On the Steiner Radial Number of Graphs

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Abstract. The Steiner n-radial graph of a graph G on p vertices, denoted by $SR_n(G)$, has the vertex set as in G and $n(2 \leq n \leq p)$ vertices are mutually adjacent in $SR_n(G)$ if and only if they are n-radial in G. While G is disconnected, any n vertices are mutually adjacent in $SR_n(G)$ if not all of them are in the same component. When $n = 2$, $SR_n(G)$ coincides with the radial graph $R(G)$. For a pair of graphs G and H on p vertices, the least positive integer n such that $SR_n(G) \cong H$, is called the Steiner completion number of G over H. When $H = K_p$, the Steiner completion number of G over H is called the Steiner radial number of G. In this paper, we determine 3-radial graph of some classes of graphs, Steiner radial number for some standard graphs and the Steiner radial number for any tree. For any pair of positive integers n and p with $2 \leq n \leq p$, we prove the existence of a graph on p vertices whose Steiner radial number is n.

Keywords: n-radius, n-diameter, Steiner n-radial graph, Steiner completion number, Steiner radial number.

1 Introduction

Throughout this paper, we consider finite undirected graphs without multiple edges and loops. Let G be a graph on p vertices and S a set of vertices of G. In [2], the Steiner distance of S in G, denoted by $d_G(S)$, is defined as the minimum number of edges in a connected subgraph of G that contains S. Such a subgraph is necessarily a tree and is called a Steiner tree for S in G. The Steiner n-eccentricity $e_n(v)$ of a vertex v in a graph G is defined as $e_n(v) = \max\{d_G(S) : S \subseteq V(G)$ with $v \in S$ and $|S| = n\}$. The n-radius $rad_n(G)$ of G is defined as the smallest Steiner n-eccentricity among the vertices of G, and the n-diameter $diam_n(G)$ of G is the largest Steiner n-eccentricity. The concept of Steiner distance was further developed in [3,6,5].

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In [4], KM. Kathiresan and G. Marimuthu introduced the concept of radial graphs. Two vertices of a graph G are said to be radial to each other if the distance between them is equal to the radius of the graph. The radial graph of a graph G, denoted by $R(G)$, has the vertex set as in G and two vertices are adjacent in $R(G)$ if and only if they are radial in G. If G is disconnected, then two vertices are adjacent in $R(G)$ if they belong to different components of G.

Any n vertices of a graph G are said to be n-radial to each other if the Steiner distance between them is equal to the n-radius of the graph G. The Steiner n-radial graph of a graph G, denoted by $SR_n(G)$, has the vertex set as in G and $n(2 \leq n \leq p)$ vertices are mutually adjacent in $SR_n(G)$ if and only if they are n-radial in G. If G is disconnected, any n vertices are mutually adjacent in $SR_n(G)$ if not all of them are in the same component. For the edge set of $SR_n(G)$, draw K_n corresponding to each set of n-radial vertices. By taking $n = 2, SR_n(G)$ coincides with $R(G)$. Consider the graph G given in Figure 1.

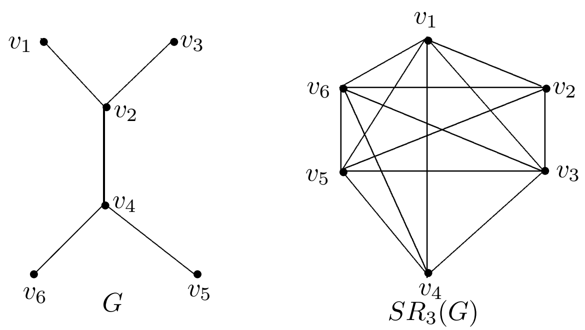

Fig. 1.

If we let $n = 3$, we get that $rad_3(G) = 3$ and that $S_1 = \{v_1, v_2, v_5\}$, $S_2 = \{v_1, v_2, v_6\}$, $S_3 = \{v_1, v_3, v_4\}, S_4 = \{v_1, v_4, v_5\}$, $S_5 = \{v_1, v_4, v_6\}$, $S_6 = \{v_2, v_3, v_5\}$, $S_7 = \{v_2, v_3, v_6\}, S_8 = \{v_2, v_5, v_6\}$, $S_9 = \{v_3, v_4, v_5\}$ and $S_{10} = \{v_3, v_4, v_6\}$ are the sets of 3-radial vertices of G. Hence the Steiner 3-radial graph of G is as shown in Figure 1. A graph G is called a Steiner 3-radial graph if $SR_3(H) \cong G$ for some graph H. If G does not contain K_3 as a subgraph, then G is not a Steiner 3-radial graph. The converse of this statement is not true. For example, the graph G given in Figure 2 is not a Steiner 3-radial graph.

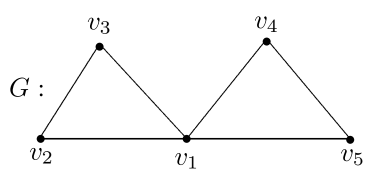

Fig. 2.

Let G and H be two graphs on p vertices. If there exists a positive integer n such that $SR_n(G) \cong H$, then H is called a Steiner completion of G. The positive integer n is said to be Steiner completion of G over H if n is the least positive integer such that $SR_n(G) \cong H$. For example, the Steiner completion number of bistar B_{p_1,p_2} over $K_{p_1+p_2+2}-e$ is p_1+p_2. If there is no n such that $SR_n(G) \cong H$, then the Steiner completion number of G over H is ∞. The Steiner completion number of G over H is not necessarily equal to the Steiner completion number of H over G. For the graphs G and H given in Figure 3, the Steiner completion number of G over H is 3 but the Steiner completion number of H over G is ∞.

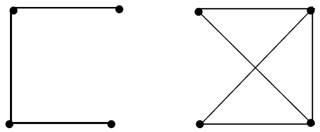

Fig. 3.

When $H = K_p$, the Steiner completion number of G over H is called the Steiner radial number of G. That is, the Steiner radial number $r_S(G)$ of a graph G is the least positive integer n such that the Steiner n-radial of G is complete. In this paper, we determine the Steiner radial number for some classes of graphs and obtain the Steiner radial number for any tree. Also we prove that for every pair of integers n and p with $2 \leq n \leq p$, there exists a graph on p vertices whose Steiner radial number is n. For graph theoretic terminology we follow [1].

2 Steiner 3-Radial Graphs of Some Classes of Graphs

Proposition 1. *Let P_p be any path on $p \geq 3$ vertices. Then $SR_3(P_p) = K_2 + \overline{K_{p-2}}$ where K_{p-2} is the complete graph on $p-2$ internal vertices of P_p.*

Proof. Let $P_p : v_1v_2 \cdots v_{p-1}v_p$ by any path on $p \geq 3$ vertices. Then the Steiner 3-eccentricity of each vertex of P_p is $p-1$ and hence $rad_3(P_p) = p-1$. Now for every vertex $v_i, 2 \leq i \leq p-1$, we have $d(\{v_i, v_1, v_p\}) = p-1$. Hence $\{v_i, v_1, v_p\}$ where $2 \leq i \leq p-1$ forms a K_3. So assume $p \geq 4$. Also for every pair of vertices v_i and $v_j, i \neq j$ and $2 \leq i, j \leq p-1$, there exists no $v_k, 1 \leq k \leq p, k \neq i$ and $k \neq j$ such that $d(\{v_i, v_j, v_k\}) = p-1$ and hence there is no edge between v_i and $v_j, i \neq j$ and $2 \leq i, j \leq p-1$. Therefore $SR_3(P_p) = K_2 + \overline{K_{p-2}}$ where K_{p-2} is the complete graph on $p-2$ internal vertices of P_p.

Proposition 2. *Let C_p be any cycle of length $p \geq 5$. Then*

$$SR_3(C_p) = \begin{cases} C_p(k) & \textit{if } p = 3k \\ C_p(k, k+1) & \textit{if } p = 3k+2 \\ C_p(k-1, k, k+1) & \textit{if } p = 3k+1. \end{cases}$$

Where the circulant graph $C_p(n_1, n_2, \ldots, n_l)$ is obtained from a cycle on p vertices by joining each vertex $v_i, 1 \le i \le p$ with the vertices v_{i-n_j} and v_{i+n_j}, the subscripts being taken modulo p, for $1 \le j \le l$. When $p = 3$, $SR_3(C_p) = K_3$ and when $p = 4, SR_3(C_p) = K_4$.

Proof. Clearly $SR_3(C_p) = K_3$ or K_4 when $p = 3, 4$ respectively.

Let $C_p : v_0, v_1, v_2, \ldots, v_{p-1}, v_0$ be any cycle of length $p \ge 5$.

Case 1. $p = 3k$.

In this case v_{i-k} and v_{i+k} are Steiner 3-eccentric vertices of v_i where the subscripts are taken modulo p. Also Steiner 3-eccentricity of a vertex v_i is $2k$ for every i and hence $rad_3(C_p) = 2k$. Thus v_{i-k} and v_{i+k} are the only vertices which are at a 3-radius distance with v_i. So that $\{v_i, v_{i-k}, v_{i+k}\}$ forms a K_3 in the corresponding Steiner 3-radial graph. Hence v_i is adjacent to v_{i-k} and v_{i+k} only. This is true for every i. Therefore we get $SR_3(C_p) = C_p(k)$.

Case 2. $p = 3k + 2$.

In this case we have three sets of Steiner 3-eccentric vertices of v_i namely $S_1 = \{v_{i+(k+1)}, v_{i-k}\}, S_2 = \{v_{i+k}, v_{i-(k+1)}\}$ and $S_3 = \{v_{i+(k+1)}, v_{i-(k+1)}\}$, where the subscripts are taken modulo p. Also their distance with v_i is $2k+1$. Thus $v_{i+(k+1)}$ and v_{i-k} are vertices which are at a 3-radius distance with v_i. So S_1 forms a K_3 in the corresponding $SR_3(G)$. Similarly each set of Steiner 3-eccentric vertices forms a K_3. Hence v_i is adjacent to $v_{i+k}, v_{i-k}, v_{i+(k+1)}$ and $v_{i-(k+1)}$. Therefore $SR_3(C_p) = C_p(k, k + 1)$.

Case 3. $p = 3k + 1$.

In this case we have six sets of Steiner 3-eccentric vertices of v_i namely, $\{v_{i+(k+1)}, v_{i-k}\}, \{v_{i+(k+1)}, v_{i-(k+1)}\}, \{v_{i+(k+1)}, v_{i-(k-1)}\}, \{v_{i+k}, v_{i-(k+1)}\}, \{v_{i+k}, v_{i-k}\}$ and $\{v_{i+(k-1)}, v_{i-(k+1)}\}$, where the subscripts are taken modulo p. Also their distance with v_i is $2k$. If we proceed as in the proof of Case 2, we get v_i is adjacent to $v_{i+k}, v_{i-k}, v_{i+(k+1)}, v_{i-(k+1)}, v_{i+(k-1)}$ and $v_{i-(k-1)}$ and hence $SR_3(C_p) = C_p(k - 1, k, k + 1)$.

Proposition 3. *Let G be any cycle C_p of length $p \ge 5$. Then $SR_3(\overline{G}) = C_p(1, 2)$ where $\overline{G}$ is the complement of G. When $p = 3, 4, SR_3(\overline{G}) = K_3, K_4$ respectively.*

Proof. If $G \cong C_3, C_4$ then $rad_3(\overline{G}) = \infty$. Hence we get $SR_3(\overline{G}) = K_3, K_4$ respectively. Let G be any cycle C_p of length $p \ge 5$ having the vertices $v_0, v_1, \cdots, v_{p-1}$. Then any vertex v_i in $\overline{G}$ is adjacent to all the vertices of $\overline{G}$ except v_{i-1} and v_{i+1}. Here $\{v_{i-1}, v_{i+1}\}, \{v_{i+1}, v_{i+2}\}$ and $\{v_{i-1}, v_{i-2}\}$ are the sets of Steiner 3-eccentric vertices of v_i. Also their distance with v_i is 3. That is in $\overline{G}$ $d(\{v_i, v_{i-1}, v_{i+1}\}) = d(\{v_i, v_{i+1}, v_{i+2}\}) = d(\{v_i, v_{i-1}, v_{i-2}\}) = 3$. So $e_3(v_i) = 3$ for all i and hence $rad_3(\overline{G}) = 3$. Thus $\{v_i, v_{i-1}, v_{i+1}\}, \{v_i, v_{i+1}, v_{i+2}\}$ and $\{v_i, v_{i-1}, v_{i-2}\}$ each forms a K_3 in the corresponding $SR_3(\overline{G})$. Similarly, each set of Steiner 3-eccentric vertices forms a K_3. Therefore v_i is adjacent to $v_{i-1}, v_{i+1}, v_{i+2}$ and v_{i-2}. Hence we get $SR_3(\overline{G}) = C_p(1, 2)$ for $p \ge 5$.

Proposition 4. *For $p_1 \leq p_2$,*

$$SR_3(K_{p_1,p_2}) = \begin{cases} K_{1+p_2} & \text{if } p_1 = 1 \\ K_{2+p_2} & \text{if } p_1 = 2 \\ K_{p_1} \cup K_{p_2} & \text{if } p_1 \geq 3. \end{cases}$$

Proof. Let $X = \{x_1, x_2, \ldots, x_{p_1}\}$ and $Y = \{y_1, y_2, \ldots, y_{p_2}\}$ be the bipartition of K_{p_1,p_2}.

Case 1. $p_1 = 1$.

Then $d(\{x_1, y_i, y_j\}) = 2$. We have $e_3(x_1) = 2$ and $rad_3(G) = 2$. So in the corresponding Steiner 3-radial graph, $\{x_1, y_i, y_j\}$ forms a K_3. Since y_i and y_j are arbitrary vertices of G, we have $SR_3(K_{1,p_2}) = K_{1+p_2}$.

Case 2. $p_1 = 2$.

Here $|X| = 2$ and $|Y| \geq 2$. Then $e_3(x_1) = e_3(x_2) = 2$ and hence $rad_3(K_{p_1,p_2}) = 2$. Thus for every pair of vertices $v_i, v_j \in V(K_{p_1,p_2})$, there exists a vertex v_k such that $d(\{v_i, v_j, v_k\}) = 2$. Therefore any two vertices are adjacent in $SR_3(K_{p_1,p_2})$. Hence we get $SR_3(K_{p_1,p_2}) = K_{2+p_2}$.

Case 3. $p_1 \geq 3$.

Here $|X| \geq 3$ and $|Y| \geq 3$. Let x_i be any vertex of X. Then $e_3(x_i) = 3$ and every two vertices in X different from x_i are the Steiner 3-eccentric vertices of x_i. Similarly for $y_i \in Y, e_3(y_i) = 3$ and every two vertices in Y different from y_i are the Steiner 3-eccentric vertices of y_i. Therefore $rad_3(K_{p_1,p_2}) = 3$ and hence $SR_3(K_{p_1,p_2}) = K_{p_1} \cup K_{p_2}$.

Theorem 5. *If G is a disconnected graph of order $p \geq 3$, then $SR_3(G) \cong K_p$.*

Proof. Let G be a disconnected graph with two components say G_1 and G_2. Then every vertex in G_1 is Steiner 3-eccentric with a vertex in G_2. Thus $e_3(v_i) = \infty$ for all i and hence $rad_3(G) = \infty$. Therefore for every two vertices v_i and v_j, there exists a vertex v_k in G such that $d(\{v_i, v_j, v_k\}) = \infty$. Hence we get $SR_3(G) = K_p$.

Theorem 6. *For any integer $n \geq 2$, there exists a graph G such that $rad_3(G) = n$.*

Proof. Let $n \geq 2$ be any integer. Construct a graph G by adding the vertices u, v, x and y with a path $P_{n-1} : v_1 v_2 \cdots v_{n-1}$ and join u, v to v_1 and x, y to v_{n-1} as shown below.

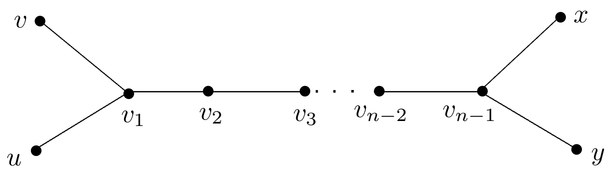

Fig. 4.

Then $e_3(v_i) = n$ for every $v_i \in V(P_{n-1})$. Also $e_3(u) = n + 1$. Similarly $e_3(v) = e_3(x) = e_3(y) = n + 1$. Then $rad_3(G) = \min\{e_3(v_i) : v_i \in V(P_{n-1}), e_3(u), e_3(v), e_3(x), e_3(y)\} = n$.

Problem 7. *Characterize all Steiner 3-radial graphs.*

3 Steiner Radial Number

Observation 8 *It follows from the definition that for any connected graph G on p vertices, $2 \leq r_S(G) \leq p$.*

Proposition 9. *If $r_S(G) = n$, then K_p is the only Steiner m-radial graph for G for $m \geq n$.*

Proof. For a graph G, let $r_S(G) = n$ and let r be the n-radius of G. Then there exists a vertex v in $V(G)$ such that $e_n(v) = r$. Let N be a n-element set containing v with Steiner distance r. Consider the set $N \cup \{x\}$, where $x \in V(G)-N$. Since Steiner n-eccentricity of any n-element set containing v is atmost r, the set $N \cup \{x\}$ is of Steiner distance either r or $r+1$ for any $x \in V(G) - N$. Otherwise a n-element subset of $N \cup \{x\}$ with v is of Steiner distance more than r.

By the same argument, $e_n(u) \leq e_{n+1}(u)$ for all $u \in V(G)$. The result follows when $(n+1)$-radius of G is r. If $(n+1)$-radius of G is $r+1$, then the vertex v in $V(G)$ has the minimum Steiner $(n+1)$-eccentricity $r+1$. Let v_i and v_j be any two vertices of G. Since v_i and v_j are adjacent in Steiner n-radial of G, there exists an n-element set S with Steiner distance r. If $r+1 = p-1$, then any set of $n+1$ elements containing v_i and v_j has the Steiner distance $r+1$. Suppose $r+1 < p-1$. If v does not belong to the Steiner tree containing S, then $S \cup \{v\}$ has Steiner distance $r+1$. If $v \in S$, then also we adjoint a vertex w in S which does not belong to the Steiner tree containing S such that the Steiner distance of $S \cup \{w\}$ is $r+1$. Hence any two vertices v_i and v_j are adjacent in Steiner $(n+1)$-radial of G. Hence the result follows.

Theorem 10. *$r_S(G) = 2$ if and only if G is either complete or totally disconnected.*

Proof. When G is complete (respectively a totally disconnected graph), 2-radius is 1 (respectively ∞) and any pair of vertices has Steiner distance 1 (respectively ∞). Hence $r_S(G) = 2$.

Suppose $r_S(G) = 2$. If G is not complete, then it has a pair of non-adjacent vertices u and v with $d(\{u,v\}) \geq 2$. If the 2-radius of G is 1, u and v are not adjacent in the Steiner 2-radial of G, a contradiction to $r_S(G) = 2$. If the 2-radius of G is ≥ 2, then we have $d(\{x,y\}) = 1$ for all $x, y \in V(G)$ where $(x,y) \in E(G)$, hence x and y are not adjacent in the Steiner 2-radial graph of G, so the edge-set must be empty.

Proposition 11. *For any star graph with p vertices,*

$$r_S(K_{1,p-1}) = \begin{cases} 2 & \text{for } p = 2 \\ 3 & \text{for } p \geq 3. \end{cases}$$

Proof. The case $p = 2$ follows directly from Theorem 10 as $K_{1,1} = K_2$. When $p = 3$, 2-radius of $K_{1,2}$ is 1 and Steiner 2-radial of $K_{1,2}$ is not complete. Also 3-radius of $K_{1,2}$ is 2 and Steiner 3-radial of $K_{1,2}$ is K_3. For $p \geq 4$, let v_1 be the vertex of degree $p-1$ and $v_2, v_3, \ldots, v_p$ be the pendant vertices of $K_{1,p-1}$. By Theorem 10, $r_S(K_{1,p-1})$ can not be 2. The 3-radius of $K_{1,p-1}$ is 2, since $e_3(v_1) = 2$ and $e_3(v_i) = 3, 2 \leq i \leq p$. In Steiner 3-radial of G, v_1 is adjacent to each vertex $v_i, 2 \leq i \leq p$, since the set $\{v_1, v_i, v_j (j \neq 1, i)\}$ has the Steiner distance 2. Also v_i is adjacent to v_j for $2 \leq i, j \leq p$ and $i \neq j$, since the set $\{v_1, v_i, v_j\}$ has the Steiner distance 2.

Proposition 12. *For any complete bipartite graph K_{p_1,p_2} with $p_1 \leq p_2$ and $p_1 \neq 1, r_S(K_{p_1,p_2}) = p_1 + 1$.*

Proof. Let $\{u_1, u_2, \ldots, u_{p_1}\}$ and $\{v_1, v_2, \ldots, v_{p_2}\}$ be the two partitions of K_{p_1,p_2}. When $n \leq p_1, e_n(u_i) = n, 1 \leq i \leq p_1$ and $e_n(v_i) = n, 1 \leq i \leq p_2$. Hence $rad_n(K_{p_1,p_2}) = n$. In Steiner n-radial of G, u_i is not adjacent to v_j, since the n-element sets containing u_i and v_j have only the Steiner distance $n-1$. Therefore, $r_S(K_{p_1,p_2}) > p_1$. When $n > p_1, e_n(u_i) = n-1, 1 \leq i \leq p_1$ and $e_n(v_i) \geq n-1, 1 \leq i \leq p_2$. Hence $rad_n(K_{p_1,p_2}) = n-1$.

In Steiner (p_1+1)-radial of G, u_i is adjacent to u_j for $1 \leq i, j \leq p_1$, u_i is adjacent to v_j for all $1 \leq i \leq p_1, 1 \leq j \leq p_2$ and v_i is adjacent to v_j for all $1 \leq i, j \leq p_2$, since each of the sets $\{u_1, u_2, \ldots, u_{p_1}, v_j\}, \{u_1, u_2, \ldots, u_{p_1}, v_j\}$ and $\{v_i, v_j, u_2, u_3, \ldots, u_{p_1}\}$ have the Steiner distance p_1 respectively. Hence Steiner (p_1+1)-radial of K_{p_1,p_2} is $K_{p_1+p_2}$.

Theorem 13. *For every tree T with $m(\neq p-1)$ pendant vertices $r_S(T) = m+2$.*

Proof. Let T be a tree with m pendant vertices $x_1, x_2, \ldots, x_m$ and the remaining vertices be $v_1, v_2, \ldots, v_{p-m}$. Then $e_n(v_i) = p-1$ for $n = m+1$ and hence $(m+1)$-radius is $p-1$. If v_iv_j is a non-pendant edge in T, then the set $\{v_i, v_j\} \cup X$, where $X \subseteq \{x_1, x_2, \ldots, x_m\}$ with $|X| = m-1$, has Steiner distance $< p-1$. Therefore, v_i is not adjacent with v_j in Steiner $(m+1)$-radial of G. Since $(m+2)$-radius is $p-1$ and any set $\{v_i, v_j, x_1, x_2, \ldots, x_m\}$ has Steiner distance $p-1$ for $1 \leq i, j \leq p-m$, Steiner $(m+2)$-radial of G is K_p.

Corollary 14 *For every positive integer $k \geq 2$, there exists a graph having Steiner radial number k.*

Proposition 15. *For any wheel,* $r_S(W_p) = \begin{cases} 2 & \text{for } p = 4 \\ 3 & \text{for } p \geq 5. \end{cases}$

Proof. When $p = 4$, the result follows from Theorem 10. So assume $p \geq 5$. Let v_1 be vertex of degree $p-1$ in W_p and $v_2, v_3, \ldots, v_p$ be the vertices on the cycle of W_p. Since W_p is not complete by Theorem 10, $r_S(W_p) > 2$. Since $e_3(v_1) = 2$ and $e_3(v_i) = 3, 2 \leq i \leq p, rad_3(G) = 2$. For $2 \leq i, j \leq p$ and $i \neq j$, the set $\{v_1, v_i, v_j\}$ has the Steiner distance 2 and hence the Steiner 3-radial of W_p is complete.

Theorem 16. *For any pair of integers n and p with $2 \leq n \leq p$, there exists a graph on p vertices whose Steiner radial number is n.*

Proof. When $p = 2$, the result is obvious. When $p = 3$, the only connected graph on 3 vertices are P_3 and K_3 in which $r_S(P_3) = 3$ and $r_S(K_3) = 2$. When $p = 4, r_S(K_4) = 2, r_S(C_4) = 3$ and $r_S(P_4) = 4$. When $p \geq 5, r_S(W_p) = 3$ by Proposition 15. Also $r_S(K_p) = 2$ and $r_S(T) = m + 2$ where m is the number of pendant vertices in T and $2 \leq m \leq p - 2$.

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