Degree Associated Edge Reconstruction Number

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Abstract. An edge-deleted subgraph of a graph G is called an *ecard* of G. An ecard of G with which the degree of the deleted edge is also given is called a *degree associated ecard* (or *da-ecard*) of G. The *edeck* (*da-edeck*) of a graph G is its collection of ecards (da-ecards). The *degree associated edge reconstruction number*, *dern*(G), of a graph G is the size of the smallest collection of ecards of G uniquely determines G. *The adversary degree associated edge reconstruction number*, *adern*(G), of a graph G is the minimum number k such that every collection of k da-ecards of G uniquely determines G. We prove that dern(G)= adern(G)=1 for any regular graph G or any bidegreed graph G with exactly one vertex of different degree, which differs by at least three. We determine dern and adern for all complete bipartite graphs except $K_{1,3}$. We also prove that dern$(G)\leq 2$ and adern$(G)\leq 3$ for any complete 3-partite graph G with n vertices in which all partite sets are equal in size as possible and a few other results.

Keywords: reconstruction number, edge reconstruction number, card, dacard.

1 Introduction

All graphs considered are nonempty, simple, finite and undirected. We shall mostly follow the graph theoretic terminology of [1]. Graphs whose vertices all have one of two possible degrees are called *bidegreed graphs.* A *balanced complete m-partite graph* of order n, denoted by $T_{m,n}$, is one whose vertex set can be partitioned into m subsets $V_1, V_2, ..., V_m$ (called partite sets) such that each vertex in V_i is adjacent to every vertex in V_j if and only if $i \neq j$ and $||V_i| - |V_j|| \leq 1$. A tree T is a *bistar* if it contains exactly two vertices that are not endvertices. The bistar with central vertices of degrees $m+1$ and $n+1$ is denoted by $D_{m,n}$. A *vertex-deleted subgraph* or *card* $G-v$ of a graph G is the unlabeled graph obtained from G by deleting the vertex v and all edges incident to v. The ordered pair $(d(v), G-v)$ is called a *degree associated card* or *dacard* of the graph G, where $d(v)$ is the degree of v in G. The *deck* (*dadeck*) of a graph G is its

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collection of cards (dacards). Following the formulation in [2], a graph G is *reconstructible* if it can be uniquely determined from its deck.

For a reconstructible graph G, Harary and Plantholt [3] have defined the *reconstruction number* $rn(G)$ to be the minimum number of vertex-deleted subgraphs which can only belong to its deck and not to the deck of any other graph, thus uniquely identifying the graph G. Myrvold [4] has studied, for a reconstructible graph G, the *adversary reconstruction number*, which is the minimum number k such that every collection of k cards of G is not contained in the deck of any other graph H, $H \ncong G$. For a reconstructible graph G from it's dadeck, Ramachandran [6] has defined the *degree associated reconstruction number* $drn(G)$ of a graph G to be the size of the smallest subcollection of the dadeck of G which is not contained in the dadeck of any other graph H, $H \ncong G$. The *edge reconstruction number, degree associated edge reconstruction number* and *adversary degree associated edge reconstruction number* of a graph are defined similarly with edge deletions instead of vertex deletions.

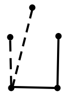
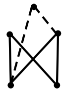
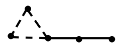
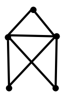
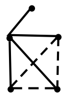
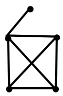
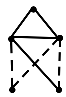
The *degree* of an edge e, denoted by $d(e)$, is the number of edges adjacent to e. That is, if $e = uv$ is an edge, then $d(e) = d(u) + d(v) - 2$. An *edge-deleted subgraph* (or *ecard*) $G - e$ of a graph G is the unlabeled graph obtained from G by deleting the edge e. The ordered pair $(d(e), G - e)$ is called a *degree associated ecard* or *da-ecard* of the graph G. The *edeck* (*da-edeck*) of a graph G is its collection of ecards (da-ecards). For an edge reconstructible graph G, the *edge reconstruction number* $ern(G)$ is defined to be the size of the smallest subcollection of the edeck of G which is not contained in the edeck of any other graph H, $H \ncong G$. For an edge reconstructible graph G from its da-edeck, the *degree associated edge reconstruction number* of a graph G, denoted by $dern(G)$, is the size of the smallest subcollection of the da-edeck of G which is not contained in the da-edeck of any other graph H, $H \ncong G$. The *adversary degree associated edge reconstruction number* of a graph G, $adern(G)$, is the minimum number k such that every collection of k da-ecards of G is not contained in the da-edeck of any other graph H, $H \ncong G$.

In this paper, we prove that $\text{dern}(G) = \text{adern}(G) = 1$ for any regular graph G or any bidegreed graph G with exactly one vertex of different degree, which differs by at least three. We also determine dern and adern for all complete bipartite graphs (except $K_{1,3}$), paths, wheels and bistars. Finally, we prove that $\text{dern}(G) \leq 2$ and $\text{adern}(G) \leq 3$ for any balanced complete 3-partite graph G.

2 dern and adern of Regular and Bidegreed Graphs

An *s-blocking* set of G is a family $\mathbb{F}$ of graphs such that $G \notin \mathbb{F}$ and each collection of s da-ecards of G will also appear in the da-edeck of some graph of $\mathbb{F}$. A graph non-isomorphic to G but having s da-ecards in common with G is called an *s-adversary-blocking graph* of G. The graphs $K_{1,3}$ and $K_3 \cup K_1$ are not edge reconstructible from their da-edeck. All other graphs G with $n \leq 4$ vertices have dern(G)=adern(G)=1. The graphs $K_{1,3} \cup K_1$ and $K_3 \cup 2K_1$ are not edge reconstructible from their da-edeck. Most other graphs G with $n = 5$

Table 1. Graphs G on 5 vertices with $dern(G) = 3$ or $adern(G) = 4$

G	dern(G)	adern(G)	2-blocking set	3-adversary-blocking graph
$C_4 \cup K_1$	3			
$K_{2,3}$	3			
$K_{2,3} - e$		4		
		4		$K_{2,3} - e$
		4		
		4		

vertices have dern(G)=adern(G)=1. The exceptions are given in Table 1; the dashed edges of graphs given in the table denote the edges correspond to the common da-ecards.

A *generator* of a da-ecard $(d(e), G - e)$ of G is a graph obtained from the da-ecard by adding a new edge which joins two nonadjacent vertices whose degree sum is $d(e) - 2$ and it is denoted by $H(d(e), G - e)$.

For a graph G, to prove $dern(G) = k$ ($adern(G) = k$), we proceed as follows.

(i) First find the da-edeck of G.
(ii) Determine next all possible generators of every da-ecard of G.
(iii) Finally, show that at least one generator other than G (every generator other than G) has at most $k - 1$ da-ecards in common with those of G, and that at least one generator has precisely $k - 1$ da-ecards in common with those of G.

Theorem 1. *If G is a bidegreed graph with exactly one vertex of different degree, which differs by at least three, then $dern(G) = adern(G) = 1$.*

Proof. Let G be a bidegreed graph of order n; let G have $n-1$ vertices of degree r and one vertex of degree s. Then G has $\frac{(n-1)r-s}{2}$ da-ecards with associated edge degree $2r-2$ and s da-ecards with associated edge degree $r+s-2$. If we join the vertices, which are the ends of the removed edge of the graph G, in the da-ecard $(2r-2, G-e)$, then $H(2r-2, G-e) \cong G$. To get a generator non-isomorphic to G, at least one of the two vertices to be joined must be different from these two ends. But then the degree sum of the two vertices to be joined is one of the four values namely $2r, 2r-1, r+s$ and $r+s-1$. Therefore, any graph non-isomorphic to G does not have the da-ecard taken as one of its da-ecards. Similarly, it can be easily proved that any da-ecard of G with associated edge degree $r+s-2$ uniquely determines G, which completes the proof.

Theorem 2. *If G is an r-regular non-empty graph, then $adern(G) = dern(G) = 1$.*

Proof. Each da-ecard of G is of the form $(2r-2, G-e)$. If we join the vertices, which are the ends of the removed edge of the graph G, in the da-ecard $(2r-2, G-e)$, then $H(2r-2, G-e) \cong G$. To get a generator non-isomorphic to G, at least one of the two vertices to be joined must be different from these two ends. But then the degree sum of the two vertices to be joined is either $2r-1$ or $2r$. Therefore, any graph non-isomorphic to G does not have the da-ecard taken as one of its da-ecards. Thus, $dern(G) = 1$ and $adern(G) = 1$.

Corollary 1. *If $G \cong K_n$ or C_n, then $adern(G) = dern(G) = 1$ for $n > 1$.*

We now determine dern and adern for paths, wheels, bistars and complete bipartite graphs.

Theorem 3. *If P_n is the path with n vertices, then $dern(P_n) = 1$ and* $adern(P_n) = \begin{cases} 1 & \text{if } n \leq 4 \\ 3 & \text{if } n > 4 \end{cases}$.

Proof. Since all graphs G (except $K_{1,3}$ and $K_3 \cup K_1$) of order at most four have $\text{dern}(G) = \text{adern}(G) = 1$ (Table 1), we assume that $n > 4$. Clearly, the generator $H(1, P_1 \cup P_{n-1})$ is isomorphic to P_n. The da-ecard of P_n with associated edge degree 2 has two components. If we join the vertices of different components, then the generator is isomorphic to P_n. If we join the vertices of same component (this is possible for $n > 4$), then the generator has two da-ecards in common with those of P_n with associated edge degree 2.

Theorem 4. *If W_n is the wheel with n (≥ 4) vertices, then $dern(W_n) = 1$ and* $adern(W_n) = \begin{cases} 3 & \text{if } n = 6 \\ 1 & \text{otherwise} \end{cases}$.

Proof. For $n = 4$, the wheel W_n is isomorphic to K_n and $adern(W_n) = dern(W_n) = 1$. So, let us take that $n \geq 5$. Then there are $n-1$ isomorphic da-ecards of the form $(4, W_n - e)$ and there are $n-1$ isomorphic da-ecards of the form $(n, W_n - e)$. There is one vertex of degree $n-2$ (≥ 3), $n-2$ vertices of degree 3, and one vertex of degree 2 in the da-ecard $(n, W_n - e)$. Since the $(n-2)$-vertex is adjacent to every other vertex except the 2-vertex, the two vertices to be joined must be different from the $(n-2)$-vertex. Therefore, the sum of degrees of the two vertices to be joined is 5 or 6. Thus, only for $n = 5$ or $= 6$, a graph non-isomorphic to W_n may have the da-ecard taken as one of its da-ecards. Therefore, it suffices to consider these two cases. When $n = 5$, there is only one 3-vertex non-adjacent to the 2-vertex in the da-ecard taken. If we join these two vertices, then $H(n, W_n - e)$ is isomorphic to W_n. When $n = 6$, the only graphs that have exactly one and two da-ecards in common with those of W_n are, respectively, H_1 and H_2 shown in Fig. 1. There are only one $(n-1)$-vertex, two 2-vertices and $n-3$ (≥ 2) vertices of degree 3 in the da-ecard $(4, W_n - e)$. Here the sum of degrees of the two 2-vertices is 4 and the generator $H(4, W_n - e)$ is isomorphic to W_n. The degree sum of all other two vertices is greater than 4. Hence, for $n \geq 5$, any graph non-isomorphic to W_n does not have the da-ecard in common with that of W_n with associated edge degree 4, which completes the proof.

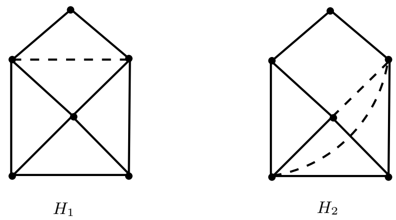


Fig. 1. The graphs H_1 and H_2

Theorem 5. *For a bistar* $D_{m,n}$ $(1 \leq m \leq n)$, $dern(D_{m,n}) = 1$ *and*

$$adern(D_{m,n}) = \begin{cases} 3 & \text{if } n = m+2, m = 2 \text{ or } n = 2 \\ 1 & \text{otherwise} \end{cases}.$$

Proof. Denote the bistar $D_{m,n}$ simply by G. We consider two cases according to whether $m = n$ or not.

Case 1. $m = n$

The graph G has $2n$ isomorphic da-ecards with associated edge degree n and one da-ecard with associated edge degree $2n$. The da-ecard $(n, G - e)$ has only one isolated vertex. To get a generator non-isomorphic to G, join two vertices different from the isolated vertex; this is possible only for $n = 2$. When $n = 2$, two endvertices are joined. If the two endvertices considered have a common neighbor in the da-ecard $(n, G - e)$, then the generator $H(n, G - e)$ has only one da-ecard in common with that of G with associated edge degree 2; Otherwise, the generator has exactly two da-ecards in common with those of G with associated edge degree 2.

The da-ecard $(2n, G-e)$ has two isomorphic components, each of which is $K_{1,n}$. If we join the two vertices of degree n, then the generator $H(2n, G-e)$ is isomorphic to G. If at least one vertex considered in the da-ecard has degree different from n, then the degree sum of the two vertices is less than $2n$. Thus, no graph non-isomorphic to G has any da-ecard in common with that of G with associated edge degree $2n$. Thus,

$$dern(D_{n,n}) = 1 \text{ and } adern(D_{n,n}) = \begin{cases} 3 & \text{if } n = 2 \\ 1 & \text{otherwise} \end{cases}.$$

Case 2. $m \neq n$

The bistar G has m isomorphic da-ecards with associated edge degree m, n isomorphic da-ecards with associated edge degree n, one da-ecard with associated edge degree $m+n$. For $m = 2$ only, there exists a graph non-isomorphic to G having maximum of two da-ecards in common with those of G with associated edge degree 2 as similar to the case of $m = n$. The da-ecard $(n, G-e)$ has an isolated vertex. Suppose the da-ecard has a unique n-vertex. If the isolated vertex is joined with the n-vertex, then $H(n, G-e)$ is isomorphic to G. Suppose the da-ecard has two n-vertices (This is possible when $m+1 = n$). If the isolated vertex is joined with any n-vertex, then the generator is isomorphic to G. To get a generator non-isomorphic to G, join two vertices different from the isolated vertex. This is possible only for two cases, namely $n = 2$ and $n = m+2$. When $n = 2$ (here $m = 1$ as $m < n$), the two endvertices are joined. The generator has two components, one of which is a 4-cycle and the other one is an isolated vertex. The da-ecard corresponding to each edge of the generator is a da-ecard of G with associated edge degree 2. Thus, the generator has two da-ecards in common with those of G with associated edge degree 2. When $n = m+2$, an endvertex is joined with the vertex of degree $m+1$. The generator is disconnected with two components, one of which is an isolated vertex and the other has a triangle. Since no da-ecard of G has a triangle, the da-ecard of the generator corresponding to the edge of the triangle can only be in common with that of G. Here the da-ecards of the generator corresponding to the two edges other than the edge whose ends are of equal degree in the generator are in common with those of G with associated edge degree n. It is clear that no graph non-isomorphic to G has a da-ecard in common with that of G with associated edge degree $m+n$. Hence,

$$dern(D_{m,n}) = 1 \text{ and } adern(D_{m,n}) = \begin{cases} 3 & \text{if } n = m+2, m = 2 \text{ or } n = 2 \\ 1 & \text{otherwise} \end{cases}.$$

Theorem 6. *If* $G = K_{m,n}, 1 \leq m \leq n,$ *then*

$$adern(G) = dern(G) = \begin{cases} 3 & \textit{if } m = 2, n = 3 \\ 2 & \textit{if } m \geq 3, n = m+1 \\ 2 & \textit{if } m \geq 2, n = m+2 \\ 1 & \textit{otherwise (except when } m = 1 \textit{ and } n = 3) \end{cases}.$$

Proof. Since all the da-ecards are isomorphic, $adern(K_{m,n}) = dern(K_{m,n})$. Let (A, B) be the bipartition of G, where $|A| = m$ and $|B| = n$. The graph G has mn da-ecards, all are isomorphic to $(m+n-2, K_{m,n} - e)$. In any generator of the da-ecard $(m+n-2, K_{m,n}-e)$, it holds that $m+n-2 = 2n-1, m+n-2 = 2n, m+n-2 = 2m-1$ or $m+n-2 = 2m$. Since $m \leq n$, it reduces to the two cases namely, $n = m+1$ or $n = m+2$.

Case 1. $n = m + 1$
In this case, a vertex of degree m in B is joined with the vertex of degree $m-1$ in B. Clearly, $m \geq 2$ (as otherwise the generator is isomorphic to G). Also, if $m = 2$, then G is isomorphic to $K_{2,3}$ and $dern(G) = 3$ (Table 1). Since all the da-ecards of $K_{2,3}$ are isomorphic, it follows that dern(G)= adern(G) = 3. Thus, we assume that $m \geq 3$. Now the generator has $m-1$ (≥ 2) triangles with each triangle has the newly added edge as the base. Therefore, removal of the newly added edge can only give a da-ecard in common with that of G. Thus, the generator has only one da-ecard in common with that of G.

Case 2. $n = m + 2$
In this case, the two vertices of degree m in B are joined in the da-ecard taken. If $m = 1$, then $n = 3$, which is excluded in the hypothesis of the theorem. So, we assume that $m \geq 2$. Now, the generator has m (≥ 2) triangles such that each triangle has the newly added edge as the base. Therefore, removal of the newly added edge can only give a da-ecard in common with that of G. Thus, the generator has only one da-ecard in common with that of G, which completes the proof.

3 *dern* and *adern* of Balanced Complete Tripartite Graphs

Theorem 7. *(i) For* $n = 3m$, $dern(T_{3,n}) = adern(T_{3,n}) = 1$.

(ii) For $n = 3m+1$, $dern(T_{3,n}) = \begin{cases} 1 & \text{if } m = 1 \\ 2 & \text{if } m \geq 2 \end{cases}.$

and $adern(T_{3,n}) = \begin{cases} 1 & \text{if } m = 1 \\ 3 & \text{if } m = 2 \\ 2 & \text{if } m \geq 3 \end{cases}.$

(iii) For $n = 3m+2$, $dern(T_{3,n}) = 1$ *and* $adern(T_{3,n}) = \begin{cases} 1 & \text{if } m = 1 \\ 3 & \text{if } m = 2 \\ 2 & \text{if } m \geq 3 \end{cases}.$

Proof. We denote $T_{3,n}$ simply by G; we consider three cases depending on the fact that $n \equiv 0, 1, 2 \pmod 3$.

(i) $n \equiv 0 \pmod 3$

In this case, $n = 3m$ for some m and the graph is a $2m$-regular graph, and hence, by Theorem 2, $dern(T_{3,n}) = 1$ and $adern(T_{3,n}) = 1$.

(ii) $n \equiv 1 \pmod 3$

Now $n = 3m+1$ for some integer m. Let (A, B, C) be the tripartition of G, where $|A| = |C| = m$ and $|B| = m+1$. The graph has $2m^2 + 2m$ isomorphic da-ecards with associated edge degree $4m-1$ and m^2 isomorphic da-ecards with associated edge degree $4m$. When $m = 1$, the generator $H(4m, G-e)$ is isomorphic to G. So, we take that $m \geq 2$. If we join the two vertices of degree $2m$, each one is adjacent to none of the $m-1$ vertices of degree $2m+1$, then the generator is isomorphic to G. To get a graph non-isomorphic to G, join two vertices different from these vertices. We join two vertices of degree $2m$ from set B and let the newly added edge be x. Clearly the da-ecard of the generator $H(4m, G-e)$ corresponding to the edge x is a da-ecard of G with associated edge degree $4m$. Any other da-ecard of H corresponding to the edge of degree $4m$ is not a da-ecard of G, since in the da-ecard of G, there are two non-adjacent $2m$-vertices having no common $(2m+1)$-neighbor, whereas the da-ecard of $H(4m, G-e)$ is not so. Also no da-ecard of H corresponding to the edge of degree $4m-1$ is a da-ecard of G, since the $(2m-1)$-vertex of the da-ecard of G is adjacent to each of the $(2m+1)$-vertices of the da-ecard, whereas the da-ecard of $H(4m, G-e)$ is not so. Thus, for $m \geq 2$, $H(4m, G-e)$ has only one da-ecard in common with that of G with associated edge degree $4m$.

When $m = 1$, the generator $H(4m-1, G-e)$ is isomorphic to G. When $m \geq 2$, the da-ecard $(4m-1, G-e)$ has only one $(2m-1)$-vertex and it is adjacent to every $(2m+1)$-vertex in the da-ecard and all the $m+1$ vertices of degree $2m$ induce a $K_{1,m}$. If we join the unique $(2m-1)$-vertex with the $2m$-vertex, which is non-adjacent to exactly $m-1$ vertices of degree $2m+1$, then the generator is isomorphic to G. To get a generator non-isomorphic to G, join the $(2m-1)$-vertex with a $2m$-vertex different from the vertex selected above. When $m = 2$, the generator $H(4m-1, G-e)$ (Fig. 2; the dashed edges correspond to the common da-ecards) has exactly two da-ecards, corresponding to the newly added edge and an edge adjacent with this edge, in common with those of G with associated edge degree $4m-1$. Any other da-ecard corresponding to the edge of degree $4m-1$ contains a $(2m-1)$-vertex adjacent with a $2m$-vertex or all the $2m$-vertices of the da-ecard are mutually adjacent, but which does not hold in the da-ecard of G.

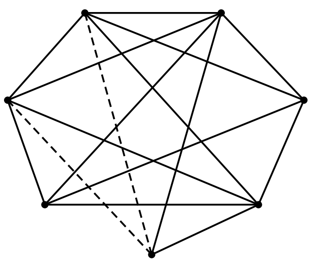


Fig. 2. The generator $H(4m-1, G-e)$

When $m > 2$, the generator has only one da-ecard in common with that of G with associated edge degree $4m - 1$. Since any graph non-isomorphic to G having a da-ecard in common with that of G with associated edge degree $4m$ does not have any da-ecard in common with that of G with associated edge degree $4m-1$, it follows that any graph non-isomorphic to G having a da-ecard in common with that of G with associated edge degree $4m - 1$ cannot have any da-ecard in common with that of G with associated edge degree $4m$. Hence

$$dern(T_{3,n}) = \begin{cases} 1 & \text{if } m = 1 \\ 2 & \text{if } m \geq 2 \end{cases} \text{ and } adern(T_{3,n}) = \begin{cases} 1 & \text{if } m = 1 \\ 3 & \text{if } m = 2 \\ 2 & \text{if } m \geq 3 \end{cases}.$$

(iii) $n \equiv 2 \pmod 3$

Here $n = 3m + 2$ for some integer m. Let (A, B, C) be the tripartition of the graph, where $|A| = |C| = m + 1$ and $|B| = m$. The graph has $m^2 + 2m + 1$ isomorphic da-ecards with associated edge degree $4m$ and $2m^2 + 2m$ isomorphic da-ecards with associated edge degree $4m+1$. Clearly, the generator $H(4m, G-e)$ is isomorphic to G, since in the da-ecard exactly two non-adjacent vertices with each one is of degree $2m$ and all other vertices are of degree greater than $2m$. Thus, it follows that no graph non-isomorphic to G has a da-ecard in common with that of G with associated edge degree $4m$. ...(E1)

If $m = 1$, then the generator $H(4m + 1, G - e)$ is isomorphic to G.

When $m = 2$, there is only one $2m$-vertex in the da-ecard $(4m+1, G-e)$ and all the $(2m + 1)$-vertices induce a tripartite subgraph $K_{m,m-1,m+1}$. Let D, E and F denote the set of $m, m - 1$ and $m + 1$ vertices in the tripartition of the induced subgraph, respectively. Then the $2m$-vertex is adjacent to all the vertices of the da-ecard $(4m + 1, G - e)$ except the vertices of E and F. The unique $(2m + 2)$-vertex in the da-ecard taken is adjacent to all the vertices of the da-ecard except the vertices of E. If we join the $2m$-vertex with the $(2m + 1)$-vertex, which is non-adjacent to that $(2m + 2)$-vertex, then the generator $H(4m + 1, G - e)$ is isomorphic to G. If we join the $2m$-vertex with any other $(2m + 1)$-vertex, then the generator has exactly two da-ecards in common with those of G with associated edge degree $4m + 1$.

If $m \geq 3$, then there is only one $2m$-vertex in the da-ecard $(4m + 1, G - e)$ and all the $(2m + 1)$-vertices induce a tripartite subgraph $K_{m,1,m+1}$. Let D, E and F denote, respectively, the set of m, 1 and $m + 1$ vertices in the tripartition of the induced subgraph. The $2m$-vertex is adjacent to all the vertices of the da-ecard $(4m + 1, G - e)$ except the vertices of E and F. Each $(2m + 2)$-vertex is adjacent to no other $(2m + 2)$-vertex. Also each $(2m + 2)$-vertex is adjacent to every other vertex except the $(2m + 1)$-vertex of the set E. If we join the $2m$-vertex with the $(2m + 1)$-vertex, non-adjacent to each of the $(2m + 2)$-vertices, then $H(4m + 1, G - e)$ is isomorphic to G. If we join the $2m$-vertex with any other $(2m + 1)$-vertex, then the generator has only one da-ecard in common with that of G with associated edge degree $4m + 1$ corresponding to the newly added edge. By (E1), when $m \geq 2$, these generators (non-isomorphic to G) have no da-ecard in common with that of G with associated edge degree $4m$. Hence,

dern($T_{3,n}$)= 1 and adern($T_{3,n}$)= $\begin{cases} 1 & \text{if } m = 1 \\ 3 & \text{if } m = 2 \\ 2 & \text{if } m \geq 3 \end{cases}$.

4 Conclusion

It follows, from their definitions, that dern(G) $\leq$ min {ern(G), adern(G)}. However, ern(G) and adern(G) are not comparable in general. For instance, adern($C_4 \cup 2K_1$) $= 3 =$ ern($C_4 \cup 2K_1$), adern($K_{1,4} \cup K_1$) $= 1 < 2 =$ ern($K_{1,4} \cup K_1$) and adern($K_3 \cup K_2 \cup K_1$) $= 4 > 2 =$ ern($K_3 \cup K_2 \cup K_1$). Moreover, if all the da-ecards of a graph G are isomorphic, then it is clear that $dern(G) = adern(G)$. But the condition is not necessary. For instance, the graph $G = P_4 \cup 2K_1$ has non-isomorphic da-ecards and dern(G)=adern(G)=1.

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