An Open Queueing Network with Temporarily Non-active Customers and Rounds

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Abstract. An open queueing network with partly non-active customers is considered. Non-active customers are in a system queue and do not get service. Customers can pass from non-active state into state, when they can get their service and vice versa. The form of stationary distribution and conditions of stationary distribution existence are obtained.

Keywords: queueing network, temporarily non-active customers, rounds, stationary distribution.

1 Introduction

Nowadays queueing networks with partly non-active customers become actual to a marked degree. Non-active customers are in a system queue and do not get service. We consider network, where customers may partly loose their capacity for service. Customers can pass from non-active condition into condition, when they can get their service and vice versa.

In paper [1] G. Tsitsiashvili and M. Osipova have observed an open queueing network with non-active customers and have established the form of stationary distribution. This paper generalizes results for network from [1]. We consider model with temporarily non-active customers and rounds of queueing systems. We have researched the form of stationary distribution and have established the criterion of stationary distribution existence.

2 An Open Queueing Network with Temporarily Non-active Customers and Rounds

Consider an open queueing network with set of systems $J = \{1, 2, \ldots, N\}$. Customers arrive at the network according to Poisson processes at rates λ_i, $i \in J$. There are input Poisson flows of signals at rates ν_i and φ_i, $i \in J$. When arriving at the system $i \in J$ the signal at rate ν_i induces an ordinary customer at system, if any, to become non-active. When arriving at the system $i \in J$ the signal at rate φ_i induces an non-active customer, if any, to become an ordinary. Non-active customers are in a system queue and can not get service. Signals do not need service.

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Service times are independent exponentially distributed random values with parameters μ_i, $i \in J$. When arriving at the system i customer queues up to the system with the probability f_i and with the probability $1 - f_i$ the customer goes round the system $i \in J$ (such customer is considered to be served). After finishing of service process at system $i \in J$ customer is routed to system $j \in J$ with the probability $p_{i,j}$ and with the probability $p_{i,0}$ is removed from network $(\sum_{j=1}^{N} p_{i,j} + p_{i,0} = 1)$, $i \in J$. Let $p_{i,i} = 0$, $i \in J$. Let $n_i(t), n_i'(t)$ are numbers of ordinary and non-active customers at system $i \in J$ at time t accordingly. Consider $X(t) = \Big((n_1(t), n_1'(t)), \ldots, (n_N(t), n_N'(t))\Big)$. $X(t)$ is a continuous-time Markov chain. States space for process $X(t)$ is $Z = \{((n_1, n_1'), \ldots, (n_N, n_N')) | \, n_i, \, n_i' \geq 0, \, i \in J\}$.

A traffic equations system is:

$$\varepsilon_i = \lambda_i + \sum_{j=1}^{N} \varepsilon_j p_{j,i}, \quad i \in J. \tag{1}$$

One can prove that under certain conditions traffic equations system has unique non-trivial solution.

Theorem 1. *Under conditions of ergodicity:*

$$\varepsilon_i f_i < \mu_i, \tag{2}$$

$$\varepsilon_i f_i \nu_i < \mu_i \varphi_i, \; i = 1, \ldots, N, \tag{3}$$

$X(t)$ has stationary distribution:

$$\pi(n, n') = \pi_1(n_1, n_1')\pi_2(n_2, n_2') \ldots \pi_N(n_N, n_N'), \tag{4}$$

where

$$\pi_i(n_i, n_i') = \left(1 - \frac{\varepsilon_i f_i}{\mu_i}\right)\left(1 - \frac{\varepsilon_i f_i \nu_i}{\mu_i \varphi_i}\right)\left(\frac{\varepsilon_i f_i}{\mu_i}\right)^{n_i}\left(\frac{\varepsilon_i f_i \nu_i}{\mu_i \varphi_i}\right)^{n_i'}, \tag{5}$$

here ε_i, $i \in J$ – is a traffic equations system solution.

Proof. Consider the following events:

1. A customer sent to the system $i \in J$, will not change the state of the network. The probability of this event denote by ψ_i.
2. A customer sent to the system $i \in J$, will be served by system $j \in J$ first time. The probability of this event denote by $\psi_{i,j}$.
3. A customer served by system $i \in J$, will not change the state of the network. The probability of this event denote by β_i.
4. A customer served by system $i \in J$, will be served by system $j \in J$ first time. The probability of this event denote by $\beta_{i,j}$.

It has been obtained in [5], that

$$\psi_i = (1 - f_i)(p_{i,0} + \sum_{j=1}^{N} \psi_j p_{i,j}); \tag{6}$$

$$\psi_{i,j} = f_i \delta_{i,j} + (1 - f_i) \sum_{k=1}^{N} p_{i,k} \psi_{k,j}; \tag{7}$$

$$\beta_i = p_{i,0} + \sum_{j=1}^{N} p_{i,j} \psi_j; \tag{8}$$

$$\beta_{i,j} = \sum_{k=1}^{N} p_{i,k} \psi_{k,j}. \tag{9}$$

Herewith

$$\psi_i + \sum_{j=1}^{N} \psi_{i,j} = 1; \tag{10}$$

$$\beta_i + \sum_{j=1}^{N} \beta_{i,j} = 1; \tag{11}$$

here $\delta_{i,j}$ – is Kronecker symbol.

It has been proved in [5], that traffic equations system solution satisfies generalized traffic equations system:

$$f_i \varepsilon_i = \sum_{k=1}^{N} \lambda_k \psi_{k,i} + \sum_{j=1}^{N} f_j \varepsilon_j \beta_{j,i}. \tag{12}$$

Intensities of transitions for Markov process $X(t)$ are

$$q(n, n + e_i) = \sum_{j=1}^{N} \lambda_j \psi_{j,i};$$

$$q(n, n - e_i) = \mu_i \beta_i I_{n_i > 0};$$

$$q(n, n + e_i - e_i') = \varphi_i I_{n_i' > 0};$$

$$q(n, n - e_i + e_i') = \nu_i I_{n_i > 0};$$

$$q(n, n - e_i + e_j) = \mu_i \beta_{i,j} I_{n_i > 0}.$$

It is obvious, that under conditions (2) Markov process $X(t)$ is ergodic, therefore unique stationary distribution $\pi(n)$, $n \in Z$ exists.

Global balance equations are:

$$\sum_{i \in J} \Big(\sum_{j \in J} \lambda_j \psi_{j,i} + \mu_i \beta_i I_{n_i > 0} + \varphi_i I_{n_i' > 0} + \nu_i I_{n_i > 0} + \sum_{j \in J} \mu_i \beta_{i,j} I_{n_i > 0} \Big) \pi(n) =$$

$$= \sum_{i \in J} \Big(\pi(n - e_i) \sum_{j=1}^{N} \lambda_j \psi_{i,j} I_{n_i > 0} + \pi(n + e_i) \mu_i \beta_i +$$

$$+ \pi(n - e_i + e_i') \varphi_i I_{n_i > 0} + \pi(n + e_i - e_i') \nu_i I_{n_i' > 0} +$$

$$+ \sum_{j \in J} \pi(n + e_i - e_j) \mu_i \beta_{i,j} I_{n_j > 0} \Big), \; n \in Z.$$

It is easy to show, that with foregoing intensities of transitions Markov process $X(t)$ is reversible.

Substituting $\pi(n)$, determined by means of (4), (5) into global balance equations, considering (6) - (11), traffic equation system (1) and generalized traffic equations system (12), we obtain identity. □

3 Conclusion

We have considered an open queueing network with temporarily non-active customers and rounds. Customers could partly loose their capacity for service. Customers could pass from non-active condition into condition, when they can get their service and vice versa. Conditions of ergodicity have been established. The form of stationary distribution and conditions of stationary distribution existence have been obtained.

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