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On Present Logico-Methodological Challenges to Fuzzy Systems

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69.1 Introduction

Today fuzzy systems play an important role in the quantitative research. This is due to their novel approach to reasoning and concept formation. On the other hand, fuzzy systems have been controversial in particular among the philosophers and mathematicians. Hence, the dissemination of these new ideas has encountered many problems. Additional criticism has arisen because various logico-methodological approaches to the fuzzy systems are adopted as well as we still have quite many unresolved problems in our basic theories.

Below we consider some basic problems which still arise in the theories of fuzzy systems. We also sketch some tentative resolutions for them.

69.2 In the Beginning There Was Fuzziness

The first problem arises at the core of fuzzy systems, viz. the meaning of the concept *fuzzy*. From the etymological standpoint, this expression probably stems from the old German word *fussig* (spongy). Today researchers assume that fuzziness at least means imprecision. However, other concepts are also used, and prior to the fuzzy era, philosophers used such concepts as *vague* or even *inexact* instead [6, 14]. Many researchers also use the concept *uncertain* in this context.

If we assume that fuzziness in fact means imprecision, the mainstream interpretation is that fuzziness is included in certain linguistic expressions, and we consider them in the light of the extensional semantics. Hence, we may establish that a linguistic expression is imprecise, or fuzzy, if it refers to such entities (e.g., sets or relations) which have borderline cases. For example, *young person* is imprecise if its corresponding extension, the set of young persons, includes borderline cases [14]. In addition to this approach, we may consider imprecision from the linguistic standpoints of intensional semantics, syntax or pragmatism. Outside linguistics, epistemological and ontological approaches are also available [14]. This jungle of interpretations already provides us with various possible confusions. The concept of vagueness, in turn, is quite close to imprecision, but if we like to draw a distinction between these two, we may assume that vagueness also includes generality.

Uncertainty, with its various meaning components, is still fairly widely used within the fuzzy systems as a synonym for *imprecision*. In a strict sense, uncertainty

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is nevertheless an epistemological concept, i.e., related to our knowledge, whereas imprecision mainly belongs to semantics. Hence, this distinction should be clear. However, we have the everlasting debate on the meanings of imprecision and uncertainty within the fuzzy systems, and no consensus seem to appear [9].

Uncertainty is also examined in its "correct" form within the probability theory. In such approaches we may apply imprecise variables and probability distributions. Examples are the expressions *probability is fairly high* or *approximately normally distributed* [23]. The possibility theories, in turn, are also at least partially related to uncertainty and probability. On some occasions they are regarded as being the preliminary stages of probability [23].

Even though the linguistic aspects of *fuzziness* or *imprecision* have aroused discussions, the corresponding quantitative interpretations have been almost unanimous. They all apply traditional mathematics, and fuzziness is examined according to the membership functions. Hence, if the extension of an expression includes borderline cases, from the mathematical standpoint this means that some objects have only partial degrees of membership to its extension. This idea is a generalization of the characteristic function, and in traditional mathematics an isomorphism holds between these functions and the crisp sets. Hence, within fuzzy systems, we are not operating with fuzzy entities directly, but rather with the corresponding membership functions in our theory formation and model construction.

Since our research work is in practice based on such mathematical entities as membership functions of fuzzy sets and intensities of fuzzy relations, we have encountered some problems for finding a full correspondence between this logico-mathematical world and the linguistic world containing imprecise expressions. In addition, we should also find a correspondence between these and the real world. We will consider next this subject matter.

69.3 Correspondence between Fuzzy Systems and the Real World

In brief, within fuzzy systems we should find correspondences between the linguistic, logico-mathematical and real worlds. The linguistic world contains vocabularies as well as syntactic, semantic and pragmatic entities of languages. Within fuzzy systems our goal has been to specify such formal or quasi-natural languages which correspond well with the natural languages, and Lotfi Zadeh has performed a valuable work in this area [22, 19, 20, 21, 23, 24, 25]. A typical fuzzy quasi-natural language contains linguistic variables with such values as primitive terms, linguistic modifiers (hedges), connectives, quantifiers and qualifiers. Examples of such expressions are *young*, *very young*, *not old*, *young or very young*, *most fuzzy pioneers are old*, or *fuzzy systems very likely replace many traditional models in engineering*. At the core we usually apply Osgood scales for linguistic values in which case our linguistic scales are such as

$$P \text{ – more or less } P \text{ – neither } P \text{ nor } Q \text{ (neutral value) – more or less } Q \text{ – } Q$$

in which Q is the antonym of P. These scales usually contain an odd number of values (mostly five or seven). For example, given the linguistic variable Age, P and Q may be *young* and *old*, respectively. A more challenging task is nevertheless to integrate our linguistic framework in the logical structures, and this issue is considered next.

69.3.1 Reasoning and the Real World

In the worlds of fuzzy logics, we should operate fluently with our formal language, logical operators and inference rules and simultaneously our logics should mimic well the true human reasoning. From the mathematical standpoint, our mathematical operations should meet the foregoing challenges and still base on simple calculations. Typical crucial problems in this area are related to truth valuation and quantification, and we will consider them briefly.

In truth valuation the fuzzy community usually applies explicitly or implicitly the correspondence theory of truth, and this idea is also maintained in Alfred Tarski's well-known definition that [6, 9]

expression *X is P* is true if and only if X is P.

Hence, we assume that truth manifests the relationship between the linguistic and real world. For example, the linguistic expression *Snow is white* is true if and only if snow is white in the real world. However, various interpretations on this idea are available in fuzzy systems, in particular, when truth valuations are specified in practice.

Within fuzzy systems we also have to bear in mind that we apply many-valued logic. However, one crucial problem seems to be that confusions still prevail when non-true truth values are considered. In this context we should notice that within fuzzy logics there is a clear distinction between the values *true*, *false*, *not true* and *not false*. Unlike in the bivalent logics, *true* is now distinct from *not false* and *false* is distinct from *not true*. This is due to the fact that *not true* means anything else but true and *not false* means anything else but false. Hence, *not true* includes *false* and *not false* includes *true*.

We should thus apply these metarules when the truth values are assigned (iff $=_{df}$ if and only if) [14]:

1. *X is P* is true iff X is P.
2. *X is P* is false iff X is the antonym of P.
3. *X is P* is not true iff X is not P.
4. *X is P* is not false iff X is not the antonym of P.

For example,

1. *John is young* is true iff John is young.
2. *John is young* is false iff John is old.
3. *John is young* is not true iff John is not young.
4. *John is young* is not false iff X is not old.

If we are unable to find an appropriate antonym for the expression P, we may use its negation. Another challenge is to apply modified values in an appropriate manner. For example, if *X is more or less P* is true, what is the truth value of the expression *X is P*?

The author has also applied the idea of the Osgood scale in this context, and hence, also bearing in mind the foregoing metarules, we may evaluate truth according to the degrees of similarity between the given expressions and their true counterparts. True expressions have maximal and false expressions minimal degrees of similarity, respectively. In other words, we consider the similarity between P and R when

X is P, provided that X is R.

in which R is the true counterpart of P. The higher the degree of similarity between P and R, the closer our truth value is to truth. Hence, with true expressions it holds that $P = R$. As regards the modified expressions, we may establish, for example,

1. *John is more or less young* is fairly true, provided that John is young.
2. *John is middle-aged* is neither true nor false ("half-true"), provided that John is young.
3. *John is more or less old* is fairly false, provided that John is young.
4. *John is old* is false, provided that John is young.

This idea may also be applied to fuzzy quantifiers even though in this context we still have various unresolved problems. If our one extreme quantifier value is *all*, the other may be *none*. Hence, for example, we may apply the Osgood scale

none – some – many – most – all

In fuzzy numbers they could mean approximately 0 %, 25 %, 50 %, 75 % and 100 %, respectively. In this case the traditional existential quantifier, $\exists$, means *not none*. In truth valuation we should now combine the foregoing metarules with our quantifier rules [6, 14]. Hence, we may start with the Tarskian-type metarule

All Swedes are tall is true iff all Swedes are tall.

However, semantic and pragmatic problems arise when we should evaluate the truths of such expressions as

1. *All Swedes are tall*, provided that none of the Swedes are tall.
2. *All Swedes are tall*, provided that all Swedes are short.
3. *All Swedes are tall*, provided that all Swedes are more or less tall.
4. *All Swedes are tall*, provided that all Swedes are more or less short.
5. *Most Swedes are tall*, provided that some Swedes are tall.
6. *Most Swedes are tall*, provided that some Swedes are more or less tall.
7. *Some Swedes are more or less tall*, provided that many Swedes are more or less short.

Hence, certain basic problems should be resolved in an appropriate manner when fuzzy quasi-natural languages and their semantics are formulated.

The foregoing situation becomes even more problematic when the corresponding quantitative meanings, i.e., fuzzy set-theoretic and logical operations, should be assigned to the linguistic entities. The extension principle is widely used in this context [1], but it often yields inappropriate outcomes from the standpoint of actual human reasoning and our linguistic formulations. In particular in quantification we still expect plausible operations for connecting the linguistic entities to fuzzy set-theoretic entities.

This problem also applies to fuzzy syllogisms. From the linguistic and intuitional standpoints such typical fuzzy syllogisms as the fuzzy modus ponens and modus tollens seem plausible, but when we operate with their quantitative meanings in model construction, our outcomes do not correspond well with the linguistic framework. For example, the mainstream inference methods in a computer environment, the Mamdani and Takagi-Sugeno reasoning, do not correspond sufficiently well with the actual human reasoning even though they are good universal approximators in computer modeling. In fact, they are mathematical models for interpolation with fuzzy sets and useful in this sense. Hence, we still lack such inference engines in which both the inputs and outputs are plausible and normalized fuzzy sets.

69.3.2 General Methodology and the Real World

Resolutions in fuzzy reasoning provide us a basis for general methodological issues. Below we focus briefly on probability theory, hypothesis verification and approximate theories and explanations.

If we consider uncertainty in its mainstream sense as an epistemological issue, we should at least draw a distinction between objective and epistemic approaches. In the former case we assume that uncertainty may be an intrinsic property of entities or phenomena of the real world and thus independent of our knowledge. An example of this is the frequency approach to probability theory. In the epistemic approaches, such as subjective uncertainty, we assume that uncertainty, and probability, exist in the human minds and are thus dependent upon our knowledge. The degrees of uncertainty may now vary among persons.

As was already implicitly assumed above, probability theories usually provide us with useful methods for considering uncertainty. When an objective approach is adopted, we have viable fuzzy resolutions principally suggested by Zadeh [23]. These include the examinations on fuzzy probability variables and distributions [5, 10]. However, still more studies should be performed in particular in statistical analysis, for example, in statistical tests, reasoning and hypothesis verification. In the long term, fuzzy statistical systems should be included fluently in the traditional statistics and thus they may enhance these methods for coping with non-parametric and non-linear systems. Examples of these applications are cluster analysis, discriminant analysis, analysis of variance, analysis of covariance, time series analyses and various regression analyses. Special attention should be devoted to the novel dimension reduction methods because these are essential in both fuzzy and traditional modeling and the prevailing principal component and factor analyses are only appropriate to linear models.

The fuzzy epistemic approaches to uncertainty and probability are more challenging because then we often consider the probabilities of the unique occurrences on the psychological and purely logical grounds. For example, what is the probability that there is life on Mars? Hence, the resolutions provided by the experts may vary. In fact, in this context we evaluate the degrees of confirmation between our hypotheses and the given evidence [2, 17]. Unfortunately, the objective and epistemic approaches to probability are sometimes confused and this has led to the considerations of quasi-problems. Within the epistemic approaches the fuzzy systems are valuable if we apply their conceptions on truth to hypothesis verification. Hence, instead of only accepting or rejecting the hypotheses, we may assess their degrees of acceptance or rejection in a formal and logical manner. The application of the fuzzified modus ponens and modus tollens may open new prospects for this subject matter [11].

Another new frontier would comprise studies on approximate theories and scientific explanations. Today we apply these ideas more or less implicitly in the conduct of inquiry [15], but such novel approaches as Zadeh's fuzzy extended logic [25], would provide a more formal and consequential basis for these. If we consider the structures of theories from the standpoint of theory formation and their role in the conduct of inquiry, we usually examine the relationship between the theories and the real world. Hence, our theories may have truth values and their contents are more or less expected to correspond with the facts of the real world, in particular in scientific realism [4], [7], [8], [16], [18]. In this sense, approximate theories and explanations are in the neighborhood of their true counterparts [9, 12, 16, 18, 25]. For example, many precise theories are only true in idealized conditions and thus they only more or less approximately correspond with the real world. Hence, in fact we operate with their non-true counterparts in practice. In particular, this applies to many mathematical and statistical theories. The planets in our Solar System do not have exact elliptical orbits, or empirical statistical data sets are never exactly normally distributed. We also encounter this situation within the scientific explanations. They may only be true in the idealized conditions, and thus their non-true approximations are only available in practice.

As above, we may consider the truths of the theories and explanations in the light of their true counterparts. The closer they are to their true counterparts, the closer they are to truth, and Zadeh's fuzzy extended logic is a good roadmap for this examination [12, 13, 25]. However, much further studies should still be performed in this area.

The qualitative research, which is widely applied to the human sciences, also encounters many of foregoing logico-methodological problems. In addition, even though they principally apply imprecise entities and approximate reasoning, most of their research work is still based on manual work [3]. Hence, fuzzy systems may provide them with usable computer models if their linguistic models are transformed into fuzzy quasi-natural languages and fuzzy reasoning is also applied. Examples of qualitative research are content analysis, discourse analysis, action research, ethnographic research, phenomenographic research and various case studies. The most challenging tasks in this area are the modeling of the human interpretation and intentional behavior in a computer environment [3,18]. For example, how we can interpret well a given document in an automated manner and even write an abstract of it, or how we can also apply teleological explanations when we model the intentions, motives and other underlying factors which affect on person's behavior. Hence, in the qualitative research fuzzy systems still await for their golden age.

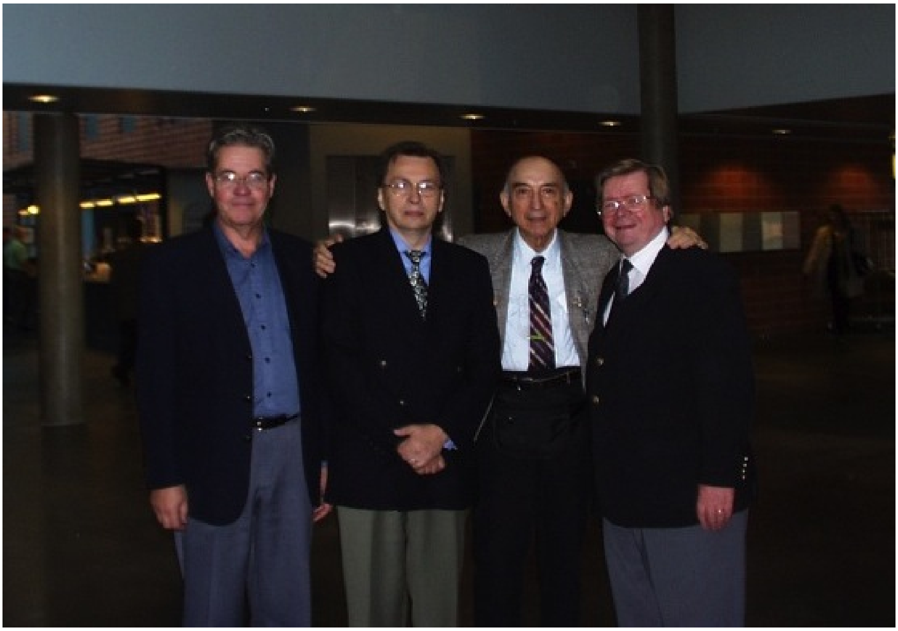

Fig. 69.1. From left to right: Erkki Oja, Vesa A. Niskanen, Lotfi Zadeh and Teuvo Kohonen in August 2000 at the Helsinki summer school on Soft Computing "Top Learning on Top of Europe"

69.4 Conclusions

We considered fuzzy systems from the logico-methodological standpoint. The fuzzy systems with such novel results as Zadeh's fuzzy extended logic open new prospects for the conduct of inquiry. They enable us to consider better such subject matters as scientific reasoning, theory formation, model construction, hypothesis verification and scientific explanation. Prior to the fuzzy systems, imprecise entities were only considered in an informal manner although their existence was already recognized in the scientific community.

However, we still encounter many problems and meet various challenges within the basic logico-methodological principles of fuzzy systems. Examples of these were truth valuation, quantification, fuzzy reasoning, probability modeling and approximate theories and explanations. These issues aroused problems at linguistic, logical and computer-modeling levels.

By resolving these problems we may provide a firm basis for our future studies as well as apply more intelligible methods to the conduct of inquiry.

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