

# Voting and Power

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## 1 Introduction

Since the seminal contributions of Penrose (1946), Shapley and Shubik (1953) and Banzhaf (1965), the question of the ‘voting power’ in voting situations has received attention from many researchers. Variations, different characterizations and alternative interpretations of the seminal concepts, their normative implications for the design of collective decision making procedures and innumerable applications can be found in the game theoretic and social choice literature.<sup>1</sup> Nevertheless, even half a century later and despite the proliferation of contributions in the field, one cannot speak of consensus in the scientific community about the soundness of the foundations of this body of knowledge and consequently about the normative value of the (often contradictory) practical recommendations

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An earlier version of this chapter has been published in *Essays in Honor of Hannu Nurmi (Homo Oeconomicus 26, 2009)*, ed. by M.J. Holler and M. Widgrén. This research is supported by the Spanish Ministerio de Ciencia e Innovación under project ECO2009-11213, co-funded by the ERDF, and by Basque Government funding to Grupo Consolidado GIC07/146-IT-377-07.

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<sup>1</sup> We do not give a comprehensive or even a summary list of the contributions in the field as we believe it to be unnecessary for the purpose of this chapter.

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that stem from it. Because of this lack of consensus it would be exaggerating to speak of a voting power ‘paradigm’<sup>2</sup> in the sense of Kuhn (1970).

Our purpose here is not to survey discrepancies between competing approaches within this field but to make some suggestions for providing more convincing foundations, still within the a priori point of view but widening the conceptual framework. The ideas summarized in this chapter are developed in detail in Laruelle and Valenciano (2008c).

The rest of the chapter is organized as follows. In Sect. 2, we give summarily our way of approaching the question and convey the main ideas. In the next two sections we concentrate on each of the two basic scenarios in which a committee may have to make decisions. In Sect. 3 we consider ‘take-it-or-leave-it’ situations, and in Sect. 4 we consider ‘bargaining committees’ situations. In Sect. 5 we summarize the main conclusions.

## 2 Suggestions for Clearer Foundations

The specification of a voting rule for making dichotomous choices (acceptance/rejection) neither involves nor requires the description of its users or ‘players’. It suffices to specify the vote configurations that would mean acceptance and those that would mean rejection. The same voting rule can be used by different sets of users to make decisions about different types of issue. But describing a voting rule as a simple TU game<sup>3</sup> falls into the trap of producing a game where there are no players. When the (then) recently introduced Shapley (1953) value was applied to this game, the Shapley and Shubik (1954) index resulted. Shapley and Shubik interpret their index as an evaluation of a priori ‘voting power’ in the committee. As the marginal contribution of a player to a coalition in this game can only be 0 or 1, and is 1 only when the presence/absence of a player in a coalition makes it winning/losing, they also propose an interpretation in terms of likelihood of being pivotal or decisive. Hence the seminal duality or ambiguity:

Q.1: *Is the Shapley-Shubik index a ‘value’, that is, an expected payoff in a sort of bargaining situation, or an assessment of the likelihood of being decisive?*

Later Banzhaf (1965) takes the point of view of power as decisiveness, and criticizes the Shapley–Shubik index in view of the unnatural probability model underlying its probabilistic interpretation in the context of voting. If power means

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<sup>2</sup> It may possibly be more accurate to use the term ‘preparadigm’ to refer to a set of concepts and ideas accepted and shared by a certain number of researchers within the academic world, related to the power indices literature, which is basically surveyed and systematized in Felsenthal and Machover’s (1998). In fact, their book can be seen as the closest thing to an embodiment of that (pre)paradigm. By contrast, another group in the profession dismisses voting power literature altogether as irrelevant in view of its weak foundations.

<sup>3</sup> By assigning ‘worth’ 1 to all ‘winning coalitions’ and 0 to the others.

being decisive, then a measure of power is given by the probability of being so. Thus an a priori evaluation of power, if all vote configurations are equally probable a priori, is the probability of being decisive under this assumption. Note that Penrose in 1946 independently reached basically the same conclusion in a narrower formal setting. In fact Banzhaf only ‘almost’ says so, as he destroys this clean probabilistic interpretation by ‘normalizing’ the vector (i.e. dividing the vector of probabilities by its norm so as to make its coordinates add up to 1). So the old dispute is served up:

Q.2: *Which is better as a measure of voting power: the Shapley-Shubik index or the Banzhaf index? What are more relevant: axioms or probabilities?*

It is our view that in order to solve these dilemmas and dissipate ambiguities a more basic issue must be addressed: *What are we talking about?* Instead of starting with abstract terms such as ‘power’ related to an exceedingly broad class of situations (any collective body that makes decisions by vote) and getting entangled prematurely in big words, we think it wiser to

(i) *start by setting the analysis of voting situations as the central goal.*

Collective decision-making by vote may include an extremely wide, heterogeneous constellation of voting situations: law-making in a parliament, a parliament vote for the endorsement of a government, a referendum, a presidential election, decision-making in a governmental cabinet, a shareholders’ meeting, an international, intergovernmental or other council, etc. By setting the analysis of voting situations as the central goal instead of the abstraction ‘voting power’, we mean to

(ii) *start from clear-cut models of well specified clear-cut voting situations,*

instead of starting from words denoting poorly-specified abstractions in poorly-specified situations. For instance, a committee capable of bargaining a proposal before voting is not the same as one only allowed to accept or reject proposals by vote. Millions of voters are not the same as a few, etc.

A dichotomy consistent with the above principles should distinguish between two types of voting situations or committees which make decisions under a voting rule: ‘take-it-or-leave-it’ committees and ‘bargaining’ committees.

(ii.1) A ‘take-it-or-leave-it’ committee votes on different independent proposals over time, which are submitted to the committee by an external agency, so that the committee can only accept or reject proposals, but cannot modify them.

(ii.2) A ‘bargaining’ committee deals with different issues over time, so that for each issue a different configuration of preferences emerges among its members over the set of feasible agreements, and the committee bargains about each issue in search of an agreement, to attain which it is entitled to adjust the proposal.

Though in reality it is often the case that a same committee acts sometimes as a ‘take-it-or-leave-it’ committee, and sometimes as a ‘bargaining’ committee, or even as something in between, this crisp differentiation of two clear-cut types of situation provides benchmarks for a better understanding of many mixed real world situations. As shown in the next two sections, they require different models and raise different questions with different answers which give rise to different

recommendations. This neat differentiation and the analysis therein also sheds some light on the old ambiguities.

An ingredient common to both types of committee is the voting rule that governs decisions. In order to proceed a minimum of notation is needed. If  $n$  voters, labelled by  $N = \{1, 2, \dots, n\}$  are asked to vote ‘yes’ or ‘no’ on an issue, any result of a vote, or *vote configuration*, can be summarized by the subset of voters who vote ‘yes’:  $S \subseteq N$ . An  $N$ -*voting rule* is then specified by a set  $W \subseteq 2^N$  of *winning* vote configurations such that (i)  $N \in W$ ; (ii)  $\emptyset \notin W$ ; (iii) If  $S \in W$ , then  $T \in W$  for any  $T$  containing  $S$ ; and (iv) If  $S \in W$  then  $N \setminus S \notin W$ .

### 3 ‘Take-it-or-Leave-it’ Committees

#### 3.1 The ‘Take-it-or-Leave-it’ Environment

A pure ‘take-it-or-leave-it’ environment is that of a committee that makes dichotomous decisions (acceptance/rejection) by vote under the following conditions: (i) the committee votes on different independent proposals over time; (ii) proposals are submitted to the committee by an external agency; (iii) the committee can only accept or reject each proposal, but cannot modify them; and (iv) a proposal is accepted if a winning vote configuration according to the specifications of the voting rule emerges.

In real world committees these conditions are seldom all satisfied. Nevertheless, for a sound analysis it is necessary to make explicit and precise assumptions about the environment, and this is the only way to have clear conclusions. Indeed, traditional power indices and the credibility of voting power theory is undermined by the lack of clarity about the precise specification of the underlying collective decision-making situation.

Under the above conditions, it seems clear that, except in the case of indifference about the outcome, each voter’s vote is determined by his/her preferences. In particular these conditions leave no margin for bargaining. The impossibility of modifying proposals, their independence over time, etc., rule out the possibility of bargaining and consequently of strategic behavior. In other words, decision-making in a take-it-or-leave-it committee is *not* a game situation, therefore *in a pure ‘take-it-or-leave-it’ environment game-theoretic considerations are out of place.*

#### 3.2 The Basic Issue in a ‘Take-it-or-it-Leave-it’ Environment

As briefly commented in the introduction, there are certain ambiguities at the very foundations of traditional voting power theory, concerning the precise conditions

under which the collective decision-making process takes place, and concerning the interpretation of some power indices. Either explicitly or implicitly the notion of power as decisiveness, i.e. the likelihood of one's vote being in a position to decide the outcome, underlies most traditional voting power literature.

Were it not for the weight of theoretical inertia it would hardly be necessary to argue about the irrelevance of this notion in a pure 'take-it-or-leave-it' environment, where voting behavior immediately follows preferences.<sup>4</sup> *Decisiveness can be a form or, more precisely, a source of power only in a situation in which there is room for negotiation and the possibility of using it with this purpose. But the conditions that specify a 'take-it-or-leave-it' environment preclude that possibility.* For instance, voting on an issue against one's preferences in exchange for someone else doing the same in one's favor on a different issue is not possible in case of strict independence between proposals, as assumed.

The interest of any voter lies in obtaining the desired outcome, and in a 'take-it-or-leave-it' situation nothing better can be done in order to achieve that end than just voting accordingly. Thus, *having success or satisfaction (i.e. winning the vote) is the central issue in this kind of voting situation.* If so the assessment of a voting situation of this type with normative purposes requires us to assess the likelihood of each voter having his/her way. For an a priori assessment it seems natural to assume all configurations of preferences or equivalently (at least if no indifference occurs) all vote configurations as being equally probable. This assumption underlies a variety of power indices in the literature, that is, the probability of every vote configuration  $S$  is  $\frac{1}{2^n}$ .

Assuming this probabilistic model, the probability of a voter  $i$  being successful in a vote under a voting rule  $W$  is given by

$$Succ_i(W) := \text{Prob}(i \text{ is successful}) = \sum_{S:i \in S \in W} \frac{1}{2^n} + \sum_{S:i \notin S \notin W} \frac{1}{2^n}. \quad (1)$$

As commented above, more attention has been paid to the probability of being decisive under the same probabilistic model, given by

$$Dec_i(W) := \text{Prob}(i \text{ is decisive}) = \sum_{\substack{S:i \in S \in W \\ S \setminus i \notin W}} \frac{1}{2^n} + \sum_{\substack{S:i \notin S \notin W \\ S \cup i \in W}} \frac{1}{2^n}. \quad (2)$$

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<sup>4</sup> Rae (1969), Brams and Lake (1978), Barry (1980) (from whom we take the term 'success'), Straffin, Davis, and Brams (1982), and more recently König and Bräuninger (1998) all pay attention to the notion of success or satisfaction, but the ambiguity has remained unsolved due to the lack of definite clarification about the underlying voting situation. In Laruelle, Martínez and Valenciano (2006) we argue in support of success as the relevant notion in a 'take-it-or-leave-it' committee.

Other conditional variants can be considered.<sup>5</sup> See Laruelle and Valenciano (2005).

### 3.3 Normative Recommendations

If the relevant issue in a ‘take-it-or-leave-it’ environment is the likelihood of success, then that should be the basis for normative recommendations. The question arises of what recommendations can be made for the choice of the voting rule in a committee of this type in which each member acts on behalf of a group of a different size. There are two basic points of view for the basis of such recommendations: egalitarian (equalizing the expected utility of all individuals represented) and utilitarian (maximizing the aggregated expected utility of the individuals represented). The implementation of either principle with respect to the people represented requires some assumption about the influence of those individuals on the decisions of the committee, *and* about the voters’ utilities at stake. The well-known idealized two-stage decision process assuming that each representative follows the majority opinion of his/her group on every issue can be neatly modeled by a composite rule in which decisions are made directly by the people represented. As to the voters’ utilities, a symmetry or anonymity assumption seems the most natural for a normative approach. This approach is taken in Laruelle and Valenciano (2008c), and the conclusions are the following.

#### 3.3.1 Egalitarianism and the Square Root Rule

The egalitarian principle, according to which all individuals should have an equal expected utility, would be implemented<sup>6</sup> by a voting rule in the committee for which the Banzhaf index of each representative is proportional to the square root of the size of the group that s/he represents.

But this is the well-known ‘square root rule’ (SQRR) that appears in voting power literature,<sup>7</sup> where power is understood as the probability of being decisive, as a means of equalizing ‘voting power’. So, are we back to the old recommendation? Yes and no, but mainly no. The important difference is the following: in the voting power approach, where the likelihood of being decisive is what matters, the SQRR is the way of implementing the egalitarian principle *in terms of power*

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<sup>5</sup> Actually some of them have been considered in the literature, but those that we consider most relevant have so far been overlooked, namely, the probabilities of a voter being successful *conditional* either upon his/her voting ‘yes’ or upon his/her voting ‘not’. They are calculated in Laruelle, Martínez, and Valenciano (2006) for some voting rules in the Council of Ministers of the EU with interesting results.

<sup>6</sup> With close approximation if all the groups are large enough.

<sup>7</sup> Conjectured by Morriss (1987) and rigorously proved by Felsenthal and Machover (1999).

*so understood.* But in the utility-based approach it is merely a sufficient condition for (approximately) equalizing the expected utility of all the individuals represented. Usually the SQRR can only be implemented approximately, so exact fulfillment is exceptional. Herein lies the crucial difference between the two approaches: according to the traditional approach the differences between the Banzhaf indices of individuals from different groups are seen as differences in the substantive notion to be equalized (i.e. power as decisiveness), while in the utility-based approach the substantive differences are in utilities. It turns out that when groups are big enough the *expected utilities of all individuals are very close, whatever the voting rule in the committee.*

### 3.3.2 Utilitarianism and the 2nd Square Root Rule

The utilitarian principle, according to which the sum of all individuals expected utilities should be maximized, would be implemented by a weighted voting rule that assigns to each representative a weight proportional to the square root of the group sizes, and the quota is half the sum of the weights.<sup>8</sup>

Again we are back to a well-known recommendation: the ‘second square root rule’ (2nd SQRR). Nevertheless it is worth remarking that the underlying justifications are different and the differences are important. As in the case of the recommendation based on the egalitarian point of view, the utilitarian recommendation has a clear justification only for this special type of committee, while for the more complex case of bargaining committees both recommendations lack a clear basis. The lack of a precise specification of the voting situation in the traditional analysis has so far concealed this important point. As we will see in a bargaining committee these recommendations lack foundations and this has important consequences for applications.

## 4 Bargaining Committees

### 4.1 The Bargaining Environment

As soon as any of the conditions specifying what has been called a ‘take-it-or-leave-it’ environment is relaxed the situation changes drastically. For instance, if decisions on different issues cease to be independent, bargaining over votes of the form ‘I will vote on this issue against my preferences in exchange for your doing

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<sup>8</sup> With close approximation if all the groups are large enough, and it is assumed that all vote configurations are equally probable and that the utility of winning a vote is the same in case of acceptance as in the case of rejection (if voters place different values on having the desired result when they support approval and when they support rejection, the quota should be adjusted).

the same in my favor on that issue' becomes possible. Also, if the committee can modify proposals negotiation prior to voting is to be expected. In fact, outside the rather constrained environment specified as 'take-it-or-leave-it' there are many possibilities.

Thus the '*non*-take-it-or-leave-it' scenario is susceptible to a variety of specifications that can be seen as variations of the 'bargaining' environment. The analysis is possible only if the conditions under which negotiation takes place are specified. To fix ideas, although others are possible, here we discuss the following specification of a bargaining committee that makes decisions using a voting rule under the following conditions: (i) the committee deals with different issues over time; (ii) for each issue a different configuration of preferences emerges among the members of the committee over the set of feasible agreements concerning the issue at stake; (iii) the committee bargains about each issue in search of a consensus, to which end it is entitled to adjust the proposal; and (iv) any winning coalition<sup>9</sup> can enforce any agreement.

Now the situation is much more complicated. The environment permits bargaining among the members of the committee, and consequently *behavior no longer trivially follows preferences. Now game-theoretic considerations are in order because the situation is inherently game-theoretic.*

## 4.2 *The Basic Issues in a Bargaining Environment*

First note that *in a bargaining situation the basic issue is that of the outcome* of negotiations. That is, given a preference profile of the members of the committee and a bargaining environment, what will the outcome be? Or at least, what outcome can reasonably be expected? It should be remarked that only if this basic question is answered can other relevant issues be addressed, e.g. the question of the influence of the voting rule on the outcome of negotiations and the question of the 'power' that the voting rule gives to each member. In particular, the meaning of the term 'power' in this context can only become clear when one has an answer to the first basic question.

In order to provide an answer to the central question a formal model of a bargaining committee as specified is needed. A model of such a bargaining committee should incorporate at least the following information: the voting rule under which negotiation takes place, and the preferences of the *players* (the usual game-theoretic term is now appropriate). Other elements need to be included for a more realistic model, but it is best to start with as simple a model as possible to see what conclusions can be drawn from it. In Laruelle and Valenciano (2007) a model of an *n*-person bargaining committee incorporating these two ingredients is introduced. The

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<sup>9</sup> Notice that, unlike what happens in the case of a take-it-or-leave-it situation, in this context the old traditional game-flavored term "winning coalition" is appropriate.



first element is just the  $n$ -person voting rule  $W$ . As to the second, the players' preferences, under the same assumptions as in Nash (1950), i.e. assuming that they are expected utility or von Neumann-Morgenstern (1944) preferences, they can be represented à la Nash by a pair  $B = (D, d)$ , where  $D$  is the set<sup>10</sup> of feasible utility vectors or 'payoffs', and  $d$  is the vector of utilities in case of disagreement.

In this two-ingredient setting the question of rational expectations about the outcome of negotiations can be addressed from two different game-theoretic points of view: the cooperative and the noncooperative approaches. The cooperative method consists of ignoring details concerning the way in which negotiations take place, and 'guessing' the outcome of negotiations between ideally rational players by assuming reasonable properties of the map that maps 'problems'  $(B, W)$  into payoffs  $\Phi(B, W) \in R^N$ . The most influential paradigm of the cooperative approach is Nash's (1950) bargaining solution. In Laruelle and Valenciano (2007) Nash's classical approach is extended or adapted to this two-ingredient setting, *assuming that players in a bargaining committee bargain in search of unanimous agreement*. In this way, by assuming adequate adaptation to our setting of some reasonable conditions to expect for a bargaining outcome (efficiency, anonymity, independence of irrelevant alternatives, invariance w.r.t. affine transformations, and null player), it is proved that a general 'solution' (i.e. an  $N$ -vector valued map  $(B, W) \mapsto \Phi(B, W)$ ) should take the form

$$\Phi(B, W) = Nash^{\varphi(W)}(B) = \arg \max_{x \in D_d} \prod_{i \in N} (x_i - d_i)^{\varphi_i(W)}. \tag{3}$$

That is to say: a reasonable outcome of negotiations is given by the weighted Nash bargaining solution (Kalai 1977) where the weights are a function  $\varphi(W)$  of the voting rule. Moreover this function must satisfy anonymity and null-player. Note also that these two properties are the most compelling ones concerning 'power indices'.<sup>11</sup> Thus formula (3) sets the 'contest' between power indices candidates to replace  $\varphi(W)$  in (3) in a new setting and provides a new interpretation of them in terms of 'bargaining power' in the precise game theoretic sense.<sup>12</sup> In the same chapter it is shown how adding an adaptation of Dubey's (1975) lattice property to the other conditions singles out the Shapley-Shubik index in (3), i.e. in that case the solution is

$$\Phi(B, W) = Nash^{Sh(W)}(B) = \arg \max_{x \in D_d} \prod_{i \in N} (x_i - d_i)^{Sh_i(W)}. \tag{4}$$

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<sup>10</sup>  $D$  is a closed, convex and comprehensive (i.e.,  $x \leq y \in D \Rightarrow x \in D$ ) set in  $R^N$  containing  $d$ , such that there exist  $x \in D$  s.t.  $x > d$ , and such that the set  $D_d := \{x \in D : x \geq d\}$  is bounded.

<sup>11</sup> These properties are satisfied by the two most popular power indices, but also by all semivalues (Dubey, Neyman and Weber (1981), see also Laruelle and Valenciano (2001, 2002, 2003)).

<sup>12</sup> In game-theoretic terms, the weight associated with each player for an asymmetric Nash bargaining solution is called 'bargaining power', as reflecting the relative advantage or disadvantage that the environment gives to each player.

It is also interesting to remark that, as shown in Laruelle and Valenciano (2007), when the bargaining element, i.e. the preference profile in the committee summarized by  $B = (D, d)$ , is *transferable utility*-like, that is,

$$B = \Lambda := (\Delta, 0), \quad \text{where } \Delta := \left\{ x \in \mathbb{R}^N : \sum_{i \in N} x_i \leq 1 \right\}$$

then we have that (3) and (4) become respectively:

$$\Phi(A, W) = \text{Nash}^{\varphi(W)}(A) = \bar{\varphi}(W) \tag{5}$$

$$\Phi(A, W) = \text{Nash}^{\text{Sh}(W)}(A) = \text{Sh}(W), \tag{6}$$

where  $\bar{\varphi}(W)$  denotes the normalization of vector  $\varphi(W)$ , which as commented in the conclusions solves the power/payoff dilemma mentioned in Sect. 2.

As mentioned above there is also the noncooperative approach, in which the model should specify with some detail the way in which negotiations take place. This is neither simple nor obvious in a situation of which the only ingredients so far are the voting rule that specifies what sets of members of the committee have the capacity to enforce any agreement, and the voters' preferences. A noncooperative modeling must necessarily choose a 'protocol' to reach any conclusion. The question arises whether (3) or (4) have a noncooperative foundation. In Laruelle and Valenciano (2008a) this problem is addressed and noncooperative foundations to (3) and (4) are provided. Assuming complete information, a family of noncooperative bargaining protocols is modeled based on the voting rule that provides noncooperative foundations for (3), which appear in this light as limit cases. The results based on the noncooperative model evidence the impact of the details of protocol on the outcome, and explain the lack of definite arguments (i.e. axioms compelling beyond argument) to go further than (3). Nevertheless (4) emerges associated with a very simple protocol also based on the voting rule under which negotiations take place, thus providing some sort of focal appeal as a reference term for the Shapley–Shubik index as an a priori measure of bargaining power.

### 4.3 Normative Recommendations

The model summarized in the previous section can be taken as a base for addressing the normative question of the most adequate voting rule in a committee of representatives. Namely, if a voting rule is to be chosen for a committee that is going to make decisions in a bargaining environment as described: what rule is the most appropriate if each member acts on behalf of a different sized group?

Laruelle and Valenciano (2008b) addresses this problem, which is tricky because for each issue a different configuration of preferences emerges in the population represented by the members of the committee. Thus if by ‘appropriate’ we mean fair in some sense, nothing can be said unless some form or other of relation about the preferences within each group is assumed. By ‘fair’ we mean *neutral* in the following sense: a neutral voting rule for the committee is one such that all those represented are indifferent between bargaining directly (ideal and unfeasible, but theoretically tractable according to Nash’s classical bargaining solution<sup>13</sup>) and leaving it in the hands of a committee of representatives. This is obviously utopian, but it can be proved to be implementable under some ideal symmetry conditions for the preferences within each group. In real world situations this condition may well fail to occur in most cases, but this idealization seems a reasonable term of reference if a voting rule is to be chosen. In fact if certain conditions of symmetry within each group are assumed the following recommendation arises:

**A neutral voting rule** (Laruelle and Valenciano 2008b): *A neutral voting rule in a bargaining committee of representatives is one that gives each member a bargaining power proportional to the size of the group that he/she represents.*

Note that this recommendation is based on (3), i.e. it does not presuppose which is the right  $\varphi$  in formula (3), but notice all the same the difference from the square root rule recommendation. The neutral voting rule would be one for which:

$$\frac{\varphi_i(W)}{m_i} = \frac{\varphi_j(W)}{m_j} \quad (\forall i, j \in N),$$

while according to the square root rule the fair voting rule is one for which

$$\frac{Bz_i(W)}{\sqrt{m_i}} = \frac{Bz_j(W)}{\sqrt{m_j}} \quad (\forall i, j \in N),$$

where  $Bz_i$  denotes voter  $i$ ’s Banzhaf index.

## 5 Conclusions

Thus we have several conclusions. Consider Q.1 and Q.2 raised in Sect. 2. The mere statements of Q.1 and Q.2 now look confusing in themselves. The reason is the narrowness of the framework in which they were formulated.

First, in the light of the conceptual and formal framework summarized above it can be said that in a ‘take-it-or-leave-it’ committee the Banzhaf index seems an appropriate measure of a priori decisiveness founded on probabilistic terms,

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<sup>13</sup> Note also that the Nash bargaining solution is a compromise between egalitarianism and utilitarianism.

although in such committees the relevant notion is that of success. Thus, in the light of the above analysis the popular square root rule, which enjoys ample support in view of its providing a priori equal chances of being decisive, appears ill-founded in two ways: first, it should be the goal of equalizing the likelihood of success that justifies it, but for this purpose the rule hardly matters; and, second and more importantly, *it is only in the context of 'take-it-or-leave-it' committees that this recommendation makes sense*. Most often supporters of this choice apply it to real world committees that make decisions in an environment closer to that of a bargaining committee than to that of a 'take-it-or-leave-it' committee.

Moreover, in the case of bargaining environments as specified here the Shapley–Shubik index, and indeed any other power index, when seen through the lens of formulae (4) or (3), appear as *the 'bargaining power' in the precise game-theoretic sense* (i.e. the weights of an asymmetric Nash bargaining solution) *that the voting rule gives to each member of the committee*. Note that this bargaining power is related to decisiveness, but when the preference profile is TU-like the bargaining power of each player coincides with his/her expected payoff as given by (5) and (6). Note that this solves the dilemma of Q.1. Among all those indices or measures  $\varphi$  which fit formula (3) the Shapley–Shubik index appears as a remarkable candidate for measuring the a priori bargaining power in such committees. Note also that in this case the support is not probabilistic but either cooperative-axiomatic or noncooperative game-theoretic (as a limit case).

Finally, there is the question of the quite different recommendations for the choice of voting rule in a committee of representatives depending on whether it is a 'take-it-or-leave-it' or bargaining committee. The different recommendations obtained from the analysis summarily described above seem rather disturbing, especially considering that real-world committees often act in an intermediate environment between the two *pure* types considered here. This does not invalidate these recommendations: on the contrary, a clear, sound conceptual founding only sets clear limits on the validity of the conclusions that one may get from formal models, while unclear situations underlying models and conceptual vagueness at the base of theory definitely blur the sense and validity of any conclusion.

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