structure*

$$Q = \begin{pmatrix} Q_{0,0} & Q_{0,1} & Q_{0,2} & \dots & Q_{0,R} & Q_{0,R+1} \\ Q_{1,0} & Q_{1,1} & Q_{1,2} & \dots & Q_{1,R} & Q_{1,R+1} \\ O & Q_{2,1} & Q_{2,2} & \dots & Q_{2,R} & Q_{2,R+1} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ O & O & O & \dots & Q_{R,R} & Q_{R,R+1} \\ O & O & O & \dots & Q_{R+1,R} & Q_{R+1,R+1} \end{pmatrix},$$

where non-zero blocks $Q_{i,j}$ are defined by

$$Q_{i,i} = \begin{pmatrix} C^{(i)}_{0,0} & C_{0,1} & O & \dots & O & O \\ C_{1,0} & C^{(i)}_{1,1} & C_{1,2} & \dots & O & O \\ O & C_{2,1} & C^{(i)}_{2,2} & \dots & O & O \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ O & O & O & \dots & C^{(i)}_{N+M-1,N+M-1} & C_{N+M-1,N+M} \\ O & O & O & \dots & C_{N+M,N+M-1} & C^{(i)}_{N+M,N+M} \end{pmatrix}, \ i = \overline{0, R+1},$$

$$Q_{i,i+l} = \begin{cases} I_{(N+M+1)(K+1)} \otimes D_l, & 0 < l < R-i+1, \\ I_{(N+M+1)(K+1)} \otimes \sum\limits_{j=l}^{\infty} D_j, & l = R-i+1 \end{cases}, \ i = \overline{0, R}.$$

$$Q_{i,i-1} = Q^- = \eta(qI_{(N+M+1)(K+1)\bar{W}} + (1-q)E^+_{N+M} \otimes E^+_K \otimes I_{\bar{W}}), i = \overline{1, R+1}.$$

Here

- $C^{(i)}_{n,n} = A - (1-\delta_{i,0})\eta I_{\bar{K}} + I_{K+1} \otimes [D_0 + \delta_{i,R+1}(D(1) - D_0)] - \min\{n, N\}\mu I_{\bar{K}}$, $n = \overline{0, N+M-1}$, $i = \overline{0, R+1}$;
- $C^{(i)}_{N+M,N+M} = A - (1-\delta_{i,0})\eta I_{\bar{K}} + I_{K+1} \otimes [D_0 + \delta_{i,R+1}(D(1) - D_0)] - N\mu I_{\bar{K}} + \gamma^+(1-p)C + \gamma^+ p(\tilde{C}E^-) \otimes I_{\bar{W}}$, $i = \overline{0, R+1}$;
- $C_{n,n+1} = \gamma^+ C$, $n = \overline{0, N+M-1}$,
- $C_{n,n-1} = \min\{n, N\}\mu I_{\bar{K}}$, $n = \overline{1, N+M}$.

Proof of the lemma consists of analysis of the Markov chain ξ_t, $t \geq 0$, transitions during the infinitesimal interval of time and further combining corresponding transition intensities into the matrix blocks. Value γ^- is the intensity of tokens releasing due to the finish of the session arrival, γ^+ is the intensity of new customers arrival in the session.

Since the four-dimensional Markov chain $\xi_t = \{i_t, n_t, k_t, \nu_t\}$, $t \geq 0$, is the irreducible and regular and has the finite state space, the following limits (stationary probabilities) exist:

$$\pi(i, n, k, \nu) = \lim_{t \to \infty} P\{i_t = i,\ n_t = n,\ k_t = k,\ \nu_t = \nu\},$$

$$i = \overline{0, R+1},\ n = \overline{0, N+M},\ k = \overline{0, K},\ \nu = \overline{0, W}.$$

Let us combine these probabilities into the row-vectors

$$\boldsymbol{\pi}(i,n,k) = (\pi(i,n,k,0), \pi(i,n,k,1), \ldots, \pi(i,n,k,W)),\ k = \overline{0,K},$$

$$\boldsymbol{\pi}(i,n) = (\boldsymbol{\pi}(i,n,0), \boldsymbol{\pi}(i,n,1), \ldots, \boldsymbol{\pi}(i,n,K)),\ n = \overline{0,N+M},$$

$$\boldsymbol{\pi}_i = (\boldsymbol{\pi}(i,0), \boldsymbol{\pi}(i,1), \ldots, \boldsymbol{\pi}(i,N+M)),\ i = \overline{0,R+1}.$$

It is well known that the vector $(\boldsymbol{\pi}_0, \ldots, \boldsymbol{\pi}_{R+1})$ is the unique solution to the following system of linear algebraic equations:

$$(\boldsymbol{\pi}_0, \ldots, \boldsymbol{\pi}_{R+1})Q = \mathbf{0},\ \ (\boldsymbol{\pi}_0, \ldots, \boldsymbol{\pi}_{R+1})\mathbf{e} = 1.$$

This system can be solved on computer directly (”by brute force”). Alternatively, the following numerically stable algorithm for solving this system, which takes into account the special structure of the generator Q, can be applied.

Step 1. Compute the matrices $P_{i,j}$ recurrently:

$$P_{i,R+1} = -Q_{i,R+1}(Q_{R+1,R+1})^{-1},\ i = \overline{0,R},$$

$$P_{i,j} = -(Q_{i,j} + P_{i,j+1}Q^-)(Q_{j,j} + P_{j,j+1}Q^-)^{-1},\ i = \overline{0,j-1},\ j = R, R-1, \ldots, 1.$$

Step 2. Calculate the matrices Φ_j, $j = \overline{0,R+1}$:

$$\Phi_0 = I,\ \Phi_j = \sum_{i=0}^{j-1} \Phi_i P_{i,j},\ j = \overline{1,R+1}.$$

Step 3. Calculate the vector $\boldsymbol{\pi}_0$ as the unique solution to the following system of linear algebraic equations:

$$\boldsymbol{\pi}_0(Q_{0,0} + P_{0,1}Q^-) = \boldsymbol{\pi}_0,\ \ \boldsymbol{\pi}_0 \sum_{j=0}^{R+1} \Phi_j \mathbf{e} = 1.$$

Step 4. Calculate the vectors $\boldsymbol{\pi}_j$: $\boldsymbol{\pi}_j = \boldsymbol{\pi}_0 \Phi_j,\ j = \overline{1,R+1}$.

4 Performance Measures

As soon as the vectors $\boldsymbol{\pi}_i,\ i = \overline{0,R+1}$, have been calculated, we are able to find various performance measures of the system under consideration.

The average number of sessions at the first stage is calculated as

$$L^{(1)} = \sum_{i=1}^{R+1} i\boldsymbol{\pi}_i\mathbf{e}.$$

The average number of customers at the second stage is calculated as

$$L^{(2)} = \sum_{i=0}^{R+1} \sum_{n=1}^{N+M} n\boldsymbol{\pi}(i,n)\mathbf{e}.$$

The average number of sessions in the buffer at the first stage is calculated as

$$N_{buffer}^{(1)} = \sum_{i=2}^{R+1} (i-1)\boldsymbol{\pi}_i \mathbf{e}.$$

Theaverage number of customers in the buffer at the second stage is calculated as

$$N_{buffer}^{(2)} = \sum_{i=0}^{R+1} \sum_{n=N+1}^{N+M} (n-N)\boldsymbol{\pi}(i,n)\mathbf{e}.$$

The average number of busy servers at the second stage is calculated as

$$N_{server} = \sum_{i=0}^{R+1} (\sum_{n=1}^{N} n\boldsymbol{\pi}(i,n)\mathbf{e} + N \sum_{n=N+1}^{N+M} \boldsymbol{\pi}(i,n)\mathbf{e}).$$

The intensity of flow of sessions, which get the service at the first stage, is calculated as

$$\lambda_{out}^{(1)} = \eta(1 - \boldsymbol{\pi}_0 \mathbf{e}).$$

The intensity of flow of customers, which get the service in the system, is calculated as

$$\lambda_{out}^{(2)} = \mu N_{server}.$$

The loss probability of whole group of sessions at the entrance to the first stage due to buffer overflow is calculated as

$$P^{(ent-loss)} = \lambda_b^{-1} \boldsymbol{\pi}_{R+1} (\mathbf{e} \otimes \sum_{k=1}^{\infty} D_k \mathbf{e}).$$

The average number of sessions at the second stage is computed as

$$B = \sum_{i=0}^{R+1} \sum_{n=0}^{N+M} \sum_{k=1}^{K} k\boldsymbol{\pi}(i,n,k)\mathbf{e}.$$

The loss probability of arbitrary session at the first stage is calculated as

$$P_1^{(session-loss)} = \lambda^{-1} \sum_{i=0}^{R+1} \boldsymbol{\pi}_i (\mathbf{e} \otimes \sum_{k=R-i+2}^{\infty} (i+k-R-1) D_k \mathbf{e}).$$

The probability $P_s^{(loss)}$ of an arbitrary session rejection upon arrival at the second stage is computed by

$$P_2^{(session-loss)} = \frac{\eta}{\lambda_{out}^{(1)}} \sum_{i=1}^{R+1} \left(\sum_{n=0}^{N+M-1} \boldsymbol{\pi}(i,n,K) + \sum_{k=0}^{K} \boldsymbol{\pi}(i,N+M,k) \right) \mathbf{e}.$$

The probability $P_c^{(loss)}$ of an arbitrary customer from admitted session rejection is computed by

$$P_c^{(loss)} = \sum_{i=0}^{R+1} \frac{\sum\limits_{k=1}^{K} k\gamma^+ \boldsymbol{\pi}(i, N+M, k)\mathbf{e}}{\sum\limits_{k=1}^{K} \sum\limits_{n=0}^{N+M} k\gamma^+ \boldsymbol{\pi}(i, n, k)\mathbf{e}}.$$

5 Conclusion

A tandem queueing system with batch session arrivals is investigated. The system underlying process is constructed. The stable algorithm for calculation of the stationary distribution of system states is presented. The key system performance measures are computed.

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