STUDIES IN *FUZZINESS*
AND *SOFT COMPUTING*

Rudolf Seising
Enric Trillas
Claudio Moraga
Settimo Termini (Eds.)

On Fuzziness

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and Settimo Termini (Eds.)

On Fuzziness

A Homage to Lotfi A. Zadeh – Volume 2

Springer

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Lotfi Zadeh in his office at the University of California at Berkeley,
Soda Hall in August 2011.

A Foreword

When I started looking at the material in this awesome volume, the first thought that came to mind was "social network." While this term has been greatly overused nowadays by the media, this volume is clearly a social network with Lotfi Zadeh at the center. The term is even more appropriate in the case of Zadeh, who in addition to being a thinker of historical note, is an extremely social human being. In addition to providing inspiring technical ideas that have allowed many people in this network to carve out impressive careers of their own, Lotfi has often provided advice on matters both professional and personal to members of this network. Lotfi was never too busy to listen to the problems of others. I often observed that Lotfi had more patience listening to other's social problems than technical matters. These pieces help to provide views of Zadeh as if looking into a big house through different windows.

This volume, in addition to providing insights to the individual contributors' experiences with Lotfi either socially or technically, even more interestingly it provides the opportunity to experience in many cases, another dimension of the contributors. While I have known most of the contributors to this volume for many years, this is one of few, if not only occasion, I have had to read their writings on a non-technical and more personal subject. In many cases, I found this to be a rewarding and an eye opening experience as I am sure other readers of this book will find.

The inclusion of pictures tremendously enhances the pleasure of this volume. Not only are there pictures of Lotfi but enjoyable pictures of other members of the community. The pictures in this volume almost span the life of the idea of fuzziness. They include black and white pictures vintage pictures from the pre-digital days that are almost invaluable. These pictures inspire warm memories. For me, it was quite notable to observe the consistency of Lotfi's physical appearance over the long history that these pictures cover.

Lotfi spent many years on the outside trying to convince people of the value of his idea of fuzzy sets before the successful applications in Japan showed its usefulness. It is worth noting that these pioneering applications in Japan occurred at a time when Japan was a rapidly raising star in the world's technological and economic order; a fact that amplified and accelerated the worlds appreciation for fuzzy sets. In many ways Lotfi is still an outsider, in this case in his own fuzzy set community. Most of the applications of fuzzy sets are based on the Mamdani-Sugeno model. This paradigm is a kind of disjunctive approach, as we get more information we add possibilities. Zadeh's perspective, as conveyed with his paradigm of restriction-based semantics, is a kind of conjunctive approach, as we get more information we reduce possibilities.

Even now as he marches into his nineties and is unable to attend conferences and interact with fellow attendees as he so enjoys, Lotfi continues to build a social network. This time using the latest technology, the Internet, he has built a social network around his inspired idea of the Berkeley Initiative in Soft Computing. Every day I receive messages from people around the world via this network of interrelated scholars. These messages usually involve interesting ideas rather then simply being announcements of conferences as is the case with some other groups. The most interesting and challenging are those that come from Lotfi, particularly those related to his attempt to deal with the issue of causality.

The editors Rudolf Seising, Enric Trillas, Claudio Moraga and Settimo Termini are to be congratulated for coming up with such a wonderful idea to help celebrate a life as rich and human as Zadeh's in this manner.

Ronald R. Yager
New York City, July, 2012

Fig. 0.1. Lotfi A. Zadeh and Ron R. Yager talking in the living room of Zadeh's house in Berkeley, CA, on September 12, 2011. This talk is printed in *Mathware & Soft Computing – The magazine of the European Society for Fuzzy Logic and Technology*, vol 18 (1), December 2011, pp. 4-14.

Foreword by the Editors

It is without any kind of doubt that the work of Professor Lotfi A. Zadeh is of a great relevance in both the scientific and technological sides. His is a work that not only meant an important departure, but also a clear cut, from some old views on which thinking and research were anchored. He also spread his work all around the world through many lectures in many countries, and offered and continues to offer exciting new views to open-eyed people wishing to do research without being blocked by some old formal ways of looking at some theoretic or practical problems, impeding to pose them in a path allowing for its treatment.

It could be said that Zadeh opened a new paradigm in, at least, Science and Technology that was initially marked by the then surprising possibility of controlling physical systems whose behavior is empirically described by a set of linguistic rules with imprecise terms, but not by an 'exact' system of differential equations, when it exists, being usually computationally difficult to solve to obtain good enough values of its outputs. A new wave of researchers arose that, today, is followed by hundreds of researchers and engineers located in almost all parts of the world. Of this wave those authors contributing to this book are but a sample.

At the very beginning of the 'fuzzy adventure', just the presentation of the idea of 'Fuzziness' not only provoked violent oppositions, as it is often the case for innovative ideas, but also triggered, and, in a sense 'forced', a rethinking of a few crucial and debated problems aroused at the beginning of last Century in the field of the foundations of Mathematics and Logic. Also some papers tried to axiomatize the notion of 'fuzzy set' in order to take it as the starting point of a subsequent building of Mathematics. Besides remembering these first reactions to the then new emerging notion, we can today certainly affirm that the notion of Fuzziness stands as one of the *really* new concepts that have recently enriched the world of Science in the same good company of the ones of Computation, Information and Complexity, but also of Bohr's Complementarity. Science grows not only through technical and formal advances on one side and useful applications on the other side, but also by introducing and assimilating new concepts in its corpus. These, in turn, produce new developments and applications. Fuzziness has done all these things and will remain as one of the few new concepts aroused in the XX Century.

It is not usual that the founder of a new line of research can see in his life both the theoretical growing of it, as well as the success of some technological applications actually important from both the economical and the business points of view, as those coming from Fuzzy Logic and Soft Computing. This is the case with Professor Zadeh, an electrical engineer passionated by posing problems in mathematical terms that not only introduced the theoretical basis of Fuzzy Logic, but who also contributed a lot in its technological side with the insights coming from some of his many and single authored papers. Zadeh is not only well credited as the introducer of Fuzzy Logic and Soft Computing, but also and around twelve years ago, of the new field of 'Computing with Words' from which a new frontier for Computer Sciences is clearly visible and that can allow to afford yet unanswered questions in Philosophy, Linguistics, Science, Sociology, Technology and, last but not least, Industry. To qualify Zadeh as the 'father of fuzzy logic' is a good short description of its personality.

It is well known how broad is the spectrum covered by the work of Lotfi Zadeh. In a way, this seems to be reflected in the variety of contributions building this book. Some authors that contribute to this book chose to speak of personal meetings with Lotfi; others, about how particular papers of Zadeh opened for them a new research horizon. There are contributions documenting results obtained following ideas of Zadeh, thus implicitly acknowledging the inspiration he gave for those achievements. Finally, there are contributions of several 'third generation fuzzysists/softies' who were first lead into the world of Fuzziness by a disciple of Lotfi Zadeh, who, following his example, took care of opening for them a new road in science.

This book just aims at homaging both Professor Lotfi A. Zadeh's personality and work, once he surpassed his ninety years and is happily creative. His gentle attitude towards all people he met, as well as his wide tolerance with those that tried to contradict, and sometimes to blame, his contributions, is a characteristic of him that helped to approach many people to his ideas. Zadeh never refused the contact with and the offer of advise to young or yet inexpert people; never refused to gentle discuss in either a public or a private space on his thinking, and this in both spoken or written form. Zadeh is always in the opposite side of those kinds of great researchers who like to be distant and elevated from other people. For short, Zadeh is a nice human being who, aside of taking an exquisite care of his creature, likes to be in a close intellectual contact with people of all conditions.

The four editors of this book in homage to Professor Zadeh, all of them working from time ago in different areas of Fuzzy Logic or Soft Computing, would like to thank the multitude of authors contributing to its two volumes. This amount of people is a clear signal of the world-wide recognition reached by Zadeh's ideas.

Rudolf Seising, Enric Trillas, Claudio Moraga, Settimo Termini,
Mieres (Asturias, Spain), and Palermo (Italy),
October, the 30th, 2012

Genesis of the Book

I. Pre-history

When we started planning this book, born from discussions by the editors at the *European Centre for Soft Computing* (ECSC), we wrote the following letter to more than 500 scientists in the field of Soft Computing whose e-mail addresses we knew :

> Dear colleagues,
>
> In 2012 it will be 50 years that Professor Lotfi A. Zadeh used the word "fuzzy" for the first time in a scientific paper:
>
> "..., we need a radically different kind of mathematics, the mathematics of fuzzy or cloudy quantities which are not describable in terms of probability distributions."[1]
>
> It is also not to be forgotten that in about three and a half years, the theory of Fuzzy Sets and Systems (FSS) will be 50 years old, and that in this year 2011 its founder Lotfi A. Zadeh celebrated his 90th anniversary! It is our opinion that this 50 years long development of a now well-known theory that is used in technology, economics and other fields should have a mirror in the scientific literature. To this end we would like to edit a book entitled "On Fuzziness".
>
> At this remarkable point of time we think that it is important to have a printed collection of documents showing the history, the present stage and the future expectations from the own views of the protagonists.
>
> We will publish this documentation in a book and we invite you as well as other protagonists in the field of FSS, to contribute to this "homage" to the life-long work of Lotfi A. Zadeh. Furthermore, we also would like to invite and encourage scientists and researchers who have not been enthusiastic with FSS but who accompanied with their criticisms the genesis and the development of that field to participate in this book project, since, without their contribution, both the history and the prospect for its future would remain incomplete.

[1] Zadeh, Lotfi A.: From Circuit Theory to System Theory, *Proceedings of the IRE*, May 1962, pp. 856-865: 857.

Hence, we ask you to contribute with a short paper "on fuzziness" (about five (5) pages) from your personal point of view. We would like to ask you to mention in this non-technical contribution to the book how you did arrive to the field of FSS and to present your views and expectations "on fuzziness".

We also kindly ask you to include, if available, one or two photographs from the times that you will mention in your contribution.

We do not want to publish papers glorifying Lotfi A. Zadeh, because no one likes this kind of papers, nor he would like to see such a book.

We hope that you will contribute to this book and that you will help us to create a very good document on the history, the presence and the future views on our area of science and technology.

Please, send us your contribution as a Word-file before January 15, 2012!

When the first reactions appeared, we did not expect that we would have to create a two-volumes-book, but after some weeks it became clear that we would have to work with the manuscript of a collection of many pages. At the end of this procedure we had to distribute all the contributions on two volumes and it was almost impossible to find reasonable partitions of the different paper types. As a most sensible and fair solution we chose the alphabetical order relating to the first authors of each contribution. To have two volumes of almost the same size the first includes the papers "A - Ma" and the second includes the papers "Me – Z" and a *Postscriptum* of four special papers (see below).

II. Historical Troubles

Already one of the first examples that Lotfi Zadeh used in his seminal article "Fuzzy Sets" was the "class of all real numbers which are much greater than 1" – others were as we all know the "class of all beautiful women" and the "class of all tall men". He wrote that these classes "were not classes or sets in the usual mathematical sense of these terms" and "that it was a fact that such imprecisely defined 'classes' played an important role in human thinking, especially in the fields of pattern recognition, communication of information and abstraction."[2]

Today we know that they also play an important role in finishing book manuscripts. Most authors wrote that they would send their manuscripts "before the end of [x]" where $x \in \{$ January, February, March, ..., December $\}$ and also the year could have been 2011 or 2012. Some authors asked for waiting some time by using fuzzy concepts as the following examples show: "Give me a couple of days please." or "I need few more days." or "Certainly 10 days should be enough." or "Please wait for me. This weekend I will finish."

[2] Zadeh, Lotfi A.: Fuzzy Sets and Systems. In: Fox, Jerome (Ed.): *System Theory*, Microwave Research Institute Symposia, Series XV. Broooklyn, New York: Polytechnic Press, 1965. pp. 29–37: 29.

We got e-mails including the sentence “I will try to finish mine before he finishes his :-).” – And until we worked with this book manuscript for over one year, we are sure that the meaning of the following sentence is pretty fuzzy: “I will do my best.”

Concerning the requested contribution of “about five (5) pages” we got – indeed – papers of 5 pages but as the reader of the book will notice very quickly, there are also a couple of shorter papers and there are many longer papers. We cede it to our interested readers to find the right membership function of the class of papers of “about five (5) pages” in these two volumes.

III. More Historical Troubles

There are always exceptions! For some of the submitted contributions to this book we would not find anybody who would say that it has “about five (5) pages”. Thus, the membership values of these papers as an element to the set of “about five (5) pages’-papers” is almost zero. Even one of the editors used to think that fuzzified on page-numbers! We considered that these papers deserved not to be reduced, since they represent a comprehensive review of the past/present and a dream of the future. How was to handle these contributions? – We decided to have a “Postscriptum” at the end of this book (volume II) and we put these four contributions into this part.

IV. Figures and Photographs

There are two kinds of figures or pictures in these two volumes: usually authors of scientific papers use pictures, paintings, statistics, etc. to illustrate their findings and results in figures. Consequently, there are many of those figures in this book but we also asked the authors to look for old photographs that show themselves with Lotfi Zadeh and/or with other protagonists of the fuzzy community. Many of the authors went into cellars, attics, garages or any other crawl space where they assumed that they have such pictures – lost from view. They opened boxes, folders, binders, photo albums and yearbooks – may be for the first time since many years or decades – and therefore we received a huge amount of unknown pictures.

We are very glad that we can publish such photographs in these two volumes because some of them are important contemporary documents or at least nice memorabilia. Most of the photograph are privately owened by the authors and we publish them with their courtesy. Other photographs we have taken from the archive of one of the editors.[3]

[3] Figs. 0.1, 83.1, 86.1, 89.2 and 102.1 as well as the photographs that show Lotfi Zadeh page 7 of volume I (Thanks to Lotfi Zadeh for this gift!) and the one on page 7 of volume II of this book.

V. Additional Thanks

The editors are most thankful to the authors for their willingness to write their papers, to Prof. Dr. Janusz Kacprzyk for accepting the book in his series *Studies in Fuzziness and Soft Computing*, to Prof. Ron R. Yager for writing the Foreword, and last but not least to the Springer Verlag (Heidelberg) and in particular to Dr. Thomas Ditzinger, Leontina Di Cecco, and Holger Schäpe for helping this edition find its way to the publisher's list.

We thank the reviewers of the papers very much, particularly for their help we thank Luis Argüelles, Christian Borgelt, Lluis Godo, and Alejandro Sobrino; special thanks for proofreading a big number of contributions go to Brian R. Gaines!

VI. End

Finally, after having survived to all that without a single nervous attack, the last pending paper arrived, the last pictures were selected, and the editors could exclaim 'Good heavens! The book is ended!'. But then one of them, in low voice, added 'Not yet. The last section deserves a few lines with wishes for Lotfi'. Thus,

In the name of all those who contributed to this book, the editors would like to finally add: 'Long life to Professor Zadeh!'

Fig. 0.2. The editors of this book at the *Second Saturday's Scientific Conversations* (SSC) in Palazzo Steri, Palermo, Sicily, May 14, 2011. May be in this moment they were agreed to prepare the book in hand!

RS+ET+CM+ST,
Mieres (Asturias, Spain), and Palermo (Italy),
October, the 30th, 2012

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Part II

On Fuzziness, Me – Z

63

Interval Type-2 Fuzzy Logic in Hybrid Neural Pattern Recognition Systems

Patricia Melin

Abstract. We describe in this paper an overview of new methods that we have been working on for building intelligent systems for pattern recogni-tion using type-2 fuzzy logic and soft computing techniques. Soft Computing (SC) consists of several computing paradigms, including type-1 fuzzy logic, neural networks, and genetic algorithms, which can be used to create powerful hybrid intelligent systems. In this paper, we are reviewing the use of a higher order fuzzy logic, which is called type-2 fuzzy logic. Combining type-2 fuzzy logic with traditional SC techniques, we are able to build powerful hybrid intelligent systems that can use the advantages that each technique offers in solving pattern recognition problems.

63.1 Introduction

Fuzzy logic is an area of soft computing that enables a computer system to reason with uncertainty [2]. A fuzzy inference system consists of a set of if-then rules defined over fuzzy sets. Fuzzy sets generalize the concept of a traditional set by allowing the membership degree to be any value between 0 and 1. This corresponds, in the real world, to many situations where it is difficult to decide in an unambiguous manner if something belongs or not to a specific class. The main disadvantage of fuzzy systems is that they can't adapt to changing situations. For this reason, it is a good idea to combine fuzzy logic with neural networks or genetic algorithms, because either one of these last two methodologies could give adaptability to the fuzzy system. On the other hand, the knowledge that is used to build these fuzzy rules is uncertain. Such uncertainty leads to rules whose antecedents or consequents are uncertain, which translates into uncertain antecedent or consequent membership functions. Type-1 fuzzy systems, like the ones mentioned above, whose membership functions are type-1 fuzzy sets, are unable to directly handle such uncertainties. We also consider in this paper, type-2 fuzzy systems, in which the antecedent or consequent membership functions are type-2 fuzzy sets. Such sets are fuzzy sets whose membership grades themselves are type-1 fuzzy sets; they are very useful in circumstances where it is difficult to determine an exact membership function for a fuzzy set. Type-2 fuzzy systems have been applied with relative success in many real-world applications, like in control, time series prediction, classification and decision, diagnosis, and pattern recognition. Uncertainty is an inherent part of intelligent systems used in real-world applications. The use of new methods for handling incomplete information is of fundamental importance [1]. Type-1 fuzzy sets used in

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conventional fuzzy systems cannot fully handle the uncertainties present in intelligent systems. Type-2 fuzzy sets that are used in type-2 fuzzy systems can handle such uncertainties in a better way because they provide us with more parameters. Neural networks are computational models with learning (or adaptive) characteristics that model the human brain [3]. Neural networks can be classified in supervised and unsupervised. The main difference is that in the case of the supervised neural networks the learning algorithm uses input-output training data to model the dynamic system, on the other hand, in the case of unsupervised neural networks only the input data is given. In the case of an unsupervised network, the input data is used to make representative clusters of all the data. It has been shown, that neural networks are universal approximators, in the sense that they can model a continuous and bounded function to a specified accuracy and for this reason neural networks have been applied to problems of system identification, control, diagnosis, time series prediction, and pattern recognition. We have worked on special structures called modular and ensemble neural networks. Basically, a modular or ensemble neural network uses several monolithic neural networks to solve a specific problem. The basic idea is that combining the results of several simple neural networks we will achieve a better overall result in terms of accuracy and also learning can be done faster and fuzzy logic is the best approach to combine or aggregate the outputs of the modules Genetic algorithms and evolutionary methods are optimization methodologies based on principles of nature [4]. Both methodologies can also be viewed as searching algorithms because they explore a space using heuristics inspired by nature. Genetic algorithms are based on the ideas of evolution and the biological process that occur at the DNA level. Basically, a genetic algorithm uses a population of individuals, which are modified by using genetic operators in such a way as to eventually obtain the fittest individual. Any optimization problem has to be represented by using chromosomes, which are a codified representation of the real values of the variables in the problem. Both, genetic algorithms and evolutionary methods can be used to optimize a general objective function. In particular, evolutionary methods can be used to optimize the structure and parameters of neural networks and fuzzy systems, which is required in applications to achieve optimal results.

63.2 Type-2 Fuzzy Logic Applications in Pattern Recognition

One approach that we have worked on for face recognition uses modular neural networks with a fuzzy logic method for response integration [4]. The method for achieving response integration is based on the fuzzy Sugeno integral and type-2 fuzzy logic. Response integration is required to combine the outputs of all the modules in the modular network. We have applied the new approach for face recognition with a real database of faces from students and professors of our institution. The results of the modular neural network approach gives excellent performance overall and also in comparison with the monolithic approach. Also, the method for achieving response integration is based on the fuzzy Sugeno integral. Response integration is required to combine the outputs of all the modules in the modular network. Another approach has been the use of neural networks, fuzzy logic and genetic algorithms for voice recognition [4]. In particular, we have considered the case of speaker recognition by analyzing the sound signals with the help of intelligent techniques, such as the neural networks and fuzzy systems. We use the neural networks for analyzing the sound signal of an unknown speaker, and after this first step, a set of type-2 fuzzy rules is used for decision making. We need to use fuzzy logic due to the uncertainty of the decision process. We also use genetic algorithms to optimize the architecture of the neural networks. We have also considered the use of three modular neural networks as systems for recognizing persons based on the iris biometric measurement of humans [5]. In these systems, the human iris database is enhanced with image processing methods, and the coordinates of the center and radius of the iris are obtained to make a cut of the area of interest by removing the noise around the iris. The inputs to the modular neural networks are the processed iris images and the output is the number of the person identified. We also have worked on human recognition from ear images as bio-metric using modular neural networks with preprocessing ear images as network inputs [5]. In this case, we have proposed a modular neural network composed of twelve modules, in order to simplify the problem making it smaller. Comparing with other biometrics, ear recognition has one of the best performances, even when it has not received much attention. The Recognition results achieved with this approach were excellent.

Fig. 63.1. Prof. Zadeh with the Mexican hat at the IFSA 2007 Congress

We have also proposed a new approach for human recognition using as information the combination of three biometric measures, iris, ear, and voice of a person [5]. Now we have considered the integration of these three biometric measures to improve the accuracy of human recognition. The new approach integrates the information from three main modules, one for each of the three biometric measures. The new approach consists in a modular structure that contains three basic modules: iris, ear, and voice. The final decision is based on the results of the three modules and uses type-2 fuzzy logic to take into account the uncertainty of the outputs of the modules.

Fig. 63.2. Prof. Zadeh receiving an Award during the IFSA 2007 banquet

63.3 Motivation by Prof. Zadeh's Work

The inspiring ideas and research work of Prof. Zadeh have been fundamental in my own work [6], [7], [8]. He has always supported my research group's work and kindly accepted our invitation to offer a keynote lecture at the World IFSA 2007 Congress that was held in Cancun, Mexico in 2007 (in which I was Program Chair), which was a very important lecture, especially for Latin America and Mexico. In Figure 63.1 we show a photo of Prof. Zadeh with the classical Mexican hat during the banquet of IFSA 2007. Also, in Figure 63.2 Prof. Zadeh is receiving an Award from Prof. Melin during the banquet of IFSA 2007.

63.4 Conclusions

In this paper an overview of new methods for building intelligent sys-tems for pattern recognition using type-2 fuzzy logic and soft computing techniques was presented. Type-2 fuzzy logic is of fun-damental importance in the area of pattern recognition as a way to manage uncertainty in decision making and will be used with more frequency in the future. We are grateful to Prof. Zadeh's original work in this area and also for his support to work along this line of research.

References

1. Castillo, O., Melin, P.: Soft Computing and Fractal Theory for Intelligent Manufacturing. Physica-Verlag, Heidelberg (2003)
2. Castillo, O., Melin, P.: Type-2 Fuzzy Logic: Theory and Applications. STUDFUZZ, vol. 223. Springer, Heidelberg (2008)
3. Melin, P., Castillo, O.: Modelling, Simulation and Control of Non-Linear Dynamical Systems. Taylor and Francis, London (2002)
4. Melin, P., Castillo, O.: Hybrid Intelligent Systems for Pattern Recognition. STUDFUZZ, vol. 172. Springer, Heidelberg (2005)
5. Melin, P.: Modular Neural Networks and Type-2 Fuzzy Systems for Pattern Recognition. STUDFUZZ, vol. 389. Springer, Heidelberg (2012)
6. Zadeh, L.A.: Fuzzy Logic = Computing with Words. IEEE Transactions on Fuzzy Systems 4(2), 103 (1996)
7. Zadeh, L.A.: Knowledge Representation in Fuzzy Logic. IEEE Transactions on Knowledge Data Engineering 1, 89 (1989)
8. Zadeh, L.A.: Fuzzy Logic. Computer 1(4), 83–93 (1998)

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Type-2 Fuzzy Sets and Beyond

Jerry M. Mendel

64.1 Introduction

This chapter explains why I started working on type-2 fuzzy sets and systems and the contributions made by my students, colleagues and myself to this field. It also predicts a very rosy outlook for this field.

64.2 Entry into Type-2 Fuzzy Sets

After working on type-1 fuzzy sets (T1 FSs) for close to a decade (see [18] for some history about how I started working on T1 FSs and some of our contributions) I shifted my attention to type-2 fuzzy sets (T2 FSs). Why? My earlier background in estimation theory and statistical signal processing moved me in the direction of trying to understand how a fuzzy logic system (FLS) could handle uncertainties. It seemed to me that the fuzzy sets that everyone was using did not have enough flexibility for them to incorporate an uncertainty such as non-stationary noise, or a histogram of consequents, as might be established from a group of subjects all of whom did not provide the same consequents for a rule. In Zadeh's 1975 three-part magnum opus [26] he introduced fuzzy sets of type-2 (now called T2 FSs) in which membership grades were themselves fuzzy sets. One could call such fuzzy sets "fuzzy-fuzzy sets." A T2 FS was exactly the kind of fuzzy set that we needed in order to continue our work [17], [19].

Another reason that I moved into T2 FSs may be amusing to the reader. Between 1994 and 1995 I supervised a sophomore undergraduate student, Matt Martin, who wanted to do some research on fuzzy sets and systems. We agreed that by the end of the academic year we would write an article that explained fuzzy logic (FL) to high school students. It very quickly became apparent that we needed a really good application. After some brainstorming, we agreed that flirtation would be an excellent application, because it would be of great interest to high school boys and girls, and because of the familiar flirtation adage "I'm getting mixed signals," and the potential for this to be explained in terms of firing of more than one rule and aggregating their outputs, all using FL. It was during this work that we carried out surveys of groups of students and encountered three situations that we felt could not be handled by T1 FSs: (1) survey data about rule consequents led to a histogram of consequent possibilities for each rule; (2) survey data about the relative importance of the indicators of flirtation also led to a histogram of weight possibilities for each indicator; and,

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(3) survey data about membership function values for the terms used to describe the indicators of flirtation, as well as flirtation, demonstrated that there was serious disagreement about this information. All of this led us to examine type-2 fuzzy sets. The report [14] that we wrote about this work can be downloaded from my website (http://sipi.usc.edu/ mendel). Many people have told me that they enjoyed learning about FL by reading this report.

64.3 Historical Background of Our Early T2Works

A[1] T2 FS[2] was introduced by Zadeh in [26]; however, until the works of Karnik and Mendel [5] no one had extended a T1 FLS (e.g., [25]) to a T2 FLS. In retrospect, the major obstacles to doing this were: (1) characterization of T2 FSs; (2) performing operations with T2 FSs; (3) inferencing with T2 FSs; and (4) going from the output of a T2 inference engine (see Fig. 1), which is a T2 FS, to a defuzzified value, which is a type-0 set. All of these obstacles were overcome with the introduction of some new concepts, which are briefly described next.

Characterizing a T2 FS is not as easy as characterizing a T1 FS. Instead of being two-dimensional, as a T1 FS is, a T2 FS is three-dimensional[3]. It is this additional dimension that lets uncertainty be handled within the framework of FL. The concept of a *footprint of uncertainty*,[4] along with the associated concepts of *lower and upper membership functions* (first described in [16]) lets us easily characterize T2 FSs. The concept of an *active branch* (first described in [11]) of a lower or upper membership function lets us design an interval T2 FLS-IT2 FLS (i.e., a T2 FLS whose T2 FSs use interval sets to characterize their fuzziness[5]).

[1] Portions of this section are taken from Section 1.3 of [17].

[2] A type-2 fuzzy set can be thought of as a type-1 fuzzy set on steroids. Its membership function no longer has a single value at each value of the primary variable, but instead is a blurred version of that function, i.e., at each value of the primary variable the membership is itself a function, called a secondary membership function. When the secondary membership function is a constant equal to 1, the type-2 fuzzy set is called an interval type-2 fuzzy set or an interval-valued fuzzy set; otherwise, it is called a general type-2 fuzzy set.

[3] The membership function of a T2 FS is three-dimensional, with x-axis called the primary variable, y-axis called the secondary variable (or primary membership) and z-axis called the MF value (or secondary MF value). A vertical slice is a plane that is parallel to the MF-value axis.

[4] The FOU of a T2 FS lies on the x-y plane (i.e., the primary and secondary variable plane) and includes all points on that plane for which the MF value is non-zero. It is the 2D-domain on which sit the secondary membership values. The term FOU does not appear in the earliest works of Karnik and Mendel [5], [6], [7]. It was coined by Mendel as a simple way to verbalize and describe the two-dimensional domain of support for a T2 FS's membership function, and appears for the first time in [10] and [8].

[5] Because an IT2 FS is a T2 FS all of whose secondary MF values equal one, there is no information contained in those secondary MF values, and so an IT2 FS is characterized just by its FOU.

We who use T1 FS theory are so used to performing the common operations of union, intersection, and complement that we take them for granted. How to perform these operations is covered in every book about (T1) FSs and logic. Operating with T2 FSs-obtaining their union, intersection, and complement-is another matter. Although some work on how to do this existed in the literature before the works of Karnik and Mendel (e.g., [8]), it had not been developed far enough to be very practical. By focusing on a very special but very useful kind of T2 FS-the interval T2 set (also called an interval-valued FS)-and using the concepts of lower and upper membership functions, it is very easy to perform all of these operations.

The *sup-star* composition is the fundamental mapping from the fuzzy input sets that excite the inference mechanism (Fig. 64.1) to its output. All T1 FLSs make use of it, and it can be viewed as a nonlinear mapping of a T1 input fuzzy set into another T1 output set. A comparable result for T2 FSs needed to be developed. This was done by using Zadeh's Extension Principle, and is called the *extended sup-star composition*.[6] All type-2 FLSs make use of it, and it can be viewed as a nonlinear mapping of a T2 input fuzzy set into another T2 output fuzzy set. To perform the calculations associated with the extended sup-star composition one needs to use the operations of union and intersection for T2 FSs; so, the developments of practical algorithms for these operations came in quite handy.

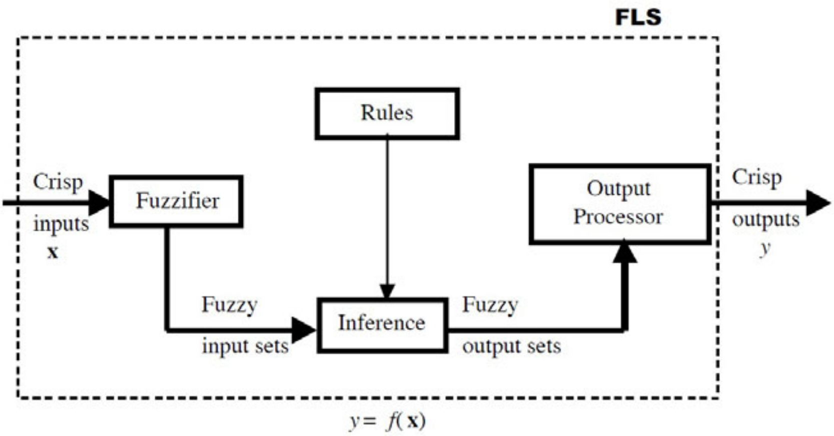

Fig. 64.1. Fuzzy logic system

Going from the output of a T2 inference engine (see Fig. 64.1), which is a T2 FS, to a defuzzified value, which is a type-0 set, was virgin territory. Inspired by what we do in a T1 FLS, when we defuzzify the (combined) output of the inference engine

[6] Dubois and Prade [2], [3] gave a formula for the composition of type-2 relations, using the minimum t-norm as an extension of the type-1 sup-min composition. Karnik and Mendel [5], [6], [7] demonstrated the validity of their formula for product as well as minimum t-norms.

using a variety of defuzzification methods many of which do some sort of centroid calculation, it became clear that the concept of the *centroid of a T2 FS* was needed. Using the Extension Principle, Karnik and Mendel [6], [9] defined the centroid of a T2 FS; it is a T1 FS. Associated with this new concept are the related new concepts of *embedded T2 and T1 FSs*[7] (first described in [6]). These sets are easy to visualize on the FOU of a T2 FS, and let us interpret a T2 FLS as a collection of T1 FLSs.

Computing the centroid of a general T2 FS can be very intensive, because the number of its embedded T2 FSs can be enormous; however, for an interval T2 FS (IT2 FS), an exact method for computing its centroid was developed [6], [9]. This was possible because the centroid of an IT2 FS is an IT1 FS. Interval sets are completely characterized by their left- and right-end points; hence, computing the centroid of an IT2 FS only requires computing the centroid of two embedded T1 FSs one each for the left- and right-end points of this centroid. The method for computing the centroid is encapsulated in two algorithms, called the *Karnik-Mendel* (or *KM*) *Algorithms*. These algorithms are iterative; however, they converge very quickly [13], [20]. One of them computes the left-end of the centroid and the other computes the right-end of the centroid.

Returning to the output of the inference engine in a T2 FLS, it is a T2 FS. In fact, one gets one such set for each rule that is fired by the input to the inference engine, and, in general, (just as in a T1 FLS) more than one rule will be fired. These T2 FSs can be combined in different ways, just as they can be in a T1 FLS. The result is another T2 FS. The operation that maps this T2 FS into a T1 FS is called type-reduction (first described in [5], [6], [7]), which was also a new concept.

Just as there are many different kinds of centroid-based defuzzifiers, there are many different and comparable type-reducers, but all are based on computing a centroid. Type-reduction is easy for IT2 FSs, and leads to an IT1 FS. Going from it to a defuzzified output for the IT2 FLS is simple-just average the end-points of the interval type-reduced set.

Putting all of these new concepts together lets us mathematically describe an IT2 FLS, just as we can mathematically describe a T1 FLS. Doing this then lets us develop design procedures for IT2 FLSs that are analogous to those that have already been developed for T1 FLSs.

Extensions of these results from IT2 FLSs to general T2 FLSs are now well underway (e.g., [12], [22], [24]). Some contributors to the T2 literature are shown in Fig. 64.2.

64.4 Is Type-Reduction Needed?

Type-reduction may be an obstacle to using T2 FLSs in real-time applications, because of the iterative nature of the KM algorithms, which may introduce an unacceptable time-delay into a system. For non-real time applications TR is not a problem.

[7] An embedded T2 FS sits on a T1 FS that is contained within the FOU (and is also called an embedded T1 FS) and has a non-zero MF value that sits atop that T1 FS. So, it is a very simple T2 FS. The FOU is covered by the embedded T1 FSs of the T2 FS.

All sorts of ways have been developed to bypass TR. Instead of describing them, I would like to focus on the more fundamental question: *Is TR needed?*

Karnik and Mendel [5], [6] introduced the following *fundamental design requirement* for a T2 FLS: When all sources of [membership function] uncertainty disappear, a T2 FLS must reduce to a comparable T1 FLS. This design requirement is analogous to what happens to a probability density function when random uncertainties disappear. In that case, the variance of the pdf goes to zero, and a probability analysis reduces to a deterministic analysis. So, just as the capability for a deterministic analysis is embedded within a probability analysis, the capability for a T1 FLS is embedded within a T2 FLS.

Fig. 64.2. Photo taken at FUZZ-IEEE 2005, Reno, Nevada. Front row (right-to-left): Woei Wan Tan, Jerry M. Mendel, and Dongrui Wu (who had just received the best student papers award), and Salang Musikasuwan. Back row (right-to-left): Jon Garibaldi, Bob John, Simon Coupland and Chris Lynch (student of Hani Hagras who is not in the photo).

All of Karnik and Mendel's TR methods satisfy this fundamental design requirement, because when all sources of MF uncertainty disappear, each of the TR methods reduces to its comparable T1 defuzzification method.

Interestingly enough, TR became burned into the architecture of a T2 FLS because Karnik and Mendel first developed all of their T2 concepts and calculations for a general T2 FS and FLS. Because TR was so simple for an IT2 FLS it was kept in the architecture of the much simpler IT2 FLS [11], [21]. There is nothing wrong with doing this; however, in retrospect we may have been blind-sided by the need for TR in a general T2 FLS from asking the question "Is TR really needed in an IT2 FLS?"

In fact, there are many ways to go from an IT2 FLS to a number that bypass TR and still satisfy the fundamental design requirement.

A student in a class that I taught some years ago asked: "Instead of performing TR, why can't we just use a combination of two T1 FLSs, one that uses only the lower membership functions and the other that uses only the upper membership functions?" My answer at that time was: "You can't do this because each end-point of the type-reduced set uses a mixture of lower and upper membership function information." While my answer was technically correct, it was predicated on using type-reduction, rather than on what the student had suggested. My answer today would be: "You can do what you are suggesting, and this can be done in different ways; however, by bypassing TR you may not be able to provide a measure of the uncertainties that have flowed through all of the IT2 FLS computations (analogous to a standard deviation)." For example, you could begin with the architecture of an IT2 FLS as a linear combination of two T1 FLSs, as in [1], or as the centroid of the average of the lower and upper membership functions of the aggregated rule fired sets [23]. More ways for using a mixture of the lower and upper membership functions can be found in [4]. All of these IT2 FLSs go directly to the defuzzified output value and they all satisfy the fundamental design requirement.

64.5 Looking into the Future

The outlook for T2 FSs, both IT2 and general T2, as well as for IT2 FLSs and general T2 FLSs is very good. As of December 15, 2011, searches on Google Scholar under "type-2 fuzzy" revealed about 1,390,000 results, "type-2 fuzzy sets" revealed about 403,000 results, and "type-2 fuzzy systems" revealed about 316,000 result, whereas a search under[8] "fuzzy sets" revealed about 2,210,000 results, and "fuzzy systems" revealed about 2,390,000 results.

Much has already occurred for all kinds of T2 FSs and T2 FLSs with major advancements made in their theories, computation and applications. For a light-hearted history of the T2 FSs and systems, see [15]. Much research is now occurring about general T2 FSs and FLSs, because a general T2 FS can be expressed, loosely speaking, as the union of IT2 FSs over a parameter called either alpha [12], [22] or zed [24]. Everything that has been learned about IT2 FSs and IT2 FLSs can now be used for general T2 FSs and FLSs.

Will we go beyond T2? Why not? Once we run out of things to do with general T2 FSs and we can demonstrate that e.g., a T3 FS is a better uncertainty model than a T2 FS, then I see no reason why we will not take the next step up to a T3 FS. Perhaps, lurking in the bushes is a general linguistic uncertainty decomposition that is in terms of T1, T2, T3, etc. FSs. I would love to explore such bushes, but I am

[8] "Fuzzy" is too ambiguous a term, and so it is not included here.

afraid that this will have to be done by younger and more adventurous explorers. I do believe, though that it will be done.

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References

1. Biglarbegian, M., Melek, W.W., Mendel, J.M.: On the Stability of Interval Type-2 TSK Fuzzy Logic Control Systems. IEEE Transactions on Systems Man and Cybernetics, Part B-Cybernetics 40(3), 798–818 (2010)
2. Dubois, D., Prade, H.: Operations in a Fuzzy-valued Logic. Information and Control 43, 224–240 (1979)
3. Dubois, D., Prade, H.: Fuzzy Sets and Systems: Theory and Applications. Academic Press, New York (1980)
4. Gorzalczany, M.B.: A Method of Inference in Approximate Reasoning Based on Interval-valued Fuzzy ets. Fuzzy Sets and Systems 21, 1–17 (1987)
5. Karnik, N.N., Mendel, J.M.: Introduction to Type-2 Fuzzy Logic Systems. In: Proceedings of the 1998 IEEE FUZZ Conference, Anchorage, AK, pp. 915–920 (May 1998)
6. Karnik, N.N., Mendel, J.M.: An Introduction to Type-2 Fuzzy Logic Systems. University of Southern California, Los Angeles (1998), http://sipi.usc.edu/~mendel/report
7. Karnik, N.N., Mendel, J.M.: Type-2 Fuzzy Logic Systems: Type-reduction. In: Proceedings of the IEEE Conference on Systems, Man and Cybernetics, San Diego CA, pp. 2046–2051 (October 1998)
8. Karnik, N.N., Mendel, J.M.: Operations on Type-2 Fuzzy Sets. Fuzzy Sets and Systems 122, 327–348 (2001)
9. Karnik, N.N., Mendel, J.M.: Centroid of a Type-2 Fuzzy Set. Information Sciences 132, 195–220 (2001)
10. Karnik, N.N., Mendel, J.M., Liang, Q.: Type-2 Fuzzy Logic Systems. IEEE Transactions on Fuzzy Systems 7, 643–658 (1999)
11. Liang, Q., Mendel, J.M.: Interval Type-2 Fuzzy Logic Systems. In: Proceedings of the FUZZ-IEEE 2000, San Antonio, TX (May 2000)
12. Liu, F.: An Efficient Centroid Type-reduction Strategy for General Type-2 Fuzzy Logic System. Information Sciences 178, 2224–2236 (2008)
13. Liu, X., Mendel, J.M.: Connect Karnik-Mendel Algorithms to Root-Finding for Computing the Centroid of an Interval Type-2 Fuzzy Set. IEEE Transactions on Fuzzy Systems 19, 652–665 (2011)
14. Martin, M.A., Mendel, J.M.: Flirtation, a Very Fuzzy Prospect: a Flirtation Advisor, USC Report (1995), http://sipi.usc.edu/~mendel
15. Mendel, J.M.: Type-2 Fuzzy Sets: a Tribal Parody. IEEE Computational Intelligence Magazine 5, 24–27 (2010)

16. Mendel, J.M., Liang, Q.: Pictorial Comparisons of Type-1 and Type-2 Fuzzy Logic Systems. In: Proceedings of the IASTED Int'l Conference on Intelligent Systems & Control, Santa Barbara, CA (October 1999)
17. Mendel, J.M.: Uncertain Rule-Based Fuzzy Logic Systems: Introduction and New Directions. Prentice-Hall, Upper-Saddle River (2001)
18. Mendel, J.M.: Reflections on Some Important Contributions Made by Lotfi A. Zadeh that Have Impacted my own Research. Scientia Iranica 18, 549–553 (2011)
19. Mendel, J.M., John, R.I.: Type-2 Fuzzy Sets Made Simple. IEEE Transactions on Fuzzy Systems 10, 117–127 (2002)
20. Mendel, J.M., Liu, F.: Super-Exponential Convergence of the Karnik-Mendel Algorithms for Computing the Centroid of an Interval Type-2 Fuzzy Set. IEEE Transactions on Fuzzy Systems 15, 309–320 (2007)
21. Mendel, J.M., John, R.I., Liu, F.: Interval type-2 Fuzzy Logic Systems Made Simple. IEEE Transactions on Fuzzy Systems 14, 808–821 (2006)
22. Mendel, J.M., Liu, F., Zhai, D.: Alpha-plane Representation for Type-2 Fuzzy Sets: Theory and Applications. IEEE Transactions on Fuzzy Systems 17, 1189–1207 (2009)
23. Nie, M., Tan, W.W.: Towards an Efficient Type-reduction Method for Interval Type-2 Fuzzy Logic Systems. In: Proceedings of the IEEE International Conference on Fuzzy Systems, Hong Kong, China, pp. 1427–1434 (2008)
24. Wagner, C., Hagras, H.: Towards General Type-2 Fuzzy Logic Systems Based on zSlices. IEEE Transactions on Fuzzy Systems 18, 637–660 (2010)
25. Zadeh, L.A.: Outline of a New Approach to the Analysis of Complex Systems and Decision Processes. IEEE Transactions on Systems, Man, and Cybernetics SMC-33, 28–44 (1973)
26. Zadeh, L.A.: The Concept of a Linguistic Variable and its Application to Approximate Reasoning. Information Sciences 8, Part I, 199–249; 8, Part II, 301–357; 9, Part III, 43–80 (1975)

65

Memories of a Crisp Engineer

Claudio Moraga

65.1 The Fuzzification

I think that the first time I heard a talk on fuzzy sets was at one of the early International Symposiums on Multiple-valued Logic, (ISMVL), and the speaker was Prof. Lotfi A. Zadeh. Of course, I felt that *the message* was very interesting, but at that time, I was fully concentrated on multiple-valued switching theory, where by definition, everything was *precise* and *crisp*, i.e., as far from fuzziness as only possible. Things however were meant to change. A few years later, (in 1980) at the International Symposium on Multiple-valued Logic, in Evanston, Illinois, I met Prof. Enric Trillas, and that was the beginning of a deep, long lasting and "most dangerous" friendship: I started to be systematically exposed to the world of fuzzy, in a series of fuzzy related Seminars and Summer Schools organized by Prof. Trillas at different places in Spain, to which I was always invited. In one of the early Seminars he gave me as a present his recently published book on Fuzzy Sets [15]. I had no longer an excuse and I started to learn fuzzy sets and fuzzy logic. When in 1986 I became a Professor at the Department of Computer Science in the University of Dortmund, Germany, I introduced for the first time at the Department, seminars on fuzzy systems, which paved the way for my later Courses on "Intelligent Systems", (there is no accepted German equivalent for "Soft Computing"), as well as for several Master and Ph.D. Theses in the area.

When I first attended a lecture on fuzzy sets, as mentioned above, Prof. Zadeh was a *regular participant* at that ISMVL. In 1995, the year of the 30th Anniversary of the seminal first paper on fuzzy sets [18], I had the honour of chairing the Special Session at that year's ISMVL, hosted by the University of Indiana, where Prof. Zadeh delivered the Main Lecture as a *Keynote Speaker*. I had later the privilege of attending a sequence of Main Lectures of Prof. Zadeh at the series of "Dortmund Fuzzy Days" Conferences in Dortmund, as well as at several other conferences. Moreover I had the unique experience of attending his Special Lectures, both when he received the Honorary Doctorate from the University of Dortmund (1993), and later, from the Technical University of Madrid (2007).

65.2 Neuro-Fuzzy

At the time when I was finishing my Ph.D., Threshold Logic became popular for the design of digital circuits. Since my Thesis was on Ternary Switching Theory,

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it was natural that after obtaining my Degree, I would work on developing aspects of a "Ternary Threshold Logic" [4], later generalized to Multiple-valued Threshold Logic. It is easy to understand that, with that background, as soon as neural networks had their re-birth in the middle of the 80's, I gladly moved to that area and, particularly I started being interested in neuro-fuzzy systems [5]. I knew that I would not be the only one with the idea of exploiting synergy by combining the most relevant features of both systems: I had read of on going research in Japan and in France. What I did not know was, that at that time Jyh-Shing Roger Jang was writing his Ph.D. Thesis in Berkeley on a neuro-fuzzy system, and his advisor was Prof. Zadeh! Yes, Jang successfully obtained sound results and produced the system ANFIS [3], which became a worldwide known neuro-fuzzy system, that following the training constraints for a neural network, was able to learn from data, the parameters of a Takagi-Sugeno rule-based model of the corresponding original system. Having been interested in *compensating* systems, which are not properly modelled with fuzzy rule bases operating with t-norms and t-conorms, but with aggregation functions or linear and non-linear combinations of t-norms and t-conorms, I succeeded to show that the front end of ANFIS could be used and adapted to learn *e.g.* the Gamma Operator (Zimmermann and Zysno, 1983) [8], [10] or the Weighted Operators (Dubois, 1983) [10]. In his Dissertation at the University of Granada (1998), José Manuel Benítez obtained a very nice result, showing that the hidden processors of a feedforward neural network using a sigmoid as activation function could have a formal transformation allowing to deduce an equivalent if-then fuzzy rule, based on the symmetric summation (introduced by Silbert in 1971). The rule would have an expression as a symmetric sum of sigmoids, each applied to a different single input. This result was published in [1]. With my Colleague Karl-Heinz Temme (University of Dortmund) we extended the result of Benítez et al. to a family of functions, which we called "S-functions". A function $f : R \to (0,1)$ is said to be an S-Activation (or simply S-function) if it satisfies the following conditions: f is continuous, strictly monotonously increasing; for any x in R, if $f(x) = y$ then $f^{-1}(y)$ is well defined and returns x; for all x in R, $f(-x) = 1 - f(x)$; finally, $lim_{x\to-\infty} f(x) = 0$ and $lim_{x\to\infty} f(x) = 1$. If f is an S-function, the symmetric sum of $y_1 = f(x_1)$ and $y_2 = f(x_2)$ is given by $f(f^{-1}(y_1) + f^{-1}(y_2))$. A sigmoid is obviously an S-function, but among others, the Elliot function divided by 2, the function $1/(1+a^{-x})$ with a real and larger than one [14], and some trigonometric functions are also S-functions. Furthermore the graphic representation of an S-function shows that it can be interpreted as a (half open) fuzzy set. At the same time, the value of $S(w_i x_i)$ gives the membership degree of the i-th argument to the fuzzy set represented by S with parameter w_i. We interpreted the obtained rule analog to a Takagi-Sugeno rule of order 0 and the conclusion of the rule was calculated as the symmetric sum induced by f, of the membership degrees of the arguments to their corresponding parameterized S-functions. The premises of the rule were connected as usual, by *and*, but we did not give this connective an unnatural interpretation based on the symmetric summation, other than the natural linguistic interpretation that the premises are satisfied (to a corresponding degree) *at the same time* [7].

65.3 Evo-Fuzzy

When I started to search for synergy between fuzzy systems and evolutionary algorithms, there were already quite a lot of works devoted to applications of evolutionary algorithms to optimize fuzzy if-then rules and even to optimize full fuzzy rule bases. Possibly [2] represents a first important reference book on the subject. My first steps in the evo-fuzzy world were *the other way around*: I discussed genetic algorithms with "fuzzy fitness", borrowing from fuzzy control the idea of using the knowledge of experienced workers in the area of the problem to linguistically judge the quality, i.e. the fitness, of evolving individuals [6]. From the Trillas' School I had learnt that a fuzzy set representation of a linguistic term had to be *designed*. One possible scenario considers asking the users, in which interval of a given universe they would all agree "without but" that the predicate which representation is being designed, is satisfied. This would give the core of the corresponding fuzzy set. Similarly the users would be asked, when would they all agree that the predicate is not at all satisfied. That would give the co-support of the fuzzy set. Now, our experience as human beings when thinking of predicates is that the transitions between not-at-all and indeed-yes are continuous and monotone. *If no further information is available* (and this is quite frequently the case at the linguistic level), linear transitions are a reasonable first choice. This leads to a trapezoidal representation of the fuzzy sets corresponding to given predicates. When data driven modeling of a system based on if-then fuzzy rules is considered, the input-output samples of data *may contain the missing information* to determine the shape of the sides of the otherwise trapeziums used to represent the linguistic terms. With Rodrigo Salas, then a Ph.D. Student at the Technical University Federico Santa María, Valparaíso, Chile, we followed this hypothesis. We parameterized the sides of trapezoidal linguistic terms by using order automorphisms to introduce nonlinearities, but preserving continuity and monotonicity. The parameters were adjusted with simulated annealing. A lower mean square approximation error on a non-trivial test problem was obtained [9]. One last degree of freedom, (more precisely, *degree of responsibility*), when using if-then fuzzy rules is the choice, or better, *the design*, of the corresponding operations for the *and* conjunction of the premises and the *then* transfer to the conclusion. The case of choosing operations has been carefully analyzed in a series of works, beginning with the last section of [15] and the latest being [17]. In a joint work with Michio Sugeno and Enric Trillas, operations of a fuzzy rule base for approximation of the same test problem mentioned earlier were "data-driven designed" by means of parameterized order automorphisms applied to the product, and adjusted with distributed genetic algorithms. The mean square approximation error was smaller than the one obtained when using the (non-adapted) product, the t-norm of Łukasiewicz or the minimum [11].

65.4 Computing with Words

My first approach to the area of Computing with Words was in 2003, when right after my retirement from the University of Dortmund, I received an invitation of the

Ministry of Education of Spain, to become a Visiting Researcher at the Department of Computer Science of the Technical University of Madrid, and work with Prof. Enric Trillas, who at that time was interested (among other things, of course) in the representation and properties of antonyms within the formalism of fuzzy sets. We had lots of interesting discussions on the subject, which finally lead to [12]. On March 2006 I joined the newly founded European Centre for Soft Computing (Mieres, Asturias, Spain) and I started doing work on fuzzy formal languages. A few months later, Prof. Enric Trillas also joined the Centre and he soon started an internal weekly Seminar on Computing with Words. This Seminar was the right environment to clarify concepts, exchange ideas and gain a better understanding of the challenges and possibilities in this new area. Moreover, since Prof. Zadeh was the Chairman of the Scientific Committee that advices and controls the research work of the Centre, when he was in Mieres for a Meeting of the Committee, we had the privilege of attending an always motivating Lecture of him, related to his latest reflections on Computing with Words. At the IFSA/Eusflat Conference 2009, a result of a joint work with Prof. Trillas was presented, showing an analogy between linguistic terms and linguistic modifiers on the one side, and classes of sentences and their semantic modifiers, on the other [13]. This was followed by an essay on the linguistic interpretation of Mamdani Systems [13].

65.5 Closing Remark

From the late 70's until today I have had the privilege of receiving from the right people, at the right time, the right motivation to dedicate research efforts to contribute to the further development of fuzzy systems, followed by soft computing systems. It has been a most rewarding *fuzzinating* experience!

References

1. Manuel, B.J., Luis, C.J., Requena, I.: Are Neural Networks Black Boxes? IEEE Transactions on Neural Networks 8, 1156–1163 (1997)
2. Herrera, F., Verdegay, J.L.: Genetic Algorithms and Soft Computing. Physica-Verlag, Heidelberg (1996)
3. Jang, J.-S.R.: ANFIS: Adaptive Network based Fuzzy Inference System. IEEE Transactions on Systems, Man and Cybernetics 23(3), 665–685 (1993)
4. Moraga, C.: A Monograph on Ternary Threshold Logic. In: Rine, D.C. (ed.) Computer Science and Multiple-Valued Logic, pp. 355–394. North-Holland, Amsterdam (1977,1984)
5. Moraga, C.: Redes Neuronales y Lógica Borrosa: un desafío y una promesa. In: Trillas, E., Gutiérrez, J. (eds.) Teoría y Aplicaciones de la Lógica Borrosa, Editorial del Consejo Superior de Investigaciones Científicas, Madrid (1992)
6. Moraga, C., Bexten, E.M.Z.: Fuzzy-Knowledge-based Genetic Algorithms. Information Sciences 103, 101–114 (1997)

7. Moraga, C., Temme, K.-H.: Functional Equivalence Netween S-neural Networks and Fuzzy Models. In: Bouchon-Meunier, B., Gutiérrez Ríos, J., Magdalena, L., Yager, R.R. (eds.) Technologies for Constructing Intelligent Systems 2: Tools, pp. 355–363. Physica-Verlag, Heidelberg (2002)
8. Moraga, C.: Data Driven Fuzzy Modelling with Neural Networks. In: Negoita, M.G., Reusch, B. (eds.) Real World Applications of Computational Intelligence, pp. 153–159. Springer, Heidelberg (2005)
9. Moraga, C., Salas, R.: A New Aspect for the Optimization of Fuzzy If-then Rules. In: Proceedings of the 35th International Symposium on Multiple-Valued Logic (ISMVL 2005), pp. 160–165. IEEE-CS-Press (2005)
10. Moraga, C.: Compensating systems: A Challenge for Neuro-fuzzy Modeling. International Journal of Systemics, Cybernetics and Informatics 1(1), 16–20 (2006)
11. Moraga, C., Michio, S., Trillas, E.: Optimization of Fuzzy If-then Rule Bases by Evolutionary Tuning of the Operations. In: Proceedings of the 39th International Symposium on Multiple-Valued Logic, pp. 221–226. IEEE-CS-Press (2009)
12. Moraga, C., Trillas, E.: A Computing with Words Path from Fuzzy Logic to Natural Language. In: Carvalho, J.P., Dubois, D., Kaymak, U., da Costa Sousa, J.M. (eds.) Proceedings of the Joint 2009 International Fuzzy Systems Association World Congress and 2009 European Society of Fuzzy Logic and Technology Conference, Lisbon, Portugal, July 20-24, pp. 687–692 (2009)
13. Moraga, C.: An Essay on the Interpretability of Mamdani Systems. In: Trillas, E., Bonissone, P.P., Magdalena, L., Kacprzyk, J. (eds.) Combining Experimentation and Theory. STUDFUZZ, vol. 271, pp. 61–72. Springer, Heidelberg (2011)
14. Karl-Heinz, T., Ralph, H., Moraga, C.: Generalized Neural Networks for Fuzzy Modeling. In: Mayor, G., Suñer, J. (eds.) Proceedings of the 1999 EUSFLAT-ESTYLF Joint Conference, September 22-25, pp. 469–472. Servei de Publicacions i Intercanvi Científic, Universitat de les Illes Balears, Palma de Mallorca (1999)
15. Enric, T.: Conjuntos Borrosos. Editorial Vicens-Vives, Barcelona (1980)
16. Trillas, E., Moraga, C., Guadarrama, S., Susana, C., Castiñeira, E.: Computing with Antonyms. STUDFUZZ, vol. 217, pp. 133–153. Springer, Heidelberg (2007)
17. Trillas, E., García-Honrado, I.: A Reflection on Fuzzy Conditionals. In: Trillas, E., Bonissone, P.P., Magdalena, L., Kacprzyk, J. (eds.) Combining Experimentation and Theory. STUDFUZZ, vol. 271, pp. 391–406. Springer, Heidelberg (2011)
18. Zadeh, L.A.: Fuzzy Sets. Information and Control 8, 338–353 (1965)

On Fuzziness in Mathematics

John N. Mordeson

I received my Ph. D. in mathematics from Iowa State University in 1963. My specialty was in abstract algebra. That same year I obtained a position in the Mathematics Department at Creighton University. For the first 20 years or so, I carried out a research program in traditional mathematics. In an attempt to reach out to other disciplines, I began collaborating with members of the Health Sciences at Creighton. It was then that I began to feel that some form of mathematics (not probability) dealing with imprecision would be useful in the health sciences. I found a book on fuzzy logic in the library. It seemed to me that this was exactly what I was looking for. However, I did not pursue the matter further at this time. It was later that I more fully appreciated the importance of the ideas of Lotfi Zadeh regarding uncertainty [1].

The classical mathematical theories, by which certain types of certainty can be expressed, are the classical set theory and probability theory. In terms of set theory, uncertainty is expressed by any given set of possible alternatives in situations where only one of the alternatives may actually happen. Uncertainty expressed in terms of sets of alternatives results from the nonspecificity inherent in each set. Probability theory expresses uncertainty in terms of a classical measure on subsets of a given set of alternatives. The set theory, introduced by Zadeh, presents the notion that membership in a given subset is a matter of degree rather than that of totally in or totally out. This concept is captured in [1] by defining a fuzzy subset of a universal set X to be a function from X into the closed interval $[0,1]$.

Another broad framework for dealing with uncertainty is the fuzzy measure theory founded by Sugeno [2], [3]. The fuzzy measure theory replaces the classical measure theory by replacing the additivity requirement with the weaker requirements of monotonicity, with respect to set inclusion and the continuity or semicontinuity of fuzzy measures. The earliest challenge to classical measure theory was by C. Choquet when he developed a theory in 1954 called the theory of capacities [4]. Other major contributions were by Dempster [5] and Shafer [6] and by Zadeh's introduction of possibility theory [7].

As noted in [8], the evolution of the fuzzification of mathematics can be broken into four stages: (1) Straightforward fuzzification during the sixties and seventies; (2) Exploration of numerous possible choices in the generalization process during the eighties; (3) Standardization, axiomatization and L-fuzzification in the nineties; (4) Deeper development of many areas of fuzzy mathematics in the 21st century. The years 2001-2009 found an expansion of the frontier of many areas in fuzzy

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mathematics, such as axiomatics of structures and concepts, fuzzy logic in the narrow sense, interval valued fuzzy sets and fuzzification of mathematical disciplines.

During the sixties and seventies nearly every domain of pure mathematics was fuzzified. The definition of a suitable fuzzification of the classical notions was undertaken, i.e., suitable in the sense that when the extension is applied to the classical case, the classical notion is obtained. During the eighties many possible choices in the generalization process occurred. The discovery of triangular norms and conforms that had been introduced to probabilistic metric spaces influenced the process of generalization. There also occurred a study of the alternative operations of disjunction, conjunction, implication in logic as well as union, intersection, and inclusion in fuzzy set theory. Pawlak introduced the concept of a rough set in 1982 [9]. This concept is fundamental to the examination of granularity in knowledge. The main themes of research in the nineties in fuzzy mathematics were the standardization, axiomatization, L-fuzzification and a comparison of the fuzzy model to other recently developed models for the representation and manipulation of imprecision and uncertainty. A comparison occurred between the fuzzy model and other models such as rough sets, subdefinite sets, and intuitionistic fuzzy sets .

The development of fuzzy set theory has been described many places. It is worth mentioning that Krassimir Atanassov introduced the notion of the degree of nonmembership in the definition of a fuzzy subset [10]. This idea is incorporated with the notion of membership in a fuzzy subset and the resulting structures is called an intuitionistic fuzzy set. H"ohle showed that large parts of fuzzy set theory are subfields of sheaf theory. Consequently, fuzzy set theory is closer to mainstream mathematics than one might think. Fuzzy logic was first invented as a representation scheme and calculus for uncertain or vague notions. It is an infinite-valued logic that allows more human-like interpretations and reasoning. With fuzzy set theory, one obtains a logic in which statements may be true or false to different degrees rather than the bivalent situation of being true or false. Consequently, certain laws of bivalent logic do not hold, e.g., the law of the excluded middle and the law of contradiction. This resulted in an enriched scientific methodology.

C. L. Chang introduced the notion of a fuzzy topology of a set in 1968 [11]. Much of the early work in fuzzy topology was based on its similarity with another branch of topology on a lattice, namely local theory. Methods of local theory were used to study problems not involving points in fuzzy topology. Many problems on fuzzy topological spaces must involve the notion of point. A summary of the work can be found in the book by Liu and Luo [12].

Dr. Malik, a member of the Mathematics Department at Creighton, showed me the seminal paper [13] on fuzzy group theory by Professor Azriel Rosenfeld. Rosenfeld was inspired by the work of Chang on fuzzy topological spaces. Rosenfeld wrote the entire paper while on a flight to a conference. This paper opened a whole new area in fuzzy mathematics, namely fuzzy abstract algebra. Dr. Malik and I began a long lasting research association in fuzzy abstract algebra. We created a center for research in fuzzy mathematics. Dr. George and Mrs. Sally Haddix believed strongly enough in the goals of the center to provide it with a generous endowment. The goals of the center were to support the paradigm shift in the sciences with respect to uncertainty and to support our colleagues in countries overseas working in fuzzy mathematics. To these ends, the center hosted visiting scholars from China, India, Korea, Japan, and Saudi Arabia. Members of the center collaborated with members of the departments of Neurology and Psychiatry at the University of Illinois, College of Medicine at Chicago and also the Center for Research in Osteoporosis at Creighton. Members of the center and members of the Department of Education at Creighton have carried out joint research with the staff of Omaha Hearing School and Madonna School, a school devoted to children with special needs. We responded to the requests of many graduate students from the University of Baghdad for reprints of papers on fuzzy mathematics. Dr. Sen, a visiting scholar from India, together with Dr. Malik and I became immersed in fuzzy algebraic automata theory. Our work led Dr. Malik and I to write a book in the area. In fact, members of the center have written over 100 papers and 10 books on various areas in fuzzy mathematics. Members of the center attended conferences on fuzzy theory and technology where they met and were influenced by leaders in the field such as Paul Wang, George Klir, Azriel Rosenfeld, and Lotfi Zadeh himself.

In another attempt to reach out to other departments at Creighton, I contacted Dr. Clark of the Political Science Department in 2005. Dr. Wierman of the Computer Science Department joined us soon after. We produced a research agenda joining formal theoretical work with empirical research. We felt that standard mathematics is too precise to model human thinking and action. One of the most important developments of our collaboration is the involvement of students in our research. We work with students from political science, mathematics, and economics. Our collaboration has led me to develop an honors course that focuses on using fuzzy mathematics to model global issues. Some of the global issues we have tackled with students are nuclear stability, children with special needs, economic freedom, smart power, political stability, cooperative threat reduction, failed states, economic stability, creative economy, quality of life, solar energy, wind energy, college freshman weight gain, population management of Sub-Saharan Africa, safe skies, remittances, health care, nuclear deterrence, developmental disorders in children, human development, and globalization. However, the main project of the research group deals with spatial modeling. Spatial models in political science typically assume that political actors possess an ideal preference that is mapped in n-dimensional space as a single point. Fuzzy set theory permits scholars to assume instead that actors are indifferent over a large number of alternatives that are mapped as regions within which all options are equally preferred. The goals of the fuzzy spatial modeling project are (1) to improve the capacity of models to predict stable outcomes and (2) to improve the empirical validity of those predictions. In so doing, the project illustrates the ability of fuzzy mathematics to deal with vagueness in human thinking and provides a touchstone for further applications of fuzzy mathematics in the social sciences.

The potential for the future application of fuzzy mathematics looks to be quite promising. One such possible application is causality. Albert Einstein stated that development of Western science is based on two great achievements: the invention of the formal logic system and the discovery of the possibility to determine causal relationships by systematic experiment. Judea Pearl states that in the last decade (the 1990s) owing partly to advances in graphical models, causality has undergone a major transformation. Practical problems relying on causal information that long were regarded as unmanageable can now be solved using elementary mathematics. Pearl [14] stresses that basic concepts of probability theory and graph theory is all that is needed for one to begin solving causal problems that are too complex for the unaided intellect. Many support the notion that problems of causality can be studied profitably by the use of mathematics uncertainty and basic concepts from graph theory.

The interested reader may find the papers [8], [15], and [16] pertinent to this paper.

References

1. Zadeh, L.A.: Fuzzy Sets. Information and Control 8, 338–353 (1965)
2. Sugeno, M.: Theory of Fuzzy Integrals and Its Applications. PhD dissertation, Tokyo Institute of Technology (1974)
3. Sugeno, M.: Fuzzy Measures and Fuzzy Integrals – A Survey. In: Gupta, M.M., Saridis, G.N., Gaines, B.R. (eds.) Fuzzy Automata and Decision Processes, pp. 89–102. North-Holland, New York (1977)
4. Choquet, G.: Theory of Capacities. Annales de l'Institut Fourier 5, 131–295 (1954)
5. Dempster, A.P.: Upper and lower Probabilities induced by Multi-valued Mappings. Annals of Mathematical Statistics 38, 325–329 (1967)
6. Shafer, G.: A Mathematical Theory of Evidence. Princeton University Press, Princeton (1976)
7. Zadeh, L.A.: Fuzzy Sets as a Basis for a Theory of Possibility. Fuzzy Sets and Systems 1, 3–28 (1978)
8. Kerre, E.E., Mordeson, J.N.: Historical Overview of Fuzzy Mathematics. New Mathematics and Natural Computation 1, 1–26 (2005)
9. Pawlak, Z.: Rough Sets. International Journal of Computer and Information Sciences 11, 341–356 (1982)
10. Atanassov, K.T.: Intuitionistic Fuzzy Sets. STUDFUZZ, vol. 35. Physica-Verlag, Heidelberg (1999)
11. Chang, C.L.: Fuzzy Topological Spaces. Journal of Mathematical Analysis and Applications 24, 182–190 (1980)
12. Liu, Y.-M., Luo, M.-K.: Fuzzy Topology, Advances in Fuzzy Systems – Applications and Theory, vol. 9. World Scientific, Singapore (1997)
13. Rosenfeld, A.: Fuzzy Groups. Journal of Mathematical Analysis and Applications 35, 512–517 (1971)
14. Pearl, J.: Causality: Models, Reasoning, and Inference. Cambridge University Press, Cambridge (2000)
15. Kerre, E.E.: On the Evolution of the Mathematics of Fuzziness. In: Ruan, D. (ed.) Fuzzy Set Theory and Advanced Mathematical Applications, pp. 1–34. Kluwer Academic Publishers, Boston (1995)
16. Mordeson, J.N.: Zadeh's Influence on Mathematics. Scientia Iranica, Transactions: Electrical and Computer Engineering (2011)

67

A Mathematician's Naive Perspective on Fuzzy Sets and Fuzzy Logic

Takehiko Nakama

I imagine that when Enric Trillas and Rudolf Seising asked me to contribute to this book, they expected me to provide a naive perspective on fuzzy sets and fuzzy logic from a mathematician's point of view—they know that I am a mathematician and that I am a novice in this field. As indicated by the title, I shall attempt to comply with their expectation. At the European Center for Soft Computing (ECSC), I continually engage in thought-provoking discussions on various aspects of fuzzy sets and fuzzy logic with prominent researchers in the field, and my perspective on the subject matter continues to evolve. In this essay, I report my nascent thoughts.

I do find fuzzy set theory and fuzzy logic fascinating. Although they have been successfully applied to a variety of real-world problems, their applications have very little to do with my interests in them; it is the theories, not their applications, that intrigue me intellectually. (Perhaps I should develop a better appreciation for ingenious applications.) Having said that, I believe that the fuzzy theories provide effective mathematical formulations of human reasoning or behavior, and I intuitively understand why their applications have been hugely successful in various areas. For instance, humans are remarkably capable of controlling a number of different devices, so it is wise to use the fuzzy theories to develop formal characterizations of how we control them and to implement them in machines.

It seems that, even today, some people are still strongly dismissive of the fuzzy theories; my colleagues at ECSC occasionally tell me about their experiences with such people. On the one hand, I understand the criticisms directed toward studies that simply apply fuzzy concepts to various practical problems without attempting to develop any theoretical basis or framework for their methodologies. On the other hand, I do not understand those who do not accept fuzzy set theory and fuzzy logic by claiming that they are not logically or mathematically well-founded. To me, representing sets not only by characteristic functions (also known as indicator functions) but also by other functions is a rather natural and important progression in generalizing the classical set theory. Similarly, I consider many-valued logics a natural generalization of the classical two-valued logic. Just as we can establish connections (isomorphisms) among the classical set theory, propositional logic, and boolean algebra, we can establish analogous connections between fuzzy set theory and fuzzy logic. Hence I do not believe that their theoretical foundation is weaker than that of the classical set theory or two-valued logic. (Russell, Whitehead, and Gödel demonstrated that even the foundation of the classical mathematical logic is, unfortunately, rather "shaky" or "incomplete".) One of the criticisms that I have observed is that

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fuzzy set theory and fuzzy logic are flawed because they lack the law of excluded middle and the law of contradiction. (It is also mysterious that those who support the argument do not seem to be particularly bothered by the lack of some other instances of tautology, such as those used as inference rules, in the fuzzy theories.) First of all, Enric Trillas has recently elucidated conditions under which the two laws hold in fuzzy logic. Second of all, in mathematics, one can find results that hold in one domain but not in another domain (for example, examine results in Real Analysis and Complex Analysis), but it would be considered absurd to claim that mathematics must exclude a domain of analysis for that reason. A healthy dose of skepticism should always be appreciated (think about how Russell identified some fundamental problems in mathematical logic, for instance), but some of the criticisms against the fuzzy theories seem to indicate blind unreceptiveness.

After all, it has been "only" about 50 years since fuzzy set theory was founded by Zadeh. In mathematics, some of the concepts that seem indispensable, self-explanatory, or highly valuable today took a lot of time to gain acceptance. One of the primary examples of such concepts is the imaginary number, $\sqrt{-1}$, which took several hundreds of years to get fully incorporated in mathematics. (Other "unnatural" numbers, such as zero and negative numbers, also have interesting histories.) Thus one could say that fuzzy sets and fuzzy logic have gained wide acceptance rather quickly. Extending the field of real numbers to include complex numbers should be considered one of the most important generalizations in mathematics, and it led to innumerable theoretical and practical breakthroughs. (Hadamard is often quoted as saying that the shortest path between two truths in the real domain passes through the complex domain.) The inclusion of fuzzy sets in set theory, which dealt with only "crisp" sets for many years, can be considered somewhat analogous to the inclusion of complex numbers in mathematics. I believe that incorporating fuzzy sets in set theory is important to develop the theory to its fullest; limiting it to crisp sets would be terribly detrimental to its progress.

Extending an existing framework or theory to include fuzzy sets often requires a stimulating intellectual challenge for mathematicians. A prime example can be found in statistics. In classical hypothesis-testing procedures such as t-tests and analysis of variance, real-valued random variables are used to express statistical models. To extend these procedures to analyze fuzzy data (i.e., data consisting of fuzzy sets), Hilbert space-valued random variables are often used to express statistical models. Thus, to establish analogous statistical procedures for fuzzy data, we need to extend classical theorems in probability theory, such as laws of large numbers and central limit theorems, to Hilbert space-valued random variables. Hence mathematicians can find an abundance of problems to solve by trying to extend existing mathematical theories to fuzzy sets. Many research fields have emerged as a result of extending existing mathematical theories to fuzzy sets—fuzzy group theory, fuzzy topology, fuzzy game theory, and fuzzy graph theory, to name a few. I expect that more and more existing mathematical theories will be extended to incorporate fuzzy sets. Consequently, the complexity of the theories will increase substantially. More efforts to simplify them by removing superfluities, to unify them, and to gain insightful perspectives that help better understand them will have to be made to counteract the trend.

Some engineers seem to think that mathematicians are making the theories more and more complex just because they can, without any meaningful objective in mind. I have two slightly conflicting thoughts on this matter. On the one hand, I believe that it is important for researchers, including mathematicians, to examine the significance of their own research as they engage in it. Here I do not think that there is any universal measure for significance in research; different researchers have different criteria for evaluating it (for instance, the most valuable attribute may be practicality for application-oriented engineers but conceptual elegance or rigor for mathematicians), and I consider the diversity important for enriching a research field. Despite the lack of universal criteria, I believe that trying to identify important problems and to conduct studies that address them can result in quality research.

On the other hand, I believe that it is critical that complete freedom be warranted in theoretical research because it is often difficult, or virtually impossible, to figure out what concept turns out to be truly significant, especially in mathematics. For instance, RSA, which revolutionized cryptography and secure communication on the Internet, is based on a handful of old results in number theory (Euclid's algorithm, Euler's totient function, and the Fermat-Euler theorem) that did not seem to have anything to do with cryptography; it is safe to say that Euclid, Fermat, and Euler never thought about contributing to cryptography or secure communication on the Internet when they established the results. Some mathematical ideas that appear to have no practical use initially can turn out to be tremendously useful for solving real-world problems in the distant future. Thus I believe that it is terribly detrimental for mathematical research to restrict endeavors or initiatives to immediate practicality; it will hinder progress not only in theory but also in applications. Let me conclude this essay with the following remark to those who are critical of mathematicians for not producing practical results: Let mathematicians pursue mathematical truths freely; though you might have to wait for many years, you could benefit enormously from the fruits of their labor.

Acknowledgement. The author thanks Mary Kathleen Kemp for helpful discussions.

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On Fuzziness and Ordinary Reasoning

María G. Navarro

In 1685, in *The Art of Discovery*, Leibniz set down an extraordinary idea: "The only way to rectify our reasonings is to make them as tangible as those of the Mathematicians, so that we can find our error at a glance, and when there are disputes among persons, we can simply say: Let us calculate [calculemus], without further ado, to see who is right." *Calculemus*. Much has been written about that celebrated expression, but if I had to remember the moment when the famous Leibnizian motto once again brought back to mind, in a way, artefacts of the present and the future, that moment would be connected with a seminar organised by Verónica Sanz at the Philosophy Institute of the Spanish Council for Scientific Research (CSIC), when she was the coordinator of the Seminario Internacional de Jóvenes Investigadores (the International Seminar for Young Researchers). At that seminar, Sergio Guadarrama presented the challenge of computing with words. It was then, if I was not mistaken and I really understood what was being explained to me, that I discovered that after all, Leibniz had something to do with a man named Lotfi A. Zadeh. I liked this, because it meant that the problem of formalising the modes of reasoning we all use was so important that many reputable researchers wanted to help the world to calculate. Alerting others to the importance of calculating and being aware of the effective realisation of a calculation when it is reasoned, is not the same as offering answers about how we can achieve this individually and even collectively, in the physical world and in the virtual one.

When it is not misinterpreted, we all tend to like Leibniz' expression; everyone likes the idea of calculating with words. Calculating by reasoning is something we normally do. A constant calculation which unites us with everyone else in a kind of endless mathematical operation, but which we accept as finished at certain moments, is an invitation to imagine ourselves as really complex creatures. Enric Trillas showed me more things about the problem of ordinary reasoning in the first "Alfredo Deaño" Seminar on Ordinary Reasoning organised by the Foundation for the Advancement of Soft Computing and the *European Centre of Soft Computing* (ECSC) in 2011. Here different research projects were discussed, relating to the challenge of Soft Computing, and the meaning of our reasoning in everyday life. The concept which I found most enthralling was the 'conjecture' which Enric Trillas spoke about.

One could say his entire discourse was imbued with the spirit of the *Novum Organon Renovatum* which led William Whewell to state that deduction is a necessary part of induction. At first glance, it is as paradoxical for a mathematician to sustain and develop a philosophical discourse based on this thesis, as it is for a

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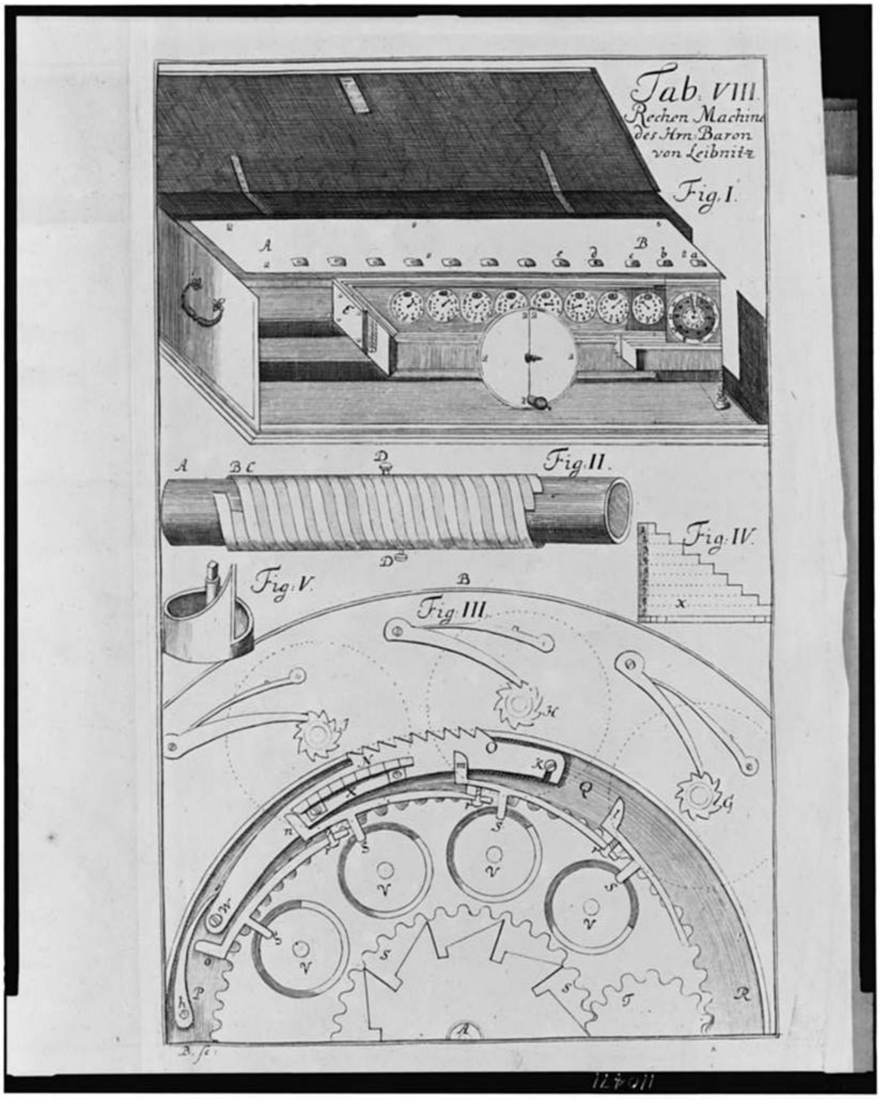

Fig. 68.1. Details of the mechanisms of the Leibniz calculator, the most advanced of its time. Illustration in *"Theatrum arithmetico-geometricum" das ist Schau-Platz der Rechnen- und Mess-Kunst...*, [1].

philosopher to exclaim "let us calculate", advising us to learn to weight our reasoning appropriately when we enter into disputes which can be resolved if we gradually find out how to present them more tangibly. It struck me that Trillas' adherence to this thesis was leading him to criticise the prejudice shared by many philosophers that all objects can be precisely defined. Trillas shares this concern with Zadeh, who has defined himself as a fervent believer in the power of mathematics. Both for Trillas and for Zadeh, it is a mistake of classical mathematics to think you can divide objects into two sub-classes: that of the objects which are examples of the concept, and that of those which are not. If we think about the paradoxical expression which Leibniz invites us to always bear in mind (*calculemus*) perhaps it is not so strange to see two fervent believers in mathematics inviting us to look at the world exactly as we all know it to be: a reality in which there are no defined (or defining) frontiers, where we understand that a pinch of salt cannot be replaced by an exact amount, or that the meaning of *better than*, *good* or *high* cannot be defined according to the classical pattern mentioned above.

There is no precise definition dividing into two the class of all objects, nor is there a deduction mechanism based on the rules of inference operating on the meaning of utterances; only on their abstract form. This is what makes so attractive the fact that human beings, and living organisms in general, reason, aware that our conclusions are not definitive, but merely provisional. However, despite the incompleteness, not only of our reasoning but of the theories we construct with them, neither can we conclude that our inferences lack informational value. We use powerful systems to represent the world in which we manipulate our beliefs, so that many people refuse to believe that this has anything to do with logical calculation. Do we calculate? Do we rebuild information without adding new semantic content? How do semantic representations, or representations of content, affect formal mechanisms to produce inferences? Answering these questions does not look easy, but it may be an interesting strategy to ask why we do things this way.

Let us suppose our semantic representations influence the formal mechanism to produce inferences. What use to us is it to do it this way? One of the explanations which have been given is that circumstances usually oblige us to make decisions and/or to act long before we know all the relevant facts. Not all the information is available to us, because the world has not finished happening. This is the idea underlying the concept of goal-directed reasoning: we all establish reasoning which goes from the premise to the conclusion, which does not prevent us returning immediately from the conclusion to the premise. The inference rules we use justify the beliefs we select and adopt. But what for? A common answer is that it is to be able to act and live in time. In a way this is like saying it is in our interests to be able to think this way (in two directions) because our reasoning is goal-directed: it is in our interests to reason this way. However, it seems paradoxical that our interest in reasoning following a model of reasoning directed to interest makes us select an imperfect model of reasoning. This option is certainly the most consistent one if we bear in mind that, as Zadeh reminds us, we do not live in a world in which we divide objects into two sub-classes: that of the objects which are examples of the concept, and that of those which are not. If we read this idea in relation with the subject of reasoning, the result is that, through reasoning, we can live in a world where we can recant our inferences. Something so apparently simple ends up being very useful: ordinary goal-directed reasoning invites us to examine the theories of epistemic justification (externalism, internalism, contextualism, reliabilism, etc.). Perhaps this analysis of epistemic justification still does not answer the question of why (why are we normally unable to restructure information without adding new semantic content?), but it offers different answers to the question of how.

Do all people share the same notion of inferential validity? If a self-description ('I think it's red'; 'Zadeh says yes'; 'I like eating with other people!') means the immediate production of a given context for the expression of states of consciousness – and this is the context which enables us to understand those expressed thoughts – it is pertinent to ask if we all share the same notion of inferential validity. Something may lead us to think, to begin with, that we do not. We have all experienced disagreement and misunderstanding. We often have experience as to whether or not we get things right when reproducing a given context to make states of consciousness and representational content understandable, or if we do this successfully, but then fail to infer other representational content, for example, the intentions, plans, beliefs, judgments or commitments which make up collective attitudes.

To understand more about what 'inferential validity' means and why this process is susceptible to every kind of cognitive bias, we must refer to the discoveries in the psychology of reasoning of Peter C. Wason. The results of Wason's experimental research are a paradigm of how concepts of inferential validity do not satisfactorily account for the phenomena of ordinary reasoning. In 1960 some experimental psychologists began to take an interest in the nature of human reasoning. A series of experiments led them to conclude that most human beings ordinarily make basic mistakes in deductive order in their inferences. This research led to the emergence of the concept of 'inferential competence'.

The definition of inferential competence proposed by the experimental psychologists was not shaped by all the principles and rules of classical logic. This research would later be enriched by the application of a tentative hypothesis according to which the experiment subjects made more formally consistent inferences when presented with a concrete version of the inferential tasks, i.e., when they were asked to resolve these tasks while still using natural language, and not through formalisations in an object language.

The research of these experimental psychologists chimes with the discovery of cognitive bias or systematic errors when making inferences. From the point of view of cognitive psychology, cognitive biases are one of the core concepts of the psychology of reasoning, because they let us infer that there is a certain type of context, conditions and situations in which a cognitive mechanism (with inferential or inductive effects) produces cognitive results which are not correct.

This cognitive heuristic itself represents an example in which we see that inferential and representational processes are dealt with in ordinary reasoning based on interpretive components and processes. Thus, in the case of epistemic biases, the previous interpretation of the contexts in which cognitive mechanisms (e.g., an inference, a representation) is what guides the subject when producing justified or acceptable cognitive mechanisms.

In their studies on defeasible inferences authors such as Keith Stenning and Michiel Van Lambalgen have drawn our attention to how little importance is usually given to the three dimensions present in interpretation (the logical, semantic and pragmatic dimensions) in the mastery of logic and the philosophy of language.

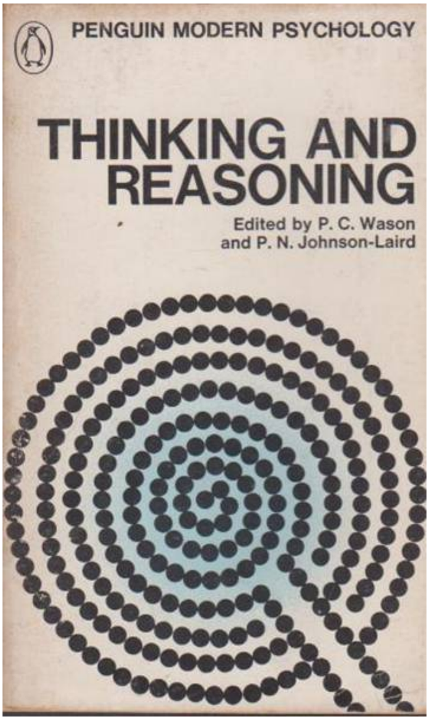

Fig. 68.2. Cover of the book edited by Peter C. Wason and P. N. Johnson-Laird, [2]

Defeasible inferences are the opposite of deductive arguments, which are not defeasible. If a conclusion follows deductively from a set of premises *P*, it can never be valid if *P* is increased, or in other words, an inference cannot be valid if, among other aspects, more information is obtained based on the inference.

For example, defeasible inferences are constantly used in historical research and reasoning. In fact, the functions which some historians who call themselves reconstructionists confer on historical reasoning are not in agreement with the nature of the inferences used in developing the reasoning: they construct historic reasoning based on a deductive conception of this reasoning. However, we find that in history, there is no place for deduction; history is an eminently defeasible space.

In everyday life too, our inferences are defeasible inferences. We can see this, for example, in some of the most important revocable or defeasible inferences, such as conditional inferences, within which we can highlight so-called conversational implicatures, as well as abductive inference, which I regard as the logical pattern inherent in the cognitive process called ‘interpretation’.

Ordinary reasoning enables us to have a closer relationship with reality (because it is eminently fuzzy), and fuzzy logic - as far as I have been able to understand until now - establishes the conditions of possibility to carry out the sophisticated calculation through words with which Lotfi A. Zadeh invites us to contemplate ourselves.

References

1. Leupold, J.: “Theatrum arithmetico-geometricum" das ist Schau-Platz der Rechnen- und Mess-Kunst... Christoph Zunkel, Leipzig (1727)
2. Wason, P.C., Johnson-Laird, P.N. (eds.): Thinking and Reasoning. Penguin, Harmondsworth (1968)

69

On Present Logico-Methodological Challenges to Fuzzy Systems

Vesa A. Niskanen

69.1 Introduction

Today fuzzy systems play an important role in the quantitative research. This is due to their novel approach to reasoning and concept formation. On the other hand, fuzzy systems have been controversial in particular among the philosophers and mathematicians. Hence, the dissemination of these new ideas has encountered many problems. Additional criticism has arisen because various logico-methodological approaches to the fuzzy systems are adopted as well as we still have quite many unresolved problems in our basic theories.

Below we consider some basic problems which still arise in the theories of fuzzy systems. We also sketch some tentative resolutions for them.

69.2 In the Beginning There Was Fuzziness

The first problem arises at the core of fuzzy systems, viz. the meaning of the concept *fuzzy*. From the etymological standpoint, this expression probably stems from the old German word *fussig* (spongy). Today researchers assume that fuzziness at least means imprecision. However, other concepts are also used, and prior to the fuzzy era, philosophers used such concepts as *vague* or even *inexact* instead [6, 14]. Many researchers also use the concept *uncertain* in this context.

If we assume that fuzziness in fact means imprecision, the mainstream interpretation is that fuzziness is included in certain linguistic expressions, and we consider them in the light of the extensional semantics. Hence, we may establish that a linguistic expression is imprecise, or fuzzy, if it refers to such entities (e.g., sets or relations) which have borderline cases. For example, *young person* is imprecise if its corresponding extension, the set of young persons, includes borderline cases [14]. In addition to this approach, we may consider imprecision from the linguistic standpoints of intensional semantics, syntax or pragmatism. Outside linguistics, epistemological and ontological approaches are also available [14]. This jungle of interpretations already provides us with various possible confusions. The concept of vagueness, in turn, is quite close to imprecision, but if we like to draw a distinction between these two, we may assume that vagueness also includes generality.

Uncertainty, with its various meaning components, is still fairly widely used within the fuzzy systems as a synonym for *imprecision*. In a strict sense, uncertainty

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is nevertheless an epistemological concept, i.e., related to our knowledge, whereas imprecision mainly belongs to semantics. Hence, this distinction should be clear. However, we have the everlasting debate on the meanings of imprecision and uncertainty within the fuzzy systems, and no consensus seem to appear [9].

Uncertainty is also examined in its "correct" form within the probability theory. In such approaches we may apply imprecise variables and probability distributions. Examples are the expressions *probability is fairly high* or *approximately normally distributed* [23]. The possibility theories, in turn, are also at least partially related to uncertainty and probability. On some occasions they are regarded as being the preliminary stages of probability [23].

Even though the linguistic aspects of *fuzziness* or *imprecision* have aroused discussions, the corresponding quantitative interpretations have been almost unanimous. They all apply traditional mathematics, and fuzziness is examined according to the membership functions. Hence, if the extension of an expression includes borderline cases, from the mathematical standpoint this means that some objects have only partial degrees of membership to its extension. This idea is a generalization of the characteristic function, and in traditional mathematics an isomorphism holds between these functions and the crisp sets. Hence, within fuzzy systems, we are not operating with fuzzy entities directly, but rather with the corresponding membership functions in our theory formation and model construction.

Since our research work is in practice based on such mathematical entities as membership functions of fuzzy sets and intensities of fuzzy relations, we have encountered some problems for finding a full correspondence between this logico-mathematical world and the linguistic world containing imprecise expressions. In addition, we should also find a correspondence between these and the real world. We will consider next this subject matter.

69.3 Correspondence between Fuzzy Systems and the Real World

In brief, within fuzzy systems we should find correspondences between the linguistic, logico-mathematical and real worlds. The linguistic world contains vocabularies as well as syntactic, semantic and pragmatic entities of languages. Within fuzzy systems our goal has been to specify such formal or quasi-natural languages which correspond well with the natural languages, and Lotfi Zadeh has performed a valuable work in this area [22, 19, 20, 21, 23, 24, 25]. A typical fuzzy quasi-natural language contains linguistic variables with such values as primitive terms, linguistic modifiers (hedges), connectives, quantifiers and qualifiers. Examples of such expressions are *young*, *very young*, *not old*, *young or very young*, *most fuzzy pioneers are old*, or *fuzzy systems very likely replace many traditional models in engineering*. At the core we usually apply Osgood scales for linguistic values in which case our linguistic scales are such as

$$P \text{ – more or less } P \text{ – neither } P \text{ nor } Q \text{ (neutral value) – more or less } Q \text{ – } Q$$

in which Q is the antonym of P. These scales usually contain an odd number of values (mostly five or seven). For example, given the linguistic variable Age, P and Q may be *young* and *old*, respectively. A more challenging task is nevertheless to integrate our linguistic framework in the logical structures, and this issue is considered next.

69.3.1 Reasoning and the Real World

In the worlds of fuzzy logics, we should operate fluently with our formal language, logical operators and inference rules and simultaneously our logics should mimic well the true human reasoning. From the mathematical standpoint, our mathematical operations should meet the foregoing challenges and still base on simple calculations. Typical crucial problems in this area are related to truth valuation and quantification, and we will consider them briefly.

In truth valuation the fuzzy community usually applies explicitly or implicitly the correspondence theory of truth, and this idea is also maintained in Alfred Tarski's well-known definition that [6,9]

expression *X is P* is true if and only if X is P.

Hence, we assume that truth manifests the relationship between the linguistic and real world. For example, the linguistic expression *Snow is white* is true if and only if snow is white in the real world. However, various interpretations on this idea are available in fuzzy systems, in particular, when truth valuations are specified in practice.

Within fuzzy systems we also have to bear in mind that we apply many-valued logic. However, one crucial problem seems to be that confusions still prevail when non-true truth values are considered. In this context we should notice that within fuzzy logics there is a clear distinction between the values *true*, *false*, *not true* and *not false*. Unlike in the bivalent logics, *true* is now distinct from *not false* and *false* is distinct from *not true*. This is due to the fact that *not true* means anything else but true and *not false* means anything else but false. Hence, *not true* includes *false* and *not false* includes *true*.

We should thus apply these metarules when the truth values are assigned (iff $=_{df}$ if and only if) [14]:

1. *X is P* is true iff X is P.
2. *X is P* is false iff X is the antonym of P.
3. *X is P* is not true iff X is not P.
4. *X is P* is not false iff X is not the antonym of P.

For example,

1. *John is young* is true iff John is young.
2. *John is young* is false iff John is old.
3. *John is young* is not true iff John is not young.
4. *John is young* is not false iff X is not old.

If we are unable to find an appropriate antonym for the expression P, we may use its negation. Another challenge is to apply modified values in an appropriate manner. For example, if *X is more or less P* is true, what is the truth value of the expression *X is P*?

The author has also applied the idea of the Osgood scale in this context, and hence, also bearing in mind the foregoing metarules, we may evaluate truth according to the degrees of similarity between the given expressions and their true counterparts. True expressions have maximal and false expressions minimal degrees of similarity, respectively. In other words, we consider the similarity between P and R when

X is P, provided that X is R.

in which R is the true counterpart of P. The higher the degree of similarity between P and R, the closer our truth value is to truth. Hence, with true expressions it holds that $P = R$. As regards the modified expressions, we may establish, for example,

1. *John is more or less young* is fairly true, provided that John is young.
2. *John is middle-aged* is neither true nor false ("half-true"), provided that John is young.
3. *John is more or less old* is fairly false, provided that John is young.
4. *John is old* is false, provided that John is young.

This idea may also be applied to fuzzy quantifiers even though in this context we still have various unresolved problems. If our one extreme quantifier value is *all*, the other may be *none*. Hence, for example, we may apply the Osgood scale

none – some – many – most – all

In fuzzy numbers they could mean approximately 0 %, 25 %, 50 %, 75 % and 100 %, respectively. In this case the traditional existential quantifier, $\exists$, means *not none*. In truth valuation we should now combine the foregoing metarules with our quantifier rules [6, 14]. Hence, we may start with the Tarskian-type metarule

All Swedes are tall is true iff all Swedes are tall.

However, semantic and pragmatic problems arise when we should evaluate the truths of such expressions as

1. *All Swedes are tall*, provided that none of the Swedes are tall.
2. *All Swedes are tall*, provided that all Swedes are short.
3. *All Swedes are tall*, provided that all Swedes are more or less tall.
4. *All Swedes are tall*, provided that all Swedes are more or less short.
5. *Most Swedes are tall*, provided that some Swedes are tall.
6. *Most Swedes are tall*, provided that some Swedes are more or less tall.
7. *Some Swedes are more or less tall*, provided that many Swedes are more or less short.

Hence, certain basic problems should be resolved in an appropriate manner when fuzzy quasi-natural languages and their semantics are formulated.

The foregoing situation becomes even more problematic when the corresponding quantitative meanings, i.e., fuzzy set-theoretic and logical operations, should be assigned to the linguistic entities. The extension principle is widely used in this context [1], but it often yields inappropriate outcomes from the standpoint of actual human reasoning and our linguistic formulations. In particular in quantification we still expect plausible operations for connecting the linguistic entities to fuzzy set-theoretic entities.

This problem also applies to fuzzy syllogisms. From the linguistic and intuitional standpoints such typical fuzzy syllogisms as the fuzzy modus ponens and modus tollens seem plausible, but when we operate with their quantitative meanings in model construction, our outcomes do not correspond well with the linguistic framework. For example, the mainstream inference methods in a computer environment, the Mamdani and Takagi-Sugeno reasoning, do not correspond sufficiently well with the actual human reasoning even though they are good universal approximators in computer modeling. In fact, they are mathematical models for interpolation with fuzzy sets and useful in this sense. Hence, we still lack such inference engines in which both the inputs and outputs are plausible and normalized fuzzy sets.

69.3.2 General Methodology and the Real World

Resolutions in fuzzy reasoning provide us a basis for general methodological issues. Below we focus briefly on probability theory, hypothesis verification and approximate theories and explanations.

If we consider uncertainty in its mainstream sense as an epistemological issue, we should at least draw a distinction between objective and epistemic approaches. In the former case we assume that uncertainty may be an intrinsic property of entities or phenomena of the real world and thus independent of our knowledge. An example of this is the frequency approach to probability theory. In the epistemic approaches, such as subjective uncertainty, we assume that uncertainty, and probability, exist in the human minds and are thus dependent upon our knowledge. The degrees of uncertainty may now vary among persons.

As was already implicitly assumed above, probability theories usually provide us with useful methods for considering uncertainty. When an objective approach is adopted, we have viable fuzzy resolutions principally suggested by Zadeh [23]. These include the examinations on fuzzy probability variables and distributions [5, 10]. However, still more studies should be performed in particular in statistical analysis, for example, in statistical tests, reasoning and hypothesis verification. In the long term, fuzzy statistical systems should be included fluently in the traditional statistics and thus they may enhance these methods for coping with non-parametric and non-linear systems. Examples of these applications are cluster analysis, discriminant analysis, analysis of variance, analysis of covariance, time series analyses and various regression analyses. Special attention should be devoted to the novel dimension reduction methods because these are essential in both fuzzy and traditional modeling and the prevailing principal component and factor analyses are only appropriate to linear models.

The fuzzy epistemic approaches to uncertainty and probability are more challenging because then we often consider the probabilities of the unique occurrences on the psychological and purely logical grounds. For example, what is the probability that there is life on Mars? Hence, the resolutions provided by the experts may vary. In fact, in this context we evaluate the degrees of confirmation between our hypotheses and the given evidence [2, 17]. Unfortunately, the objective and epistemic approaches to probability are sometimes confused and this has led to the considerations of quasi-problems. Within the epistemic approaches the fuzzy systems are valuable if we apply their conceptions on truth to hypothesis verification. Hence, instead of only accepting or rejecting the hypotheses, we may assess their degrees of acceptance or rejection in a formal and logical manner. The application of the fuzzified modus ponens and modus tollens may open new prospects for this subject matter [11].

Another new frontier would comprise studies on approximate theories and scientific explanations. Today we apply these ideas more or less implicitly in the conduct of inquiry [15], but such novel approaches as Zadeh's fuzzy extended logic [25], would provide a more formal and consequential basis for these. If we consider the structures of theories from the standpoint of theory formation and their role in the conduct of inquiry, we usually examine the relationship between the theories and the real world. Hence, our theories may have truth values and their contents are more or less expected to correspond with the facts of the real world, in particular in scientific realism [4], [7], [8], [16], [18]. In this sense, approximate theories and explanations are in the neighborhood of their true counterparts [9, 12, 16, 18, 25]. For example, many precise theories are only true in idealized conditions and thus they only more or less approximately correspond with the real world. Hence, in fact we operate with their non-true counterparts in practice. In particular, this applies to many mathematical and statistical theories. The planets in our Solar System do not have exact elliptical orbits, or empirical statistical data sets are never exactly normally distributed. We also encounter this situation within the scientific explanations. They may only be true in the idealized conditions, and thus their non-true approximations are only available in practice.

As above, we may consider the truths of the theories and explanations in the light of their true counterparts. The closer they are to their true counterparts, the closer they are to truth, and Zadeh's fuzzy extended logic is a good roadmap for this examination [12, 13, 25]. However, much further studies should still be performed in this area.

The qualitative research, which is widely applied to the human sciences, also encounters many of foregoing logico-methodological problems. In addition, even though they principally apply imprecise entities and approximate reasoning, most of their research work is still based on manual work [3]. Hence, fuzzy systems may provide them with usable computer models if their linguistic models are transformed into fuzzy quasi-natural languages and fuzzy reasoning is also applied. Examples of qualitative research are content analysis, discourse analysis, action research, ethnographic research, phenomenographic research and various case studies. The most challenging tasks in this area are the modeling of the human interpretation and intentional behavior in a computer environment [3,18]. For example, how we can interpret well a given document in an automated manner and even write an abstract of it, or how we can also apply teleological explanations when we model the intentions, motives and other underlying factors which affect on person's behavior. Hence, in the qualitative research fuzzy systems still await for their golden age.

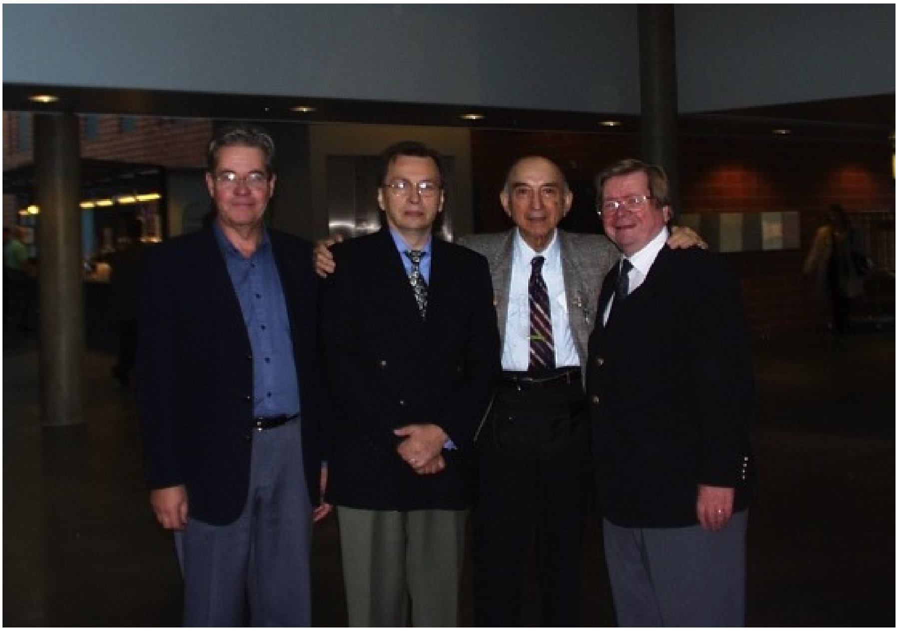

Fig. 69.1. From left to right: Erkki Oja, Vesa A. Niskanen, Lotfi Zadeh and Teuvo Kohonen in August 2000 at the Helsinki summer school on Soft Computing "Top Learning on Top of Europe"

69.4 Conclusions

We considered fuzzy systems from the logico-methodological standpoint. The fuzzy systems with such novel results as Zadeh's fuzzy extended logic open new prospects for the conduct of inquiry. They enable us to consider better such subject matters as scientific reasoning, theory formation, model construction, hypothesis verification and scientific explanation. Prior to the fuzzy systems, imprecise entities were only considered in an informal manner although their existence was already recognized in the scientific community.

However, we still encounter many problems and meet various challenges within the basic logico-methodological principles of fuzzy systems. Examples of these were truth valuation, quantification, fuzzy reasoning, probability modeling and approximate theories and explanations. These issues aroused problems at linguistic, logical and computer-modeling levels.

By resolving these problems we may provide a firm basis for our future studies as well as apply more intelligible methods to the conduct of inquiry.

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References

1. Bandemer, H., Näther, W.: Fuzzy Data Analysis. Theory and Decision Library, Series B: Mathematical and Statistical Methods. Kluwer, Boston (1992)
2. Carnap, R.: The Logical Foundations of Probability. The University of Chicago Press, Chicago (1962)
3. Denzin, N.K., Lincoln, Y.S.: Handbook of Qualitative Research. Sage, Thousand Oaks (1994)
4. Fetzer, J.: Philosophy of Science. Paragon House, New York (1993)
5. Grzegorzewski, P., Hryniewicz, O., Gil, M.-Á. (eds.): Soft Methods in Probability, Statistics and Data Analysis. Physica-Verlag, Heidelberg (2002)
6. Haack, S.: Philosophy of Logics. Cambridge University Press, Cambridge (1978)
7. Hempel, C.G.: Philosophy of Natural Science. Prentice Hall, Englewood Cliffs (1966)
8. Nagel, E.: The Structure of Science. Routledge & Kegan Paul, London (1961)
9. Niiniluoto, I.: Truthlikeness. Reidel, Dordrecht (1987)
10. Niskanen, V.A.: A Meta-level Approach to Approximate Probability. In: Setchi, R., Jordanov, I., Howlett, R.J., Jain, L.C. (eds.) KES 2010, Part IV. LNCS, vol. 6279, pp. 116–123. Springer, Heidelberg (2010)
11. Niskanen, V.A.: Application of Approximate Reasoning to Hypothesis Verification. Journal of Intelligent & Fuzzy Systems 21(5), 331–339 (2010)
12. Niskanen, V.A.: Application of Zadeh's Impossibility Principle to Approximate Explanation. In: Proceedings of the IFSA 2009 Conference, Lisbon, pp. 352–360 (2009)
13. Niskanen, V.A.: Fuzzy Systems and Scientific Method – Meta-level Reflections and Prospects. In: Seising, R. (ed.) Views on Fuzzy Sets and Systems from Different Perspectives. STUDFUZZ, vol. 243, pp. 37–65. Springer, Heidelberg (2009)

14. Niskanen, V.A.: Soft Computing Methods in Human Sciences. STUDFUZZ, vol. 134. Springer, Heidelberg (2004)
15. Popper, K.R.: The Logic of Scientific Discovery. Hutchinson, London (1959)
16. Rescher, N.: Scientific Explanation. The Free Press, New York (1970)
17. Wright, V., Henrik, G.: The Logical Foundations of Probability. Blackwell, Oxford (1957)
18. Von Wright, G.H.: Explanation and Understanding. Cornell University Press, Cornell (1971)
19. Zadeh, L.A.: Fuzzy Logic and Approximate Reasoning. Synthese 30, 407–428 (1975)
20. Zadeh, L.A.: Fuzzy Logic = Computing with Words. IEEE Transactions on Fuzzy Systems 2, 103–111 (1996)
21. Zadeh, L.A.: Toward a Theory of Fuzzy Information Granulation and Its Centrality in Human Reasoning and Fuzzy Logic. Fuzzy Sets and Systems 90(2), 111–127 (1997)
22. Zadeh, L.A.: From Computing with Numbers to Computing with Words – From Manipulation of Measurements to Manipulation of Perceptions. IEEE Transactions on Circuits and Systems 45, 105–119 (1999)
23. Zadeh, L.A.: Toward a Perception-Based Theory of Probabilistic Reasoning with Imprecise Probabilities. Journal of Statistical Planning and Inference 105(2), 233–264 (2002)
24. Zadeh, L.A.: From Search Engines to Question Answering Systems? The Problems of World Knowledge, Relevance, Deduction and Precisiation. In: Sanchez, E. (ed.) Fuzzy Logic and the Semantic Web. Elsevier, Amsterdam (2006)
25. Zadeh, L.A.: Toward Extended Fuzzy Logic – A First Step. Fuzzy Sets and Systems 160, 3175–3181 (2009)

70

How Ideas of L.A. Zadeh Gave Rise to Mathematical Fuzzy Logic

Vilém Novák

70.1 Introduction

Everybody who heard Zadeh's plenary talks in various conferences could realize that Lotfi never gave the same talk twice, despite the fact that he was giving each year about 60 lectures all over the world. And even when repeating something, he always explained the matter from different points of view or shed a different light on it. His talks are never too detailed, but they are always full of inspiration, suggesting new ideas, new concepts. Though one cannot always agree with his way of solution of the given problem, his suggestions and ideas sooner or later found fertile soil and made us think about our problems from a new perspective. His work, personality, way of life, permanent good and positive mood are never ending source of inspiration for which we are grateful to him.

In this short paper, I want to underline that Lotfi Zadeh's ideas stand also behind one of the noble mathematical theories of today — the *mathematical fuzzy logic* (MFL). This is a deeply elaborated theory capable of modeling the vagueness phenomenon and having high potential for applications. Moreover, MFL originated in the Czech Republic and so, I will also mention Zadeh's relations to this country.

70.2 L.A. Zadeh in the Czech Republic

Zadeh's work became known in the Czech Republic very early since the fuzzy set theory was recognized as an interesting mathematical theory providing a working model of the vagueness phenomenon (in a way similar to probability theory that provides the same for uncertainty). This was interesting especially for logicians and so, his work soon found its followers. In 1976, Czech mathematician J. Pavelka defended a PhD thesis devoted to rigorous mathematical elaboration of fuzzy logic. Besides the early Zadeh's papers, e.g. [26, 27] it was also strongly influenced by the paper [6] written by the J. A. Goguen. The thesis was published in a sequence of three seminal papers [24] and it can be taken as one of the turning points for the development of MFL.

The popularity of fuzzy set theory in the Czech Republic was also positively affected by the book [12] written by the author of this paper. This book was quite successful and so, it was published as revised English edition [13] and then again in

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Fig. 70.1. L. A. Zadeh and I. Perfilieva at IFSA'97 World Congress in Prague listening to the concert of participants

Czech [15]. Let us emphasize that fuzzy set theory was for the first time explained in this book in a unified way from the point of view of MFL. Moreover, the book also underlined the role of natural language in connection with fuzzy sets for construction of more realistic models.

One of the reasons for popularity of the the work of L. A. Zadeh was the fact that he recognized the power of one of the distinguished features of natural language — the vagueness of its semantics. Zadeh proposed interesting way how vagueness of semantics of natural language can be captured using a simple mathematical tool — the fuzzy set theory. Since there is a long tradition in the Czech linguistic school[1], his ideas could not be left unnoticed.

Fig. 70.2. L. A. Zadeh with his friends, I. Perfilieva, R. R. Yager and D. Filev at IFSA'97 World Congress in Prague

Because of the popularity of fuzzy sets, each visit of L. A. Zadeh in the Czech Republic was eagerly expected. He visited it four times during the past 30 years. His first visit was at International Conference Coling'82 which took place in Prague in 1982. This was also occasion for the author of this paper to get acquainted with L. A. Zadeh. He was from the very beginning extremely friendly and we talked as if being already old friends.

[1] Famous Czech linguists in seventies and eighties are P. Sgall, E. Hajičová and J. Panevová. One of their most important publications is the book [25].

The second visit was again in Prague at IFSA'97 World Congress 1997. This congress was very successful with over 350 participants. Two memories on this congress are in Figures 70.1 and 70.2.

But Czech Republic is not only Prague. My native town, Ostrava, became during years important center both of the theory as well as of various kinds of applications of fuzzy logic and related techniques. A very important event in its history was the award of the title Doctor Honoris Causa given to L. A. Zadeh by the University of Ostrava in 1998. Actually, it was his 10^{th} Dr. H. C. title and the first honorary doctorate given to foreign scientist by the University of Ostrava. As always, Lotfi came to Ostrava full of joy and energy. Though being the top world scientist, he remained modest and spent many hours in discussions with people from our institute, most of them being very young. His ceremonial speech was very impressive, because he not only presented his road to fuzzy logic but also mentioned serious political and social concerns and warned our young democracy against various bad phenomena of the "advanced capitalism", such as advertisements, bureaucracy, etc.

Fig. 70.3. 10^{th} Dr.H.C. title given to L. A. Zadeh by the University of Ostrava in the Czech Republic

The last visit of Lotfi in the Czech Republic was in 2007 at the occasion of EUSFLAT 2007 Ostrava International Conference which was organized by our institute. One remembrance to this conference is in Figure 70.4.

Fig. 70.4. L. A. Zadeh and I. Perfilieva at EUSFLAT 2007 International Conference in Ostrava

70.3 On the Concept of Fuzzy Logic

Though the concept of fuzzy logic was inherent in the early Zadeh's papers, it is not clear, who and when indeed used this term for the first time. In early seventies, though, it was used in many papers written not only by Zadeh but also by other authors. Moreover, this term from the very beginning was ambiguous.

In 1976, Gaines in his paper [5] distinguished three meanings of fuzzy logic:

(a) *Fuzzy logic as a basis for reasoning with vague or imprecise statements*. Gaines argues that this is consistent with the colloquial use of the term "fuzzy" so that this term replaces previous usage of terms such as "inexact" or "vague".
(b) *Fuzzy logic as a basis for reasoning with imprecise statements using fuzzy sets theory for the fuzzification of logical structures*. This is more specific version of (a) consistent with many papers of Zadeh.
(c) *A multivalued logic in which truth values lay in* $[0,1]$. Fuzzy logic is thus taken as a generalization of classical mathematical logic.

It should be emphasized that Zadeh himself did not consider fuzzy logic in the sense (c) but mainly in the sense (b) and introduced several now well known concepts: fuzzy IF-THEN rule, compositional rule of inference, linguistic hedges modeled using special operations over fuzzy sets, fuzzy quantifiers (see [28,29,27,31] and newly also [32, 33]). All these ideas fertilized the grounds for developing of fuzzy logic in the sense (c). Unfortunately, during years the meaning of the term "fuzzy logic" was still more and more widened up to denoting, in fact, all kinds of applications that use fuzzy sets. To avoid various misunderstandings special terms were introduced instead. The former wide meaning is sometimes called *fuzzy logic in wide sense*. But in fact, it is preferable in this case not to speak about fuzzy logic at all.

Fuzzy logic in the sense (c) is developed as a *mathematical fuzzy logic* and now called *fuzzy logic in narrow sense* (FLn). Note that Goguen in his seminal paper [6] replaced the term fuzzy logic by the "logic of inexact concepts". Following Goguen's and Zadeh's ideas Pavelka in his PhD thesis introduced a deeply developed formal system of FLn which is now called *fuzzy logic with evaluated syntax* $\mathrm{Ev}_{Ł}$. The papers [24] contain full description of the formal system of propositional $\mathrm{Ev}_{Ł}$ including algebraic analysis of possible structures of its truth values, its metatheory and a complicated algebraic proof of syntactico-semantical completeness of $\mathrm{Ev}_{Ł}$. We can say without exaggeration that Pavelka's work started the development of FLn as a noble mathematical theory. It is important to note that this logic goes thoroughly along the principal concept of fuzziness by assuming not only truth values between 0 and 1 but enabling also axioms not to be fully convincing, i.e. a formula A can be axiom only in some (arbitrary) degree. As a consequence, we arrive at the concept of *provability degree* of a formula A. A very important is the metatheorem stating that if we consider a residuated lattice on $[0,1]$ and the implication (residuation) operation is not continuous then it is *not possible to form a syntactico-semantically complete fuzzy logic with evaluated syntax*. Consequently, the only plausible structure of truth values on $[0,1]$ for this logic are the standard Łukasiewicz MV-algebra and its isomorphs. The author of this paper followed the ideas of J. Pavelka and developed a predicate version of $\mathrm{Ev}_{Ł}$ in his PhD thesis defended in 1988 (see [13, 15, 14]). The $\mathrm{Ev}_{Ł}$ logic is in detail presented in the book [23].

A great impulse for further development of FLn came from the Czech logician P. Hájek who in his book [8] presented MFL to great depth and significantly helped fuzzy logic to be recognized also by mathematicians coming from outside fuzzy community. At present, many top mathematicians all over the world contribute to MFL (cf. [1, 7] and the citations therein). Unified efforts of them crowned Zadeh's and Goguen's great ideas, besides others, by proving that all systems of FLn are complete, i.e. that *a formula A is provable in a theory T iff it is true in the degree 1 in all models of T*. In case of predicate $\mathrm{Ev}_{Ł}$, the following nice generalization of the Gödel-Henkin completeness theorem holds:

$$T \vdash_a A \iff T \models_a A$$

for all fuzzy theories T and formulas A where by $T \vdash_a A$ we mean that a formula A is *provable in a degree a* in T and by $T \models_a A$, that it is *true in a degree a*.

Zadeh's seminal ideas concerning fuzzy logic in the sense (b) above led to establishing the paradigm of *fuzzy logic in the broader sense* (FLb-logic) in [16]. This logic is an extension of FLn and so, it also belongs to mathematical fuzzy logic. Its program is to develop a *formal theory of natural human reasoning, which is* *characterized by the use of natural language*. At present, FLb-logic consists of the following special theories:

(a) A formal theory of evaluative linguistic expressions ([19, 18]).
(b) A formal theory of fuzzy/linguistic IF-THEN rules and reasoning based on them ([4, 17, 21, 22]).
(c) A formal theory of intermediate and generalized quantifiers ([3, 9, 11, 20]).

It should be noted that the paradigm of FLb-logic partially overlaps with the paradigm of commonsense reasoning proposed by J. McCarthy in [10]. The main drawback of these formalizations of commonsense reasoning is neglecting vagueness present in the meaning of natural language expressions (cf. [2] and the citations therein). Therefore, it is notable that the possibility to use fuzzy logic in commonsense reasoning was proposed by Zadeh already in 1983 in the paper [30].

70.4 Conclusion

We may conclude that the ideas of L. A. Zadeh inspired J. A. Goguen to its "logic of inexact concepts". Ensuing deep elaboration especially by the Czech mathematicians led to establishing mathematical fuzzy logic both in narrow as well as in broader sense. While the former nontrivially generalizes classical mathematical logic, the latter is a thorough mathematical elaboration of Zadeh's ideas to make fuzzy logic as a basis for reasoning with imprecise statements using fuzzy set theory for the fuzzification of logical structures. It should be emphasized that Zadeh showed that the theories proposed by him can have a lot of various kinds of applications and his ideas touched many disciplines ranging from abstract theory, such as philosophical logic through linguistics up to engineering.

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References

1. Cintula, P., Hájek, P., Horčík, R.: Formal Systems of Fuzzy Logic and Their Fragments. Annals of Pure and Applied Logic 150, 40–65 (2007)
2. Davis, E., Morgenstern, L.: Introduction: Progress in formal commonsense reasoning. Artifical Intelligence 153, 1–12 (2004)
3. Dvořák, A., Holčapek, M.: L-fuzzy Quantifiers of the Type $\langle 1 \rangle$ Determined by Measures. Fuzzy Sets and Systems 160, 3425–3452 (2009)
4. Dvořák, A., Novák, V.: Formal Theories and Linguistic Descriptions. Fuzzy Sets and Systems 143, 169–188 (2004)
5. Gaines, B.R.: Foundations of Fuzzy Reasoning. International Journal of Man-Machine Studies 8, 623–668 (1976)
6. Goguen, J.A.: The Logic of Inexact Concepts. Synthese 19, 325–373 (1968-1969)

7. Gottwald, S.: Mathematical Fuzzy Logics. Bulletin of Symbolic Logic 14(2), 210–239 (2008)
8. Hájek, P.: Metamathematics of Fuzzy Logic. Kluwer, Dordrecht (1998)
9. Holčapek, M.: Monadic L-Fuzzy Quantifiers of the Type $\langle 1^n, 1\rangle$. Fuzzy Sets and Systems 159, 1811–1835 (2008)
10. McCarthy, J.: Programs with Common Sense. In: Proceedings of the Teddington Conference on the Mechanization of Thought Processes, pp. 75–91. Her Majesty's Stationary Office, London (1959)
11. Murinová, P., Novák, V.: A Formal Theory of Generalized Intermediate Syllogisms. Fuzzy Sets and Systems 186, 47–80 (2012)
12. Novák, V.: Fuzzy Sets and Their Applications. SNTL, Praha (1986) (in Czech)
13. Novák, V.: Fuzzy Sets and Their Applications. Adam Hilger, Bristol (1989)
14. Novák, V.: On the Syntactico-semantical Completeness of First-order Fuzzy logic I, II. Kybernetika 26, 47–66, 134–154 (1990)
15. Novák, V.: Fuzzy Sets and Their Applications. SNTL, 2nd edn., Praha (1990) (in Czech)
16. Novák, V.: Towards Formalized Integrated Theory of Fuzzy Logic. In: Bien, Z., Min, K.C. (eds.) Fuzzy Logic and Its Applications to Engineering, Information Sciences, and Intelligent Systems, pp. 353–363. Kluwer, Dordrecht (1995)
17. Novák, V.: Perception-based Logical Deduction. In: Reusch, B. (ed.) Computational Intelligence, Theory and Applications, pp. 237–250. Springer, Berlin (2005)
18. Novák, V.: Mathematical Fuzzy Logic in Modeling of Natural Language Semantics. In: Wang, P., Ruan, D., Kerre, E. (eds.) Fuzzy Logic – A Spectrum of Theoretical & Practical Issues, pp. 145–182. Berlin, Elsevier (2007)
19. Novák, V.: A Comprehensive Theory of Trichotomous Evaluative Linguistic Expressions. Fuzzy Sets and Systems 159(22), 2939–2969 (2008)
20. Novák, V.: A Formal Theory of Intermediate Quantifiers. Fuzzy Sets and Systems 159(10), 1229–1246 (2008)
21. Novák, V., Lehmke, S.: Logical Structure of Fuzzy IF-THEN Rules. Fuzzy Sets and Systems 157, 2003–2029 (2006)
22. Novák, V., Perfilieva, I.: On the Semantics of Perception-based Fuzzy Logic Deduction. International Journal of Intelligent Systems 19, 1007–1031 (2004)
23. Novák, V., Perfilieva, I., Močkoř, J.: Mathematical Principles of Fuzzy Logic. Kluwer, Boston (1999)
24. Pavelka, J.: On Fuzzy Logic I, II, III. Zeitschrift für Mathematische Logik und Grundlagen der Mathematik 25, 45–52, 119–134, 447–464 (1979)
25. Sgall, P., Hajičová, E., Panevová, J.: The Meaning of the Sentence in Its Syntactic and Pragmatic Aspects. D. Reidel, Dordrecht (1986)
26. Zadeh, L.A.: Fuzzy Sets. Information and Control 8, 338–353 (1965)
27. Zadeh, L.A.: Outline of a New Approach to the Analysis of Complex Systems and Decision Processes. IEEE Transations on Systems, Man, and Cybernetics SMC-3, 28–44 (1973)
28. Zadeh, L.A.: Quantitative Fuzzy Semantics. Information Sciences 3, 159–176 (1973)
29. Zadeh, L.A.: The Concept of a Linguistic Variable and its Application to Approximate Reasoning I, II, III. Information Sciences 8-9, 199–257, 301–357, 43–80 (1975)
30. Zadeh, L.A.: Commonsense Knowledge Representation Based on Fuzzy Logic. IEEE Computer Magazine 16, 61–65 (1983)
31. Zadeh, L.A.: A Computational Approach to Fuzzy Quantifiers in Natural Languages. Computers and Mathematics 9, 149–184 (1983)
32. Zadeh, L.A.: Fuzzy Logic = Computing with Words. IEEE Transactions on Fuzzy Systems 4, 103–111 (1996)
33. Zadeh, L.A.: Precisiated Natural Language. AI Magazine 25, 74–91 (2004)

71

Fuzzy Sets Seemed to Work

Hannu Nurmi

71.1 The First Encounter

Perhaps it is appropriate – given the nature of the subject matter – that it is difficult for me to date my first encounter with fuzzy sets in any degree of certainty. Some bounds for the time frame can, however, be given. I presented my doctoral thesis in 1974 and, while systems theory played a role in it, fuzzy sets were not dealt with [6]. My next book-length work was published in 1977 by *Societas Scientiarum Fennica* [8]. It was a collection of essays and one of them unmistakably refers to fuzzy sets: „On Fuzzy Games“. On closer inspection it turns out that in a paper published in 1975 I already dealt with fuzzy sets, albeit in a very cursory manner ([7], p. 242-243) The paper was published in proceedings of the Second European Meeting on Cybernetics and Systems Research held in 1974 in Vienna. The series of meetings was held immediately after Easter on even-numbered years starting 1972. This puts my first exposure to fuzzy sets to the spring of 1974.

The notion of fuzzy sets seemed to open possibilities for addressing issues that I had been uncomfortable with from the beginning of my post-graduate work. More specifically, the formal apparatus used in social sciences in dealing with complexity appeared somehow inadequate *inter alia* since it made intuitively too sharp distinctions between categories. The ways to deal with imprecision were mostly derived from probability theory and tended to equate impreciseness with randomness which intuitively struck me – and many others – as inappropriate. Ours was, rather, a view that social imprecision does not evaporate after "the experiment" (e.g. tossing a coin) has been conducted, but is inherent in the events themselves. A set concept applicable to the characterization of notions like "substantial increase in energy consumption", "highly esteemed colleague" etc. looked to me very promising especially from the view point of the social sciences. Therefore, Zadeh's concept of fuzzy set was welcomed with open arms. [21]

71.2 Modes of Impreciseness

In his well-known article Weaver made a distinction between degrees of complexity dealt with in history of science. [20] Problems of simplicity involve basically two-variable relationships. That was the domain in which – according to Weaver – physical science operated till the end of the 19'th century. Problems of disorganized complexity emerged also in physics when it became necessary to deal with

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large numbers of variables. The techniques of probability theory and statistical mechanics were devised to tackle these problems. Weaver's third category, organized complexity, in a sense falls between simplicity and disorganized complexity. To wit, these are problems involving "a sizable number of factors which are interrelated into an organic whole." [20] Arguably fuzzy sets can help solving problems in each of the three categories since it seems that impreciseness that takes on degrees is simply missing in Weaver's characterization.

The last statement brings us to the subject of my talk at the Conference on Subjective Probability, Utility and Decision Making (SPUDM) in Warsaw in 1977.[1] By then I had studied Bellman and Zadeh's seminal work on individual decision making in fuzzy environments. [1] What I wanted to explain in my talk were the conceptual, epistemic and methodological differences between probability and fuzziness in contexts that are relevant for decision theory. I found the concept of verisimilitude (truthlikeness) particularly useful in this endeavor. Intuitively, verisimilitude is a matter of degree in the same way as membership in a fuzzy set. An yet somewhat confusingly, the word for verisimilitude in many languages, Wahrscheinlichkeit (German), sannolikhet (Swedish), todennäköisyys (Finnish), is interpreted as probability. To be sure, there are instances in which fuzziness and probability co-exist. It may, for example, be the case that the perceived degree of strength of a speech condemning a certain act – i.e. the degree of its membership in the set of condemning speeches or stands – can be derived from the relative frequency ("empirical probability") of condemning utterances in the speech, provided that the latter can be unambiguously determined. Indeed, I found it (and still do) futile to try to replace either probability with fuzziness or vice versa. They just seem to refer to different things. So, a reasonable way to go is to take full advantage of the potential that these two notions and their underlying methodologies have.

71.3 Fuzzy Games

After defending my thesis on causality and complexity, I embarked on a long journey to the world of decision and game theory. In preparing lectures on the subject I encountered the well-known game of Prisoner's Dilemma.[2] By now it is taught to first-year students in social sciences, but in mid-1970's it was still a challenging novelty and seemed to present a paradox of sorts in showing that a dominant strategy choice on the part of players leads to a (Pareto) non-optimal outcome. Thus, a clear conflict between individual and collective rationality emerges. This is the normative aspect of Prisoner's Dilemma. Empirical observations suggest that it is not the case that when called upon to play a Prisoner's Dilemma game people would in general choose the dominating, non-cooperative strategy. Hence, the predictive accuracy of the strongest principle of rationality is called into question. Since many social

[1] The talk was much later published in the conference proceedings [15].

[2] The lectures eventually developed into a Finnish elementary textbook [10]. A seminal account of Prisoner's Dilemma is [19]. See also [18]. My early overview of "solutions" to Prisoner's Dilemma is given in [9].

situations can be described as Prisoner's Dilemmas, it makes sense to look at the underlying parameters of the game. In similar vein, it would also seem plausible to approach Prisoner's Dilemma from the viewpoint of fuzzy sets.

Fuzzy sets can be introduced to game models in several ways. One obvious way is to generalize the payoffs. Instead of sums of money, one could invoke fuzzy goal sets. Another way is to deem the strategies as fuzzy sets. In Prisoner's Dilemma, the degree of cooperativeness of a strategy would seem a plausible way to proceed. A combination of these two ways results in a fuzzy game where each strategy has a membership degree in the cooperativeness subset and each strategy combination of players leads to a non-fuzzy outcome with its associated membership degrees in the players' fuzzy goal sets. This is the basic idea underlying the notion of fuzzy Prisoner's Dilemma.

Another approach to fuzzy games resorts to a representation of games as fuzzy automata [11]. A finite deterministic automaton is defined by means of a state transition function which, for each of the finite number of (present) states and each input, assigns the (next) state to which the automaton moves. A non-deterministic automaton, in turn, specifies, again for any given state and any input, a subset of states. When considering games, the inputs are the strategies of players. In Zadeh's definition the next state is fuzzy, i.e. characterizable by a membership function [22]. This apparatus is well-suited for representation of repeated two-person games.

71.4 Fuzzy Social Choice

In the 1970's the literature of social choice theory started to grow rapidly. One of the main motivations of the theory was voting. The present writer's interest in fuzzy voting games was due to its potential in accounting for heretofore unexplained empirical findings. Roughly simultaneously I started to work on solution concepts for fuzzy voting games. In these works the concept of fuzzy goal set and fuzzy preference relation devised by Zadeh was of crucial importance [21].

71.4.1 Explaining Experimental Puzzles

The voting game experiments conducted by Fiorina and Plott in late 1970's resulted in several puzzling observations [4]. The experiments focused on voting behavior in spatial context where each experimental subject was given an ideal point and (circular) indifference curves in a two-dimensional Euclidean space. The outcomes of the voting game were determined by pairwise majority votes between proposed alternatives in the space. The ideal point of each voter represents the maximum payoff for this voter and the further – in terms of Euclidean distance measure – away from the voter's ideal point a proposal lies, the smaller is its associated payoff for the voter. The purpose of the experiment was to evaluate the effects of various parameters of the voter positions and payoffs to the outcomes ensuing from the pairwise majority voting. In particular the experimenters wanted to assess the predictive accuracy of a solution concept known as the core, i.e. the set (typically singleton or empty) of

Fig. 71.1. Mysterious photo perhaps from 1975; in the middle: Lotfi A. Zadeh sitting next to Ellen Hisdal; behind them: Christer Carlsson and Hannu Nurmi

majority un-dominated points in the space. Several puzzling observations were made by Fiorina and Plott. Firstly, the core point was a better predictor of voting outcomes when the payoffs were relatively larger than when they were small. Secondly, the theoretical equilibrium – the core – was not stable, but in some experiments the core alternative was defeated by another alternative. Thirdly, some experiments ended up with Pareto-suboptimal outcomes. Fourthly, when a slight modification was made in the ideal point configuration so that the core became empty, the outcomes still tended to concentrate in the neighborhood of the point where the core existed before the modification.

The second and third observations were particularly disturbing since they call into question the very rationality of the experimental subjects. By fuzzifying the setting suitably – i.e. by introducing "thick" indifference curves and assuming that the experimental subjects' preferences were fuzzy – it turned out possible to accommodate most of the experimental anomalies encountered in the non-fuzzy setting [12]. Admittedly, since no data on the subjects' fuzzy preferences and subjective indifference regions was available, the explanations in terms of these notions remain conjectural.[3]

[3] The same holds for the solutions to paradoxes encountered in individual decision making (e.g.Allais' paradox) [14].

71.4.2 Fuzzy Solutions to Voting Games

Could a social choice theory, then, be built on the assumption that the individuals are endowed with fuzzy rather than crisp preference relations? This idea was first pursued by Blin and Winston and somewhat later by Bezdek et al. [3], [2]. Another line of research sought to introduce fuzzy analogs for crisp solution concepts [13]. Thus, for example, the set of α-consensus winners, i.e. alternatives preferred to any other alternative with a preference degree of at least α, is a simple generalization of the notion of unanimity winner. Similarly many other solutions were introduced and related to each other [13]. In similar vein, some tournament solution concepts have been transformed to suit fuzzy environments [17]. Hence, fuzzy social choice theory is not only possible, but it already exists, albeit in a rudimentary form. But is it needed? Perhaps it is, since quite a few paradoxes in crisp choice theory can be solved in the fuzzy framework [16]. Thus, perhaps we should agree with Hillinger who argues that in the crisp social choice theory we are faced with "... a new 'paradox of voting': It is theorists' fixation on a context dependent and ordinal preference scale; the most primitive scale imaginable and the mother of all paradoxes." [5].

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References

1. Bellman, R.E., Zadeh, L.A.: Decision Making in a Fuzzy Environment. Management Science 17, B141–B164 (1970)
2. Bezdek, J., Bonnie, S., Spillman, R.: Fuzzy Relation Space for Group Decision Theory. Fuzzy Sets and Systems 1, 255–268 (1973)
3. Jean-Marie, B., Whinston, A.B.: Fuzzy Sets and Social Choice. Journal of Cybernetics 4, 17–22 (1973)
4. Fiorina, M.P., Plott, C.R.: Committee Decisions under Majority Rule: An Experimental Study. American Political Science Review 72, 575–595 (1978)
5. Hillinger, C.: The Case for Utilitarian Voting. Homo Oeconomicus 22(3) (2005)
6. Nurmi, H.: Causality and Complexity (Series B, Tom. 131). Annales Universitatis Turkuensis (1974)
7. Nurmi, H.: Some Lessons from the Study of Biological Systems for the Modelling of Social Systems. In: Trappl, R., Pichler, F.R. (eds.) Progress in Cybernetics and Systems Research, vol. I. Hemisphere Publishing Corporation (1975)
8. Nurmi, H.: Rationality and Public Goods – Essays in Analytic Political Theory (Commentationes Scientiarum Socialium 9). Societas Scientiarum Fennica (1977)
9. Nurmi, H.: Ways out of the Prisoner's Dilemma. Quality and Quantity 11, 135–165 (1977)
10. Nurmi, H.: Johdatus päätös- ja peliteoriaan. Gaudeamus (1978)
11. Nurmi, H.: On Fuzzy Games. In: Robert, T. (ed.) Progress in Cybernetics and Systems Research, vol. IV. Hemisphere Publishing (1978)
12. Nurmi, H.: A Fuzzy Solution to a Majority Voting Game. Fuzzy Sets and Systems 5, 187–198 (1981)

13. Nurmi, H.: Approaches to Collective Decision Making with Fuzzy Preference Relations. Fuzzy Sets and Systems 6, 249–259 (1981)
14. Nurmi, H.: Imprecise Notions in Individual and Group Decision Theory: Resolutions of Allais Paradox and Related Problems. Stochastica VI, 283–303 (1982)
15. Nurmi, H.: Probability and Fuzziness: Some Methodological Considerations. In: Sjöberg, L., Tyszka, T., Wise, J.A. (eds.) Human Decision Making, Doxa (1983)
16. Nurmi, H.: Resolving Group Choice Paradoxes Using Probabilistic and Fuzzy Concepts. Group Decision and Negotiation 10, 177–198 (2001)
17. Nurmi, H., Kacprzyk, J.: On Fuzzy Tournaments and Their Solution Concepts in Group Decision Making. European Journal of Operational Research 51, 223–232 (1991)
18. Rapoport, A.: Prisoner's Dilemma – Recollections and Observations. In: Rapoport, A. (ed.) Game Theory as a Theory of Conflict Resolution. D. Reidel (1974)
19. Rapoport, A., Chammah, A.M.: Prisoner's Dilemma. University of Michigan Press (1965)
20. Weaver, W.: Science and Complexity. American Scientist 36, 536–544 (1948)
21. Zadeh, L.A.: Fuzzy Sets. Information and Control 8, 338–353 (1965)
22. Zadeh, L.A.: Toward a Theory of Fuzzy Systems. In: Kálmán, R.E., DeClaris, N. (eds.) Aspects of Network and Systems Theory. Holt, Rinehart and Winston, New York (1971)

72

From Fuzzy Deformable Prototypes to Fuzzy Web Search

José A. Olivas

Abstract. It is presented a resume of my scientific relationship with Prof. Zadeh and the BISC group, starting with the way of reasoning using patterns and prototypes (in the framework of AI) and going to fuzzy Web search and text understanding (semantics).

72.1 Fuzzy Deformable Prototypes

I had the pleasure of meeting Prof. Zadeh at the ISMVL conference (organized by Senen Barro, Alberto Bugarín and Alejandro Sobrino), held in Santiago de Compostela, Spain, in 1996 (see Picture 72.1). My early conversations with Prof. Zadeh were about how people make inferences and reasoning based in patterns, adapting the experience acquired patterns (or "prototypes") to the real situations that a human can approach. As an example, we can observe the act of driving a car in a hailstorm: If it starts to hail while we are driving, we adjust to our "driving under potentially dangerous conditions" scheme. In other words, we associate a fact or a set of facts with a paradigm so that the paradigm interprets the situation and the actions we carry out depend on it. To generalize, many of the actions we carry out in our daily life depend on our forming an interpretation, on our finding the most similar paradigm or prototype for the circumstances of the problem. In AI developments, many times we try to simulate the expert's capacity for interpreting the situations and finding the prototype or prototypes of the observed phenomena that is most appropriate in the current conditions.

So I worked in those ideas and I presented my PhD. Thesis in 2000 [6], Prof. E. Trillas was my advisor, introducing the Fuzzy Deformable Prototypes (from now on FDPs), that can provide an adequate formal framework for working with this idea. FDPs come from the confluence of two interesting approaches to the concept of prototype: the "deformable prototypes" of Bremermann [2], introduced in the late seventies from the field of pattern recognition, and the fuzzy prototypes of Zadeh [15], result of a controversy with cognitive psychologists [7]. Below is a brief description of both concepts. In the framework of 'deformable prototypes' a real element is classified according to the minimum energies required for physically deforming the closest prototype. In turn, a fuzzy prototype is not an element – usually the best representative of a set or class –, but a reunion of good, bad and borderline elements of a category. Zadeh mentioned the classical prototype theories from the point of view

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of psychology, criticizing precisely what we have just pointed out: that these theories do not fit the function that a prototype should have. Zadeh's approach to what must be taken as a prototype is less intuitive than the conceptions of psychological theories but is more rational and closer to the meaning of a prototypical concept displayed in a more detailed examination. In general, we have observed that Zadeh's idea suggests a concept that encompasses a set of prototypes, which represent the high, medium, or low compatibility of the samples with the concept.

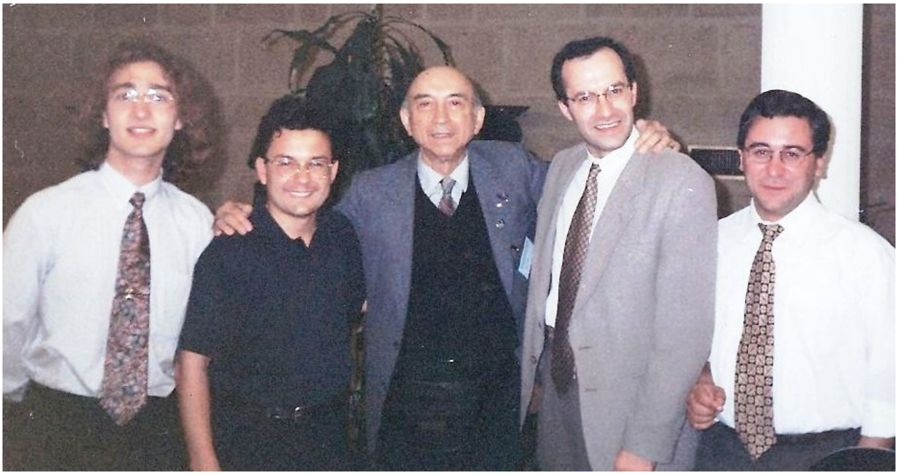

Fig. 72.1. Santiago Fernández-Lanza, Alejandro Sobrino, Lotfi A. Zadeh, Senén Barro and José A. Olivas, ISMVL 1996, Santiago de Compostela, Spain

Taking into account these approaches, FDP can be defined as a linear combination of Fuzzy Prototypical Categories (described as tables of attributes) able to be adapted to any real situation, where the coefficients are the degrees of membership to each of these Fuzzy Prototypical Categories. Broadening the combination described in the concept of Deformable Prototype to the case of affinity with more than one Fuzzy Prototypical Category, the definition of a real situation would be:

$$C_{real}(w_1 \ldots w_n) = \left|\sum \propto p_i(v_1 \ldots v_n)\right| \qquad (72.1)$$

where:

C_{real}	Real case.
$(w_1, \ldots, w_n)$	Parameters describing the real case.
p_i	Degrees of compatibility with Fuzzy Prototypical Categories different to 0.
$(v_1, \ldots, v_n)$	Parameters of these Fuzzy Prototypical Categories.

Many applications of this approach had been accomplished, such as the ones related to Information Systems and Software Engineering applications ([5], [8]), traffic control [1], health documents management [10], social sciences [12] or Information Retrieval and Web Search.

72.2 Fuzzy Web Search

I did my postdoc visiting scholarship at Berkeley's BISC. I interacted with Prof. Zadeh and many other BISC visitors, mainly with Tomohiro Takagi, Tru Cao and Andreas Nürnberger (72.2), together with Marcin Detyniecki and Mori Anvari, working in the organization of the first conference on Fuzzy Logic and Internet, FLINT 2001 (72.3). Due to these interesting interactions, when I came back to my University in Spain, I established the SMILe (Soft Management of Internet and Learning) research team.

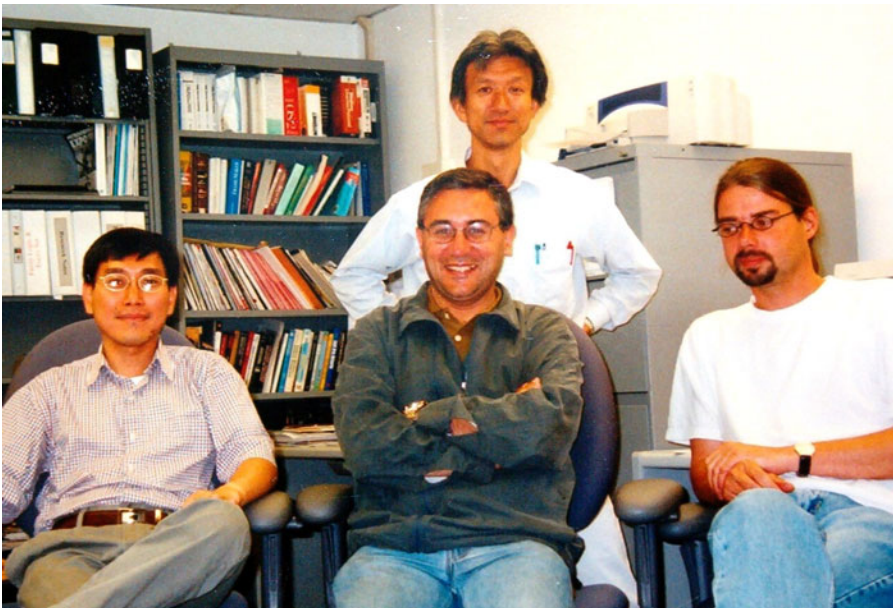

Fig. 72.2. Tru Cao, Tomohiro Takagi, José A. Olivas and Andreas Nürnberger at Berkeley's BISC office in summer 2001

My conclusion was that nowadays there are not commercial Fuzzy Searchers and Soft computing and Fuzzy Logic could play an interesting role in Web Search and Meta-search engines. So some relevant topics could be improved with Soft computing techniques:

72.3 The Role That Nowadays Fuzzy Logic Plays in Search and Meta-search Engines

If it is done a search in Google with the words *"fuzzy searcher"* many results appear. Selected those that we consider to be relevant, we observe that the fuzzy character that some commercial searchers assume is based exclusively on the use of a syntactic fuzzy matching; this is, in a fitting of the word included and possibly badly messed with another from a dictionary that the searcher contains or the searcher accedes and which is correctly written. The result is that the searcher sends a sign of notice (written text) that says: *Did you mean?...* Evidently, though the utility of this function is, to name a searcher as *"fuzzy"* for implementing it is excessive. The proposal is that a searcher will be fuzzy when it implements approximate semantic searches; this is, when it includes in the searches semantic approximate criteria, not only syntactic ones. Some aspects that there would be considered are presented.

Fig. 72.3. Ana García-Serrano, Lotfi A. Zadeh, José A. Olivas, Andreas Nürnberger, Marcin Detyniecki, María José Martín-Bautista and Mori Anvari, BISC FLINT 2001 in Berkeley

72.3.1 The Use of a Dictionary of Synonymous and Thesaurus (Ontology)

When a user searches for a single word, the search can be facilitated by the use of a dictionary of synonyms. The dictionary will allow searches not only for the source word, but for its synonyms and will make possible to calculate the synonymy degree, having to contemplate this degree in the relevancy of the retrieved pages as response to the source terms. The search can also be improved using thesaurus and ontologies. Nowadays, there are many ontologies referring to different domains, that improve several aspects in some applications, but they all have been hand-made following different methodologies. On the other hand, automatic ontology building is a focus in current research where the results hitherto have not been too satisfactory. As an example, we developed FIS-CRM [4] as a model for representing the concepts contained in any kind of document. It can be considered an extension of the vector space model (VSM) [11]. Its main characteristic is that it is fed on the information stored in a fuzzy synonymy dictionary [3] and several fuzzy thematic ontologies. The dictionary stores the synonymy degree between every pair of recognized synonyms. The ontology stores the generality degree between every word and its more general words. The way of calculating this value is the one proposed by Widyantoro and Yen at FLINT 2001 [14]. The key of the FIS-CRM model is first to construct the base vectors of the documents considering the number of occurrences of the terms (what we call VSM vectors) and afterwards readjust the vector weights in order to represent concept occurrences, using for this purpose the information stored in the dictionary and the ontologies. The readjusting process involves sharing the occurrences of a concept among the synonyms which converge to the concept and give a weight to the words that represent a more general concept than the contained ones.

Fig. 72.4. Lotfi A. Zadeh, Vesa Niskanen and José A. Olivas, Berkeley's bar in summer 2001

72.3.2 Sentences Search and Deduction Capabilities

If the search includes sentences, besides the dictionary of synonymous, thesaurus and ontologies, suitable fuzzy connectives should be used, to discriminate for example between a search "*A* and *B*" where *A* and *B* have common information, of the search "*A* and *B*" where *A* and *B* are completely independent. Something similar can happen with the relation "*A* or *B*". Another desirable aspect is that the searcher keeps the meaning of the words in mind under the synonymy relation, to choose the best similarity function. But the problem can be bigger in the case of "causal" relations. First, it is very difficult to detect a causal relation in a written sentence (a query or a text). For example, the text could be: "stormy and dark", that could be understood by a person as: "If the weather is stormy, the sky gets dark". How can a search engine distinguishes the conjunctive 'and' and the causal one? Nowadays is rather impossible, even if there is some knowledge about the context. Second, it is very difficult to find the most adequate implication function to represent the sentence (it is well known that there is a huge variety of fuzzy implications). The detection and management of causal relations (interesting conversations with Prof. Zadeh and V. Niskanen, see picture 72.4) could be very important for developing *Question Answering Systems*. To detect the causal relationships that exist in a collection of documents, a starting point could be to detect conditional phrases. Nevertheless, this is not an easy task. Descartes could not have possibly imagined that to propose his famous phrase "I think, therefore I am", would have given birth to so many conjectures and interpretations for centuries after. In reality, what did he want to say, "First I think and after I am a person", or "As I am capable of thought, I am a person". To sum up, even on this occasion the intention of Descartes seems clear when he expressed his maxim, it is not easy to interpret and format the information expressed in natural language, especially when it involves complex sentences with complicated turns. With the aim of detecting conditional phrases, some basic systems of detecting structures and a classification of sentences have been developed [9] which allow to locate, in terms of basic components (verb tenses, adverbs, linguistic turns, etc.), some causal forms. To accomplish the grammatical analysis, it is observed on the one hand, that it is possible to separate certain causal relationships based on the verb form used, while on the other hand it is possible to separate others based on the adverbs used in the sentences. Both analyses give rise to some causal rules that can be used to make an automatic extraction of knowledge. In the same way, every structure is subdivided into two structures which correspond to the antecedent and consequence of the causal relationship, and a parameter that measures the degree of certainty, conjecture, or compliance of the said causal relationship. In other words, it is not the same to form a sentence such as: "If I win the lottery, I will buy a car", in which there is no doubt that if the antecedent comes true the consequence will come true, as to form the sentence "If we had bought a ticket in Sacramento, we could have won the lottery" which leaves many more doubts and conjectures, in which you cannot be sure that the completion of the antecedent guarantees the consequences. But this is still a Natural Language Processing complex problem. There are some other very interesting approximations, such as the one of Trillas [13] (see picture 72.5, with other SMILe members) for representing conditional sentences with fuzzy implications. On the other hand, an approach based on PNL and protoforms could be a promising work line, such as Prof. Zadeh proposes:[1]

[1] Seminar: Web Intelligence, World Knowledge and Fuzzy Logic. Lotfi A. Zadeh, September 14; 2004, University of California, Berkeley.

"Existing search engines – with Google at the top-have many remarkable capabilities; but what is not among them is deduction capability – the capability to synthesize an answer to a query from bodies of information which reside in various parts of the knowledge base."

"In recent years, impressive progress has been made in enhancing performance of search engines through the use of methods based on bivalent logic and bivalent-logic-based probability theory. But can such methods be used to add nontrivial deduction capability to search engines, that is, to upgrade search engines to question-answering systems? A view which is articulated in this note is that the answer is'No´."

"The problem is rooted in the nature of world knowledge, the kind of knowledge that humans acquire through experience and education."

"It is widely recognized that world knowledge plays an essential role in assessment of relevance, summarization, search and deduction. But a basic issue which is not addressed is that much of world knowledge is perception-based, e.g., "it is hard to find parking in Paris," "most professors are not rich," and "it is unlikely to rain in midsummer in San Francisco." The problem is that (a) perception-based information is intrinsically fuzzy; and (b) bivalent logic is intrinsically unsuited to deal with fuzziness and partial truth."

"To come to grips with the fuzziness of world knowledge, new tools are needed. The principal new tool – a tool which is briefly described in this note – is Precisiated Natural Language (PNL). PNL is based on fuzzy logic and has the capability to deal with partiality of certainty, partiality of possibility and partiality of truth. These are the capabilities that are needed to be able to draw on world knowledge for assessment of relevance, and for summarization, search and deduction."

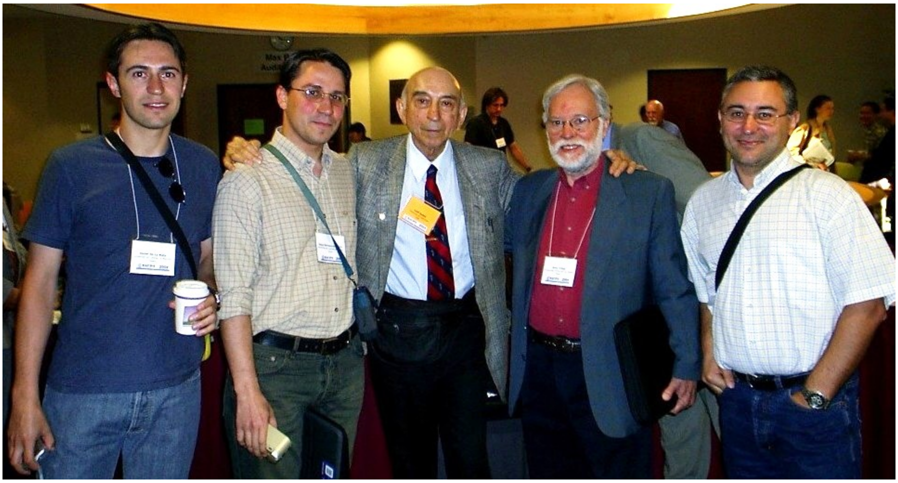

Fig. 72.5. Javier de la Mata, Jesús Serrano-Guerrero, Lotfi A. Zadeh, Enric Trillas and José A. Olivas, NAFIPS 2004 in Banff (Canada)

72.3.3 Combination of Fuzzy Values, Fuzzy Clustering and Web Meta-search Engines Architecture

A Meta-searcher has to carry out a combination of logics (the algorithms that every searcher uses) in order to combine the local similarities in a global similarity or final order. But the local similarities are not based on fuzzy criteria. Therefore, the orders of relevant pages are not approximate. Usually Meta-searchers consider the searchers according to the prestige that they grant for the question that the users do and, depending on it, they qualify its results to incorporate them into the final list. In this case, the approximation comes for a criterion of market, but, again, not for a linguistic criterion. It would be interesting to apply this criterion, first, as it is previously indicated, doing that searchers make fuzzy semantic searches. Later, achieving that Meta-searchers arrange the pages in a final list combining the relevancy of the searchers with the confidence degrees associated with every result of its local obtained list, not only with the word got in the search box, but also using its related (synonyms…) words, the measures of similarity used in the calculation of the degree of linguistic relations and the fuzzy operators used in searches with sentences. Each of these searches would answer, therefore, to a fuzzy logic used by the searchers, which the Meta-searcher would have to combine to provide the final order. The use of the links that provides the order that the Meta-searcher gives might be useful as a test bench to check hypothesis on the combination of fuzzy logics. Another important problem appears when it is necessary to aggregate several different fuzzy values from various sources. Two words (concepts) can have more than one linguistic relations (each one with its fuzzy value), such as hyponymy and synonymy. For example "football" and "soccer" are synonyms but the first is also more general than the last. A causal relation can also exist between both words (concepts). Moreover, a fuzzy relation based on the physical distance (same sentence, paragraph, chapter…) could be considered. Then, it is necessary to join all these different fuzzy values into only one, to be applied in representation and search tasks. How to aggregate these fuzzy values is still an open problem.

Document classification or text categorization (as used in information retrieval context) is the process of assigning a document to a predefined set of categories based on the document content. However, the predefined categories are unknown in a real repository of documents. Text clustering methods can be applied to structure the resulting set of documents, so they can be interactively browsed by the user. Therefore, using a clustering process, it is possible to achieve the splitting up of the collection of documents in a reduced number of groups made up of documents with enough conceptual similarity. There are a lot of fuzzy clustering and classification proposals.

Fuzzy logic could also play a fundamental role in an agent based architecture, mainly in the task of joining the information from different sources (agents) and managing the results in an efficient and satisfactory way.

72.4 Conclusions

It was very interesting and productive for me my interaction with Prof. Zadeh and the BISC members. Taking into account these presented reflections, among others, would make possible to have really fuzzy searchers, or what is the same, searchers that do searches in terms of approximate meanings. The main focus of these engines must be the Web, not for general search artefacts but for Meta-search tools, because they use General Web Search engines as a basis. Having fuzzy searchers would offer the possibility to do interesting tests and experiments. The Artificial Intelligence is an area of mixture of logics, because the approaches in the formal analysis of a sentence can be very different. Then, the logical form of the following phrase, a bit long, but not strange: "I suppose that you believe that I will pick you up a little bit earlier", implies using different logics: belief, non monotonic, fuzzy, temporal, ... But the problem is more complex yet, because, for the words with vague meaning, there can be also several modalities of fuzzy logics. The election has not been studied too much. Meta-searchers could provide a useful frame, restricted by the language that it lets, to research on the variety of formalisms that fuzzy logic provides. Using user profiles in Web Meta-search engines could provide some advantages to improve the search. The user profile can be another parameter to take into account for expanding the query (with profile-related concepts: synonyms, broader than...), for selecting the search engines and adapting the queries to them and for choosing and ranking the results of the search. Soft computing techniques can help in learning and representation tasks.

Meta Question-answering Systems?, perhaps the next goal to achieve would be Meta Web Question-answering Systems, which analyze the user question and generate a set of precise queries (expanded queries) to the more suitable major Search engines and Directories, to get the correct answer to the query. Soft computing and mainly fuzzy logic, as tools closer to human expression nature, can play an essential role for detecting the human user correct meaning and intention.

References

1. Angulo, E., Romero, F.P., García, R., Serrano-Guerrero, J., Olivas, J.A.: An Adaptive Approach to Enhanced Traffic Signal Optimization by Using Soft-computing Techniques. Expert Systems with Applications 38(3), 2235–2247 (2011)
2. Bremermann, H.: Pattern Recognition. In: Hartmut, B., Klaczko, S., Müller, N. (eds.) Systems Theory in the Social Sciences, pp. 116–159. Birkhäuser Verlag, Basel (1976)
3. Fernandez, S.: A Contribution to the Automatic Processing of the Synonymy Using Prolog. PhD Thesis, University of Santiago de Compostela, Spain (2001)
4. Garcés, P.J., Olivas, J.A., Romero, F.P.: Concept-matching IR Systems Versus Word-matching IR Systems: Considering Fuzzy Interrelations for Indexing Web Pages. Journal of the American Society for Information Science and Technology (JASIST) 57(4), 564–576 (2006)
5. Genero, M., Olivas, J.A., Piattini, M., Romero, F.: Using Metrics to Predict OO Information Systems Maintainability. In: Dittrich, K.R., Geppert, A., Norrie, M. (eds.) CAiSE 2001. LNCS, vol. 2068, p. 388. Springer, Heidelberg (2001)

6. Olivas, J.A.: A Contribution to the Experimental Study of Prediction Based on Fuzzy Deformable Categories. PhD Thesis, University of Castilla-La Mancha (2000)
7. Osheron Daniel, N., Smith, E.E.: On the Adequacy of Prototype Theory of Concepts. Cognition 9, 35–58 (1981)
8. Peralta, A., Romero, F.P., Olivas, J.A., Polo, M.: Knowledge Extraction of the Behaviour of Software Developers by the Analysis of Time Recording Logs. In: Proceedings of the 2010 IEEE International Conference on Fuzzy Systems, FUZZ-IEEE 2010 (2010)
9. Puente, C., Sobrino, A., Olivas, J.A.: Extraction, Analysis and Representation of Imperfect Conditional and Causal Sentences by Means of a Semi-Automatic Process. In: Proceedings of the 2010 IEEE International Conference on Fuzzy Systems, FUZZ-IEEE 2010 (2010)
10. Romero, F.P., Caballero, I., Serrano-Guerrero, J., Olivas, J.A.: An Approach to Web-based Personal Health Records Filtering Using Fuzzy Prototypes and Data Quality Criteria. Information Processing & Management (in press) (available online: February 26, 2011)
11. Salton, G., Fox, E.A., Wu, H.: Extended Boolean Information Retrieval. Communications of the ACM 26(11), 1022–1036 (1983)
12. Sobrino, A., Olivas, J.A., Puente, C.: Causality and Imperfect Causality from exts: A Frame for Causality in Social Sciences. In: Proceedings of the 2010 IEEE International Conference on Fuzzy Systems, FUZZ-IEEE 2010 (2010)
13. Trillas, E.: Lógica borrosa y narrativa: un párrafo de Vila-Matas. In: Actas del XII Congreso Español sobre Tecnologías y Lógica Fuzzy ESTYLF 2004, Universidad de Jaen, Spain, pp. 15–20 (2004)
14. Widyantoro, D.H., Yen, J.: Incorporating Fuzzy Ontology of Term Relations in a Search Engine. In: Proceedings of the 1st BISC International Workshop on Fuzzy Logic and the Internet, UC Berkeley, USA, pp. 155–160 (2001)
15. Zadeh, L.A.: A Note on Prototype Set theory and Fuzzy Sets. Cognition 12, 291–297 (1982)

73

My Journey to Fuzziness in Berkeley

Sergei Ovchinnikov

73.1 Moscow

Fuzzy set theory has been well-known and accepted in the Soviet Union since Lotfi Zadeh presented a talk at the Cybernetics Congress in the Soviet Union in 1965. In fact, it was the first presentation of his new brainchild. It was not until 1976 that I became really interested in the theory after reading Zadeh's monograph *The Concept of a Linguistic Variable and its Application to Approximate Reasoning* published in Russian [1]. The monograph was a translation of three Lotfi's preprints that were published about the same time as papers in *Information Sciences*. It immediately became apparent to me that the methodology of fuzzy set theory could be useful in the work that I was engaged in with my PhD student V.B. Kuz'min. I was right and by 1978 we wrote three papers in which fuzzy set theory was successfully applied to various decision problems. Two of these papers [2,3] were submitted to the journal *Fuzzy Sets and Systems* that was lounched in the same year. My own studies in the area of fuzzy sets and systems were greatly influenced by the pioneering Zadeh's paper *Similarity relations and fuzzy orderings* [4]. In September of 1979, I submitted a paper on fuzzy binary relations [5] to *Fuzzy Sets and Systems*. During 1979, I was in the process of waiting for permission to leave the Soviet Union and could not submit the paper through the official channels. The paper was smuggled to Hans-Juergen Zimmermann who was the Editor-in-Chief of the journal at the time.

For the last three years of my life in the former Soviet Union, my research work was mainly in the area of fuzzy sets. I recall many seminars and colloquia were talks on the subject were presented and numerous discussions about applications of fuzzy set theory that I witnessed. The theory remains popular in Russia today. This subject is part of curricula in many universities and other institutions of higher education. Lotfi's 1965 talk at the Cybernetics Congress had a great impact on the development of fuzzy set theory and its applications in Russia.

73.2 Vienna

On sunny winter morning of January 25, my small family (my wife, our daughter, and I) arrived in Vienna on an Airflot flight from Moscow. We lived in Vienna in a small communal apartment until our departure to the United States on May 14, 1980. My main activity during this tough time was writing job application letters to various academic and research institutions in the US. Naturally, my first letter was

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sent to Lotfi Zadeh. Very soon I received an invitation from Hans Zimmermann to visit RWTH Aachen (Aachen Institute of Technology, Operations Research). It took some time to get appropriate refugee documents so I could travel to Germany. Upon my arrival in Aachen I was greeted by a charming gentleman smoking a pipe. Hans introduced me to his team of young engineers some of them working in the area of fuzzy sets. I spent a productive week in Aachen enjoying numerous conversations with Hans. Just before my trip to Aachen I received a letter from Lotfi in which he told me that he cannot help me while I am in Vienna but could try to do something if I would be in the Bay Area. When I told Zimmermann about this letter, he strongly advised me to go to San Francisco. Lotfi is a very helpful man, he told me. I remain very thankful to Hans for his support and advice that I needed so much during that difficult transition into a new life.

73.3 Berkeley

Tolstoy Foundation—the organization that sponsored our immigration to the US—settled us in a motel on Polk street in San Francisco. One day later, I received a telephone call from Lotfi who was informed by the Tolstoy Foundation that I arrived in San Francisco. After that call everything started developing very fast. That day Lotfi took us to his house in Berkeley where we were greeted by his wife Fay. We were then taken to a Chinese restaurant (of course). Very soon Lotfi told me that he managed to secure support for me from some grants to work at UC Berkeley for one year. We moved to the Shuttack Hotel in Berkeley and then to a nice furnished apartment near the UC campus that Fay Zadeh found for us. It is impossible to underestimate how much help and support my family received from Lotfi and Fay during our first years in Berkeley. We will be always indebted to the Zadehs for all the good things that they have done for us.

On the first day of my employment, I recall it was June 1, 1980, I came to Lotfi's office and asked him to describe my duties. In turn, he inquired if I have any problems to work on. I replied that I do have a list of problems. Then Lotfi plainly told me that working on these problems is my duty. I knew already from my conversations with Zimmermann that there is no "fuzzy lab" in Berkeley and that Lotfi is working alone. However, there were students working under Lotfi's supervision and, most importantly, visitors doing research work in fuzzy set theory. My first collaborator was Teresa Riera[1], a graduate student from Barcelona, with whom we wrote a paper on fuzzy classifications that she presented at the *International Congress on Applied Systems Research & Cybernetics*, Acapulco, December 1980.

I was very lucky to meet Enric Trillas in Berkeley in the fall of 1980. Our long conversations had a great influence on my work in the area of fuzzy sets. Several papers which I wrote later on logical connectives were influenced by Enric's own research. Our collaboration continued for many years thereafter. I visited Spain

[1] Presently, Teresa Riera Madurell is a member of the European Parliament.

on numerous occasions during the 80s and early 90s and established good working relations with Spanish "fuzzy community". My last paper published in *Fuzzy Sets and Systems* was written with Llorenç Valverde[2], a former student of Enric Trillas.

Fig. 73.1. Enric Trillas, Lotfi Zadeh, Sergei Ovchinnikov, and Elie Sanchez in the living room of Lotfi's house

My active interactions with the fuzzy community started in 1981 when I attended two conferences in the United States. In May, I presented a talk at the *11th International Symposium on Multiple-Valued Logic*, Oklahoma City, Oklahoma, where for the first time I met Ronald Yager. There was also a group of young Spanish scientists headed by Enric Trillas. That fall, Elie Sanchez stayed in Berkeley with his family. In December of 1981, I drove our two families to San Diego where Elie and I attended the 20th IEEE Conference on Decisions and Control. As far as I remember the photo in Fig. 73.1 was taken in the fall of 1981.

For more than three decades I have participated in various capacities in many activities of the international "fuzzy community". Initiating meetings that I attended include NAFIPS'84 meeting in Hawaii, the first IFSA Congress in Mallorca (1985), and IPMU'86 in Paris. In 1988 I organized NAFIPS Conference in San Francisco and was awarded the K.S. Fu Certificate of Appreciation. I am a proud Fellow of the International Fuzzy Systems Association (IFSA).

I remember that during my first year in Berkeley it came as a big surprise to me the attitude of most professors in UC Berkeley and other universities towards fuzzy

[2] Presently Dr. Llorenç Valverde i Garcia is Vice President of the *Universitat Oberta de Catalunya*.

theory that was ranging from ignorance to hatred. During the next several years I observed a strong contrast of this attitude in the United States with worldwide acceptance of the theory. I recall that in 1982 I had to “smuggle” my presentation at the seminar ran by Kenneth Arrow (Nobel Prize in economics, 1972) by removing every appearance of the word “fuzzy” in the summary of my talk. Of course, the term surfaced during my presentation making some people clearly unhappy. The situation is not very much different today despite exponentially growing body of evidence supporting soft computing, computing with words, and fuzzy set theory as a backbone of these applied areas. In my humble opinion, fuzzy set theory and methodologies based on it are here to stay for a long time if not forever. Those who disbelieve in these theories remind me of the people in caves from Plato’s Allegory of the Cave. These people are unable to free themselves from the cave and to step out to see the reality.

References

1. Zadeh, L.A.: The Concept of a Linguistic Variable and its Application to Approximate Reasoning. Mir, Moscow (1976) (in Russian)
2. Kuz’min, V.B., Ovchinnikov, S.V.: Group decisions I. In Arbitrary Spaces of Fuzzy Binary Relations. Fuzzy Sets and Systems 4, 53–62 (1980)
3. Kuz’min, V.B., Ovchinnikov, S.V.: Design of Group Decisions II. In Spaces of Partial Order Fuzzy Relations. Fuzzy Sets and Systems 4, 153–165 (1980)
4. Zadeh, L.A.: Similarity Relations and Fuzzy Orderings. Information Sciences 3, 177–200 (1971)
5. Ovchinnikov, S.V.: Structure of Fuzzy Binary Relations. Fuzzy Sets and Systems 6, 169–195 (1981)

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Encounters with Fuzziness and Ambiguity in Patterns – A Memorable Journey

Sankar K. Pal

74.1 Background

I joined the Indian Statistical Institute (ISI), Calcutta on March 01, 1975 to work on a PhD in the area of pattern recognition and man-machine communication by voice as a CSIR Senior Research Fellow of the Government of India. I had no idea about pattern recognition and neither were there any text books on this subject; only a few edited volumes, mostly by Prof. K.S. Fu, were available in our library or in the market. From these, I started to pick up the basics of sequential pattern recognition using statistical approaches. One day my thesis advisor, Prof. D. Dutta Majumder, gave me a typewritten note, of about four pages titled something like "Pattern classification with property sets", written by Prof. Ramesh Jain who had presented the concept in a seminar in Calcutta. From it, I got some idea of the fuzzy properties of a pattern and the concept of its multi-class belonging based on them - which appealed to me very much as it seemed to be very natural for decision-making in real-life problems. There, I also got the reference of Prof. Zadeh's famous paper, "Outline of a New Approach to the Analysis of Complex Systems and Decision Processes", which had appeared in the *IEEE Transactions on SMC* (vol. 3, pp. 28–44), in 1973.

At that time my own institute (ISI) library was not subscribing to almost any IEEE transaction, as Electrical Engineering or Computer science was not considered to be a core subject of research there. However, I collected a copy from the Institute of Radiophysics and Electronics, Calcutta University where I had done my B.Tech and M.Tech, and then subsequently got the other two seminal papers of Zadeh e.g., Information Control (1965) and Information Sciences (1967). Apart from these three papers, I also bought an edited volume from my research grant, which I consulted often. It was titled "Fuzzy Sets and their Applications to Cognitive and Decision Processes" by Zadeh, Fu, Tanaka and Shimura, published by Academic Press in 1975. The Institute library subsequently acquired another book on fuzzy sets, by A. Kaufmann, titled "Introduction to Theory of Fuzzy Subsets: Fundamental and Theoretical Elements", Academic Press, 1975, though only in the later part of my doctoral work. These were the only information sources on fuzzy sets to begin with, in the early part of my research career.

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74.2 Visualizing Fuzziness

"Fuzzy sets are NOT fuzzy!"

In the following, I shall explain how I encountered fuzziness/ ambiguities in various problems of pattern recognition and image analysis, got motivated to handle them with fuzzy set theory, and subsequently continued to develop from time to time different hybrid technologies in the soft computing framework, as and when required to meet the then need. While doing so, I shall also describe the road map that I followed, acknowledging some persons concerned, and the different situations I faced on the way.

A pattern recognition system (PRS) has basically three blocks, namely, measurement space, feature space and decision space. Uncertainties arise from deficiencies of information available in a situation. Deficiencies may result from incomplete, imprecise, ill-defined, not fully reliable, vague, contradictory information in various stages of a PRS. For example, vagueness can occur in the measurement space due to experimental error, limitation in instrument/ measurement to go for finer details, and availability of input in linguistic form. Sometimes, it may be convenient and appropriate to express the input feature value in interval form, or having one side of the interval unknown or even both sides fuzzy. In case of handwritten characters, for instance, vagueness comes from badness in writing, not from randomness. Accordingly, the resulting classes in the decision space may become non-convex, elongated, and overlapping, thereby making them intractable. These necessitate the design of classifiers with the capability of generating linear to highly nonlinear boundaries, or modeling overlapping class boundaries efficiently. In case of overlapping boundaries, it is natural and appropriate to make a multi-valued or fuzzy decision on an unknown pattern, i.e., a pattern has the possibility of belonging to more than a class with a graded membership. Depending on the distribution of membership values over different classes, the output decision on a pattern with respect to a class may therefore be *soft* and linguistically quantified as - "definitely belongs", "definitely does not belong", "combined choice", "2nd choice" etc. For a doubtful pattern it is always better to say "doubtful", rather than misclassifying it. In that case the aforesaid multi-valued decisions have at least an opportunity to get the pattern correctly classified with some higher level information (e.g., syntactic, semantic), if available. This, in turn, dictates that a good classifier should be able to restrict the misclassified samples within less number of classes. Dispersion Index (*Pattern Recognition*, 45, pp. 2690–2707, 2012) quantifies this characteristic.

I was interested in working on speech and speaker recognition problems. Speech, being patterns of biological origin, their characteristics depend greatly on speakers' health, sex, age, temperament, spirit and mind; thereby resulting in considerable amount of fuzziness in them and overlapping among the classes. For example, the same word uttered by a speaker at different times in a day may have different characteristic features. Accordingly, I started developing methodologies for nonparametric classification and recognition, and published my first IEEE paper - "Fuzzy Sets and Decision-making Approaches in Vowel and Speaker Recognition", *IEEE Transactions on SMC*, vol. 7, pp. 625–629, 1977. Subsequently, I published on plosive recognition and self-supervised adaptive recognition systems, and submitted my PhD thesis to Calcutta University in 1978 titled "Some Studies on Pattern Recognition and Man-machine Communication by Voice with Fuzzy Set Theoretic Approach". The foreign examiner of my thesis was Prof. K. S. Fu, the father of pattern recognition, Purdue University, USA. While appreciating the work as a pioneering contribution, he envisaged a future problem to study the sensitivity of different fuzzifiers and hedges, which were used in the distance function and similarity measure, on recognition performance. Prof. Zadeh mentioned in the Foreword of my 1992 IEEE Press book, co-edited with Jim Bezdek, that "S. K. Pal first applied fuzzy sets to the speech recognition problems in 1977".

Meanwhile, I started realizing that the processing of gray images could be another good candidate area for fuzzy set theory application. Since it is gray, the basic concepts of image regions, segments, edges, skeletons and relations among them etc, do not lend themselves to precise definition. For example, a question like – "Where is the boundary?" – has no precise answer. Whatever hard decision that one may make for extracting those features/ primitives would always lead to an uncertainty. In other words, it is appropriate and also natural to consider the various tasks of processing of a gray image to be fuzzy, NOT hard, to manage the associated uncertainty in processing as well as in recognizing the content. Again, in an image recognition or vision system, once an uncertainty is caused in edge detection, segmentation, skeleton extraction etc. on account of the application of hard decisions (0 or 1) at the processing stage, it is likely to propagate further to the primitive extraction stage and may finally affect the decision-making process where one needs to identify the image contents. This further justifies the significance of fuzzy processing whereby the uncertainty can be minimized at the final stage of a vision system by retaining the gray information in the preceding stages as much as possible, and the ultimate output will not then be biased/ affected much by lower level decisions. One may note that gray information is very informative and expensive too; once they are made crisp by a threshold, the information is lost and can no way be retrieved. At the point of final decision-making at the highest level, one can always make them binary.

I was then looking for an opportunity to work in image processing. At that time, labs with complete software and hardware facilities for working in (gray) image processing were not readily available in many universities/ institutes, not even in the developed nations. Luckily, I got a Commonwealth Scholarship to study at Imperial College, London in 1979. (Though there was a possibility to work at Purdue University, USA, as a Post-doc Fellow with Prof. K. S. Fu, I chose to go to Imperial College.) The digitized image data (in paper tape) was collected from Philips Research Laboratory, Redhill, Surrey. I then started developing various algorithms for enhancement including image definition, edge detection, primitive extraction and image entropy measures using fuzzy sets, and publishing them in IEEE Transactions and Electronics Letters, and obtained another PhD in early 1982 in the area of fuzzy image processing. In a gray image there are two types of ambiguities, namely, grayness ambiguity and spatial ambiguity. The former is concerned with whether a pixel can be considered to be black or white, and depends only on the gray value, whereas, the latter is concerned with both the gray level and location of pixels characterizing the geometry of image subsets. In the process of formulating the algorithms, Zadeh's contrast enhancement operator (INT), S & π membership functions, max & min operators, index of fuzziness, and entropy of fuzzy sets were used. With contrast enhancement of a fuzzy image around a fixed cross-over point, the difficulty in deciding whether a pixel is black or white reduces, and accordingly the values of its index of fuzziness and entropy decrease (*IEEE Transactions on PAMI*, vol. 4, pp. 204–208,1982). Similarly, given a set of fuzzified versions of an image, the one with minimum index of fuzziness or entropy gives the best segmented output for object extraction.

As an application, we choose the problem of identifying different stages of skeletal maturity (growth) with age from x-ray images of radius and ulna of wrist. The problem is significant from the point of determining the various stages of malnutrition of babies. We collected the image data from Prof. L.F. Turner, Institute of Sick Children, London. Here the shapes of radius and ulna at several stages of growth have overlapping character, i.e., they look alike. Accordingly, these were handled with fuzzy syntactic recognition approach, where both the primitives (e.g., vertical, horizontal & oblique lines, and curves) and the relations among them were considered to be fuzzy in developing the unambiguous grammars using production rules. Since the same set of production rules with different membership functions characterizes more than a class, the number of rules required is less, as compared to those using deterministic rules (*IEEE Transactions on SMC*, vol. 16, pp. 657–667, 1986.)

It may be mentioned here that my thesis advisor Dr. Robert A. King, Department of Electrical Engg., was basically an expert in the area of signal processing, and had not worked earlier on fuzzy set theory or image processing, till I joined him. However, he was convinced about my ideas and allowed me to work independently to develop the subject. One may further note that there was another pioneering group on fuzzy image processing led by Prof. Azriel Rosenfeld, Univ. of Maryland, College Park, father of image processing, working since late seventies, particularly in fuzzy geometry, connectedness and topology on image subspace, among others.

After returning to India in May 1983, I started developing, with my students, multi-valued recognition systems with linguistic input, fuzzy syntactic recognition methods, and various entropy measures and image segmentation algorithms, among others. We have defined correlation between fuzzy sets, and fuzzy operators using ordinary sets. Problems like estimating the entire class from a set of few sampled patterns, selected randomly, were dealt with fuzzy sets. In the area of image analysis, we have given various definitions of image entropy based on exponential gain function, and other quantitative indices for image processing tasks. The exponential gain function relies on the fact that a better measure of ignorance is $(1 - p_i)$ rather than $1/p_i$ (as used by Shannon), where p_i is the probability in receiving the ith event (*IEEE Transactions on SMC*, vol. 21, pp.1260–1270, 1991). Accordingly, we have defined higher order fuzzy entropy, image entropy and hybrid entropy. As the order of image entropy increases, the validity of the segmented outputs, with respect to minimizing uncertainty, becomes more meaningful and valid. Hybrid entropy takes care of both probabilistic and fuzzy entropy and has significance in digital communication, particularly in noisy environment, where the concern is whether a bit is transmitted or not in a noisy channel and if its exceeds a threshold or not.

During 1986-1987, I visited the University of California, Berkeley and the University of Maryland, College Park as a Fulbright Fellow. That was the first time I met Prof. Lotfi A. Zadeh and Prof. Azriel Rosenfeld in person. Among the several characteristics of Lotfi, two features that appeared to be unusual and thus impressed me are - when I wanted to write a paper with him, Lotfi told me that he loves to work alone (showing his list of publications), and advised me not to put his name; and he never discussed fuzzy sets when we were together, whether in a car, or a restaurant or at his house.

Fig. 74.1. Lotfi Zadeh and his wife (Fay) at his residence in December 1986

My reminiscences would remain incomplete, if I do not mention the criticism that I received often, like many other fuzzy researchers, from my colleagues when delivering lectures or seminars within my Institute and outside. The situation can be felt easily considering that I have been in an organization named, Indian Statistical Institute, surrounded by probabilists and statisticians. However, we have always viewed it as follows:

- Fuzzy set theoretic approach supplements the probabilistic approach and it is not a competitor, rather provides enrichment.

- We find a better solution to a crisp problem by looking at a larger space at first, which has different (usually less) constraints and therefore allows the algorithm more freedom to avoid errors by commission to hard answers in intermediate stages – *notion of embedding*.

74.3 Neuro-Fuzzy and Rough-Fuzzy Computing

In the late nineteen-eighties, I got interested in neural networks, and started developing various neuro-fuzzy models mainly for classification, clustering, rule generation and connectionist knowledge base systems. The idea of synergistic integration was to enable ANNs to accept linguistic input (*low*, *medium*, *high*, *missing features*) in addition to numerical input; exploit the ANN characteristics like adaptivity, robustness, ruggedness, speed via massive parallelism, optimality and capability in generating highly nonlinear boundary; and uncertainty handling capability of fuzzy sets in the input, output and during training. This greatly enhances the application domain of ANNs. Our article - "Multi-layer perceptron, fuzzy sets and classification" (*IEEE Transactions on TNN*, vol. 3, pp. 683–697, 1992) received the *Outstanding paper award from IEEE Neural Networks Council.* We have developed a series of generic models and demonstrated their applications to noisy/overlapping fingerprint identification, speech recognition, atmospheric science, image processing etc. Through integration, it has also been possible to make a layered network, which is usually used as supervised classifier, act as an unsupervised classifier using the index of fuzziness and fuzzy entropy as error detectors.

To enhance the computational intelligence characteristics of the said fuzzy networks, particularly for mining large data sets, we then started integrating the merits of rough sets and genetic algorithms into them. I had become interested in rough sets (RS) and genetic algorithms (GA) when I was visiting the NASA Johnson Space Center, Houston, TX during 1990-92 and 1994 as an NRC Senior Research Associate. I had attended several seminars on rough sets organized in the Software Technology Branch, Information Technology Division. Two features of rough sets, namely, granular computing with information rules and uncertainty analysis with lower and upper approximations drew my attention. Since RS has the capability in extracting the domain knowledge, whether supervised or unsupervised, with reduced dimension in the form of information granules/rules, these can be encoded as initial network parameters for reducing its learning time significantly. Similarly, GA based learning (with chromosomes based on the network parameters, and modified genetic parameters) can replace the traditional gradient descent search technique which is slow and often gets stuck at local minima. The aforesaid synergistic integration of the four tools in the soft computing paradigm results in gain in terms of performance, computation time and compactness of the network, among others (*IEEE Transactions on KDE*, vol. 15, pp.14–25, 2003). So, it has wide application in mining data sets with large dimension and size, and in knowledge discovery.

Fig. 74.2. IIZUKA-96, Fukuoka, Japan: (L-R) Sushmita Mitra, Lotfi Zadeh, Elie Sanchez and Sankar Pal

Meanwhile I also realized that since both fuzzy sets and rough sets provide algorithms for different kinds of uncertainty, why not integrate them to have a much stronger paradigm for uncertainty handling than either of them. In 1997 I visited Prof. Andrzej Skowron, Warsaw University, Poland under an INDO-POLISH collaborative project; there I met Prof. Z. Pawlak, father of rough sets. Andrzej and I edited a volume – *Rough-Fuzzy Hybridization: A New Trend in Decision Making*, Springer, Singapore, 1999, which is the first of its kind.

One may note that Pawlak's rough set theory is based on the concept of crisp set and crisp granules, and provides a framework of handling uncertainty arising from granularity in the domain of discourse or limited discernibility of objects,. However, in real life problems, one or both of them may be fuzzy. A gray image is such an example where the set (e.g., object region) can be fuzzy and the granules (e.g., pixel windows) may be overlapping. In order to model this, we have recently defined generalized rough sets (*IEEE Transactions on SMC*, vol. B-39, pp. 117-128, 2009), where the set and granules could be crisp as well as fuzzy. Accordingly one could use "granular *fuzzy computing*" or "*fuzzy granular* computing" depending on the application.

For example, in an image nearby gray levels have limited discernibility, i.e., nearby gray levels *roughly resemble* each other and the values at nearby pixels have *rough resemblance*. Therefore, in the rough-fuzzy computing framework image ambiguity may be viewed as resulting from fuzzy boundaries of regions + rough resemblance between nearby gray levels + rough resemblance between nearby pixels. Accordingly, we defined generalized rough-fuzzy entropy based on lower and upper approximations. Its merits over fuzzy entropy and the significance of fuzzy granules have been demonstrated, for example, on image segmentation problems.

Fig. 74.3. BISC International Workshop Computational Intelligence in Bioinformatics and Cybersearch (FLINT-CIBI-03), Berkeley: Sankar Pal with Lotfi and his wife in the Banquet in Dec 2003

Merits of rough sets and fuzzy sets have also been integrated judiciously in clustering problems where rough sets deal with vagueness and incompleteness in class definition, and fuzzy sets enable handling of overlapping partitions. Each cluster here is represented by a cluster prototype, a crisp core (lower approximation) and a fuzzy boundary. Membership values are unity for the objects in the crisp core region, and are in $[0,1]$ for those in the fuzzy boundary region. In other words, rough-fuzzy clustering provides a balanced mixture between *restrictive partition* of hard clustering and *descriptive partition* of fuzzy clustering. Therefore, it is faster than fuzzy clustering and is capable of better uncertainty handling/ performance (*IEEE Transactions on SMC*-B, vol. 37, pp. 1529-1540, 2007). Thus, wherever fuzzy c-means or c-medoids have been found to be useful in the past four decades, rough-fuzzy clustering would be superior.

74.4 CWW and Z-Numbers: Future Work

Let us now consider one of our current research problems of computing with words (CWW) and the significance of the Z numbers, as recently explained by Zadeh (*Information Sciences*, vol. 181, pp. 2923–2932, 2011). The CWW paradigm is inspired by the astounding ability of the human brain to perform tasks on the basis of concepts encoded in the words and phrases that frame natural language statements. It aspires to induce this amazing decision-making ability in the computer - a step towards evoking M-IQ in a computer. CWW is imperative when a) the information to be conveyed lacks numeric precision; b) the situational imprecision can be exploited to arrive at robust, low-cost solutions; c) numeric computing principles cannot be applied; and d) words express a lot more than numbers. Potential areas of application of CWW are in semantic-web searching, linguistic summarization of text samples or complex phenomenon, and subjective decision-making.

We envision the paradigm as a model of the natural, intuitionistic process of comprehension; and aspire to apply its principles to the development of systems capable of formulating subjective judgments on the basis of natural language descriptions of related events. The systems should ideally be working in real-time and incorporate the 'emotional' element and recognize 'behavioral aspects' of natural language. As such the prime areas that we have begun addressing are:

- Modeling word-perceptions, where words are inherently 'uncertain' –
  a) The same adjective may convey different (even if subtle) meanings across individuals [inter-uncertainty];
  b) The same adjective may bring in different interpretations across time – at any time instant [intra-uncertainty], as well as with experience;
  c) A single word may have multiple meanings – that depend on the context;
  d) A word may have multiple syntactic forms – tense-forms, pluralization, parts of speech;
  e) Words are often not used literally.
- Word-sense disambiguation from synonyms or from words along the spatial locality of reference - evaluation of the perception of the sentence in its entirety.
- Adapting the machine vocabulary to include new words and concepts pertaining to a context.
- Identification of the context-relevance of natural language statements and appropriate handling of absolutely irrelevant statements.
- Formulation of the rules of computation on the basis of the word-perception model and the antecedent-consequent relations within the context.
- Granulation of the sentences into sub-contexts for faster processing and simulation of the human process of cognition.

The Interval-Type 2 fuzzy set has long been proved as being capable of modeling the intra-uncertainty and the inter-uncertainty of word-perceptions. We believe that the concepts of fuzzy-grammar, fuzzy-entropy and fuzzy-granules, are essential for the purpose of text annotation, context-relevance measures and granulating sentences into sub-contexts.

An investigative insight into the concept of the Z-number, proposed by Zadeh in 2011, depicts the Z-number as to being able to provide the perfect environment of amalgamation of all the concepts mentioned. The methodology moreover, incorporates a parameter that indicates the ’uncertainty’ of the information conveyed by the sentences - which could be exploited to merge CWW and behavioral computing. Using the Z-number as a framework for CWW is our current line of thought.

Prof. Lotfi Zadeh visited the Indian Statistical Institute, Calcutta in February 2006 when I was the Director, and he was conferred with *Honorary Doctorate* degree to acknowledge his everlasting contribution to science. He also interacted with our students and faculty members of the Center for Soft Computing Research regarding their activities.

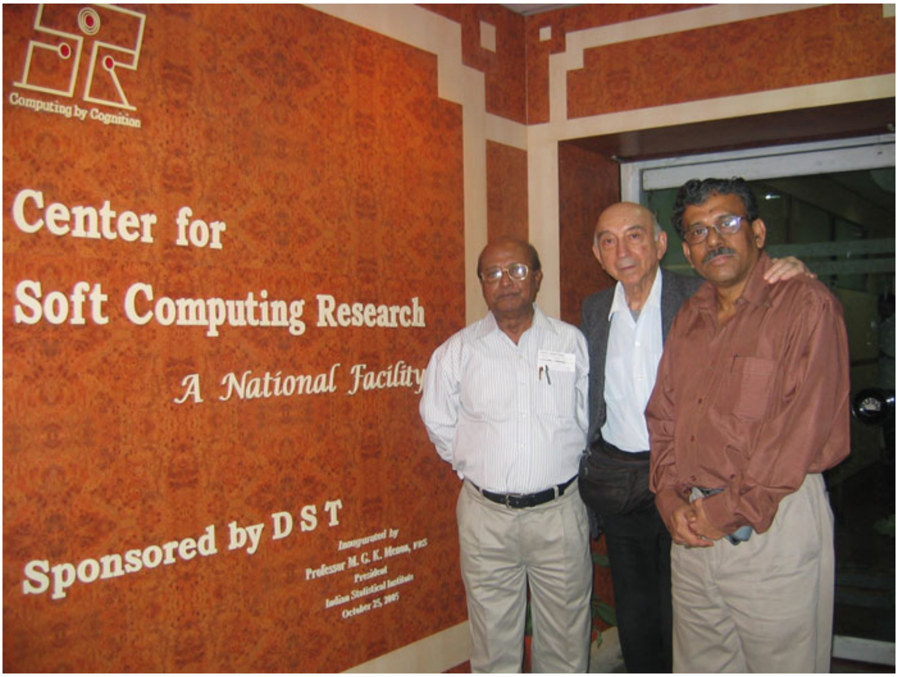

Fig. 74.4. Visiting CSCR, Indian Statistical Institute, Calcutta in February 2006: (L to R) Dwijesh Dutta Majumder, Lotfi Zadeh and Sankar Pal

References

1. Pal, S.K., Dutta Majumder, D.K.: Fuzzy Mathematical Approach to Pattern Recognition. John Wiley & Sons (Halsted), New York (1986)
2. Pal, S.K., Mitra, S.: Neuro-Fuzzy Pattern Recognition: Methods in Soft Computing. John Wiley, New York (1999)
3. Pal, S.K., Shiu, S.C.K.: Foundations of Soft Case-Based Reasoning. John Wiley, New York (2004)

4. Pal, S.K., Mitra, P.: Pattern Recognition Algorithms for Data Mining. CRC Press, Boca Raton (2004)
5. Bandyopadhyay, S., Pal, S.K.: Classification and Learning Using Genetic Algorithms: Applications in Bioinformatics and Web Intelligence. Springer, Heidelberg (2007)
6. Maji, P., Sankar, K.: Rough-Fuzzy Pattern Recognition: Application in Bioinformatics and Medical Imaging. Wiley, New York (2012)

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My Way to Fuzzy Control

Rainer Palm

In the 80's of the last century I was member of the robotic Lab of the academy of sciences in East-Berlin. Our task was to develop robot skills for industrial manipulators mainly equipped with tactile sensors, intelligent grippers and force/torque sensors. The main application of our robot skills were technological tasks like welding, grinding, glueing tasks etc. performed on the basis of contour following algorithms.

The mixture of pure control theoretic aspects and technological expert knowledge makes the control of such tasks quite cumbersome. So we were looking for means to overcome these difficulties.

At that time within the academy several symposia were organized with pure mathematical character and also with topics with engineering aspects. One of those symposia was held in 1985 in Leipzig where Steffen Bocklisch from the University of Karl-Marx-Stadt (now Chemnitz) gave a talk on a strange but interesting area named *Fuzzy Logic*. This scope caught my attention from the beginning, and I went back to Berlin with the will to learn more about it, the theoretical bases and practical consequences for my work in robotics.

In the mid 80's textbooks on fuzzy logic/control were quite rare. So I mostly read editorials – for example the book "Foundations of Fuzzy Reasoning" published by Elsevier, 1977, edited by B. R.Gaines. This book gave me some insight into the ideas of fuzzy reasoning, the construction of fuzzy relations and fuzzy rules and some hints to build fuzzy controllers. The advantage of an editorial is that – in the absence of good textbooks – it shines a light on a scientific field from different prospectives.

In the next period of time Steffen Bocklisch and also Siegfried Gottwald from the University of Leipzig organized meetings on fuzzy logic to one of which I was invited to give a talk on my special field: Application of fuzzy control to the field of robotics. At this meeting a polish scientist was also invited to give a talk. It was Witold Pedrycz – an internationally known expert in fuzzy logic. To my pleasure he liked my talk and encouraged me to submit a paper to "Fuzzy Sets and Systems" which was accepted and published under the title "Fuzzy controller for a sensor guided robot manipulator" Volume 31 Issue 2, 26 June 1989.

Shortly after a precipitating event changed our life especially in the East drastically: The fall of the wall. This made a close collaboration between the academy and the Fraunhofer Society in West-Berlin possible which worked on similar problems in the field of robotics.

Initiated by the upcoming "fuzzy boom" in the late 80's especially in Japan and the US, Siemens R & D started to build up a so-called 'fuzzy task force' in

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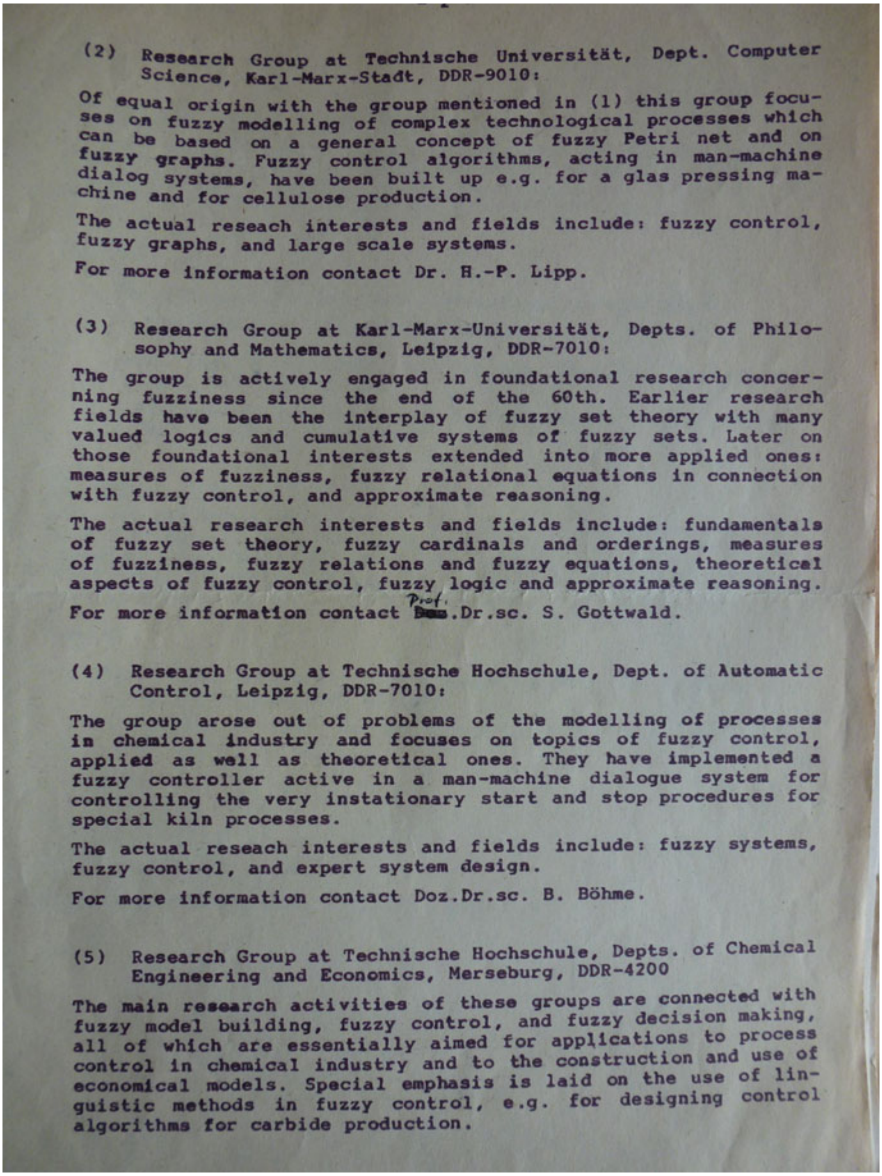

(2) Research Group at Technische Universität, Dept. Computer Science, Karl-Marx-Stadt, DDR-9010:

Of equal origin with the group mentioned in (1) this group focuses on fuzzy modelling of complex technological processes which can be based on a general concept of fuzzy Petri net and on fuzzy graphs. Fuzzy control algorithms, acting in man-machine dialog systems, have been built up e.g. for a glas pressing machine and for cellulose production.

The actual reseach interests and fields include: fuzzy control, fuzzy graphs, and large scale systems.

For more information contact Dr. H.-P. Lipp.

(3) Research Group at Karl-Marx-Universität, Depts. of Philosophy and Mathematics, Leipzig, DDR-7010:

The group is actively engaged in foundational research concerning fuzziness since the end of the 60th. Earlier research fields have been the interplay of fuzzy set theory with many valued logics and cumulative systems of fuzzy sets. Later on those foundational interests extended into more applied ones: measures of fuzziness, fuzzy relational equations in connection with fuzzy control, and approximate reasoning.

The actual research interests and fields include: fundamentals of fuzzy set theory, fuzzy cardinals and orderings, measures of fuzziness, fuzzy relations and fuzzy equations, theoretical aspects of fuzzy control, fuzzy logic and approximate reasoning.

For more information contact Prof..Dr.sc. S. Gottwald.

(4) Research Group at Technische Hochschule, Dept. of Automatic Control, Leipzig, DDR-7010:

The group arose out of problems of the modelling of processes in chemical industry and focuses on topics of fuzzy control, applied as well as theoretical ones. They have implemented a fuzzy controller active in a man-machine dialogue system for controlling the very instationary start and stop procedures for special kiln processes.

The actual reseach interests and fields include: fuzzy systems, fuzzy control, and expert system design.

For more information contact Doz.Dr.sc. B. Böhme.

(5) Research Group at Technische Hochschule, Depts. of Chemical Engineering and Economics, Merseburg, DDR-4200

The main research activities of these groups are connected with fuzzy model building, fuzzy control, and fuzzy decision making, all of which are essentially aimed for applications to process control in chemical industry and to the construction and use of economical models. Special emphasis is laid on the use of linguistic methods in fuzzy control, e.g. for designing control algorithms for carbide production.

Fig. 75.1. Examples of Fuzzy groups and their programs in East Germany

Fig. 75.2. The Fuzzy group at Siemens R & D, 1995, from left to right: K. Heesche, Uli Rehfuss, Th. Runkler, R, Berstecher, J. Hollatz, R. Palm, P. T. Pilgram

Munich, Germany, in order not to fall behind their competitors in this field of research and application. At the end of the year 1990 I got a call from Michel Reinfrank from Munich in which he asked me whether I wouldn't like to join his group. After some serious consideration I agreed and joined his group in spring 1991. Apart from M. Reinfrank this group started with a just a few members: Erwin Gerstorfer (Austria), Paul-Theo Pilgram (Germany), Hans Hellendoorn (The Netherlands) and myself. After a while this relatively small group attracted people from several places in Europe and the US. Our focus was directed both to theoretical problems of fuzzy logic/control and practical applications like fuzzy washing machine, vacuum cleaner, and exhaust measuring systems. Theoretical aspect were fuzzy sliding mode, defuzzification methods and – last but not least – the development of a fuzzy tool TILShell 3.0 (later SieFuzzy 1.0) a mutual development between Siemens and Togai InfraLogic located in Irvine CA, USA.

The first fuzzy conferences brought us together with the leading scientists in the field: Hans Zimmermann, Siegfried Gottwald, Witold Pedrycz, Rudolf Kruse, Peter Klement, Ron Yager, Jim Bezdek, Jim Keller, Didier Dubois and Henri Prade, and many others and, of course, the founder of fuzzy logic: Lotfi Zadeh.

Lotfi's interest in our work was mainly focussed on Siemens applications of fuzzy logic. On a fuzzy symposium in Munich 1992 organized by Siemens R & D he

Fig. 75.3. Lotfi Zadeh visiting Siemens Fuzzy Group, 1992, from left to right: E. Gerstorfer, R. Palm, L. Zadeh, H. Hellendoorn

visited our Lab to inform himself about our progress in the fields of theory and practical applications .

A very close collaboration took place especially with Dimiter Driankov from Linkoeping University in Sweden who visited us for several years. He initiated the publication of many Journal and conference papers and books e.g. “Introduction to fuzzy control” (1993/96) in collaboration with spanish and australian scientists and later the book “Model based fuzzy control” (1996).

After a few years the fuzzy group merged with our neural net group led by Bernd Schuermann. The fuzzy aspect did not play a prominent role any more but appeared to be a useful and recognized tool for many industrial solutions and projects. In the field of fuzzy control the modeling and identification aspect got more and more into the focus. As a modeling tool fuzzy clustering was represented by Thomas Runkler who leads the Lab since a couple of years.

Since my retirement in 2004 I work as an adjunct professor and guest scientist at the Oerebro University/Sweden at a Robotic Lab led by D. Driankov.

Although my main research interest went back to my old field robotics, fuzzy modeling and control remain a field of research and a scientific attraction.

Fig. 75.4. R. Felix, H. Hellendoorn, D. Driankov and son Stefan (from left to right) at FuzzIEEE in Orlando 1994

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The Role of Fuzzy Sets in Information Retrieval

Gabriella Pasi and Gloria Bordogna

76.1 Introduction

Our journey in the land of Fuzzy Logic started in 1990, at the Italian National Council of Research (CNR), where we were young researchers, who, coming from research experiences in distinct groups, were excited at the idea to start an independent and challenging research activity. We came across the world of Information Retrieval (before Web search engines' birth) not by chance, but because we were both working at a project aimed at defining and implementing an IR system (IRS) for managing the research papers produced by the researchers of CNR.

In the phase of understanding the complexity of the IR task as well as its open problems, we focused on the central role of the concept of *relevance* of documents to users' information needs, and on the potential application of Fuzzy Set Theory to model it. In analyzing this topic we had the opportunity of reading some papers written by Tadeusz Radecki and, subsequently, by Donald Kraft. In his seminal paper of 1979, titled *"Fuzzy set theoretical approach to document retrieval"*, Radecki proposed a first generalization of the Boolean retrieval model (the first formal model of an Information Retrieval System) [11]; this proposal was later followed by several others. The main issue underlying this generalization was that Information Retrieval is a decision making activity which relies on *subjective* and *approximate* needs that an automatic system may only guess, and that it shall somehow model to produce possibly meaningful results. We discovered that Fuzzy Set Theory (FST) and Fuzzy Logic had been proposed by several researchers such as Donald H. Kraft, Abraham Bookstein, Ducan Buell, Rita de Caluwe, Etienne Kerre, as an adequate formal framework for modeling document's relevance as a gradual property of documents, and for shaping IR systems that can rank documents based on their estimated relevance to user queries [4] , [7], [8].

This approach fascinated us at such a point that (jointly with our colleague Paola Carrara) we developed our first contribution, that was published in the Information Processing and Management Journal, in 1991 (*Query term weights as constraints in fuzzy information retrieval*) [2]. But what we remember as the most important opportunity to explore the potentials of FST in the context of modeling systems for information access, as well as the true initiation to the fuzzy community was the *First International Conference on Fuzzy Systems* (Fuzz-IEEE) in San Diego, in 1992. We presented at that time a paper titled *"Extending Boolean Information Retrieval: a Fuzzy Model based on Linguistic Variables"*. This event was fundamental in determining a big part of the research activities we did undertake in subsequent years.

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DOI: 10.1007/978-3-642-35644-5_76

At that conference we met people with whom we started beautiful personal and professional relationships.

We met Donald H. Kraft, who was the chair of our session at the conference, and with whom we had an interesting working lunch in which we talked about the content of our proposals as well as the links to his works. We remember the presentation on OWA operators given by Ronald Yager with whom we had later the chance to talk; as a consequence of the discussions with him, we envisioned the usefulness of OWA operators for defining flexible query languages for IR systems, that we first proposed in 1995 in our paper titled *Linguistic Aggregation Operators of selection Criteria in fuzzy information retrieval* [3]. In San Diego we also met Patrick Bosc, who invited us to submit a paper in his and Janusz Kacprzyk's seminal edited volume on fuzzy databases; we met at that conference Didier Dubois and Henry Prade, Jim Bezdek, Piero Bonissone and several other people who we would have seen and met in numerous subsequent events. Last but not least, we met Lotfi Zadeh, a guru for us, who became an important reference for the evolution of our research experience. What especially surprised us was his kindness, availability and curiosity in talking to us, in giving us advices. Overall we found a greater openness and enthusiasm of this scientific community with respect to other scientific communities. Several years have passed from that conference, and with the birth of the World Wide Web the problem of defining systems able to provide users with an easy access to information relevant to specific needs has become still more crucial: a huge dynamic data/information repository acquires worth if its contents are accessible as effective answer to specific needs.

Fig. 76.1. Lotfi Zadeh, Gabriella Pasi and Rachel Yager at the reception of the 11th IPMU Conference on July 2-7, 2006 in Paris, France

However, retrieval is still a quite difficult task, and the tradeoff between systems' efficiency and their effectiveness has resulted in the definition of various though *"simple"* models of both documents (basic retrievable units of information), and query languages. The main issue related to the IR task still concerns the fact that the complexity of natural languages cannot be fully captured by a computer application; the uncertain and vague nature of the IR task has been deeply exploited from 1990 to now, and several approaches based on probability theory, machine learning techniques, and natural language analysis have been proposed. Since then, also Fuzzy Set Theory has proven to be a valuable means to define flexible approaches in this context, ranging from the definition of flexible query languages, fuzzy associative mechanisms, adaptive document's representations, and personalized systems [5], [6], [9], [10].

However, despite of the big developments underlying the technologies for managing and accessing information of interest to specific needs, state of the art and commercial search engines are still mainly based on keyword-based processing, with seldom use of knowledge resources to face the problem of word disambiguation. The complexity of natural languages, with their nuances and their subjective usage is still far to be effectively captured by computer applications.

An interesting computational approach, with a big potential, but which has not been sufficiently exploited in this context is offered by the "computing with words" paradigm, as well outlined by Lotfi Zadeh in [12], [13], [14].

In the next section we shortly overview the history of Fuzzy Set Theory in IR.

76.2 A Synthesis of the Main Approaches to Model Fuzziness in IR

The application of Fuzzy Set Theory to Information Retrieval was mainly aimed to the definition of retrieval techniques capable of modeling, at least to some extent, the subjectivity, vagueness and imprecision that is intrinsic to the process of locating information relevant to users' needs. In particular, Fuzzy Set Theory has been applied in the context of IR to the following main purposes:

- to deal with the imprecision and subjectivity that characterize the document indexing process, i.e., the process by which the content of a text is automatically synthesized and represented on the basis of keywords extracted from (or associated with) it;
- to capture and manage vagueness in query formulation: user requests are usually expressed by a few terms, whose intended semantics shall be disambiguated depending on both the user and query context, and whose importance in characterizing the search topics varies depending on both their specificity, position within the query string, collection searched and other factors;
- to "soften" associative mechanisms, such as thesauri and algorithms for documents' clustering, which are often employed to extend the functionalities of an IRS. Within classic thesauri the considered relations between pairs of terms are

crisp, while in natural languages they are not Boolean; they are more adequately represented by fuzzy relations, where a degree in $[0,1]$ expresses the strength of association of two terms/concepts. Traditional clustering techniques partition documents into disjoint sets to reflect the distinct topics they are centered on; however a crisp partitioning hardly reflects the variety of topics dealt by texts;
- to define flexible decision strategies in meta search engines and in distributed IR for fusing the ranked lists of documents retrieved by several search engines, or for selecting the most adequate sources of information to query based on their contents, reputation, user needs;
- to represent and inquiry semi-structured information (XML document repositories) by allowing users to specify degrees of preference on the documents subparts they consider as the most significant to their needs, as well as flexible constraints on both document structure and content.

As previously outlined, the first approaches that applied Fuzzy Set Theory to Information Retrieval were aimed at generalizing the Boolean IR model to the main purpose of overcoming the binary modeling of the concept or document's relevance to a user's query. Within the fuzzy framework relevance was conceived as a gradual property of documents with respect to a user's query, with the consequence of making an Information Retrieval System able to produce a document ranking [2], [11]. In a fuzzy IR model a document is formally represented as a fuzzy subset of index terms (the membership value associated with a term represents its index term weight, computed based on statistic models of terms significance in document's texts, first studied by Luhn in 1950s). By means of Fuzzy Set Theory two main generalizations of the Boolean query language have been proposed: the introduction of query term weights and a generalization of the aggregation operators (the connectives AND and OR in the Boolean query language).

A query term weights is interpreted as the specification of the importance of that term as a descriptor of the user's needs, and it is formally defined as a flexible constraint on the fuzzy document representation. By such an extension, the structure of a Boolean query is maintained, by allowing weighted query terms to be aggregated by the AND, OR connectives and negated by the NOT operator. In this way the exact matching of the Boolean model is relaxed to a partial matching, where the matching degree of a document to a query (the so called Retrieval Status Value of a document) is computed by the evaluation of the flexible constraints on the document representation. In the context of Fuzzy Set Theory the connectives AND and OR are defined as aggregation operators belonging to the classes of T-norms and T-conorms respectively. Usually, the AND is defined as the min (minimum) aggregation operator, and the OR as the max (maximum) aggregation operator.

In the first fuzzy models query term weights were defined as numeric values in the range $[0,1]$. The flexible constraint identified by a query term weight depends on its semantics; several semantics have been proposed for query term weights, corresponding to distinct fuzzy generalizations of the Boolean model (distinct fuzzy IR models) [4], [7]. The three main semantics for query term weights are:

the *relative importance* semantics (query weights express the relative importance of pairs of terms in a query), the *threshold* semantics (a query weight expresses a threshold on index term weights), and the *ideal index term weight* semantics (a query weight expresses the *"perfect"* index term weight). The choice of one out of the three proposed query weight semantics implies a distinct modeling of the retrieval function evaluating a query against documents' representations.

Fig. 76.2. Gloria Bordogna and Gabriella Pasi, Lotfi Zadeh, Maria Amparo Vila, Carlos Molina, Nicolás Marín, and at the right Janusz Kacprzyk at the 2005 IFSA Conference in Bejing

It is important to notice that among the IR models that proposed the use of query term weights, the Fuzzy IR models were the only ones to consider the problem of the possible distinct semantics of the query term weights.

As the association of a numeric value forces the user to quantify the qualitative concept of importance of query terms, some late models proposed in the literature have introduced linguistic extensions of the Boolean query language, based on the concept of linguistic variable [1], [4]. By using linguistic query weights, query terms can then be labeled by words such as *important*, *very important*. Analogously to the evaluation of numeric query term weights, also linguistic query term weights express flexible constraints on index term weights.

Fig. 76.3. Lotfi Zadeh, Gabriella Pasi and Fay Zadeh at the Welcome reception to the World Conference on Soft Computing in San Francisco, 2011

A second kind of generalization of the Boolean query language has concerned the definition of *soft* aggregation operators (to improve query formulation with respect to the use of the AND and OR Boolean aggregation operators) [3]. In fact, when the AND is used for aggregating N (weighted) keywords in a user query, a document indexed by all keywords but one is not retrieved, thus causing the possible rejection of useful items. The opposite behavior characterizes the aggregation by OR. To express more flexible aggregations, the use of linguistic quantifiers (formally defined within Fuzzy Set Theory) was proposed. Linguistic quantifiers, such as *at least 2* and *most*, specify more flexible document selection strategies. Linguistic quantifiers have been formally defined as averaging aggregation operators, the behavior of which lies between the behavior of the AND and the OR connectives, which correspond to the *all* and the *at least one* linguistic quantifiers. Notice that our proposal, formulated in 1995 [3] to use linguistic quantifiers as soft aggregations of query terms makes it possible to rank documents retrieved by taking into account not only the significance of terms representing documents, but also the number of the query terms that index a retrieved document. This feature is now one of the criteria used by search engines to rank documents, so that in the first positions appear documents containing most of the query terms, and in the lower positions documents with only a few terms.

Another historical application of Fuzzy Set Theory to IR has concerned the definition of fuzzy associative retrieval mechanisms, which were first proposed in 1983 [9];

they are based on the concept of fuzzy association [1], [4], [6]. In Information Retrieval, different kinds of fuzzy associations can be modeled; examples of fuzzy associative mechanisms are offered by fuzzy pseudo-thesauri, and fuzzy clustering techniques. Fuzzy associative mechanisms based on thesauri or clustering techniques have been employed to cope with the incompleteness characterizing either the representation of documents or the users' queries. In the former case their usage allows to expand the index terms of documents with those terms that are more strongly associated with them in the fuzzy thesaurus or pseudo-thesaurus.

76.3 Conclusions

To summarize, Fuzzy Set Theory has provided early ideas for modeling important features of current IR systems, that later have been independently proposed and implemented within ad hoc heuristic models and proved their effectiveness.

We think that fuzzy set theory can still be a promising framework for shaping search engines of the future. In this respect the future of Information Retrieval cannot be conceived by disregarding the Semantic Web.

An interesting and promising field for the definition and development of a new generation of search engines is constituted by the research on meme identification and tracking.

The word "meme" has been coined in 1976 by Richard Dawkins to indicate "the basic unit of cultural transmission or imitation": as well outlined by Susan Blackmore [1]: *"Memes, like genes, are replicators, that is, they are information that is copied with variation and selection. Because only some of the variants survive, memes (and hence human cultures) evolve".*

As it may be easily figured out, the Web constitutes a huge information repository, and several Internet based-services and applications represent a quite rich soil for memes' growth and evolution. Let us think about blogs, e-mails, news resources and social networks applications (like Twitter and Facebook). In the context of the Web, the concept of Internet Meme has been introduced as a unit of information (idea) replicated and propagated through the Web by one or more of the previously mentioned applications. An interesting and critical aspect related to the various and easy means for information spreading and replication on the Internet is related to a potential lack of human control in the memes evolutionary process.

To the aim of both studying memes on the Internet (e-meme) and defining the future search engines capable of identifying e-memes and of tracking their spread and evolution over the Internet, we think that the "computing with words" paradigm, coupled with fuzzy associative mechanisms can provide a suitable and feasible approach.

This will enable future search engine to retrieve not just isolated web pages, but e-memes, i.e., connected Web pages that refelect the meme spreading and evolution over the Web.

References

1. Blackmore, S.J.: The Meme Machine. Oxford University Press, Oxford (1999)
2. Bordogna, G., Carrara, P., Pasi, G.: Query Term Weights as Constraints in Fuzzy Information Retrieval. Information Processing & Management 27(1), 15–26 (1991)
3. Bordogna, G., Pasi, G.: Linguistic Aggregation Operators of Selection Criteria in Fuzzy Information Retrieval. International Journal of Intelligent Systems 10, 233–248 (1995)
4. Bordogna, G., Pasi, G.: Modeling Vagueness in Information Retrieval. In: Agosti, M., Crestani, F., Pasi, G. (eds.) ESSIR 2000. LNCS, vol. 1980, pp. 207–241. Springer, Heidelberg (2001)
5. Bordogna, G., Pasi, G.: Personalized Indexing and Retrieval of Heterogeneous Structured Documents. Information Retrieval 8(2), 301–318 (2005)
6. Bordogna, G., Gabriella, P., Yager, R.R.: Soft Approaches to Distributed Information Retrieval. International Journal of Approximate Reasoning 34(2-3), 105–120 (2003)
7. Kraft, D.H., Bordogna, G., Pasi, G.: Fuzzy Set Techniques in Information Retrieval. In: Bezdek, J.C., Dubois, D., Prade, H. (eds.) Fuzzy Sets in Approximate Reasoning and Information Systems. The Handbooks of Fuzzy Sets Series, pp. 469–510. Kluwer Academic, Norwell (1999)
8. Pasi, G.: Fuzzy Sets in Information Retrieval: State of the Art and Research Trends. In: Bustince, H., Herrera, F., Montero, J. (eds.) Fuzzy Sets and Their Extensions: Representation, Aggregation and Models, Intelligent Systems from Decision Making to Data Mining, Web Intelligence and Computer Vision. STUDFUZZ, pp. 517–535. Springer, Berlin (2008)
9. Miyamoto, S., Teruhisa, M., Nakayama, K.: Generation of a Pseudo-thesaurus for Information Retrieval Based on Cooccurrences and Fuzzy Set Operations. IEEE Transactions of Systems, Man, and Cybernetics SMC 13(1), 62–70 (1983)
10. Miyamoto, S.: Fuzzy Sets in Information Retrieval and Cluster Analysis. Kluwer Academic, Dordrecht (1990)
11. Radecki, T.: Fuzzy Set Theoretical Approach to Document Retrieval. Information Processing and Management 15(5), 247–260 (1979)
12. Zadeh, L.A.: Fuzzy Sets. Information and Control 8, 338–353 (1965)
13. Zadeh, L.A.: From Search Engines to Question-answering Systems – The Need for New Tools. In: Proceedings of the EUSFLAT Conference 2003, pp. 3–5 (2003)
14. Zadeh, L.A.: Computing with Words and Perceptions - A Paradigm Shift in Computing and Decision Analysis and Machine Intelligence. In: Proceedings of the ICMLA Conference 2003, pp. 3–5 (2003)

77

Fuzzy Sets: A Brief Retrospect and Beyond

Witold Pedrycz

In this brief note, my intent is to offer some personal views at fuzzy sets - a paradigm that has brought an innovative and fresh perspective to numerous endeavors to numerous disciplines in sciences, humanities, and engineering. Obviously, any historical notes of this nature are sketchy, incomplete, and highly subjective. By no means they pretend to be a fully reflective of the rich history, landscape and people forming the fuzzy set community.

77.1 My Way to Fuzzy Sets

Fuzzy sets appeared as my research area quite early almost after I finished my MSc in 1977 and started working at the Silesian University of Technology, Gliwice, Poland. At that time Poland was a part of the Eastern block and the iron curtain had a visible impact on all aspects of our everyday life including academic affairs. There was a shortage of current literature, journals were scarce, international interaction quite limited and in many areas almost non-existent. I was fortunate in some sense as having an opportunity to visit Japan for about 2 months and staying with the electronic company (Anritsu) while at the same time taking part in some research seminars. During one of them I met Kaoru Hirota, who at that time was completing his PhD on probabilistic sets – a topic very closely linked with fuzzy sets [1]. This was perhaps the first time I got acquainted with the world of fuzzy sets. A few months later, in Poland, it was late Professor Ernest Czogala who brought the ideas of fuzzy sets, especially fuzzy control and fuzzy controllers. Ernest was not only an outstanding researcher bursting with brilliant ideas. His enthusiasm was contagious. The support he offered was paramount. In 1978, I started my PhD in the area of fuzzy relational equations and fuzzy systems completed two years later in 1980. Then there was a one-year postdoctoral fellowship at the Delft University of Technology – a very fortunate event, which crystallized main directions of my research in fuzzy sets. Delft offered the very best what the academic environment could ever provide: the best tradition of solid, creative and applied research done in a highly collaborative environment. It was the first time that I started to appreciate the term applicability (not applications) of research ideas. There were outstanding mentors, just to mention Professors Dijkman, van Naute Lemke, Backer [4], and Lootsma. At the early of the 80ties there were very few regular publications in fuzzy sets. Professor Dijkman and his group published a series of interesting internal reports covering various areas of

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fuzzy sets. From these days I can recall some interesting meeting with Walter Kickert (who at that time was at the University of Nijmegen). He was somewhat moving away from fuzzy sets but had developed interesting views about the area.

The fuzzy set research community in Poland and Eastern Europe was very supportive and, in spite of obvious limitations, led to a number of long-lasting research interactions and friendships. Janusz Kacprzyk, Siegfried Gottwald, Laszlo Koczy, Arkady Borisov, Vilem Novak – I have been fortunate having known them for many years.

77.2 First Meetings with Lotfi

I can vividly recall my early meetings with Lotfi. The first one was at the IFSA Congress in Tokyo. His plenary talk was inspiring with so lucidly posed arguments – for myself this opened new perspectives and helped me position the research known from the literature in a completely new, very much enriched perspective. The time was hectic and Lotfi surrounded by the crowds was not easily accessible. We had a brief conversation. What I found striking was his ability to listen, offer advise, and encourage. From a broader perspective, this second IFSA Congress was successful indeed and very much important to the developments of fuzzy sets worldwide. Japan was booming at this time; fuzzy sets were on a rise, the term *fuzzy* was *en vogue*, a plethora of applications of fuzzy sets was embraced by the Japanese industry (Sendai railway system, home appliances, to recall the most visible examples). The presentations made by the pioneers and eminent Japanese researchers and engineers were highly attractive and influential - for the first time what had been known in the literature about fuzzy controllers was showed in its full galore through a large number of experimental setups so much enjoyed by the participants of the Congress. The Congress was a turning point for the academia and industry – an event loaded with enthusiasm, dedication, and ingenuity of the young and rapidly growing community.

The next meeting with Lotfi took place in a quite different environment and happened during my postdoctoral research stay in the Netherlands. Lotfi came to Delft to give a seminar; this must have been one of the stops during his busy trip to Europe, I suppose. The talk was again enlightening and there was more time for discussion in a far more relaxed environment than the one during the IFSA Congress. I think it was the time Lotfi was very much preoccupied by the ideas of PRUF (used for test-score semantics for natural languages and meaning representation) and this framework was at the center of our discussion.

77.3 Fuzzy Sets – A Retrospective

Going back several decades, it is interesting and informative to put some developments in fuzzy sets in a broader perspective.

77.3.1 Fuzzy Modeling

Fuzzy models and fuzzy modeling emerged at the very early stages of fuzzy sets as a natural consequence of some papers of Lotfi along with the highly influential principle of incompatibility. Interestingly enough, the studies were mostly focused on the conceptual level addressing the burning question about the new quality and modeling horizons fuzzy sets could bring to the area. Not surprising, there were far less developments at the level of detailed estimation procedures, learning, validation of the models. Just the idea was the most attractive and the detailed design was left. I can clearly recall early works by Wenstop [7], Kickert [3], van Naute Lemke [2], Kandel [5], and Tong [6], to mention only a few names. The term *linguistic* modeling seemed to be quite visible and influential and had started to become associated with fuzzy sets as one of its most visible landmarks. It has various dimensions not only by forming a backbone of system modeling with fuzzy sets, but contributing to the realization of structures of human decision-making and pattern recognition.

77.3.2 Fuzzy Control

Fuzzy control or being more specific, fuzzy controllers, were one of these concepts that became very much visible and became a flagship of applied fuzzy sets. For the first time, the omnipresent and some entrenched dogma of control theory and control engineering has been challenged. Control strategies were inherently tied with a model. At some point it was somewhat overlooked or forgotten that control does not concern system control (which is tied with reality) but *model* control. Likewise a control performance index (better be a quadratic one) is a must so that control problem translates into an optimization task. The underlying principle of fuzzy controller is definitely attractive: the rules capture both the performance index and a feasible control strategy. The problems of system identification were bypassed, the control started to become more in rapport with the particular needs of the problem. Yet, on the more formal note, there were no comprehensive tools to analyze the closed-loop control behavior of the system, discuss stability, come up with a suite of thorough design practices. Even today, time-to-time we witness some critical comments coming from the camp of "classic" control even though a lot of progress has been done. Perhaps still some points call for better clarification and more dialogue with the control community.

77.3.3 Computational Intelligence and Soft Computing

There was an important development in the discipline of intelligent systems and fuzzy sets. The eighties have brought a flurry of new ideas under the banner of Computational Intelligence or Soft Computing (whether correct or not, these two names are used quite interchangeably). Computational Intelligence stressed a synergy of the contributing technologies including neurocomputing, fuzzy sets and evolutionary optimization- a tendency that was important and of significant relevance and was, to a significant extent, inevitable. We have seen hybrids: neurofuzzy systems, evolutionary -fuzzy architectures. New, sometimes difficult to decipher, acronyms appeared. Fuzzy sets gained some new capabilities (especially at the learning end), expanded a scope of algorithmic pursuits and applications. There was, however, a visible shift from emphasis on knowledge representation (which is profoundly associated with fuzzy sets) to striving to achieve high accuracy of the resulting fuzzy set constructs.

77.4 The Prospects and Challenges

Fuzzy sets, after almost 50 years, achieved a remarkable level of maturity and assumed a visible and well-recognized position on the arena of the fundamentals and technology of intelligent systems. How could the future is going to look like and what could we envision in the years to come? Nobody has a crystal ball however some main directions can be envisioned:

mastering, abstracting and fostering the development of the well-established concepts and algorithms of fuzzy sets. This is a natural tendency – one tries to build upon what has been established and has already shown some interesting applications. Undoubtedly, the pursuits in this realm will continue by building a body of knowledge and contribute to the progression of the area.

mathematization of the area and a silent abandonment of the fundamental principles of fuzzy sets. This tendency has already appeared. Fuzzy sets are used in a mechanical fashion. They become merely mathematical constructs loosing their semantics. We have witnessed constructs with 20+ fuzzy sets defined in a universe of discourse. It becomes difficult to imagine that such fuzzy sets could carry any meaning. Fuzzy sets are over-precisiated. Fuzzy sets are generalized – no matter whether such generalizations do make sense, are justified or any applicability of such constructs could be envisioned. Fuzzy control becomes an area of mathematical manipulation of symbols - we are back to square one- we require a model (nowadays it is a fuzzy one) and a performance index to do any number crunching.

further progression of the concepts and building upon the fundamentals of fuzzy sets. This tendency leads to the significant expansion of the area and. Here Granular Computing (as introduced by Lotfi [8] and just recently lucidly elaborated in his paper [9]) comes as a representative and useful example along this line. It is very likely we will be witnessing a substantial progress in this domain.

77.5 Closing Thoughts

We have traveled a long way but it looks that many things happened just yesterday. No doubt, a lot of new exciting developments are awaiting us in the years to come. We will be seeing intriguing, surprising and completely unanticipated twists, which could direct fuzzy sets in new, uncharted territories and unexpected application domains. Undoubtedly, we will be seeing some re-discoveries. What remains certain, though, is the visibility and importance of the main paradigm of fuzzy sets and its human-centricity facets forming an overarching principle, over which numerous intelligent systems are built.

References

1. Hirota, K.: Concepts of Probabilistic Sets. Fuzzy Sets and Systems 5, 31–46 (1981)
2. Kickert, W.J.M., van Nauta Lemke, H.R.: Application of a Fuzzy Controller in a Warm Water Plant. Automatica 12(4), 301–308 (1976)
3. Kickert, W.J.M.: Towards an Analysis of Linguistic Modeling. Fuzzy Sets and Systems 2, 293–307 (1979)
4. Simon, J.C., Backer, E., Sallantin, J.: A Structural Approach of Pattern Recognition. Signal Processing 2(1), 5–22 (1980)
5. Rickman, S.M., Kandel, A.: Column Table Approach for the Minimization of Fuzzy Functions. Information Sciences 12(2), 111–128 (1977)
6. Tong, R.M., Beck, M.B., Latten, A.: Fuzzy Control of the Activated Sludge Wastewater Treatment Process. Automatica 16(6), 695–701 (1980)
7. Wenstøp, F.: Quantitative Analysis with Linguistic Values. Fuzzy Sets and Systems 4(2), 99–115 (1980)
8. Zadeh, L.A.: Towards a Theory of Fuzzy Information Granulation and its Centrality in Human Reasoning and Fuzzy Logic. Fuzzy Sets and Systems 90, 111–117 (1997)
9. Zadeh, L.A.: A Note on Z-numbers. Information Sciences 181(14,15), 2923–2932 (2011)

Fuzzy Relational Equations – From Theory to Software and Applications

Ketty Peeva

78.1 Motivation

In 1977 I finished my PhD Thesis on categories of stochastic machines. The main attention was paid on computing behavior, establishing equivalence of states and equivalence of stochastic machines, as well as on reduction and minimization. All these problems were solved for stochastic machines, using linear algebra: they require solving linear system of equations with traditional algebraic operations, establishing linear dependence or linear independence of vectors, Noetherian property. After 1980 I was interested in similar problems, but for finite fuzzy machines.

Finite fuzzy machines were proposed and first studied by Santos [23]. Santos set equivalence, reduction and minimization problems for finite max-min fuzzy machines and for finite max-product fuzzy machines [24], [25]. Most of the results presented in [24], [25] have counterparts in the theory of stochastic sequential machines [1]. Nevertheless that stochastic machine seems to be similar to fuzzy machine, the main obstacle was that linear algebra and fuzzy algebras are completely unrelated. In order to investigate fuzzy machines, two types of algebras, called max-min algebra and max-product algebra have been developed [25], [26]. The role played by these algebras in the theory of max-min and max-product fuzzy machines is supposed to be the same as that played by linear algebra in the theory of stochastic machines. But these algebras propose tremendous manipulations for solving problems for fuzzy machines. Supplementary, there are several issues in [24], that are either incomplete or confused [6]. This motivated the author to develop theory, algorithms and software for finite fuzzy machines (FFMs). Obviously developing theory and software for solving problems for FFMs requires first to develop theory and software for fuzzy relational calculus and then to implement it for FFMs. At this time these problems were open and I intended first to develop method, algorithm and software for solving FLSEs when the composition is max-min, then to implement it to FFMs. In 1980 I had no idea whether the solution exists and how long it will take me to solve the problems. Just now, after 30 years, I have positive answers to all these questions. My first results for FLSEs were published in [8], [10] (1985-1992), but in fact their natural development and extension is given in the monograph [12] and in papers [4], [5], [11], [15], presenting analytical methods, algorithms and software for solving FLSEs in fuzzy algebras with applications in various subjects, in which I worked during the years with my bachelor, master and PhD students: fuzzy machines [6], [9], [16], linear dependence [18], [19], optimization [14], [17], [20], [21], artificial intelligence and expert systems [3], [12], [13].

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78.2 Fuzzy Relational Equations

Publications for FLSEs demonstrate long period of investigations for discovering methods and procedures to solve them. The traditional linear algebra methods cannot be used - operations in fuzzy algebras are different from classical addition and multiplication. First the following max-min and min-max fuzzy linear systems of equations were studied:

$$A \bullet X = B \tag{78.1}$$

$$A \circ X = B \tag{78.2}$$

where $A = (a_{ij})_{m\times n}$ stands for the matrix of coefficients, $X = (x_j)_{n\times 1}$ – for the matrix of unknowns, $B = (b_i)_{m\times i}$ is the right-hand side and the composition "•" for 78.1 is max-min, while "∘" is min-max for 78.2.

The main problems (like in linear algebra) are: is the FLSEs consistent or not, and if it is consistent – how to find its solution(s).

For 78.1 and 78.2 the first and most essential were Sanchez results [22]: formulas that permit to determine the potential maximal (minimal, resp.) solution in case of max-min (min-max, resp.) composition law. Then in references attention was paid on the complete solution set. Various methods were proposed to find solutions: algebraic-logical approach, characteristic matrix, covering, binding variables, partitions and irreducible paths, solution based matrix, etc. The main ideas in [10] – to distinguish three categories of coefficients (greater, equal or less than right-hand side term of the equation) and to order equations in FLSEs helped the algebraic-logical approach from [8], [10] and provided the first software [10], [28]. This was also the theoretical background for the next step with list operations: to propose general approach, universal and exact method, algorithms and software for solving FLSEs in some BL-algebras (in Gödel algebra, when the composition is max-min or min-max, and in Goguen algebra in case of max-product composition): the potential extremal solution of the FLSEs is used to obtain its consistency and to list the coefficients that contribute to its solvability. Then list operations are used to drop redundant branches and to reduce the complexity (the time complexity of the problem is exponential [2]) of the exhaustive search. If the system is inconsistent, we obtain the equations that cannot be satisfied by potential extremal solution. Computational and memory complexity are also analyzed. The software realization is available [27]. It is worth to mention that up to now there does not exist other software for solving FLSEs. The packages are created by Zl. Zahariev [27] and Y. Kyosev [28], both of them – PhD students of the Technical University of Sofia.

78.3 Linear Dependency

It is almost obvious that establishing linear dependence or linear independence of vectors in the above fuzzy algebras (Gödel, Goguen, etc.) means to solve FLSEs. We investigate these problems with my PhD and Master Students in [14], [18], [19], [20].

Fig. 78.1. PhD and Master Students I. Atanasov, D. Petrov, Zl. Zahariev, G. Pantaleeva 35th International Conference Applications of Mathematics in Engineering and Economics, Sozopol, Bulgaria, 2009

78.4 Linear Optimization Problem

Solving linear optimization problem under FLSEs constraint is provided by the solution of FLSEs. The problem is to optimize the linear objective function

$$Z = \sum_{j=1}^{n} c_j x_j, \;\; c_j \in \Re, \;\; 0 \leq x_j \leq 1, 1 \leq j \leq n, \tag{78.3}$$

with traditional addition and multiplication, if $c = (c_1, \ldots, c_n)$ is the cost vector, subject to the FLSEs as constraint. In [4], [14], [17], [20], [21] we present the solution when the constraint is FLSEs in various fuzzy algebras: max-min, min-max, max-product, some BL-algebras. My PhD and Master students Zl. Zahariev and I. Atanasov won the *second students' price* on *Fourth International IEEE Conference on Intelligent Systems* (2008) for optimization of linear objective function with fuzzy relational constraint [21].

78.5 Finite Fuzzy Machines

In order to investigate FFMs, suitable methods and algorithms in fuzzy relation calculus were developed for direct and for inverse problem resolution: direct problem

Fig. 78.2. I.Atanasov, K. Peeva, J. Kazprzyk, Zl. Zahariev and G. Klir Fourth International IEEE Conference on Intelligent Systems, Sept. 2008, Varna, Bulgaria

resolution [12], [26], [27] provides computing the behavior of FFMs; solving FLSEs provides equivalence, reduction and minimization of FFMs. The most essential theoretical results [4], [16] include: computing the behavior matrix, establishing equivalence of states and solving reduction and minimization problems (reduction of the number of states, finding reduced machine, minimization with respect to the number of states, finding minimal machine, etc.). Software created by Zl. Zahariev (the only one on the subject [26]) is proposed in [28] for computing behavior, for solving equivalence, reduction and minimization problems. The results are valid for finite max-min, min-max and max-product fuzzy machines.

78.6 Software

There exist some software packages for fuzziness [26], but only two for fuzzy relation calculus [27] and [28]. They are created to solve direct and inverse problems in fuzzy relation calculus. The first package [28] deals mainly with max-min composition and its applications, the second [27], named Fuzzy Calculus Core (or FC^{2}ore) is focused on working with various fuzzy algebras. This package can also solve related problems in fuzzy optimization and for finite fuzzy machines. In both packages the software is developed in (and for) MATLAB environment and is supposed to be used for solving concrete problems as well as to be used as a base of other applications.

Fig. 78.3. Technical University of Sofia – Senate – the Vice Rector Prof. Dr. R. Pranchov congratulates I. Atanasov and Zl Zahariev on their success

78.7 Other Applications

Other applications of FLSE are developed in team with students (bachelor, master, PhD) and colleagues from the Technical University of Sofia in some artificial intelligence areas – diagnosis, prediction and decision making in textile engineering and chemistry [3], [12], [13], syntactic pattern recognition [7], [12].

78.8 Conclusions

Next investigations for all these problems will be on generalization of the results for t-norms and t-conorms.

References

1. Carlyle, J.W.: Reduced Forms for Stochastic Sequential Machines. Journal of Mathematical Analysis and Applications 7(2), 167–175 (1963)
2. Chen, L., Wang, P.P.: Fuzzy Relational Equations (I): The General and Specialized Solving Algorithms. Soft Computing 6, 428–435 (2002)
3. Kyosev, Y., Peeva, K., Reinbach, I., Reinbach, I., Gries, T.: Max-product Fuzzy Relational Equations as Inference Engine for Prediction of Textile Yarn Properties. In: Reusch, B. (ed.) Proceedings of the International Conference 9th Fuzzy Days on Computational Intelligence, Theory and Applications, part 4, Dortmund, Germany, September 18-20. Advances in Soft Computing, pp. 93–104. Springer, Heidelberg (2006)
4. Peeva, K.: Resolution of Fuzzy Relational Equations – Method, Algorithm and Software with Applications. Information Sciences (2011) (special Issue), doi:10.1016/j.ins.2011.04.011
5. Peeva, K.: Universal Algorithm for Solving Fuzzy Relational Equations. Italian Journal of Pure and Applied Mathematics 19, 169–188 (2006)
6. Peeva, K.: Finite L-Fuzzy Machines. Fuzzy Sets and Systems 141(3), 415–437 (2004)
7. Peeva, K.: Finite L-Fuzzy Acceptors, Regular L-Fuzzy Grammars and Syntactic Pattern Recognition. International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems 12(1), 89–104 (2004)
8. Peeva, K.: Fuzzy Linear Systems. Fuzzy Sets and Systems 49, 339–355 (1992)
9. Peeva, K.: Behavior, Reduction and Minimization of Finite L-Automata. Fuzzy Sets and Systems 28(2), 171–181 (1988)
10. Peeva, K.: Systems of Linear Equations Over a Bounded Chain. Acta Cybernetica 7(2), 195–202 (1985)
11. Peeva, K., Kyosev, Y.: Algorithm for Solving Max-product Fuzzy Relational Equations. Soft Computing – A Fusion of Foundations, Methodologies and Applications 11(7), 593–605 (2007)
12. Peeva, K., Kyosev, Y.: Fuzzy Relational Calculus-Theory, Applications and Software (with CD-ROM). Advances in Fuzzy Systems – Applications and Theory, vol. 22. World Scientific Publishing Company, New Jersey (2004)
13. Peeva, K., Kyosev, Y.: Construction, Validation and Optimization of Fuzzy Inference Engine in Expert System for Textile Engineering. In: Marinov, M.S., Venkov, G. (eds.) Proceedings of the 29th International Conference Applications of Mathematics in Engineering and Economics AMEE 2003, Sozopol June 9-14, Bulvest 2000, Sofia 2004, pp. 271–276 (2003)
14. Peeva, K., Petrov, D.: Optimization of Linear Objective Function Under Fuzzy Equation Constraint in BL-Algebras – Theory, Algorithm and Software. In: Sgurev, V., Hadjiski, M., Kacprzyk, J. (eds.) Intelligent Systems: From Theory to Practice. SCI, vol. 299, pp. 417–431. Springer, Heidelberg (2010)
15. Peeva, K., Petrov, D.: Algorithm and Software for Solving Fuzzy Relational Equations in some BL-Algebras. In: IVth International IEEE Conference Intelligent Systems, Varna, vol. 1, pp. 2-63–2-68 (September 2008)
16. Peeva, K., Zahariev, Z.: Computing Behavior of Finite Fuzzy Machines – Algorithm and its Application to Reduction and Minimization. Information Sciences 178(21), 4152–4165 (2008)
17. Peeva, K., Zahariev, Z.: Optimization of Linear Cost Function With Fuzzy Max-product Relational Equation Constraints. In: Marinov, M.S., Todorov, D.M. (eds.) Proceedings of 32th International Conference AMME, Sozopol June 2006, Softtrade, Sofia, pp. 261–272 (2006,2007)

18. Peeva, K., Zahariev, Z.: Linear Dependence in Fuzzy Algebra. In: Proceedings of 31th International Conference AMME, Sozopol June 2005, Softrade, Sofia 2006, pp. 71–83 (2005,2006)
19. Peeva, K., Zahariev, Z.: Software for Testing Linear Dependence in Fuzzy Algebra. In: Second International Scientific Conference Computer Science, Part I, Chalkidiki, September 30-October 2, pp. 294–299 (2005)
20. Peeva, K., Zahariev, Z., Atanasov, I.: Optimization of Linear Objective Function Under Max-product Fuzzy Relational Constraint. In: 9th WSEAS Intern. Conference on Fuzzy Systems (FS 2008), Sofia, Bulgaria, May 2-4, pp. 132–137 (2008)
21. Peeva, K., Zahariev, Z., Atanasov, I.: Software for Optimization of Linear Objective Function With Fuzzy Relational Constraint. In: Fourth International IEEE Conference on Intelligent Systems, Varna, vol. 3, pp. 18–19 (September 2008)
22. Sanchez, E.: Resolution of Composite Fuzzy Relation Equations. Information and Control 30, 38–48 (1976)
23. Santos, E.S.: Maximin Automata. Information and Control 13, 363–377 (1968)
24. Santos, E.S.: On Reduction of Maxi-min Machines. Journal of Mathematical Analysis and Applications 40, 60–78 (1972)
25. Santos, E.S.: Max-product Machines. Journal of Mathematical Analysis and Applications 37, 677–686 (1972)
26. Zahariev, Z.: Software Package and API in MATLAB for Working With Algebras. In: Venkov, G., Kovacheva, R., Pasheva, V. (eds.) Proceedings of the 35rd International Conference AMEE, Sozopol. American Institute of Physics: AIP Conference Proceedings, vol. 1184, pp. 341–348 (June 2009)
27. `http://www.mathworks.com/matlabcentral/fleexchange/27046-fuzzy-calculus-core-fc2ore`
28. `http://www.mathworks.com/matlabcentral/fleexchange/6214-fuzzy-relational-calculus-toolbox`

79

Fuzzy Set Theory Utility for Database and Information Systems

Frederick E. Petry

79.1 Introduction

Information systems are often thought of as encompassing database systems as well as information retrieval systems [4]. Fuzzy set theory has had a significant impact in all of these systems. For both areas various fuzzy set representations for imprecise data have been utilized. Additionally fuzzy matching approaches have been applied in database querying and information retrieval.

79.2 Development of Similarity – Based Fuzzy Databases

Our earliest approach to fuzzy databases involved the use of fuzzy similarity relations in a database. It generalized the concept of null and multiple-valued domains where the crisp relational database remained a special case [1]. An example of the values for such a domain is a set of linguistic terms and the fuzzy model uses a similarity relationship to allow the comparison of these linguistic terms.

The identity relation used in ordinary relational databases induces equivalence classes (most frequently singleton sets) over a domain, D, which affects the results of certain operations and the removal of redundant tuples. The identity relation is replaced in the fuzzy similarity database by an explicitly declared similarity relation of which the identity relation is a special case.

A key aspect of most fuzzy relational databases is that domain values need not be atomic. A domain value, d_i, where i is the index of the attribute in the tuple, is defined to be a subset of its domain base set, D_i, that is, any member of the power set $P(D_i)$. So a fuzzy relation R is a subset of the set cross product $P(D_1) \times P(D_2) \times \cdots \times P(D_m)$. Membership in a specific relation, r, is determined by the underlying semantics of the relation. A fuzzy tuple, t, is any member of both r and $P(D_1) \times P(D_2) \times \cdots \times P(D_m)$. An arbitrary tuple is then of the form $t_i = [d_{i1}, d_{i2}, ..., d_{im}]$ where $d_{ij} \subseteq D_j$.

An interpretation $\alpha = [a_1, a_2, \ldots, a_m]$ of a tuple $t_i = [d_{i1}, d_{i2}, ..., d_{im}]$ is any value assignment for the relation's attributes such that $a_j \in d_{ij}$ for all j. Note that in an ordinary relational database a tuple is equivalent to its interpretation.

A domain value, d_{ij}, consists of one or more elements from the domain base set, D_j. That is, $d_{ij} \subseteq D_j$ where $i = 1, \ldots ..., n$, is the tuple index and $j = 1, \subseteq, m$, is the domain index. Given a domain, D_j, in a relation, the similarity threshold is defined

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to be: $Thres(D_i) = min_i\{min[s(x,y)]\}; x,y \in d_{ij}$. Note that in a crisp database, the cardinality of $d_{i_j} = 1$ and $s(x,x) = 1$, so $Thres(D_j) = 1$ for all j. A very significant concept is that a minimal threshold value given a priori can be used to determine which tuples may be combined by direct set union of the respective domain values.

In a crisp database, a tuple is redundant if it is exactly the same as another tuple. Any operation over a nonfuzzy relation at least implicitly entails removing redundant tuples. That is, any interpretation of the domains can be found in at most one tuple in the relation. In a fuzzy database, a tuple is redundant if it can be merged with another through the set union of corresponding domain values. The merging of tuples, however, is subject to constraints on the similarity thresholds. So tuples $t_i = [d_{i1}, d_{i2}, \cdots, d_{im}]$ and $t_k = [d_{k1}, d_{k2}, \cdots ..., d_{km}]$ are redundant if $Level(D_j) \leq min[s(x,y)], x,y \in d_{ij} \cup d_{kj}$ for $j = 1, \ldots, m$ and a $Level(D_j)$ given a priori.

An important property for this database is related to redundant tuples. The lack of redundant tuples in an ordinary database is equivalent to the absence of multiple occurrences of the same interpretation and so for any interpretation of the domains, a fuzzy relation should contain at most one tuple with that interpretation. This is significant as it avoids the possibility of creating anomalies during updating.

We were then able to prove that in our approach, if a fuzzy relation has no redundant tuples then $T_i \cap T_j = \oslash$ if $i \neq j$, where T_i is the set of possible interpretations for tuple t_i. The converse of this was also true, if no two tuples can be interpreted in an identical manner, then there exist level values for the domains under which no two tuples are redundant. Additionally the desirable property of the removal of redundancy having only one outcome was also proven, i.e. a fuzzy relation derived by merging redundant tuples is unique.

A related important early development for fuzzy similarity databases was our approach to information-theoretic characterizations which could measure the overall uncertainty in an entire relation [2]. Fuzzy entropy may be measured as a function of a domain value or as a function of a relation. Intuitively, the uncertainty of a domain value increases as its cardinality $|d_{i_j}|$ increases or when the similarity $s_j(x,y)$ decreases. So if a domain value in a relational scheme, d_{i_j}, consisting of a single element represents exact information and multiple elements are a result of fuzziness, then this uncertainty can be represented by entropy. Adapting the DeLuca and Termini entropy to a fuzzy database, the entropy $H_{f_z}(d_{ij})$, for a domain value $d_{ij} \subseteq D_j$ $((x,y \in d_{ij}))$ would be

$$H_{f_z}(d_{ij}) = -\sum_{x,y}[s_j(x,y)log_2(s_j(x,y)) + (1 - s_j(x,y))log_2(1 - s_j(x,y))] \qquad (79.1)$$

Note that $H_{f_z}(d_{ij})$ is directly proportional to $|d_{ij}|$ and inversely proportional to $s_j(x,y) > 0.5$.

This definition could not be directly extended to tuples, so a Shannon probabilistic entropy measure was needed for an entire tuple. Let α_i be the number of possible interpretations for the ith tuple, t_i, i.e., the cardinality of the cross product of the domain values, $|d_{i1} \times d_{i2} \times \cdots \times d_{im}|$. Viewing all interpretations as a priori equally likely, the entropy of tuple t_i could be defined as

$$H_{p_b}(t_i) = -\sum_{k=1}^{\alpha_i} (1/\alpha_i) log_2(1/\alpha_i) = -log_2(\alpha_i)$$

For a nonfuzzy database, clearly $\alpha_i = 1$ and $H_{p_b}(t_i) = 0$.

If the choice of a tuple in a relation r is independent of the interpretation of the tuple, the joint probabilistic entropy $H_{p_b}(r,t)$ of a relation can be expressed as

$$H_{p_b}(r,t) = -\sum_{i=1}^{n} \sum_{k=1}^{\alpha_i} (n\alpha_i)^{-1} log_2[(n\alpha_i)^{-1}]$$

where there are n tuples.

Since the domains in a fuzzy database may be both ordinary and fuzzy sets, some combined information estimate is desirable. One possible approach would be an entropy combining Shannon information and fuzzy information.

Fig. 79.1. FUZZ-IEEE / IFES '95 – March 1995 – Yokohama Japan From the left: Walter Karplus and wife; back row – Fred Petry, Bob Marks, Jim Bezdek Photo courtesy Kaori Yoshida, Kyushu Institute of Technology

79.3 Background of My Research

A 1974 survey paper in the journal *Science* which described fuzzy sets was my first exposure to the ideas and concepts of fuzzy set theory. I retained a copy of this

Fig. 79.2. IFSA' 97 Prague - June 25-29, 1997 at University of Economics On far right are Fred Petry and 2 of his graduate students: From the left: Y. Prada; Fred Petry (back to camera – with backpack); Ashley Morris

paper in my files from the time I first read it when I was on the Ohio State faculty. A few years later at the University of Alabama in Huntsville, I introduced several possible fuzzy set applications for projects in a computer science course. This led to the development of the similarity fuzzy database with Bill Buckles in the late 1970s. This work then first appeared in *Fuzzy Sets and Systems* in 1982 [1] and was also presented at the first NAFIPS conference (1982) in Logan Utah. It was at this NAFIPS conference that I first encountered Lotfi Zadeh and in discussions with him was greatly encouraged to continue my research in this area.

Subsequently I participated in about 15 Nafips conferences and several IFSAs and FUZZ-IEEEs (see Figures 79.1, 79.2. A particular notable opportunity occurred in 1984 when I was a participant in the NSF sponsored, Sino-American Symposium on Fuzzy Methodology and its Applications, Beijing, China which had 9 participants from the US including Zadeh, Yager, Bezdek, Ruspini and Paul Wang (see the photograph in figure 79.3). Other significant events related to fuzzy sets in which I was involved include the time when I was General Chair of the 1996 Fuzz-IEEE and two NAFIPS (1986 and 2002) all held in New Orleans, US.

Fig. 79.3. Sino-American Symposium on Fuzzy Methodology and its Applications, Beijing, China, July 1984. From the left: P. Wang, R. Yager, J. Bezdek, S. Bedrosian, L. McAllister, D. Buell, F. Petry, L. Zadeh, E. Ruspini.

79.4 Other Research Areas and Future Directions

Other developments in the database area in which I was involved was the development of an object-oriented fuzzy database [9] again using similarity relations for representation of class hierarchies. An alternative database representation for imprecise data using a combination of fuzzy set and rough set theory [11] has also been an on-going topic of interest of mine and several colleagues. Additionally early applications of fuzzy sets in genetic algorithms were developed for applications to both image recognition [3] and information retrieval [7]. In the mid 1990s I became very interested in the use of fuzzy sets to manage the complexities found in dealing with spatial data [5], [8]. This area has complex issues that will continue in the future. Another topic that also should continue to attract research interest is the application of fuzzy sets in data mining [6], [10].

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References

1. Buckles, B.P., Petry, F.E.: A Fuzzy Representation for Relational Data Bases. Fuzzy Sets and Systems 7(3), 213–226 (1982)
2. Buckles, B.P., Petry, F.E.: Information Theoretic Characterization of Fuzzy Relational Data Bases. IEEE Transactions on Systems Man and Cybernetics 13(1), 74–77 (1983)
3. Ankebrandt, C., Buckles, B.P., Petry, F.E.: Scene Recognition Using Genetic Algorithms with Semantic Nets. Pattern Recognition Letters 11(4), 285–293 (1990)
4. Petry, F.E.: Fuzzy Databases: Principles and Applications. Kluwer Academic Publishers, Dordrecht (1996)
5. Petry, F.E., Robinson, V.B., Cobb, M.A. (eds.): Fuzzy Modeling with Spatial Information for Geographic Problems. Springer, Heidelberg (2005)
6. Petry, F.E., Zhao, L.: Data Mining by Attribute Generalization with Fuzzy Hierarchies in Fuzzy Databases. Fuzzy Sets and Systems 160(15), 2206–2223 (2009)
7. Petry, F.E., Buckles, B.P., Kraft, D., Prabhu, D.: Genetic Algorithms for Fuzzy Boolean Information Retrieval. In: Proceedings of NAFIPS Conf., Allentown PA, pp. 52–62 (1993)
8. Cobb, M., Petry, F.E.: Modeling Spatial Data within a Fuzzy Framework. Journal of the American Society for Information Science 49(3), 253–266 (1998)
9. George, R., Buckles, B.P., Petry, F.E.: Modeling Class Hierarchies in the Fuzzy Object-Oriented Data Model. Fuzzy Sets and Systems 60(3), 259–272 (1993)
10. Ladner, R., Petry, F.E., Cobb, M.: Fuzzy Set Approaches to Spatial Data Mining of Association Rules. Transactions on GIS 7(1), 123–138 (2003)
11. Beaubouef, T., Petry, F.E.: Fuzzy-Rough Set Techniques for Uncertainty Processing in a Relational Database. International Journal of Intelligent Systems 15(5), 389–424 (2000)

80

Fuzzy Sets in Foundations of Quantum Mechanics

Jarosław Pykacz

80.1 Introduction

Three times in my scientific life I fell in love. For the first time, in the middle of the Seventies of the XX Century the object of my love was (and still is, since real love never ends) the theory of quantum logics. Quantum logics are mathematical structures that are encountered in the very foundations of quantum mechanics. Formally, they are order-theoretic structures more general than Boolean algebras: orthomodular partially ordered sets or lattices, that are believed to represent properties of quantum objects in the same way as Boolean algebras represent properties of objects that conform to laws of classical physics.

The object of my second scientific love was fuzzy set theory. From the very beginning of my acquaintance with this theory the very idea of a set the boundaries of which are not sharp but vanish gradually was for me so visible and beautiful that I could not resist. Specially, that very soon it occurred that the objects of my first and second scientific love intertwine or, more precisely, the second one embraces the first. But this needs more detailed explanation.

80.2 Mączyński's Functions and Giles Weakly Disjoint Sets

Professor Maciej Mączyński, the supervisor of my PhD Thesis, proved in 1973 [1] that any quantum logic possessing so called ordering set of probability measures (only such quantum logics are interesting from the physical point of view!) can be isomorphically represented as a family of $[0,1]$-valued functions L such that:

a) 0 (the null function) belongs to L,
b) if f belongs to L, then $1-f$ also belongs to L,
c) for any (finite or countable) sequence f_i of functions that belong to L such that $f_i + f_j \leq 1$ (such functions were called in [1] *pairwise orthogonal*), pointwise algebraic sum of these functions belongs to L,

and, conversely, any family of functions fulfilling conditions a) - c) is a quantum logic in the traditional, order-theoretic sense.

Obviously, I was well acquainted with Mączyński's Functional Representation Theorem, so as soon as I learned the rudiments of fuzzy set theory I noticed that Mączyński's functions could be treated as membership functions of fuzzy sets and that out of his three conditions that characterize any quantum logic in its functional

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form, the first two conditions could be in a straightforward way expressed in the language of fuzzy set theory. Some time later I noticed that also a part of the third of Mączyński's conditions is easily expressible in the language of fuzzy sets: pairwise orthogonality of functions defined by the condition $f_i + f_j \leq 1$ is in fact equivalent to the condition: $f_i \cap f_j = 0$, where $\cap$ denotes intersection of fuzzy sets defined by $f \cap g = max(f + g - 1, 0)$ This intersection was called *bold intersection* by Giles [2] who was the first to study it in the domain of fuzzy sets. Nowadays it is usually called *Łukasiewicz intersection*. Since Giles called in [2] *weakly disjoint* two fuzzy sets f and g such that $f \cap g = 0$, Mączyński's conditions can be expressed as follows:

a') the empty set belongs to L,
b') if f belongs to L, then its fuzzy complement also belongs to L,
c') for any (finite or countable) sequence of pairwise weakly disjoint sets that belong to L, their pointwise algebraic sum also belongs to L.

As we can see, after such reformulation only the second part of the third condition is not expressed by standard fuzzy set operations. Therefore, my clear aim was to replace algebraic sum that appeared in the condition c') by a "genuine" fuzzy set operation. In the meantime, the results stated above were announced at the Second International Fuzzy Systems Association Congress held in Tokyo in July 1987 [3].

80.3 Finally, Only Łukasiewicz Operations

Expressing quantum logic entirely in terms of "genuine" fuzzy set operations was not an easy task and it took me a couple of years before I made it. Finally, it occurred that it is De Morgan triple consisting of the standard fuzzy complement and Łukasiewicz operations (union being defined via De Morgan law: $f \& g = (f' \cap g')' = min(f + g, 1)$) that solves the problem. However, it occurred that in order to replace pointwise algebraic sum appearing in the condition c') by Łukasiewicz union, it was necessary to add one more, fortunately very natural, condition. The result, announced for the first time in [4], was as follows:

Any quantum logic with an ordering set of probability measures can be isomorphically represented as a family L of fuzzy sets such that:

a") the empty set belongs to L,
b") if f belongs to L, then its fuzzy complement also belongs to L,
c") for any (finite or countable) sequence of pairwise weakly disjoint sets that belong to L, their Łukasiewicz union also belongs to L.
d") the empty set is the only set in L that is weakly disjoint with itself,

and, conversely, any family of fuzzy sets satisfying a") - d") is a quantum logic in the traditional, order-theoretic sense. Let us note that the condition c") is obviously fulfilled in any family of crisp sets when fuzzy operations are degenerated to traditional set-theoretic operations.

80.4 Philosophical Consequences

Elements of quantum logics represent properties of studied physical systems (I shall keep the word "quantum" by an abuse of language since these structures represent also properties of classical physical systems). On the other hand in the developed fuzzy set representation of quantum logics these elements are represented by fuzzy subsets of a set of all pure states (*phase space*) of a studied physical system. If a physical system obeys deterministic laws of classical physics, then, whatever is a state of this system, it either possesses or does not possess each of its properties. This implies that a subset of a phase space representing a property is a crisp set and it occurs that a "quantum logic" of such a system is a Boolean algebra.

Quantum mechanics is not a deterministic theory. According to the nowadays most popular interpretation its laws allow only to calculate *probabilities* of results of future experiments, in particular experiments designed to check whether a quantum system possesses or not any of its properties. In the developed fuzzy set representation these probabilities are reinterpreted as degrees to which a quantum system that is in a specific pure state possesses all its properties even before they are measured.

It should be mentioned that according to the orthodox Copenhagen interpretation of quantum mechanics one is not even allowed to say that a quantum system possesses or not any of its properties before a suitable experiment is carried out, which gave rise to the famous statement: *Unperformed experiments have no results* [5]. According to the propounded "fuzzy interpretation" of quantum mechanics one should replace this statement by a statement: *Unperformed experiments have all their possible results, each of them to the degree defined by suitable quantum-mechanical calculations*.

It is nothing strange, of course, that according to the propounded interpretation a quantum system can possess a property to the degree, say, a, and simultaneously does not possess it to the degree $1-a$. For example a photon that is in a state of linear polarization oriented under the angle α to the direction of a polarization filter possesses the property of *being able to pass through the filter* to the degree $cos^2\alpha$ and simultaneously it possesses the property of *not being able to pass through the filter* to the degree $1-cos^2\alpha = sin^2\alpha$. This results in the fact that when an experiment is repeated many times, the fraction $cos^2\alpha$ of identically prepared photons passes through the filter and the fraction $sin^2\alpha$ does not.

80.5 Possible Mathematical Consequences

Another argument that the statement *unperformed experiments have no results* is true, seems to come from mathematics. Already in 1964 J. S. Bell [6] proved that an attempt to endow quantum objects with well-defined sharp properties *before they are measured* in some cases leads to numerical results not compatible with quantum-mechanical calculations that were many times confirmed by experiments. These results took form of the famous Bell inequalities which are in some cases violated by quantum objects. Obviously, Bell's considerations were based on the classical paradigm according to which a property of an object can be only either entirely possessed or entirely not possessed by the object, and on utilizing the classical Kolmogorovian probability calculus based on the Boolean algebra of crisp random events. Therefore, basing these considerations on fuzzy set concepts, in particular on the concept that an object can possess a property "partially", should allow to annihilate the apparent discrepancy between the "common sense" considerations and experimental results.

Moreover, it is possible to show that Bell-type inequalities do not have to hold in some versions of fuzzy probability calculus [7], which again indicates that the mathematical formalism of quantum mechanics should be rather based on fuzzy mathematics than traditional crisp mathematics.

Another problem is the problem of the possibility of constructing phase-space representation of quantum mechanics considered already by E. Wigner in 1932 [8] with the aim of making quantum mechanics "more similar" to classical statistical mechanics. Although Wigner succeeded in constructing such representation, his "pseudo-probability distribution" is cursed with one unacceptable feature: it unavoidably becomes negative in some regions of the phase space. In my opinion this might be an artifact caused by unjustified use of traditional crisp mathematics in the area where fuzzy mathematics is a proper tool.

80.6 Prospects for the Future

According to the results reported in the previous Sections of this note, fuzzy set ideas, like the idea that a physical object can possess its properties only partially, seem to be better suited for description of quantum objects than traditional, crisp ones. I do hope that changing the language of description of quantum objects from the traditional language based on crisp sets to the language based on fuzzy sets would allow to avoid numerous paradoxes that plague quantum mechanics and will make this theory more comprehensible.

References

1. Mączyński, M.J.: The Orthogonality Postulate in Quantum Mechanics. International Journal of Theoretical Physics 8, 353–360 (1973)
2. Giles, R.: Łukasiewicz Logic and Fuzzy Set Theory. International Journal of Man-Machine Studies 67, 313–327 (1976)
3. Pykacz, J.: Quantum Logics as Families of Fuzzy Subsets of the Set of Physical States. Preprints of the Second IFSA Congress, Tokyo, July 20-25, vol. 2, pp. 437–440 (1987)
4. Pykacz, J.: Fuzzy Quantum Logics and Infinite-valued Łukasiewicz Logic. International Journal of Theoretical Physics 33, 1403–1416 (1994)
5. Peres, A.: Unperformed Experiments Have no Results. American Journal of Physics 46, 745–747 (1978)
6. Bell, J.S.: On the Einstein Podolsky Rosen Paradox. Physics 1, 195–200 (1964)

7. Pykacz, J., D'Hooghe, B.: Bell-type Inequalities in Fuzzy Probability Calculus. International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems 9, 263–275 (2001)
8. Wigner, E.: On the Quantum Correction for Thermodynamic Equilibrium. Physical Review E 40, 749–759 (1932)

81

Real-Valued Realizations of Boolean Algebras Are a Natural Frame for Consistent Fuzzy Logic

Dragan Radojevic

Fifty years ago, Lotfi Zadeh, as a visionary, realized a need for radically different mathematics to deal effectively with the systems which were generally orders of magnitude more complex than man-made systems [1]. Soon afterwards, he ingeniously introduced the notion of gradation in the theory of fuzzy sets [2] and in fuzzy logic in a wider sense. The use of fuzzy logic, based on the notion of gradation, first of all, enables a drastic reduction of the complexity immanent to the classical mathematical approaches in real problems. Thus, accordingly, fuzzy approaches are much more feasible due to their lower complexity compared to the classical approaches. Actually, a graded approach is much more descriptive than a classical two-valued (black and white) approach. From my point of view, the main drawback of the conventional fuzzy logics, in a wider sense, based on the truth functional principle, is the fact that they are not within the Boolean framework and hence these approaches are not Boolean consistent generalizations of the classical techniques.

81.1 Fuzzy Logic in a Boolean Framework

Realizing fuzzy logic in the Boolean framework means that all Boolean axioms and theorems are valid in the general case and in the case of gradation. Since the corresponding classical techniques are based on Boolean algebras, i.e. on their two-valued realizations, a consistent generalization or a consistent fuzzy case should be based on real-valued realizations of Boolean algebras. The real-valued realization of a finite or atomic Boolean algebra [4,5,6] is briefly illustrated here.

The main problem of the conventional approaches is the fact that they are based on the truth-functionality principle, which is taken from the two-valued logics. According to the Boolean algebraic point of view, this principle can only be adequate or valid in the classical two-valued case. The reason is very simple: Boolean functions, in the general case, have a vector nature, but in the classical case the attention is paid only to one component (which is determined by the $0-1$ values of free variables). In the general case, such as the treatment of gradation, it is necessary to include in computation more than one or even all components of the vector immanent to the Boolean function. In order to illustrate the main idea, we use the Boolean

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function of two free variables x and y, from the famous Boole's paper dated from 1848 [3]:

$$\begin{aligned}\phi(x,y) = \phi(1,1)xy + \phi(1,0)x(1-y) + \\ + \phi(0,1)(1-x)y + \phi(0,0)(1-x)(1-y).\end{aligned} \tag{81.1}$$

Actually, this equation is a special case of the Boolean polynomial [4]:

$$\begin{aligned}\phi^{\otimes}(x,y) = \phi(1,1)x\otimes y + \phi(1,0)(x - x\otimes y) + \\ + \phi(0,1)(y - x\otimes y) + \phi(0,0)(1 - x - y + x\otimes y).\end{aligned} \tag{81.2}$$

Free variables in the general case take the values from the unit interval x, $y \in [0,1]$. $\otimes$ is a generalized product [4] or t-norm with the following property:

$$max(x+y-1,0) \leq x\otimes y \leq min(x,y).$$

81.1.1 Boolean Polynomials

The real valued realization of a finite (atomic) Boolean algebra is based on Boolean polynomials. Any Boolean function can be uniquely transformed into the corresponding Boolean polynomial [6].

Example 1: *Using equation 81.2 relations of equivalence, exclusive disjunction and implication are, respectively*:

a. $\phi(x,y) =_{def} x \Leftrightarrow y$
$\phi(1,1) = 1; \qquad \phi(1,0) = 0; \qquad \phi(0,1) = 0; \qquad \phi(0,0) = 1;$

$\boxed{x \Leftrightarrow y = 1 - x - y + 2x\otimes y}$.

b. $\phi(x,y) =_{def} x\underline{\vee}y$
$\phi(1,1) = 0; \qquad \phi(1,0) = 1; \qquad \phi(0,1) = 1; \qquad \phi(0,0) = 0;$

$\boxed{x\underline{\vee}y = x + y - 2x\otimes y}$.

c. $\phi(x,y) =_{def} x \Rightarrow y$
$\phi(1,1) = 1; \qquad \phi(1,0) = 0; \qquad \phi(0,1) = 1; \qquad \phi(0,0) = 1;$

$\boxed{x \Rightarrow y = 1 - x + x\otimes y}$.

The finite (atomic) Boolean algebra BA(Ω) = $P(P(\Omega))$, is generated by the set of free variables $\Omega = \{x_1, \ldots x_n\}$, where: $P(\Omega)$ is a power set of Ω (a set of all subsets of Ω). The atomic elements of the analyzed Boolean algebra BA(Ω) are [6]:

$$\alpha(S)(\{x_1,\ldots x_n\}) = \bigwedge_{x_i \in S} x_i \bigwedge_{x_j \in \Omega \setminus S} \bar{x}_j, \qquad S \in P(\Omega). \tag{81.3}$$

The Atomic Boolean Polynomials $\alpha^{\otimes}(S)(x_1,\ldots x_n)$ uniquely correspond to the atomic elements $\alpha(S)(x_1,\ldots x_n)$ and they are defined by the following equations [6]:

$$\alpha^{\otimes}(S)(x_1,\ldots x_n) = \sum_{C \in P(\Omega \setminus S)} (-1)^{|C|} \bigotimes_{x_i \in C \cup S} x_i, \qquad S \in P(\Omega). \tag{81.4}$$

Example 2: The atomic Boolean polynomials for the Boolean algebra generated by $\Omega = \{x,y\}$ are:

$$\begin{aligned}
\alpha^{\otimes}(\{x,y\}) &= x \otimes y;\\
\alpha^{\otimes}(\{x\}) &= x - x \otimes y;\\
\alpha^{\otimes}(\{y\}) &= y - x \otimes y;\\
\alpha^{\otimes}(\{\ \}) &= 1 - x - y + x \otimes y.
\end{aligned}$$

The values of the atomic polynomials in the real valued case are non-negative $\alpha^{\otimes}(S)(x_1,\ldots,x_n) \in [0,1]$, $S \in P(\Omega)$, but their sum is identically equal to 1. In the example described by equation 81.2 for $x, y \in [0,1]$:

$$x \otimes y + (x - x \otimes y) + (y - x \otimes y) + (1 - x - y + x \otimes y) \equiv 1.$$

The classical two-valued case is the only special case which also satisfies this fundamental identity, since only the value of one atom is equal to 1 and all others are identical to 0.

Any Boolean function, an element of the analyzed Boolean algebra, $\phi(x_1,\ldots, x_n) \in \mathrm{BA}(\Omega)$ can be uniquely represented in the disjunctive canonical form (disjunction of relevant atomic elements):

$$\phi(x_1,\ldots,x_n) = \bigvee_{S \in P(\Omega)} \sigma_\phi(S)\alpha(S)(x_1,\ldots,x_n). \tag{81.5}$$

Where: $\sigma_\phi(S)$, $(S \in P(\Omega))$ is a relation of the inclusion of the corresponding atom $\alpha(S)(x_1,\ldots,x_n)$ in the analyzed Boolean function $\phi(x_1,\ldots,x_n)$, defined in the following way:

$$\sigma_\phi(S) =_{def} \phi(\chi_S(x_i) \mid i = 1,\ldots,n), \quad \left(\chi_S(x_i) =_{def} \begin{cases} 1, & x_i \in S \\ 0, & x_i \notin S \end{cases}\right), (S \in P(\Omega)). \tag{81.6}$$

Inclusion relationships define which atoms are included in the analyzed Boolean function

$$\sigma_\phi(S) = \begin{cases} 1, & \alpha(S)(x_1,\ldots,x_n) \subset \phi(x_1,\ldots,x_n) \\ 0, & \alpha(S)(x_1,\ldots,x_n) \not\subset \phi(x_1,\ldots,x_n) \end{cases}, (S \in P(\Omega)). \tag{81.7}$$

(values of the corresponding relation of the inclusion equal to 1) and which are not included (values of the relation of the inclusion equal to 0).

A Boolean polynomial corresponds unequally to any Boolean function, as a figure of 81.6:

$$\phi^{\otimes}(x_1,\ldots,x_n) = \sum_{S\in P(\Omega)} \sigma_{\phi}(S)\alpha^{\otimes}(S)(x_1,\ldots,x_n), \qquad (x_1,\ldots,x_n \in [0,1]). \quad (81.8)$$

A Boolean polynomial, equation 81.8, can be presented as a scalar product of two vectors:

$$\boxed{\phi^{\otimes}(x_1,\ldots,x_n) = \boldsymbol{\sigma}_{\phi}\boldsymbol{\alpha}^{\otimes}(x_1,\ldots,x_n)}, \qquad x_1,\ldots,x_n \in [0,1]). \quad (81.9)$$

- $\boldsymbol{\sigma}_{\phi} = \left[\sigma_{\phi}(S) \mid S \in P(\Omega)\right]$ is a structure of the analyzed Boolean function $\phi(x_1,\ldots,x_n) \in \mathrm{BA}(\Omega)$, i.e. a vector of relations of the inclusion of the atomic functions in it.
- $\boldsymbol{\alpha}^{\otimes}(x_1,\ldots,x_n) = [\alpha(S)(x_1,\ldots,x_n) \mid S \in P(\Omega)]^T$ is a vector of the atomic polynomials of the analyzed finite Boolean algebra $\mathrm{BA}(\Omega)$

Example 3: *Structures of the analyzed Boolean functions from Example1 are*

$$\boldsymbol{\sigma}_{x\Leftrightarrow y} = [1\;0\;0\;1\,], \qquad \boldsymbol{\sigma}_{x\underline{\vee}y} = [0\;1\;1\;0\,], \qquad \boldsymbol{\sigma}_{x\Rightarrow y} = [1\;0\;1\;1\,].$$

and vector of the atomic polynomials of two variables is:

$$\boldsymbol{\alpha}^{\otimes}(x_1,x_2) = [x_1\otimes x_2 \qquad x_1 - x_1\otimes x_2 \qquad x_2 - x_1\otimes x_2 \qquad 1 - x_1 - x_2 + x_1\otimes x_2]^T.$$

Structural Functionality Principle: The structure of any combined Boolean function can be calculated directly on the basis of its component structures using following identities:

$$\boldsymbol{\sigma}_{\phi\wedge\psi} = \boldsymbol{\sigma}_{\phi}\wedge\boldsymbol{\sigma}_{\psi} \qquad \boldsymbol{\sigma}_{\phi\vee\psi} = \boldsymbol{\sigma}_{\phi}\vee\boldsymbol{\sigma}_{\psi} \qquad \boldsymbol{\sigma}_{\neg\phi} = \neg\boldsymbol{\sigma}_{\phi} = 1-\sigma_{\phi}$$

The famous truth functionality principle is a figure of the structural functionality on the value level only in the case of a two-valued realization. In the general case (multi-valued or real-valued realization), the truth functionality principle is not able to preserve all Boolean algebraic properties. This is the reason why the fuzzy approaches based on the truth functionality principle cannot live in the Boolean framework.

The structures of the Boolean functions preserve all Boolean algebraic laws [6]:

Monotone Laws

Associativity

$$\begin{aligned}\boldsymbol{\sigma}_{\phi}\vee(\boldsymbol{\sigma}_{\psi}\vee\boldsymbol{\sigma}_{\zeta}) &= (\boldsymbol{\sigma}_{\phi}\vee\boldsymbol{\sigma}_{\psi})\vee\boldsymbol{\sigma}_{\zeta};\\ \boldsymbol{\sigma}_{\phi}\wedge(\boldsymbol{\sigma}_{\psi}\wedge\boldsymbol{\sigma}_{\zeta}) &= (\boldsymbol{\sigma}_{\phi}\wedge\boldsymbol{\sigma}_{\psi})\wedge\boldsymbol{\sigma}_{\zeta}\end{aligned} \quad (81.10)$$

Commutativity

$$\begin{aligned} \boldsymbol{\sigma}_{\phi\vee\psi} = \boldsymbol{\sigma}_{\psi\vee\phi}, \qquad \boldsymbol{\sigma}_{\psi} \vee \boldsymbol{\sigma}_{\phi} &= \boldsymbol{\sigma}_{\phi} \vee \boldsymbol{\sigma}_{\psi}; \\ \boldsymbol{\sigma}_{\phi\wedge\psi} = \boldsymbol{\sigma}_{\psi\wedge\phi}, \qquad \boldsymbol{\sigma}_{\psi} \wedge \boldsymbol{\sigma}_{\phi} &= \boldsymbol{\sigma}_{\phi} \wedge \boldsymbol{\sigma}_{\psi} \end{aligned} \tag{81.11}$$

Distributivity

$$\begin{aligned} \boldsymbol{\sigma}_{\phi} \wedge (\boldsymbol{\sigma}_{\psi} \vee \boldsymbol{\sigma}_{\zeta}) = (\boldsymbol{\sigma}_{\phi} \vee \boldsymbol{\sigma}_{\psi}) \wedge (\boldsymbol{\sigma}_{\phi} \vee \boldsymbol{\sigma}_{\zeta}) \\ \boldsymbol{\sigma}_{\phi} \vee (\boldsymbol{\sigma}_{\psi} \wedge \boldsymbol{\sigma}_{\zeta}) = (\boldsymbol{\sigma}_{\phi} \vee \boldsymbol{\sigma}_{\psi}) \wedge (\boldsymbol{\sigma}_{\phi} \vee \boldsymbol{\sigma}_{\zeta}) \end{aligned} \tag{81.12}$$

Identity

$$\boldsymbol{\sigma}_{\phi} \wedge 0 = 0; \qquad \boldsymbol{\sigma}_{\phi} \wedge 1 = \boldsymbol{\sigma}_{\phi}; \qquad \boldsymbol{\sigma}_{\phi} \vee 0 = \boldsymbol{\sigma}_{\phi}; \qquad \boldsymbol{\sigma}_{\phi} \vee 1 = 1 \tag{81.13}$$

Idempotence

$$\boldsymbol{\sigma}_{\phi} \vee \boldsymbol{\sigma}_{\phi} = \boldsymbol{\sigma}_{\phi}; \qquad \boldsymbol{\sigma}_{\phi} \wedge \boldsymbol{\sigma}_{\phi} = \boldsymbol{\sigma}_{\phi} \tag{81.14}$$

Absorption

$$\boldsymbol{\sigma}_{\phi} \wedge (\boldsymbol{\sigma}_{\phi} \vee \boldsymbol{\sigma}_{\zeta}) = \boldsymbol{\sigma}_{\phi}; \qquad \boldsymbol{\sigma}_{\phi} \vee (\boldsymbol{\sigma}_{\phi} \wedge \boldsymbol{\sigma}_{\zeta}) = \boldsymbol{\sigma}_{\phi} \tag{81.15}$$

Non-monotone Laws

Complementation

$$\boldsymbol{\sigma}_{\phi} \vee \boldsymbol{\sigma}_{\neg\phi} = 1; \qquad \boldsymbol{\sigma}_{\phi} \wedge \boldsymbol{\sigma}_{\neg\phi} = 0. \tag{81.16}$$

De Morgan laws

$$\neg(\boldsymbol{\sigma}_{\phi} \wedge \boldsymbol{\sigma}_{\psi}) = \boldsymbol{\sigma}_{\neg\phi} \vee \boldsymbol{\sigma}_{\neg\psi}; \qquad \neg(\boldsymbol{\sigma}_{\phi} \vee \boldsymbol{\sigma}_{\psi}) = \boldsymbol{\sigma}_{\neg\phi} \wedge \boldsymbol{\sigma}_{\neg\psi}. \tag{81.17}$$

According to this approach, all Boolean algebraic laws are preserved in any valued realization (from the classical two-valued until the real-valued realizations), independently of the chosen generalized products.

Complementation or non-contradiction and excluded middle are also valid in the general case! The classical definition of the excluded middle and non-contradiction are correct only in the classical two-valued case. However, in the general case of the real-valued realization or in the Boolean consistent fuzzy logic, one proposition can simultaneously have both a property and its opposed property in such a way that the sum of their intensities is identical to 1. In the Boolean consistent fuzzy set theory, one element can have the analyzed property with some intensity and then it must have a complementary property with the complementary intensity so that the sum of their intensities is identical to 1. In the general case, non-contradiction means that there is nothing in common between the analyzed property and its complementary property and the excluded middle means that anything that is not contained in the analyzed property is contained in its complementary property. Actually, for an arbitrary property, the excluded middle and the non-contradiction principles uniquely define its complementary property and consequently these laws are fundamental and unavoidable for cognition in general.

This can be illustrated with the simple example of a glass of water. In the setting of the classical two-valued case, the glass can only be either full or empty. Being empty is the complement of being full and vice-versa. In the general case, a glass can simultaneously be partially full and then it is simultaneously empty with a complementary intensity, so that the sum of the intensities of “full” and “empty” is identical to 1. It is clear that, besides the fact that properties “empty” and “full” do not have anything in common, they simultaneously apply to the same glass.

A Boolean function of the analyzed finite or atomic Boolean algebra either contains relevant atoms or can be represented as a union of relevant atoms – disjunctive canonical form. A complementary Boolean function contains all the atoms that are not contained in the analyzed Boolean function – excluded middle –, and there is no atom that is common to the analyzed Boolean function and to its complementary function – non contradiction. In the classical case, only one atom has a value equal to 1 and all others equal to 0 and, consequently, if an analyzed Boolean function has a value 1 then its complementary function is equal to 0 or vice versa. In the general case of the gradation, all atoms can have non-negative values but their sum is identical to 1. Since the intersection of the analyzed Boolean function and its complement doesn’t have any atom, it is always identical to 0, and their union contains all atoms and, consequently it is always identical to 1.

Introducing intensity of the realization – gradation of Boolean variables and functions –,finite Boolean algebras are adequate for all real problems thanks to the descriptiveness of the gradations. Any classical theory based on a finite Boolean algebra using the fuzzy logic in the Boolean framework can be generalized immediately [6]. This is very important for many interesting applications which are logically much more complex, such as: AI, mathematical cognition, theory of prototypes in psychology, theory of general concepts, etc.

So, with real-valued realizations of Boolean algebras, it is immediately possible to generalize all classical results based on two-valued realizations of finite Boolean algebras, and besides Aristotle, in the Boolean framework there is enough space for Zadeh’s ideas as well. [5].

References

1. Zadeh, L.A.: From Circuit Theory to System Theory. Proceedings of the IRE 50, 856–865 (1962)
2. Zadeh, L.A.: Fuzzy Sets. Information and Control 8(3), 338–352 (1965)
3. Boole, G.: The Calculus of Logic. The Cambridge and Dublin Mathematical Journal 3, 183–198 (1848)
4. Radojevic, D.: New [0,1]-valued logic: A natural generalization of Boolean logic. Yugoslav Journal of Operational Research - YUJOR 10(2), 185–216 (2000)

5. Radojevic, D.: There is Enough Room for Zadeh's Ideas, Besides Aristotle's in a Boolean Frame. In: 2nd International Workshop on Soft Computing Applications (SOFA 2007), Oradea (2007)
6. Radojevic, D.: Interpolative Realization of Boolean Algebra as a Consistent Frame for Gradation and/or Fuzziness. In: Nikravesh, M., Kacprzyk, J., Zadeh, L.A. (eds.) Forging New Frontiers: Fuzzy Pioneers II. STUDFUZZ, vol. 218, pp. 295–317. Springer, Heidelberg (2008)

82

The Meeting with Fuzzy Mathematics as the Great Adventure of My Life

Elisabeth Rakus-Andersson

82.1 Introduction

It has passed over twenty five years since I encountered fuzzy mathematics for the first time but I can still feel this excitement, which I have experienced during that meeting with an unknown mathematical domain. This was different from other classical mathematical fields but I immediately felt that the concept of imprecision would be expected to have a fine future. Since that day in 1987, when I held in my hand the first available paper on the medical applications [9] of fuzzy relations it was obvious for me that I could devote my time to investigate the topic much deeper.

I made an acquaintance with the creator of fuzzy mathematics, Professor Lotfi Zadeh, in Budapest in 1999. I expected to be confronted with a self-confident and outstanding scientist who was not going to talk to unknown people. Instead I saw a very nice and modest man speaking to everybody in a very friendly manner, especially to new members of the fuzzy society. His behavior awoke in me sympathy and gratitude for his kindness and I wanted to meet him again.

The idea of editing a book, based on our memories and experiences concerning fuzzy mathematics, is really great. Let me thus tell you about my own scientific carrier in which fuzzy mathematics has played a dominant role.

82.2 The Promising Start with Applications of Fuzzy Systems

During the 80ties of the twentieth century I was employed as a consult of mathematics in the Medical Academy of Łódź in Poland. Mostly I used statistics to help medi-cal researchers to make proper conclusions assisting their clinical data samples. After some years I found this occupation to be mechanic and dull; therefore I wanted to learn something more, which could inspire me to do another kind of medical applica-tions. The problem of selecting the most probable diagnosis was promoted to be solved at that time. Accidentally, I came into contact with a mathematician who had just returned from an international conference. He gave me a paper proposing the solution of the diagnostic choice by means of a strange theory called fuzzy set theory. After reading this dissertation I had the impression that I should extend my knowledge about this new subject. It was not so easy since the access

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to international literature was rather poor. We could buy books [1], [3] written in Polish. Apart from these two basic encyclopedias some conference papers and publications from Fuzzy Sets and Systems were delivered to us by friendly colleagues. The mathematician who showed me the paper on medical applications, my younger colleague and I constituted a group of enthusiasts who wanted to know more about fuzzy set theory. We studied the papers available during our leisure to explain their contents to each other. In that way we were continually extending the number of new definitions concerning fuzzy sets [10], fuzzy relations [1], [3], fuzzy numbers [2] and other concepts to make the first application trials. In 1988 I managed to reveal the effects of a composition of fuzzy relations, adapted by me to the medical diagnosis purpose, during some seminars at the Medical Academy. The expectations to diagnose a patient in the controversial way proposed by fuzzy set theory were huge, thus the leading professor in parasitological sciences provided me with clinical data to check her primarily stated diagnoses. By means of fuzzy relations her diagnoses were confirmed in the very substantial percent. This made me proud of yielding the efficient solution of the diagnostic task to the medical staff. Diagnosis estimation together with evaluation of medicine level action was included in the main part of my doctoral dissertation in fuzzy sets, which was the first one composed in the field of fuzziness in my home city Łódź in 1991.

82.3 Further Developments of Applicable Fuzzy Set Theory

In spite of the growing interest in applications of fuzzy set theory in medicine in Po-land I had to interrupt my scientific investigations between 1993 and 1995 for the sake of personal reasons. I married a Swedish citizen and moved to Sweden. At the short time after my arrival to another country I got employments at Swedish universi-ties as a lecturer in mathematics. Mostly I taught the classical mathematical subjects to young students but I was still keen on studying the advances in fuzzy set theory. It was not easy to combine a lot of teaching with research. Nevertheless, I sent new papers to international conferences during the 90ties. The access to internet publica-tions was already essentially expanded that resulted in getting more information about the latest progresses. The number of my papers published by well known international sources grew rapidly. This fact allowed me obtaining the Associate Professor competence in 2001. In Sweden fuzzy set theory was not so very popular at the end of the twentieth century and I, as each pioneer, had to arrange seminars to explain the main ideas of fuzzy sets treated by some researchers as other kinds of probability distributions. I did not bother about these attitudes and I prepared new papers to solve the classification of internet protocols [5] and the choice of the most efficacious medicines made by fuzzy decision making [5]. I also found a space of verbal fuzzy numbers [4] and the algorithm of generating the least eigen fuzzy set of a fuzzy matrix [5]. At that time about 2004 my students noticed new applicable possibilities of fuzzy logic (they called fuzzy mathematics in this way). This interest inspired me in preparing the material to the course in fuzzy mathematics and its applications, which was held for the first time in 2005. The course gathered many adherents of the theory among Master of Science students and doctoral students.

To keep the contact with the fuzzy society members I participated in each international conference of the substantial importance. I met plenty of nice and engaged people who became my friends. We have kept warm contacts during many years by cooperating in the arrangement of special sessions and common invited lectures. Even the exchange of important information about new conferences and other events let me be updated in the fuzzy subject in my new country. I remember the very kind and friendly atmosphere among the conference participants when attending lecture sessions or organizing common excursions to sightsee new places (see Figure 82.1).

Each international meeting furnished me with a new power to continue my research and encouraged me in my efforts to fight for spreading the achievements of fuzzy set theory among my university colleagues and students. I was supported in my engagement by funds granted by the Royal Swedish Academy of Sciences.

82.4 The Latest Progress

At last the success came in 2007. After collecting my research results in the book [5] I could successfully apply for the professor position. This gave me more freedom in planning my research. Together with doctoral students I developed my own version of fuzzy games to prove them in medicine and technical sciences [7]. The new concept of fuzzy probability of continuous events was found by me in 2010 [8]. From 2008 up to now I have been granted by the county hospital in Blekinge, Sweden, for testing fuzzy techniques to find new approaches to the estimation of survival length and the surgery decision. I have proved control systems and different hybrids of fuzzy systems combined with neural networks, evolutionary algorithms and immune methods to give answers to questions posed by physicians when setting the clinical data as input data. A large number of publications have followed the achieved results (see all publications on http://www.bth.se/tek/amn/era.nsf).

Together with other prominent professors I edited a book in decision making systems in 2009 [6].

Another course in computational intelligence and fuzzy systems was established at my university by me in 2010.

Even if I have been rewarded with prestigious prizes for my scientific progress I am mostly satisfied with giving a chance to young people to do research in fuzzy set theory. Many students have chosen fuzzy mathematics topics to discuss their genuine applications in Master of Science and doctoral theses. Together with the students I am still presenting the results of current research at international conferences (see Figure 82.2).

Fig. 82.1. The ICAICS 2002 Conference in Zakopane, Poland. From the left: Janusz Kacprzyk, Lotfi Zadeh, Elisabeth Rakus-Andersson, Eulalia Szmidt.

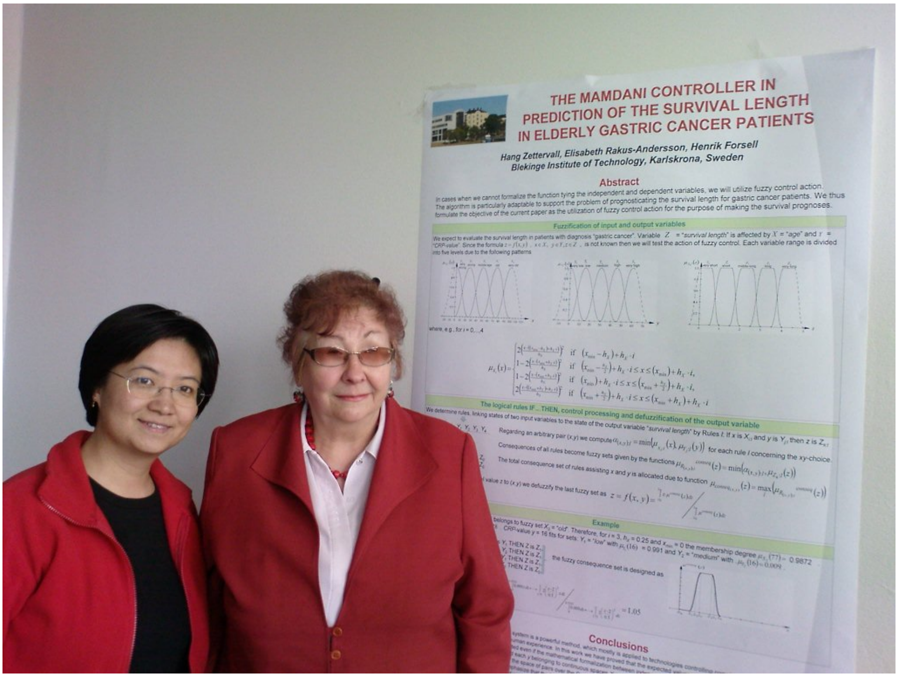

Fig. 82.2. The presentation of results at The Bioinformatics 2011 Conference, Rome, Italy. From the left: Hang Zettervall, Elisabeth Rakus-Andersson.

82.5 Conclusions

After going through the events that have been important for the development of my achievement range I should come to the main conclusion. Namely, I could not have done such broad and successful research without meeting fuzzy set theory. It was my life chance to engage myself in these individual studies to learn more and more from accessible sources. Since I am fond of applications then I can help other researchers to find solutions of problems unsolvable by methods of classical mathematics. The representatives of new generations help me in this effort, which means that someone will continue my work to create more models in fuzzy set theory.

Who provided me and others with such essential ability to expand Professor Zadeh's concepts? Professor Zadeh himself stimulated us to be involved in his new ideas which he, as a very skilful teacher, portrayed to us during conference plenary speeches. I can only express my highest respect and gratitude for this researcher who gave us fuzzy set theory.

References

1. Czogała, E., Pedrycz, W.: Elements and Methods of Fuzzy Set Theory. PWN, Warsaw (1985) (in Polish)
2. Dubois, D., Prade, H.: Operations on Fuzzy Numbers. International Journal of Systems Science 9(6), 613–626 (1978)
3. Kacprzyk, J.: Fuzzy Sets in System Analysis. PWN, Warsaw (1986) (in Polish)
4. Rakus-Andersson, E.: Factor Analysis with Qualitative Factors and Yager's Probability in the Lattice of Verbal Fuzzy Numbers. Issues in Soft Computing Theory and Applications, pp. 241–260. EXIT – the Publishing House of Polish Academy of Sciences (2005)
5. Rakus-Andersson, E.: Fuzzy and Rough Techniques in Medical Diagnosis and Medication. Springer, Heidelberg (2007)
6. Rakus-Andersson, E., Yager, R.R., Jain, L. (eds.): Recent Advances in Intelligent Decision Support System. Springer, Heidelberg (2009)
7. Rakus-Andersson, E., Salomonsson, M., Zettervall, H.: Prioritisation of Weighted Strategies in Multiplayer Games with Fuzzy Entries of the Payoff Matrix. International Journal of General Systems 39(3), 291–304 (2010)
8. Rakus-Andersson, E.: Fuzzy Probabilities of Biological Continuous Fuzzy Events. International Journal of Computational Intelligence Studies 1(2), 191–204 (2010)
9. Sanchez, E.: Resolution of Eigen Fuzzy Set Equations. Fuzzy Sets and Systems 1, 69–74 (1978)
10. Zadeh, L.A.: Fuzzy Sets. Information and Control 8, 338–353 (1965)

83

Uncertainty and Knowledge Repositories in the Web of Data

Marek Z. Reformat

83.1 Uncertainty and Web Information

Millions of people relay on the Internet to do their work, to discover new things, to learn what is happening around them, or to find entertainment for themselves.

The utilization of the web should provoke such questions as: how much can we relay on the web to discover new things? how many sources of data are trustworthy? how much imprecision and incorrectness is out there? All this becomes very relevant when we take into consideration the fact that the users' involvement in creating and shaping the web is growing. Tweets, posts, blogs, e-mails, pieces of text, and documents are examples of the users' direct, spontaneous, and uncontrolled contributions.

There is no doubt that uncertainty is an integral component of information and knowledge. As stated by Lotfi Zadeh, many concepts we deal with are without precise definitions, or with unknown facts, missing or inaccurate data [13]. The Internet is not different – the users should be aware that imprecision and ambiguity are present on the web. The web is a large uncensored network and anyone can contribute to it by providing truthful or false information. In general, information acquired from websites is equipped with some degree of uncertainty. We can talk about two contributors to uncertainty: trust in data sources, and quality of information.

Trust in Data Sources. Not every data source on the web is equally trustworthy. There are a number of research activities that are focused on assigning trust values to different sources, as well as methods dedicated to aggregation and inference of trust values. The following trust strategies have been proposed to rationale about trust: optimistic, pessimistic, centralized, trust investigation, and trust transitivity [6]. Each of these approaches deals with uncertainty and tries to discover aspects of the environment that are relevant to reduce uncertainty.

Quality of Information. The quality of information relates to the amount of missing or ambiguous information. The quality-based information uncertainty can be divided into three categories [5, p. 1] *non-specificity (imprecision)* – manifested when two or more pieces of information are left unspecified, this may be the result of generalization, simplification, imprecision, or simply time constraints imposed on information collecting processes; 2) *fuzziness (vagueness)* – characterized by the lack of definite

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or sharp distinction among pieces of information and may result from vagueness or any variety of indecisiveness. In some cases, especially for linguistic-based knowledge representation, terms and facts can be ambiguous due to differences in meaning as perceived by authors of the information; and 3) *strife or discord* – characterized by disagreement in a selection process among pieces of information, this may happen due to dissonance, incongruity, discrepancy, and conflict.

This indicate not only overwhelming existence of uncertainty, but also points to positive influence the uncertainty could and should play in building knowledge. In this context, the importance of uncertainty can be expressed via the following facts:

- **uncertainty triggers learning:** a state of ambiguity forces an individual to search for more information and facts to resolve the vagueness;
- **uncertainty enables adaptability:** a constant state of not being sure means that an individual has to be prepared for a possible change of his/her opinion, in such a case it is easier to accept a change;
- **uncertainty prevents misjudgment:** processes of induction and deduction of new facts should have the ability to deal with situations which are not clearly true or false, it is not desirable to simplify everything to those two values;
- **uncertainty leads to more accurate models of reality:** the real world is not just black and white, it is full of gray areas, i.e., vagueness and ambiguity – any models real phenomena should be able to accommodate uncertainty.

This short paper provides a brief description of one of possible views on uncertainty as a key component influencing searching for, processing, analysis, and assimilation of web information for the purpose of building users' personal knowledge repositories. It gives a glimpse on a new web paradigm called *Web of Data*, and postulates that uncertainty and methods of handling it, developed around fuzziness and possibility theory [11] [12], will thrive in creating knowledge repositories in the environment of the *Web of Data*.

83.2 The Web of Data

An ultimate contribution of the Semantic Web [4] is utilization of ontology as the knowledge representation form. Resource Description Framework (RDF) [3] is introduced as an underlying framework for using ontology in the web environment. The RDF data model treats each piece of information as a triple: subject-property-object [3].

In the last few years, the RDF as a data representation format has become a very popular way of representing data on the web [7]. Over time, the term Linked Data (LD), or more general the *Web of Data*, has been used to describe the network of data sources using RDF triples as information representation [1]. The power of this new web paradigm, in contrary to hypertext web, is that entities from different sources and locations are linked to other related entities on the web. This enables one to view the web as a single global data space [1]. In other words, hypertext web connects documents in a naive way – links point to documents. However, in the *Web of Data* single information items are connected – links point to other pieces of information stored at different physical locations.

As stated, pieces of information are expressed as RDFs, i.e., triples: subject, property, and object, where each one of these entities is represented by a single URI. This means a process of finding a specific piece of information on the *Web of Data* is facilitated with the help of interpretable URIs, and there are no restrictions regarding their locations.

83.3 Uncertainty as Necessary Component of Learning and Exploration

In the current web, documents or web pages are treated as units of information. This is very different in the *Web of Data*. The granularity of information units is much smaller – the smallest piece of information is a triple. Multiple triples are "connected" via mutual entities, i.e., a single entity can be a part of many triples. All triples constitute a structure built from stars of triples: each star is a bunch of triples with a common entity, and describing this entity. In other words, all triples are linked together, and the information in the *Web of Data* is composed of interconnected stars.

The fact that we are able to deal with individual triples creates an opportunity to develop different, and hopefully better, ways for searching and collecting information. The most important benefit of this information representation is the ability to closely "relate" single pieces of information with their levels of uncertainty, and use this uncertainty as a key factor influencing processes of looking for new information, and assimilating it.

The idea presented here is based on the application of multiple agents continuously crawling the web and looking for new information on the user's behalf. As a result, a user's personal knowledge repository is built based on the information triples found by the agents. The repository reflects the user's state of knowledge. In order to truly represent the user's level of understanding of collected information, the agents have to be equipped with methods suitable for:

- determining a degree of novelty of information found on the web when compared with the information already known; done via application of similarity evaluation methods based on possibility theory [2];
- integrating a new information with the user's personal knowledge repository, and estimating confidence in the user's information after the integration; done via fuzzy- and possibility-based aggregation mechanisms applied to stars of triples;
- assessing levels of compatibility between information in the user's repository and the web contents; done with mechanisms of fuzziness and possibility theory used for comparison of stars of triples representing descriptions of entities and frequencies of their occurrence.

The personal knowledge repository, created and maintained in such a way, contains the information and facts experienced by the user – her agents – on the web, together with measures of its quality. We can assert that the respiratory reflects the user's perception of trust and her confidence in the information found on the web.

For knowledge building processes, uncertainty can be also linked with novelty of information – new information introduces and modifies uncertainty. In this context, we can distinguish three scenarios how different processes of assimilating web information contribute to the user's knowledge repository and how they influence levels of uncertainty.

- Updating existing knowledge – change of confidence in facts already existing in the user's repository; uncertainty associated with correctness of known information is updated; pieces of information used for this update have to be evaluated and their uncertainty determined.
- Modifying existing knowledge – change of facts existing in the repository; the acceptance of changes requires knowledge of confidence in assimilated information; a range of modifications depend on estimated levels of uncertainty.
- Increasing existing knowledge – addition of new facts to the repository; assimilation procedures capable of handling uncertainty; for example, regulations are required to determine up to what degree of uncertainty new pieces of information can be accepted.

All this indicates that we can see uncertainty as a driving force behind finding information on the *Web of Data*, and creating knowledge based on this information. In such a context, knowledge is not just a collection of information pieces (triples), but an integrated network of such pieces and levels of confidence in their correctness. Construction of such a repository would not be possible without recognizing and a proper handling of uncertainty associated with individual triples.

The capability to "recognize" uncertainty at such low levels of granularity has also a significant impact on the way a new information is searched for. For example, once we determine the levels of truthfulness associated with individual information triples, we can initiate a verification process that involves sending agents to search for pieces of evidence proving or disproving facts contained in the repository.

Once the repository is built, it can constitute a base for information analysis and learning processes. The structure of the repository – a network of interconnected triples – allows us to treat stars of triples as definitions of entities. This means that definitions of entities, and concepts represented by these entities, can be expressed as fuzzy sets, like in [10]. Different fuzzy sets operations can be used for comparing these definitions, and constructing fuzzy relations based on them. Further, we can use operations of fuzzy relations to infer about their characteristics.

The interconnected entities and their descriptions can be seen as a environment suitable for application of participatory learning mechanisms [8] [9]. If we treat information triples as propositions and the uncertainty levels associated with as weights, the approximate reasoning inference mechanism can be applied to generate new facts to ensure consistency of the repository, and its compatibility with newly found information.

83.4 Opportunity and Benefits

The usefulness of the concept of fuzziness and uncertainty in the process of creating personal knowledge repositories based on pieces of information found on the *Web of Data* is obvious.

The utilization of fuzzy sets allows for bringing mechanisms of fuzzy set operations. For example, they can be used to: 1) construct fuzzy sets based on information triples found on the *Web of Data* that represent different concepts; 2) determine degrees of belonging of information pieces to constructed fuzzy sets (concepts); 3) determine relations between multiple concepts; and 4) make projections at different dimensions to investigate different views of concepts. In other words, thanks to the application of fuzzy-based technologies we will be able to better explore multi-value and multidimensional nature of information collected on the web.

It is anticipated that the results of applications of techniques and methods related to fuzziness and possibility theory will allow for building human-centric systems able to harvest knowledge from the web and analyze it, and to decrease our burden of dealing with unreliable and faulted information.

Fig. 83.1. Fltr: Asli Celikyilmaz, Lotfi Zadeh, Farley Nobre and Marek Reformat at the 2012 Annual Meeting of the North American Fuzzy Information Processing Society, NAFIPS 2012, Berkeley, CA, August 6-8, 2012

References

1. Bizer, C., Heath, T., Berners-Lee, T.: Linked Data – the Story so Far. International Journal on Semantic Web and Information Systems 4, 1–22 (2009)
2. Hossein-Zadeh, P.D., Marek, Z.: Reformat: Context-aware Similarity Assessment within Semantic Space Formed in Linked Data. Journal of Ambient Intelligence and Humanized Computing (to appear)
3. Lassila, O., Swick, R.R.: Resource Description Framework (RDF) Model and Syntax Specification, W3C Tech. Reports and Publications (1999), http://www.w3.org/TR/1999/REC-rdf-syntax-19990222
4. Berners-Lee, T., Hendler, J., Lassila, O.: The Semantic Web. Scientific American 284, 34–43 (2001)
5. Mccauley-Bell, P.: Intelligent Agent Characterization and Uncertainty Management with Fuzzy Set Theory: A Tool to Support Early Supplier Integration. Journal of Intelligent Manufacturing 10, 135–147 (1999)
6. O'Hara, K., Alani, H., Kalfoglou, Y., Shadbolt, N.: Trust Strategies for the Semantic Web. In: Proceedings of the Trust, Security and Reputation Workshop at the ISWC 2004, pp. 78–85, Hiroshima, Japan (2004)
7. Shadbolt, N., Hall, W., Berners-Lee, T.: The Semantic Web Revisited. IEEE Intelligent Systems 21, 96–101 (2006)
8. Yager, R.R.: Model for Participatory Learning. IEEE Transactions on Systems, Man and Cybernetics 20, 1229–1234 (1990)
9. Yager, R.R.: Participatory Learning of Propositional Knowledge. IEEE Transactions on Fuzzy Systems 20, 715–727 (2012)
10. Yager, R.R., Marek, Z.: Reformat: Looking for Like-minded Individuals in Social Networks Using Tagging and Fuzzy Sets. IEEE Transactions on Fuzzy Systems (to appear)
11. Zadeh, L.A.: Fuzzy sets. Information and Control 8, 338–353 (1965)
12. Zadeh, L.A.: Fuzzy Sets as a Basis for a Theory of Possibility. Fuzzy Sets and Systems 1, 3–28 (1978)
13. Zadeh, L.A.: Toward a Perception-based Theory of Probabilistic Reasoning with Imprecise Probabilities. Journal of Statistical Planning and Inference 105, 233–264 (2002)

The Influence of Lotfi Zadeh on *Informatik I* in Dortmund

Bernd Reusch

> "Around 1970 I met Lotfi for the first time — young, innocent, and as completely unknown as I was. When I reached his office, there was a note for me, 'Sorry, forgot my birthday. Please meet me at (some Bay-Shore restaurant)'. Meeting him at his birthday party I was even more impressed by him than by the amount of shrimps I was able to swallow. He has always been that way, open to new ideas and people, helpful in the extreme, and a human in the best sense of the word. Thank you Lotfi, and thank you Fay for maintaining him so well."

I wrote this for publication in [1]! Very recently, I have sent Lotfi a fax related to the BISC-discussion on "causality". Here is the central part of it: "From a philosophical point of view we cannot be sure, that something like 'causality' or even 'natural law' exists at all (Kant, Kritik der reinen Vernunft). All we have are observations or better statements on observations. In some case we have 'observation A was made earlier than observation B".

Of course, over thousands of years, we have a huge amount of observations (but at any time "only" finitely many). It seems human to wish to "organize" this set of observations (better: subsets) more or less elegantly.

The ancient Babylonians recorded their observations concerning sun, moon and stars and discovered regularities in their "time-series". They were able to predict situations concerning sun, moon and stars more accurately than the followers of Greek science for more than 2000 years! With no assumption of natural laws.

What is "correct": do we live in a helio-centric or geo-centric world? In both theories we are able to describe our world sufficiently precise. We choose the helio-centric version only because it is simpler (Okham's razor). "Natural laws" may change: recently some scientists claim to have discovered particles that move quicker then light! This will have impact on basic natural laws (although I do not see how they solve the problem of "simultaneousness" which is basic to Einstein's "Gedanken-experiment"). What I want to say, that 'causality', 'natural laws' etc. are not true or false. They are only more or less useful."

It is my "strong conjecture" that Lotfi did not like this. He is an engineer! And a very very good one as he proved in the pre-fuzzy-age. Please forgive me.

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Well, what happened between 1970 and 2012? I left GMD (Gesellschaft für Mathematik und Datenverarbeitung) in 1972 to co-found the Department of Computer Science in Dortmund and take responsibility for Chair Informatik I.

This chair was (and still is) one of two theory-oriented groups in the department. We worked on Boolean Algebras, Finite state machines, Petri-nets, many-valued logics etc. But from the beginning we were engaged in projects to apply this knowledge to CAD of Micro-electronics together with institutions all over Europe and also with Industry.

At this time, Computer Science was not well established in Germany. We had to work hard to find our own definition, did quite well and still have some aspects in our curriculum that are unique (at least in Germany).

Therefore I forgot almost completely what I had learned about fuzzy-logic, until for some forgotten reason, Prof. Gisbert Dittrich introduced a course on this topic in 1990 (or 1989?). Dittrich was a member of Chair I as well as Prof. Claudio Moraga, one of the editors of this volume. Later, after his retirement in East Berlin Prof. Helmut Thiele also joined the group. In the best tradition of the chair, this group worked successful in theory, but also in application of fuzzy logic. These activities are well documented in [2]. Emphasis there is layed on theory (see references from [2]), but the authors also state:

"Scientific activities of the Dortmund researchers in the fuzzy logic area were realized in the framework of several projects and applied in cooperation with some industry units. We mention only some of these application activities. They can be roughly splitted in to the following areas:

1. **Expert Systems Using Fuzzy Logic Rules**
   (in cooperation with the mechanical engineering departments of the Universities of Dortmund and Bochum, as well as with the chemical engineering department of the University of Dortmund).
   - Applications in the design of composite materials
   - Special composites made of metal and ceramics

2. **Optimization of Fuzzy Expert Systems**
   - Modeling of 1D and2D functions using fuzzy controllers
   - mprovement of the rule set
   - Evolutionary concepts for the improvement of the performance

3. **Fuzzy Logic in Industrial Image Processing**
   (in cooperation with industry partners Mannesmann, Demag)
   - Development of operators for image processing tasks
   - Evolutionary optimization of digital filter kernels
   - Development of a new way of describing colors: Fuzzy color processing
   - Estimation of 3D features using stereo camera systems

4. **Evolution Strategies for the Optimization of Fuzzy Systems**
   (in cooperation with Mannesmann, Degussa)
   - Optimization of the fuzzy rules
   - Optimization of membership functions
   - Applications in industry

5. **Fuzzy Logic and Robot Soccer**
   - Embedded in the FIRA robot systems
   - Development of robots
   - Fuzzy logic for the control of the robots and the estimation of the current situation on the playfield

6. **Fuzzy Logic and Medicine**
   (in cooperation with the University of Essen and University of Witten/Herdecke and University of Bochum)
   - Fuzzy logic based descriptions of human tissues
   - Fuzzy image segmentation
   - Fuzzy based diagnosis"

After Dittrich's course we came up with a Conference Series: The Dortmund Fuzzy Days. This was just 21 years after I met Lotfi for the first time ! There have been 9 Dortmund Fuzzy Days until 2006, the year of my retirement. The next quote is taken from my introduction 2006.

> "For the 9th time since 1991 we invite researchers to participate in the Dortmund Fuzzy-Days. I am very glad that our conference has established itself as an international forum for the discussion of new results in the field of Computational Intelligence. Again all papers had to undergo a thorough review: each one was judged by five referees to guarantee a solid quality of the programme.
> From the beginning of the fuzzy-Days on Lotfi A. Zadeh felt associated with the conference. I would like to express my gratitude for his encouragement and support and I am particularly glad that he once again delivers a keynote speech. Much to my pleasure Ewa Orlowska, Radko Mesiar together with Vilém Novák, Ernesto Damiani together with Tharam Dillon and Nik Kasabov have also agreed to present new results of their work as keynote speakers.
> Many thanks go to my friends Janusz Kacprzyk and Enric Trillas who together with Lotfi Zadeh again served as honorary chairmen."

I may add that invited sessions were presented by Mario Fedrizzi, Krassimir T. Atanassow, Ernesto Damiani, Enric Trillas, Igor Aizenberg, Arkady Borisov, Janos,

Fig. 84.1. From left to right: Prof. Dr. Konrad Zuse (First Dr. h. c. at our department) Prof. Dr. Bernd Reusch (Chair Computer Science I), Prof. Dr. Lotfi A. Zadeh and Dr. Helmut Kohls (at that time CEO of Sparkasse Dortmund) during the evening banquet in the festival room of the Sparkasse Dortmund

Fodor, Irina Perfilieva, Vilém Novák. Together with the people serving at the programme committee this almost reads like "who-is-who in fuzzy logic".

I confess to be proud of this series of conferences. All proceedings, except the first one, were published by Springer. And Lotfi is the only person (except locals like me) who attended all 9 conferences!

Usually, my friends attending were invited to our house one evening. These meetings were very nice on the personal level, but also very stimulating on a scientific level. Lotfi obviously always played the central role. I want to stress, that these conference as well as the successful working group in Dortmund would never have happened without the support and stimulating interest of Lotfi. Thank you for all you did to us!

Lotfi's commitment to Dortmund was officially recognized in 1993 when we made him one of still only three "Dr. h. c." of our department. H. c. really means "honoris causis", not "humoris causis" as in some other cases.

P.S.: This text was typed by Mrs. Lippe, my former secretary. "Dear Ulrike" as Lotfi usually addresses her.

Fig. 84.2. Prof. Dr. Lotfi A. Zadeh and Prof. Dr. P. Marwedel (at that time Chairman of the Computer Science Department at the University of Dortmund) during the award

References

1. Zadeh, F.: My Life and Travels with the Father of Fuzzy Logic. TSI Press, Albuquerque (1998)
2. Mesiar, R., Novák, V.: Fifteen Years of Fuzzy Logic in Dortmund. In: [3]
3. Reusch, B. (ed.): Computational Intelligence, Theory and Applications, 9th Fuzzy Days in Dortmund. Advances in Soft Computing. Springer, Berlin (2006)
4. Reusch, B., Dascalu, D. (eds.): Real World Applications of Intelligent Technologies National Institute for Research and Development in Microelectronics (2000)
5. Reusch, B. (ed.): Fuzzy Days 2001. LNCS, vol. 2206. Springer, Heidelberg (2001)

85

A Tribute to Lotfi Zadeh with Personal Recollections

John T. (Terry) Rickard, Janet Aisbett, and Greg Gibbon

Isaac Newton once wrote to Robert Hooke that (with slight paraphrasing) "If we have seen further, it is by standing on the shoulders of Giants." Life has an interesting way of presenting opportunities for collaboration between us lesser mortals who are catalyzed by such giants. Certainly that has been our experience in our own collaborations over the past several years, with the "giant" in this case being Professor Lotfi Zadeh. Lotfi's pervasive inquiries into the nature of human reasoning, and his assemblage of a sophisticated mathematical apparatus enabling humble machines to mimic this marvel of cognition, however imperfectly, stand as one of the towering intellectual accomplishments of the past century. In this tribute, we are pleased to recount some of our experiences that have benefitted from a larger view of the horizon provided by standing upon Lotfi's broad shoulders.

Terry was introduced to fuzzy logic early on as a graduate student in the early '70s by attending a seminar given by Lotfi at UC San Diego. At this time, the elegant and beautiful theories of probability were the reigning paradigm for dealing with uncertainties of all types. The fundamental notion of the impreciseness of knowledge, as contrasted to probabilistic uncertainty, had yet to gain much of a foothold. It was only later, when Terry began grappling with information processing problems above and beyond classical "signal plus noise" that the true power of fuzzy logic became apparent. This led him to the adoption of a fuzzy perspective on the world of engineering that has informed his approach to many real-world problems to this day.

In 1999 Professor Peter Gardenförs, a cognitive scientist from the Lund University Philosophy Department, published a book called *Conceptual Spaces: The Geometry of Thought*. He represented concepts in terms of properties defined on geometric domains, and he stressed the importance of similarity in reasoning. The book roamed over cognitive science, linguistics and psychology but did not present a sound mathematical formulation on which to base an implementation.

Terry was by that time a Senior Fellow for Lockheed Martin, concerned with data fusion applications, and had been working with Ron Yager on fuzzy graph similarity measures (*IEEE Transactions on Fuzzy Systems*, vol. 15, 2007). Gardenförs' work interested him for its emphasis on similarity and for its acknowledgement that not only individual properties but their co-occurrence could distinguish a concept.

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Terry therefore represented concepts as matrices of fuzzy properties and their co-occurrences, and recast observations as pseudo-concepts to enable their comparison in classification tasks.

Meanwhile, Janet Aisbett and Greg Gibbon, information technology academics with pure mathematics backgrounds, had independently begun formalising the theory of conceptual spaces while on sabbatical with Peter Gardenförs. Their formalism modelled concepts and properties as functions on metric spaces, and provided a process model for tasks such as classification. Three years later Janet and Greg spent some months with a cognitive science group at the University of Indiana in which they reformulated their work in terms of the basic units of cognition: memory, working memory, attention and probes (the input) (*Cognitive Systems Research*, vol. 6, 2005). In this representation, memories were functions from a bounded subspace of the plane into the unit interval – i.e. fuzzy sets. A neat aspect of the theory was that concepts and properties had the same form.

Fig. 85.1. Concepts and Similarity – Terry at right (photos from the Bahamas and Papua New Guinea)

Through a serendipitous coincidence, Terry came across Janet and Greg's formalism of conceptual spaces as metric spaces, and after making contact via email they began collaborating on a blending of their theories. Properties and concepts were defined as fuzzy sets, and similarity was defined using subsethood (*Information Sciences*, vol. 177, 2007). Working with real world data soon motivated the extension of the theory to fuzzy concepts, observations and properties. This opened up some of the intricacies of type-2 and higher order modelling to the three researchers, including computations of type-2 and higher order subsethood, unions and intersections. They investigated type-2 fuzzy sets in terms of Computing with Words and in terms of classification of fuzzy observations. They showed that any type-2 fuzzy set can be constructed using what Bellman and Zadeh called fuzzy compatibility of a reference (an observation) with a linguistic value (a class) (*Fuzzy Sets and Systems*, vol. 163, 2010).

The researchers next turned to the aggregation functions used to draw together a set of observations into a complex ranking or categorisation task. Following work of Jozo Dujmović, they investigated fuzzy weighted power means. Terry applied this in a new online service known as Discovery Investing Scoreboard (DIS) (www.discoveryboard.com), which represents the first commercial application of Computing With Words. The problem addressed by this system is the lack of standardized means for investment evaluation of companies that have discovered resource assets or technologies having large future potential, but which lack the revenues and/or trading history to enable the traditional forms of fundamental or technical analysis used by the financial community. DIS uses type-2 fuzzy scoring of the 10 factors illustrated in the figure below, along with their importance and designation as mandatory or merely desirable, to compile a composite score for a company on a 0-10 scale using weighted power means and conjunctive partial absorption operators. Apart from this online service, Terry has also applied type-2 fuzzy set representations and manipulations in a wide range of consulting work, from mining to finance (see Figure 85.2).

On the theoretical side, Janet and Terry are looking at other forms of aggregation. Recently, they have derived new classes of aggregation operators based upon the Tsallis q-exponential function, which exhibit thresholding behaviors mimicking an important trait of human reasoning. As well, they believe type-2 fuzzy sets have an interesting role in cognitive modelling, particularly in representing working memory (material directly accessible to cognitive processes).

It thus goes without saying that we owe a great debt to Lofti Zadeh for being the giant on whose shoulders we have stood. In our view, his most important and inspired contribution is the Extension Principle, which enables the propagation of fuzzy membership values through arbitrarily complex mathematical operations. Perhaps his only legacy with which we've sometimes struggled has been the occasionally cumbersome fuzzy set notation, which we believe raises an unnecessary barrier between fuzzy practitioners and a broader scientific community. In an effort to lower this barrier, Terry and Janet published a translation of this notation into the more concise and widespread language of functions and spaces (*IEEE Transactions on Fuzzy Systems*, vol. 18, 2010). However, this by no means detracts from the sheer genius of Lotfi's creation. He embodies the most exceptional qualities of a scholar and philosopher, and we shall always be in his debt. His diminutive physical shoulders belie his broad and powerful intellectual shoulders, upon which we and so many of his colleagues stand.

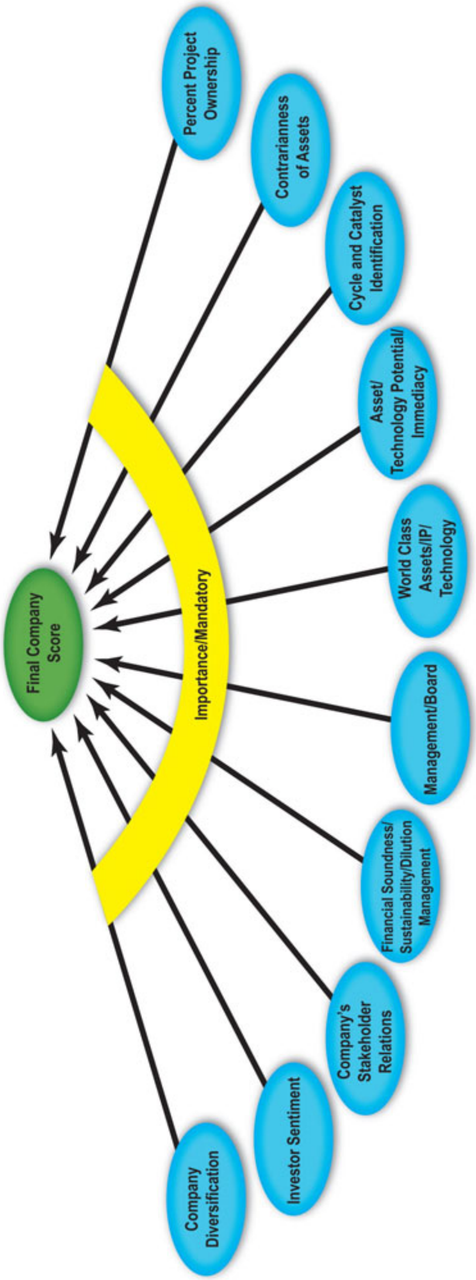

Fig. 85.2. Fuzzy evaluation factors of the Discovery Investing Scoreboard

Fig. 85.3. Aggregation – Janet above, Terry below with Lotfi and Ron Yager (photos from the memorable Zadeh tribute dinner at the World Conference on Soft Computing, 2011)

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Neither Concepts Nor Lotfi Zadeh are Fuzzy Sets

Eleanor Rosch

In the 1960s and '70s at the University of California, Berkeley, two people at opposite ends of the campus were doing work that challenged applications of classical set theory. In the Department of Computer Science, Lotfi Zadeh noticed that phenomena in the world do not necessarily come in the all-or-none packages required by classical sets (for example, a man may be tall to a degree), and he set out to devise a mathematical calculus by which matters of degree could be encompassed within a variant of classical logic. He gave it the name *fuzzy logic*. In the Department of Psychology, Eleanor Rosch (who happens to be myself) was doing empirical psychological research on the nature of concepts and categorization, matters that, from the time of the Greeks, had been assumed to have the basic form of classical sets. The prevailing belief was that humans categorized by identifying necessary and sufficient conditions for an item to belong to a category; once such criteria were met, all items that belonged to the category were equivalent with respect to membership; and the meaning of conceptual combinations could be explained by the operations of classical logic. My research indicated that nothing about this model was true either for the representation, processing or use of natural language concepts and categorizations. One of the ways in which people differ from the model is that they see category membership as a matter of degree (e.g. *apple* is judged a better *fruit* than *plum*, *blueberry*, etc.), a finding that is psychologically important because degree of membership predicts the other psychological operations on categories. It is also a finding that might, on the surface, appear to unite this research with fuzzy logic.

The work of Zadeh and Rosch developed independently; in fact, both were unaware of each other's existence until the advent of an interdisciplinary cognitive science program. Even then, despite developing a friendship, neither of us viewed the other's work as a basis for collaborative thought, and the two fields continued to develop independently. By now, fuzzy logic is a multifaceted international area of mathematics with many technological, but not psychological, applications. And the study of concepts and categorization has become a thriving field in psychology and cognitive science that contains, among other types of research, many rival mathematical models, but none of them explicitly based on fuzzy logic. Recently, however, a movement has arisen to seriously consider concepts in the context of fuzzy logic [1]. With a gentle bow to Lotfi Zadeh, I would like to indicate some of the issues that arise with respect to the possible rapprochement between these two very different fields of study. (For a more detailed and inclusive exposition of these matters, see [10].)

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Are concepts logically fuzzy? To answer "yes," requires two conditions:

1. That degree of membership in conceptual categories be numerically and psychologically meaningful, and
2. That it be possible to perform deductive operations on those degrees of membership – that is, once one has defined the appropriate form of the input and of operations (such as intersection or union) on that input, the conclusion should follow without having to introduce extra material from knowledge about the world into each case.

In short, concepts need to be fuzzy sets on which one can perform the mathematical operations of fuzzy logic. Are they?

After close to 40 years of research (my own and that of many others), the first condition seems clearly to be met. Judgments of goodness of category membership are readily scalable, and those scales correlate with all the major dependent variables used in psychological experiments: speed of processing, ease and speed of learning, expectation, mental representation, associative strength, probability judgments, inference, judgments of conceptual similarity and distance, and a host of language measures. Of course, if one explicitly asks for formal definitions, people will struggle to give the necessary and sufficient criteria that constitute a classical definition such as we have in dictionaries, but this is not a problem for fuzzy set theory since it includes classical sets as crisp sets.

What of the second condition: can the deductive operations of fuzzy logic be applied to concepts? Here is where we run into trouble. Concepts don't occur in isolation but in a context that includes both the circumstances in which the conceiver finds himself and everything in his knowledge base: his internal dictionary, encyclopedia, sensory knowledge, repertoire of habits and skills, emotions, beliefs, autobiographical memories, and everything else comprising the vast pastiche of the human mind. Given all of this, can we lay out rules for logical operators by which the meaning and/or goodness of example rating of a conceptual combination can be deduced from the meaning etc. of the two or more concepts being combined? Here are some of the difficulties that arise (note that most of the research has been done on the operation of *intersection*):

1. *Even classical operations don't work with conceptual intersection.* If you have classically defined groups of material objects, say balls and blocks, each of which is red or green, then the set of red balls will be unproblematically the logical intersection of balls and red things. But now let's assume that all of the concepts in the following example are classical – i.e. that each one in itself has necessary and sufficient criteria for membership, its boundaries are clear cut, and all of its members are equally good with respect to membership. Is there anything in the meaning of the concepts themselves, each taken separately, that will tell us that: "corporate stationery is *used* by the corporation and has the corporate logo on it;

a corporate account is an account that is *charged* to the corporation; a corporate car is *owned by* a corporation and used for business travel; a corporate building is *where* the business is carried out; a corporate lawyer is one who *works for* a corporation; and a corporate donor is a donor who is a corporation [7, p. 450]." It is not the fuzziness of these concepts that make it impossible for us to know the meaning of the combinations from each word alone; it is that we have to know a great deal more than just each word.

2. *Emergent attributes: World knowledge of many sorts enters ad hoc*. Emergent attributes are those in which a conjoined concept has attributes possessed by neither of the concepts separately, and yet these are what determine meaning and goodness of example. Here are a few examples (you can think of what a person needs to know for each to be the case): The attribute *long* is neither a property of *peas* nor of *unshelled things*, but is given as an attribute of *unshelled peas* [6]. *Talks* is not listed as an attribute for either *pet* or *bird*, but shows up for *pet bird* [3]. And a *typewriter table*, unlike either of its constituents, must be of a proper height for typing and must have side space on which to put one's papers; how else would one know this except from a functional relationship with typing? The point of all this is that a deductive logic of any kind is not designed to incorporate memories or creative ideas from outside the system brought in *ad hoc* for each individual case in order to yield the proper results, yet this is what is needed to produce emergent properties in conjoined concepts.

3. *Non-logical and non-consistent results for intersections, unions, negation and transitivity*. A clock is furniture, Big Ben is a clock, but Big Ben is not furniture. From experiments using ingeniously worded phrases, Hampton (see summaries in [4], [5]) provides much evidence for the *ad hoc* intrusion of world knowledge and associations into reasoning about fuzzy concepts. The result is that subjects' endorse mutually inconsistent or non-logical statements. Furthermore, the boundaries of concepts, i.e. what is or is not judged as a member, can vary with context of use. It's easy to see the reasoning behind any given result after the fact, but hard to see how operations of a deductive fuzzy logic could encompass all such vagaries in a useful way.

4. *Models of the intersection of fuzzy concepts need more than degree of membership to produce their results*. *Guppy* is a poor example of pets and a poor example of fish, but a very good example of *pet fish* [8], [9].[1] Since such anomalies were pointed out there have been a number of attempts to model the goodness of example of conceptual combinations (see [5] and [7] Chapter 12 for summaries). In each model, there is a good deal of machinery besides goodness of example ratings needed to characterize the two concepts of the combination (such as schemas in which dimensions of variation and attributes interact). But filling the slots in the schemas and prescribing how the machinery is to function are

[1] Paper [8] was actually a peculiarly misguided attack on fuzzy logic and, through that, on the relevance of graded structure and prototypes to concepts (see [2] and [10] for critiques), but it did launch a stream of research papers on conceptual combination.

dependent upon world knowledge, much of it related to the interaction of the two concepts so that it must be evoked uniquely for each conceptual pair. And even with all this "cheating," none of the models covers more than a limited number of cases.

86.1 Conclusions

Where have we gotten? Research to date has shown that concepts do fulfill the criteria of fuzzy sets, but that the deductive operations of fuzzy logic (or of classical logic) are insufficient for explaining or generating conceptual combinations. The conceptual system appears to need to be creatively porous in each instance to what humans know, perceive, and do in order to understand and perform operations on concepts.

Such a conclusion leads to deeper questions. Is it reasonable or is it misleading to call concepts *fuzzy sets* if only degrees of membership apply to them, but they cannot be deductively manipulated by the operations of fuzzy logic? After all, the word *fuzzy* in common language has negative connotations, and if applied to concepts without the full context of fuzzy logic, it could easily lose its mathematical reference and become an epithet. My inclination is not to use the word loosely. More importantly, the question arises as to whether there is anything further that the mathematics of fuzziness has to offer the psychology of concepts? Through its alliance with statistics, psychology is already reasonably sophisticated about scaling and other operations on distributions of numbers, and research on degree of membership in concepts began and has been proceeding without need to refer to the mathematical work in fuzzy logic. Can this work be further enriched by technical input from the mathematics of fuzzy sets?

Perhaps, but probably not in the way psychologists might immediately think, i.e. not by enabling yet more mathematical models. Mathematical models as such have a poor track record in psychology. Try the following thought experiment: think of as many important theories and/or impactful experiments in psychology as you can, and then ask how many of them were the product of a mathematical model. The usual answer is none. Such models do not seem to have the appropriate level of abstraction (not too much, not too little) or the connection to psychological reality that is generative of new knowledge in the field.

What fuzzy logic has been good at is helping solve particular technological problems in cases that involve graded rather than all-or-none structures. But these are not the kind of questions that most psychologists have asked about concepts. Rather, ever since the fall of the classical view, the quest seems to have been for a general theory of concepts that will replace the alleged virtues of necessary and sufficient conditions. That is, researchers have demanded that any factor proposed in the functioning of concepts provide a stable and lasting meaning for concepts beyond the flux of experience and usage, a meaning that will furthermore account for conceptual combinations. (For example, these were the demands made of my own theory of prototypes even though it was explicitly designed as counter to the requirements of the classical view.) But neither fuzzy logic nor anything else will provide this kind of account because it is the wrong question to ask and the wrong demand to make (see [10] for the full argument as to why). However fuzzy logic may be able to help at another level entirely, that of modeling and control for very specific situations of conceptual meaning or change.

There's more. Paradoxically the real reason why there may yet be fruitful interaction between fuzzy logic and the psychology of concepts is the same reason for its present failure at conceptual combinations. That reason is the vast interactivity and creativity of the human mind. And that is why the mind of Lotfi Zadeh who initiated fuzzy logic is greater than and cannot be bound by the structures of fuzzy sets and fuzzy logic. With all respect and good wishes to Lotfi.

Fig. 86.1. Eleanor Rosch during the discussion after her plenary talk for the conference NAFIPS 2012 in the Shattuck Hotel in Berkeley, California. In the audience on this picture among others, fltr: Karin Hutflötz, Thomas Whalen, Sergio Guadarrama, José Angel Olivas, Daniel Sanchez, Enric Trillas, Lotfi Zadeh, Shahnaz Shahbazova, Nick Pizzi and others.

References

1. Belohlavek, R., Klir, G.J. (eds.): Concepts and Fuzzy Logic. MIT Press, Cambridge (2011)
2. Belohlavek, R., Klir, G.K.: Fallacious Perceptions of Fuzzy Logic in the Psychology of Concepts. In: [1], pp. 121–148
3. Hampton, J.A.: Inheritance of Attributes in Natural Concept Conjunctions. Memory and Cognition 15, 55–71
4. Hampton, J.A.: Concepts and Natural Language. In: [1], pp. 233–258
5. Hampton, J.A.: Conceptual Combinations and Fuzzy Logic. In: [1], pp. 209–232
6. Murphy, G.L.: Comprehending Complex Concepts. Cognitive Science 12, 529–562 (1988)
7. Murphy, G.L.: The Big Book of Concepts. MIT Press, Cambridge (2002)
8. Osherson, D.N., Smith, E.E.: On the Adequacy of Prototype Theory as a Theory of Concepts. Cognition 9, 35–58 (1981)
9. Osherson, D.N., Smith, E.E.: Gradedness and Conceptual Conjunction. Cognition 12, 299–318 (1982)
10. Rosch, E.: Slow Lettuce: Categories, Concepts, Fuzzy Sets, and Logical Deduction. In: [1], pp. 89–120

87

On the Meaning of Fuzziness

Enrique H. Ruspini

Abstract. Interpretation of the basic structures of fuzzy logic, notably that of possibility distribution, is an essential requirement to clarify their value as an important tool in automated reasoning. Despite its evident value, as shown by the multiplicity and importance of its applications, much of the discussion to this day about such notions as "possibility" or the "fuzzy paradigm" typically relies on illustrative examples while failing to provide clarifying insights into important conceptual matters. In this paper we review an interpretation of the basic notions of fuzzy logic in terms of metric and utilitarian notions such as similarity and utility that clarifies major semantic issues while establishing links to existing formal frameworks such as the notion of possible worlds and the theory of metric spaces. We remark that these ideas were present, albeit in an implicit fashion, in the pioneering applications of fuzzy logic to system control. We review also recent extensions of similarity-based interpretations to fuzzy evidence.

87.1 Introduction

Ever since Zadeh's seminal paper [35] the theory of fuzzy sets and its subsequent extensions to approximate-reasoning methods [6, 37, 39, 38] there has been considerable skepticism about the nature of its basic concepts and structures, their possible relations to probabilistic approaches, and, ultimately, the need for such a formalism. As pointed out by Zadeh [40] many of these misgivings could be traced to the common, pejorative, usage of the word "fuzzy" to denote poorly conceived concepts or ideas. As originally formulated, however, basic notions such as fuzzy sets, possibility distributions, and fuzzy inferential methods were precisely defined making the formalism clear and unambiguous.

In other cases, reluctance can be traced to misgivings about the potential probabilistic nature of fuzzy logic. The related claims and statements claims are often made without the benefit of any argument, let alone one that is cogent and convincing. In the few cases where arguments have been advanced they have relied on axiomatic frameworks purported to show that probability theory is the only framework capable of representing uncertainty [3, 20]. As has been pointed out by several authors [7, 17, 16] these formalisms are either non-rigorous or are based on questionable assumptions regarding the requirements that must be satisfied by any approach to the representation of uncertainty. In yet another twist of flawed logical argumentation it has been suggested, and purportedly proved, that fuzzy logics are formalisms

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that are inconsistent with the possibility of multiple degrees of truth beyond those of classical logic [9]. Once again these arguments were quickly found to be based on incorrect assumptions about the nature of fuzzy logic [8, 25]. Other skeptics have also argued against the formalism on philosophical grounds [15] or, more often, on brief, harsh, assertions where the argument of the critic is usually one of authority rather than reason (as by D. Scott [15, p. 230]).

It is not the purpose of this paper, however, to review or categorize criticisms of fuzzy logic.[1] Rather, we argue that much of these criticisms stems from lack of understanding about the conceptual relations between the basic structures of fuzzy logic and fundamental and philosophical bases of other scientific frameworks. As pointed out by Bunge [1] one of the major traits distinguishing science from pseudoscience is the inability of the latter to integrate its epistemological frameworks and domains with those of established scientific knowledge. In this work we aim to discuss this essential connection while, at the same time, remarking on the essentially different nature of probabilistic and possibilistic approaches to the representation and manipulation of uncertainty. In this regard, however, it is not our intention to deprecate the former to the advantage of the latter but, rather, to provide the bases to understand that both are valid approaches with their own distinguishing characteristics and advantages.

Our discussion will be based on an interpretation of the basic structures of fuzzy logic on the basis of the notion of similarity.[2] This interpretation, formulated initially in 1991 [24], has been expanded since then to encompass various formalizations of the notion of approximate truth [14, 13].

This characterization is closely related to the notion of *truthlikeness* [21, 22]. Informally, the idea of likeness-to-truth approaches is to provide frameworks that are capable of measuring the degree to which statements, possibly false, differ from that describing the true state of the world. Stating, for example, that the population of Japan is 120,000,000 people is "less false" than stating that it is 50,000,000. In the context of practical reasoning applications we formally link the utilization of approximate-reasoning statements in inferential arguments to the potential errors that may arise in terms of incorrect conclusions, or equivalently, to the utility of approximately-true premises in the derivation of inferential conclusions

The format of this paper precludes a more thorough discussion of the semantic models underlying this view of fuzzy concepts and structures. Interested readers may consult our previous work for further details [24, 23]. We shall endeavor, nonetheless, to emphasize the formal mathematical and logical correctness of the approach while noting its relevance to the solution of reasoning problems as found in numerous applications throughout science. It is our contention that the ability to utilize these approximate, yet valid, inferential formalisms is the reason for the success of fuzzy-logic schemes as effective tools to solve these problems.

[1] A more thorough review of these issues can be found in the work of Entemann [10].

[2] For other interpretations and for a discussion of their relations, see, for example, Ruspini and Esteva [27].

Fig. 87.1. Fltr: Hamid Berenji, Elie Sanchez, Piero Bonissone, Enrique Ruspini, Lotfi Zadeh and two other participants of the Second IEEE International Conference on Fuzzy Systems (FUZZ-IEEE'93), San Francisco, California, 1993

87.2 Similarities and Preferences in a Logical Framework

Our point of departure is the well known concept of similarity relation, introduced by Zadeh [36] as a generalization of the classical notion of equivalence relation. This notion is important from various conceptual and practical perspectives.

First, the notion of similarity or, indistinguishability, relation is directly related to that of dissimilarity or distance. From a mathematical perspective, similarities are the complement of metrics defined in a universe of discourse **X**, which take values between 0 and 1, with 1 corresponding to maximum similarity, or zero distance, and 0 corresponding to maximum dissimilarity or distance. From a logical perspective these relations provide a valuable tool to model the extent to which statements that are true in some situation are valid in another.

A key feature of similarity relations is that of extended transitivity, which generalizes the corresponding property of classical equivalence relations:

$$S(x,y) \geq S(x,z) \circledast S(z,y)\,, \text{ for all } x,y,z \text{ in } \mathbf{X}$$

where $\circledast$ is a continuous triangular norm, or t-norm [19, 30]. Its importance lies on its ability to generalize the transitivity property of classical inferential schemes

$$(A \to B\,, B \to C) \to (A \to C)\,.$$

This desirable attribute has no similar counterpart in probabilistic schemes where knowledge that the conditional probabilities $P(B|A)$ and $P(C|B)$ have high values does not permit, in general, make any conclusion about the value of the conditional probability $P(C|A)$.

Similarity relations are also important from a conceptual viewpoint because of their relations to the utilitarian notion of preference [33]. Informally, these conceptual relations provide a mechanism to define two situations as similar if they are equally preferable from various perspectives that gauge their acceptability as solutions of a problem (for example as desirable control actions that steer a dynamical system into suitable behaviors). If, on the other hand, some action, situation, or outcome is significantly preferred to another from some relevant perspective, then these actions are not similar. Preference functions, typically modeled as a fuzzy order relations, are themselves closely related to the notion of utility functions: mappings from the universe of discourse $\mathbf{X}$ into the $[0,1]$ interval of the real line that assign to each element of $\mathbf{X}$ a number indicating its degree of desirability as a state of the system being modeled or that of actions influencing its behavior. Modeling of outcomes of control actions as generalized, elastic, constraints described by utility functions has, in fact, been the bases for the success of the application of fuzzy logic to practical problems since the early development of fuzzy-control systems [34].

Finally, it is important to note that similarity-based reasoning is a major cognitive tool that permits to conclude, by analogy, the applicability of knowledge about certain situations in similar contexts [32, 31]. Similarity-based interpretations of fuzzy logic provide this form of reasoning, generally thought as not being a correct inferential method, a validity based on proper understanding and quantification of the extent to which knowledge applicable in some case can be extended to another.

87.3 The Similarity Model

The similarity-based interpretation of fuzzy logic is based on the consideration of similarity relations defined in a logical framework inspired by the notion of possible world as introduced by Carnap [2] to relate logic and probability theory. In our discussion, the framework conceived by Carnap in the context of first-order predicate logic will be simplified by confining ourselves to propositional logic.

87.3.1 The Carnapian Universe

The central structures of the similarity-based interpretation of fuzzy logic are a language, or collection of sentences $\mathscr{L}$ representing assertions about the state of a system and a nonempty set $\mathscr{U}$ representing possible states, situations, or behaviors of that system.

Sentences in the language $\mathscr{L}$ are formed by combining, as is customary in propositional logic frameworks, by combining letters of an alphabet $\mathscr{A}$ with the logical operators $\neg, \vee, \wedge, \rightarrow$, and $\leftrightarrow$, according to the usual rules.

Elements of the set $\mathscr{U}$, called the Carnapian universe, or universe for short, are called *possible worlds*. We will assume that there exists a non-empty, conventional subset $\mathscr{E}$ of the universe $\mathscr{U}$, called the *evidential set*. This subset models the possible states of the system that are consistent with available knowledge or information about it. If no information exists, then $\mathscr{E} = \mathscr{U}$, that is, any possible world may correspond, for all we know, to the actual state of the system being modeled.

We will also assume that there exists a function, called a *valuation*, that assigns one and only one of the truth values **T** (*true*) or **F** (*false*) to the every possible world w and every sentence ϕ in the language $\mathscr{L}$. Abusing the language we will, in the rest of this paper, use the same notation to denote a subset of possible worlds and the proposition that is true in all worlds in that subset.

We will augment this framework by addition of a similarity relation S in the universe $\mathscr{U}$ that assigns a number between 0 and 1 to every pair of possible worlds w and w'.

This function satisfies the axioms:

1. $S(w,w') = 1$, if and only if $w = w'$,
2. $S(w,w') = S(w',w)$, for all w,w' in $\mathscr{U}$,
3. $S(w,w') \geq S(w,w'') \circledast S(w'',w')$, for all w,w',w'' in $\mathscr{U}$,

where $\circledast$ is a continuous t-norm.

The similarity function S induces a metric structure in the universe $\mathscr{U}$ since its complement $\delta = 1 - S$ is a distance satisfying the generalized triangular inequality:

$$\delta(w,w') \leq \delta(w,w'') \oplus \delta(w'',w'),$$

for all w,w',w'' in $\mathscr{U}$, and where $\oplus$ is a continuous triangular comorm.[3]

The similarity relation S induces also a graded modal logic structure in $\mathscr{U}$ since members of the family of crisp relations

$$R_\alpha(w,w') \text{ if and only if } S(w,w') \geq \alpha,$$

where $0 \leq \alpha \leq 1$, are accessibility relations in the sense of modal logic [18].

In our scheme, the accessibility relations R_α model the degree to which what is true in a world w is true in another world w' in the same sense that, statements like "If q then p is possible," and "If q then p is necessary," are interpreted in modal logic. Having now available, however, an infinite family of accessibility relations we can describe relations of inclusions between metric neighborhoods of subsets of the universe $\mathscr{U}$.

Similarity and metric structures are often found and formally characterized in a variety of fields, notably in pattern recognition, information science, psychology, and sociology. The ability to characterize the extent to which statements about one object, situation, or event can be said to apply to another is central to any form of analogical reasoning.

[3] When $\oplus \equiv +$ this inequality is the triangular inequality satisfied by classical distance functions.

In applications of fuzzy logic, however, these structures have been usually defined in an implicit, indirect, fashion through previous definition of utility structures. In early, pioneering, applications of fuzzy control [34], for example, the adequacy of control actions was measured employing a number of performance indexes (comfort, safety, etc.) that ranked the relative desirability of certain actions. Similar ideas have also been applied to the control of robots [28] and teams of robots [29]. The underlying notion behind this characterization of similarity is that, whenever the utilities of two different actions are nearly or equally useful from every significant respect, then these actions are nearly or completely similar. The work of Valverde [33] exemplifies successful efforts to relate the notions of utility (as a fuzzy order) and similarity relations. Today, however, a generalized theory of similarities paralleling that of probability theory with its conditional, joint, and marginal distributions is yet to be formulated.

87.3.2 Probability and Possibility

Introduction of the Carnapian universe as a set of the possible worlds that correspond to possible states of a system permits to make a clearer distinction between the probabilistic and possibility to the treatment of uncertainty.

This uncertainty arises when a particular proposition, or hypothesis, $\mathscr{H}$ cannot be logically inferred to be true from the knowledge provided by the evidence $\mathscr{E}$.

From the perspective of the universe of discourse $\mathscr{U}$ this is equivalent to say that the subsets corresponding to the propositions $\mathscr{H} \wedge \mathscr{E}$ and $\neg\mathscr{H} \wedge \mathscr{E}$ are both nonempty.

Probabilistic methods endeavor to quantify this state of affairs by quantifying the relations between a set measure P of those sets:[4]

$$P(\mathscr{H} \wedge \mathscr{E}) \,:\, P(\neg\mathscr{H} \wedge \mathscr{E}),$$

or, equivalently, by the values of the related conditional probabilities $P(\mathscr{H}|\mathscr{E})$ and $P(\neg\mathscr{H}|\mathscr{E})$. The characterization thus provided is one of relative weight of sets from set measures related to the measured or perceived likelihood of the corresponding events.

By contrast possibilistic structures, as interpreted by similarity-based interpretations, seek to determine how far a hypothesis $\mathscr{H}$ should be generalized, or "stretched," to encompass the evidence $\mathscr{E}$. The resulting framework generalizes the transitive structures of classical logic by generalizing the conventional relation of set inclusion to that of inclusion between their metric neighborhoods.

87.4 Similarities between Subsets

The generalization of the notion of similarity as a relation between pairs of points in the universe $\mathscr{U}$ to a relation between subsets of that universe is based on the

[4] Once again, we abuse the language by using the same notation to denote subsets of $\mathscr{U}$ and their corresponding propositions

well-known method to extend a distance defined on a set to the Hausdorff distance between subsets of that set [4]. In our case, we will apply the same idea to similarity relations, which as discussed before, are the dual of distance measures taking values between 0 and 1. The extended measure permits to measure how a subset q of $\mathscr{U}$ must be enlarged to encompass another set p.

87.4.1 Degree of Implication

The first step in defining similarities between subsets of $\mathscr{U}$, along the lines suggested by the Hausdorff metric, is to characterize the extent to which a set is included in a metric neighborhood of another by means of the notion of degree of implication.

Definition: The *degree of implication* of p by q is the binary relation in the power set $\mathscr{P}(\mathscr{U})$, that is,

$$\mathbf{I}\colon\ \mathscr{P}(\mathscr{U}) \times \mathscr{P}(\mathscr{U}) \longmapsto [0,1],$$

given by the expression

$$\mathbf{I}(p \mid q) = \inf_{w' \in q}\ \sup_{w \in p}\ S(w, w').$$

The degree of implication is a $\circledast$-transitive relation if the similarity relation S is. Furthermore this relation has a number of properties that are important to explain the major inferential operation of fuzzy logic: the generalized modus ponens:

1. $\mathbf{I}(p \mid q) \geq 0$, for all subsets p and q of $\mathscr{U}$,
2. $\mathbf{I}(p \mid q) = 1$, if and only if $q \subseteq p$,
3. $\mathbf{I}(p \mid q) \geq \mathbf{I}(p \mid r) \circledast \mathbf{I}(r \mid q)$, for all subsets p, q and r of $\mathscr{U}$,
4. $\mathbf{I}(p \mid q) = \sup_{r \subseteq \mathscr{U}} \big(\mathbf{I}(p \mid r) \circledast \mathbf{I}(r \mid q)\big)$, for all subsets p and q of $\mathscr{U}$.

From the degree of implication it is possible to define a $\circledast$-transitive similarity function $\hat{S}$ in $\mathscr{P}(\mathscr{U})$ that extends the similarity relation S in $\mathscr{U}$:

$$\hat{S}(p,q) = \min\big(\mathbf{I}(p \mid q), \mathbf{I}(q \mid p)\big).$$

87.4.2 The Generalized Modus Ponens

The degree of implication permits to characterize the basic possibilistic structures of fuzzy logic and the inferential rule known as the generalized modus ponens or compositional rule of inference of Zadeh [39].

This characterization is based on the formal definition of the dual concepts of necessity and possibility distributions. A necessity distribution $\mathbf{N}_{\mathscr{E}}(p)$ is a lower bound of the similarity between *any* world in the evidential set $\mathscr{E}$ and some possible world where p is true.

Definition: If $\mathscr{E}$ is a nonempty subset of $\mathscr{U}$, then a function $\mathbf{N}_{\mathscr{E}}(\cdot)$, defined over subsets of the universe $\mathscr{U}$, is called an *necessity distribution* for $\mathscr{E}$ if

$$\mathbf{N}_{\mathscr{E}}(p) \leq \mathbf{I}(p \mid \mathscr{E}).$$

Intuitively, the necessity distribution value $\mathbf{N}_{\mathscr{E}}(p)$ is a lower bound of how similar is *every* world in the evidential set $\mathscr{E}$ from some world in p, or, in other words, the extent to which p *must* be true in any world in $\mathscr{E}$.

Possibility distributions, based a dual notion of the degree of implication called the *degree of consistence*, are upper bounds of the similarities between any evidential world in $\mathscr{E}$ and any world in p. Intuitively, a possibility distribution is an upper bound of the similarity between *some* world in $\mathscr{E}$ to some world in p, or, in other words, the extent to which p *may* be true in $\mathscr{E}$.

In what follows, we will limit, for reasons of space, our discussion to necessity distributions omitting the dual characterization of possibility distributions. Interested readers may found the full development and associated proofs in our original work [24].

The notion of conditional independence is defined by means of the *pseudoinverse* function associated with a triangular norm $\circledast$ [33]:

Definition: If $\circledast$ is a triangular norm, its *pseudoinverse* $\oslash$ is the function defined over pairs of numbers in the unit interval of the real line by the expression $a \oslash b = \sup\{\, c : \; b \circledast c \leq a \,\}$.

A conditional necessity function measures the proximity of all worlds in the evidential set $\mathscr{E}$ to some world in p *relative* to their proximity to worlds that satisfy a conditioning proposition q.

Definition: Let $\mathscr{E}$ be a nonempty subset of $\mathscr{U}$. A function $\mathbf{N}_{\mathscr{E}}(\cdot|\cdot)$ mapping pairs of subsets of $\mathscr{U}$ into $[0,1]$ is called a *conditional necessity distribution* for $\mathscr{E}$ if

$$\mathbf{N}_{\mathscr{E}}(p|q) \leq \inf_{w \in \mathscr{E}} \left[\mathbf{I}(p \mid w) \oslash \mathbf{I}(q \mid w) \right],$$

for any subsets p and q.

We are in a position now to state the fundamental inferential operation of fuzzy logic in terms of unconditioned and conditional necessity distributions. This result, which generalizes [12,11] the original formulation of Ruspini [24], makes use of the notion of partition of $\mathscr{U}$:

Definition (Partition of $\mathscr{U}$): If $\mathbf{P} = \{p_i \text{ in } \mathscr{U}, i \text{ in } I\}$ is a collection of subsets of possible worlds that satisfy $\cup_I p_i = \mathscr{U}$, then $\mathbf{P}$ is called a *partition* of $\mathscr{U}$.

Employing this definition we may now state the generalized modus ponens in terms of two non-empty subsets $\mathscr{E}$ and $\mathscr{F}$ of the universe $\mathscr{U}$.

Theorem: *(Generalized Modus Ponens for Necessity Distributions)*: Let $\{p_i, i \text{ in I}\}$ be a partition of $\mathscr{U}$ and let $\mathscr{E}$ and $\mathscr{F}$ be nonempty subsets of $\mathscr{U}$. Then, it is

$$\sup_I \left[\mathbf{N}_{\mathscr{F}}(q|p_i) \circledast \mathbf{N}_{\mathscr{E}}(p_i) \right] \leq \mathbf{N}_{\mathscr{E} \cap \mathscr{F}}(q).$$

This result is the formal validation of the following inferential scheme:

$$\begin{array}{l} \text{If } w \text{ is in } \mathscr{E}\text{, then } w \text{ is necessarily similar to } p, \\ \text{If } w \text{ is in } \mathscr{F}\text{, then} \\ \quad \text{if } w \text{ is necessarily similar to } p, \\ \quad \text{then it is necessarily similar to } q, \\ \hline \text{If } w \text{ is in } \mathscr{E} \cap \mathscr{F}\text{, then } w \text{ is necessarily similar to } q. \end{array}$$

Note that this result includes two evidential sources $\mathscr{E}$ and $\mathscr{F}$. The evidential set $\mathscr{E}$ models available factual evidence while the set $\mathscr{F}$ represents conditional knowledge (i.e., the rules of a knowledge-based system). The result estimates the extent to which a statement is true if both evidential sources are true.

87.4.3 Extensions to Fuzzy Evidence

Our developments so far have been based on the study the similarity relations between classical, or crisp, subsets of the universe $\mathscr{U}$. Recently [26] previous results have been extended to similarity relations between fuzzy subsets of $\mathscr{U}$.

In what follows we shall assume that the universe $\mathscr{U}$ is finite. We will denote by $\mathscr{F}(\mathscr{U})$ the fuzzy power set of $\mathscr{U}$, that is, the set of all fuzzy subsets of $\mathscr{U}$.

Definition: The *fuzzy degree of implication* induced by a $\circledast$-transitive similarity relation S is the function $\mathbf{I}^F$ mapping pairs (p,q) of members of the fuzzy power set $\mathscr{F}(\mathscr{U})$ into numbers in the $[0,1]$ interval of the real line given by the expression:

$$\mathbf{I}^F\,(p \mid q) = \min_{w'}\ \max_{w}\ \left[\, S(w,w') \circledast \big(p(w) \oslash q(w')\big)\,\right],$$

where the scope of the max and min operators are all the elements w, w' in $\mathscr{U}$, respectively.

The fuzzy degree of implication $\mathbf{I}^F$ generalizes the degree of implication $\mathbf{I}$, that is, if p and q are crisp sets, then $\mathbf{I}^F\,(p \mid q) = \mathbf{I}(p \mid q)$. In addition, the degree of implication $\mathbf{I}^F$ has the following properties:

1. The equation $\mathbf{I}^F\,(p \mid q) = 1$ is true if and only if $q(w) \le p(w)$ for all w in $\mathscr{U}$, that is if the fuzzy set q is a subset of the fuzzy set p.
2. If $p \ge p'$ and if $q \le q'$, then it is

$$\mathbf{I}^F\,(p \mid q) \ge \mathbf{I}^F\,(p' \mid q)\,,$$
$$\mathbf{I}^F\,(p \mid q) \ge \mathbf{I}^F\,(p \mid q')\,.$$

3. If S is a $\circledast$-transitive similarity relation, then the degree of implication $\mathbf{I}^F$ induced by S is a $\circledast$-transitive relation in $\mathscr{F}(\mathscr{U})$.

4. $\mathbf{I}^F(p \mid q) = \sup_r \left[\mathbf{I}^F(p \mid r) \circledast \mathbf{I}^F(r \mid q) \right]$.

5. The binary relation $\widehat{S}$ defined by the expression

$$\widehat{S}(p,q) = min\left(\mathbf{I}^F(p \mid q), \mathbf{I}^F(q \mid p) \right),$$

   is a $\circledast$-similarity in $\mathscr{F}(\mathscr{U})$.

The properties of the fuzzy degree of implication $\mathbf{I}^F$ permit to extend the generalized inferential results presented in Section 87.4.2 to cases where evidence and knowledge are fuzzy. Furthermore, the extended interpretation provides new bases to examine the relations of similarity-based interpretations with other formalisms such as possibilistic logic [6,5].

Fig. 87.2. Fltr: Mayuka F. Kawaguchi, Lotfi Zadeh, Annamária R. Várkonyi-Kóczy, another participant, and Enrique Ruspini at the 2000 IEEE International Conference on Fuzzy Systems, San Antonio, Texas, May 2000

87.5 Conclusion

The seminal concepts introduced by Zadeh with his theories of fuzzy set and fuzzy logic have led to the development of numerous concepts and applications based on those formalisms. The extent and importance of applications of these ideas in applied science is ample evidence of their importance in the solution of problems characterized by conditions of imprecision and uncertainty.

The usefulness and validity of these methods would be enhanced and their acceptability would be wider if the underlying concepts were related to other scientific concepts and their own underlying constructs. Similarity-based interpretations of fuzzy logic characterizing major concepts and results in terms of metric structures defined over a logic-based universe—a set of possible worlds as conceived by Carnap—provides the required connections between fuzzy-logic concepts and those supporting classical logical frameworks, the theory of metric spaces, and utility theory. Furthermore, the resulting formal structure permits to make a clear, understandable, distinction between probabilistic and possibilistic approaches as complementary methodologies for the treatment of uncertainty.

References

1. Bunge, M.: What is Pseudoscience? The Skeptical Inquirer 9, 36–46 (1984)
2. Carnap, R.: Logical Foundations of Probability. University of Chicago Press, Chicago (1950)
3. Cox, R.T.: Probability, Frequency, and Reasonable Expectation. American Journal of Physics 14(1), 1–13 (1946)
4. Dieudonné, J.A.E.: Foundations of Modern Analysis. Academic Press, New York (1960)
5. Dubois, D., Esteva, F., García, P., Godó, L., Prade, H.: A Logical Approach to Interpolation based on Similarity Relations. International Journal of Approximate Reasoning 17, 1–36 (1997)
6. Dubois, D., Lang, J., Prade, H.: Possibilistic Logic. In: Gabbay, D.M., Hogger, C.J., Robinson, J.A. (eds.) Handbook of Logic in AI and Logic Programming, vol. 3. Oxford University Press (1994)
7. Dubois, D., Prade, H.: The Logical View of Conditioning and its Application to Possibility and Evidence Theories. International Journal of Approximate Reasoning 4(1), 23–46 (1990)
8. Dubois, D., Prade, H., Smets, P.: Responses to Elkan (Didier Dubois, Henri Prade, Philippe Smets). IEEE Expert 9(4), 15–19 (1994)
9. Elkan, C.: The Paradoxical Success of Fuzzy Logic. IEEE Expert 9, 3–8 (1994)
10. Entemann, C.W.: Fuzzy logic: Misconceptions and Clarifications. Artificial Intelligence Review 17(1), 65–84 (2002)
11. Esteva, F., García, P., Godó, L.: Relating and Extending Approaches to Possibilistic Reasoning. International Journal of Approximate Reasoning 10(4), 311–344 (1994)
12. Esteva, F., García, P., Godó, L., Valverde, L., Ruspini, E.H.: On Similarity Logic and the Generalized Modus Ponens. In: Proceedings of the 1994 IEEE International Conference on Fuzzy Systems, pp. 1423–1427. IEEE Press, Orlando (1994)
13. Godó, L., Esteva, F., García, P., Aguist-Cullel, J.: A Formal Semantic Approach to Fuzzy Logic. In: Proceedings of the 21st International Symposium on Multiple-Valued Logic, pp. 72–79. IEEE Computer Society Press (1991)

14. Godó, L., Rodríguez, R.O.: Logical Approaches to Fuzzy Similarity-based Reasoning: An Overview. In: Della Riccia, G., Dubois, D., Kruse, R., Lenz, H.-J. (eds.) Preferences and Similarities. CISM Courses and Lectures, vol. 504, pp. 75–128. Springer (2008)
15. Haack, S.: Deviant Logic, Fuzzy Logic: Beyond the Formalism. The University of Chicago Press, Chicago (1996)
16. Halpern, J.Y.: A Counter xample to Theorems of Cox and Fine. Journal of Artificial Intelligence Research 10, 67–85 (1999)
17. Halpern, J.Y.: Cox's Theorem Revisited. Journal of Artificial Intelligence Research 11, 429–435 (1999)
18. Hughes, G.E., Maxwell, J.C.: A New Introduction to Modal Logic. Routledge, London (1996)
19. Klement, E.P., Mesiar, R., Pap, E.: Triangular Norms, Position paper I: Basic Analytical and Algebraic properties. Fuzzy Sets and Systems 143(1), 5–26 (2004)
20. Lindley, D.V.: Scoring Rules and the Inevitability of Probability. International Statistical Review 50, 1–26 (1982)
21. Niiniluoto, I.: Truthlikeness. Reidel, Dordrecht (1987)
22. Oddie, G.: Likeness to Truth. Western Ontario Series on Philosophy of Science, vol. 30. Dordrecht, Reidel (1987)
23. Ruspini, E.H.: Approximate Reasoning: Past, Present, Future. Information Sciences 57/58, 297–317 (1991)
24. Ruspini, E.H.: On the Semantics of Fuzzy Logic. International Journal of Approximate Reasoning 5, 45–88 (1991)
25. Ruspini, E.H.: Responses to Elkan (Enrique H. Ruspini). IEEE Expert 9(4), 32–33 (1994)
26. Ruspini, E.H.: Similarity and Implication Between Fuzzy Sets. In: Trillas, E., Bonissone, P.P., Magdalena, L., Kacprzyk, J. (eds.) Combining Experimentation and Theory. STUDFUZZ, vol. 271, pp. 237–246. Springer, Heidelberg (2011)
27. Ruspini, E.H., Esteva, F.: Interpretations of Fuzzy Sets. In: Ruspini, E.H., Bonissone, P.P., Pedrycz, W. (eds.) Handbook of Fuzzy Computation. Oxford University Press, Bristol (1998)
28. Saffiotti, A., Konolige, K., Ruspini, E.H.: A Multivalued-logic Approach to Integrating Planning and Control. Artificial Intelligence 76(1-2), 481–526 (1995)
29. Saffiotti, A., Ruspini, E.H.: Global Team Coordination by Local Computation. In: Proceedings of the European Control Conference (ECC), Porto, Portugal (2001), http://www.aass.oru.se/˜asaffio/
30. Schweizer, B., Sklar, A.: Associative Functions and Abstract Semigroups. Publicationes Mathematicae Debrecen 10, 69–81 (1983)
31. Tversky, A.: Features of Similarity. Psychological Review 84, 327–352 (1977)
32. Tversky, A., Kahneman, D.: Judgment under Uncertainty: Heuristics and Biases. Science 185(4157), 1124–1131 (1974)
33. Valverde, L.: On the Structure of f-indistinguishability Operators. Fuzzy Sets and Systems 17, 313–328 (1985)
34. Yasunobu, S., Miyamoto, S.: Automatic Train Operation by Predictive Fuzzy Control. In: Sugeno, M. (ed.) Industrial Applications of Fuzzy Logic. Elsevier Science Publishers B.V. (North-Holland), Amsterdam (1985)
35. Zadeh, L.A.: Fuzzy Sets. Information and Control 8, 338–353 (1965)
36. Zadeh, L.A.: Similarity Relations and Fuzzy Orderings. Information Sciences 3, 177–200 (1971)

37. Zadeh, L.A.: Outline of a new Approach to the Analysis of Complex Systems and Decision processes. IEEE Transactions on Systems Man and Cybernetics SMC-3 (1), 28–44 (1973)
38. Zadeh, L.A.: Fuzzy Logic and its Application to Approximate Reasoning. In: Information Processing 74, Proceedings of IFIP Congress 74, Stockholm, Sweden, August 5-10, pp. 591–594. North-Holland (1974)
39. Zadeh, L.A.: A Theory of Approximate Reasoning. In: Hayes, J., Michie, D., Mikulich, L.I. (eds.) Machine Intelligence, pp. 149–194. Wiley (1979)
40. Zadeh, L.A.: Fuzzy Logic. Scholarpedia 3(3), 1766 (2007)

88

Fuzzy Control: From Heuristic Rules to Optimization on Thousands of Decision Variables

Antonio Sala

88.1 Introduction

One of the earliest applications of fuzzy logic was fuzzy control. Indeed, early Lofti Zadeh's background was in systems and control engineering and, hence, his way of thinking was close to that of control engineers, the group of people I belong to.

My personal trajectory in the last 20 years follows a path of formalisation akin to that of fuzzy control in the a bit more than 40 years that have elapsed since its original inception.

Indeed, fuzzy control started as a way to incorporate heuristic expert knowledge in the control loop but nowadays it has evolved into a formal control discipline with optimization, robustness, stability proofs, etc.

The following sections will present a somehow critical view on the current status of fuzzy control from its historic perspective. The discussion will only discuss the "IF-THEN" fuzzy controllers, and hence, will be incomplete: "intelligent" control is more than fuzzy control (adaptive, dynamic programming, ...) but my biased background and the book's motivation made me focus only on such class of controllers. Due to my background, adaptive fuzzy control issues will not be discussed, either, even if there are intriguing questions, such as the difference between "adaptive" control and "learning" control: is proving *uniform ultimate boundedness* of an error signal anything related to intelligence or learning?.

88.2 The Original Fuzzy Idea in Control: Heuristic Fuzzy Control

Somethimes great engineering application results come from a strike of genius by a clever theorist who gives rise to a new branch on a particular discipline which gets down to revolutionary changes in applications. However, it is a fact that many times theoretical calculations justifies what engineers have *already* made work: the Wright brothers flew their plane without finite-element Navier-Stokes supercomputer simulations. In this latter case, theorists come *a posteriori*, giving refinements and clearly stating the conditions of applicability of the engineering methodology.

In the specific history of control engineering, there existed both kinds of "clever ideas" (clever practical ideas which inspire theorists, clever theoretical ideas which

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inspire practitioners) shaping the current status of the discipline. In the clever "practical" ones, we have PID control, frequency response of amplifiers and, in my opinion "fuzzy control" (understood as interpolation between binary logic rules, see later). The clever "theoretical" milestones in control are the differential equation and Lyapunov theory, convex optimization, generalised-plant robust control, etc. which have shaped mainstream control in the last five decades.

88.2.1 Who Invented Fuzzy Control?

The early ideas originated heuristic fuzzy control. Basically, fuzzy rules were crafted encoding "expert knowledge" to control a given industrial process. This heuristic idea is not what Zadeh's 1972 paper on fuzzy control (Systems, Man and Cybernetics) was envisaging, which was more about (fuzzy) set-valued mappings. However, although Zadeh had a unique theoretical insight when proposing fuzzy sets, and some theoretical considerations on fuzzy control were also posed by him, the actually-working idea in many control applications was more practical and prosaic: a nice interface for *interpolation*. No stability proofs or similar were posed in any early fuzzy control paper.

Actually, in control, the original rulebases might say nothing that many other people may have inadvertently used even before the formal "birth" of fuzzy sets: I'm sure many people will have "interpolated" binary decision rules in order to avoid continuous "switching" and hard discontinuities in their logic controllers. Indeed, switching is a daunted enemy in "practical" control (unless used to our advantage, i.e., sliding-mode control) so dead-zones and interpolators are routinely used to mitigate its occurrence in an heuristic way. In fact, proportional control may be considered to be a smooth interpolation between lower and upper saturation limits of a control signal, *i.e.*, being exaggerated, we might claim that proportional control fuzzified preexisting all-or-nothing controllers. "*Gain scheduling*" is the magic word in control terms, which was coined and used a few years before fuzziness was born and, in fact, referred to interpolation of controller parameters. However, the analog electronic technology in the 1960's didn't allow for easy interpolation over more than one variable and, hence, multi-dimensional interpolation in industrial practice had to wait for 15 more years; by then, fuzzy was already there and people called their controllers "fuzzy"... Had the term not been coined, maybe fuzzy controllers might have been called *multi-dimensional gain-scheduling*.

88.2.2 Fuzzy Control: Revolution or User Interface?

Once the ideas of "fuzzy expert control system" and the like were coined, people flocked to set up (by hand and trial-and-error) rulebases to control various processes. The feeling in late 1970s and 80s was that fuzzy control could apply to almost anything and it was a revolutionary, so-much-easier way of controlling than other alternatives... and almost math-free because it was "reasoning"!

Jumping onto the bandwagon of the then incipient "artificial intelligence" community allowed many low-profile researchers (weak in math, physics) to pretend they were doing “intelligent control” and claiming that they were doing something “revolutionary”. Computer scientists, rushing to expand the realm of application of artificial intelligence, quickly welcomed these “control people” to their club.

However, saying that “I succeeded in hand-tuning some rules for controlling this system” does not justify the assertion “anything can be controlled with the same methodology” or “this is a revolutionary idea which will change forever the theory and practice of control”. Relying into the justification of “expert” knowledge provides also a very easy explanation of failures: “if it does not work, it’s because you were not a clever enough expert when hand-tuning”: you are not so “intelligent”, and intelligent persons “know” how to do things, they don’t need theorems.

However, these kind of universal assertions are not too scientifically sound or useful: if you just say that your success is due to “your expert knowledge” then that is not too useful to the research community. That is not the same as saying it is “useless”: indeed, your expert “knowledge” may be very valuable on a particular industrial problem, but scientific research requires methodology, explanations and repeatability. Physical “science” means excruciating the generality from particular cases, proposing theories that apply to not-yet seen cases. And then, there it comes Mathematics: axioms and proofs are needed to rigorously determine the power and the limits of your ideas... fortunately, later interpretations of fuzzy control are better in these aspects, see below.

In fact, the actual truth on heuristic fuzzy control is that although it gave a convenient user interface to account for some subtleties of some systems (introducing “low” and “high” concepts in the computer, instead of complicated first-principle concepts such as exponentials of temperatures, trigonometric functions in mechanical systems, ...), it stopped at that in academic research. In fact, the only widely-applicable rules were, oh surprise!, the fuzzy PD, fuzzy PI and alike ones which are not philosophically very different to the PD, PI regulators they try to replace with “intelligent” ones: 90% of rulebases in fuzzy-control literature are basically *the same*!, with only a couple of error and error-increment scaling factors as adjustable parameters (for instance, as in the industrial controller in Figure 88.1). In summary, although elaborate rulebases did succeed in some industrial control applications, and they indeed deserve recognition, it is also true that “fuzzy control” was a convenient term for (some) people to fake research on plain-old hand-tuned PIDs while climbing the academic ladder.

In summary, for an engineer’s conception, its first aim is “make things work, no matter why / how”: the end application justifies the means; in this perspective, fuzzy control is a clear success and industrial applications abound. However, not being able to tell why or how you achieved your “thing” to work is a kind of “alchemist”-like conception of technology: getting controllers to work due to your “expert knowledge” is not too different to the “secret formulae” of middle-age incipient explorers in the world of chemistry. In this context, heuristic fuzzy control quickly got exhausted as a research topic, even if it still appears recurrently in low-level conferences or in non-control journals where somebody used a fuzzy PD rulebase to solve a control problem.

Fig. 88.1. Omron™ E5AF: An industrial fuzzy controller of the early 1990's

88.3 Takagi-Sugeno's Approach

Another very idolatred concept in fuzzy control are Takagi-Sugeno fuzzy models. Indeed, the 1985 Takagi-Sugeno paper in IEEE Trans. Systems, Man and Cybernetics has almost 10000 cites[1], and most current fuzzy control designs are based in Takagi-Sugeno models. However, Takagi's paper, apart from puting forward the possibility of such models, does not actually propose any rigorous way of designing controllers for them (the proposed model was static, i.e., no differential or difference-equations were considered).

TS models were considered as interpolations of linear system so controllers were designed for each system and then, it was checked whether the design was lucky enough to work via extensive simulations.

88.3.1 Present-Day Situation

It's only after the Linear Matrix Inequality (LMI, in the rest of the chapter) convex optimization framework was made popular in the nineties (first, in the non-fuzzy gain scheduling community) that Tanaka and Wang's papers were actually seminal in the approach of controlling nonlinear systems via fuzzy TS models: for me this is the true birth of fuzzy control as it is today.

So, even if Tanaka/Wang humbly root their work in Takagi-Sugeno and Zadeh's ideas, the fact is that most of the gain-scheduling community never read Zadeh's or Sugeno's papers and Tanaka/Wang's seminal line today would have been equally fruitful without the supposedly seminal ideas from Sugeno and Zadeh.

[1] Isaac Newton's *principia mathematica* has only 1300 citations, for instance. Takagi-Sugeno's paper has 15% more cites than Albert Einstein's most cited paper (data from Google Scholar, January 2012). Of course, these numbers are not "scientific" as Google doesn't fetch many old non-digital documents.

The only thing is that, hadn't Tanaka/Wang included Sugeno and Zadeh in their citations LMI fuzzy control people would be considered to be plain "non-inteligent" gain-scheduling control but, due to the fuzzy link we are considered to research on "intelligent" control, whatever it is, for good or bad.

88.3.2 Are Convex-Optimization-Based Developments "Intelligent" Control?

Defining what is to be understood as "intelligent" is a tough question but the answer might be negative according to many interpretations.

Indeed, no akin to reasoning and to linguistic interpretability of the results is present in the latest developments and, furthermore, such linguistic and reasoning aspects are intentionally avoided (some of the results, even for simple academic examples involve optimization with hundreds or thousands of decision variables and constraints... Taylor-series polynomial fuzzy systems exacerbate the computational needs).

Also, no "learning" is present (in terms of discovering concepts and structures in the data reaching the controller, in terms of optimising its behaviour based on reward from the environment), except at most gradient-based adaptation of some controller parameters.

In summary, which is the interest of LMI semidefinite-programming and Lyapunov functions to the artificial intelligence community? I don't think it's too high... however, traditionally, we are considered one of them.

88.4 Fuzzy Control's Place in the (Academic) World

From the previous ideas on the lack of current relevance of the heuristic approach and the lack of reasoning / intelligent-like designs in semidefinite-programming Lyapunov-based approach, it is not hard to understand that the fuzzy-control community feels somehow out of place in many circles.

Indeed, only a handful of current "fuzzy control" researchers contribute meaningful content to top-tier control journals. Major control conferences (IFAC World Congres, CDC) only receive a small percentage of "fuzzy control" papers once substandard "heuristic control" ones are rejected. However, if you delve away of the major events, the percentage of substandard fuzzy control papers that get through (somebody has to attend the event) reaches worrisome levels, and contributes to the bad overall reputation of fuzzy things in the core control community. Hence, fuzzy control's relevance in the control community is scarce.

Conversely, fuzzy "control" plays a marginal role in key fuzzy "Artificial Intelligence" conferences, because no relevant new ideas are usually contributed in "reasoning", which is the main topic of interest to computer scientists and also no relevant content from the control side is contributed to "fuzzy set theory" which is the topic of interest of most of the mathematicians attending such fuzzy conferences. Hence, fuzzy control's relevance in the computer-science artificial intelligence community is also scarce, as well as in the set-theorists arena...

In summary, “serious” fuzzy control researchers find themselves being in the bottom ranks of “topic” popularity (number of paper by keyword) both in the control conferences (because of being “fuzzy”) and in the fuzzy conferences (because of being “control”).

88.5 Conclusions

This personal view has expressed a quite critical perspective on the history of fuzzy control which maybe I wouldn’t have dared to express some time ago. The root of the criticism is based on two ideas. *First*: even if wildly cited, the current relevance of original Zadeh’s and Takagi-Sugeno ideas in automatic control has, as we control people like to say, decayed in an exponential way, even if routinely cited. *Second*: many substandard *I-saw-this-rulebase-hundreds-of-times* papers keep pouring on too many places so it’s hard to find another control “speciality” with lower reputation among mainline control people.

The more relevant ideas in fuzzy control nowadays come from the linear-matrix-inequality paradigm and polytopic uncertainty / gain scheduling developments of people who do not consider themselves related whatsoever to the “intelligent control” community.

Fuzzy control researchers are in a “purgatory” between “hardcore control” and AI. It has the advantage of being a bridge between both worlds, it has the disadvantage of not being totally “at home” in any of them.

Anyway, some of my points of view are intentionally exaggerated, giving room for discussion, and trying to make the book in which this chapter appears something different to a self-laudatory ode. In fact, current top fuzzy control research is reasonably well regarded and a handful of its higher-ranked individuals are active in journal editorial boards as well as national and international scientific committees.

Perspectives are uncertain because, once heavy maths comes in, the practical relevance of many complex results (delay+uncertain+stochastic+fault modes+. . .) with largely intractable LMIs is scarce (after all, I’m an engineer)... but I guess that happens everywhere: if it were certain that following a certain track would yield sure key new results, such a topic would be exhausted in a couple of years!.

89

A Grain in the Heap

Daniel Sánchez

89.1 An Homage to Professor Lotfi A. Zadeh

Fuzzy Set Theory owe to Prof. L. A. Zadeh not only its existence, but a sustained effort in guiding and inspiring its development and application. This effort continues nowadays, Prof. Zadeh providing motivating ideas and discussions that indicate some of the main paths to follow in this area. There is no doubt that, to a greater or lesser extent, Prof. Zadeh's ideas and personal dedication have influenced all the researchers on Fuzzy Sets. At least, I can say that this is my case. My modest contribution to this book conceived as an homage to Prof. L. A. Zadeh, is to give some insight into his influence in my personal history with, research about, and view on, Fuzzy Set Theory (despite the fact that, like Prof. Zadeh, I really feel uncomfortable when I write about myself).

The title of this contribution will bring to the reader's mind the well-known *Sorites paradox*, for which Fuzzy Sets offer a convenient solution. It also refers to my own personal research history with Fuzzy Sets as one of the smaller grains of sand in the heap of the Fuzzy Set Community; a heap of sand grains having at its core the foundation rock put by Prof. Zadeh in 1965, and which is still growing with his contributions and those of hundreds of researchers all around the world.

89.2 Getting into the Fuzzy Set Community

I started working with fuzzy sets in 1995 when I entered the Ph.D. programme at the Department of Computer Science and Artificial Intelligence (DECSAI) of the University of Granada, under the supervision of Profs. Miguel Delgado and María-Amparo Vila. I had finished my MSc. in Computer Science the same year, where I had my first contact with Zadeh's Fuzzy Set Theory in the course "Knowledge Engineering" taught by Miguel Delgado.

From September 1995 to July 1997 I attended the courses of the Ph.D. Programme, many of them devoted to fundamentals and applications of fuzzy sets and fuzzy logic, taught by DECSAI members Amparo Vila, Miguel Delgado, Antonio González, Juan Luis Castro, Francisco Herrera, Serafín Moral, Jose Luis Verdegay, Ignacio Requena, Luis Miguel de Campos, and María Teresa Lamata, among others.

Eventually, I became assistant lecturer at DECSAI in February 1996. Two important events gave me the opportunity to meet Prof. L.A. Zadeh in Granada that same year, for the very first time. First, in June 28, 1996, I attended his investiture as

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Dr. Honoris Causa by the University of Granada, promoted by DECSAI, being Prof. Miguel Delgado the Sponsor. The picture in figure 89.1 was taken on that occasion. Prof. Zadeh's speech, dedicated to Prof. Enric Trillas and the fuzzy logic community in Spain, was the first time I heard him talking about Soft Computing, Granularity, and Computing with Words, including his well-known examples involving among others the tallness of Swedes.

Shortly afterwards, July 1-5, 1996, the IPMU conference took place in Granada, organized by DECSAI. This was my first *fuzzy* conference ever, though I only participated by helping the organizers and as attendee. The remarkable anecdote for me at that conference came after I was commissioned with my colleague José Manuel Benítez to pick up Prof. Zadeh at his hotel. As we arrived back to the Congress Centre, Prof. Zadeh's friends and most important researchers in fuzzy logic came to greet him, and he kindly introduced us to all of them, one after another, as "Prof. Benítez and Prof. Sánchez, from University of Granada". I was not known at all but to my colleagues in Granada, and I had no paper published at that time! Who can imagine a better way to get introduced to the "fuzzy" research community? Now seriously, what I really appreciated in that occasion is something well known for all the researchers in Fuzzy Sets: Prof. Zadeh's kind and warm attention to everyone that approaches him.

89.3 Fuzzy Quantification and Computing with Words

My research topic as a Ph.D. student was about fuzzy data mining. More specifically, the objective was to study the extension of association rules and approximate dependencies to the case of fuzzy data. For this purpose it is necessary to calculate counts of items in transactions in order to measure their frequency, that is, the cardinality of the set of transactions that contain certain itemsets. Since, in the case of fuzzy data, the membership of items to transactions is a matter of degree, I had to study the issue of cardinality of fuzzy sets and the strongly related problem of fuzzy quantified sentences.

Prof. Zadeh was one of the first contributors to both topics. Fuzzy quantification is about assessing the accomplishment degree in [0,1] of sentences of the form "Q of D are A", where D and A are fuzzy subsets of the same reference set X, and Q is a linguistic fuzzy quantifier defined as a fuzzy subset of the non-negative integers (absolute quantifiers, e.g. "Approximately between 3 and 8") or a fuzzy subset of [0,1] (relative quantifiers, e.g. "Around 40%", "Most"). Zadeh's proposal, dating back to the 80's, and which he uses still in his talks and papers, is to calculate the sigma-count of the fuzzy set (the classical scalar cardinality obtained by adding the membership degrees of all the elements), and then to calculate the membership of the sigma-count to the fuzzy quantifier. During the following years, and up to 1995, there have been other proposals and studies by Ronald Yager, Janusz Kacprzyk, Didier Dubois and Henri Prade, Patrick Bosc et al., and a joint proposal by DECSAI members Juan-Carlos Cubero, Juan-Miguel Medina, Olga Pons, and María-Amparo Vila, among others.

Fig. 89.1. Picture taken in June 28, 1996 on the occasion of the Honoris Causa Doctorate of L. A. Zadeh by the University of Granada. Left to right: Rafael Molina, Antonio González, María-Amparo Vila, Javier Montero, José-Luis Verdegay, Francesc Esteva, L. A. Zadeh, María-Teresa Lamata, Miguel Delgado, Ramón López de Mántaras, María-Ángeles Gil, Serafín Moral, and Enric Trillas.

Fuzzy quantification was the topic of my first research papers, presented in the Spanish conferences about fuzzy logic and technology (ESTYLF) of the years 1997 and 1998. In September 1999 I presented my first two papers in international conferences: one at EUFIT'99 about rules and dependencies, and a second one at the first EUSFLAT conference that took place, jointly with ESTYLF, in Palma de Mallorca. This second paper was about fuzzy quantification again, and gave me for the first time the opportunity to present my research having Prof. Zadeh in the audience.

I was nervous during the presentation, but not only because it was my second international conference, and I had to present in English, but because my paper included some criticisms to Zadeh's approach to quantification. In my Ph.D., which I defended in December of the same year, I had proposed a collection of theoretical properties that any method for evaluating quantified sentences should satisfy and, apart from concluding that no existing method for quantification verified all the properties, I had also proposed some new methods as more suitable alternatives. One of the main criticisms was the very strict behaviour of Zadeh's approach with crisp quantifiers. The example I proposed was more or less the following: let Q_u be the crisp relative quantifier "Strictly greater than u", with $u > 0$, represented by $Q_u(x) = 0$ if $x < u$, and 1 otherwise. Let us consider the quantified sentence "Q_u of X are A" with $|X| = n$ and sigma-count$(A)/n = u$. Then, the evaluation is $Q_u(u) = 1$. Now, diminishing the membership of any element in the support of A by any amount $\varepsilon > 0$, no matter how small, gives a sigma-count $u' = u - (\varepsilon/n) < u$ and the evaluation is $Q_u(u') = 0$, so arbitrarily small changes in the cardinality turn the fulfilment of the sentence from 1 to 0 and vice versa.

As I expected, after my presentation, Prof. Zadeh raised his hand to make a question. But to my surprise, he made no reference to my criticisms about his method. His question was about how my alternative proposals could be employed in a specific Computing with Words task, and was similar to the following: *Let us suppose that given a set X we are told that it is partitioned so that* around half *of the elements in X satisfy A,* a few *satisfy B, and* most *of the rest satisfy C. How many elements are left?*

I had no answer. I was interested in quantification only for the purpose of assessing patterns in fuzzy data mining. But this question made me think. My advisors Miguel and Amparo, that were also present, agreed that it was a really interesting question, and in the years following my Ph.D. and up to this date, fuzzy quantification and how to use it in the setting of Computing with Words has been one of our main joint research lines. A research line that took us eventually beyond fuzzy set theory, as I will explain later, and was motivated by Prof. Zadeh's provocative question.

Much later, I could see that my criticisms to Prof. Zadeh's method were nothing compared to the concerns that several researchers from the scientific community had written, or even declared publicly in his presence, about Fuzzy Set Theory. Prof. Zadeh himself recalls that the first comments on his 1965 paper were "skeptical or hostile", and after a presentation in which Prof. Zadeh was explaining his new notion of Linguistic Variable, he had to hear that his proposals "could be severely, ferociously, even brutally criticized" [1]. Prof. Zadeh's answers to all critics have always been restrained and educated, according to his high human quality.

89.4 An Alternative View of Fuzziness

After several years it became apparent for us that the properties that had been proposed by several researchers for the evaluation of quantified sentences, including those of Ingo Glöckner, Alberto Bugarín et al., and my own proposals, could only be satisfied simultaneously if the logic operations for fuzzy sets verify all the properties of Boolean logic. For instance if we are told that from ten objects, four are of type A, one intuitively expects that six are of type $\neg A$ (excluded middle). If $A = B$ one expects that the amount of objects that are $A \wedge B$ is the same that the amount of objects that are A (idempotency), etc. However, at least in the case of standard Fuzzy Set Theories (FST), it is well known that this is not possible. For instance, Dubois and Prade showed that no standard FST can satisfy idempotency (as well as mutual distributivity) together with the laws of excluded middle and non-contradiction. Zadeh's method based on the sigma-count is able to avoid some of the problems since sigma-count(A)+sigma-count($\neg A$)=$|X|$ using the standard negation, but has other drawbacks.

This problem led us to work with an alternative representation of fuzzy concepts. Since our evaluation methods rely on the use of alpha-cuts, a possible solution was to represent fuzzy concepts by a collection of alpha-cuts whose logical operations had a Boolean structure. We proposed to perform Boolean operation in each level independently. In this *representation and operation by levels*, the result of operations is not a fuzzy set in general, even if we take as input fuzzy sets represented by alpha-cuts. Hence, fuzzy sets can be seen as a particular case of representation by levels, but not closed under operations by levels.

We found that other authors have proposed similar schemes for different purposes. For instance, Dubois and Prade had proposed the notion of *fuzzy elements* and *gradual sets*, which are identical to our scheme, as also the idea of representing a fuzzy set as a *sheaf of sets* proposed by Ulrich Höhle, among other proposals. Perhaps the main novelty in our proposal is not to restrict the representation to be a particular view or representation of fuzzy sets, but going beyond, and seeing it as a representation of fuzzy concepts, fuzzy sets being a particular case.

However, we are not leaving behind fuzzy sets. We think that fuzzy sets are the kind of representations that humans are able to provide and understand. Hence, our scheme is: ask concepts to users as fuzzy sets, operate by levels, and try to show the result as a fuzzy set. For the later purpose, a fuzzy set is viewed as a kind of measure obtained from a random set interpretation of the representation by levels, using the single-point coverage function. From another point of view, the idea is that (as also well known), in order to have a Boolean structure we have to lose the truth-functionality, so concepts are represented by a structure, operations are performed between structures, and fuzzy memberships (as probability values in the case of probability measures) are computed from the structure. Summarizing, a fuzzy set is the collection of values of a measure of "membership" (nothing to do with fuzzy entropy), computed from the representation by levels, for the possible singletons in the universe. Remarkably, when representing fuzzy sets by levels, union and intersection correspond to minimum and maximum (as it is well known), and complement yields a representation which does not correspond to a fuzzy set, but whose single point coverage function is the standard negation of fuzzy sets. Hence, the operations agree with the standard notions of individual operations with fuzzy sets, proposed by Prof. Zadeh. This proposal, which we published in [2], is a way to have a representation of fuzzy concepts with an associated Boolean structure.

Using this proposal we could develop easily a method for evaluating quantified sentences, satisfying all the Boolean properties. The proposal allows to have an element satisfying a concept and its negation to some degree at the same time. Other types of applications needing a Boolean structure can benefit from it and, additionally, extension of crisp operations and definitions to the fuzzy case are straightforward. As a final point in this development, we submitted a paper to NAFIPS 2012, that will take place in Berkeley, showing how this proposal can be used in order to answer questions like that Prof. Zadeh posed to us thirteen years ago. As a final point in this development, I had the opportunity and pleasure to present some of these results to Prof. Zadeh in the occasion of NAFIPS 2012, held in Berkeley, showing how this proposal can be used in order to answer questions like that Prof. Zadeh posed to us thirteen years ago.

89.5 At the European Centre for Soft Computing

Prof. Zadeh's contribution to fuzzy sets and the broader field of Soft Computing includes the promotion or inspiration of several initiatives, like the Berkeley Initiative in Soft Computing (BISC) at the Electrical Engineering and Computer Sciences Department of the University of California. He also suggested to Prof. Enric Trillas to create a research centre in Europe for the promotion of research and development in Soft Computing. This led to the creation of the European Centre for Soft Computing (ECSC) in 2006. Prof. Zadeh was in the first Scientific Committee of the Centre, giving his support to the initiative, and is nowadays its Honorary President. In October 2010 I joined the Centre, working as associate researcher at the Computing with Perceptions Research Unit led by Dr. Gracian Trivino. This unit has the specific mission to work in the development and practical application of the Computational Theory of Perceptions, one of the more recent proposals of Prof. Zadeh. Hence, I owe to him also the opportunity to work and share time and ideas with researchers at the ECSC, which is being so much fruitful for me at the research but also at the personal level.

The ideas of Prof. Zadeh about Computing with Words and perceptions are, in my opinion, one of the main lines of present and future research in Soft Computing. In my view, fuzzy concepts come exclusively from the human brain, and we use them in order to describe our perceptions and to communicate with others by means of natural language. Most of the words and expressions we use daily are affected by

Fig. 89.2. At the conference Banquet of NAFIPS 2012 in the Berkeley City Club: Daniel Sanchez and Lotfi A. Zadeh

fuzziness, among other sources of uncertainty which are independent from it, like randomness. Hence, fuzziness is a key point in the design of intelligent systems able to communicate with human beings, and to represent information the way we perceive it. The representation of fuzziness allows to fill the semantic gap between the precise, numerical representation of information in computers, and the concepts and linguistic expressions that we use.

I am collaborating with my colleagues at the ECSC and at DECSAI in researching on these topics, working on the definition of the use of semantic concepts relative to images and data by means of representations of fuzziness, and in how to employ them in order to obtain linguistic descriptions of different kinds of data for practical purposes. Together with all of them, and following the path inspired by Prof. Zadeh, I hope to leave a small grain of sand in this heap.

Acknowledgement. I would like to thank the Editors of this book, Enric Trillas, Rudolf Seising, Claudio Moraga, and Settimo Termini, for their kind invitation to contribute, that gave me the opportunity to pay my modest homage to Prof. L. A. Zadeh.

References

1. Zadeh, L.A.: My Life and Work – A Retrospective View. Applied and Computational Mathematics, Special Issue on Fuzzy Set Theory and Applications 10(1), 4–9 (2011)
2. Sánchez, D., Delgado, M., Vila, M.A., Chamorro-Martínez, J.: On a Non-nested Level-based Representation of Fuzziness. Fuzzy Sets and Systems 192, 159–175 (2012)

90

The Robot and the Butterfly

Elie Sanchez

90.1 What It Is All About

This is a non-technical personal view paper describing how I arrived to the field of fuzzy sets and systems, getting around on the roads opened by Professor Zadeh. Then it is followed by some comments on fuzziness and on the future.

90.2 Discovering and Investigating Fuzzy Sets Theory

The first time I heard of fuzzy sets was during a seminar of Arnold Kaufmann (later on I also read an interview of L.A. Zadeh, called “Les ensembles Flous – un concept précis”, in l’Informatique, 1970) while he was presenting Zadeh’s theory and his own research. He was carrying with him the manuscript of his first book on the subject. The book was entitled “Introduction à la Théorie des Sous Ensembles Flous. Tome I, Eléments Théoriques de Base” (Masson, Paris, 1973). He called them “sous ensembles flous”, i.e. fuzzy subsets, for the class of points, or objects, on which membership functions were defined was not fuzzy. This manuscript was written by hand in India ink on tracing papers. So it was one of my early influences. When the book appeared, it greatly inspired fuzzy-to-become French people (who also read Zadeh’s first papers), European researchers too, but particularly after the English translation of his book: *Introduction to the Theory of Fuzzy Subsets: Fundamental theoretical elements*, Academic Press, 1975. Kaufmann was very prolific and he took especially care of his readers. He wanted them to simply follow his presentations and proofs with a piece of paper and a pencil, so his books were very easy to read. Moreover the concepts and properties he presented were fully illustrated with elaborated examples.

Figure 90.1 is a reproduction of a transparency that I draw for my early lectures when I introduced fuzzy sets in China (following Kaufmann’s lectures) and in Japan. The baldness of the human was meant to recall somebody.

One of my early research works was related to Boolean (then in ternary logic), matrix equations: “*Matrices et Fonctions en Logique Symbolique* (Ph.D. in Mathematics from the Faculty of Sciences of Marseille, 1972). Then, as indicated above, when I discovered fuzzy sets (the term ’fuzzy-logic’ was not used until 1973-74) I naturally found and developed an extension of these works to fuzzy relation equations: “*Equations de Relations Floues*”, with a proposed application to medical diagnosis assistance (Ph.D. in Human Biology from the Faculty of Medicine of Marseille,

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1974) This work was still related to Zadeh's original fuzzy sets with [0,1]-valued membership functions. Then I extended it to L-fuzzy sets, where L was a complete Brouwerian lattice. The aim was to propose the largest class of lattice-valued fuzzy sets to which the resolution methodology could apply. The results appeared in a 1976 paper I wrote as *Resolution of Composite Fuzzy Relation Equations* (Information and Control, 30, 1, 38-48, 1976).

Fig. 90.1. Human reasoning versus computer reasoning

Figure 90.2 is a picture of Prof. Zadeh and me. It was taken during the IEEE Conference on Decision and Control, at New Orleans, Louisiana, on December 1977.

Two years after my first sabbatical research visit to UC Berkeley, following a preliminary lecture at the National Computer Conference (1976) and a memorandum at the Electronics Research Laboratory (1977), I wrote a paper which appeared in the very first issue of the new Journal *Fuzzy Sets and Systems*, vol. 1, 1978, pp. 69–74. The paper was *Resolution of eigen fuzzy sets equations* and it was followed by a much extended version *Eigen fuzzy sets and fuzzy relations*, which appeared in the *Journal of Mathematical Analysis and Applications*, vol. 81 (2), 1981, pp. 399–421. It is worth noticing that it was in this very first issue that Zadeh published his important seminal paper *Fuzzy sets as a basis for a theory of possibility*, *FSS*, vol. 1, 1978, pp. 3–28.

At that time we knew nearly all the papers on fuzzy sets, and personally most of the people working in the field. Since the beginning I believed in the strong potential of this theory, and Zadeh opened many research directions, in a multitude of application fields which otherwise would have been very separate from each other. In fact, we can say that creativity in science comes from bringing together, or complementing, two or more domains, or concepts, that originally were distant and so, provoking the emergence of a new field or of genuine improvements, for example soft computing, fuzzy logic control, fuzzy logic and neural networks, fuzzy logic and genetic algorithms, uncertainty reasoning, etc.

Fig. 90.2. L. A. Zadeh and E. Sanchez, New Orleans, Dec. 1977

90.3 On Fuzziness

When I draw figure 90.3, I thought of simply representing how fuzziness, at least partially, can be illustrated. It consists of a humanoid robot hand gently holding - not an inverted pendulum - but a delicate butterfly.

Any field can be fuzzified and hence generalized. As expressed by Zadeh, what is gained through fuzzification is greater generality, higher expressive power, and enhanced ability to model real-world problems. But, especially from the beginning, this fact contributed to multiple criticisms, founded or not. Sometimes, novel and successful applications emerged, but sometimes almost trivial results were presented. It depends on what one fuzzifies: it is natural to do it with data, facts, inputs of a system etc., corresponding to real-world information and for a model or a system, for example fuzzy logic control contributed to the credit of fuzzy logic. Let me take another example from medical diagnosis assistance (Elie Sanchez, Fuzzy logic and inflammatory protein variations, *Clinica Chimica Acta*, vol. 270 (1), 9 February 1998, pp. 31–42). It is related to Zadeh's relational facet, among the four facets, of fuzzy logic (L.A. Zadeh, plenary lecture *Toward a Restructuring of the Foundations of Fuzzy Logic*, Actes 3èmes Rencontres Francophones sur la Logique Floue

Fig. 90.3. The robot and the butterfly

et ses Applications, Lyon, France, déc. 1997, pp. i–ii, Cépaduès Editions, ISBN 2-85428.466.6.)

The model is of special interest in the processing of borderline cases, allowing a graded assignment of diagnoses to patients. Interpretation of biological analyses suffers from some arbitrariness, particularly at the boundaries of the quantities that are measured, or evaluated. But biologists are familiar with normal variation ranges that are a prerequisite to a proper interpretation of all laboratory tests. So, it is natural to consider propositions like *Haptoglobin is very elevated*, where '*very elevated*' is the result of the fuzzification of a crisp interval. Then, a model can be implemented with fuzzy if-then rules, involving linguistic variables (as defined by Zadeh and it is a notion that exists only in fuzzy logic) – see L.A. Zadeh, *The Concept of a Linguistic Variable and its Application to Approximate Reasoning, II*, Information Sciences, vol. 8, 1975, pp. 301–357. When information on medical knowledge, from symptoms to diagnoses, is established, diagnosis assistance for patients can be processed using possibility measures (also introduced by Zadeh), yielding many important developments. These possibility measures are again a fuzzification of a crisp process. So, fuzzification can be natural or not, depending on problems.

Figure 90.4 is a picture where I am with professors Takeshi Yamakawa and Lotfi Zadeh. It was taken at Kurashiki, Japan, in 1989, during the first annual meeting of the Biomedical Fuzzy Systems Association (BMFSA). Note that T. Yamakawa was the first to design and implement a fuzzy chip, he presented it at a conference in Hawaii in 1984.

Finally, in the picture of figure 90.5, I am with professors Toshio Terano and Lotfi Zadeh, it was taken during the International Conference on Fuzzy Logic & Neural

Networks, Iizuka, Japan, 1990 (organized by T. Yamakawa, chairman of FLSI, the Fuzzy Logic Systems Institute). T. Terano was the director of the Laboratory for International Fuzzy Engineering (LIFE) in Japan.

90.4 About the Future

"It is difficult to make predictions, especially about the future."
Niels Bohr, but also Mark Twain, Albert Einstein, George Bernard Shaw, Winston Churchill, Groucho Marx, and it is a golden rule for economists

In his paper *From Search Engines to Question Answering Systems – The Problems of World Knowledge, Relevance, Deduction and Precisiation* (in: Sanchez, E. (ed.): *Fuzzy Logic and the Semantic Web*, 2006, pp. 163–210), Zadeh expressed:

> *If I were asked, "What is the most challenging problem in the realm of information science and technology?" my unequivocal answer would be: conception and design of question-answering systems. And if I were asked what is likely to be the most important application area for fuzzy logic in coming years, my answer would be (a) improvement of performance of search engines; and (b) upgrading search engines to question-answering systems, or Q/A systems for short.*

This quotation is related to information science and technology, but there are many other domains exhibiting rich developments on fuzziness, in theory as well as in practice. In general, people make predictions in their own research environment, which is natural. So that it is difficult to bring out a general prediction. Who could have predicted the successful applications of fuzzy logic control in the 60's? Moreover new domains of application of fuzzy logic will surely appear in the future.

As pointed out by Zadeh, fuzzy logic has many facets (see above, but also: L. A. Zadeh, Is there a need for fuzzy logic? *Information Sciences*, vol. 178, 2008, pp. 2751–2779). The principal facets are: (a) the logical facet, (b) the fuzzy-set-theoretic facet, (c) the epistemic facet and (d) the relational facet. Most of the practical applications of fuzzy logic are associated with its relational facet. The logical facet may be viewed as a generalization of multivalued logic. The fuzzy-set-theoretic facet is focused on fuzzy sets. The epistemic facet is concerned with knowledge representation, semantics of natural languages and information analysis. The relational facet is focused on fuzzy relations and, more generally, on fuzzy dependencies. The concept of a linguistic variable – and the associated calculi of fuzzy if-then rules – play pivotal roles in almost all applications of fuzzy logic (cf. the biomedical example above). So, predictions can be made within the four facets.

Numerous applications are to emerge in the social sciences, in politics, in medical instrumentation, in economics, in semantic information retrieval, in machine intelligence, in the Semantic Web, etc. The future of fuzzy logic or soft computing will be closely associated with systems with high capabilities in intelligent human-like reasoning. But this statement is too general and vague. In fact we think that the future of fuzzy logic will not be encapsulated in a box, there is no limit to where it can go.

Fig. 90.4. T. Yamakawa, E. Sanchez and L. A. Zadeh, Kurashiki, Japan, 1989

Fig. 90.5. T. Terano, L. A. Zadeh and E.Sanchez, Iizuka, Japan, 1990

91

The Membership of a Fuzzy Set as Coherent Conditional Probability

Romano Scozzafava

91.1 Introduction

A well–known view supported by Zadeh concerns the inadequateness of probability to capture what is usually treated by fuzzy theory. In particular, in his 2002's paper [15] he refers to PT – standard Probability Theory – as not being fit to offer solutions for many simple problems in which a key role is played by (what he calls) a "perception–based information". I agree with Zadeh's position, inasmuch he specifies that by PT he means *"standard probability theory of the kind found in textbooks and taught in courses"*.

Actually, many traditional aspects of probability theory are not so essential as they are usually considered; for example, a strict frequentist interpretation, which unnecessarily restricts the domain of applicability, or the requirement that the set of all possible "outcomes" should be endowed with a beforehand given algebraic structure – such as a Boolean algebra or σ–algebra – or the aim at getting, for these outcomes, *uniqueness* of their probability values, with the ensuing introduction of suitable relevant assumptions (such as σ-additivity, maximum entropy, conditional independence,...).

So our starting point is a synthesis of the available information, expressed by one or more *events*: to this purpose, the concept of event must be given its more general meaning, *i.e. it must not be looked on just as a possible outcome* – a subset of the so–called "sample space", as it is usually done in PT – but expressed by a *proposition*.

The aim of this paper is to show the embedding of fuzzy set theory – and related concepts – in a coherent conditional probability scenario, as done in a series of papers (see, e.g., Coletti & Scozzafava [1], [2], [3], [4], [5] and the book by the same authors [6]). This allows to deal with perception–based information and with a rigorous treatment of the concept of likelihood.

A coherent conditional probability is looked on as a general non-additive "uncertainty" measure $m(\cdot) = P(E|\cdot)$ of the conditioning events. This gives rise to a clear, precise and rigorous mathematical frame, which allows to define fuzzy subsets and to introduce in a very natural way the counterparts of the basic continuous T-norms and the corresponding dual T-conorms, bound to the former by *coherence*, a concept that goes back to de Finetti [10].

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91.2 Coherent Conditional Probability

An *event* can be singled-out by a (nonambiguous) statement E, that is a (Boolean) *proposition* that can be either *true* or *false* (corresponding to the two "values" 1 or 0 of the *indicator* I_E of E).

The "logic of certainty" deals with *true* and *false* as final, and *not asserted*, answers concerning a *possible* event, while two particular cases are the *certain* event Ω (that is always true) and the *impossible* event $\emptyset$ (that is always false): notice that only in these two particular cases the relevant propositions correspond to an assertion. To make an assertion, we need to say something extra-logical, such as "we know that E is false" (so that $E = \emptyset$).

As far as conditional events are concerned, we generalize the idea of de Finetti of looking at a conditional event $E|H$, with $H \neq \emptyset$, as a 3–valued logical entity, which is *true* when both E and H are true, *false* when H is true and E is false, "undetermined" when H is false, by letting instead the third value $t(E|H)$ *suitably depend on the given ordered pair* (E,H) and not being just an undetermined *common value* for all pairs.

It turns out that the above function $t(E|H)$ is a measure of the degree of belief in the conditional event $E|H$, which under suitable – and natural – conditions is a $(\oplus,\odot)$-decomposable conditional measure (see [7]).

In the case that $\oplus,\odot$ are, respectively, the usual sum and product, $t(E|H)$ is a conditional probability (in the sense of de Finetti [9], see also [6], [11], [13],)

Definition 1. *Let $\mathscr{E} = \mathscr{B} \times \mathscr{H}$, with $\mathscr{B}$ a Boolean algebra and $\mathscr{H}$ an additive set (i.e., closed with respect to finite logical sums) not containing $\emptyset$. The function P : $\mathscr{E} \to [0,1]$ is a conditional probability if the following conditions hold:*

(C1) $P(H|H) = 1$, for every $H \in \mathscr{H}$,
(C2) for any $H \in \mathscr{H}$ the function $P(\cdot|H)$ is is a (finitely additive) probability on $\mathscr{B}$,
(C3) for every $A \in \mathscr{B}$, $E \wedge H \in \mathscr{H}$,

$$P(E \wedge A|H) = P(E|H)P(A|E \wedge H).$$

We recall also an easy consequence of the above axioms, i.e. the *disintegration formula* for the probability of an event $E|H$ with respect to a partition of an event H

$$P(E|H) = \sum_{k=1}^{N} P(H_k|H)P(E|H_k) \tag{91.1}$$

The above definition of (conditional) probability is strictly based on the Boolean structure of the domains. Actually, in real problems, logical conditions on the domain can be unrealistic: in fact, the expert (or decision maker) usually has information and interest only on a bunch of (conditional) events.

The concept of coherence, introduced by de Finetti in probability theory, has the fundamental role to manage partial assessments and its enlargements, i.e. it is a tool to check whether a function defined on an arbitrary set of (conditional) events is consistent with a probability and to rule extensions of this function to new conditional events.

Definition 2. *Given an arbitrary set $\mathscr{F} = \{E_i|H_i\}$ of conditional events, a real function P on $\mathscr{F}$ is a coherent assessment if there exists a conditional probability $P'(\cdot|\cdot)$ extending P on $\mathscr{E} = \mathscr{B} \times \mathscr{H}$, with $\mathscr{B}$ the Boolean algebra spanned by the events $\{E_i, H_i\}$ and $\mathscr{H}$ the additive set spanned by the events $\{H_i\}$.*

We recall also the following fundamental result for conditional probability:

Theorem 1. *Let $\mathscr{C}$ be any family of conditional events, and take an* arbitrary *family $\mathscr{K} \supseteq \mathscr{C}$. Let P be an assessment on $\mathscr{C}$; then there exists a (possibly not unique) coherent extension of P to $\mathscr{K}$* if and only if *P is coherent on $\mathscr{C}$.*

Notice that what is usually emphasized in the relevant literature – when a conditional probability $P(E|H)$ is taken into account – is only the fact that $P(\cdot|H)$ *is a probability for any given H*: this is a very restrictive – and misleading – view of conditional probability, corresponding trivially to just a modification of the so-called "sample space" Ω. It is instead essential to regard also the conditioning event H as a "variable", *i.e.* the "status" of H in $E|H$ is not just that of something representing a given *fact*, but that of an uncertain *event* – like E – for which the knowledge of its truth value is not required. In other words, even if beliefs may come from various sources, they can be treated in the same way, since the relevant *conditioning* events – including both *statistical data* and any *perception–based information* – can always be considered as being *assumed* propositions: this means, using a terminology due to Koopman [12], that H must be looked on – even if *asserted* – as being *contemplated*.

Moreover, due to its *direct* assignment as a whole, knowledge – or assessment – of "joint" and "marginal" unconditional probabilities $P(E \wedge H)$ and $P(H)$ are not required, and the *conditioning* event H – which *must* be a *possible* event – may have *zero probability*. So conditioning in a coherent setting gives rise to a general scenario that makes the classic Radon–Nikodym procedure – and the relevant concept of *regularity* – neither necessary nor significant (see [4]).

91.3 Membership Function as Coherent Conditional Probability

Let φ_X be any *property* – in the sequel, to simplify notation we will write simply φ in place of φ_X – related to a random quantity X: notice that a *property*, even if expressed by a statement, does not single–out an *event*, since the latter needs to be expressed by a *nonambiguous* proposition that can be either *true* or *false*. Consider now the **event** E_φ = "You claim φ" and a coherent conditional probability $P(E_\varphi|A_x)$, looked on as a real function $\propto_\varphi(x) = P(E_\varphi|A_x)$ defined on C_X.

Since the events A_x are incompatible, then – by Theorem 5 of the book [6], p.89 – every $\propto_\varphi(x)$ with values in $[0,1]$ is a coherent conditional probability. So we can *define* a fuzzy subset in the following way.

Definition 3. – *Given a random quantity X with range C_X and a related property φ, a fuzzy subset E^*_φ of C_X is the pair*

$$E^*_\varphi = \{E_\varphi\,,\,\propto_\varphi\},$$

with $\propto_\varphi(x) = P(E_\varphi|A_x)$ for every $x \in C_X$.

So a coherent conditional probability $P(E_\varphi|A_x)$ is clearly a measure of how much You, given the event $A_x = \{X = x\}$, are willing to *claim* the property φ, and it plays the role of the membership function of the fuzzy subset E^*_φ.

Notice also that the significance of the conditional event $E_\varphi|A_x$ is reinforced by looking on it as "a whole", avoiding a separate consideration of the two propositions E_φ and A_x.

A fuzzy subset E^*_φ is a *crisp set* when the *only* coherent assessment $\propto_\varphi(x) = P(E_\varphi|A_x)$ has range $\{0,1\}$. Then, by Theorem 20 of the book [6], p.225, a fuzzy subset E^*_φ is a crisp set when the property φ is such that, for every $x \in C_X$, either $E_\varphi \wedge A_x = \emptyset$ or $A_x \subseteq E_\varphi$.

Remark – Let us emphasize that in our context the concept of *fuzzy event*, as introduced by Zadeh [14], is nothing else than a proposition, *i.e.*, an ordinary event, of the kind "You claim the property φ". So, according to the rules of conditional probability – in particular, the "disintegration" formula 91.1, often called in the relevant literature "theorem of total probability" – we can easily compute its probability as

$$P(E_\varphi) = \sum_x P(A_x)P(E_\varphi|A_x) = \sum_x P(A_x)\propto_\varphi(x) \quad , \qquad (91.2)$$

which coincides with Zadeh's *definition* of the probability of (what he calls) a "fuzzy" event.

Notice that this result is only a *trivial consequence of probability rules* and *not* a definition. It puts also under the right perspective the subjective nature of a membership function, showing once again that our approach to probability goes beyond – both syntactically and semantically – the traditional one, denoted PT by Zadeh.

It is now possible to introduce in a very natural way the counterparts of the basic continuous T-norms and the corresponding dual T-conorms, bound to the former by *coherence*. These results and some relevant applications are expounded in the paper by Coletti & Vantaggi [8] in this book.

References

1. Coletti, G., Scozzafava, R.: Conditional Subjective Probability and Fuzzy Theory. In: Proceedings of 18th NAFIPS International Conference, pp. 77–80. IEEE, New York (1999)
2. Coletti, G., Scozzafava, R.: Fuzzy Sets as Conditional Probabilities: Which Meaningful Operations can be Defined? In: Proceedings of 20th NAFIPS International Conference, pp. 1892–1895. IEEE, Vancouver (2001)
3. Coletti, G., Scozzafava, R.: Conditional Probability, Fuzzy Sets, and Possibility: A Unifying View. Fuzzy Sets and Systems 144, 227–249 (2004)
4. Coletti, G., Scozzafava, R.: Conditioning in a Coherent Setting: Theory and Applications. Fuzzy Sets and Systems 155, 26–49 (2005)
5. Coletti, G., Scozzafava, R.: Conditional Probability and Fuzzy Information. Computational Statistics & Data Analysis 51, 115–132 (2006)
6. Coletti, G., Scozzafava, R.: Probabilistic Logic in a Coherent Setting (Trends in logic nr. 15). Kluwer, Dordrecht (2002)

7. Coletti, G., Scozzafava, R.: From Conditional Events to Conditional Measures: A New Axiomatic Approach. Annals of Mathematics and Artificial Intelligence 32, 373–392 (2001)
8. Coletti, G., Vantaggi, B.: Inference With Probabilistic and Fuzzy Information (in this issue)
9. de Finetti, B.: Sull'impostazione assiomatica del calcolo delle probabilità. Annali Triestini 19, 3–55 (1949); Engl. transl. In: Probability, Induction, Statistics, ch. 5. Wiley, London (1972)
10. de Finetti, B.: Teoria della probabilità, Einaudi, Torino (1970); Engl. transl.: Theory of Probability, vol. 1, 2. Wiley, Chichester (1974)
11. Dubins, L.E.: Finitely Additive Conditional Probabilities, Conglomerability and Disintegration. The Annals of Probability 3, 89–99 (1975)
12. Koopman, B.O.: The Axioms and Algebra of Intuitive Probability. Annals of Mathematics 41, 269–292 (1940)
13. Krauss, P.H.: Representation of Conditional Probability Measures on Boolean Algebras. Acta Mathematica Academiae Scientiarum Hungaricae 19, 229–241 (1968)
14. Zadeh, L.A.: Probability Measures of Fuzzy Events. Journal of Mathematical Analysis and Applications 23, 421–427 (1968)
15. Zadeh, L.A.: Toward a Perception-based Theory of Probabilistic Reasoning With Imprecise Probabilities. Journal of Statistical Planning and Inference 105, 233–264 (2002)

Some of My Experiences and Views on Zadeh's Fuzzy Logic

Alejandro Sobrino

The first time I met Lotfi Zadeh was in the *XVI Symposium of Multiple-Valued Logic*, meeting that I co-organized with my colleagues at the University of Santiago de Compostela. Professor Zadeh gave the plenary lecture and I had the opportunity to personally taste his wisdom and kindness. Zadeh impressed me by his naturalness: I still have a napkin written by him suggesting me to pursue a topic while taking a coffee in the airport, before boarding. Some years before, when I began to conduct my doctoral dissertation, I posted him asking for some papers that he kindly sent me in a few weeks. A note of the Zadeh's work that deserves to be highlighted is the clarity of the ideas and the elegance with which he writes them. Reading his papers, we find easily that we are facing new and genuine thoughts that introduce us a virgin field.

My first encounter with fuzzy logic was a Spanish book of logic entitled: 'Introducción a la lógica formal' (*Introduction to Formal Logic*), 1974, by the Spanish philosopher and logician, Alfredo Deaño [1]. The book included the section 'Mas allá de este libro' (Beyond this book) and Zadeh's Fuzzy logic was one of the topics addressed in it. The text approached fuzzy logic in three pages mainly following the G. Lakoff's decription of semantic hedges. Of those pages, the phrase that caught my greatest attention was not inserted in the text, but appeared as a footnote. The footnote was a explanation of why the author refer to this logic as 'the logic of vague predicates' and not as 'fuzzy logic'. He argues that what is fuzzy is not the logic, but the subject of this logic; i. e., the vague predicates. Using a very lucky metaphor, he explains his choise saying that, "*In other words, this would be like sttutering a theory on the causes of stutter*". This sentence intrigued me and troubled me at the same time and caused my calling for fuzzy logic. Fuzzy logic was a crisp logic of fuzziness, a precise logic of imprecision. It would be a few years later when Prof. Trillas, the introducer of fuzzy logic in Spain, helped me to understand this statement in a more informed way, providing me the basics of the Zadeh's fuzzy logic [7].

My approach to fuzzy logic, as happened with most of people from a humanistic field, came from the discontent with the excessive constraints that traditional logic enforces to the natural language and the support for the logical formalism as a way to clarify our thoughts. Perhaps that is an inevitable tension for someone who likes the logic of common language.

Zadeh's Fuzzy logic was a breath of fresh air in an academic environment where classical logic, its principles and metalogical properties, are unquestioned. Frequently, Philosophy' students show certain tendency to reject the accuracy, the

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Fig. 92.1. Alejandro Sobriño and Enric Trillas, in the early 1990s or so

immobility, the eternal truths of a discipline that felt far away from their most pressing concerns. The majority of books on formal logic cover examples from mathematics, but only a few ones include examples on the logical analysis of ordinary language (I remember that Copi's *Symbolic Logic* was perhaps an exception) and even less of texts including philosophical arguments. Classical logic seemed far from philosophical issues, even in its more friendly presentations, as natural deduction calculus. It frequently causes indifference or aversion to Philosophy students. Indifference to those who thought that Philosophical contents exceed the constraints of formal logic; aversion to those who think that Logic induces a kind of Philosophy, the Scientific Philosophy, very related to Science and technology and far from the humanism that have been the mark of Philosophy from centuries.

The rigidity of classical logic favors the attraction for Zadeh's fuzzy logic. In fuzzy logic, as occurs in worldly affairs expressed in natural language, truth is contextual rather than universal, imprecise rather than precise and metalogical properties, as axiomatizability, decidability, consistency and completeness, peripheral than central, as L. Zadeh pointed out in [2]. Fuzzy logic provides models to represent and manage imprecise statements and approximate reasoning, offering inferences that perhaps are not completely necessary, but persuasive, as are the majority of which occur in philosophical reasonings. Philosophy is not, of course, deterministic and neccesary, but free and indeterministic, posing problems that evolve throught History, resolving old opositions in new thesis, in a dialectical movement that seems to be endless. In this regard, Zadeh's fuzzy logic meant for some philosophers an 'open door', as it approaches ordinary reasoning with flexibility, performed by the alternatives in the representation of a vague predicate. However, this enthusiasm soon tempered.

The sense of freedom associated with Zadeh's fuzzy logic attracted many students of Philosophy, but many of them did not remain long under the spell, due to what Deaño emphasized: fuzzy logic is the logic of vague predicates, but it is not a vague logic, as it is guided by precise rules. Fuzzy Logic, as occurs with Probability Theory and the uncertainty, is a formal theory of vagueness or imprecision and, as such, shows a high a degree of determinism about the indeterminism. The illusion to see fuzzy logic as a vague logic, as the logic coupled with our intimate rational desires, vanishes. Fuzzy logic has rules, more flexible and adaptable than classical rules, but rules in the end.

Flexibility and relative normativity are two keys that make Zadeh's fuzzy logic an atractive tool for representing and managing approximate reasoning.

Flexibility because defining truth as 'local truth' it allows incorporate the context to the logical analysis. In fuzzy logic, the context is provided by the universe of discourse. If we evaluate in what degree a 'number is small', we must know in advance to what set of numbers we refer. As is obvious, 4 is small with a different degree in the set $0,\ldots,10$ or in the set $0,\ldots,100$. Thus, vague predicates are contextual and in order to represent them adequately we need to define the universe of discurse, which serves of reference. Usually the universe will be local, adapted to the specific problem analysed and far from the universes of classical logic, frequently related with big, universal and numerical sets, as N, Z, R and so on.

Relative normativity as not everything is permitted. For example, fuzzy logic provides several ways to perform the conjuction of two propositions. If the conjuncts show no-interaction, the *min* election is the right choice. But if the conjuncts show interaction, the product seems to mix better their content. The t-norm of Lukasiewicz ($x \wedge y = max\,\{0, x+y-1\}$) is still other election. A metatheoretic finding shows that, although there are several ways to conjunct, the most common solutions will fall between the *min* (the bigger t-norm) and the Lukasiewic's connective (one of the smaller t-norms). In fuzzy logic, connectives show flexibility, suited to semantic peculiarities of the addressed problem, but the flexibility is not totally open; it is bounded. There are freedom, but with margins. In my view, this result represents a suggestive contribution not sufficiently exploited by philosophers.

Another fascination caused by Zadeh's fuzzy logic comes from the applications. I think it's a privilege to live the time in which emerge the applications from a loved theory. Relating classical logic, a theory that I love too, I saw the birth of computers as domestic devices. Although Philosophy is not very concerned with applications, the emergence of computers provoked new fields of reflection, as Philosophy of Mind, Body-Mind interaction, Philosophy of Technology or Philosophy of AI. For anyone interested in logic, it was fascinating to make a tour from truth-values to logic gates and circuits. In the same way, interested in fuzzy logic as I was, it was exciting to found in newspapers advertisements of all types of appliances with the label 'fuzzy logic controlled', 'fuzzy logic powered' or, even better, to know about the first human voice-controlled helicopter, the prototype developed by Yamaha I had the opportunity to see. Despite a promising start, I think that fuzzy logic applications have stalled in its most successful version: the control of industrial processes. In that field there is a general consensus that fuzzy logic has got a notable success. The achievements in control came from using few rules in order to manage complex situations in scenarios that, if approached by classical methods, need a lot of resources to reach a similar result. But we must go a step further, guessing a problem in which Zadeh's fuzzy logic was essential; an application in which Zadeh's logic is inevitable for solving it. In some popular applications, fuzzy logic has been useful but not essential: the same could be achieved with more means using other tools. Fuzzy logic is essential to manage vague language; threfore, a genuine application will be some in which vague language has a inevitable and irreplaceable role. Is not easy to find a 'crucial' application, but I guess it would be good for the advancement of fuzzy logic.

Successful theories have shady areas, and fuzzy logic is no exception. Encouraged by theoretical and applied progress, many researchers embarked on the road of carry out a massive fuzzyfication of unexpected fields. But even if a subject admits to be tackled from a fuzzy perspective, it does not mean that it is interesting to do so. The extensive fuzzification contributes to the discredit of fuzzy logic. Some problems are visited by fuzzy logic in an irrelevant way, both for the problem and for the fuzzy logic: for the problem if it doesn't demand a fuzzy solution; for fuzzy logic if the provided solution is too local. As we previously said, fuzzy logic is local in the universe, but not in the rules definition. The exaggerations in the fuzzification and the mistake confusing local truth with *ad hoc* solutions led some logicians to speak about fuzzy logic as a 'techno-logic' instead of a true 'logic'. Fuzzy logic is both: there is nothing wrong with logic to be applied if it has local applications based on theoretical foundations.

It is usual to read in the Zadeh's papers that vagueness is a common feature of ordinary language and that it plays a relevant role in the common sense reasoning. That is undoubtly a matter of fact, but it deserves to ask why. Traditionally, linguistic vagueness has been associated to negative connotations and precision to positive ones. Thus, a gesture of displeasure often accompanied an expression as "terribly vague", but a face of satisfaction is presumably shown whenever someone concludes something with 'absolute precision'. If vagueness is negative and precision positive, why natural language is full of vague language instead of precise words seems to be a pertinent question. I have not any answer to this question but I think it would be fruitful to inquiring on it. Perhaps it will be advisable to refer the answer to this question both in a biological and in a cultural framework. Maybe that we use profusely imprecise language because our neural or cognitive configuration is better suited to the use of such words. Or maybe that we abundantly use vague words because we obtain benefits or advantages in the communication process that could not otherwise achieve. Perhaps both of these are the head and the tail of the same coin: a brain can benefit imprecise communication and the success of vague words in imprecise communication favors the neuronal structure that generates it, in a continuous feedback that contributes to the better adaptation of the human species.

It is known the Zeki's paper on the neurology of ambiguity [8], but I don't know any similar work on the neurology of vagueness, only a small and peripheral paragraph from W. Calvin who, in his coauthored book with D. Bickerton – *Lingua ex Machina. Reconciling Darwin and Chomsky with the Human Brain* –, wrote: "*nature seems to like fuzzy edges, at least at the cellular level of organization. Precision is accomplished with large committees redundantly trying to do the same task; precision is often an emergent property of enough imprecise neurons*". Unfortunately, he did not elaborate more on this topic. I guess that in future neurobiology will provide keys to progress in the question of why the vague language is so abundant. Meanwhile, another way is to approach this issue from the advantages that the utility of vague terms involves for communication. As R. Parikh showed in [5], even though people does't share the same extension of a vague predicate, its usage can benefit communication, as their use significantly reduces the answer time to a verbal request. Vague language, far from blocking or impeding the dialogue between speakers, often becomes a ductile and useful strategy for the exchanging of messages. Considering both cooperative and noncooperative games, several researchers show that choosing a vague expression when communicating with the hearer is adapative. There are situations in communication in which vagueness has a higher utility than precision. Vagueness is perhaps frequent because it satisfyes some role in language, either as a representative system or as a communication system. Vagueness is characteristic of our cognitive and perceptive resources, helping to shape us as a species. Cognitively, linguistic vagueness is often associated both with the limitations of our memory and perceptive abilities. In general, humans are unable to remember many things because we have a quite fragmented and fragile memory. Anyone could be the actor of this story: Suppose you have read in a newspaper the exact number of victims caused by the recent tsunami at Japan: 10.321. Perhaps for a moment you can remember the exact figure, but surely after a few minutes, if no extra motivation appear, that number would be distorted or simply wrong if someone request, again, the exact information. Perhaps you would be able to approach it in terms of thousands (10.000) or simply saying 'many'. Regarding perception, perhaps you are able to remember an unfamiliar face a few moments after having seen it, but not perhaps a few days later. Human memory and perception are quite limited and seem to have an unstoppable tendency to the economy, to name approximately what is said or seen, probably to make room for other things that are convenient to remember avoiding the collapse. So, we can conclude that vagueness is a common and relevant factor of our cognition, memory and perception.

As previously said, if vagueness is consubstantial to our cognitive and perceptual apparatus, it should show utility. This happens, perhaps, when we have no well-defined metrics or the metric are not participated equally, with similar skill, by the agents of communication. Expressions such as 'high rate of oxigen saturation' do more than generalize the information contained in a precise measurement as '81%': the word 'high', easely understood by anyone, call for 'caution', 'danger', 'riskiness'. If someone hears that a biological parameter is 'high', he does not need to be a doctor to know that his health needs care. In the dialogic process of communication, not always a crisp figure from an expert carries usefulness for an ordinary person;

in many cases, a vague word says much more about a situation and the actions to be undertaken than the corresponding precise one. Vagueness, far from blocking communication, favors it, avoiding the complexity that involves the use of a precise metric (available equipment, calibration, optimal conditions of measurement, etc...), perhaps do not required in order to satisfy a goal. F. ex. in the context of deciding the amount of medication that should be prescribed to a patient, it is often redundant to know his exact weight. It is enough to discriminate if he is an adult or a child. The use of a generic and unspecific word as 'adult' has cognitive savings without any penalization in the action, i.e., in the correct prescribed dose.

The relevance of vagueness in natural language justified the opportunity of Zadeh's fuzzy logic as a tool for modelling imprecise sentences and managing approximate reasoning. Nevertheless, the concern about the vagueness is not from Zadeh. It can be traced back to Aristotle who, in his *Prior Analytics*, said that a sentence as 'Tomorrow will be a naval battle' is today neither true nor false. Although this sentence was mainly involved with modal logic instead of fuzzy logic, it questioned the Bivalence Principle and, thus, it was considered the origin of the multiple-valued logics, the germ of fuzzy logic. In the last centuty, was Russell in [6] who argued in favor of the logical analysis of vagueness and Black [3] the first who made specific contributions to its measurement with his consistency profiles, according to Dubois & Prade [4], "the ancestors of the membership functions". Zadeh was not the first that dealt with vagueness, but was the pioner suggesting a measure of vagueness, in both senses of quantifying the degree to which an element is a member of a fuzzy set and how vague is that measure. This gave the chance to study vagueness differently, i.e., as a subject about which is possible to provide accurate models and compare them in order to choose the most appropriate. In fuzzy logic the vague phenomenon is approached as fuzziness and, since them, two traditions in the study of the imprecision emerges: the vagueness approach, followed by philosophers critics with fuzzy logic as it disputes the clasical principles of Bivalence, LEM and so on; and the fuzziness aproach, that promotes the study of vagueness as a matter of experimental research. This approach is essentially followed by computer scientists and for some philosophers do not tied by ontic presuppositions. In both traditions, either vagueness or fuzziness, Zadeh played a key role.

Another marked feature of Professor Zadeh is the ability to baptize new fields. 'Fuzzy' was certainly a lucky word to describe, so brief, the logic of vague predicates. It's a short and sonorous label. He promoted also, with great success, the name 'Soft Computing' to denote the set of methodologies that share the family resemblance of being models to represent and manage the imprecision and uncertainty in daily language. More recently, he has avanced 'Computing with words' to address a new model of computation, using words instead of numbers; linguistic labels instead of aritmetic functions. This is the latest but surely not the last invent from Lotfi. Let me to make finally some comments about the name of this field.

'Computing with words' is a sonorous label used by Zadeh to refer "*a methodology in which the objects of computation are words and propositions drawn from a natural language, e.g., small, large, far, heavy, not very likely, the price of gas is low and declining, Berkeley is near San Francisco, it is very unlikely that there will be a* *significant increase in the price of oil in the near future, etc*". Note that Zadeh's examples focuse preferently on vague words or vague sentences. If vagueness is exclusively what is adressed, we should employ more properly the expression 'computing with *vague* words' than 'computing with words' to name this new field. Computing with vague words is a part of computing with words, although probably the main part, because the presence of vagueness in natural language is overwhelming. But natural language includes more than vague words. Anaphora, presuppostion, ambiguities, ellipsis, focus,..., are neither mainly crisp nor vague; have their own idiosyncrasies and requires its own tools for representing and managing the sentences in which they appear. In language, the dichotomy crisp/vague is too thick.

A not uncommon order in natural language is, f. ex., 'safely close the valve presuposes to close before the previous one'. This sentence does not include vague words, only time and presuppositional aspects, but yet we can reasonably say that, if we represent this sentence in a way amenable by a computer, we are doing 'computing with words'. Computing with vague words is a specific task for fuzzy logic, but the language, in its attempt to denote reality -whatever it is-, uses everything at hand (vagueness, time, presupposition,...) with no veto to any particular lexico. Computing with vague words will be a main part of computing with words, but computing with words is not limited exclusively to computing with vague words.

Finally, let me make a brief sketch of Zadeh. Lotfi Zadeh has been a revolutionary. He has changed the face of logic and computation and he has introduced vagueness in the contemporary philosophical debate. His contributions are part of modern Artificial Intelligence and his thoughts a legacy of creativity and depth. I'm very honored for contributing with this short paper to highlith this qualities and I thanks to the editors of this book for making me a partner of this initiative.

Fig. 92.2. Santiago Fernández Lanza, Alejandro Sobrino, Lofti A. Zadeh, Senén Barro and José Angel Olivas in the ISMVL '96, held in Santiago de Compostela, Spain

References

1. Deaño, A.: Introducción a la lógica formal. Alianza Editorial, Madrid (1974)
2. Bellman, R., Zadeh, L.A.: Local and Fuzzy Logic. In: Dunn, J.M., Epstein, G. (eds.) Modern Uses of Multiple-Value Logics, pp. 105–151. Reidel, Dordrecht (1977)
3. Black, M.: Vagueness. An Exercise in Logical Analysis. Philosophy of Science 4, 427–455 (1937)
4. Dubois, D., Prade, H.: Fuzzy Sets and Systems. Theory and Applications. Academic Press, New York (1980)
5. Parikh, R.J.: Vague Predicates and Language Games. Theoria 11(27), 97–107 (1996)
6. Russell, B.: Vagueness. The Australasian Journal of Psychology and Philosophy 1, 84–92 (1923)
7. Trillas, E., Alsina, C., Terricabras, J.M.: Introducción a la lógica borrosa. Ariel, Barcelona (1995)
8. Zeki, S.: The Neurology of Ambiguity. Consciousness and Cognition 13, 173–196 (2004)

93

What Is the Source of Fuzziness?

John F. Sowa

Fuzziness is characteristic of the way people use natural languages. Over the centuries, philosophers, linguists, and logicians independently discovered and commented on many aspects of fuzziness, but without a common foundation for organizing and relating their discoveries. In their historical survey, Dubois, Ostasiewicz, and Prade [2] cited numerous examples:

> Looking back in time, what is really amazing is the diversity of fields, where intuitions about fuzziness were expressed and more or less formalized, and the number of scientists who participated to the emergence of the fuzzy set concept. Also it is surprising to see how long it took before such a simple, although powerful, idea of graded membership, could be cast into a proper, widely accepted mathematical model, due to the far-ranged vision, the tenacity, and the numerous seminal papers of Lotfi Zadeh.

Dubois et al. presented a thorough survey of the mathematical methods for quantifying and computing with and about fuzziness. Zadeh [14] identified fuzzy logic and "computing with words" (CWW). Mendel, Zadeh, and others [6] discussed the challenge of relating the CWW methodology to the semantic issues in linguistics and the technology for natural language processing (NLP). This article surveys the issues and suggests some ways for relating them.

93.1 Fuzziness in Language

According to Heraclitus, *panta rhei* – everything is in flux. But what gives that flux its form is the logos – the words or signs that enable us to perceive patterns in the flux, remember them, talk about them, and take action upon them even while we ourselves are part of the flux we are acting in and on. Modern physics is essentially a theory of flux in which the ultimate building blocks of matter maintain some semblance of stability only because of conservation laws of energy, momentum, spin, charge, and more exotic notions like charm and strangeness. Meanwhile, the concepts of everyday life are derived from experience with objects and processes that are measured and classified by comparisons with the human body, its parts, and its typical movements. Yet despite the vast differences in sizes, speeds, and time scale, the languages and counting systems of our stone-age ancestors have been successfully adapted to describe, analyze, and predict the behavior of everything from subatomic particles to clusters of galaxies that span the universe.

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With such a vast range of topics, no language with a finite vocabulary can have a one-to-one mapping of words to every aspect of every topic. Vagueness is not only inevitable, it is necessary for language to be robust, flexible, and extensible. Dubois et al. cited the logician, philosopher, and scientist Charles Sanders Peirce as "one of the first scholars in the modern age" to point out the importance of vagueness. Peirce wrote a succinct summary of the issues:

> "It is easy to speak with precision upon a general theme. Only, one must commonly surrender all ambition to be certain. It is equally easy to be certain. One has only to be sufficiently vague. It is not so difficult to be pretty precise and fairly certain at once about a very narrow subject." [8, 4.237]

The narrow subjects for which precision is possible are ones that the speakers or authors selected for a specific purpose. In writing dictionary definitions, lexicographers start by defining the most typical examples, such as a chair with a back and four legs. Then they list exceptions that deviate from the type for various reasons. To illustrate that practice, Lehmann and Cohn [5] drew egg-yolk diagrams such as Figure 93.1. Typical chairs are shown in the yolk, unusual chairs are in the egg white, and things that might be used as chairs are just outside the egg.

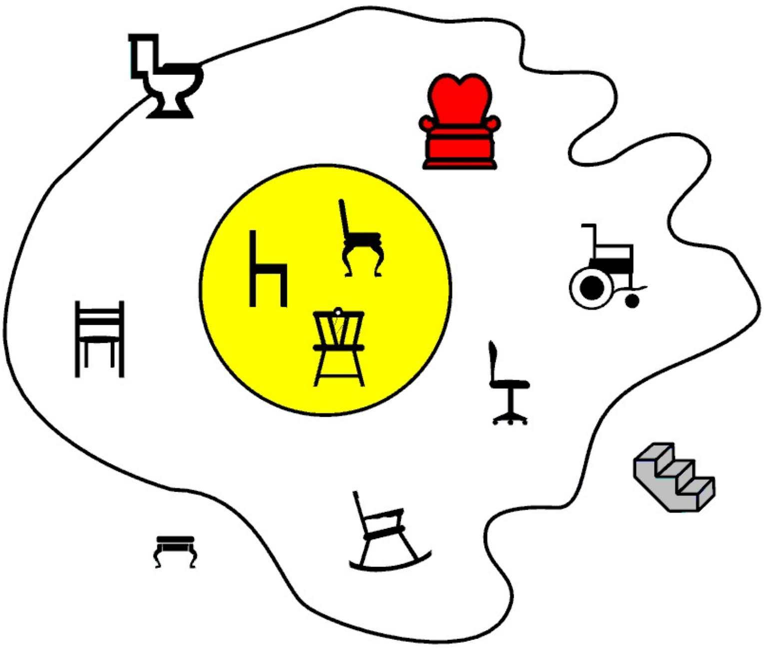

Fig. 93.1. An egg-yolk diagram for the word chair

The boundaries of the egg and egg-yolk of Figure 93.1 resemble the *level cuts* by Bandler and Kohout [1]. The level 0.9, for example, could be chosen for the boundary of the yolk, which partitions the most typical chairs from the ones that omit or modify some typical characteristics. The level 0.6 could be chosen for the outer edge of the egg. The toilet on the edge of the egg in Figure 93.1 would have that value. The footstool and the stairs, which are just outside the egg, would have values slightly less than 0.6. Yet those numbers by themselves cannot distinguish the significant differences between a folding chair, a rocking chair, a wheelchair, and a chair that has wheels at the bottom of a pod. The numbers are important for computing with words, but the reasons why those chairs differ from typical chairs are also important. Mendel et al. [6] noted “Numbers alone may not activate the CWW engine.”

In the 19th century, William Whewell and John Stuart Mill debated the methods for representing and reasoning about variability. Whewell [10] described the practice of biologists, who base their classifications on a type specimen for each species and a type species for each genus:

> Natural groups are given by Type, not by Definition. And this consideration accounts for that indefiniteness and indecision which we frequently find in the descriptions of such groups, and which must appear so strange and inconsistent to anyone who does not suppose these descriptions to assume any deeper ground of connection than an arbitrary choice of the botanist. Thus in the family of the rose tree, we are told that the ovules are very rarely erect, the stigmata usually simple. Of what use, it might be asked, can such loose accounts be? To which the answer is, that they are not inserted to distinguish the species, but in order to describe the family, and the total relations of the ovules and the stigmata of the family are better known by this general statement....
> Though in a Natural group of objects a definition can no longer be of any use as a regulative principle, classes are not therefore left quite loose, without any certain standard or guide. The class is steadily fixed, though not precisely limited; it is given, though not circumscribed; it is determined, not by a boundary line without, but by a central point within; not by what it strictly excludes, but by what it eminently includes; by an example, not by a precept; in short, instead of a Definition we have a Type for our director. [10, vol. 2, pp. 120–122]

Mill [7] dropped the assumption of necessary and sufficient conditions, but he still assumed that types are defined by a set of features or characters stated in words. He weakened the requirements to a preponderance of defining characters:

> Whatever resembles the genus Rose more than it resembles any other genus, does so because it possesses a greater number of the characters of that genus, than of the characters of any other genus. Nor can there be the smallest difficulty in representing, by an enumeration of characters, the nature and degree of the resemblance which is strictly sufficient to include any object

> in the class. There are always some properties common to all things which are included. Others there often are, to which some things, which are nevertheless included, are exceptions. But the objects which are exceptions to one character are not exceptions to another: the resemblance which fails in some particulars must be made up for in others. The class, therefore, is constituted by the possession of all the characters which are universal, and most of those which admit of exceptions. [7, p. 277]

Both Whewell and Mill assume a range of variability in nature, but they propose different ways of measuring it. Instead of "a boundary line without," Whewell suggested "a central point within." But that criterion would require some measure of the distance between any instance and the type specimen. Instead of using a specimen, Mill defined his measure of similarity by enumerating the "characters" of a definition. In theory, Whewell's method is closer to nature, since it is based on a specimen taken from nature. In practice, both methods are based on words. Whewell uses descriptions of specimens, and Mill uses definitions abstracted from the descriptions. Whewell's method is one step closer to nature, but it depends on the words that biologists choose to describe nature.

The psychologist Eleanor Rosch wrote her bachelor's thesis on Wittgenstein's classification by family resemblance and her PhD dissertation on its psychological basis. Rosch and Mervis [9] concluded that family resemblances characterize "prototype formation as part of the general process by which categories themselves are formed." They cited Zadeh [13], but their analysis is closer to Whewell and Mill. They agree with Whewell that prototypes are the basis for classification. But they also give some support to Mill because the prototypes that people naturally choose are the ones that have the largest number of attributes or resemblances that characterize the category. These observations suggest that the cognitive basis for classification is a fuzzy kind of similarity, not rigid definitions or identity conditions. But if human thought is ultimately fuzzy, how is precise reasoning possible in science and mathematics?

Unlike Rosch and Mervis, who searched for a cognitive source of fuzziness, Immanuel Kant [4] maintained that the open-ended variability of nature is the cause of fuzziness:

> Since the synthesis of empirical concepts is not arbitrary but based on experience, and as such can never be complete (for in experience ever new characteristics of the concept can be discovered), empirical concepts cannot be defined. Thus only arbitrarily made concepts can be defined synthetically. Such definitions ... could also be called declarations, since in them one declares one's thoughts or renders account of what one understands by a word. This is the case with mathematicians. [4, §103, p. 219], [3, p. 142]

In short, a precise definition is only possible when the author has complete control over the subject matter. But all authors control their subject to some extent. The critical questions are how, why, and to what extent.

93.2 Mathematical Language

Most mathematicians and logicians pay little attention to vagueness in ordinary language because their language is not vague. They are careful to use consistent definitions within a single document, but they often use different definitions in different documents. Therefore, mathematicians cite or restate the critical definitions and assumptions in every publication. Even the word *number*, the most fundamental in all of mathematics, has a long history of definitions that evolved over the centuries. Figure 93.2 shows an egg-yolk diagram for the many meanings of the word *number*.

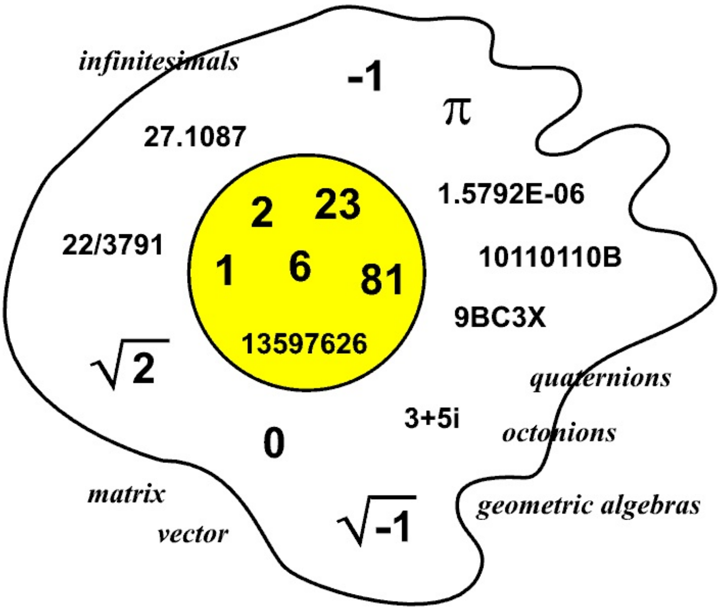

Fig. 93.2. An egg-yolk diagram for the word *number*

The yolk of Figure 93.2 shows the positive integers, which were discovered or invented by our-stone age ancestors. The egg white includes generalizations that can be mixed with the integers in the common arithmetic operations: rational numbers, irrational numbers, zero, negative numbers, and various encodings designed for computers. On the border or outside the egg are mathematical systems that use some of the mathematical operators, but with more variations. All these things called numbers share a family resemblance, as Wittgenstein [11] said:

> Why do we call something a "number"? Well, perhaps because it has a direct relationship with several things that have hitherto been called number; and this can be said to give it an indirect relationship to other things we call by the same name. And we extend our concept of number as in spinning

> a thread we twist fibre on fibre. And the strength of the thread does not reside in the fact that some one fibre runs through its whole length, but in the overlapping of many fibres. [11, §67]

Wittgenstein used the word *Sprachspiel* (language game) for various ways of using language. He compared the words of language to the pieces in a game of chess. The rules of chess are as precise as any version of mathematics, but some people define new rules that use the same pieces in a different way. In mathematics, the oldest games with numbers are counting, simple arithmetic, bookkeeping, and banking. But mathematicians have used the same symbols for different games, as Figure 93.2 illustrates. In each game, precision is possible because all the players agree to a fixed set of rules for using a fixed set of symbols or pieces. The word *number* has a precise meaning in each game, but when taken out of context, the word is ambiguous.

93.3 Relating Patterns to Patterns

When words are used to express novel experiences, they acquire new meanings or senses. But words seldom occur in isolation. They normally occur in larger patterns in which the senses of multiple words shift in a systematic way. Telephones, for example, led to new patterns for the words *talk*, *call*, and *conversation*. Cell phones enabled new patterns of activities, which led to further shifts in the senses of the words that express them. Smart phones combine those patterns with modified patterns of words for activities related to cameras, computers, GPS location, maps, games, television, and shopping. At each stage, old words are used in novel combinations, such as *cell phone* and *smart phone*. But even words that occur in the old lexical patterns acquire new senses from the novel activities they express.

In science, collections of patterns form theories. In other fields, they are called models, blueprints, project plans, or syndromes. Whatever they're called, collections of patterns are expressed in notations for which precision is important. Yet scientists are always aware of the experimental error, which they try to limit by carefully controlled experiments. Engineers express their frustration in a pithy slogan: All models are wrong, but some are useful. To bridge the gap between theories and the world, Figure 93.3 shows a model as a Janus-like structure, with an engineering side facing the world and an abstract side facing the theories. On the left is a picture of the physical world, which contains more detail and complexity than any humanly conceivable model or theory can represent. In the middle is a mathematical model that represents a domain of individuals **D** and a set of relations **R** over individuals in **D**. If the world had a unique decomposition into discrete objects and relations, the world itself would be a universal model, of which all accurate models would be subsets. But the selection of a domain and its decomposition into objects depend on the intentions of some agent and the limitations of the agent's measuring instruments. Even the best models are approximations to a limited aspect of the world for a specific purpose.

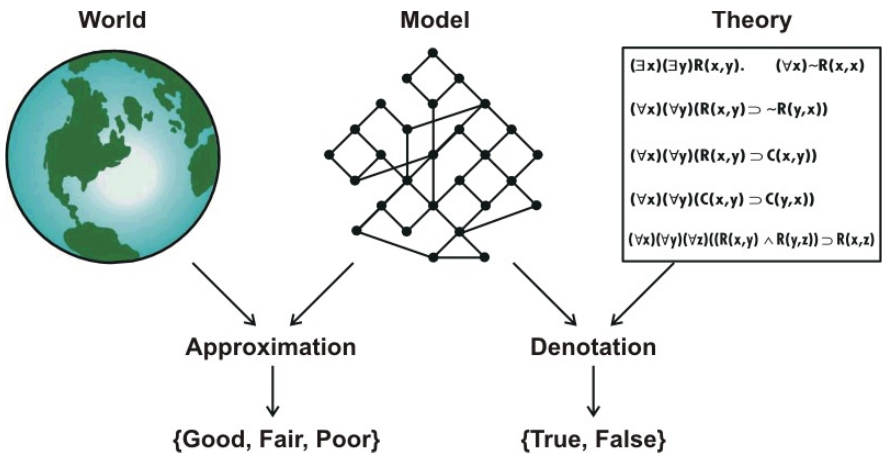

Fig. 93.3. Relating a theory to the world

The two-stage mapping from theories to models to the world can reconcile a Tarski-style model theory with the fuzzy methods pioneered by Lotfi Zadeh. In Tarski's models, each sentence has only two possible truth values: true, false. In fuzzy logic, a sentence can have a continuous range of values from 0.0 for certainly false to 1.0 for certainly true. Hedging terms, such as likely, unlikely, very nearly true, or almost certainly false, represent intermediate values. The two-stage mapping of Figure 93.3 makes room for both kinds of reasoning: a rigorous two-valued logic for evaluating the truth of a mathematical theory in terms of a model; and a continuum of fuzzy values that measure the suitability of a particular model for a specific application. Such two-stage mappings have long been used in science and engineering: a strict two-valued logic for mathematical reasoning, and a continuum of values for quantifying experimental error and degree of approximation.

As Peirce said, "Logicians have too much neglected the study of vagueness, not suspecting the important part it plays in mathematical thought" ([8], 5.505). In that same section, he said that the defining characteristic of a vague sentence is a violation of the law of contradiction: if the sentence *s* is vague, both *s* and *not s* can be true. Zadeh drew the following distinction [6]:

> Fuzzy relates to un-sharpness of class boundaries, while vagueness relates to insufficient specificity. As an illustration, "I'll be back in a few minutes" is fuzzy, but not vague. While "I'll be back sometime" is both fuzzy and vague... Usually, what is vague is fuzzy, but not vice-versa.

In practice, the word *sometime* often becomes *never*. With that qualification, Zadeh's examples are consistent with Peirce's criterion. But Peirce also distinguished vagueness from generality. For example, the general word *animal* is underspecified in comparison to *raccoon or beaver*, but it's not vague.

In summary, Lotfi Zadeh should be congratulated for introducing a fruitful paradigm that has stimulated a large body of research with many valuable applications.

The CWW methodology has introduced new ways of analyzing language and applying computable algorithms. But the discussions in the article by Mendel et al. [6] show that CWW is unrelated to current linguistic research. More collaboration could help both fields clarify the sources of fuzziness.

References

1. Bandler, W., Kohout, L.J.: Special Properties, Closures and Interiors of Crisp and Fuzzy Relations. Fuzzy Sets and Systems 26, 317–332 (1988)
2. Dubois, D., Ostasiewicz, W., Prade, H.: Fuzzy Sets, History and Basic Notions. In: Dubois, D., Prade, H. (eds.) Fundamentals of Fuzzy Sets. Kluwer, Dordrecht (1999)
3. Hartmann, R.S., Schwarz, W.: Logic. Dover, New York (1988)
4. Kant, I.: Logik: Ein Handbuch zu Vorlesungen, Königsberg 1800, edited by Jäsche, G.B., English translation: [3] and [12]
5. Lehmann, F., Cohn, A.G.: The Egg-yolk Reliability Hierarchy: Semantic Data Integration Using Sorts with Prototypes. In: Proceedings of the Conference on Information and Knowledge Management, CIKM 1994, Gaithersburg, Maryland, November 29-December 2, pp. 272–279. ACM Press, New York (1994)
6. Mendel, J.M., Zadeh, L.A., Trillas, E., Yager, R., Lawry, J., Hagras, H., Guadarrama, S.: What Computing with Words Means to me. IEEE Computational Intelligence Magazine, 20–26 (February 2010)
7. Mill, J.S.: A System of Logic. Longmans, London (1865)
8. Peirce, C.S.: Collected Papers of C. S. Peirce. In: Hartshorne, C., Weiss, P., Burks, A.W. (eds.), vol. 8, pp. 1931–1958. Harvard University Press, Cambridge (1958)
9. Rosch, E., Mervis, C.B.: Family Resemblances: Studies in the Internal Structure of Categories. Cognitive Psychology 7, 573–605 (1975)
10. Whewell, W.: History of Scientific Ideas. J. W. Parker & Son, London (1858)
11. Wittgenstein, L.: Philosophical Investigations. Basil Blackwell, Oxford (1953)
12. Young, J.M.: Lectures on Logic. Cambridge University Press, Cambridge (1992)
13. Zadeh, L.A.: Fuzzy Sets. Information and Control 8, 338–353 (1965)
14. Zadeh, L.A.: Fuzzy Logic = Computing With Words. IEEE Transactions on Fuzzy Systems 4(2), 103–111 (1996)

94

Vague Computing Is the Natural Way to Compute!

Apostolos Syropoulos

94.1 Introduction

Typically, when a computer performs a task, it can be seen as a calculation or a reckoning. For example, consider a simple arcade video game where the machine continuously gets input from the user and computes the new position of some "characters" that move on a board, etc. A particularly interesting aspect of computation is that the majority of people understand it as an exact function. Nevertheless, this is an excessive expectation or requirement, depending on how one perceives computation. In particular, users expect computer programs to deliver exact results while computer programmers work under the assumption that everything is precisely defined and no vagueness arises anywhere. But is this a reasonable assumption?

The answer to this question is not a simple one. For instance, we are successfully using computers that operate in a precise manner for many years and we have achieved much with these devices. Or is this an oversimplification of what actually happens in reality, whatever this may mean? Obviously, digital computers execute software in the expected way as long as hardware operates within some tolerance range. So one may be tempted to say that everything related to computers is based on an illusion or a rough assumption. One may argue that this is an exaggeration, nevertheless, it is a view that may help us understand things differently.

Vagueness is widely accepted to characterize terms that, to some extent, have borderline cases, that is, a case in which it seems impossible either to apply or not to apply a vague term. The Sorites Paradox, which was introduced by Eubulides of Miletus, is a typical example of an argument that shows what it is meant by borderline cases. The term sorites derives from the Greek word soros, which means "heap." The paradox is about the number of grains of wheat that make a heap. All agree that a single grain of wheat does not comprise a heap. The same applies for two grains of wheat, three grains of wheat, etc. However, there is a point from which the number of grains becomes large enough to be called a heap, but there is no general agreement as to where this occurs, hence the paradox.

In general, there are everyday objects and activities that seem to be exact, yet they are vague! For example, "[e]xperience has shown that no measurement, however carefully made, can be completely free of uncertainties" [15, p. 3]. Remarks like this one may have some "unexpected" consequences. For instance, one might go as far as to argue that vagueness is the norm and exactness the exception! If this is not an exaggeration, which is not as I will show later on, then one could reasonably argue

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that many, if not most, things are vague by definition. Thus, one should be able to employ vagueness in computation or, even, she should to be able to perform truly *vague* computations, whatever that may mean.

94.2 What Is Vagueness?

Bertrand Russell [9] was perhaps the first thinker who had given a definition of vagueness: "*Per contra*, a representation is *vague* when the relation of the representing system to the represented system is not one-one, but one-many." According to this view, a photograph that is so smudged that it might equally represent three different persons is vague. Building on Russell's ideas Max Black [2] had argued that most scientific theories, computability theory included, are "ostensibly expressed in terms of objects never encountered in experience." Black [2] proposed as a definition of vagueness the one given by Charles Sanders Peirce: "A proposition is vague when there are possible states of things concerning which it is intrinsically uncertain whether, had they been contemplated by the speaker, he would have regarded them as excluded or allowed by the proposition. By intrinsically uncertain we mean not uncertain in consequence of any ignorance of the interpreter, but because the speaker's habits of language were indeterminate." According to Black, the word *chair* demonstrates the suitability of this definition. But it is the "variety of applications to objects differing in size, shape and material" that "should not be confused with the vagueness of the word." In different words, vagueness should not be confused with *generality*. Russell and Black had argued against this misconception. A term or phrase is *ambiguous* if it has at least two specific meanings that make sense in context. Thus, one should not confuse ambiguity with vagueness.

It is widely accepted that there are three different expressions of vagueness [11]:

Many-Valued Logics and Fuzziness. Borderline statements are assigned truth- values that are between absolute truth and absolute falsehood. In the case of fuzziness, truth-values are usually drawn from the unit interval.
Supervaluationism. The idea that borderline statements lack a truth value.
Contextualism. The truth value of a proposition depends on its context (i.e., a person may be tall relative to American men but short relative to NBA players).

94.3 From Exact Computing to Fuzzy Computing

Conceptual computing devices are idealizations of tools that can perform computations. However, these idealizations tend to overlook details concerning the process of computation. This is exactly where vagueness, in general, and fuzziness, in particular, comes into play. I will try to be more specific by presenting two exact models of computation, namely Turing machines and P systems, and how one can easily fuzzify these models. Let me start with Turing machines, which are considered to be the archetypal model of computation.

Turing machines were introduced by Alan Mathison Turing [16] in order to give a formal definition of the notion of computation. In addition, the machine was used in order to give an answer to the *entscheidungsproblem* posed by David Hilbert (i.e., a problem that can be answer with yes or no, in different words a decision problem). Typically, a Turing machine consists of an infinite tape, a controlling device, and a scanning head. The tape is divided into an infinite number of cells. The scanning head can read and write symbols in each cell. The symbols are elements of some set Σ. At any moment, the machine is in a state q_i, which is a member of a finite set Q. What should happen next depends on the symbol just read and the current state and this is hardwired into the controlling device. If no action has been specified for a particular combination of state and symbol, the machine halts. Tuples that conditionally describe the next action are called configurations.

At this point, it is rather interesting to note that Carole E. Cleland [3] has concluded that "Turing machines may be characterized as providing procedure schemas, i.e., temporally ordered frameworks for procedure." In addition, she has claimed that "Turing machine instructions cannot be said to prescribe actions, let alone *precisely* describe them." Based on these one could argue that Turing machines are not computing devices. Surely, this is an exaggeration, nevertheless, it clearly shows that this model of computation is not as well-thought-of as it was always considered to be. Furthermore, Cleland has argued against the idea that Turing machine "symbols" are genuine symbols.

Such remarks and conclusions clearly show that the Turing machine model of computation is implicitly vague. Thus, it does make sense to explicitly introduce vagueness into this model. Indeed, first Lotfi A. Zadeh [17] *vaguely* described a fuzzy Turing machine where configurations form a fuzzy subset. Based on Zadeh's ideas, Eugene S. Santos [10] had formally defined fuzzy Turing machines. The evolution of fuzzy Turing machine, in particular, and fuzzy computing devices, in general, is described in a forthcoming book by this author [13].

P systems is a model of computation that was introduced and popularized by Gheorghe Păun [8]. P systems are conceptual computing devices made up of nested compartments surrounded by porous membranes that define and confine these compartments. Initially, each compartment contains a number of possibly repeated objects, that is, a multiset of objects. When "computation" commences, compartments exchange objects according to a number of multiset processing rules that are associated with each compartment. The activity stops when no rule can be applied. The result of the computation is equal to the number of objects that reside in a designated compartment called the *output membrane*.

As in the case for Turing machines, one can easily see that vagueness is part of this machinery. First, one can never be sure that membranes contain exact copies of some object—it is more reasonable to expect copies to be similar. Also, one may argue that the rules should not be exact, but should give an "outline" of what may happen. These and other aspects of P systems have been studied by this author [12, 14].

94.4 The Need for Fuzzy Computing

Unfortunately, the notion of *uncertainty* is considered by many to be almost equivalent to vagueness, which, of course, is wrong. This is one reason why there is a debate over the superiority of either fuzzy set theory or probability theory to represent vagueness. Clearly, this debate is far from settled. Basically, there are three views—one that naturally claims that fuzzy set theory has nothing new to offer, one that advocates that fuzzy sets and probabilities are two facets of uncertainty (e.g., see [18]), and one that assumes that fuzziness is a fundamental property of our world. There is no question that the first view is deeply flawed. The second view is also problematic, since it considers vagueness and uncertainty to be the same thing. The third view, is, in my eyes, the most reasonable approach. In particular, when I say more fundamental, I mean that most, if not all, natural processes can be characterized as vague, while probabilities are "theoretical quantities which, once the sets and the measure functional on those sets are chosen ('the model'), are capable of being calculated exactly and are perfectly definite (real) numbers which contain no reference to chance" [4, p. 45]. Last, but certainly not least, Bart Kosko [7] has also convincingly argued that fuzzy set theory is more fundamental than probability theory.

One reasonable question that may pop on one's mind is the following: If vagueness is a fundamental property of our world, how should this affect the way we compute? First, let me stress that until now vagueness was not taken under consideration by any computing device. Engineers have employed various techniques in order to ensure that a "digital logic" is correctly implemented, yet they did so using *vague* constituents! Next, one could argue that just like probabilities are employed in ordinary (aka crisp) computer programs, one could analogously use fuzziness in crisp programs. Indeed, one can implement fuzzy databases, fuzzy programming languages, etc. [6]. Nevertheless, this approach has a serious drawback—it implicitly implies that vague computing can be implemented in machines that operate in an "exact" manner. So, if vagueness is a fundamental property of our world, why should we add an artificial layer to perform vague computational tasks? The answer is not easy, but the reason for this apparent disparity lies in the way we have learned to think. From ancient times, people tried to think in terms of pure and precisely defined objects. In addition, simple things such as reckoning were considered precise operations. For example, two plus two equals four since when one has two sheep and gets two more sheep, she has four sheep in the end. However, what happens when one exchanges two really well-fed animals with two starving animals? In principle, she still has four animals, but they are not the same! Thus, one can argue that even arithmetic is the result of an oversimplification. In different words, exactness should be considered as a limit case and vagueness the norm and not the other way around!

From the discussion so far one may wrongly deduce that there is no fuzzy hardware when, in fact, a good number of researchers is working in the design and construction of real fuzzy hardware.[1] Although there is fuzzy hardware, there is nothing

[1] For example, see [5] for a not so up-to-date account of fuzzy hardware.

that can be classified as a general purpose fuzzy computer. Nevertheless, it is more than necessary to build such a machine in order to be able to fully exploit vagueness in computing. I expect that such machines will be able to solve more easily *every-day* problems that concern *ordinary* people. Since such machines should be equipped with the analog of an operating system and the corresponding tools for programming, editing, etc., more research on fuzzy programming and computing should be carried out. For instance, the work on the definition of a fuzzy version of the λ-calculus by Daniel Sánchez Alvarez and Antonio F. Gómez Skarmeta [1] can be seen as step towards this goal.

Fig. 94.1. Artistic impression of a fuzzy computer. Original drawing by Nikos Amiridis; post-processing with gimp by Apostolos Syropoulos.

References

1. Sánchez Álvarez, D., Gómez Skarmeta, A.F.: A Fuzzy Language. Fuzzy Sets and Systems 141, 335–390 (2002)
2. Black, M.: Vagueness. An Exercise in Logical Analysis. Philosophy of Science 4(4), 427–455 (1937)
3. Cleland, C.E.: On Effective Procedures. Minds and Machines 12, 159–179 (2002)
4. Cook, D.B.: Probability and Schrödinger's Mechanics. World Scientific, Singapore (2002)
5. Kandel, A., Langholz, G. (eds.): Fuzzy Hardware: Architectures and Applications. Kluwer Academic Publishers, Dordrecht (1997)
6. Klir, G.J., Yuan, B.: Fuzzy Sets and Fuzzy Logic: Theory and Applications. Prentice Hall (Sd) (1995)
7. Kosko, B.: Fuzziness vs. Probability. International Journal of General Systems 17(2), 211–240 (1990)
8. Păun, G.: Membrane Computing: An Introduction. Springer, Berlin (2002)
9. Russell, B.: Vagueness. Australasian Journal of Philosophy 1(2), 84–92 (1923)
10. Santos, E.S.: Fuzzy Algorithms. Information and Control 17, 326–339 (1970)

11. Sorensen, R.: Vagueness. In: Zalta, E.N. (ed.) The Stanford Encyclopedia of Philosophy, Fall 2008 edition (2008)
12. Syropoulos, A.: Fuzzifying P Systems. The Computer Journal 49(5), 619–628 (2006)
13. Syropoulos, A.: Fuzzy Computability. Book in preparation (2012)
14. Syropoulos, A.: On Generalized Fuzzy Multisets and their Use in Computation. To appear in the Iranian Journal of Fuzzy Systems (2012)
15. Taylor, J.R.: An Introduction to Error Analysis: The Study of Uncertainties in Physical Measurements, 2nd edn. University Science Books, Sausalito (1997)
16. Turing, A.M.: On Computable Numbers, With an Application to the Entscheidungsproblem. Proceedings of the London Mathematical Society 42, 230–265 (1936)
17. Zadeh, L.A.: Fuzzy Algorithms. Information and Control 12, 94–102 (1968)
18. Zadeh, L.A.: Discussion: Probability Theory and Fuzzy Logic Are Complementary Rather Than Competitive. Technometrics 37(3), 271–276 (1995)

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Fuzziness: Came for the View, Stayed for the Same

Marco Elio Tabacchi

95.1 How I Met Fuzziness

The first time I came across Fuzzy Sets, my first point of contact with Soft Computing, I was a young and inexperienced student of Computer Science in my hometown University. I had just completed the first semester, and as an aside of an image processing curriculum a loud-spoken, very kinetic lecturer spent a couple of hours introducing us to the mysteries of the idea of degree and Fuzzy arithmetic. The whole thing had a sort of secret sect bent, something like the secret tools that the powers to be don't want you to know and use – a probably unwitting attitude I have seen many times in the community and that has harmed more than helped, but that on an unexperienced and young student as myself undeniably had a major fascination. Such fascination was further built up the same day, when during a routinary trip to the main street bookstores I found on the Popular Science shelf an italian copy of Kosko's *Fuzzy Thinking* [2]. The coincidence seemed odd, so I bought the book which is in my bookshelf yet and whose first two chapters I still use as student's material for my AI course, skimmed from the occasional psychobabble here and there – but hey! what should I have expected from something published by Disney Books? I swallowed the book whole and in the following days I tackled the lecturer and expressed my enthusiasm for the field and my appreciation for the elements of oriental philosophy behind the idea. He slapped me in the head, and with the soft voice of thunder explained me that the real deal was in the applications, and how the new concept was perfect for describing phenomena that were much more complicated to describe otherwise and for operating on them without resorting to the intricacies of calculus. To demonstrate the point he told me his ideas for a picture search engine based on fuzzy distances – we were years from google image back then, or even google itself. This idea resonated with my passion for images, as in a previous professional life I was a fine artist, and from this meeting started my first collaboration to a published scientific paper. My work with the boisterous Vito Di Gesù continued for the following ten years, first as his student and then as a collaborator, and all of our common research was focused on applying soft computing methodologies to classification problems in arts and cognitive science, the label under which I had then switched my research.

In the meantime I become the Scientific Director at *Istituto Nazionale di Ricerche Demopolis*, and once again Soft Computing showed its bacon-saving properties. In demoscopic research one of the classic "huge pains" is the analysis of so called open modalities, or the kind of response to a question where the subject can freely express

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opinions using natural language and is not restricted to a serie of choices. This freedom brings, as in many other language related problems, a thick layer of complexity, only aggravated by the sheer multiplicity of contexts – as any subject can choose to use adjectives, phrasal verbs, entire wikipedia entries and the like – and the cardinality effect – a typical poll during election weeks ranges between 20k and 30k subjects. We were able to build a tool that fully exploits Fuzzy Sets Theory's power of representation to discover at a glance powerful connections between opinions expressed in natural language for a wide range of topics.

The untimely demise of Vito left me without a Fuzzy context for a little while, but proving that sometimes lightnings can really strike twice, Settimo Termini of Fuzzy Entropy fame [1] took me as a collaborator. Settimo is a physicist by training, as Vito was, but his familiarity with both the "two words" of human knowledge and his sensibility toward the foundational problems of uncertainty have added a new dimension to my research in Fuzzy Sets. Along the scientific guidance, which is (appropriately) unmeasurable, Settimo also introduced me to a number of very interesting people, confirming that you only can do science you like with people you like. Any list would make omnifarious omissions, but I like to mention Enric Trillas and Claudio Moraga, Rudi Seising, pictured in Figure 1 along Settimo during the second edition of the Saturday's Scientific Conversations, and through him the Californian wave, people such Ron Yager. Plus I got the honour of being introduced for the first time to the grandpa of the science himself, Lotfi Zadeh – twice!

I now see applications of Fuzzy Set Theory, such as the work we have recently authored on Esthetic evaluation and Fuzziness, more as a step toward a more coherent theory of uncertainty, which considers not only degrees of ownership and composition thereof, but the inherent imprecision of (our representations of) things, and the pervasiveness of such concept not only in the more traditional human science, where this is somehow taken for granted, (a malicious person here would specify that this is not true, and that the recent trend of trying to fit the so-called exact sciences' models to humanities has brought disgrace and discredit to both – we state confidently that 33.3% of people like Brand X and then are surprised when opinion polls botch elections after elections, but I don't feel evil at the moment) but also in hard science such as engineering and computing. These years spent in Fuzziness research have surely widened my views on uncertainty, and changed my expectations about what precise measurement can really yield -and why this can be a blessing. This is the message I think is worth passing along to students.

95.2 Where Is Fuzziness Going...

The view from my window on Fuzziness is exactly as vast in scope and exciting in perspective as the one I see when approached this world for the first time. What fifteen years of research in Soft Computing, countless discussions with mentors, colleagues and students, and even more conferences brought to the plate is a more general approach to the problem. Some reflections of Settimo Termini [7], on which I have recently collaborated [9], are worth briefly mentioning here.

Fig. 95.1. SSC 2011 - Left to right: Seising, Trillas, Termini, Moraga. I am the camera.

95.2.1 A Shift from Fuzziness to Vagueness and Uncertainty

Some changes in our ontologies have forced us to consider something new: Fuzzy Sets cannot, alone, model vague predicates in a general sense. Terricabras and Trillas have nailed the complete aptness of Fuzzy Sets for representing vagueness in extensional terms using traditional mathematical means [6], but Pultr [4] has demonstrated that while Fuzzy Sets theory is enriched with a non trivial representation, it is not powerful enough to really represent vagueness and Uncertainty in an universal fashion. As such, in the next years we should see a shift from the concept of membership to more constructive approaches such as Vopenka's AST [10] or Beeson's PST [8].

95.2.2 The Formalization of a Rigorous Notion of Vagueness

We must look for new, different ways of abstracting away: ways that allow the notion of vague predicates to work as well as it informally works in natural languages, and in everyday routine of scientists, without, if possible, losing the representational power of Fuzzy Sets.

95.2.3 The Re-weakening of Borders between Disciplines

Seising has shown how the "the absence of strict boundaries" has influenced the development of disciplines in the last century [5]. Such boundaries have been unfortunately rebuilt, especially between hard and soft sciences, but we continually find analogies between art and technology, showing that the similarities between technological devices and products of art can be stronger than the ones between science and technology [3]. In the coming years the search for even more similarities and analogies should definitively raze such borders to the ground.

95.3 ...and What We Should Do to Bring It There

Many things, obviously. In this brief recollection of ideas I would like to concentrate only on two complementary points, which in my opinion really should be discussed more and brought to the front of the actual debate on how to find the right place for Soft Computing, Fuzzy Sets Theory and the other methodologies pertaining uncertainty in science. It may seem a bit contradictory to foretell going beyond Fuzziness while at the same time hoping for it to be taught everywhere, but the contradiction is only apparent. In my opinion, only by creating in the minds of the people at large a familiarity with the basic concepts of Fuzziness we can then have further advancements at the high end of the research spectrum.

95.3.1 Teaching Fuzzy Logic in School

The introduction of Set Theory in schools have certainly changed the way children, and then adults, perceive the basic mathematical concepts, and have decisively facilitated for more and more people the grasp of basic elements of logic, something which really makes a difference in the way lives are lead. Now it would probably be the right time to supplement this knowledge with a small Fuzziness module. A deep understanding of Fuzzy Sets and Logic seem really to be easy to attain for children: all basic concepts are easy to grasp due to their natural derivation, and even easier to illustrate graphically (the great power of representation of Fuzzy Sets has always been one of the strong suits of the field). Even Computing with Words can probably be simplified enough to become easily transferrable to children. Such approach should be preceded by ample testing on both adults and children but I am optimistic this could be done with success in a reasonable amount of time: in an unpublished pilot research study in 2002 we have worked on testing the ability of naive youngsters to understand the basic concept of Fuzzy Logic and its operators using different representations, with promising results.

Fig. 95.2. Marco Tabacchi presenting at the 8th WILF International Workshop in Fuzzy Logic 2009, Palermo, Italy, June 9-12, 2009

95.3.2 Establishing University Courses in Fuzzy Logic, Fuzzy Sets Theory and Uncertainty

Even in a context such as Academia, where Fuzzy Logic is born and breed, Fuzziness is relegated to a sideline, often being nothing more than an addendum to other courses. Electronic engineers deal with standard controls and then introduce fuzzy control as an improvement, advanced courses in logic spare some hour for Fuzzy Logic, the occasional Computational Methods course has a couple of lectures dealing with Soft Computing, Quantum Fuzziness is sometimes taught in Quantum Computing curricula. This is really a pity – today the study of uncertainty from a logic and computational standpoint is a mature science, with a heavy load of applications, a sound theoretical foundation and a bright future in expansion. Any CS, EE or VSSP degree would clearly profit of a specific course dealing with uncertainty. When I researched this paper, I could only find one University level course in Italy specifically dedicated to Fuzziness (at Milano's Statale), and no-one in Spain (though this may be in part alleviated by ECSC and its links with Oviedo and other universities). For what else I know situation in other european countries such as France is not that different. Promoting courses which explicitly deal with uncertainty would prepare a new class of degree holders with instruments to tackle a lot of real world problems in a more natural way, and to better deal with the part of our surrounding complexity due not to the our inability to measure precisely, but to the intrinsic impossibility to measure. In the hyper-technologic era we are living trough, this would certainly be a good thing.

References

1. De Luca, A., Termini, S.: A Definition of a Nonprobabilistic Entropy in the Setting of Fuzzy Sets Theory. Information and Control 20(4), 301–312 (1972)
2. Kosko, B.: Fuzzy Thinking. The New Science of Fuzzy Logic. Hyperion Books, New York (1993)
3. Tabacchi, M.E., Termini, S.: Measures of Fuzziness and Information: Some Challenges from Reflections on Aesthetic Experience. In: Proceedings of WConSC 2011 (2011)
4. Pultr, A.: Fuzziness and Fuzzy Equality. Commentationes Mathematicae Universitatis Carolinae 23(2), 249–267 (1982)
5. Seising, R.: On the Absence of Strict Boundaries – Vagueness, Haziness, and Fuzziness in Philosophy, Science, and Medicine. Applied Soft Computing 8(3) (2008)
6. Terricabras, J.M., Trillas, E.: Some Remarks on Vague Predicates. Theoria 4(1), 1–12 (1988)
7. Termini, S.: On Some Vagaries of Vagueness and Information. Annals of Mathematics and Artificial Intelligence 35(1-4) (2002)
8. Beeson, M.: Towards a Computation System Based on Set Theory. Theoretical Computer Science 60(3), 297–340 (1988)
9. Tabacchi, M.E., Termini, S.: Varieties of Vagueness, Fuzziness and a Few Foundational (and Ontological) Questions. In: Proceedings of EusFLAT 2011. Advances in Intelligent Systems Research, pp. 578–583. Atlantis Press (2011)
10. Vopenka, P.: The Philosophical Foundations of Alternative Set-Theory. International Journal of General Systems 20(1), 115–126 (1991)

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On Fuzziness in Complex Fuzzy Systems

Dan E. Tamir, Mark Last, and Abraham Kandel

96.1 Prologue

Abraham Kandel, one of the pioneers of fuzzy logic research, has numerous theoretical and practical contributions in the field of fuzzy systems. Mark Last's main contribution to the field of fuzzy logic includes the introduction of fuzzy based automatic perception and info fuzzy networks. Dan Tamir has been active in the area of formalization of axiomatic fuzzy logic and in applications of fuzzy logic in pattern recognition. One of the recent research threads pursued by Kandel, Last, and Tamir is complex fuzzy logic. This chapter provides a brief review of the contribution of the authors to the field of fuzzy logic as well as a survey on current state of the theory of complex fuzzy sets, complex fuzzy classes, and complex fuzzy logic.

In 1973 Kandel has published his work on minimization of fuzzy functions and the formal definition of the fuzzy algebra as an extension to the classical Boolean algebra [13]. That paper has been the initial stage of investigating the subject of fuzzy switching and automata [15]. Later, the theory established in [13], [15] has resulted in the applicability of the theoretical developments discussed in [14,16,17]. In 1978 the proceedings of the IEEE published his paper on fuzzy Sets [14]. This has been the first time that this archival journal has accepted any work in the subject of fuzzy sets and/or fuzzy logic. References [16] and [17] followed in 1982 and 1986, respectively, representing Kandel's work on fuzzy techniques in pattern recognition. Reference [17] has also been used in many graduate courses and short seminars around the world. The educational efforts using reference [17] aimed to show how fuzzy mathematics and its applications – from the simplest notions of fuzzy sets and fuzzy logic to the complex ideas of fuzzy functions, fuzzy statistics, fuzzy relations and their applications as developed by Kandel – is indeed a great source of joy and practicality. In the prologue to reference [17] Professor Lotfi Zadeh says: "Although there is extensive literature on the theory of fuzzy sets and its applications, it is difficult for one who wishes to acquire a familiarity with the theory to find a text that both provides a readable introduction and presents an up-to-date exposition of some of the main applications of the theory. Professor Kandel's text serves this purpose with authority, and his treatment of the subject matter reflects his many important contributions to both the theory and its applications." At the time references [13,15,14,16,17] were published the theory of fuzzy sets has been controversial, to say the least. Thus, reference [17] has been highly influential in what Professor Zadeh refers to as the "acid test" in the prologue [17]: "In coming years, the ability of the theory of fuzzy sets to provide a basis for a formalization of commonsense reasoning, may well be an acid

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test of its usefulness in artificial intelligence. It is my conviction that the test will be passed and the theory of fuzzy sets will eventually become a standard tool for the management of uncertainty in expert systems. Many of the techniques needed for this purpose are the techniques described in Professor Kandel's book." In 1984 Maria Zemankova and Kandel published their work on fuzzy relational databases [50]; which has been groundbreaking and together with reference [12] on fuzzy linear regression provided concrete evidence about the applicability of the "fuzzy concept." In 1978 Professor Kandel was one of the seven researchers who came up with the idea of creating the first journal in fuzzy logic, "The International Journal of Fuzzy Sets and Systems." This initiative had tremendous impact on the field. One must look back at the beginning of fuzzy logic and understand that for a long time after Zadeh's first paper in 1965, the field of fuzzy logic had no journals, no conferences dedicated to the subject and very little support from the academic community. In 1992 Kandel initiated the research on fuzzy expert systems followed by the work on fuzzy control systems in 1994 [18], [19]. An important work on that subject, investigating the genetic-based approach to fuzzy reasoning and its applications in fuzzy control, is summarized in [35]. Three years later that work lead to the well-recognized coverage of compensatory neurofuzzy systems [51]. During the last decade Professor Kandel has performed research on fuzzy dynamical systems, fuzzy differential equations, complex fuzzy logic [34], and the application of fuzzy logic and info-fuzzy techniques to anomaly detection algorithms and terrorist detection systems [23].

In 1999 Mark Last has introduced a novel fuzzy logic based method for automating the human perceptions of visualized data [20]. The method, called "Automated Perceptions," is capturing the key features of manual perception. A new concept, the degree of reliability, is defined as a fuzzy measure of certainty that, from the user's point of view, the data is correct [27,28,29]. It is assumed that the relational schema is partitioned into a subset of input (completely reliable) and a subset of target (partially reliable) attributes. An oblivious decision graph model referred to as "Info-Fuzzy Network (IFN)" has been constructed to evaluate the information content of links between input and target attributes. In [21], Mark has presented an anytime algorithm for feature selection. To monitor the algorithm performance, he has introduced a new measure: Fuzzy Information Gain. Fuzzy set theory is used in [23] for reducing the dimensionality of a rule set discovered in real-world data, without losing its actionable meaning. The fuzzification of the rules can be utilized for hiding confidential information from unauthorized users of the published data mining model. In [22,25], Last et al., have presented a fuzzy-based approach to automating the process of detecting and isolating outliers. In [24], he has presented a fuzzy-based method for automating the process of comparing frequency histograms. The research uses type-2 fuzzy logic for representing the domain knowledge of human experts. In [26], Last et al., have improved the performance of genetic algorithms by providing a new, fuzzy based, extension of the Life Time feature. The method uses a Fuzzy Logic Controller to adapt the crossover probability as a function of the chromosomes' age.

Tamir has entered the field of fuzzy logic in 1986 considering the application of fuzzy logic to digital signal processing. One of his first contributions to the field has been the axiomatization of fuzzy sets [40]. This has been one of the first attempts to formalize fuzzy logic using the rigor of class theory. In addition, Tamir developed a new framework for Fuzzy knowledge representation and fuzzy expert systems [42]. Later, he introduced a Pattern Recognition Interpretation of fuzzy Implications [41]; and an Architecture for Rule-Based Knowledge Representation and Parallel Inference [42, 41]. In recent years, Tamir has investigated methods for discovering fuzzy quantitative association rules in databases [39]. He conducted a Comparative study of software testing using artificial neural networks and Info-Fuzzy Networks [1], and developed a Pyramid Fuzzy C-means Algorithm [43] as well as axiomatic theory of complex fuzzy logic [44, 45, 46, 47].

96.2 Fuzzy Sets and Fuzzy Logic

In 1965, L. A. Zadeh has established the theory of fuzzy sets where the degree of membership of an item in a set can get any value in the interval $[0,1]$ rather than the two values $\{\in, \notin\}$ [48]. In addition he introduced the notion of fuzzy logic [11]. Which is a multilevel extension of classical logic where propositions can get truth values in the interval $[0,1]$, and are not limited to one of the two values {True, False} (or $\{0, 1\}$) [11]. The four decades that followed the introduction of fuzzy sets and fuzzy logic by Zadeh, has shown a multitude of research work and applications related to signal processing, knowledge representation, control theory, reasoning, and data mining [7, 38, 4]. In 1975 Zadeh introduced the concept of linguistic variable and the induced concept of type-2 (type-n) fuzzy sets [49, 30]. In recent years, type-1 and type-2 along with interval type1/type-2 fuzzy logic and fuzzy systems have been applied in many areas including signal processing [30, 31], fuzzy clustering [43], data mining [20, 28], and software testing [1]. Nevertheless, many researchers have observed that the grade based membership approach is limited in its capability to deliver a concise and precise formalism for fuzzy logic. Hence, current research in fuzzy logic, fuzzy class theory, fuzzy mathematics, and its applications is based on axiomatic theory [2, 6, 32, 10, 3, 9].

96.3 Complex Fuzzy Sets and Complex Fuzzy Logic

Ramot et al., observe that the expressive power of fuzzy sets and fuzzy logic and the utility of derived applications can be significantly improved via the introduction of complex fuzzy sets [36]. Their observation is mainly motivated by fuzzy processes that contain periodical behavior such as the cycles in economic markets. In this sense, the concept can be used for fuzzy temporal logic. Generally, in these applications several fuzzy variables interact with each other in a complex way that cannot be represented effectively via simple fuzzy set operations such as union, intersection, complement, conjunction and disjunction [36]. The initial formulation of these terms stems from the definition of complex fuzzy grade of membership [36,37]. Ramot et al., propose an extension of fuzzy set theory and fuzzy logic where the range of degrees of membership and the range of truth values is the complex unit circle [36,37]. In order to capture these phenomena in reasoning, they introduce a complex grade of membership and derive the definition of complex fuzzy sets. Under this notation, a complex fuzzy set S on a universe of discourse U is defined by a complex-valued grade of membership function $\propto_S(x)$; (Ramot et al.) [36]:

$$\propto_S(x) = r_S(x)e^{(j\Phi_S(x))} \qquad (j = \sqrt{(-1)}) \tag{96.1}$$

This definition utilizes polar representation of complex numbers along with conventional fuzzy set definitions; where $r_S(x)$, the amplitude part of the grade of membership, is a fuzzy function defined in the interval $[0,1]$ and $\Phi_S(x)$ is a real number standing for the phase part of the grade of membership. In addition, Ramot et al., propose the conventional set of operations on fuzzy sets such as complement, union, intersection etc. Later, they introduce complex fuzzy logic via relations on complex fuzzy sets [37]. Their formalism, however, restricts the membership function to representation using polar coordinate system where only the radius carries fuzzy information. Dick, Kandel, Teodorescu, and several other researchers have extended the work by Ramot et al., yet their approach is also limited to polar representation with single fuzzy component [8,33,52,5]. Motivated by similar considerations, Tamir et al., extend the rigor and applicability of the formalism proposed by Ramot et al., and introduce complex class theory where both component of a complex fuzzy grade of membership carries fuzzy information [44,45,46]. They provide further generalization of the concept of complex fuzzy membership function and use a Cartesian complex fuzzy membership function where both the real part and the imaginary part are fuzzy functions. Alternatively, polar representation where both the radius and the phase values of the complex membership function convey fuzzy information, is utilized [44,45]. Furthermore, they provide a new interpretation of complex fuzzy grades of membership as a representation of a complex fuzzy class along with complex fuzzy class operations [44]. This enables reasoning about processes with multi-dimensional components where each component is carrying fuzzy information and the interaction between the components cannot be decomposed and represented via primitive, one dimensional, fuzzy set theory and fuzzy logic operations, such as conjunction, disjunction, union, and intersection. The Cartesian representation of the complex grade of membership is given in the following way:

$$\propto(V,x) = \propto_r(V) + j\propto_i(x) \tag{96.2}$$

where $\propto_r(V)$ and $\propto_i(x)$, the real and imaginary components of the complex fuzzy grade of membership, are real value fuzzy grades of membership. That is, $\propto_r(V)$ and $\propto_i(x)$ can get any value in the interval $[0,1]$. The polar representation of the complex grade of membership is given by:

$$\propto(V,x) = r(V)e^{(j\phi(x))} \tag{96.3}$$

where $r(V)$ and $\phi(x)$, the amplitude and phase components of the complex fuzzy grade of membership, are real value fuzzy grades of membership. That is, they can get any value in the interval $[0,1]$. The transformation from Cartesian to polar representation and from polar to Cartesian representation of complex predicates can be defined in a straightforward semantic preserving way [44]. Hence without loss of generality, in this chapter, we concentrate on the Cartesian representation of complex predicates.

While this formulation has several advantages over previous formalisms, it is still based on the definition of graded membership, which is a limiting factor in the ability to provide a rigorous, axiomatic based, theory. In ref. [46], Tamir et al., develop an axiomatic based propositional complex fuzzy logic theory that is independent of complex fuzzy sets, classes, and relations. In addition, they demonstrate the potential use of this formalism for inference in complex systems. The new theory is compatible with classical logic, as well as with axiomatic fuzzy logic and set theory [2, 6, 32, 10, 3, 9]. Furthermore, the new theory supports Cartesian as well as polar representation of complex logical fuzzy propositions with two components of ambiguous information. Hence, this form significantly improves the expressive power and inference capability of complex fuzzy logic.

A complex fuzzy proposition $P = p_r + jp_i$ is a composition of two propositions each of which can accept a truth value in the interval $[0,1]$. In other words, the interpretation $(i())$ of a complex fuzzy proposition is a pair of truth values from the Cartesian interval $[0,1] \times [0,1]$. Alternatively, the interpretation can be formulated as a mapping to the unit circle. Formally a fuzzy interpretation of a complex fuzzy proposition Γ is an assignment of fuzzy truth value of the form $i(\Gamma_r) + j \times i(\Gamma_i)$, or of the form $i(r(\Gamma)) \times e^{(j \times i(\Phi(\Gamma)))}$.

For example, consider a complex proposition P of the form "$x \dots A \dots B \dots$" along with the definition of linguistic variables and constants. Namely, a linguistic variable is a variable whose domain of values is formal or natural language words [49]. Generally, a linguistic variable is related to a fuzzy set; e.g., {very young male, young male, old male, very old male} and can get any value from the set. A linguistic constant has a fixed and unmodified linguistic value i.e., a single word or phrase from formal or natural language. Thus, in a proposition of the form "$x \dots A \dots B \dots$" where A and B are linguistic variables, $i(p_r)$ $(i(r(p)))$ can be assigned to the term A and $i(p_i)$ $(i(\Phi(p)))$ can be assigned to term B.

While the above formalism is general it is not best suited for integer processing units. For this end, we have presented a formalism for discrete CFL [47]. In [47] Tamir et al., propose a new complex fuzzy logic (CFL) system that is based on the extended Post system (EPS) [9] of order $P > 2$ and demonstrate its utility for reasoning with fuzzy facts and rules. Both propositional calculus and first-order predicate calculus of the EPS based CFL are developed. The application to approximate reasoning is described. The new formalism is motivated by the fact that fuzzy systems are commonly used in environments such as control, digital signal processing (DSP), embedded systems, and real time applications where generally processors consist of high performance integer arithmetical and logical units and do not have intensive floating point processing capabilities [30, 31]. In this sense, the EPS developed in [9] is an excellent tool for rigorous formalization of discrete based fuzzy logic. Nevertheless it lacks the expressive power associated with CFL. The new theory proposed in [47] bridges the gap. It can be used to formalize advanced discrete complex fuzzy logic systems. Moreover, it can be used for inference with type-2 (or higher) fuzzy systems [49, 30]. Furthermore, the introduction of complex logic can be used for analysis of periodic temporal fuzzy processes where the period is fuzzy.

96.4 Epilogue

In this chapter we have presented the pioneering work of Prof. Zadeh along with numerous branches of this work. Some of these branches represent the work of Kandel, Last, and Tamir. We have concentrated on one important and contemporary part of this work, namely Complex Fuzzy Sets (CFS) and Complex Fuzzy Logic (CFL). CFS and CFL can significantly improve the expressive power and inference capability of numerous fuzzy based systems and applications. We are convinced that the theory and practice of CFL/CFS will continue to play an important role in the field of fuzzy sciences and soft computing. In the future, we plan to extend the theory to multidimensional fuzzy propositional and predicate logic; explore the utility of the theory for fuzzy temporal logic; and further explore its potential for usage in advanced complex fuzzy logic systems as well as inference with type 2 (or higher) fuzzy sets.

References

1. Agarwal, D., Tamir, D.E., Last, M., Kandel, A.: A Comparative Study of Software Testing Using Artificial Neural Networks and Info-Fuzzy Networks. To appear in the IEEE, Transactions on Man Machine and Cybernetics, Part A (2012)
2. Baaz, M., Hajek, P., Montagna, F., Veith, H.: Complexity of t-tautologies. Annals of Pure and Applied Logic 113(1-3), 3–11 (2002)
3. Behounek, L., Cintula, P.: Fuzzy Class Theory. Fuzzy Sets and Systems 154(1), 34–55 (2005)
4. Mahua, B., Das, A.: Genetic Algorithm Based Feature Selection in a Recognition Scheme Using Adaptive Neuro Fuzzy Techniques. International Journal of Computers Communications & Control 5(4), 458–468 (2010)
5. Chen, Z., Aghakhani, S., Man, J., Dick, S.: ANCFIS: A Neuro-Fuzzy Architecture Employing Complex Fuzzy Sets. IEEE Transactions on Man Machine and Cybernetics, Part A 19(2), 305–322 (2011)
6. Cintula, P.: Weakly Implicative Fuzzy Logics. Archive for Mathematical Logic 45(6), 673–704 (2006)
7. Cordova Felisa, M., Leyton, G.: A Fuzzy Control Heuristic Applied to Non-linear Dynamic System Using a Fuzzy Knowledge Representation. International Journal of Computers Communications & Control 5(5), 664–674 (2010)
8. Dick, S.: Towards Complex Fuzzy Logic. IEEE Transaction on Fuzzy Systems 13, 405–414 (2005)
9. Di Zenzo, S.: A Many-valued Logic for Approximate Reasoning. IBM International Research & Developments 32(4), 552–565 (1998)

10. Hájek, P.: Fuzzy logic and Arithmetical Hierarchy. Fuzzy Sets and Systems 3(8), 359–363 (1995)
11. Hajek, P.: Fuzzy Logic. In: Zalta, E.N. (ed.) The Stanford Encyclopedia of Philosophy (fall 2010 edition)
12. Heshmaty, B., Kandel, A.: Fuzzy Linear Regression and its Applications to Forecasting in Uncertain Environments. Fuzzy Sets and Systems 15, 159–191 (1985)
13. Kandel, A.: On Minimization of Fuzzy Functions. IEEE Transactions on Computers C-22(9), 826–832 (1973)
14. Kandel, A., Byatt, W.J.: Fuzzy Sets, Fuzzy Algebra, and Fuzzy Statistics. Proceedings of the IEEE 66(2), 1619–1639 (1978)
15. Kandel, A., Lee, S.C.: Fuzzy Switching and Automata: Theory and Applications. Edward Arnold, New York (1979); Second Printing (1980)
16. Kandel, A.: Fuzzy Techniques in Pattern Recognition. Wiley Interscience, New York (1982)
17. Kandel, A.: Fuzzy Mathematical Techniques with Applications. Addison Wesley, Boston (1986); Second printing (1987)
18. Kandel, A. (ed.): Fuzzy Expert Systems. CRC Press (1992)
19. Kandel, A., Langholz, G. (eds.): Fuzzy Control Systems. CRC Press (1994)
20. Last, M., Kandel, A.: Automated Perceptions in Data Mining. In: IEEE International Fuzzy Systems, Part I, Seoul, Korea, pp. 190–197 (August 1999) (invited paper)
21. Last, M., Kandel, A., Maimon, O., Eberbach, E.: Anytime Algorithm for Feature Selection. In: Ziarko, W.P., Yao, Y. (eds.) RSCTC 2000. LNCS (LNAI), vol. 2005, pp. 532–539. Springer, Heidelberg (2001)
22. Last, M., Kandel, A.: Automated Detection of Outliers in Real-World Data. In: Proceedings of the Second International Conference on Intelligent Technologies, Bangkok, Thailand, November 27-29, pp. 292–301 (2001)
23. Last, M., Abraham Kandel, A.: Fuzzification and Reduction of Information-Theoretic Rule Sets. In: Kandel, A., Bunke, H., Last, M. (eds.) Data Mining and Computational Intelligence. STUDFUZZ, vol. 68, pp. 63–93. Physica-Verlag (2001)
24. Last, M., Kandel, A.: Fuzzy Comparison of Frequency Distributions. In: Grzegorzewski, P., Hryniewicz, O. (eds.) Soft Methods in Probability, Statistics, and Data Analysis. Advances in Soft Computing, pp. 219–227. Physica-Verlag (2002)
25. Last, M., Kandel, A.: Automated Quality Assurance of Continuous Data, Systematic Organization of Information in Fuzzy Systems. In: Melo-Pinto, P., Teodorescu, H.-N., Fukuda, T. (eds.) NATO ASI Series, pp. 89–104. IOS Press (2003)
26. Last, M., Eyal, S.: A Fuzzy-Based Lifetime Extension of Genetic Algorithms. Fuzzy Sets and Systems Special Issue on Fuzzy Sets in Knowledge Discovery 149(1), 131–147 (2005)
27. Maimon, O., Kandel, A., Last, M.: Fuzzy Approach to Data Reliability. In: Pons, O., Vila, M.A., Kacprzyk, J. (eds.) Knowledge Management in Fuzzy Databases, pp. 89–101. Physica-Verlag, Heidelberg (2000)
28. Maimon, O., Last, M.: Knowledge Discovery and Data Mining — The Info-Fuzzy Network (IFN) Methodology. Kluwer Academic Publishers, Boston (2000)
29. Maimon, O., Kandel, A., Last, M.: Information-Theoretic Fuzzy Approach to Data Reliability and Data Mining. Fuzzy Sets and Systems 117(2), 183–194 (2001)
30. Mendel, J.M.: Uncertainty, Fuzzy Logic, and Signal Processing. Signal Processing 80, 913–933 (2000)
31. Anis, M., Nabil, J., Collart, D.S., Etienne, C., Benrejeb, M.: Fuzzy Filtering of Sensors Signals in Manufacturing Systems with Time Constraints. International Journal of Computers Communications & Control 5(3), 362–374 (2010)

32. Montagna, F.: On the Predicate Logics of Continuous t-Norm BL-Algebras. Archives of Mathematical Logic 44, 97–114 (2005)
33. Moses, D., Degani, O., Teodorescu, H.-N., Friedman, M., Kandel, A.: Linguistic Coordinate Transformations for Complex Fuzzy Sets. In: IEEE International Conference on Fuzzy Systems, Seoul, Korea (1999)
34. Nguyen, H.T., Kandel, A., Kreinovich, V.: Complex fuzzy sets: towards new foundations. In: Proceedings, 2000 IEEE International Conference on Fuzzy Systems, San Antonio, Texas (2000)
35. Park, D., Langholz, G., Kandel, A.: Genetic-Based New Fuzzy Reasoning Models with Application to Fuzzy Control. IEEE Transactions on System, Man and Cybernetics 24(1), 39–47 (1994)
36. Ramot, D., Milo, R., Friedman, M., Kandel, A.: Complex Fuzzy Sets. IEEE Transactions on Fuzzy Systems 10(2), 171–186 (2002)
37. Ramot, D., Friedman, M., Langholz, G., Kandel, A.: Complex Fuzzy Logic. IEEE Transactions on Fuzzy Systems 11(4), 450–461 (2003)
38. Ronen, M., Shabtai, Y., Guterman, H.: Hybrid Model Building Methodology Using Unsupervised Fuzzy Clustering and Supervised Neural Networks. Biotechnology and Bioengineering 77(4), 420–429 (2002)
39. Shragai, A., Tamir, D.E., Schneider, M., Kandel, A.: High Accuracy Method for Discovering Quantitative Association Rules in Data-tables and Databases. The Journal of Fuzzy Economic Review XIV(1) (2009)
40. Tamir, D.E., Kandel, A.: An Axiomatic Approach to Fuzzy Set Theory. Information Sciences 52, 75–83 (1990)
41. Tamir, D.E., Kandel, A., Schwartz, D.G.: A Pattern Recognition Interpretation of Implications. Information Sciences 57/58, 197–215 (1990)
42. Tamir, D.E., Kandel, A.: Fuzzy Semantic Analysis and Formal Specification of Conceptual Knowledge. Information Sciences, Intelligent Systems 82(3-4), 181–196 (1995)
43. Tamir, D.E., Kandel, A.: The Pyramid Fuzzy C-means Algorithm. The International Journal of Computational Intelligence in Control (December 2010)
44. Tamir, D.E., Lu, J., Kandel, A.: A New Interpretation of Complex Membership Grade. International Journal of Intelligent Systems (2011)
45. Tamir, D.E., Last, M., Kandel, A.: Generalized Complex Fuzzy Propositional Logic. In: The World Conference on Soft Computing, San Francisco, CA (2011)
46. Tamir, D.E., Kandel, A.: Axiomatic Theory of Complex Fuzzy Logic and Complex Fuzzy Classes. International Journal of Computers, Communications & Control VI(3) (2011) (invited paper)
47. Tamir, D.E., Last, M., Teodorescu, H.-N., Kandel, A.: Discrete Complex Fuzzy Logic (submitted, 2012)
48. Zadeh, L.A.: Fuzzy Sets. Information and Control 8, 338–353 (1965)
49. Zadeh, L.A.: The Concept of a Linguistic Variable and its Application to Approximate Reasoning – Part I. Information Sciences 7 (1975)
50. Zemankova, M., Kandel, A.: Fuzzy Relational Database — A Key to Expert Systems. Verlag TV̈ Rheinland, Köln (1984)
51. Zhang, Y.-Q., Kandel, A.: Compensatory Neurofuzzy Systems with Fast Learning Algorithms. IEEE Transactions on Neural Networks 9(1), 83–105 (1998)
52. Zhang, G., Dillon, T.S., Cai, K., Ma, J., Lu, J.: Operation Properties and Delta Equalities of Complex Fuzzy Sets. International Journal on Approximate Reasoning 50(8), 1227–1249 (2009)

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Fuzzy Systems in Brazil and at QMC

Ricardo Tanscheit

97.1 Introduction

When I was asked to contribute to this book I wondered what I could tell about my personal experiences in the field of fuzzy sets. It occurred to me that perhaps it would be a good idea to let people know how fuzzy sets and fuzzy control in particular got under way in Brazil. In the mid-seventies, Brazil was not an emerging country, as it is classified nowadays, and doing research in novel subjects was much harder than it is now. I also felt it would be useful to tell something about the days I spent at Queen Mary College (QMC) in the 1980s, working in Abe Mamdani's research group, which was a very pleasant experience. As a conclusion, I return to Brazil, giving a brief view on how things stand today and what are the expectations of our scientific community.

97.2 Early Days in Brazil

In the mid-seventies, having graduated in Electrical Engineering, I enrolled in an M.Sc. programme at the Military Institute of Engineering, in Rio de Janeiro, Brazil. It must be said that such programmes in Brazil consist of one year of courses and at least another year dedicated to research in a chosen topic, which must be reported in a dissertation. After finishing the courses, I was introduced by my supervisor to a strange subject: Fuzzy Sets. In fact, what motivated us was a paper by Kickert and van Nauta Lemke on fuzzy control of a heat exchange process [1]. In those pre-internet times, in an underdeveloped country, access to foreign publications was not as easy as it is nowadays. Therefore, it took us some time to get access to the relevant publications on Fuzzy Sets, including Zadeh's seminal publication and Abe Mamdani's papers on fuzzy control ([2] [3]). To our rigid engineering minds it was at least puzzling to get acquainted with this novel approach but we became more and more interested and decided to apply the new notions in the control of a fluid mixer. Our initial purpose was to build a practical experiment but eventually several difficulties overcame us and the final work consisted of a simulation. The paper we wrote was accepted for presentation at the 1978 Brazilian Congress on Automation, which, in those times, was almost entirely dedicated to the standard approaches to Control. In the presentation, I had, of course, to give an introduction to Fuzzy Sets, which were unknown to the engineering community in Brazil. I recall the bewilderment of the audience to be introduced to such a subject, so that after the

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presentation several people asked me for references an further clarification. As far as I am aware, that paper was the first one to deal with the application of fuzzy sets and fuzzy logic to an engineering problem in Brazil.

After that I become more interested in the area and applied for a Ph.D student position with Abe Mamdani's research group at Queen Mary College (QMC), University of London.

97.3 Fuzzy Control at QMC

When I arrived at QMC, Abe Mamdani's students research room (Fig. 97.1) had a familiar picture on the wall: the famous Christ the Redeemer statue in Rio de Janeiro. Afterwards, everybody thought I had brought with me, but that was not the case.

Fig. 97.1. Research room at QMC

The pioneering work of Assilian, under Abe Mamdani's supervision, made QMC a very attractive place for researchers on Fuzzy Control. In the early 1980s the main emphasis was placed on investigating aspects and applications of the so-called Fuzzy Self-Organising Controller (SOC), devised by one of Abe Mamdani's Ph.D. students [4]. SOC was able to create a consistent rule-base, depending on the system's behavior when compared to a standard performance index. This was an interesting idea, since the available tools in those times were not the same as today regarding rule creation and tuning. The structure of SOC consisted of fixed, limited and discrete universes of discourse, identical for all variables involved. The number of linguistic terms and the shapes of membership functions were also identical for all variables. Tuning consisted of adjusting some scaling factors, which in later developments could lead to nonlinear mappings. In the lack of automatic tuning procedures, this had to be done by hand, which could become an ordeal. This approach was, and still is, very useful for practical and real-time applications, since the strategy can be summarized in a so-called decision table once all the parameters have been set.

It is interesting to note that Abe himself was moving away from Fuzzy Systems in the early 80s, preferring to invest in the field of Expert Systems. Nevertheless, some works with improved and modified versions of SOC were still carried out, also considering real world applications [5].

Fig. 97.2. Barbecue at Abe Mamdani's

Some words must be said about Abe Mamdani. His abilities as a researcher are well known, but he also had some nice personal aspects. He used to drive to College and home, instead of taking the underground and train combination,as many other lecturers did. Many times he came to our research room, which faced the main road he needed to take on his way home. When he noticed that the traffic was heavy, he would compel us all to go the pub across the road to wait for the traffic to subdue. He also liked to host Xmas parties and barbecues at his home, as shown in Figure 97.2 below. Albeit being a very busy person, he always made an effort to attend to his students invitations (Figure 97.3). It was very pleasing to work in such an environment.

Having finished my doctorate, I returned to Brazil and made some contact with the few people who were then operating in the area of fuzzy systems.

Fig. 97.3. Dinner at the house of one of Abe Mamdani's students

97.4 Computational Intelligence in Brazil

Many researchers – mainly lecturers at universities – returned to the country, after finishing their doctorates, in the 1980s and early 1990s. The area of Computational Intelligence (or Intelligent Systems, as it used to be called) was acquiring some status and courses were created even at the undergraduate level in some universities.

Two important events took place in Brazil at that time: the 1994 Brazil Japan Joint symposium in Fuzzy Systems, which took place in Campinas, São Paulo state, and the 1995 IFSA World Congress, held in the city of São Paulo. For us Brazilians those were memorable occasions and we believe that both conferences were very successful.

Diverse applications of fuzzy systems and neural networks became commonplace sice then, so that nowadays this area is well established in the country. Research on Fuzzy Control has also become widespread, either with the Mamdani-type FIS or with the TSK structures. These have always been put in the same basket, but many of us have always been uncomfortable about this. One of these days, talking to Prof. Fernando Gomide, we agreed that those two structures should not be put in the same basket. It seems quite clear that the initial idea behind fuzzy control was to emulate the actions human operators would take in the control of a given plant. This is done by the so-called Mamdani-type of controller. This should not be taken as diminishing the importance of the TSK fuzzy controller, which has his own merits, but should not be compared to its conunterpart.

I am glad that the question of interpretability has become important in the last decade. This actually takes us back to the philosophical aspect mentioned above.

The keynote talk of Prof. Luis Magdalena at the 9th International Workshop on Fuzzy Logic and Applications (WILF 2011) – Some Open Questions in Fuzzy Rule-Based Systems Design – was very stimulating regarding what has been achieved so far and was should worry us all regarding the Mamdani-type fuzzy control. There have been many successful applications in these almost 30 years of its inception, but there still are many unresolved questions regarding tuning of the several parameters.

In our research group there has been a great interest on clustering and classification problems too. There has also been an investment in the robotics area. We see evolving fuzzy systems as a strong approach to those problems and believe that they will be of great help to address structure definition.

97.5 Conclusion

The idea here was to give a very quick insight on how Fuzzy Systems an Computational Intelligence grew in Brazil, an underdeveloped country in the 1970s and an emerging one, at least, in the 21st century.

In between, I thought it would be interesting to tell something about my days at QMC, which was a fundamental place for research in fuzzy control.

References

1. Kickert, W.J.M., Van Nauta Lemke, H.R.: The Application of a Fuzzy Controller in a Warm Water Plant. Automatica 12, 301–308 (1976)
2. Mamdani, E.H.: Application of Fuzzy Algorithms for Control of Simple Dynamic Plant. Proceedings of the Institution of Electrical Engineers 121(12), 1585–1588 (1974)
3. Mamdani, E.H., Assilian, S.: An Experiment in Linguistic Synthesis With a Fuzzy Logic Controller. International Journal of Man-Machine Studies 7(1), 1–13 (1975)
4. Procyk, T.J.: A Self-Organising Controller for Dynamic Processes. Ph.D. Thesis, University of London (1977)
5. Tanscheit, R., Scharf, E.M.: Experiments With the Use of a Rule-based Self-Organising Controller for Robotics Applications. Fuzzy Sets and Systems 26(2), 195–214 (1988)

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On Fuzziness, Its Homeland and Its Neighbour

Settimo Termini

It has been frequently remembered (also in the invitation letter to contribute to the present volume) that in his 1962 paper [27] which is a sort of proto-manifesto of fuzzy sets [28], Zadeh among the possible names that could denote the new notion he was trying to introduce, basides *fuzzy*, mentioned the term *cloudy*. If he had chosen this last one as the name of the theory, today we had contributed to a volume *On Cloudiness.*

It is natural to ask the question whether a different choice of the name would had caused something different in the development of the field. More generally, perhaps, it is not trivial to investigate in which sense we should take into consideration the fact that the choice of a name for a *new* field of investigation is not a neutral and purely conventional action, but it bears on its shoulders a big bush of different problems, starting from a reassessment of the meaning (intended meaning) of other words that had been used before to indicate facts, aspects and nuances of concepts and pieces of reality which, after the official introduction of a new name, one is naturally induced to consider as pertaining and belonging to the new field.

The joke about fuzziness and cloudiness, in fact, was intended as an introduction to at least the following problems:

i) the interaction between informal concepts and their formalizations,
ii) the adequacy of a formalization to all the meaningful nuances and aspects of the informal notion,
iii) the drift of meaning intrinsic to the use of scientific concepts in a non-routine (i.e. creative) way.

The emergence of fuzziness as a true new scientific notion, morover, can be fully understood only in the realm of the epistemological questions posed by the novelty of some emerging disciplines. It happens in the midst of a not already completed scientific revolution of the last Century, one that is called with many names, among which we pick up now the one of "information sciences", but many others names have been used and are still used, Cybernetics, System Theory, Cognitive Sciences, AI, etc. All these names are not completely interchangeble, but anyway, they refer to more or less extended overlapping subdomains of the overall domain in which the scientific revolution to which we referred above, is happening. A true problem is that we do not still have a name for the whole domain. Rudi Seising [16] has provided a very interesting reconstruction of this creative chaos ponting out that we must consider the birth and development of fuzzy sets in this general setting if we

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want to understand the real import of the introduction of a new scientific concept like the one of fuzziness. A personal view on this problem can be found in [19].

But let us ask the following question: is the situation outlined above of an emergent scientific field which is still missing an overall name something usual? We could also think that since a crucial feature of the new field is interdisciplinarity, we could look at this domain as something original also from a methodological point of view. In contrast with such classical disciplines as physics which deals with inanimate world or biology which deals with living sistems, this new field could be considered as a new type of science that we could call "science archipelag", characterized by the fact that no single domain is definable, but there is a constant interaction between many subdomains which have specific features. This aspects is certainly present and is also a central and crucial feature of this field, but let us see whether we can also try to add to this aspect also something additional and hopefully conceptually new.

As, perhaps, is clear from what I wrote above, the homage to Lotfi presented in these pages will not involve any comment on personal episodes or of crucial interactions with the fuzzy community or with Lotfi himself (although, while writing these notes, I am rewieving in my mind episodes which have been relevant for me. Some of them can be found in the interview to Rudolf Seising which appeared in [20]).

I shall, instead, here try to concentrate in presenting – hopefully in a reasonable way – some (until now, only personal) attempts to analyze the notion of fuzziness *as such* in the light of (and in its interaction with) a few other innovative notions emerged in the last few decades. The starting point will be the last mentioned question: the lack of a name for an important emerging field. This field *is* the homeland of the notion of fuzziness, and here we should look for its neighbour.

98.1 Towards a "Physics" of the "Immaterial"?

The thesis I shall present now is that we can pick up a common element in the development of the scientific activities subsumed under all the names listed above (Cybernetics, Information Sciences, Cognitive Sciences, A.I., System Sciences, etc,) and this is related to the fact that, for the first time in the history of science, the crucial notions of fields which are investigated with traditional techniques of hard sciences like physics present the unusual feature that they do not refer directly to "material" aspects. All the meaningful notions in the new field of investigation are "immaterial" and not rooted into specific "concrete" things. Let me repeat this concept with slightly different words. What I propose is that a common characterizing feature of all the investigations recalled above can be found in the fact that we are always dealing with "something" immaterial.

In this sense it appears an apparent strong similarity with disciplines belonging to human or social sciences in which there are not such evident crucial parameters as "position", "velocity", and so on. In these disciplines, in fact, the central meaningful concepts are more "conventional" than in physics in which the notions referred to above, position, velocity, are "suggested" – in a sense – by Nature itself. Notwithstanding this fact, the theories of the "immaterial" look more to what is present in theoretical physics than to what happens in Sociology, for instance. *What* does make the difference?

My hypothesis is that all this have been made possible by the fact that one central notion in this field, the one of "computation", presents a very strong *stability* with respect to all possible changes we could think to make (different formalizations, different interpretations, different models, different technologies). This makes possible the "miracle" of having a field in which we can construct very strong and stable theories formalizing phenomena that are not based on "material" things. Of course, the "character of the play", the main actor, the star, is nothing more than the well known and frequently discussed *Church-Turing's Thesis*. However the interpretation proposed and the role assigned is apparently different from the usual considerations done on this classical Thesis.

From this point of view it is interesting to observe that – in the setting of information sciences (or in the terminology heralded here, in the setting of the "immaterial") a few central other notions (that do not possess the unicity of formalization, which seems to be a unicum of the notion of computation) like Kolmogorov complexity, became very interesting new scientific concepts (and the innovative concept is also the basis, the angular stone, of a new interesting *theory*), just after using some crucial results of the theory of computation.

In a sense we could affirm that it is only the possibility of using the theory of computabilty that allows us to transform into a universal strong theory a very interesting intuition and concept, which, however, without the previously mentioned support, would be deemed to remain only an interesting idea. Incidentally, let me remeber that 2012 is the Centennial of Alan Turing's birth. His seminal ideas are still capable to induce people to think along new paths.

All these flashes of course need to be worked on in more details and also other different examples should be considered.

98.2 Back to Fuzziness

Fuzziness as part of this play, introduces in the plot the possibility of modelling uncertainty in innovative ways which make also use of "linguistic" tools [30], opening the possibility of opening again a dialogue with human sciences.

More specifically, in the setting outlined above, the notion of fuzziness – as presented by Zadeh – provides a very innovative contribution (first of all of conceptual type) since it allows to look at the qualitative, informal problem of the presence of uncertainty in a fresh way, not already regimented by very sophisticated formalizations like the one of probability, after Kolmogorov axiomatization. It is this *conceptual freedom* that allows a Re-Vision of a few crucial aspects present in this field of investigation. Among them let me recall (but what follows is only a very partial list biased by my personal experience and preferences):

a) the generalization of the same notion of logical connective along the lines envisaged by von Neumann of introducing elements and tools borrowed from mathematical analysis in the previously purely combinatorial terrain of logical investigations [11, 12]; such a context allows to these developments to go far beyond the limits of the original Lukasiewicz setting although fully interpreting his motivations. This change of mind, in my view is particularly evident in the approach followed by Enric Trillas, starting from his classical classification of "negations" [1,24]. It is not a case that recently the same Trillas has been leaded to reconsider the validity of the logical principles in this extended universe [23, 25].

b) the possibility of *confronting* probabilistic approaches with Zadeh's proposal. These have a long history, but it continues to be an interesting topic [26]. It is worthwhile to remember also connections with non standard interpretation of probability like the one proposed by De Finetti [5, 6], as done by Coletti and Scozzafava ([15] in this volume and [4]). One must observe also that the connection found between coherent conditional probability and fuzziness established by Coletti and Scozzafava does not exaust all the possibilities offered by the developments of fuzzy set theory along the last decades.

c) the possibility of introducing – through the notion of *measure of fuzziness* – a more flexible tool than the measure of information provided by Shannon. The fuzzy setting allows introducing both more "measures" and more conceptual interpretations [7,8,9,21,22]. This fact helps in applying in very natural ways this notion in different fields not only in classical ones as Pattern recognition but also in unusual domains like the one of quantitative approaches to aestetic theories [18, 3, 14], in which straighforward applications of Shannon theory had produced ambiguous results, according to such scholars as Rudolf Arnheim [2].

But there is another central point which is – once again – due to Lotfi. By introducing the challenge of "computing with words" [31], he has presented a completely new path of investigation, which – according to the hypothesis done here about the special role of the notion of computation in the setting of "immaterial sciences" – poses the notion of fuzziness in a very peculiar and strict connection withe the basic roots of "immaterial sciences".

98.3 Fuzziness or Cloudiness? It Does Not Matter, But Fuzzines AND Computation Does

The question posed at the beginning about Fuzziness or Cloudiness, was obviously a joke. However every notion carries with itself a burden of suggestions, intuitions and analogies that can interfere with the technical development of a piece of investigation. So I think that the final choice of fuzziness was a good one since it stresses more the aspect of the difficulty of focusing which some concepts present instead that the one of being "cloudy".

Anyway Fuzziness is a central part of the creative chaos that in the last few decades has characterized the birth and development of new scientific notions, theories, techniques and technological innovations. Three years after publishing his seminal papers, Lotfi Zadeh touched upon the problem of Fuzzy algorithms [29]. This is not the place to reconstract the history and development of "Fuzzy computatbility". I want only mention the analysis provided by Claudio Moraga [10], who shows that a fuzzy context induces a very great flexibility. We can remain inside classical computability or go in the direction of computable reals according to the "point" in which "fuzzy elements" are introduced. When the idea of "computing with words" will show a few of its potential developments, the notion of computation – while preserving its strong stability – will be certainly enriched by many innnovative nuances. The two notions of fuzziness and computation will remain as two pillars of "immaterial sciences".

Fig. 98.1. Fltr: Teresa Riera, Xavier Domingo, NN, Alicia Casals, Francesc Esteva, Settimo Termini, Sergei Ovchinnikov, and Enric Trillas in an Indian camp close to Oklahoma City, at the Eleventh International Symposium on Multiple-Valued Logic, Oklahoma City, May 27-29, 1981

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References

1. Alsina, C., Trillas, E., Valverde, L.J.: On Some Logical Connectives for Fuzzy Sets Theory. Journal of Mathematical Analysis and Applications 93(1), 15–26 (1983)
2. Arnheim, R.: Entropy and Art. University of California Press, Berkeley (1969)
3. Cardaci, M., Di Gesú, V., Petrou, M., Tabacchi, M.E.: On the Evaluation of Images Complexity: A Fuzzy Approach. In: Bloch, I., Petrosino, A., Tettamanzi, A.G.B. (eds.) WILF 2005. LNCS (LNAI), vol. 3849, pp. 305–311. Springer, Heidelberg (2006)
4. Coletti, G., Scozzafava, R.: Probabilistic Logic in a Coherent Setting. Trends in Logic, vol. 15. Kluwer, Dordrecht (2002)
5. de Finetti, B.: La prévision: ses lois logiques, ses sources subjectives, Annales de l'Institut Henri Poincaré, vol. 7, pp. 1–68 (1937); Translation reprinted: Kyburg, H.E., Smokler, H.E.: Studies in Subjective Probability, 2nd edn., pp. 53–118. Robert Krieger, New York (1980)
6. de Finetti, B.: Theory of Probability, vol. 1. John Wiley and Sons, New York (1974)
7. De Luca, A., Termini, S.: A Definition of a Non Probabilistic Entropy. Information and Control 4, 301–312 (1972)
8. De Luca, A., Termini, S.: Entropy and Energy Measures of a Fuzzy Set. In: Gupta, M.M., Ragade, R.K., Yager, R.R. (eds.) Advances in Fuzzy Set Theory and Applications, pp. 321–338. North-Holland, Amsterdam (1979)
9. De Luca, A., Termini, S.: Entropy Measures in Fuzzy Set Theory. In: Singh, M.G. (ed.) Systems and Control Encyclopedia, pp. 1467–1473. Pergamon Press, Oxford (1988)
10. Moraga, C.: Towards a Fuzzy Computability? Mathware & Soft Computing 6(2-3), 163–172 (1999)
11. von Neumann, J.: The General and Logical Theory of Automata. In: Cerebral Mechanisms in Behaviour – The Hixon Symposium. J. Wiley, New York (1951); reprinted in [13], pp. 288–328
12. von Neumann, J.: Probabilistic Logics and the Synthesis of Reliable Organisms from Unreliable Components. In: Shannon, C.E., MacCarthy, J. (eds.) Automata Studies, pp. 329–378. Princeton University Press (1956); reprinted in [13], pp. 329–378
13. von Neumann, J.: Design of Computers, Theory of Automata and Numerical Analysis. Collected Works, vol. V. Pergamon Press, Oxford (1961)
14. Pinna, B., Tabacchi, M.E.: A Fuzzy Approach to the Role of Symmetry in Shape Formation: The Illusion of the Scalene Triangle. In: Di Gesù, V., Pal, S.K., Petrosino, A. (eds.) WILF 2009. LNCS, vol. 5571, pp. 197–204. Springer, Heidelberg (2009)
15. Scozzafava, R.: The Membership of a Fuzzy Set as Coherent Conditional Probability (In this volume)
16. Seising, R.: The Fuzzification of Systems. The Genesis of Fuzzy Set Theory and Its Initial Applications – Developments up to the 1970s. Springer, New York (2007)
17. Seising, R., Sanz, V. (eds.): Soft Computing in Humanities and Social Sciences. STUDFUZZ, vol. 273. Springer, Heidelberg (2011)

18. Tabacchi, M., Termini, S.: Measures of Fuzziness and Information: Some Challenges from Reflections on Aesthetic Experience. In: Yager, R.R., Reformat, M.Z., Shahbazova, S.N., Ovchinnikov, S. (eds.) Proceedings of the World Conference on Soft Computing, May 23-26, San Francisco State University, San Francisco (2011); (memory stick, paper no. 152)
19. Termini, S.: Remarks on the Development of Cybernetics. Scientiae Matematicae Japonicae 64(2), 461–468 (2006)
20. Termini, S. (Istituto di Cibernetica "Eduardo Caianiello" Pozzuoli (NA) Italy) – Interview by Rudolf Seising. Philosophy & Soft Computing Newsletter 2(2), 7–14 (2010), http://docs.softcomputing.es/public/NewsletterPhilosophyAndSoftComputingNumber_3.pdf
21. Trillas, E., Riera, T.: Entropies for Finite Fuzzy Sets. Information Sciences 15, 159–168 (1978)
22. Trillas, E., Alsina, C.: Sur les mesures du degré de flou. Stochastica III(1), 81–84 (1979)
23. Trillas, E.: On Exact Conditionals. Stochastica 13(1), 137–143 (1992)
24. Trillas, E.: Sobre funciones de negación en la teoría de subconjuntos difusos. Stocastica III, 47–59; an english version appeared in the volume: Barro, S., Bugarin, A., Alejandro, S. (eds.): Advances of Fuzzy Logic, pp. 31–43. Press of the Universidad de Santiago de Compostela, Spain (1998)
25. Trillas, E.: Non Contradiction, Excluded Middle, and Fuzzy Sets. In: Di Gesù, V., Pal, S.K., Petrosino, A. (eds.) WILF 2009. LNCS, vol. 5571, pp. 1–11. Springer, Heidelberg (2009)
26. Takéhiko, N., Trillas, E., Itziar, G.-H.: Axiomatic Investigation of Fuzzy Probabilities. In: [17], pp. 125–140
27. Zadeh, L.A.: From Circuit Theory to System Theory. Proceedings of the IRE 50, 856–865 (1962)
28. Zadeh, L.A.: Fuzzy Sets. Information and Control 8, 338–353 (1965)
29. Zadeh, L.A.: Fuzzy Algorithms. Information and Control 12, 99–102 (1968)
30. Zadeh, L.A.: The Concept of a Linguistic Variable and its Application to Approximate Reasoning I. Information Science 8(3), 199–249 (1975); II: Information Science 8(4), 301–357 (1975), III: 9(1), pp. 43–80 (1975)
31. Zadeh, L.A.: From Computing with Numbers to Computing with Words – From Manipulation of Measurements to Manipulation of Perceptions. International Journal of Applied Mathematics and Computer Science 12, 307–324 (2002)

99

Wittgenstein and Zadeh, Side by Side

Josep-Maria Terricabras

My first significant contact with issues concerning imprecision and vagueness was through the work of Ludwig Wittgenstein (1889-1951), when I was preparing in Germany my doctoral thesis in philosophy for over forty years. I came to Zadeh, when I read some of his key works and especially in 1985 when I had occasion to spend a few months near him at the Computer Science Division of UC Berkeley.

Vagueness and imprecision are not a new subject matter of study but a very old one. In any case, when in contemporary times we talk about vagueness and indeterminacy of sense, we are always confronted with the powerful work of Gottlob Frege (1848-1925), who is the father – now already the grandfather – of the current rigorous studies on the logic of language. According to Frege, a concept must always have “sharp boundaries” and this means that the definition of a concept must always determine with precision and without ambiguity whether a given object falls within that concept, or not. A concept that is not defined with precision is not itself a concept. Frege is assuming, of course, the propositional bivalence: since a proposition includes concepts that are precise and well determined, any proposition deserving that name will be either true or false, even if we do not know, at a given time, whatever it is.

Frege, like Wittgenstein’s *Tractatus Logico-philosophicus* (1921), is not making an empirical research but a logical, in fact a mathematical one. He looks at a concept as being a function, and a mathematical function is well defined only if its truth-value is unequivocally established for any argument. However, if a complex expression is a function of the meanings of its constituent parts, then we can only accept absolutely determined meanings, since any undetermined meaning of a constituent part would eventually infect the final complex expression. According to this view, indeterminacy should be avoided simply because it is contagious. Therefore, the definition of a concept must be complete: it has to determine unequivocally whether any object falls within a given concept, or not.

This is Frege’s thinking and is also Wittgenstein’s thinking when he writes the *Tractatus*, where Wittgenstein also accepts bivalent logic and even firstly invents a truth-table. However, facts do no seem to be in favour of this logical position: like it or not, ordinary language is full of vagueness and imprecision. Frege and Russell (1872-1970) believed imprecision to be imperfection, and in front of the imperfection of ordinary language, they considered it necessary to avoid both things: imperfections and ordinary language. So they decided that sciences should completely diverge from common: they had to dispose of an own ideal language, a rigorously logical language.

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Wittgenstein shares the idea that ordinary language presents all sort of difficulties, but he does not evaluate them, he simply states that they exist: in other words, he recognizes that ordinary language is unavoidably vague but he does not reject the language in which that vagueness is expressed, because he believes that vagueness belongs only to the surface of language; after a rigorous logical analysis we can discover the underlying structure of natural languages, and their structures are absolutely perfect, in excellent logical order. Furthermore: according to Wittgenstein, this has to be so on philosophical, metaphysical reasons. Language would in no way reveal which is the configuration of the elements of reality – by definition, specific and determined – should it not dispose of ultimate elements (names) in strict correspondence with the ultimate elements of reality (objects). At the time, Wittgenstein feels that the most important point is this: without a strict correspondence between language and reality (finally, between names and objects), without a language reflecting the precise configuration of objects, we would not be able to talk about sense, let alone about truth and falsehood. That is basically the "picture theory" of the *Tractatus*. Language describes definitely definite realities. This is why language makes sense and why ordinary language, despite its vagueness, also makes sense after being properly analyzed.

In his *Philosophical Investigations* (1953), however, Wittgenstein realizes he made a mistake: when we use language, we are not using a calculus that follows fixed rules. Rules are always there just as indicators that we can use or not, and that we can use in very different ways when we use them. Bivalence, bipolarity is an optional feature of language; vagueness is not at all contagious; vagueness and precision are given in language without necessarily having to compete against each other, without having to replace one another. Discussing whether an object is "long" or "short" may become at times a tedious task, even in the case we agreed before on the definition of those terms for a given context. To be sure, some speakers may clearly define an object according with strict criteria of "length" and, nonetheless, they can come to disagree in the way their own criteria should be applied in a different context.

"Long" or "short", as any other concepts we can imagine, do not only admit degrees in use, but also very different uses that will not be reducible to an ultimate, explanatory essence. The many different uses we cover with a unique term do not reveal the existence of a unique element common to all of them but they simply show that they remain related among them through what Wittgenstein calls "family resemblances". In fact, among the various uses of the same term it is not necessary to accept a common feature – the *possibility* of such a feature is the best proof of its *non necessity* –, but just to recognize similarities and differences which remind us of those we can find in a family. However, some particular uses resemble each other not because they belong to the same family (as if such families were ready-made constructs); the other way round is true: we say that they belong to the same family because they resemble each other. Similarities always depend also on our outlook, on our interests when we consider them. Many cases of scientific or artistic progress are due to the fact that someone, at a particular time, has managed to see some similarities or differences between realities that previously have been seen too much distant or too similar. It is also true that at a given point "a family" accepts prototypes, i.e. accepts some uses as best and more significant cases, which do not play the role of an (inexistent) essence of the whole, but simply turn into a sort of axis around which other members of the same category may find its place. Prototypes may, of course, change their positions without provoking any upset or disturbance to the family resemblance of the rest of members.

For sure, it is good not to lose foot of what we usually call "reality", though we will never consider it as "pure reality". In any case, real language shows us constantly how prejudiced it is to accept an ideal language, as if there was also to be only one ideal logic. A definition is inaccurate not because it does not meet a supposed unique ideal of accuracy, but because it does not meet the demands of understanding in a particular context. That is why indeterminacy can adopt many forms and cannot be predicted or avoided in all cases. Many of our own experiences are very fuzzy and blurred, for example, our visual experiences or memories.

The general diagnosis of Wittgenstein meets some years later Zadeh's mathematical analysis. Wittgenstein says that language is not a simple logical calculus; Zadeh says that logic is not a calculus that simple. In fact, neither Wittgenstein nor Zadeh are capable to tell us on their own whether a concept is, or is not, accurate or inaccurate, for one simple reason we have already used: accuracy or inaccuracy do not refer to concepts but to uses of terms. Wittgenstein emphasized the many possibilities of inaccuracy; Zadeh helps to calculate some of them. In fact, the very wittgensteinian concept of "family resemblance" may turn out to be too vague. But, so to speak, a concept cannot exert self-criticism, it requires to be delimitated by its real or possible uses. Wittgenstein thinks that what specifies and delimitates a term is the language game in which the term appears. Zadeh contributes to it with the technique of fuzzy logic.

A concept is useful, usable, applicable, if it is well defined in some cases, so that we know what falls within the concept and what not, but also how it falls within it. In fact, a proposition remains being a proposition despite having a vague sense, as an imprecise limit is still a limit. "Make a heap" is a clear order; "Make the smallest heap which still counts as such" is not. The first order has no limits but the concept is correct; the second one aims to have limits, but it is unknown what those limits may be because they have not been yet established. We can always set limits, although not always is necessary to set them. Concepts depend on the context in which they are explicitly examined, a framework that sometimes is deemed sufficient, sometimes, insufficient, incomplete. We now know, however, that when we say "inaccurate", "incomplete", "unfinished", we are not making a judgment about reality – as if there was an ideal reality, complete and finished, to which our actions had to adapt themselves- but we are, in general, saying that we do not like something and want to change it. On the other side, when we say that what we have is "exact", "full", "finished", we are expressing our satisfaction. (The satisfaction of a philosopher or a mathematician usually is just provisional. As it is the one of the artist, by the way.)

A central contribution of Zadeh's fuzzy logic is that it provides the technique to determine the uses that define a given concept. We will have a well defined concept, even if it is a concept supposedly vague, if we know the range $[0,1]$ for a given collection of objects and if we can establish the degree of membership of those objects relatively to the range. Once we know the universe of discourse, we can map it in a thousand different ways. The objects and their membership's degree may vary, and this means that sets stay always open to new applications and functions. Obviously, fuzzy logic includes classical logic, but in no case should be contaminated by it. In fact, classical logic has not only to be a part of fuzzy logic but it has to become a marginal form of it. In language and thought creativity is endless. It is not always true, for instance, that the less different are two fuzzy sets, the more similar have to be. This is so because "similar" and "different" are not opposites terms in a world that is not necessarily bipolar. It seems clear that neither Wittgenstein nor Zadeh want to simply promote vagueness, they are not ready to make just propaganda for it. Their goals are much more interesting.

In fact, Wittgenstein wants to fight his old vision of the absolute necessity of determinacy of sense. At the beginning of his work he had really thought that a non determined sense is not sense at all, but it is nonsense. Afterwards he realizes that the indeterminacy of sense is one way of appearing sense. In terms of the interpretation of sense this means that is not always necessary to remove doubt or disagreement in the application of an expression. A use loaded by questions, disputes and disagreements, still remains a legitimate use, a use full of sense. Obviously not with the degree of sense that will have a unanimously accepted sense. May the last have even more sense? (This is a silly question, just for the sake of joking. It may happen that I like more a use of an expression than a different use of it, but this does not mean that the use I like most has more sense than the other one. A dogmatic definition of sense is unreasonable. We have to decide about sense case by case. A vague, imprecise or careless, even erroneous, expression is not condemned to be senseless or useless.)

Zadeh agrees with that. When he says that he is convinced that in the future there will only exist applied mathematics he is not supposing that in the future mathematicians will devote themselves to mathematics and only later on they will look for applications of their findings, but that mathematics – at least the major part of it – will grow out from the sheer necessity of understanding not mathematics itself but our diverse reality which needs to be managed, measured, used, interpreted.

This is what pushes me to put Wittgenstein and Zadeh side by side. They do not coincide in what they say, but they have common concerns.

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Aggregation Operators

Vicenç Torra

100.1 Introduction

It was in 1990, when I was still studying my degree in Computer Science, when I met Prof. Lotfi Zadeh for the first time, if I do not remember wrongly. It was in a conference I was participating in Barcelona where he gave one of the plenary talks. I remember clearly that he mentioned fuzzy systems and the successful Japanese applications of the late 80es. I have always used the applications of fuzzy systems in class as an example of successful applications of artificial intelligence and related technologies.

Since then I have met Prof. Zadeh in several conferences and workshops. Among all those meetings, I have specially memories of my attendance to EFDAN 1997 and 1998, the conferences organized by Dr. R. Felix in Dortmund, where I participated with Prof. L. Godo (at IIIA-CSIC). Prof. Zadeh was also attending the conference. Being a small workshop, EFDAN permitted relaxed discussions with Prof. Zadeh as well as with the other participants.

When I heard Prof. Zadeh for the first time in the early 90s, I was being introduced into the field of artificial intelligence, where I was going to complete my PhD under the supervision of Prof. Claudi Alsina and Prof. Ulises Cortés. My topic of research was information synthesis functions for artificial intelligence.

My first research related to fuzzy sets [21] was about the aggregation of membership functions. In a paper published at the 3rd Spanish conference on fuzzy logic and technologies (1993) [11], and latter extended and published in Fuzzy Sets and Systems [12]. Since then, part of my research in the area of fuzzy sets has been related to aggregation operators, including topics of fuzzy measures and fuzzy integrals. I have also studied fuzzy clustering and used fuzzy systems. This paper gives a personal view of current research on aggregation operators.

100.2 Aggregation Operators

Techniques based on fuzzy sets and fuzzy systems have an important role when modeling decisions and in information fusion. Aggregation operators [17] permits us to express how information needs to be fused. In particular, t-norms, t-conorms and all kind of means are useful functions in this process. Fuzzy measures and integrals [10] permit us additional flexibility as they permit us to express additional information about the variables or objects being fused, their interactions and dependences. Simpler aggregation operators as the weighted mean do not have this flexibility.

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Although there are several issues related to fuzziness that are important in the aggregation process (e.g. in modeling utilities and preferences), I focus here on the aggregation functions themselves. Most of my discussion is independent of the type of data being aggregated. That is, whether data is numerical, categorical, corresponds to membership functions, and so on. For particular data types some specific issues are of rellevance, as e.g. the concept of computing with words [22] when data is categorical.

At present, a large number of aggregation functions have been defined. A few years ago, in two position papers [14, 18], I mentioned as rellevant for a research agenda three research topics. I review these topics here, splitting one of them in two.

- **Parameter Learning.** Aggregation operators usually depend on some parameters. For example, the weighed mean requires a weighting vector, and the Sugeno integral a fuzzy measure. The definition of the parameters is not always easy, and due to this, we need methods to determine the parameters from examples or from the experience that an intelligent system might have.

  In the last years, a lot of research has been done on the determination of parameters from examples. Methods developed for this purpose assume that we have a set of examples consisting of the input parameters of the aggregation operators as well as the expected output. From these examples the parameters are determined as the ones that minimize the distance between the expected outcome and the real outcome. This can be expressed in more detail as follows. Let us consider a set of examples. They are defined in terms of *(input(x), output(x))* pairs, and an operator $\mathbb{C}$ with some parameter P to fit, where $\mathscr{P}$ is the set of possibles parameters; then, the parameter is selected so that the divergence between the output of $\mathbb{C}_P(input(x))$ and *output(x)* is minimized. Expressing the divergence as the distance d between the input and the output, and the total divergence as the sum for all examples $x \in Examples$, we define the parameter learning problem as follows:

$$\arg\min_{P \in \mathscr{P}} \sum_{x \in Examples} d(\mathbb{C}_P(input(x), output(x)))$$

  Note that this definition does not entail any particular data type in the examples. That is, with appropriate definitions of d and $\mathbb{C}$, it is valid for numerical and categorical data, but also for other data types as membership functions or dendrograms. Parameter determination in this way is described in some detail in [17].

  Other research on parameter determination has focused on the use and interviews to experts. This is the case where the weights are inferred using Saaty's Analytical Hierarchy Process (AHP) [8]. A mixed approach is the one in [7] for the OWA operators [20]. In this work, an expert gives a degree of the orness, and then an optimization problem is formulated to find the optimal weights that satisfy the orness level. The optimization uses dispersion as the objective function. That is, the best solution is the one that maximizes dispersion for a given orness.

  From the perspective of machine learning, all methods mentioned so far are basically supervised methods. Unsupervised learning methods, which do not include information about the output of the system, have also been considered in the literature, although in a lesser extend (see e.g. [5] and [9]). A third approach in machine learning is reinforcement learning. Up to our knowledge, no methods exist for parameter determination that follows this approach. Reinforcement learning algorithms use reward/penalty according to good or bad behaviour of the outcome of the aggregation method.

Taking into account the machine learning perspective, an issue that is not considered in the literature on aggregation operators is the one of the bias-variance problem. This is to study the appropriate number of parameters so that the model is detailed enough but not overfitted to the data. For some of the aggregation methods, as e.g. the fuzzy integrals, the number of parameters is not a constant but can be selected according to the complexity of the problem. This is the case of k-additive measures [4] and m-dimensional distorted probabilities [16], where with $k = 1$ or $m = 1$ we have basically as much parameters as the number of inputs. In contrast, for $k = |X|$ we have $2^{|X|}$ parameters. Research is needed in this direction.

- **Function/Operator Selection.** Characterization of functions permits us to select the appropriate function for a given problem. Most usual functions for numerical data (as e.g. the weighted mean, quasi-arithmetic mean, Choquet integral) have already been characterized. In addition, families of operators have been established, and relationships between some operators and more general ones have been established. For example, it is well known that the Choquet integral is a generalization of the weighted mean or, in other words, that the weighted mean is a particular case of the Choquet integral.

  New families of operators have been defined in the last years, some of them for interval-valued fuzzy sets and other generalizations. Characterizations of these operators are needed. Families of these operators have to be studied.

  In addition, algorithms for automatic selection of the aggregation operator given a set of examples would be useful. Some research has already been done in this area. See e.g. [1]. The solution of the bias-variance problem can also help in this direction.
- **Architecture for Information Integration.** Within artificial intelligence, aggregation operators are embedded in intelligent systems. In these systems, prior to the aggregation, data have to be transformed so that the aggregation makes sense. In the same way, decisions should be made with the result of the aggregation process. All these steps define the architecture of information integration. In short, information integration includes the following four processes: data acquisition, data preprocessing, information fusion and aggregation, and the final execution of an action. The architecture explains how these elements are structured for an optimal performance. Research exist on architectures for information integration, but additional synergies with the research on aggregation functions might foster this research.
- **Hierarchical Models for Data Aggregation.** Aggregation operators can be just simple functions as the arithmetic mean or the Choquet integral, or composite models built over more simple functions. Hierarchical models (as the two-step Choquet integral [6]) are examples of such composite models. It has been proven

that models based on aggregation operators can approximate any function at the desired level of detail. See e.g. [13]. This good result is at the expenses of a major complexity: the hierarchy has to be defined, sets of functions have to be selected, and their parameters fixed. Due to this, the definition of a hierarchical model is a difficult task.

Although at present research in this area is focused on the properties of the models (as in the above mentioned work [6]), some more practical oriented results also exist (see e.g. [2, 3]). This research links with other research on composite models as hiearchical fuzzy systems [15, 19]. Further research is needed in this area.

100.3 Conclusions

In this paper I have outlined some lines of research related to aggregation operators and some lines for future research. We have mentioned the issues of parameter learning, function selection, architectures for information integration, and also hierarchical models.

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References

1. Beliakov, G., Warren, J.: Appropriate Choice of Aggregation Operators in Fuzzy Decision Support Systems. IEEE Transactions on Fuzzy Systems 9(6), 773–784 (2001)
2. Dujmovic, J.J., de Tré, F.: Multicriteria methods and logic aggregation in suitability maps. International Journal of Intelligent Systems 26(10), 971–1001 (2011)
3. Dujmovic, J.J., Nagashima, H.: LSP Method and its Use for Evaluation of Java IDEs. International Journal of Approximate Reasoning 41(41), 3–22 (2006)
4. Grabisch, M.: *k*-order Additive Discrete Fuzzy Measures and Their Representation. Fuzzy Sets and Systems 92(2), 167–189 (1997)
5. Kojadinovic, I.: Unsupervised Aggregation by the Choquet Integral Based on Entropy Functionals: Application to the Evaluation of Students. In: Torra, V., Narukawa, Y. (eds.) MDAI 2004. LNCS (LNAI), vol. 3131, pp. 163–175. Springer, Heidelberg (2004)
6. Murofushi, T., Narukawa, Y.: A Characterization of Multi-level Discrete Choquet integral Over a Finite Set. In: Proceedings of the 7th Workshop on Evaluation of Heart and Mind, pp. 33–36 (2002) (in Japanese)
7. O'Hagan, M.: Aggregating Template or Rule Antecedents in Real-time Expert Systems With Fuzzy Set Logic. In: Proceedings of the 22nd Annual IEEE Asilomar Conference Signals, Systems, Computers, Pacific Grove, CA, pp. 681–689 (1988)
8. Saaty, T.L.: The Analytic Hierarchy Process. McGraw-Hill, New York (1980)

9. Soria-Frisch, A.: Unsupervised Construction of Fuzzy Measures Through Self-organizing Feature Maps and its Application in Color Image Segmentation. International Journal of Approximate Reasoning 41, 23–42
10. Sugeno, M.: Theory of Fuzzy Integrals and its Applications, Ph. D. Dissertation, Tokyo Institute of Technology, Tokyo, Japan (1974)
11. Torra, V.: Consenso de funciones de pertenencia triangulares. Actas del Tercer Congreso Español sobre Tecnologías y Lógica Fuzzy, Santiago de Compostela, pp. 161–168 (1993)
12. Torra, V.: Combining Fuzzy Sets: The Geometric Consensus Function Family. Fuzzy Sets and Systems 74, 335–342 (1995)
13. Torra, V.: On Some Relationships Between Hierarchies of Quasi-arithmetic Means and Neural Networks. International Journal of Intelligent Systems 14(11), 1089–1098 (1999)
14. Torra, V.: Aggregation Operators and Models. Fuzzy Sets and Systems 156(3), 407–410 (2005)
15. Torra, V., Narukawa, Y.: On the Meta-knowledge Choquet integral and Related Models. International Journal of Intelligent Systems 20(10), 1017–1036
16. Torra, V., Narukawa, Y.: Fuzzy Measure and Probability Distributions: Distorted Probabilities. IEEE Transactions on Fuzzy Systems 13(5), 617–629 (2005)
17. Torra, V., Narukawa, Y.: Modeling Decisions: Information Fusion and Aggregation operators. Springer, New York (2007)
18. Torra, V., Narukawa, Y.: A View of Averaging Aggregation Operators. IEEE Transactions on Fuzzy Systems 15(6), 1063–1067 (2007)
19. Tunstel, E., de Oliveira, M.A.A., Berman, S.: Fuzzy Behavior Hierarchies for Multi-Robot Control. International Journal of Intelligent Systems 17, 449–470 (2002)
20. Yager, R.R.: On Ordered Weighted Averaging Aggregation Operators in Multi-criteria Decision Making. IEEE Transactions on Systems, Man and Cybernetics 18, 183–190 (1988)
21. Zadeh, L.A.: Fuzzy Sets. Information and Control 8, 338–353 (1965)
22. Zadeh, L.A.: From Computing With Numbers to Computing With Words – From Manipulation of Measurements to Manipulations of Perceptions. International Journal of Applied Mathematics and Computer Science 12(3), 307–324 (2002)

101

On Some Classical Tenets and Fuzzy Logic

Enric Trillas

101.1 Introduction

A fertile seed dropped by Zadeh's work into the soil of classical logic, refers to the possibility of reconsidering some of the tenets logicians seems to preserve with almost no actual debate with thinkers in the 'fuzzy' community of researchers. The flourishing potentiality of that seed is partially due to the different perspective from which fuzzy logic looks at what is its object in comparison with what the classical views maintain. Such a different perspective comes, in a first place, by considering imprecise linguistic terms, linguistic connectives, modifiers and quantifiers as the objects to be represented, instead of the classical formal ones, and commonsense reasoning processes instead of formal ones. That is, for instance, for trying to mathematically and computationally modeling the Natural Language's expressions with which some dynamical systems are described, or can only be described when no precise mathematical models of them are available. Almost always these expressions are not representable with classical sets without modifying their meaning as it is given by their use under some purpose in the corresponding context. Context-sensitive and purpose-driven meaning are typical characteristics shown by the problems fuzzy logic deals with.

Since the allowed extension for this paper does not permit a large presentation, only the following three topics will be shortly taken into account:

- The validility of some formal fuzzy laws, and the design of fuzzy models.
- The universality of the principles of non-contradiction and excluded-middle in fuzzy logic
- The necessity of fuzzy logic to consider non-deductive reasoning.

101.2 On the Validity of Some Formal (Fuzzy) Laws, and the Design of Fuzzy Models

Since one of the goals of fuzzy logic is the formal representation of natural language statements made up with both precise-boolean and imprecise terms, at least involving the linguistic connectives and, or, not, it is important to have formal frames of representation able to capture as much as possible the different properties these connectives show in language. Such frames are known as 'algebras of fuzzy sets' ,or 'fuzzy algebras', of which no a generally accepted definition is currently known in

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the same form as there are known those of, for instance, ortholattices, and De Morgan algebras. Since natural language is very complex and not static at all, a general definition of fuzzy algebra is not an easy task. For instance, neither the linguistic 'and' is always commutative, nor the linguistic 'not' is always involutive, as they are considered in the former lattices.

Additionally, since fuzzy sets are at its turn represented by functions with values in the unit interval, it is also not obvious that the functions representing the before mentioned connectives could always be functionally expressible by numerical functions. For instance, if a operation . between fuzzy sets represents the linguistic 'and', it should be distinguished if such representation is, or is not, functionally expressible in the form, $\propto \cdot \sigma = T \circ (\propto \times \sigma)$, with some numerical function $T : [0,1] \times [0,1] \to [0,1]$, as it is the case if T is a (commutative!) continuous t-norm.

Once assumed that the set of functions $[0,1]^X = \{\propto; \propto : X \to [0,1]\}$ is partially ordered by the pointwise ordering,

$$\propto \leq \sigma \text{ iff } \propto(x) \leq \sigma(x), \text{ for all } x \text{ in } X,$$

a tentative, but general enough definition of fuzzy algebra for the goal of this paper, is the following [1],

Definition 1. *A* **Basic Formal Fuzzy Algebra** *(BFFA) is a four-tuple* $([0,1]^X, \cdot, +, ')$*, where* $\cdot$ *and* $+$ *are binary operations* $[0,1]^X \times [0,1]^X \to [0,1]^X$*, and* $'$ *is a unary operation* $[0,1]^X \to [0,1]^X$*, verifying:*

1) If $\propto \leq \sigma$*, then* $\propto \cdot \lambda \leq \sigma \cdot \lambda$*, and* $\lambda \cdot \propto \leq \lambda \cdot \sigma$*,*
If $\propto \leq \sigma$*, then* $\propto + \lambda \leq \sigma + \lambda$*, and* $\lambda + \propto \leq \lambda + \sigma$*, for all* λ *in* $[0,1]^X$*.*
2) $\propto \cdot \propto_0 = \propto_0 \cdot \propto = \propto_0$*,* $\propto \cdot \propto_1 = \propto_1 \cdot \propto = \propto$*,*
$\propto + \propto_0 = \propto_0 + \propto = \propto$*,* $\propto + \propto_1 = \propto_1 + \propto = \propto_1$*,*
with $\propto_0$ *and* $\propto_1$ *the constant fuzzy sets respectivelly equal to 0 and 1.*
3) $\propto_0' = \propto_1$*, and* $\propto_1' = \propto_0$
If $\propto \leq \sigma$*, then* $\sigma' \leq \propto'$
4) If $\propto$ *and* σ *are in* $\{0,1\}^X$*, then also* $\propto \cdot \sigma$*,* $\propto + \sigma$ *and* $\propto'$*, are in* $\{0,1\}^X$*, and* $\propto \cdot \sigma = min(\propto, \sigma)$*,* $\propto + \sigma = max(\propto, \sigma)$*,* $\propto' = 1 - \propto$*.*

Axiom (4) is necessary to capture the representation of crisp terms. It is easy to prove that in all BFFA it holds:

a) $\propto \cdot \sigma \leq min(\propto, \sigma) \leq max(\propto, \sigma) \leq \propto + \sigma$;
b) The only BFFA that are lattices are the De Morgan algebras $([0,1]^X; min, max, ')$ provided $'$ is involutive;
c) No BFFA is an ortholattice, thus no one is a Boolean algebra.

Notice that it is not supposed that a BFFA is neither with . or + commutatives, nor associatives, nor that ' is involutive, nor that distributive or duality laws do hold,etc. A BFFA is functionally expressible (FE) provided there are three numerical functions $T, S : [0,1] \times [0,1] \to [0,1]$, $N : [0,1] \to [0,1]$, such that $\propto \cdot \sigma = T \circ (\propto \times \sigma)$, $\propto + \sigma = S \circ (\propto \times \sigma)$, and $\propto' = N \circ \sigma$, for all $\propto$ and σ in $[0,1]^X$. This is the case of the Standard

Fuzzy Algebras in which T is a continuous t-norm, S is a continuous t-conorm, and N is a strong negation (see [2]), that are those currently considered in both the theoretic and applied literature on fuzzy logic.

That some imprecise assertive statements can be satisfactorily represented by means of fuzzy sets, does not imply that all the laws of Boolean algebras can be applied to, for instance, shortening complex statements. This is the case with statements of the type "(x is P and y is Q) or (x is P and y is not Q)", that cannot be always taken as equivalent to "x is P", since the logical law of *perfect repartition*, $p \cdot q + p \cdot q' = p$, only holds in the setting of Boolean algebras, but neither in proper ortholattices, nor in proper De Morgan algebras, and less again in most fuzzy algebras. The problem lies in finding fuzzy algebras $([0,1]^X, \cdot, +, ')$ in which the representation of the former statement: $(\propto_P \cdot \propto_Q)(x) + (\propto_P \cdot \propto'_Q)(x)$, does coincide with $\propto_P(x)$ for all x in X. That is, the fuzzy algebras where the formal law $\propto \cdot \sigma + \propto \cdot \sigma' = \propto$, holds. The problem is completely solved [3], and also in the case the algebra is functionally expressible by means of a continuous t-norm T, a continuous t.conorm S, and a strong negation N, that is, in the particular setting of the Standard Fuzzy Algebras. The only standard algebras of fuzzy sets in which the corresponding law $\propto(x) = S(T(\propto(x), \sigma(x)), T(\propto(x), N(\sigma(x))))$ holds for all x in X, are those given by $T = Prod_\varphi$, $S = W^*_\varphi$, $N = N_\varphi$, for any order-automorphism φ of the unit interval. In these algebras, that are not lattices, neither any law of duality [1], nor several other lattice's laws hold.

The law can be used only in the contexts where *and* can be modeled by $T = Prod_\varphi$, *or* by $S = W^*_\varphi$, and *not* by N_φ, with the same φ. A situation very different of the classical-boolean, and with some distant similarity with the quantum-orthomodular. If the involved linguistic terms are imprecise, but representable by fuzzy sets, the law is only applicable provided the connectives admit the former representations.

This example shows that when imprecise statements in natural language involving linguistic connectives do be represented in fuzzy terms, it is strictly necessary to correctly choose the corresponding algebra. At its turn the algebra can force the non validity of some other laws [4], [6] that, if necessary, can be reached by adding new connectives. For instance, if in the example it were necessary that "(x is P) and (x is not P)" always does not hold, it can be considered the Pexider algebra with two 'intersections' $([0,1]^X, Prod_\varphi, W_\varphi, W^*_\varphi, N_\varphi)$ (see [5]), in which for the second conjunction holds $\propto \cdot \propto' = W_\varphi(\propto \times N \circ \propto) = \propto_0$,for all $\propto$ in $[0,1]^X$. It should be noticed that in large linguistic pieces different uses of the connectives can appear like it is the case of commutative and non-commutative uses of 'and'.

Nevertheless, in all case concerning the representation of linguistic pieces the first problem is the design or selection of the membership functions of the predicates appearing in them, that depends on the context and the purpose in which they are used. For instance, the usual representation by piecewise linear membership functions could be just erroneous if this kind of functions are not in agreement with what is known on the use of the corresponding linguistic terms. Concerning the design or selection of the linguistic connectives, their representations in fuzzy terms do be choosen accordingly with their meanings. For instance, in the case of a rule "If x is P, then y is Q' in which the negation of the antecedent does not play any role (for instance, for not having physical sense), it is possible a conjunctive-type representation of the type $T(\varphi(\propto_P(x)), \propto_Q(y))$ [16]. Notwithstanding, provided it were known that it should always be $\propto_P(x) \leq \propto_Q(y)$, the correct representation could be a residuated implication [6], [8] but not a conjunctive-type one.

In any case, before representing anything in terms of fuzzy sets and fuzzy connectives, to capture the best than possible knowledge of the system, or the best understanding of the linguistic piece and its parts, is essential. A non correct enough design can conduct to represent in fuzzy terms a problem different from the given linguistic one, and at least to eventually finding partially unrealistic solutions of the linguistically posed problem. A good knowledge on the basic mathematical theory of fuzzy algebras and fuzzy logic is actually important for the designers, and yet it will be more important in the path towards Zadeh's Computing with Words in which larger and more complex natural language's expressions will play a pivotal role.

101.3 On the Universality of the Principles of Non-contradiction and Excluded-Middle in Fuzzy Logic

The linguistic term 'impossible', Aristotle used to state the principle of Non-Contradiction (NC), was translated by 'false', by $A \cap A' = \emptyset$ in the boolean case of classical sets, and by $\propto \cdot \propto' = \propto_0$ in that of the algebras of fuzzy sets. In the standard algebras, the principle corresponds with the verification of the equation $T(\propto(x), N(\propto(x))) = 0$, for all $\propto$ in $[0,1]^X$, and all x in X. Hence, the solutions (T,N) of the functional equation $T(a,N(a)) = 0$, for all a in $[0,1]$, give the cases in which NC holds. These solutions are $T = W_\varphi$, and $N \leq N_\varphi$ (see [7]. Hence the principle NC fails in most of the standard algebras of fuzzy sets.

Although Aristotle is very opaque with respect to the principle of Excluded-Middle (EM),it is currently translated by A U A' = X with classical sets, and by $\propto + \propto' = \propto_1$ in the algebras of fuzzy sets. In the standard algebras, this corresponds with the verification of the equation $S(\propto(x), N(\propto(x))) = 1$, for all $\propto$ and all x. Hence the solutions (S,N) of the functional equation $S(a,N(a)) = 1$, give the cases in which EM holds. These solutions are $S = W_\psi^*$, and $N_\psi \leq N$ (see [8], and show that the principle EM fails in most of the standard algebras.

Hence, both NC and EM principles do hold in the standard algebras of fuzzy sets if and only if $T = W_\varphi$, $S = W_\psi^*$, and $N_\psi \leq N \leq N_\varphi$. There are algebras in which only one of the two principles hold, and both jointly fail in many, many cases.

Nevertheless, if 'impossible' is translated by 'self-contradictory', both principles do hold in all BFFA if posed by:

- NC : $\propto \cdot \propto' \leq (\propto \cdot \propto')'$, that is, $\propto \cdot \propto'$ is self-contradictory,
- EM : $(\propto + \propto')' \leq ((\propto + \propto')')'$, that is $(\propto + \propto')'$ is self-contradictory,
  and it is easy to prove that the two inequalities hold in all BFFA (see [9]).

It should be noticed that provided the operations $\cdot$ and $+$ are commutative, the negation is involutive, and the laws of duality hold, the former NC and EM inequalities reduce to the obviously valid inequality $\propto \cdot \propto' \leq \propto + \propto'$. It also should be noticed that if $\propto$ is in $\{0,1\}^X$, then NC reduces to $\propto \cdot \propto' = \propto_0$ and EM reduces to $\propto + \propto' = \propto_1$. In [10], and provided the BFFA is FE but without t-norms and t-conorms, the numerical functions giving the functional expressibility of $\cdot$ and $+$ verifying NC and EM are characterized. Hence, expressing the principles by understanding 'impossible' as 'self-contradictory' or 'absurd', both NC and EM principles are universally valid in all BFFA and, in particular, in the standard algebras of fuzzy sets, something that sheds a different light on what is usually asserted since, in fact, *fuzzy sets never violate the two Aistotle's principles*. In addition, it does be remarked that the fuzzy case opens the way towards a deductive study of the the general validity of the two principles (see [9]), perhaps against the Aristotle's view that at least NC cannot be submitted to proof.

101.4 On the Necessity of Formalizing Non-deductive Reasoning

Most of the commonsense reasonings are not deductive, but conjectural. Perhaps no more than a 25% of the totality of these reasonings are deductively made step by step and under well known rules of inference. Hence, the remaining 75% is of relevance for any methodology trying to represent Commonsense Reasoning (CR) throughout some formalization process that, hence, should necessarily take into account the non-deductive ways of reasoning. That is, abductive and inductive types of reasoning in which, and contrarily to deduction, are typical the 'jumpings' to the conclusions.

Of course, deduction does be considered the only 'safe' form of reasoning in the sense that their conclusions are as valid as premises can be, and for any deductive conclusion its negation is refused as such. Instead, abduction and induction do not give 'safe' conclusions since they are not only doubtful with respect to the given premises, but its negations can be also obtained from the same premises. If in deduction all that is concluded just deploys, or necessarily follows from what is in the premises, in abduction and induction the situation is different since their conclusions often represent something that is 'new', in the sense of not being directly deployable from the premises.

In addition, deduction is monotonic since when new premises are known, no less conclusions can be deployed. Instead, neither abduction, nor induction can be monotonic; experience shows that in these kind of reasonings new premises can easily conduct to cancel some previously reached conclusions, that is, and cautiously said, no more conclusions can be obtained.

At this respect, a first problem is how to define the concept of conjecture in such a way that deductive, abductive, and inductive conclusions, can be captured as their only particular cases. Conjectures are viewed as those elements in the frame of representation thar are just 'consistent' with the information conveyed by the premises.

With the goal of not introducing more technical complexities than those that are strictly necessary, let us try to introduce this new concept (see [11]) in the setting of the De Morgan algebra of fuzzy sets $([0,1]^X, min, max, 1-id)$, that currently is maybe the most employed one in the applications of fuzzy logic.

Definition 2. *Let $P = \{\propto_1, ..., \propto_n\} \subset [0,1]^X$, a set of premises such that $\propto_\wedge = min$ $(\propto_1, ..., \propto_n)$ is not self-contradictory, that is $\propto_\wedge \nleq 1 - \propto_\wedge$. The set of conjectures from P, is $Conj(P) = \{\sigma \in [0,1]^X; \propto_\wedge \nleq 1 - \sigma\}$.*

Obviously, it is $P \subset Conj(P)$, and

a) If $P \subset Q$, then $Conj(Q) \subset Conj(P)$. That is, the 'operator' $Conj$ is anti-monotonic.
b) The operator $C(P) = \{\sigma \in [0,1]^X; \propto_\wedge \leq \sigma\}$, is extensive: $P \subset C(P)$, monotonic: $P \subset Q \to C(P) \subset C(Q)$, a clausure: $C(C(P)) = C(P)$, and consistent: $\sigma \in C(P) \to \sigma' \notin C(P)$: C is a consistent consequence operator.
c) $C(P) \subset Conj(P)$, and $Conj(P) = \{\sigma \in [0,1]^X; \sigma' \notin C(P)\}$.
d) It cannot be neither assumed that if $\sigma \in Conj(P)$, then $\sigma' \notin Conj(P)$, nor that $\sigma \nleq \sigma'$.
e) It is supposed that $\propto_\wedge$ summarizes the information conveyed by P.

Result (c) could make to think that conjecturing should necessarily come after deduction. Nevertheless, there are operators that can be called of conjectures and that do not 'follow' from a consequence operator. For instance,

$$Conj^*(P) = \{\sigma \in [0,1]^X; \propto_\wedge \cdot \sigma \nleq (\propto_\wedge \cdot \sigma)'\},$$

which although verifying $P \subset C(P) \subset Conj^*(P)$, and being anti-monotonic, is not coming from any conjecture operator since the only possible C^* with which it can be $Conj^*(P) = \{\sigma; \sigma \notin C^*(P)\}$, is $C^*(P) = \{\sigma; \propto_\wedge \cdot \sigma' \leq (\propto_\wedge \cdot \sigma')'\}$, that is not a consequence operator. Of course, $Conj^*$ is obtained after understanding the consistency with $\propto_\wedge$ as "$\propto_\wedge \cdot \sigma$ is not self-contradictory", instead that for Conj is understood by 'σ is not contradictory with $\propto_\wedge$'. Yet also the operator

$$Conj^{**}(P) = \{\sigma \in [0,1]^X; \propto_\wedge \cdot \sigma \neq \propto_0\},$$

obtained by understanding the consistency with $\propto_\wedge$ as 'non-incompatibility´ with it, only can come from the operator $C^{**}(P) = \{\sigma; \propto_\wedge \cdot \sigma' = \propto_0\}$, that is not also a consequence operator. $Conj^{**}$ is anti-monotonic, and verifies $P \subset C(P) \subset Conj^{**}(P)$ (see [13]). Hence, it does not seem that conjecturing necessarily comes from a previous form of logical deduction even if always includes deduction as a particular case.

Which are the other types of conjectures? Since, $Conj(P) - C(P) = \{\sigma; \propto_0 < \sigma < \propto_\wedge\} \cup \{\sigma \in Conj(P); \sigma NC \propto_\wedge\}$, with NC shortening 'not comparable under the ordering $\leq$', and $\propto_0 \notin Conj(P)$ since $\propto_\wedge \leq \propto_1 = \propto_0'$, it can be defined $\{\sigma; \propto_0 < \sigma < \propto_\wedge\} = Hyp(P)$, and $\{\sigma; \sigma NC \propto_\wedge \nleq \sigma'\} = Sp(P)$, the subsets of *hypotheses* (or explanative conjectures), and of *speculations* (or lucubrative conjectures), respectivelly. In this way, since $Conj(P) = C(P) \cup Hyp(P) \cup Sp(P)$ is clearly a partition, it can be said that conjecturing just consists in deducing (obtaining consequences), abducing (obtaining hypotheses), and inducing (obtaining speculations). In addition, the set $Ref(P) = Conj(P)' = \{\sigma; \sigma' \in C(P)\}$, can be called that of the *refutations* of P.

Notice that the partition, $[0,1]^X = Conj(P) \cup Ref(P) = (C(P) \cup Ref(P)) \cup (Hyp(P) \cup Sp(P))$, shows that $C(P) \cup Ref(P)$ is the set of C-decidable elements, and Hyp(P) U Sp(P) is that of the C-undecidable elements of $[0,1]^X$.

Remarks

a) Notice that by their own definition in the presented model, to find speculations could force to make 'jumps' in the poset $([0,1]^X, \leq)$.
b) If $\sigma \in Sp(P)$, and provided $\propto_\wedge \cdot \sigma \neq \propto_0$, it is $\propto_\wedge \cdot \sigma \in Hyp(P)$, and $\propto_\wedge + \sigma \in C(P)$. Hence, once jointly taken with $\propto_\wedge$, speculations can help to obtain hypotheses and consequences [17]. Something that seems in agreement with how people sometimes reason.
c) In a slighty different way, also in $Conj^*(P)$ and in $Conj^{**}(P)$, hypotheses and speculations can be individuated by separating $C(P)$ in them see ([12] [13]).
d) All that has been presented can be *mutatis mutandis* translated to the setting of ortholattices [12] by just substituing the condition that in all P, $\propto_\wedge$ is not self-contradictory by the weaker condition that the 'intersection' of all the premises in P is not nul (in which case it is not self-contradictory, and it is only equivalent in the particular case of Boolean algebras).
e) With all that has been presented, it is possible to accept that most of reasoning could be identified with 'deducing+abducing+inducing+refuting'. Of course, if deducing and refuting are nothing else than deductive processes, there is not yet clear how the undecidable elements can be systematically obtained in human reasoning (see [14]). A way that sometimes people use, is by means of an analogy with a previously considered similar case.

101.5 Conclusion

This paper just constitutes a reflection on three topics that, differentiating the methodologies of classical and fuzzy logics, could be important for the path towards Zadeh's Computing with Words. Of course, provided this new subject is, also and additionally, viewed as an enlargement of fuzzy logic with the goal of representing more complex linguistic expressions than those considered in its current applications.

The first topic tries to support the view under which to deal with non-ambiguous expressions in Natural Language, it is necessary a correct design of all the fuzzy terms representing the linguistic elements, from membership functions and connectives to modifiers and quantifiers, since there are no universal ways to represent them. Between lines, it is also pointed out that a more general concept of what is currently understood by a fuzzy algebra does be introduced for such enterprise. The axiom of specification for classical sets (see [15]) has not an immediate translation to fuzzy sets since, for instance, imprecise predicates usually neither perfectly classify the universe of discourse, nor have the same membership function in all context.

The second topic tries to notice that there are alternative views allowing to prove as theorems what formerly has been taken either as axioms, or as failing properties, like are the important cases of the NC and EM principles. These alternatives help, for instance, to see fuzzy logic as founded in more solid grounds, and support the fact that, although its referent is imprecise, fuzzy logic offers precise ways of formally dealing with imprecision.

The third topic tries to show that there are new possibilities to go towards seeing logic as more than the study of deductive systems, by means of some mathematical formalizations of the non-deductive forms of reasoning, and once they are based on the concept of 'conjecture'. Something that seems essential for any useful setting of representation, in which larger parts of Commonsense Reasoning could be mathematically modeled by, for instance, considering most of 'people's reasoning' as the sum 'conjecturing+refuting'.

Anyway, this paper is not conclusive but, thoughout some insights it only tries to be a 'suggestive' one.

Fig. 101.1. Surrounding Lotfi Zadeh at the Eleventh IEEE International Symposium on Multiple-Valued Logic in Oklahoma City, 1981: Settimo Termini, Ronald Yager, Francesc Esteva, NN, Sergei Ovchinnikov, Teresa Riera, Lorenzo Peña, Enric Trillas.

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References

1. Trillas, E., Alsina, C., Pradera, A.: On a Class of Fuzzy Set Theories. In: Proceedings of the FUZZ-IEEE 2007, IEEE International Conference on Fuzzy Systems, Imperial College, London, UK, July 23-26, pp. 1–5 (2007)
2. Pradera, A., Trillas, E., Renedo, E.: An Overview on the Construction of Fuzzy Set Theories. New Mathematics and Natural Computation 1(3), 329–358 (2005)
3. Alsina, C.: On Connectives in Fuzzy Logic Satisfying the Condition $S(T1(x,y),T2(x,N(y))) = x$. In: Proceedings of the 6th IEEE International Conference on Fuzzy Systems (FUZZ-IEEE 1997), Barcelona, Spain, pp. 149–153 (1997)
4. Alsina, C., Trillas, E.: On Iterative Boolean-like Laws of Fuzzy Sets. In: Montseny, E., Sobrevilla, P. (eds.) Proceedings of the Joint 4th Conference of the European Society for Fuzzy Logic and Technology and the 11th Rencontres Francophones sur la Logique Floue et ses Applications (EUSFLAT-LFA 2005), Barcelona, Spain, September 7-9, pp. 389–394 (2005)
5. Alsina, C., Trillas, E., Renedo, E.: On Two Classic Laws with Fuzzy Sets. In: Proceedings of the XIII Congreso Espaõl sobre Tecnologías y Lógica Fuzzy (ESTYLF 2006), Ciudad Real, pp. 19–21 (2006)
6. Trillas, E., Guadarrama, S.: Fuzzy Representations Need a Careful Design. International Journal of General Systems 39(3), 329–346 (2010)
7. Trillas, E., Alsina, C., Pradera, A.: Searching for the Roots of Non-contradiction and Excluded-middle. International Journal of General Systems 31(5), 499–513 (2002)
8. Klir, G.J., Yuan, B.: Fuzzy Sets and Fuzzy Logic: Theory and Applications. Prentice Hall PTR, Upper Saddle River (1995)
9. Trillas, E.: Non Contradiction, Excluded Middle, and Fuzzy Sets. In: Di Gesù, V., Pal, S.K., Petrosino, A. (eds.) WILF 2009. LNCS, vol. 5571, pp. 1–11. Springer, Heidelberg (2009)
10. García-Honrado, I., Trillas, E.: Characterizing the Principles of Non Contradiction and Excluded Middle in $[0,1]$. International Journal of Uncertainty, Fuzziness and Knowledge Based Systems 2, 113–122 (2010)
11. Trillas, E.: On Conjectures in De Morgan Algebras. ECSC Research Report (available upon request) (2011)
12. García-Honrado, I., Trillas, E.: On an Attempt to Formalize Guessing. In: Seising, R., Sanz González, V. (eds.) Soft Computing in Humanities and Social Sciences. STUDFUZZ, vol. 273, pp. 237–255. Springer, Heidelberg (2012)
13. Trillas, E.: A Mathematical Model to 'Reason' With Fuzzy Sets. ECSC Research Report (available upon request) (2012)
14. Barker, S.F.: Induction and Hypotheses. Cornell University Press, Ithaca (1957)
15. Halmos, P.R.: Naive Set Theory. D. Van Nostrand Company, Princeton (1960); reprinted: Springer, New York (1974)
16. Trillas, E., Alsina, C.: From Leibniz's Shinning Theorem to the Synthesis of Rules through Mamdani-Larsen Conditionals. In: Trillas, E., Bonissone, P.P., Magdalena, L., Kacprzyk, J. (eds.) Combining Experimentation and Theory. STUDFUZZ, vol. 271, pp. 247–258. Springer, Heidelberg (2011)
17. Trillas, E., Pradera, A., Álvarez, A.: On the Reducibility of Hypotheses and Consequences. Information Sciences 179(23), 3957–3963 (2009)

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Fuzzy Regression Models Beyond Fuzzy Rule Base Models

I. Burhan Türkşen

102.1 Preface

I was introduced to Prof. Dr. Lotfi A. Zadeh in 1970 Summer by Robert Macol, then the President of Operations Research Society of America (ORSA) at a Nato Advanced Study Institute meeting in Istanbul Turkey. In 1971, one of my research students gave me a copy of Zadeh‘s first paper called Fuzzy Sets. In 1972, Prof. Zadeh came as a member of an accreditation team to visit our Department of Industrial Engineering at the University of Toronto. At our Faculty Club Lunch held during that visit, Prof. Zadeh told me that he was born in Baku Azerbaijan. From that moment on, I attended Prof Zadeh‘s lectures at various conferences for the next 5 years or so. In 1976 when I went for my first research leave, I studied Prof Zadeh's various papers including "the Concept of a Linguistic Variable". [7,8]

Later in the 80‘s I visited Prof Zadeh at Berkeley on several of my research leaves. As well, I started my research publications with Disjunctive and Conjunctive Normal Forms discovering their separation forming upper and lower bounds in their fuzzy versions. After working with fuzzy rule based systems a number of years, I have introduced Fuzzy Regression Functions in place of Fuzzy Rule Bases. Currently I am working on the generation of "Full Type-2" fuzzy system models from data analyses. I have developed an algorithm that generates FULL TYPE2, FULL TYPE3 ... system models in a recursive manner.

In most of my research in fuzzy logic, Prof. Zadeh personally and his published papers, in particular, have always provided a source of inspiration.

102.2 Abstract

We review first the two Fuzzy Rule Based System models such as Sugeno-Yasukawa and Tagaki-Sugeno approaches which are essentially extensions of Zadeh‘s fuzzy rulebase model. Next an essential structure of Türkşen‘s approach is reviewed as a foundation for Fuzzy Regression models which forms the bases of many of our recently published research papers.

102.3 Background

As a background, we review three distinct approaches which are:

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(a) Sugeno-Yasukawa [3] approach where both the right hand side and the left hand side of a fuzzy rule base are determined either by experts or by fuzzy clustering algorithms such as FCM (Bezdek, [1]).
(b) Tagaki-Sugeno [5] approach where fuzzy sets of the left hand side of a fuzzy rule base are determined either by experts or by fuzzy clustering algorithms such as FCM (Bezdek, [1]) and the right hand sides are regression functions determined by function estimation methods.
(c) Türkşen's [6] approach was further investigated by Celikyilmaz and Türkşen [2]. In these papers, a classical regression is enhanced by introduction of membership values and their transformations. This improves the regression constant. Hence, we introduce "Fuzzy Regression Models" in place of fuzzy rule bases. In these investigations, a fuzzy clustering algorithm such as FCM (Bezdek, [1]) or IFC (Celikyilmaz-Türkşen [2]) is used to determine the number of such fuzzy regressions required for an effective solution.

102.4 Fuzzy Regression Models

Türkşen [6] and Celikyilmaz-Türkşen [2] approaches are enhanced regression models with an introduction of membership values and their transformations to form *Fuzzy Regression Models with LSE, FRM-LSE*, which are improved alternatives to fuzzy rule base systems.

The generalization of LSE for Fuzzy Regression Models, called *FRM-LSE for short*, and their various versions requires that a fuzzy clustering algorithm, such as FCM (Bezdek, [1]), or IFC of Celikyilmaz-Türkşen [2] be available to determine the interactive (joint) membership values of input-output variables in each of the fuzzy clusters that can be identified for a given training data set. For this purpose:

Let (X_k, Y_k), $k = 1, \ldots, nd$, be the set of observations in a training data set, such that $X_k = \left(x_{jk} | j = 1, \ldots, nv, k = 1, \ldots nd\right)$

First, one determines the optimal (m^*, c^*) pair for a particular performance measure, i.e., a cluster validity index, with an iterative search and an application of FCM or IFC algorithm, where m is the level of fuzziness (in our experiments we usually take $m = 1.4, \ldots, 2.6$) (Ozkan and Türkşen, [4], Celikyilmaz-Türkşen [2]), and c is the number of clusters (in our experiments we usually take $c = 2, \ldots, 10$). The well known FCM algorithm can be stated as follows:

$$min\, J(U,V) = \sum_{k=1}^{nd} \sum_{i=1}^{c} (u_{ik})^m \left(\|x_k - v_i\|_A\right) \tag{102.1}$$

s.t. $\quad 0 \le u_{ik} \le 1, \quad \forall i,k, \qquad \sum_{i=1}^{c} u_{ik} = 1, \forall k, \quad 0 \le \sum_{k=1}^{nd} u_{ik} \le nd, \forall i,$

where J is the objective function to be minimized, $\|\cdot\|_A$ is a norm that specifies a distance-based similarity between the data vector x_k and a fuzzy cluster center v_i. In particular, $A = I$ is the Euclidian Norm and $A = C^{-1}$ is the Mahalonobis Norm, etc.

Once the optimal pair (m^*, c^*) is determined with the application of FCM or IFC algorithm, one next identifies the cluster centers for $m = m^*$ and $c = 1, \ldots, c^*$ as:

$$v_{\underset{m^*}{X|Y,j}} = \left(X_{1,j}^c, X_{2,j}^c, \ldots, X_{nv,j}^c, y_j^c\right) \tag{102.2}$$

From this, one identifies the cluster centers of the "input space" again for $m = m^*$ and $c = 1, \ldots, c^*$ as:

$$v_{\underset{m^*}{X|j}} = \left(X_{1,j}^c, X_{2,j}^c, \ldots, X_{nv,j}^c\right) \tag{102.3}$$

Next, one computes the normalized membership values of each data sample in the training data set:

(a) First one determines the (local) optimum membership values u_{ik}'s and then determines $\propto_{ik}$'s that are above an α-cut in order to eliminate harmonics generated by FCM or IFC as:

$$u_{ik} = \left(\sum_{j=1}^{c}\left(\frac{\|x_k - v_{X,i}\|}{\|x_k - v_{X,j}\|}\right)^{\frac{2}{m-1}}\right)^{-1}, \qquad \propto_{ik} = \{u_{ik} \geq \alpha\}, \tag{102.4}$$

where $\propto_{ik}$ denotes the membership value of the kth vector, $k = 1, \ldots, nd$, in the ith rule, $i = 1, \ldots, c^*$ and x_k denotes the kth vector and for all the input variables $j = 1, \ldots, nv$, in the input space.

(b) Next, one normalizes them as: $\gamma_{ik}(x_j) = \frac{\propto_{ij}(x_j)}{\sum_{i'=1}^{c} \propto_{i'j}(x_j)}$.

Let $\Gamma = (\gamma_{ij} | i = 1, \ldots, c^*; j = 1, \ldots, n\nu)$ be the membership values of X data sample in the ith cluster, i.e., ith rule. Next one determines as a new augmented input matrix of X for each of the clusters.

As an example of the possible augmented input matrix $X_i^{'} = [1, \Gamma_i, X]$, we develop

$Y_i = \beta_{i0} + \beta_{i1}\Gamma_i + \beta_{i2}X_{ij}$, with

$$X_i^{'} = [1, \Gamma_i, X_{ij}] = \begin{bmatrix} 1 & \gamma_{i1} & x_{ij1} \\ \vdots & \vdots & \vdots \\ 1 & \gamma_{i1} & x_{ij1} \end{bmatrix}. \tag{102.5}$$

Thus the fuzzy regression function $Y_i = \beta_{i0} + \beta_{i1}\Gamma_i + \beta_{i2}X_{ij}$, that represents the ith rule corresponding to the ith interactive (joint) cluster in the (Y_i, Γ_i, X_j) space, would be estimated with the FRM-LSE approach as follows:

The fuzzy regression function $Y_i = \beta_{i0}^* + \beta_{i1}^*\Gamma_i + \beta_{i2}^*X_{ij}$ in (Y_i, Γ_i, X_j) space with $\beta_i^* = (X_{ij}^{'T}X_{ij}^{'})^{-1}(X_{ij}^{'T}Y_i)$ would be obtained as $Y_i^* = \beta_{i0}^* + \beta_{i1}^*\Gamma_i + \beta_{i2}^*X_{ij}$. The overall output value is:

$$Y_i^* = \frac{\sum_{i'=1}^{c^*}\gamma_i Y_i^*}{\sum_{i'=1}^{c^*}\gamma_i}. \tag{102.6}$$

The overall output value is calculated using each output value one from each cluster and weighting them with their corresponding membership values.

Fig. 102.1. I. Burhan Türkşen and Madan M. Gupta among others at the first World Conference on Soft Computing (WConSC'11) in San Francisco, May 23-26, 2011. In the background: Michio Sugeno.

102.5 Conclusions

We have developed various versions of this fuzzy regression models with successful results. Such investigations can be found in Türkşen [6] and Celikyilmaz and Türkşen [2] as well as Ozkan and Türkşen [4] studies and their various versions. Currently, we are developing Full TYPE 2 fuzzy system studies with the application of our new algorithm called "Full TYPE 2 Fuzzy System Models".

References

1. Bezdek, J.C.: Pattern Recognition With Fuzzy Objective Function Algorithms. Kluwer Academic Publishers, Norwell (1981)
2. Celikyilmaz, A., Burhan Türkşen, I.: Enhanced Fuzzy System Models With Improved Fuzzy Clustering Algorithm. IEEE Transactions on Fuzzy Systems 16(3), 779–794 (2008)
3. Sugeno, M., Yasukawa, T.: A Fuzzy Logic Based Approach to Qualitative Modelling. IEEE Transactions on Fuzzy Systems 1(1), 7–31 (1993)
4. Ozkan, I., Burhan Türkşen, I.: Upper and Lower Values for the Level of Fuzziness in FCM. Information Sciences 177(23), 5143–5152 (2007)
5. Takagi, T., Sugeno, M.: Fuzzy Identification of Systems and Its Applications to Modeling and Control. IEEE Transactions on Systems, Man and Cybernetics SMC- 15(1), 116–132 (1985)
6. Burhan Türkşen, I.: Fuzzy Functions with LSE. Applied Soft Computing 8, 1178–1188 (2008)
7. Zadeh, L.A.: Fuzzy Sets. Information and Control 8, 338–353 (1965)
8. Zadeh, L.A.: The Concept of a Linguistic Variable and its Application to Approximate Reasoning. Information Sciences 8, 199–249 (1975)

On Fuzzy Sets Philosophical Foundations

Luis Adrian Urtubey

103.1 The Emergence of Fuzzy Sets

Doubtlessly the motivations for the development of fuzzy logic are deemed closely associated with technologically biased concerns. In his 1962 paper -where the term fuzzy is supposed to appear for the first time in its current usage- Zadeh was primarily attentive to the emergence and evolution of system theory as well as its impact on the field of electric engineering [9], [10]. Moreover when Zadeh uses the term fuzzy in this paper, he is not cared about inanimate systems, but he is mostly troubled above animate or biological systems, which are generally orders of magnitude more complex than man-made systems. It brought himself to claim that it appears necessary to count on a radically different class of mathematics, one that accounts for fuzzy or cloudy quantities which are not describable in terms of probability distributions. Moreover even in the case of man-made systems it turns out apparent the need of such innovation.[1]

Ironically, Zadeh also referred there to the fact that in most practical cases the a priori data as well as the criteria by which a system is judged are far from being precisely specified or having accurately known probability distributions. I say ironically, because later on one of the most frequent criticism on the application of fuzzy sets to deal with vagueness and imprecision -specially from the philosophical side- has been focussed on the very assignment of determined quantities by means of fuzzy sets membership functions.[2]

As someone coming from philosophy, I would like to address here some reflections about fuzzy sets from a philosophical standpoint, far from these criticisms. These are not about completely new issues, but I think that they will show themselves to be significant to look into other connections between philosophy and fuzziness.

103.2 What Logic Has to Do With It

This is a question that many logicians must have likely formulated when they were confronted with the increasing application of fuzzy logic. For some philosophically-minded logicians, fuzzy logic seemed to be appropriate for treating the logic of vagueness. It produced immediate reactions against fuzzy logic led by logicians

[1] Rudolf Seising considers the rise and evolution of fuzzy logic over these years. See [5].

[2] Carl W. Entemann has dealt with some of this criticisms in [2].

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who remain stubbornly faithful to classical bivalent logic.[3] One can say that many generations up to now have philosophically grew up under the influence of the unfavorable reception of fuzzy logic by some notorious philosophers and logicians. I never had sympathy for this attitude, much less if one considers for example, that it is likely to draw a parallel between the rise of Aristotle's syllogistic -according to the interpretation of some prestigious scholars- and fuzzy logic beginnings. This is one of the lines I want to pursue here.

Specialists in Aristotle's logic have paid attention to the influence of geometry and the theory of proportions in the origins of his syllogistic. In particular, Robin Smith dares say that Aristotles syllogistic theory is more properly regarded as mathematics than as logic as understood by most contemporary logicians [6].

Leaving aside most philological details, it turns out that many techniques and ideas which Aristotle makes use of in the theory of syllogism are borrowed from *harmonic theory*, i.e. from the mathematics of music. Moreover many clues split throughout Aristotles *Prior Analitics* would suggest a mathematical context much more than an argumentative one. If things went this way, while working on his syllogistic, Aristotle could have had in mind something else: a mathematical theory of epistemology, as R. Smith have called it, ultimately derived from Plato's Theory of Forms. The famous passage in Plato's *Republic* VI, where mathematics is described as in some way an image of true knowledge, had already suggested a theory of this type.

103.3 Fuzzy Sets Epistemimological Foundations

Plausibly one may claim that the epistemological grounds of fuzzy sets have not received as much attention as their logic-semantical conundrums. Philosophically the problem posed by an epistemology of fuzzy sets might be tackled from a Kantian perspective, as a consequence of the thesis that properties in itself cannot be perceived, but only the individual phenomena. Kant's *Critique* established that human search for knowledge is possible because there are phenomena, but at the same time, it introduced an insuperable dichotomy between a subject who perceives the phenomenon on the one side and the phenomenal object on the other. The property, the Kantian *think in itself*, remains impenetrable to the perceiving subject. Consequently the true perceivable phenomenon is the individuation; the split of properties and the ongoing participation an individual x has in a property P. The language, as a universal human mean of representation, aims just at representing these perceptions. Therefore, even in language, it cannot be either a truly clear-cut determination of predicates by sets of individuals because properties inexorably escape that sort of determination.

Such a conclusion is the main contempt of the philosophy of knowledge of Arthur Schopenhauer, one of the most prominent post-kantian and anti-idealist philosophers of the XIXth century. From the beginning, human search for knowledge confronts a subject with an object that marks subjects limitations. It is Schopenhauer main

[3] For a recent publication covering that issues see [3].

concern that this opposition never is over. There is no way, including dialectic, of overcoming this limitation human beings are damned to, therein the Schopenhauerian pessimistic conclusion that see human existence as striving continuously in a senseless battle [4].

Recently Manuel Tarrazo has also stressed this connection of Schopenhauer epistemic framework with the existing concept of fuzziness [7]. That being the case, a fuzzy set can be seen epistemologically as an accurate and mathematically elaborated mean to represent the split of properties in many individuals.

According to this interpretation, a fuzzy set contains infinite instances of possible individual perceptions concerning certain property. Notably all these perceptions can only involve individuals and then fuzzy sets represent properties only indirectly, by giving the individual different degrees of participation. In this way properties are implicitly included in the fuzzy representation of the world. Consequently, degrees of membership in a fuzzy set, have to be naturally explained by the impossibility of expressing properties in a direct way.

Zadeh's *computing with words and perceptions* approach introduced in the last years in soft computing, has achieved a more epistemological interpretation of fuzzy sets [12]. Notably E. Trillas has also emphasized in different places the relationship between the use of words and fuzzy sets stemming from the worldly interaction of a subject with objects and properties [8].

103.4 A Literary Digression on Borges and "The Zahir"

It is noteworthy that the story called The Zahir, by the Argentinian writer Jorge Luis Borges, has an appealing connection with the split-property-approach to fuzzy sets considered above. The story refers the existence of a fantastic object, the Zahir, which has the power of representing universal properties in a single object. In Buenos Aires at the time of the story -Borges said, not lacking in fine irony- the zahir is a twenty-cent coin that the story-teller incidentally obtains in a corner-bar-and-grocery-store: "I asked the owner for an orange gin; with the change I was given the Zahir; ...The thought struck me that there is no coin that is not the symbol of all the coins that shine endlessly down throughout history and fable". [1]

There have been many different objects which embodied the Zahir at different times all around the world. Among them, "in Gujarat, at the end of the eighteenth century, Zahir was a tiger" ... "a magic tiger that was the perdition of all who saw it". The figure of this tiger was painted in a palace: In "the jail at Nighur, there was a cell whose floor, walls and vaulted ceiling was covered by a drawing (in barbaric colors that time, before obliterating had refined) of an infinite tiger. It was a tiger composed of many tigers, in the most dizzying of ways; it was crisscrossed with tigers, striped with tigers, and contained seas and Himalayas and armies that resembled other tigers". [1]

Thinking about the story, one can also figure out a fuzzy set as a mean of expressing something like the infinite tiger of that picture. Capturing the infinite non-perfect individuals that are embraced by a property, a fuzzy set can represent the very property as it shows up.

103.5 Conclusion

As L. Zadeh used to say, Fuzzy Logic is not fuzzy. A fuzzy set is a very precise device to deal with the imprecision of human life. Fuzzy sets and Fuzzy Logic have innumerable affinities with several areas of human knowledge [11]. I have explored some relationships of fuzzy sets with mainstream epistemology and metaphysics. I have also shown that there are antecedents concerning the impact of technologically motivated developments, as may be the case of Fuzzy Logic, on theoretical underpinnings. This fact has been illustrated by appealing to such a prestigious logical theory as Aristotle's syllogistic.

Contrasting with the project of a mathematical epistemology of perfectly defined concepts, which Aristotle might have had in mind while he was elaborating his syllogistic theory, it makes sense to see Fuzzy Logic now as the endeavor for developing a mathematical epistemology of commonplace imperfect intellectual constructs.

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References

1. Borges, J.L.: Collected Fictions, Transl. by Andrew Hurley. Penguin Books U.K. (1999)
2. Entemann, C.W.: Fuzzy Logic: Misconceptions and Clarifications. Artificial Intelligence Review 17, 65–84 (2002)
3. Ronzitti, G. (ed.): Vagueness: A Guide. Springer (2011)
4. Schopenhauer, A.: The World as Will and as Representation, Transl. by E.F.J. Payne. Dover (1958)
5. Seising, R.: The Fuzzification of Systems. The Genesis of Fuzzy Set Theory and its Initial Applications – Developments up to the 1970s, 1st edn. STUDFUZZ, vol. 216. Springer, Heidelberg (2007)
6. Smith, R.: The Mathematical Origins of Aristotle's Syllogistic. Archive for History of Exact Sciences 19, 201–210 (1978)
7. Tarrazo, M.: Schopenhauer's Prolegomenon to Fuzziness. Fuzzy Optimization and Decision Making 3, 227–254 (2004)
8. Trillas, E.: On the Use of Words and Fuzzy Sets. Information Sciences 176, 1463–1487 (2006)
9. Zadeh, L.A.: From Circuit Theory to System Theory. Proceedings of the IRE 50, 856–865 (1962)
10. Zadeh, L.A.: Fuzzy Sets. Information and Control 8, 338–353 (1965)

11. Zadeh, L.A.: Coping with the Imprecision of the Real World: An Interview with Lotfi A. Zadeh. Communications of the ACM 27(4) (April 1984)
12. Zadeh, L.A.: From Computing with Numbers to Computing with Words: From Manipulation of Measurements to Manipulation of Perceptions. IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications 45(1), 105–111 (1999)

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The Two Cultures of Logic

Kees van Deemter

104.1 C. P. Snow's Two Cultures

In his famous Rede Lecture of 1959, the physicist, novellist and polymath C.P.Snow put forward his thesis of the Two Cultures [13]. Snow observed that a gulf had appeared between two academic cultures, with engineers and scientists on one side, and scholars in the humanities on the other. Snow lamented that little communication went on across the gulf, so that most representatives of each of the two academic cultures lacks the most basic understanding, and appreciation, of the other. My thesis, in this written homage to Lotfi Zadeh, is that something akin to Snow's gulf runs through mathematical logic. For even though logic is a science, there are two types of logicians: on one side of the gulf are those who insist on using True and False (and, possibly, Indeterminate or Undefined) as the only possible truth values; on the other side are those logicians who embrace a wealth of different truth values, with True and False as the extremes of a continuum; to denote the latter area of work, which covers a large variety of approaches including (but not limited to) Fuzzy Logic, we shall loosely use the term multi-valued logic.

This article is a plea for diplomacy across the gulf. My vantage point is the study of language and communication, first as a theoretical enterprise, then as an area of language engineering. The gist of my remarks will be that multi-valued logics have much to offer in both areas, but that cultural and intellectual obstacles are standing in the way.

104.2 Vagueness in Language and Communication

Vagueness looms large in two research areas in which I have been active: the theoretical study of natural language and the practical endeavour of Natural Language Generation.

104.2.1 Formal Semantics of Natural Language

A strong tradition at the intersection of linguistics and mathematics focusses on the construction of formal models of meaning, addressing questions like "Does S_1 follow from S_2", where S_1 and S_2 are (for example) English sentences [2]. Despite the rise of non-classical logics, and despite recent statistical work on textual entailment [1],

[1] See [1] for a survey.

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the bulk of this work still relies on “Boolean" logics, based on just two truth values (plus occasionally a third value for “unknown" or “undefined"). Fuzzy expressions, such as degree adjectives, pose difficult challenges to this approach. A classic illustration is the ancient *sorites* paradox. Here is a modern, scientifically enhanced version [2] based on the experimental finding that differences in amplitude of 0.5 decibel are too small to be perceived by the human ear:

Premisses: **(1)** -30dB is inaudible; **(2)** 100db is audible; **(3)** any statement of the form “if xdB is inaudible then $(x+0.5\text{dB})$ is inaudible”.

-30dB is inaudible **(1)**. Therefore, by **3**,
-29.5dB is inaudible. Therefore, by **3**,
-29dB is audible. Therefore, by **3**, ... (etc) ...
100dB is inaudible. But,
100dB is audible **(2)**. Contradiction

Boolean approaches struggle to explain the flaw in this argument because, faced with the choice between rating **(3)** as True or False, True seems to be the choice consistent with experimental findings, which makes the contradictory conclusion difficult to avoid. There is no shortage of proposed solutions[3] but a generally accepted solution remains elusive. Fuzzy logic attempts to square this circle by allowing truth values that reflect degrees of truth in between completely True (the value 1) and completely False (the value 0). These allow us to say that the statements **(3)** are *very nearly* true. Different analyses are possible, depending on how the conditional is analysed. One of the more attractive analyses uses the following definitions:

$\|\varphi\| = 1 - \|\varphi\|$
$\|\varphi \vee \psi\| = max(\|\varphi\|, \|\psi\|)$
$\|\varphi \wedge \psi\| = min(\|\varphi\|, \|\psi\|)$
If $\|\varphi\| \leq \|\psi\|$ then $\|\varphi \to \psi\| = 1$ else $\|\varphi \to \psi\| = 1 - (\|\varphi\| - \|\psi\|)$

Suppose we assess the truth value of “-10dB is inaudible” as 1 (completely true) but the truth value of “-9.5dB is inaudible” as 0.9, the value of “-9dB is inaudible” as 0.85, and so on until we arrive at “-0.5dB is inaudible” with a value of 0. This would make all the relevant conditionals nearly true, at 0.9. This would allow the Fuzzy Logician to analyse the argument as based on premisses that are nearly true, but leading to a conclusion that is completely false. This is an attractive analysis, because it explains why the sorites argument is wrong yet (at least somewhat) convincing. Yet, as we shall see below, it does leave something to be desired.

104.2.2 Natural Language Generation

Natural Language Generation (NLG) systems generate (for example) English utterances from nonlinguistic input.[4] Examples include medical decision support NLG

[2] This version of the paradox, like much else in this chapter, stems from [15].
[3] See [10] for a dated but still outstanding collection of famous papers.
[4] [12], for a general introduction.

systems that take clinical time-series data (heart rhythm, temperature, etc.) of a patient as input, and weather forecasting NLG systems convert computer-generated numbers into human-digestible text [8] [7]:

> Input: Average Windspeed(6:00-12:00) = 37 knots
> Output: "Gale force winds are expected in the morning"

In human-authored reports (in both these areas), vague expressions abound. For example, in the BT-Nurse corpus developed under the Babytalk medical informatics project [8], one nurse wrote, with vague expressions italicised by me:

> Today he managed 1.5 hours off CPAP in *about* 0.3 litres nasal prong oxygen, and was put back onto CPAP after a desaturation with bradycardia. However, over the day his oxygen requirements *generally* have come down from 30% to 25%. Oxygen saturation is *very variable*. *Usually* the desaturations are down to the 60s or 70s (...)

Their frequent use by professionals suggests that vague expressions are thought to be effective. A substantial amount of research addresses the question when and how vague expressions should be produced by an NLG system to make these systems optimally useful.

104.3 Open Questions

I believe that multi-valued approaches to logic have much to offer, both to the theoretical understanding of language and to language technology. This conviction, however, is not widely shared in these research communities. Moreover, even those who share it differ over the choice of logical system, and over the question how these systems may be grounded in data.

104.3.1 Can We Bridge the Gulf between Our Own Two Cultures?

Fuzzy and other multi-valued logics[5] have reached considerable scientific respectability and offer attractive solutions to puzzles like sorites. Yet, their influence on the study of language and communication has so far been modest. I believe that this may be best explained on sociological grounds: students of language and communication are educated in a tradition that buys lock, stock and barrel into the Boolean model. Change glimmers on the horizon though, since recent computational work on language has emphasised engineering methods based on statistics (hence real numbers, as in Fuzzy Logic). Although this has temporarily pushed logic to the background, it seems plausible that once logic reappears on the scene, this will be in an undogmatic "engineering" spirit, which is likely to be more open towards multi-valued logics.

[5] See [3] for a thorough textbook on fuzzy and multi-valued logic.

Fig. 104.1. Expressing weather data in a bygone age: Edwardian banjo barometer

104.3.2 Which Multi-valued Logic Models Vague Language Best?

Fuzzy Logic, as it stands, has certain properties that limit its value as a model of language. Let me explain, re-using an example by Dorothy Edgington. Imagine two balls, x and y, of equal size, with $\|small(x)\| = 0.5$ and $\|small(y)\| = 0.5$. Suppose x

is a much darker shade of black than y, with $\|black(x)\| = 0.9$ and $\|black(y)\| = 0.5$. If I ask you to pick up the small black ball, you will surely pick up x, because it is a better candidate for the description than y. The truth conditions cited in section 104.2.1 fail to predict this, however, since they dictate that $\|small(x) \wedge black(x)\| = min(0.5, 0.9) = 0.5$, and $\|small(y) \wedge black(y)\| = min(0.5, 0.5) = 0.5$. To fix the problem, one might suggest a different set of truth conditions, for example by multiplying the values of the conjuncts, instead of taking their minimum. But if this is done, disjunction should be examined as well. Consider a ball z that is halfway between red and pink, with $\|pink(z)\| = 0.5$ and $\|red(z)\| = 0.5$. Do we really want to say that “red or pink" is only a so-so description of y (with $\|red(z) \vee pink(z)\| = 0.5$)? Surely, “Give me the ball that’s red or pink please" is a perfectly apt way to talk about z. Once again, the truth conditions don’t seem to give us what the analysis of language requires.

The latter example brings us to truth conditionality. A logic is truth functional if the truth value of a complex expression depends functionally on the truth values of its parts. The truth conditions offered above make Fuzzy Logic truth functional, and this has unwanted consequences. Consider the conditions that make up the sorites paradox, for example: the definition cited above gives $inaudible(x) \rightarrow inaudible(x)$ a truth value of 1, which is reasonable. Assume $\|audible(x)\| = 0.5$, hence $\|inaudible(x)\| = 0.5$ likewise. Now substitute audible for inaudible in the conditional *(salva veritate)*, yielding $inaudible(x) \rightarrow audible(x)$. This sentence has a truth value of 1 again, which seems absurd. The cause, this time, is not some detail of the truth conditions, but the very mechanism of Fuzzy Logic, which wasn’t built to model *penumbral connections* [6] between expressions (such as the connection between red and pink). One is beginning to fear that the truth conditions above were designed specifically for cases where φ and ψ are closely related (as in the sorites paradox), and that they are lacking generality.

As I have argued elsewhere ([15], p.213-218.), the solution to these shortcomings is not to abandon multi-valued logic, but to re-construct it along probabilistic lines.[6] Doing so is not to de-value Fuzzy Logic but to acknowledge that some problems to which it has been applied require a different, though closely related approach.

104.3.3 Empirical Issues

When Fuzzy Logic is applied to a problem in real life, truth values need to be assigned to atomic statements. To do this in a well-founded way, careful experimentation is required.[7] Unfortunately, psychologists fail to get much unanimity when they ask subjects to what degree a vague word applies truthfully to a situation, and they find that truth degrees depend strongly on expectations [11], with different expressions being sensitive to expectations of different kinds (e.g., hearers’ or speakers’ expectations). If and when the logic implications of these findings are explored, Fuzzy Logicians could do worse than to be inspired by work in philosophical logic (in the “other" culture).

[6] This position has been defended consistently by Dorothy Edgington, e.g., [4], [5].

[7] See for example [16], chapter 14, where there are hints of such an empirical approach.

Perhaps the most important empirical question is *why* people express themselves vaguely. In the economist Lipman's words, why have we tolerated what appears to be "a world-wide several-thousand-year efficiency loss" by failing to express ourselves precisely [9]? Answers have been suggested[8], but much is still unclear. It appears to me that Fuzzy Logic could play a useful role here too. For once we understand fully why fuzzy approaches can be useful in engineering, this could help us understand why fuzziness can be useful in human communication too.

Acknowledgement. My first exposure to Fuzzy Logic came at a seminar by Lotfi Zadeh in Rotterdam around 1984, as an undergraduate student at the University of Amsterdam. I thank him for setting an inspiring example, and the editors of this volume for giving me the opportunity for expressing my thanks in published form.

References

1. Androutsopoulos, I., Malakasiotis, P.: A Survey of Paraphrasing and Textual Entailment Methods. Journal of Artificial Intelligence Research 38, 135–187 (2010)
2. van Benthem, J., ter Meulen, A. (eds.): Handbook of Logic and Language. MIT Press, Cambridge (1997)
3. Bergmann, M.: An Introduction to Many-Valued and Fuzzy Logic. Cambridge University Press, Cambridge (2008)
4. Edgington, D.: Validity, Uncertainty and Vagueness. Analysis 52, 193–204 (1992)
5. Edgington, D.: Vagueness by Degrees. In: [10]
6. Fine, K.: Vagueness, Truth and Logic. Synthese 30, 265–300 (1975)
7. Goldberg, E., Driedger, N., Kittredge, R.: Using Natural Language Processing to Produce Weather Forecasts. IEEE Expert 9(2), 45–53 (1994)
8. Hunter, J., Freer, Y., Gatt, A., Reiter, E., Sripada, S., Sykes, C., Westwater, D.: BT-Nurse: Computer Generation of Natural Language Shift Summaries from Complex Heterogeneous Medical Data. Journal of the American Medical Informatics Association (JAMIA) 18(5), 621–624 (2011)
9. Lipman, B.L.: Why is Language Vague? Working Paper. Department of Economics, Boston University (December 2006)
10. Keefe, R., Smith, P. (eds.): Theories of Vagueness. MIT Press, Cambridge (1997)
11. Moxey, L.M., Sanford, A.J.: Communicating Quantities: A Review of Psycholinguistic Evidence of How Expressions Determine Perspectives. Applied Psychology 14(3), 237–255 (2000)
12. Reiter, E., Dale, R.: Building Natural Language Generation Systems. Cambridge University Press, Cambridge (2000)
13. Snow, P.C.: The Two Cultures, 2nd edn. Cambridge University Press, Cambridge (1993)
14. van Deemter, K.: Utility and Language Generation: The Case of Vagueness. Journal of Philosophical Logic 38(6), 607–632 (2009)
15. van Deemter, K.: Not Exactly: in Praise of Vagueness. Oxford University Press, Oxford (2010)
16. Zimmermann, H.-J.: Fuzzy Set Theory and its Applications. Kluwer/Nijhoff, Boston (1985)

[8] [14], for a discussion of possible answers to Litman's question.

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Fuzzy Approaches in Anytime Systems

Annamária R. Várkonyi-Kóczy

Abstract. Nowadays practical solutions of engineering problems involve model-integrated computing. Model based approaches offer a very challenging way to integrate a priori knowledge into the procedure. Due to their flexibility, robustness, and easy interpretability, the application of soft computing (SC), in particular fuzzy models, may have an exceptional role at many fields, especially in cases where the problem to be solved is highly nonlinear or when only partial, uncertain and/or inaccurate data is available. Nevertheless, ever so advantageous their usage can be, it is still limited by their exponentially increasing computational complexity. At the same time, there are other soft computing approaches which can counteract the non-advantageous aspects of fuzzy (in general SC) techniques.

Anytime processing is the youngest member of the soft computing family. Systems based on this approach are flexible with respect to the available input data, time, and computational power. They are able to work in changing circumstances and can ensure continuous operation in recourse, data, and time insufficient conditions with guaranteed response time and known error. Thus, combining fuzzy and anytime techniques is a possible way to overcome the difficulties caused by the high and explosive complexity of the applied models and algorithms. The vagueness of the design procedure of the models in respect of the necessary complexity can be vanquished by model optimization and anytime mode of operation because the former can filter out the redundancy while the latter is able to adaptively cope with the available, usually imperfect or even missing information, the dynamically changing, possibly insufficient amount of resources and reaction time. This chapter deals with the history and advantageous aspects of anytime fuzzy systems.

105.1 Introduction

In the last five decades, new computational techniques and methodologies have been developed for the solution of of today's complex and difficult problems in engineering and science. Fuzzy logic and Anytime algorithms belong to these tools and compose, together with Neural networks, Evolutionary computations, and Chaotic systems, a new discipline called Soft Computing (SC) methodology or Computational Intelligence (CI). As a matter of fact, these methods are all at least partially inspired by some biological concepts or processes and hold some of the features of

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nature and living beings, like robustness, adaptivity, flexibility, tolerance, approximate inference, generalization, and learning ability. Many challenging problems have been solved and many never imagined, important results have been derived by a variety and mixture of CI methods and tools which opened new perspectives in engineering and science.

Fuzzy set theory [25], fuzzy algorithms [26], and fuzzy decision making [28] underly the soft computing principles where the performance of a system is optimized by criteria functions taking into account the time and accuracy as cost factors, as well [29]. Fuzzy systems [27] can be well applied in cases when no analytical knowledge (only expert knowledge and/or sample data) is available about the system, the information is uncertain or inaccurate, when the available mathematical form is too complex to be used, or the interpretation depends on the context. In system control, Takagi-Sugeno (TS) [15] and Takagi-Sugeno-Kang (TSK) fuzzy models [13] proved to be especially advantageous.

Anytime systems [31], [19] are to provide continuous operation in changing circumstances and to avoid critical breakdowns in cases of missing input data, temporary shortage of time, or computational power. The idea is that if there is a temporal shortage of computational power and/or there is a loss of some data, the actual operations should be continued to maintain the overall performance at lower price, i.e., information processing based on algorithms and models of simpler complexity should provide outputs of acceptable quality to continue the operation of the complete system. The accuracy of the processing will be temporarily lower but possibly still enough to produce data for qualitative evaluations and supporting decisions. Consequently, anytime processing provides short response time and is very flexible with respect to the available input information and computational power.

Situational models [11], a related approach to anytime concepts, have been designed for the control of complex systems where the traditional cybernetics models haven't proved to be sufficient because the characterization of the system is incomplete or ambiguous, containing unique, dynamically changing, and unforeseen situations. Typical cases are the alarm situations, structural failures, starting and stopping of plants, etc.

Embedding fuzzy models in anytime systems extends the advantages of the Soft Computing approach with the flexibility with respect to the available input information and computational power. There are mathematical tools, like Singular Value Decomposition (SVD), which offer a universal scope for handling the complexity problem by anytime operations [1].

The rest of the chapter is organized, as follows: In Section 105.2 the history, basic principles, and drawbacks of anytime systems are summarized. Section 105.3 gives a brief overview about the alternatives of anytime fuzzy systems together with the main advantages of these approaches. In Section 105.4 the conclusions are drawn.

105.2 Anytime Processing

Anytime algorithms, models, and systems [31], [19] are a special type of real-time systems used when the processing has to cope with the changing, possibly insufficient resource, time, and/or data availability. The processing can continuously be kept on by temporary using a reduced amount of processing time and data however with a burden of degrading the output quality. They are able to provide guaranteed response time and known error. The flexibility with respect to the available input data, time, and computational power makes these systems able to work in changing circumstances without critical breakdowns in the performance.

The starting of anytime processing can be traced back to the starting of Artificial Intelligence (AI) where the port of departure has been the question: how can we limit the amount of thinking of artificial agents when solving complex real-time problems (because thinking has a cost associated with it). A rational agent should find a trade-off between resource consumption and output quality. This idea led to the generalization of the standard call-return mechanism meaning the mapping from a set of input *and time allocation* into a set of output. The 'reasoning about reasoning' has been referred to as meta-reasoning (see e.g. [4], [2]). It can be used in various ways in order to improve the performance of systems by selecting the most appropriate base level reasoning procedure in any given situation or by dynamic allocation of computational resources to competing computation sequences.

A great leap forward has been the definition of (interruptible) anytime algorithms by Dean and Buddy [5]. Unfortunately, however, using only interruptible algorithms significantly limits the range of applicable (anytime) methods which has resulted in the introduction of contract algorithms [10].

The anytime concept (basically dealing only with time allocation) was further generalized to data insufficiency in 1998 [19] and a universal modular frame for contract type anytime systems has been defined in [21]. Today, contract algorithms may have a possibly even more significant role in anytime systems, however interruptible algorithms have remained very popular because of their easy handling and less information need. (A serious limitation on applying contract algorithms is that not only the resource/time need of the solution but also the error of the approximation has to be known in advance which limits the range of appropriate methods and tools (of course, the latter requirement is true in case of interruptible algorithms as well)).

Today, we can find a wide range of fields where the anytime concept can be utilized successfully (see e.g. [6]- [24]). Although, it is clear that there are still difficulties to be solved in real-life anytime applications.

105.2.1 Operational Modes of Anytime Systems

Basically, two types of algorithms/models can be used in anytime systems. Iterative algorithms/models are popular tools, because their complexity can easily and flexibly be changed. These algorithms always give some, possibly not accurate result and more and more accurate results can be obtained, if the calculations are continued.

A further advantageous aspect of iterative algorithms is that we don't have to know the time/resource-need of a certain configuration in advance. The calculations can be continued until the results are needed. Then, by simply stopping the calculations, feasible results are obtained.

Unfortunately, the usability of iterative algorithms is limited. Because of this limitation, a general technique for the application of a wide range of other types of models/ computing methods has been suggested in [21], however at the expense of lower flexibility and a need for extra planning and considerations. In this case, an anytime modular architecture is used which is composed of modules realizing the subtasks of a given problem. Each module of the system offers several implementations (characterized by different attribute-values) for a certain task. These units within a given module have uniform interface (same set of input, output, and solve the same problem) but differ in their computational need and accuracy. An expert system is monitoring the actual circumstances (tasks to complete, achievable time/resources, needed accuracy, etc.) in order to choose the adequate configuration, i.e. the units to be used.

105.2.2 Difficulties in Practical Anytime Systems

Despite their advantages, anytime systems have handicaps as well. One of the main problems in real-life systems is that the operation of the "supervisor" (responsible for monitoring, detecting the problems, and making decisions about the resource and time settings of the used algorithms) also needs time. This decreases the operational time (and thus also the output quality) of the processing itself.

Serious problems can be caused by the compilation of the actual realizations of the solution. It belongs to the NP complete problems and thus, the size of the needed storage grows exponentially with the number of modules.

A further non-negligible drawback of anytime schemes based on feedback systems is that they unavoidably suffer from transients. These well-known phenomena are due to the dynamic nature of the processing structures applied. With the spreading of time-critical, reconfigurable, and embedded systems, transient handling, covering both active and passive methods, has become an important research area (see e.g. [12]- [30]).

105.3 Fuzzy Approaches in Anytime Systems

Fuzzy approaches proved to be advantageous in nearly all areas of science and application. The first linkage between anytime systems and fuzzy (and neural network) models has been established in [20] with an indication to application areas related to signal processing, measurement, and control. This paper initiated a new research direction in signal processing and by this at all fields where signal processing tasks have to be solved (i.e., practically at the whole engineering area). The first conference dedicated to anytime and other soft computing methods in signal processing related fields was the first IEEE International Workshop on Intelligent Signal Processing (WISP'99) (Figs. 105.1 and 105.2) organized in Budapest, Hungary, recognized and supported by the IEEE Instrumentation and Measurement Society (with special issue in TIM). Professor Lotfi A. Zadeh had a significant role in the fruition and since the very beginning, he has participated and followed the event-series with attention (and has given plenary talks till 2007) (see also Figs. 105.3 and 105.4). The author of this chapter is much obliged to Professor Zadeh also for his continuous help and ideas in her research [18].

Fig. 105.1. WISP'99: Discussion in a technical session: right: Prof. Lotfi A. Zadeh

105.3.1 Alternatives of Anytime Fuzzy Algorithms and Models

In many of the cases, the used fuzzy models can be turned to anytime models and thus, can be built in anytime systems. These schemes usually apply contract type anytime fuzzy models using SVD based exact and non-exact complexity reduction. As examples, product-sum-gravity fuzzy systems with singleton consequents (PSGS), product-sum-gravity fuzzy systems with non-singleton consequents (PSGN), Takagi-Sugeno (TS) fuzzy models, and systems having extremely large rule-bases (where the size of the rule-base is greater then the available operational memory) can be mentioned (for details, see [23]). In all of these cases, SVD (in high dimensional cases Higher Order (HO) SVD) offers a formal measure to filter out the redundancy of the systems and also the weekly contributing parts.

This technique is very advantages is anytime systems: (HO)SVD ensures the best results in the given circumstances. If SVD based complexity reduction is applied to a two dimensional matrix then it can be proved that the resulting matrix of lower rank will be the best approximation of the original matrix in least-squares sense (minimum

Fig. 105.2. WISP'99: River cruise on the Danube; from left: Prof. Gábor Péceli (general co-chair), Prof. Annamária R. Várkonyi-Kóczy (general co-chair), Dr. Stephen F. Adam (honorary chair) and wife, Prof. Lotfi A. Zadeh (honorary chair), and Dr. Feng-Hui Yao

$||L_2||$ norm of the error, i.e. the reduction is "optimal"). In case of higher dimension matrices (tensors) where HOSVD is applied, the minimum property doesn't hold anymore. We can only state that the significant singular values will have the lower indices. However, in the most of the cases if there is a considerable difference among the singular values, HOSVD results in an approximation which is "close" to the optimal one.

Up till recently, iterative fuzzy models have been unknown in anytime systems, despite that the needed time/resource need could be decreased by evaluating only a subset of the rules. It is because the significance of the rules highly depends on the actual inputs and, thus, it is hard to tell which rules could be omitted if we wanted to ensure a given accuracy. Similarly, we could hardly ensure that the result of the processing is the available most accurate one. Although, in [14] a new SVD based transformation method has been described, by which PSGS fuzzy systems can be transformed into a form which matches the requirements of iterative evaluation. The transformed model can be processed rule by rule with known error bound in every step and holding the optimum (minimum error) criteria.

105.3.2 Advantages of Anytime Fuzzy Systems

Anytime fuzzy systems may offer a solution to many of the disadvantages of anytime systems. Concerning the problem of transients, it can be proved that the nature of

Fig. 105.3. WISP'2001: Prof. Lotfi A. Zadeh giving a plenary talk

the transients depends not only on the transfer function of the structures to be implemented, but also on the actual implementation of the processing structure. According to our experience, fuzzy models are advantageous from this respect and produce lower transients then other structures [22].

The complexity problem can, at least be held, by using such techniques as are used in fuzzy control, e.g. for modeling TS fuzzy modeling and for the controller design Parallel Distributed Compensation (PDC) [17]. Anytime processing can be applied on two levels in the TS fuzzy controller. First, we can reduce the complexity of the local models (local level reduction). Secondly, it is possible to reduce the complexity of the overall controller by neglecting those local controllers, which have negligible or less significant role in the control (model level reduction). Both can be applied adaptively, where we take into account the “distance” between the current position and the operating point, as well. The model granularity or the level of the iterative evaluation can cope with this distance: the more far we are the more rough control actions can be tolerated. Although, the approximated models may not guarantee the stability of the original nonlinear system, this can also be ensured by introducing robust control (see e.g. [16]).

Finally, the author longs to enumerate some interesting problems which could be solved only by applying anytime fuzzy approaches, thus proving their advances in time-critical, reconfigurable, and embedded systems:

Fig. 105.4. WISP’2001: At the Banquet Dinner; Prof. Lotfi A. Zadeh (honorary chair) in the middle

- Anytime Fuzzy Fast Fourier Transformation and Adaptive Anytime Fuzzy Fast Fourier Transformation: How can we determine the most important signal parameters *before the signal period arrives*? How can we implement fast algorithms with only negligible delay?

- Anytime Recursive Overcomplete Signal Representations: How can we minimize the channel capacity necessary for transmitting certain amount of information? How can we provide optimal and flexible on-going signal representations, on-going signal segmentations into stationary intervals, and on-going feature extractions for immediate utilization in data transmission, communication, diagnostics, or other applications if the transmission channel is overloaded and in the case of processing non-stationary signals when complete signal representations can be used only with serious limitations?
- High Dynamic Range (HDR) imaging and situational image quality improvement: How can we make the invisible details of images visible? How can we enhance the useful information of images which is significant *from the point of view of further processing*?
- Anytime control and fault diagnosis of plants: How can we produce useful results and react in crisis situations very quickly in order to avoid catastrophes? How can we increase the safely available reaction time of the (slow) human supervisor by significantly decreasing the time needed for the automatic detection and diagnosis of faults?
- CASY, an Intelligent Car Crash Analysis System: How can we build an intelligent expert system, capable to reconstruct the 3D model of crashed cars autonomously (without any human interaction) using only 2D photos; and based on it, how can it determine characteristic features of crashes like the energy absorbed by the car-body deformation, the direction of impact and the pre-crash speed of the car?

105.4 Conclusions

In modern time-critical engineering systems, the available time and resources are often not only limited, but can also change during the operation. In these cases, the so called anytime models and algorithms can be used advantageously, however with the burden of significant drawbacks, like the occurrence of transients and the system-supervision's overhead further reducing the insufficient operational time. On the other hand, while fuzzy methods are widely used in engineering systems, their usability is limited, because the lack of any universal method for the determination of the needed complexity often results in huge and redundant rule-bases. This chapter aims to demonstrate that if fuzzy approaches and anytime processing are combined then the most of the above problems can be solved. The presented issues clearly show that the fuzzy concept introduced by Professor Lotfi A. Zadeh resulted in many prospective and fruitful achievements in the field of reconfigurable, time-critical, and adaptive systems.

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References

1. Baranyi, P., Várkonyi-Kóczy, A.R., Várlaki, P., Michelberger, P., Patton, R.J.: Singular Value Based Model Approximation. In: Mastorakis, N.E. (ed.) Problems in Applied Mathematics and Computational Intelligence. Mathematics and Computers in Science and Engineering, pp. 119–124. World Scientific and Engineering Society Press, Danvers (2001)
2. Batali, J.: Reasoning About Control in Software Meta-level Architectures. In: Proceedings of the 1st Workshop on Meta-Architectures and Reflection, Sardinia (1986)
3. Corwin, D.L., Rhoades, J.D., Simunek, J.: Leaching Requirement for Soil Salinity Control: Steady-state Versus Transient Models. Agricultural Water Management 90, 165–180 (2007)
4. Davis, R.: Meta-rules: Reasoning About Control. Artificial Intelligence 15, 179–222 (1980)
5. Dean, T.L., Boddy, M.: An Analysis of Time-dependent Planning. In: Proceedigns of the 7th National Conference on Artificial Intelligence, Minneapolis, Minnesota (1988)
6. Drummond, M., Bresina, J., Kedar, S.: The Entropy Reduction Engine: Integrating Planning, Scheduling, and Control. SIGART Bulletin 2, 48–52 (1991)
7. Fischer, J., Niemann, H., Nöth, E.: A Real-Time and Any-Time Approach for a Dialog System. In: Proceedings of the International Workshop Speech and Computer, SPECOM 1998, St.-Petersburg, pp. 85–90 (1998)
8. Kitano, H., Tadokoro, S., Noda, I.: RoboCup Rescue: A Grand Challenge for Multi-agent Systems. In: Proceedings of the 4th International Conference on Multiagent Systems, Boston, USA, pp. 5–12 (2000)
9. Kywe, W.W., Fujiwara, D., Murakami, K.: Scheduling of Image Processing Using Anytime Algorithms for Real-Time System. In: Proceedings of the International Conference on Pattern Recognition, Hong Kong, pp. 1095–1098 (2006)
10. Korf, R.E.: Real-time Heuristic Search: First results. In: Proceedings of the 7th National Conference on Artificial Intelligence, Minneapolis, Minnesota, pp. 139–144 (1988)
11. Madarász, L., Andoga, R., Fozo, L., Lazar, T.: Situational Control, Modeling and Diagnostics of Large Scale Systems. In: Rudas, I.J., Fodor, J., Kacprzyk, J. (eds.) Towards Intelligent Engineering and Information Technology. SCI, vol. 243, pp. 153–164. Springer, Heidelberg (2009)
12. Robinson, P.: Architecture and Protocol for Authorized Transient Control. The Kluwer International Series in Engineering and Computer Science, pp. 113–129. Springer, Heidelberg (2006)
13. Sugeno, M., Kang, G.T.: Structure identification of fuzzy model. Fuzzy Sets and Systems 28, 15–33 (1988)
14. Takács, O., Várkonyi-Kóczy, A.R.: Iterative-type Evaluation of PSGS Fuzzy Systems for Anytime Use. IEEE Transactions on Instrumentation and Measurement 54(1), 391–397 (2005)
15. Takagi, T., Sugeno, M.: Fuzzy Identification of Systems and Its Applications to Modeling and Control. IEEE Transactions on Systems, Men, and Cybernetics 15, 116–132 (1985)
16. Tanaka, K., Taniguchi, T., Wang, H.O.: Robust and Optimal Fuzzy Control: A Linear Matrix Inequality Approach. In: Proceedings of the 1999 IFAC World Congress, Beijing, pp. 213–218 (1999)
17. Tanaka, K., Wang, H.O.: Fuzzy Control Systems Design and Analysis. John Wiley and Sons, Inc., New York (2001)
18. Personal discussions with Professor Lotfi A. Zadeh starting in 1996

19. Várkonyi-Kóczy, A.R., Kovácsházy, T.: Anytime Algorithms in Embedded Signal Processing Systems. In: Proceedings of the IX. European Signal Processing Conference, EUSIPCO 1998, Rhodes, Greece, vol. 1, pp. 169–172 (1998)
20. Várkonyi-Kóczy, A.R., Román, G.: The Role and Power of Soft Computing in Complex Measurement Systems. In: CD-ROM Proceedings of the 1998 World Automation Congress, WAC 1998, Anchorage, USA (1998)
21. Várkonyi-Kóczy, A.R., Ruano, A.E., Baranyi, P., Takács, O.: Anytime Information Processing Based on Fuzzy and Neural Network Models. In: Proceedings of the 2001 IEEE Instrumentation and Measurement Technology Conference, IMTC 2001, Budapest, Hungary, pp. 1247–1252 (2001)
22. Várkonyi-Kóczy, A.R.: Methods of Computational Intelligence for Modeling and Data Representation of Complex Systems. DSC Thesis, Hungarian Academy of Science, Budapest (2009)
23. Várkonyi-Kóczy, A.R.: Model Based Anytime Soft Computing Approaches in Engineering Applications. In: Balas, V.E., Fodor, J., Várkonyi-Kóczy, A.R. (eds.) Soft Computing Based Modeling in Intelligent Systems. SCI, vol. 196, pp. 63–92. Springer, Heidelberg (2009)
24. http://windows.microsoft.com/en-US/windows7/products/features/windows-anytime-upgradem (2009)
25. Zadeh, L.A.: Fuzzy Sets. Information and Control 8(3), 338–353 (1965)
26. Zadeh, L.A.: Fuzzy Algorithms. Information and Control 11(2), 94–102 (1968)
27. Zadeh, L.A.: Towards a Theory of Fuzzy Systems. In: Kalman, R.E., De Clairis, N. (eds.) Aspects of Networks and Systems Theory, pp. 469–490. Holt, Rinehart, and Winston, New York (1971)
28. Zadeh, L.A.: Outline of a New Approach to the Analysis of Complex Systems and Decision Processes. IEEE Transactions on Systems, Man and Cybernetics 3(1), 28–44 (1973)
29. Zadeh, L.A.: Fuzzy Logic, Neural Networks, and Soft Computing. Communications of the ACM 37(3), 77–84 (1994)
30. Zhanga, Y., Jiang, J.: Bibliographical Review on Reconfigurable Fault-tolerant Control Systems. Annual Reviews in Control 32(2), 229–252 (2008)
31. Zilberstein, S.: Using Anytime Algorithms in Intelligent Systems. AI Magazine 17(3), 73–83 (1996)

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Fuzziness in Software Engineering

Peter Vojtáš

Abstract. This paper contains my memories on how did I arrive to field of fuzziness and personal views on present stage and expectation on fuzziness. Main message is: go back to roots and start with real world problems, large scale data and solutions like L. A. Zadeh did 50 years ago in his applications which emerged into fuzzy theory. I will illustrate these on some examples from software engineering applications, especially mining user preferences (in the form of fuzzy functions) for web search. The perspective is to develop reliable measurement of fuzzy sets and operators for commercial use (control and web applications are different).

106.1 Introduction, History, How Did I Come to "Fuzziness"

In mid of seventies I have promoted at Charles University in Prague in the field of theoretical cybernetics (program founded by late prof. Petr Vopěnka, founder of Prague seminar of set theory, which in this year celebrates 50's anniversary too). Theoretical cybernetics was that time by Vopěnka's vision a combination of three areas: 1. probability and statistics 2. constructive mathematics, automata and languages, programming and 3. logic and set theory. I have worked in set-theoretic topology, Boolean algebras and set theory and my work is quoted also in respective Handbooks, [1, 2, 3]. Nevertheless, my teaching was mainly theoretical computer science subjects at P. J. Safarik University Kosice.

To be honest, my first contact with fuzziness has a negative connotation. In early 90's I was sitting in a panel of a grant agency deciding about financing of research projects. There were some mathematical proposals concerning fuzziness. These were often just straightforward generalizations from crisp models to fuzzy (taking same notions, tasks, methods, problems ...). Even today I am suspicious to automated mathematical fuzzification of anything (especially with max-min operators).

My views changed when I met Petr Hajek in mid of 90's. I have visited his lectures in Vienna. His development of fuzzy logic was a nontrivial, exiting adventure and mathematically challenging.

After velvet revolution we entered the free competitive market. The idea of Alexander von Humboldt like university education emerged - lecturing based on / unified with scientific research. I felt responsibility for teaching students skills for their future survival on the job market this way. I have started with fuzzy Prolog.... In my first "fuzzy years" I have to express thanks especially to B. Riecan and FSTA conferences [5] and all colleagues from European projects [6, 7]. When I express

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here my opinion it does not mean I claim those for my contribution, I could have been motivated by all colleagues mentioned. A more close description of these results is in my paper [4] tribute to Hajek's 70's birthday and my list of publications. Main experience from this time is that fuzzy Prolog/Datalog worked well with implicative rules, one can have a sound deduction and an approximate completeness, our model is equivalent (modulo some assumptions) with Generalized Annotation Programs (which do have continuous semantics in this case). A model of fuzzy similarity based on max-min connectives is able to describe only hierarchical refining equivalences, our models gives a much more general result (more on this in our publications with many colleagues, see http://www.ksi.mff.cuni.cz/ vojtas/). From 1998 things seemed to be for me easier in Czech Republic. First I have used hospitality of Hajek's Institute of Computer Science part time position and from 2005 I have moved permanently to Prague Department of Software Engineering at Charles University.

106.2 Present State – A Computer Science Conference Reviewer Impulse

My computer science period is connected to projects [8,9,10,11], to all colleagues I express my thanks (views are personal; tributes are detailed in our papers).

Main impulse to my work came from an anonymous referee of one of our contributions to a computer science conferences (the paper was rejected).

When I have described a solution of a problem using fuzzy logic programming, his/her question was "where do you have these rules from". And I had to admit that that time I had no answer – I just assumed I have some rule base. Nevertheless in a real world scenario, one has to describe where from rules are coming.

I have to mention what was new for me that time: Computer Science conferences are highly selective (top conferences accept only less that 20% of submissions, some even less than 10%). Claims have to be supported by an experimental tool and (often large scale benchmark) data and compared to similar solutions of others. Results published at Computer Science conferences are very often in a final form and seldom published later in a journal (the development is so fast that in a journal production time landscape of computer science changes radically).

Looking for an inductive procedure we learned that learning fuzzy rules in control is not suitable for our purpose. In a control problem (like in Figure 106.1 for an inverted pendulum, the control space are all point in a Cartesian product – all possible states of the pendulum). The rule system able to control the inverted pendulum in/to a (almost) stable position has to be able to react in any point of state space.

The situation is totally different in a web shop, where we would like to personalize the system and to offer the user most preferred products. In Figure 1 we describe a situation when products quality is correlated with price (data do not fill the whole data space) and the ideal point of user 1 is not produced (note that in the control space the ideal point is in the control space equilibrium). To our surprise, this representation came from outside the fuzzy community, a group at IBM at Almaden led by R. Fagin used fuzziness to express preference and they gave an optimal algorithm for finding top-k products without necessity of looking to all products [13].

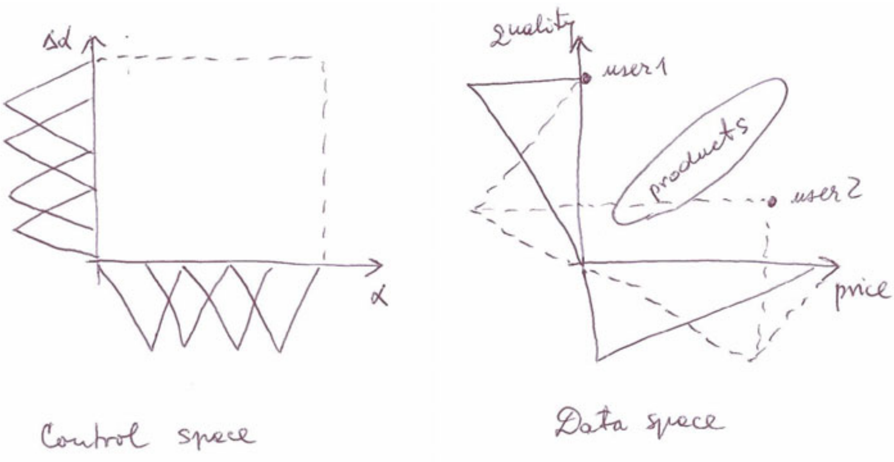

Fig. 106.1. Control space and Data space

Situation is even more difficult when we assume there is a second user with different preferences. We have developed several methods for learning user preferences from his/her explicit rating of a sample of products. There are several challenges to this – one has to work with a small number of ratings (because an average user will not rate large number of products, possibly he will not rate any and then we have to learn his/her preferences from implicit behavior, like mouse actions). Next the response has to be fast, user is assumed to wait less than a second. Further, our recommendation cannot be a black box, user usually prefers to know, why the system made decision this way.

Practically we are in a situation of a multicriterial decision making system, with large number of alternatives and large number of decision makers (users, customers). The task is to learn preferences of each user and attribute and to learn user's utility function.

Having fuzzy functions expressing attribute preferences (u_1 prefers cheap, u_2 prefers medium price etc.) the data space is translated to preference space, a power of unit interval $[0,1]$ with ideal point at $[1,1,\ldots,1]$. The problem now is to learn the user's utility function. Our acquaintance agrees with that of Zimmermann and Zysno [12], even much more general: close to ideal point user utility is OR-like and close to worst point it is AND-like (see Figure 106.2. Preference space). So by our experience, it need not be weighted average. In time pressure when answer has to be retrieved in a fraction of second, one cannot look for an analytic expression using standard mathematical functions. Our utility function is very often a Pareto closure

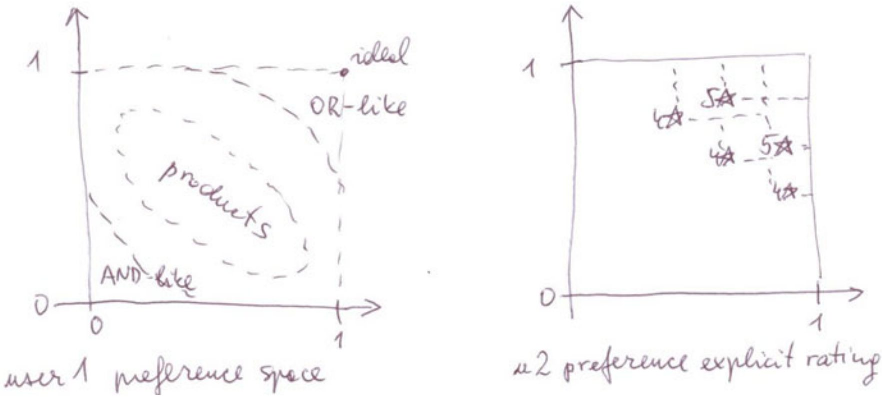

Fig. 106.2. Preference space and Zimmemann-Zysno-like connective of user rating

of rating points (see Figure 106.2. Position of rating sample products by five stars, four stars, . . .).

This way we have learned several rules of fuzzy Prolog/Datalog. First comment is that there are lots of fuzzy data created by user rating. Second is, that fuzzy valued acts here as a preference degree. Third comment is that learning a fuzzy set which has to generate user preference is not an approximation task. We are not interested in good approximation like in a case of a control function (which has to act in all points of the control space). Our function decides which are top-k objects (a typical user does not look to more than 4 pages of recommendation, so k is at most 40).

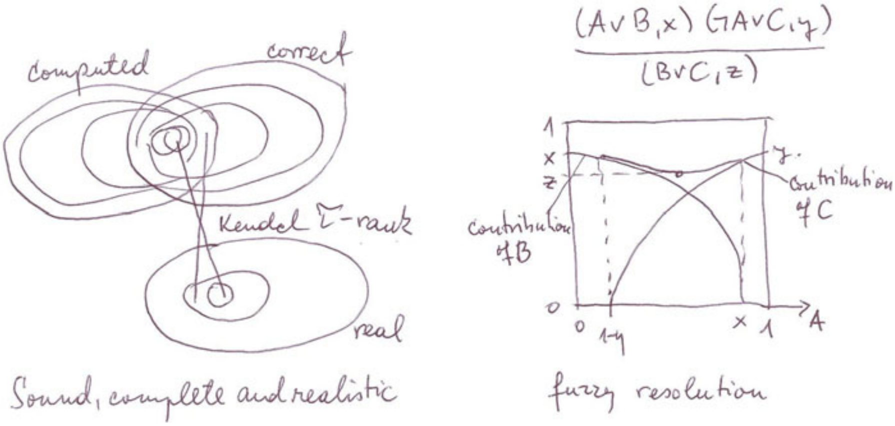

Fig. 106.3. Realistic soundness and completeness. Fuzzy resolution

Figure 106.3 depicts the situation when (having a sound and complete system of rule system deduction) one has to measure success on real data by violation of order on top-k objects (we used Kendall τ correlation coefficient). These all with a semantics of implicative rules with deduction based on a reverse usage of fuzzy modus ponens. Classical model of logic programming uses clauses and deduction is refutation. Here we did not come so far. We have developed a model of fuzzy resolution where deduction can recommend minimum of aggregation (disjunction) of minimal necessary contribution of variables B and C (see Figure 106.3).

106.3 Expectations

Our expectation is based on our experience with computer science applications (especially with software engineering, dealing with modeling of application domain and design of a solution). It can be formulated, motivated by our understanding L. A. Zadeh work 50 years ago in his applications which emerged into fuzzy theory: start with real world problems, large scale data and a clear measure of expectation on solutions.

Fig. 106.4. Peter Vojtáš, approximately 1994, the time when the author started with fuzziness

A nice example in control field is the system from [14]. It is LFLC (Linguistic Fuzzy Logic Controller) dealing with linguistic descriptions and enabling fuzzy approximation. A large-scale application of LFLC can be found in Kovohute Bridlicna, in the Czech Republic, where LFLC controls five massive aluminum furnaces. In a real life the situation of an e-shop is much more complicated than our fuzzy model (even with realistic induction of fuzzy rules). The system can be influences by marketing perspective; we do not say whether our model is/has to serve user or seller.

Performance of the system can depend on the fact whether you look for goods in a weekly period (typically free time activity) or in a yearly period (e.g. summer vacation) or once/twice a life (e.g. buying a house). We have developed a model for longer decision in phases. Ultimate evaluation should depend on the fact whether the user bought the recommended product and maybe also on the fact if he/she was satisfied also later. So far I did not find myself enough responsible to suggest an e-shop owner to use our system. Our main perspective is to develop reliable measurement of fuzzy sets and operators for commercial use.

Fig. 106.5. Peter Vojtáš, approximately 2002, after his first FSS publication

References

1. Kunen, K., Vaughan, J.E. (eds.): Handbook of Set-Theoretic Topology. Elsevier, Amsterdam (1988)
2. Bonnet, R., Koppelberg, S., Monk, R. (eds.): Handbook of Boolean Algebras, vols. 1-3. Elsevier, Amsterdam (1989)
3. Foreman, M., Kanamori, A. (eds.): Handbook of Set Theory. Springer Netherlands, Amsterdam (2010)
4. Vojtas, P.: Fuzzy logic as Hajek's Comparative Notion of Truth and Applications in Preference Modeling. In: Cintula, P., Hanikova, Z., Svejdar, V. (eds.) Witnessed Years: Essays in Honour of Petr Hajek, pp. 345–360. College Publications 2009 (2009)

5. FSTA conferences, see e.g., http://www.math.sk/fsta2012/
6. ICT COST Action 15 "Many-Valued Logics for Computer Science Applications" (1997-1999), http://www.cost.esf.org/domains_actions/ict/Actions/15
7. ICT COST Action 274 "Theory and Application of Relational Structures as Knowledge Instruments" (2001-2005), http://www.cost.eu/domains_actions/ict/Actions/274
8. Slovak Project "NAZOU – Tools for Acquisition, Organisation, and Maintenance of Knowledge in an Environment of Heterogeneous Information Resources" (2004-2008), http://nazou.fiit.stuba.sk
9. Czech project 1ET100300419 "Intelligent Models, Algorithms, Methods and Tools for the Semantic Web (realization)" (2004-2008)
10. Czech project 1ET100300517 – Methods for Intelligent Systems and Their Applications in Datamining and Natural Language Processing (2005-2009)
11. Czech project GAR P202/10/0761 "Web Semantization" (2010-2012)
12. Zimmermann, H.-J., Zysno, P.: Latent Connectives in Human decision making. Fuzzy Sets and Systems 4, 37–51 (1980)
13. Fagin, R., Lotem, A., Naor, M.: Optimal Aggregation Algorithms for Middleware. Journal of Computer and System Sciences 66, 614–656 (2003)
14. Novák, V., Kovar, J.: Linguistic IF-THEN Rules in Large Scale Application of Fuzzy Control. In: Ruan, D., Kerre, E.E. (eds.) Fuzzy If-Then Rules in Computational Intelligence, pp. 223–242. Kluwer Academic Publishers, Dordrecht (2000)

107

Some Algebraic Aspects of Fuzzy Set Theory

Carol Walker and Elbert Walker

107.1 Introduction

A **fuzzy subset** of a set X is a function $A : X \to [0,1]$. This generalizes the notion of a **subset** of X which may be considered as a function $A : X \to \{0,1\}$, with the subset corresponding to the elements that go to 1. This notion of fuzzy subset was introduced by Zadeh in [4]. The gist of it is that if one has in mind that the elements of X have some property, more or less, one may express via a function $A : X \to [0,1]$ a degree to which an element $x \in X$ has that property, namely by the image $A(x)$ of the element x. Suffice it to say that this basic notion has led to the emergence of fuzzy set theory as an important tool in practical applications. Our main interest in this topic is the underlying mathematics, and in particular in various algebraic objects that arise in its application and in its study in general. Our own work in the various mathematical areas of fuzzy theory has been motivated by the development and applications of fuzzy sets. In the following, we will comment briefly on only one aspect of the underlying mathematics of fuzzy theory, that of truth-value algebras of fuzzy sets.

107.2 The Unit Interval

Various operations are placed on the fuzzy subsets of a set X, and since X has no a priori structure, these operations come from operations on the unit interval $[0,1]$. The most common operations used are the binary operations max and min, denoted $\vee$ and $\wedge$, respectively, the unary operation $'$, where $x' = 1 - x$, and the nullary operations 0 and 1. This yields an algebra $([0,1], \vee, \wedge, ', 0, 1)$, which we recognize as a complete, bounded, distributive lattice, and in fact a chain. This algebra typically serves as the *truth value algebra* for fuzzy sets. But there are other operations on the unit interval that are used in fuzzy theory, for example, t-norms and t-conorms, and other negations besides $'$. In fact, endless combinations of lattice and arithmetic operations make possible a host of operations on [0,1], and hence on the set of fuzzy subsets of a set. Since operations on fuzzy subsets come from operations on its truth values, it is necessary to be familiar with the properties of the various algebras that arise. In particular, it would be nice to know what equations a particular algebra satisfies. This area has been studied rather extensively, and there is now quite an extensive theory of fuzzy sets, the basics involving operations on [0,1] and hence on the set of fuzzy subsets of a set. Some papers dealing with these algebras are listed in our web page *http://www.math.nmsu.edu/~elbert/#publications*.

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107.3 Interval-Valued Fuzzy Sets

There was an increasingly prevalent view that models based on $[0,1]$ were inadequate. Many believed that assigning an exact number to an element was too restrictive, and that the assignment of an interval of values was more realistic. This gave rise to algebras whose elements are intervals of the unit interval, that is, the set $\{(a,b) : a,b \in [0,1], a \leq b\}$, which is denoted $[0,1]^{[2]}$. Thus an interval-valued fuzzy subset of a set X is a mapping $A : X \to [0,1]^{[2]}$. Operations put on $[0,1]^{[2]}$ come mainly from componentwise operations on the elements of a pair (a,b). One resulting algebra is a De Morgan algebra, whose basic equational properties are well understood. Some of these details are spelled out in [1], where a framework is presented for fuzzy theory in which fuzzy values are intervals rather than points in $[0,1]$.

Many applications use interval-valued fuzzy sets rather than models based on the unit interval.

107.4 Type-2 Fuzzy Sets

The set of elements of the truth value algebra of type-2 fuzzy sets is $[0,1]^{[0,1]}$, the set of all functions from the unit interval into itself. Thus a type-2 fuzzy subset of a set X is a mapping $A : X \to [0,1]^{[0,1]}$. The set $[0,1]^{[0,1]}$ is furnished with operations that are convolutions of operations on the unit interval, yielding a rather complicated algebra. This algebra has been investigated thoroughly, and there are still unresolved problems. For example, an equational base is not known for it, and indeed whether or not a finite one exists. Several papers concerning this algebra appear in the website given above. Professor Zadeh introduced this algebra in 1975 in [5], and it generalizes ordinary fuzzy sets and interval-valued fuzzy sets. It is another example of an interesting mathematical entity arising from fuzzy concepts, and one on which much mathematical research has been done.

Acknowledgement. Of course we would like to acknowledge Professor Zadeh for his work over many years, and in particular for his initial mathematical modeling of fuzzy concepts. This cannot be overemphasized. While our own interest is in the more algebraic areas, Zadeh's introduction of fuzzy concepts has led to mathematical investigations in many areas, investigations that may or may not impinge on applications: fuzzy logic, fuzzy topology, and fuzzy measure theory, just to mention a few. A concrete example of this is the series of Linz seminars. As they say "Since their inception in 1979 the Linz Seminars on Fuzzy Sets have emphasized the development of mathematical aspects of fuzzy sets by bringing together researchers in fuzzy sets and established mathematicians whose work outside the fuzzy setting can provide direction for further research." Their thirty-third yearly meeting was held in February 2012. The whole point is that the introduction of fuzzy concepts has spurred mathematical activity in many areas.

But we owe much to others, our various coauthors, especially Mai Gehrke with whom we wrote some of our early papers on fuzzy topics. Our initial interest in this subject came from Hung Nguyen, who had spent a year at Berkeley with Professor Zadeh and who was a colleague at New Mexico State for many years until his retirement in 2011. One result of this association was a book by Professor Nguyen and Professor Elbert Walker [2] on the theory of fuzzy sets and logic, which first appeared in 1997 and now is in its third edition.

References

1. Gehrke, M., Walker, C., Walker, E.: Some Comments on Interval-valued Fuzzy Sets. International Journal of Intelligent Systems 11, 751–759 (1996)
2. Nguyen, H.T., Walker, E.: A First Course In Fuzzy Logic. CRC Press, Boca Raton (1997); Second Edition 1999; Third Edition 2006.
3. Walker, C., Walker, E.: The Algebra of Fuzzy Truth Values. Fuzzy Sets and Systems 149(2), 309–347 (2005)
4. Zadeh, L.A.: Fuzzy Sets. Information and Control 8, 338–353 (1965)
5. Zadeh, L.A.: The Concept of a Linguistic Variable and its Application to Approximate Reasoning. Information Science 8, 199–249 (1975)

108

The Path of Linguistic Random Regression to Knowledge Acquisition

Junzo Watada

108.1 Introduction

Fuzzy sets play a pivotal role in computing with words being casted in the setting of granular computing (cf. [17],zedeh2006b). "The essence of granular computing is to carry out computing that exploits information granules" [16],zedeh2006b. Information granules are regarded as collections of elements that can be perceived and treated together because of their similarity, functional properties, or spatial or temporal adjacency [2], [6], [7], [11]. In this sense, fuzzy logic becomes instrumental as an effective vehicle to manipulate information granules.

It becomes apparent that experts with much professional experiences are capable of making assessment using their intuition and experience. In such cases, judgements may be expressed by experts using linguistic terms. The difficulty in the direct measurement of certain characteristics makes their estimation highly imprecise and this situation implies the use of fuzzy sets (cf. [8], [13],zadeh1975b). There have been a number of well-documented cases in which fuzzy regression analysis has been effectively used.

To cope with linguistic variables, we define processes of vocabulary translation and vocabulary matching which convert linguistic expressions into membership functions defined in the unit interval. That is, human words can be translated (formalized) into fuzzy sets (fuzzy numbers, to be more specific) which are afterward employed in a fuzzy reasoning scheme. Fuzzy regression analysis [8], [11]- [12] is employed to deal with the mapping and assessment process [9]- [10] of experts which are realized from linguistic variables of features and characteristics of an objective into the linguistic expression articulating the total assessment.

To cope with linguistic variables, we define processes of vocabulary translation and vocabulary matching which convert linguistic expressions into membership functions defined in the unit interval, and vice versa. Fuzzy random regression analysis [1]- [2], is employed to deal with mapping and assessment process of experts which are realized from linguistic variables of features and characteristics of an objective into the linguistic expression articulating the total assessment.

108.2 Linguistic Fuzzy Random Regression Model

In making assessments regarding some objects, we use multi-attribute evaluation. The difficulty in the direct measurement of certain characteristics makes their

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estimation highly impressive and this situation results in the use of fuzzy values and linguistic values. Often, experts use a linguistic word to judge an object from various features and characteristics. And the whole process is pursued in linguistic way. For instance, although it is possible to measure numerical value, it is difficult to analytically interpret the obtained numerical value in terms of possible influence. This result might have impacted on further decision making.

Table 108.1. Linguistic values of each sample ω given by experts

sample	Input Attribute k-th Value			Output Value
	1	$\cdots$	K	Y
1	(L_{11}, p_{11})	$\cdots$	(L_{1K}, p_{1K})	(Y_1, p_1)
2	(L_{21}, p_{21})	$\cdots$	(L_{2K}, p_{2K})	(Y_2, p_2)
3	(good,0.2)	$\cdots$	(very good,0.1)	(good,0.1)
$\vdots$	$\vdots$		$\vdots$	$\vdots$
ω	$(L_{\omega 1}, p_{\omega 1})$	$\cdots$	$(L_{\omega K}, p_{\omega K})$	(Y_ω, p_ω)
$\vdots$	$\vdots$		$\vdots$	$\vdots$
N	$L_{N1}, p_{N1})$	$\cdots$	(L_{NK}, P_{NK})	(Y_N, P_N)

where $L_{\omega k}$ and Y_ω denote linguistic values of input k-th attribute and output value of ω-th sample, respectively.

In this study we built a model based on the relationship between the assessments given for different attributes and the overall assessment of the object totally. Watada *et al.* [1] propose fuzzy random regression model with confidence interval to deal with situations under hybrid uncertainty. The data given by experts are shown in Table 108.1 such as *"good," "bad," "extremely bad,"* as fuzzy random numbers.

An event has its population including the finite or infinite number of samples with probability. Generally such probability is not known clearly. We employ it by the linguistic assessment result percentage. Such as, 50 experts evaluate the object good, and 50 percentage evaluate the object very good, then the probability is 0.5, 0.5 respectively.

Then, we translate attributes from linguistic values L_i into fuzzy grades $X_L$making use of triangular membership functions:

$$X_L \equiv (a, b, c) \tag{108.1}$$

where X_L denotes the central value of the fuzzy event, a is the central value and b, c are the left-side bound and right-side bound, respectively.

The estimation of the total assessment is written by the following fuzzy assessment function:

$$Y_i = f(X_{L_{i1}}, X_{L_{i2}}, \cdots, X_{L_{iK}}) \tag{108.2}$$

where *i=1, 2...N*, is the number of experts, K is the number of the attributes of the object. Then the X_L is obtained from the vocabulary of experts. From this dictionary we can convert the linguistic words to fuzzy variable random numbers.

108.2.1 Credibility Measure

Note that credibility measure (cf. Y-K Liu and B. Liu [4]) is an average of the possibility and the necessity measure, i.e., Cr{·} = (Pos{·}+Nec{·})/2, and it is a self-dual set function, i.e., Cr{A} = 1- Cr{A^c} for any A in P(Γ). The motivation behind the introduction of the credibility measure is to develop a certain measure, which is a sound aggregate of the two extreme cases, such as the possibility (which expresses a level of overlap and is highly optimistic in this sense) and necessity (which articulates a degree of inclusion and is pessimistic in its nature). Based on credibility measure, the expected value of a fuzzy variable is presented as follows:

Let Y be a fuzzy variable. The expected value of Y is defined as

$$E[Y] = \int_0^{\infty} Cr\{Y \geq r\} dr \int_{-\infty}^{0} Cr\{Y \leq r\} dr \tag{108.3}$$

provided that at least two integrals are finite.

Let ε be a fuzzy random variable with expected value e. Then, the variance of ε is defined by $V[\varepsilon] = E[(\varepsilon - e)^2]$.

108.2.2 Regression Model

All the linguistic data have been converted to fuzzy random variable data. We need to build a fuzzy regression model for fuzzy random data, which is based on the possibilities linear model.

Fuzzy Random Regression Model with Confidence Interval: Table 2 is the format of data that come from linguistic words, where input data X_{ik} and output data Y_i, for all *i=1,2,..., n* and *k=1,2,...,K*. They are all fuzzy random variables, which defined as:

$$Y_i = \bigcup_{t=1}^{M_{Yi}} \left\{ \left(Y_i^t, Y_i^t, Y_i^t\right), p_i^t \right\}, \quad X_{ik} = \bigcup_{t=1}^{M_{Xik}} \left\{ \left(X_{ik}^t, X_{ik}^{t,l}, X_{ik}^{t,r}\right), q_{ik}^t \right\}, \tag{108.4}$$

respectively. This means that all values are given as fuzzy numbers with probabilities, where fuzzy variables (Y_i^t, Y_i^t, Y_i^t) and $\left(X_{ik}^t, X_{ik}^{t,l}, X_{ik}^{t,r}\right)$ are associated with probability p_i^t and q_{ik}^t, for $i = 1, 2, \cdots, N$, $k = 1, 2, \cdots, K$ and $t = 1, 2, \cdots$, M_{Yi} and M_{Xik} respectively.

Let us denote fuzzy linear regression model with fuzzy coefficients $\bar{A}_1, \cdots, \bar{A}_k$ as follows:

$$\bar{Y}_i = \bar{A}_1 X_{i1} + ... + \bar{A}_K X_{iK}, \tag{108.5}$$

And then we need to determine the optimal fuzzy parameters $\widetilde{A}_i$. Two optimization criteria are considered. One concerns the fitness of the fuzzy regression model, h. The other one deals with fuzziness captured by the fuzzy regression model (108.5). Let us elaborate on the detailed formulation of these criteria.

In this study, we employ the confidence-interval based inclusion, which combines the expectation and variance of fuzzy random variables and the fuzzy inclusion relation satisfied at level h, to deal with the model (108.5) as discussed in [2], [12]. There are also some other ways to define the fuzzy random inclusion relation $\supset_h$, which will yield more complicated fuzzy random regression models. For instance, in order to retain more complete information of the fuzzy random data, we can use the fuzzy inclusion relation directly for the product between a fuzzy parameter and a fuzzy value at some probability level.

First we consider the one-sigma confidence interval of each fuzzy random variable, and it is expressed as follows:

$$\begin{aligned} I(e_{X_{ik}}, \sigma_{X_{ik}}) &= [e_{X_{ik}} - \sigma_{X_{ik}}, e_{X_{ik}} + \sigma_{X_{ik}}] \\ I(e_{Y_i}, \sigma_{Y_i}) &= [e_{Y_i} - \sigma_{Y_i}, e_{Y_i} + \sigma_{Y_i}] \end{aligned} \tag{108.6}$$

Then, the new confidence-interval-based fuzzy random regression mode is built as follows:

$$\left.\begin{aligned} &\min_{\bar{A}} J(\bar{A}) = \sum_{k=1}^{K} (\bar{A}_k^r - \bar{A}_k^l) \\ &\text{subject to } \bar{A}_k^r \geq \bar{A}_k^l, \\ &\qquad \bar{A}_i = \sum_{k=1}^{K} \bar{A}_k I[e_{X_{ik}}, \sigma_{X_{ik}}] \subset_h I[e_{Y_i}, \sigma_{Y_i}] \end{aligned}\right\} \tag{108.7}$$

where $i = 1, 2, \ldots, N, k = 1, 2, \ldots, K$, and the $\supset_h$ denotes the fuzzy inclusion relation realized at level h.

After then, in order to obtain linguistic expression, we need to match the obtained fuzzy numbers to the most appropriate linguistic words (Vocabulary Matching).

108.3 Concluding Remarks

We stressed the role of experts in accumulation of domain knowledge and experience, Experts frequently express their judgements in terms of linguistic expressions rather than pure numeric entities. In this sense, the linguistic treatment of assessments becomes essential when fully reflecting the subjectivity of the judgement process.

As we all know, human experts are always involved in decision-making process. However, the judgment experience and knowledge of experts are unique to each other. Better understanding of this judgment knowledge, sometimes, we need to convert it to numerical values which can give people more ocular way to experience the whole assessment process. And at the same time, there is always linguistic assessment of an object from various attributes. Then it is difficult to get a total assessment when we have linguistic data. In this paper, our model is built to solve this kind of problem.

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References

1. Imai, S., Wang, S., Watada, J.: Regression Model Based on Fuzzy Random Variables. In: Lovrek, I., Howlett, R.J., Jain, L.C. (eds.) KES 2008, Part III. LNCS (LNAI), vol. 5179, pp. 127–135. Springer, Heidelberg (2008)
2. Li, S., Imai, S., Watada, J.: Building Linguistic Random Regression Model and its Application. In: Watada, J., et al. (eds.) Proceedings of the International Conference on Knowledge-Based Intelligent Information and Engineering Systems (KES-IDT 2012). Springer (to appear, 2012)
3. Liu, B.: Fuzzy Random Chance-constrained Programming. IEEE Transactions on Fuzzy Systems 9(5), 713–720 (2001)
4. Liu, B., Liu, Y.-K.: Expected Value of Fuzzy Variable and Fuzzy Expected Value Models. IEEE Transactions on Fuzzy Systems 10(4), 445–450 (2002)
5. Liu, Y.-K., Liu, B.: Fuzzy Random Variable: A Scalar Expected Value Operator. Fuzzy Optimization and Decision Making 2(2), 143–160 (2003)
6. Pedrycz, W. (ed.): Granular Computing: An Emerging Paradigm. Physica-Verlag, Heidelberg (2001)
7. Pedrycz, W.: Computational Intelligence as an Emerging Paradigm of Software Engineering, Keynote Speech. In: Proceedings of SEKE 2002, Ischia, Italy, July 15-19, pp. 7–14 (2002)
8. Tanaka, H., Uejima, S., Asai, K.: Linear regression analysis with fuzzy model. IEEE Transactions on Systems, Man and Cybernetics SMC-12, 903–907 (1982)
9. Watada, J.: Multiattribute Decision-making. In: Terano, T., Asai, K., Sugeno, M. (eds.) Applied Fuzzy System, pp. 244–252. Academic Press, New York (1994)
10. Watada, J.: The Thought and Model of Linguistic Regression. In: Proceedings of the 9th World Congress of International Fuzzy Systems, Association, Canada, pp. 340–346 (2001)
11. Watada, J., Pedrycz, W.: A Fuzzy Regression Approach to Acquisition of Linguistic Rurles. In: Pedrycz, W., Skowron, A., Kreinovich, V. (eds.) Handbook of Granular Computing, pp. 719–732. John Wiley & Sons, Ltd. (2008)
12. Watada, J., Wang, S., Pedrycz, W.: Building Confidence-interval-based Fuzzy Random Regression Model. IEEE Transactions on Fuzzy Systems 11(6), 1273–1283 (2009)
13. Zadeh, L.A.: Fuzzy Sets. Information and Control 8, 338–353 (1965)
14. Zadeh, L.A.: The Concept of a Linguistic Variable and its Applications to Approximate Reasoning, Part 1. Information Science 8, 199–249 (1975)
15. Zadeh, L.A.: The oncept of a Linguistic Variable and its Applications to Approximate Reasoning, Part 2. Information Science 8, 301–357 (1975)
16. Zadeh, L.A.: Fuzzy Sets and Information Granularity. In: Gupta, M.M., Ragade, R.K., Yager, R.R. (eds.) Advances in Fuzzy Set Theory and Applications, pp. 3–18. North-Holland, Amsterdam (1979)
17. Zadeh, L.A.: What is Computing with Words. In: Zadeh, L.A., Kacpruzyk, J. (eds.) Computing with Words in Information/Intelligent Systems, Foundation, pp. VIII–IX. Physica-Verlag, Heidelberg (2006)
18. Zadeh, L.A.: Fuzzy Logic:= Commputing With Words. In: Zadeh, L.A., Kacpruzyk, J. (eds.) Computing with Words in Information/Intelligent Systems, Foundation, pp. 3–23. Physica-Verlag, Heidelberg (2006)

109

Syzygy

Mark J. Wierman

> " ... the race is not to the swift, nor the battle to the strong, neither yet bread to the wise, nor yet riches to men of understanding, nor yet favour to men of skill; but time and chance happeneth to them all."
>
> Ecclesiastes 9:11

109.1 Entropy

We were having a rare meeting of the Entropy Club at Purchase College. The Entropy Club met irregularly since, by strict rule, a meeting could only occur when all four of its members just happened to be in the same place at the same time.

John, who was the physics major, was doing his senior thesis on entropy, and was currently bedeviled with Maxwell's demon. This imp controls a shutter on a stream and sends the high temperature particles to the left, and the low temperature particles to the right, creating a temperature differential that could be tapped to create electricity. This meant that the demon was basically making energy from nothing, a paradoxical result. Since I was doing my senior thesis on paradox I found this a fascinating situation to ponder.

John explained that what was really happening was that information was being transmuted into energy, and since information also has entropy, the paradox is dissolved. In physics, there are laws of conservation of information as well as conservation of energy.[1]

109.2 Information

In the old days (before the Internet!) people went to libraries to get information. When I decided to get my PhD I went to the library at Binghamton University, which is close to my home town, to investigate graduate programs. While I was there I decided to kill two birds with one stone and file an application for the Computer Science program. The Computer Science Department was housed in the Watson School of Engineering. This application was a bit premature as I had not yet received my GRE scores.

A short time later, just after my GRE scores came in, I got a call from someone from the Watson School. He told me on the phone that he was not from the

[1] One of the greatest problems in modern physics asks whether or not quantum information is destroyed by black holes.

R. Seising et al. (Eds.): *On Fuzziness: Volume 2*, STUDFUZZ 299, pp. 755–762.
DOI: 10.1007/978-3-642-35644-5_109

Computer Science Department but that he was interested in finding someone with a good mathematical background to do research in entropy–like measures of information, and that he had a grant.

Later that week and I went to the Watson School, located the Systems Science Department and met George Klir for the first time. I accepted an assistantship and began researching the mathematics of the Shannon entropy [10, 4, 1, 2].

I also signed up to take a class called *Fuzzy Set Theory* [7] and my life turned a corner.

I had taken courses in Logic and in Set Theory while getting my Bachelor's and Master's Degrees in Mathematics.[2] I had read Cantor, Frege, Russell, Łukasiewicz, Godel, ..., but I had never read anything like Zadeh! I had no idea something like this existed. And why hadn't I seen any of this in my math classes?

By the time I finished my PhD my bookshelf contained every important book on fuzzy set theory in existence. It totaled roughly ten books. I also learned three very important lessons:

1. fuzzy mathematics is *hard*,
2. fuzzy was probably *not* a good name for this theory, and
3. *serendipitous* meetings would have great impact on my life and research.

109.3 Uncertainty

When I went searching for a faculty position after graduation, Creighton University had the only opening that specifically requested a researcher in fuzzy set theory. I applied, interviewed, got an offer, and took the job.

John Mordeson was then Chairman of the Mathematics and Computer Science Department. He thought it would be nice if we could bring my thesis advisor, George Klir, to Omaha for a visit.

During the visit, John suggested that Klir and I should collaborate on a collection of lecture notes concerning Uncertainty. Creighton would then publish a small print run for the use of our graduate student.

At JCIS 1996 Klir and I ended up talking with Janusz Kacprzyk. Kacprzyk thought an improved version of the lecture notes would make a good entry in the Springer series *Studies in Fuzziness and Soft Computing*. The subsequent book [8] is still the number one result of googling my name. It has been referenced thousands of times.[3]

109.3.1 Granularity

Writing a book gives you a very good foundation for subsequent research. Around the year 2000 I decided to try my hand at an axiomatic measure of uncertainty for

[2] I mentioned that I did my Senior Thesis on *Paradox*.

[3] Mordeson subsequently published many books with Springer with various collaborators, including myself.

Evidence Theory. While most of the axioms of a measure of uncertainty are similar from one mathematical theory to the next, the crucial axiom always turns out to be the one that defines how the measure operates on product spaces.

This axiom is critical because if we pick a unique $x \in X$ and a unique $y \in Y$ we have determined a unique $\langle x, y\rangle \in X \times Y$. However, there are many more choices in $X \times Y$ than there are in X and Y together. The product space axiom is always the one that introduces a logarithmic function that characterizes most measures of information.

The product axiom for evidence theory was difficult because, in general, evidence theory acts on covers of X and Y.

I decided to start with an axiomatic measure for rough set theory, since it was structurally similar to evidence theory. Strangely enough, this paper, [12, 13] is one of my most influential.

109.4 Dissonance

Bill Tastle came to Omaha for the Student Programming Challenge of the 2004 AITP National Conference. Bill was a fellow Graduate Student in the System Science Department, as was my wife. By *coincidence*, my wife was Co-Chair of the conference and noticed his name among the conference organizers.

So I sent him an email and invited him out for a beer.

While we were having beer and pizza at the Upstream (a most excellent restaurant) Bill mentioned that he was looking for a superior measure of agreement in the venue of consensus building. He said:

> "How does a chairman know when his committee is getting close to agreement? How does he know when to call the vote? I am sure you will figure out something and send me an email tomorrow."

Well I went all glassy eyed as mathematical equations sprung into my mind; but then my wife reminded me we were entertaining and I came back to earth. Still, the next day, I wrote down a formula that just seemed right and sent it off to Bill.

So far we have written seventeen papers on the subject of measuring consensus and dissention [11, 15].

109.5 Psychohistory

John Mordeson had long been trying to involve other members of the Creighton University faculty in collaborative research. Terry D. Clark had long been frustrated by the lack of predictive results using traditional mathematical in Political Science models. For example, McKelvey [9] predicts chaos in a political system with three or more strong and independent parties. Arrow [3] predicts dictatorships when actors are rational. Clever voting agendas can defeat the majorities choice in a set of alternatives.

While Zadeh has long been interested in “Computing with words,” most applications of fuzzy set theory have taken place in the scientific and engineering community.

When Mordeson saw the need for someone with the ability to apply theory to data I was invited to join the “The Fuzzy Spatial Modeling Project.” This ever evolving group of students and faculty has produced a substantial body of work over the last six years, including a book and the first “fuzzy” paper published by the prominent journal *Public Choice*, [6,5].

109.6 Fuzzy

Eastern philosophy had long accepted that the world is not made up of absolutes. There is yin in yang and yang in yin.

The fact that Aristotle (who essentially invented logic) had found fault with logical absolutism (The Sea-Battle) was dismissed by a culture that wanted a world of black and whites. Even the overwhelming impact of probability, and the acceptance of randomness as a fact of life, did little to change the common perception of a world divided into black and white.

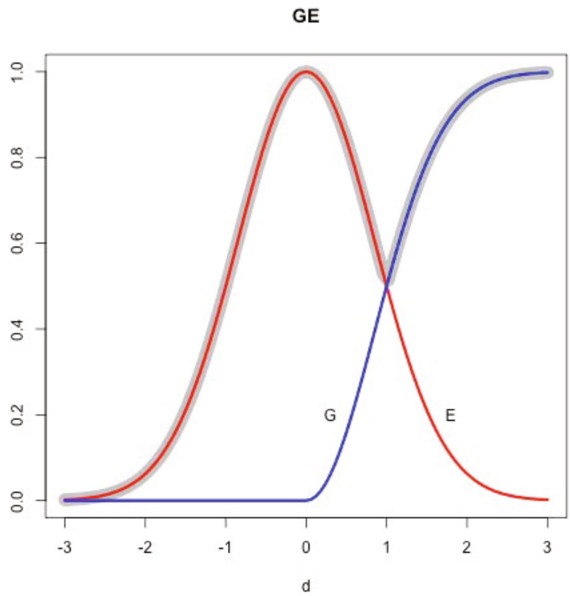

Fig. 109.1. Graphs of E (equal zero) and G (greater than zero) are monotonic but $GE = G \vee E$ (greater than or equal zero, or max $[E,G]$) is not

109.6.1 Fuzzification

For me, fuzzy set theory has forced me to question the common assumptions of every formal system. This inquisitiveness also made me question the way we are doing things in the world of fuzzy set theory.

Example 1. Let us consider two alternatives a and b and let us suppose that on some scale that a is less than or equal to b. The phrase "less than or equal" is pretty intuitive, and there is a precise mathematical definition of order.

Suppose we want to make a fuzzy analog of "equal" called E. One common approach is to use a distance between a and b. Then a function such as $E(x,y) = 2^{-|x-y|}$ will attain a value of one when the distance is zero, and hence x and y are equal, and decrease rapidly as the distance increases.

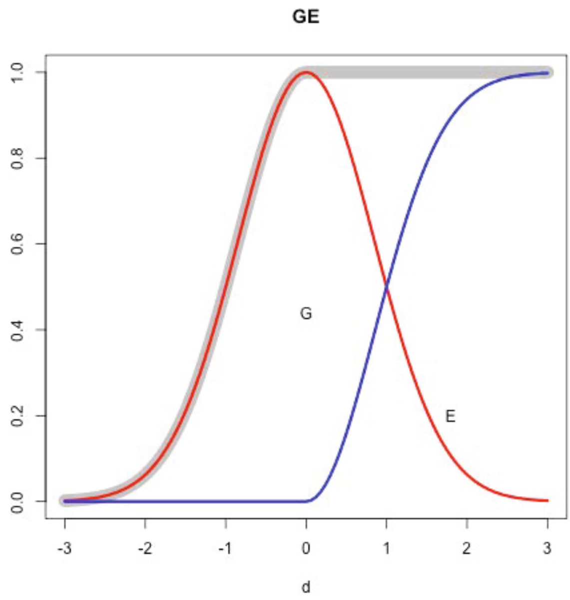

Fig. 109.2. Graphs of E (equal zero) and G (greater than zero) are monotonic. Using the Łukasiewicz t-conorm for $E \vee G$ (greater than or equal zero) produces a GE that is also monotonic.

Now suppose we want to make a fuzzy analogue of "greater than" called G. Intuitively, the farther to the right of a that b is the more b is "greater than" a. As b is moved closer to a the distance δ decreases and the degree of greater than should also decrease. When $G(b,a) = 1$ we interpret this as b being significantly more valuable than a, i.e, "much greater than". As we move b to the left the amount of "greater than" decreases monotonically until. So what do the values of zero and one–half mean about the distance between a and b for the concept "greater than?"

Does $G(b,a) = 0$ indicate b is far to the left of a or that they are coincident? Is $G(b,a) = 0.5$ a better value for coincidence or does it indicate that b is halfway between a and some (infinite?) maximum value.

Let us suppose we decide that $G(b,a) = 0$ when a and b are coincident. Then a formula for G might look like:

$$G(x,y) = \begin{cases} 1 - 2^{-(x-y)} & x > y \\ 0 & \text{otherwise} \end{cases} \tag{109.1}$$

Everything seems fine until we define "greater than or equal" as the union (logical or) of G and E. Figure 109.1 shows the result when we fix y at zero. While E (equal) and G (greater than) are monotonic the $E \vee G$ (greater than or equal, or $\max[E(x,y), G(x,y)]$) is not. When $x = 1$ we have that $E(x,0) = G(x,0) = 0.5$. And as x decreases or increases $E \cup G$ increases.

Note that we will not be better off if we build a monotonic GE, or "greater than or equal" and then define G as $G = GE \wedge \neg E$. This will produce a multimodal G.

The use of other operators for combining G and E will not help the situation. We could use the Łukasiewicz t-conorm, $s(x,y) = \min(x+y, 1)$, to combine G and E, which produces a monotone result for GE as depicted in Figure 109.2.

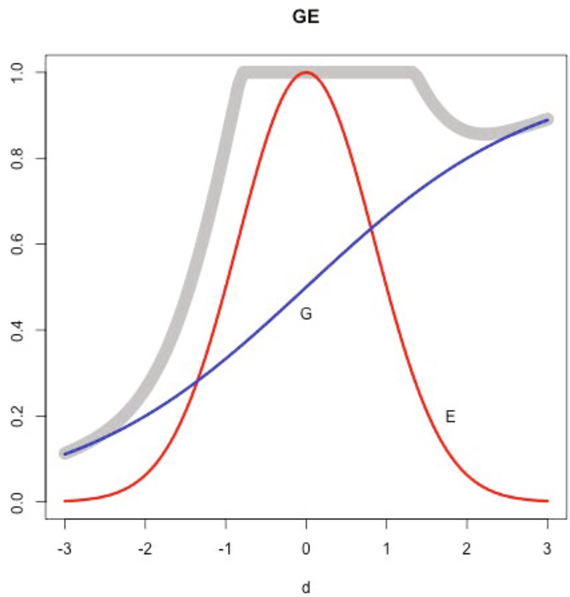

Fig. 109.3. Graphs of E (equal zero) and G (greater than zero) with G using a logistic curve. Now GE using the Łukasiewicz t-conorm is not monotonic.

However the Łukasiewicz t-conorm has a drawback. Suppose we had decided that $G(b,a) = 0.5$ is more reasonable when a and b are coincident. Then we might use a logistic function for G

$$G(x,y) = \frac{1}{1+e^{y-x}}$$

and then $G \vee E$ using the Łukasiewicz t-conorm gives a GE which is not monotone. This result is illustrated in Figure 109.3.

The point is that fuzzy mathematics is hard and we have to think about what we are doing. We cannot naively convert crisp concepts into fuzzy analogues, and if we do we will often produce counterintuitive results.

In the fuzzy world "greater than or equal" may be a concept that can not be built up from simpler components. I suspect that $a \to b$ may be another concept that indicates many different relationships between a and b which all just happen to coincide in a crisp world, but which bifurcate under Zadeh's gaze. [4]

When we finally "compute with words," our grammar is going to have a revolutionary structure. The fuzzy set community has a lot of work to do. Zadeh [16, 14], like Maxwell's demon, has only opened the gates for us.

☯

References

1. Aczél, J.: Lectures on Functional Equations and Their Applications. Academic Press, London (1966)
2. Aczél, J., Daróczy, Z.: On Measures of Information and Their Characterizations. Academic Press, New York (1975)
3. Arrow, K.J.: Social Choice and Individual Values. Wiley, New York (1951)
4. Ash, R.B.: Information Theory. Wiley Interscience, New York (1965)
5. Clark, T.D., Larson, J.M., Mordeson, J.N., Potter, J.D., Wierman, M.J.: Applying Fuzzy Mathematics to Formal Models in Comparative Politics. Springer, Berlin (2008)
6. Clark, T.D., Larson, J.M., Mordeson, J.N., Wierman, M.J.: Extension of the Portfolio Allocation Model to Surplus Majority Governments: A Fuzzy Approach. Public Choice 134(3-4), 179–199 (2008)
7. Klir, G.J., Folger, T.A.: Fuzzy Sets, Uncertainty, and Information. Prentice Hall, Englewood Cliffs (1988)
8. Klir, G.J., Wierman, M.J.: Uncertainty-based Information. Physica-Verlag, New York (1998)
9. McKelvey, R.D.: Intransitivities in Multidimensional Voting Models. Journal of Economic Theory 12, 472–482 (1976)
10. Shannon, C.E.: The Mathematical Theory of ommunication. The Bell System Technical Journal 27(3,4), 379–423, 623–656 (1948)
11. Tastle, W.J., Wierman, M.J.: Consensus and Dissension: A New Measure of Agreement. In: NAFIPS 2005, Ann Arbor, MI (2005)
12. Wierman, M.J.: Measuring Uncertainty in Rough Set Theory. International Journal of General Systems 28(4,5), 283–297 (1999)
13. Wierman, M.J.: Measuring Granularity in Evidence Theory. International Journal of General Systems 30(6), 649–660 (2001)

[4] One could examine the myriad proposals to model fuzzy implication, where even Zadeh changed his mind.

14. Wierman, M.J.: Cloud Sets as a Measure Theoretic Basis for Fuzzy set Theory. In: NAFIPS 2010, Toronto, CA (2010)
15. Wierman, M.J., Tastle, W.J.: Consensus and Dissension: Theory and Properties. In: NAFIPS 2005, Ann Arbor, MI (2005)
16. Zadeh, L.A.: Fuzzy Sets. Information and Control 8(3), 338–353 (1965)

110

Philosophy of Science, Operations Research, and Fuzzy Set Theory – Personal Observations

Hans-Jürgen Zimmermann

110.1 Introduction

Since the following are personal observations, it might be appropriate to tell the reader some relevant facts of my background. They have shaped my views, which otherwise might seem strange to some colleagues in other disciplines:

Due to several flights of approaching fronts during last world war and from eastern Germany I arrived in the West in 1949 with 16 and started a business apprenticeship in 1950. I was supervisor of an order department (ex-and import) from 1952 till 1953 during which time I attended night classes to qualify for university studies. After obtaining my master degree in engineering, one year of post graduate studies in Oxford and my PhD degree in Berlin (in Business Administration and Mathematical Economics) with a thesis on Operations Research, I went into industry again, working as Head of Department of Production Control for the European IT&T in Stuttgart. A good career in industry seemed to be ahead of me! Unexpectedly I was invited as visiting professor for Operations Research and Production Management by the University of Illinois in Urbana (USA). I stayed there - with short lecturing breaks in Berkeley and Ann Arbor- for three years. That is where I tasted and liked the academic life and research! Areas of activity were mainly Operations Research, Mathematics, Computing and Production Control. When in 1967 I accepted a new chair for Operations Research at the Aachen Institute of Technology in Aachen (Germany) I had spent 10 years in industry, focusing on solving problems and just a short time in doing academic research. Now I was eager to do research, but due to my past, mostly problem oriented and seldom for the sake of the theory I was working on. And I also took some principles of the original Operations Research seriously: 1. The justification of all methods and models to be developed or used was the problem considered and 2. Work should be done in interdisciplinary teams because most problems contain aspects not only of one but of several disciplines. For the years to come the co-workers in my chair came from between 5 and 10 disciplines (business administration, mathematics, informatics, psychology, statistics, and operations research). The beginning of such a group is rather difficult but the rewards later on are surprising! We had a good relationship with industry and the only thing that always bothered me was, that our activities were confined to operational problems (here data existed) and operations research was not accepted for the more interesting strategic decisions, that were too ill structured to be modeled by OR methods and models.

Then in 1971, when I studied the literature after my class in (cognitive) decision theory I came across the paper “Decision-making in a fuzzy environment” by

R. Seising et al. (Eds.): *On Fuzziness: Volume 2*, STUDFUZZ 299, pp. 763–774.
DOI: 10.1007/978-3-642-35644-5_110

Bellman and Zadeh [6] in Management Science 1970, and this changed my life remarkably for 2 reasons: 1. I saw chances for OR to work in less structured areas of decision making and 2. On the basis of my research and practical work in decision making I could not believe, that the word “and”, when used by humans would always have the same meaning and could be modeled by “min” or by “prod” in all contexts. I called Lotfi Zadeh in Berkeley and after a very short time I was in Berkeley, enjoying Fay’s and Lotfi’s exceptional hospitality and extremely productive discussions with Lotfi. This started all the work I have done afterwards, gave my life another turn and lead to results that nobody expected at that time.

Research – and applications – in my chair in Aachen went along two lines: 1. developing fuzzy versions of the OR tools that we were using, such as Mathematical Programming, Expert Systems, Multi Criteria Decision models, Data Mining etc. 2. Mathematical and empirical research in shapes of membership functions and in operators that really modeled human speech, inference, and decision making.

In addition I tried to increase communication and discussion between colleagues on a worldwide scale, for instance, by starting the international journal “Fuzzy Sets and Systems” in 1978 and by founding IFSA (International Fuzzy Systems Association) in Brussels and Hawaii in 1984.

110.2 Different Perspectives on Fuzzy Set Theory

To judge the “value” of a theory it is useful to first state , what one expects of a theory or what is intended by the theory. This is discussed extensively in the theory of science, a branch of philosophy. I am a friend of Popper’s views [25], who distinguishes between “formal scientific theories” and “factual scientific theories”. Former develop a formal (mathematical or logical) theory, which can be proven to be correct by formal arguments . If that has been done correctly (generally on the basis of dual logic) it is considered nomologically correct. Latter wants to make true statements about reality. According to Popper such a theory cannot be proven (because reality might change), but it is considered “corroborated” until it has been falsified. If one considers fuzzy set theory as a formal theory most of it has been proven sufficiently [11], [19], [24], [29], [30], [38], [39], [40], [42].

If, however, it is considered as a factual scientific theory, to my mind, a lot of more empirical work would have to be done to verify (try to falsify) the assumptions of this theory. Let us just consider some of the central elements of this theory: membership functions and operators: [45]

Very often membership functions are considered to be either triangular or trapezoidal. Consider the membership functions which have been determined empirically for groups of students [34], [47], [51].

About 50% of the members of the group had one shape of membership function (similar to triangles) the other half had membership functions that looked more s-shaped. Obviously the first group considered “age-groups” and the second considered aging as a process. Also in general (except in higher order fuzzy sets) membership values or functions are generally treated as being on an absolute scale level (all

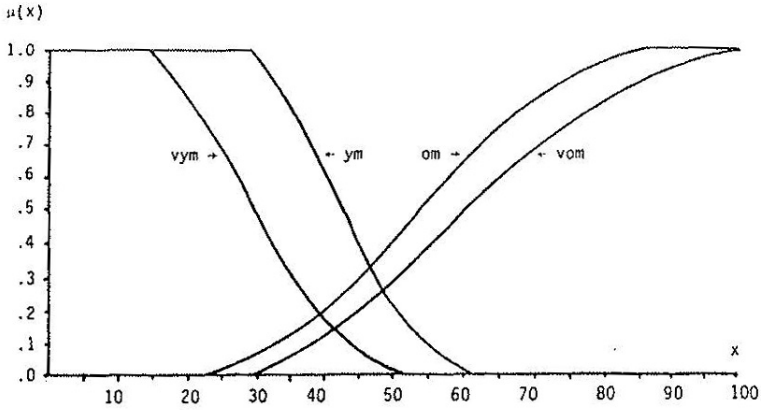

Fig. 110.1. Membership functions of 52% of the group members for the fuzzy sets "old men" and "very old men"

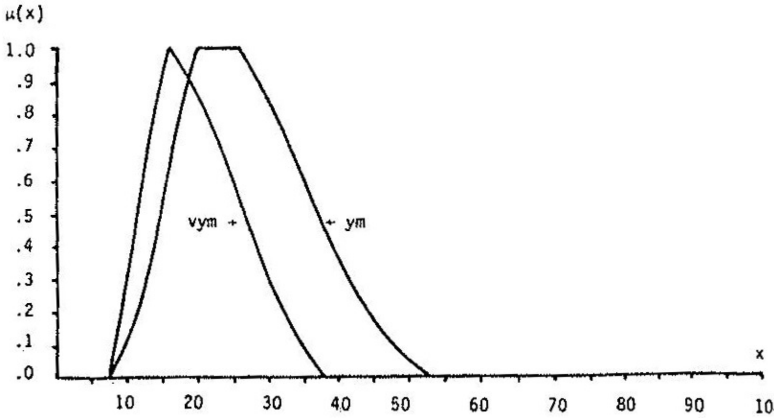

Fig. 110.2. Membership functions of 48% of the group members for the fuzzy sets "old men" and "very old men"

mathematical operations allowed) even though psychologists know, that humans can only make judgments on an ordinal scale level.

Similar holds for operators of fuzzy set theory. T-norms and T-conorms are certainly mathematically attractive. Empirically it has , however, never been shown, that humans use them when they use words like "and" and "or". Of course, I argue from the point of view, that fuzzy expressions should model human communication or inference. There are other areas, in which fuzzy set theory can be "verified" to a higher extend.

A third "view" could be, that fuzzy set theory should be used as a tool to better solve problems, i.e. for applications. The term "application" is generally used in a number of ways: The application of one theory to another (fuzzy set theory applied to linear programming, which results in fuzzy linear programming), the application of a theory to models (of problems), i.e., fuzzy set theory to inventory control models, and finally the application of theories or models to "real" problems. When I talk about applications in this paper, I will always mean the last type. A problem generally has an objective part (inventory) and a subjective part (the decision maker who considers the inventory as too high). In this function (as a modeling tool) fuzzy set theory has often been shown to be very helpful. The difficulty is just to extract the subjective part of the problem from the head of the decision maker into a fuzzy formulation.

110.3 Fuzzy Sets, Operations Research, and Decision Theories

There are essentially two views on Operations Research (OR):

The original view (from the military foundation in British military) in which OR is a problem focused way of using scientific methods in interdisciplinary teams to solve real problems better than is possible without OR [36]. Since the fifties of last century very many formal methods and application areas have been developed in OR and that led to another view of OR, which essentially comprises the mathematical methods of OR. I am a representative of the first view.

From the many areas of OR I will consider exemplarily two in optimization [10]. The two probably largest areas in optimization are mathematical programming and knowledge based systems (expert systems). In the former [18] problems are modeled as optimizing an objective function over a solution space, which is defined by constraints. Normally there is only one objective function and the constraints are crisp. Hence one can determine an optimal solution in the solution space. It becomes difficult, however, if there are several objective functions or if the solution space, as defined by the constraints, is empty. The first complication leads to multi objective decision making [2], [48] or vector optimization [26], [37], [44] and the second complication often occurs because the model constraints differ from the problem restrictions (given, that a real problem has a real solution). In both cases fuzzy sets have been applied and have helped considerably by either aggregating objective functions meaningfully or by fuzzyfying (relaxing) model constraints such that they better model the real views of decision makers, which very often are not crisp. This "fuzzy mathematical programming" has a number of additional advantages, compared to traditional crisp mathematical programming, which shall not be discussed here.

The situation is somewhat different for knowledge based systems (KBS): In traditional "expert systems" one tries to store human expertise (mostly as rules) and then deduce from these rules and the observed facts desired results. The problem is, that facts, features etc. are defined crisply and that the inference method is generally based on dual logic. Hence, these systems are actually not knowledge processing systems but rather symbol processing systems, in which truth values (0 or 1) are processed. The introduction of fuzzy sets for the description of facts and features and fuzzy logic or approximate reasoning for the inference has certainly improved these systems considerably and actually transformed the symbol processing into knowledge processing. One major problem, however, remains: If such a system is to use human views and inference procedures one has to extract them from the heads of the experts, which is rather difficult, or stick to approximations, which we believe are close to what the experts think and do.

Decision theory has always been considered as a good area for the application of fuzzy set theory. Besides statistical decision theory there exist “Decision Logic” (DL) as a formal theory, in which a decision is normally a timeless act of choice [16], and “Empirical, Cognitive” or Descriptive Decision Theory (DT), in which a decision is a time consuming, multi person, interactive information processing process, as a factual theory. As can be expected fuzzy set theory can be and has been applied repeatedly to DL. For DT we have the same problems, which were already mentioned in the last section. Since statements about real decision making are being made, to apply fuzzy set theory we would have to extract views from the heads of humans and that has not yet been done too often.

110.4 Knowledge Based Systems

Formally there exist two types of knowledge based systems (KBS) in the area of fuzzy set applications: Fuzzy Expert Systems (FES) [4], [12], [13], [43], [46] and Fuzzy Control (FC) [3], [13], [22], [32]. Both contain a knowledge base and an inference machine. FC has as inputs numbers (measurements), which are then fuzziyfied and at the end we obtain again numbers by defuzzyfication. FES normally allow crisp or fuzzy inputs and provide as outputs linguistic expressions obtained by “linguistic approximation” [35]. There is, however, to my mind, a much more basic difference between these two types of fuzzy systems: Fuzzy control, which actually started the “Fuzzy Boom” in many countries in the 80s of last century, is intended to control man made technical systems [3], [32]. The components (fuzzyfier, knowledge base, inference engine, defuzzyfier) can be calibrated until the controller functions as desired [22]. For FES this is not the case, particularly if they are intended for long term decisions ,e.g., strategic planning decisions etc. Here again the expertise of the expert has to be extracted and there is no way to verify the results when designing the system.

110.5 Data Mining and Analytics

Data Mining is an area that has existed for a long time. Since we have moved from a time of scarce data into one of an abundance of data it has become more and more important. Essentially it aims to extract meaningful information from masses of data. In classical methods (e.g clustering) methods are generally dichotomus , which very often does not model reality well enough. Hence, a fuzzyfication of these methods and the use of additional approaches (such as neural nets) [20], [21] makes a lot of sense and has been done very successfully [5], [7], [27], [28].

Different methods of fuzzy clustering have been developed and applied very successfully in a large number of areas (bus. administration, regional policy, medicine, fraud prevention, etc.) [1], [8], [9], [14], [23], [31], [49]. It can be expected, that the importance of these methods will still increase in the future. As in fuzzy control, parameters can be varied until the results of the methods are satisfactory. Recently there have even been motions to rename Operations Research and call it Business Analytics. Whether that makes sense remains to be seen.

110.6 Media and Public Interest

When I started working on fuzzy sets after my first discussions with Lotfi, colleagues smiled at me. Hardly anybody took our research seriously and I was warned (particularly in military) that hardly anybody wanted to be a “fuzzy decision maker”. There were two universities in Germany that offered any lectures on fuzzy set theory and control engineers were convinced, that they had much better methods. That changed dramatically in Germany in the 80s of last century, when a German journalist visited Japan and was impressed by the products that contained fuzzy control (video cameras, rice cookers, washing machines, etc.) and that were very fashionable in Japan. He wrote an article in a leading German journal, warning the German industries that they were losing another market to Japan (after consumer electronics from Japan had become a serious competitor to German products). That started a wave in German media on fuzzy control. Hardly any trade journal or journal in information processing dared to not publish articles on applications of “Fuzzy Logic”.

To our booth at the Hannover Trade Fair (one of the world’s largest) where we had a stand together with OMRON, hundreds of people came to ask :“What is fuzzy logic?” We had built and exhibited a fuzzy car (with about 250 rules in the FC knowledge base), that could go autonomously from A to B without hitting any obstacle between the points. Even the minister of the German research ministry came to look at it and watch it go.

Within one or two years the number of universities that offered lectures on fuzzy set theory went from 2 to 30!

The annual EUFIT (European Congresss on Fuzzy and Intelligent Systems) conferences started in Aachen with between 250 and 300 papers each time and at each of the conferences there was an exhibition in which 20 to 30 companies exhibited their “fuzzy products”. Software tools (FuzzyTech [15] for fuzzy control and DataEngine for intelligent data mining) were build and the courses on “fuzzy topics” increased remarkably in number. ERUDIT, the European Network in Uncertainty Techniques, for which I had the honor to be the president, was founded in 1994 and financed by the European Commission.

Fig. 110.3. Typical Cover Picture of a Trade Journal in Germany at the beginning of the 90s

The number of papers and books on fuzzy set theory exploded! One article in a trade journal had been more convincing to the public than many man years of academic work! After a few years the media lost interest in this topic and so was the public. Today hardly anybody, except the experts, knows about "fuzzy logic" in Germany. On reason might also be, that at that time fuzzy set theory was presented as a theory that was very simple and easy to understand and apply. That may have changed in the meantime.

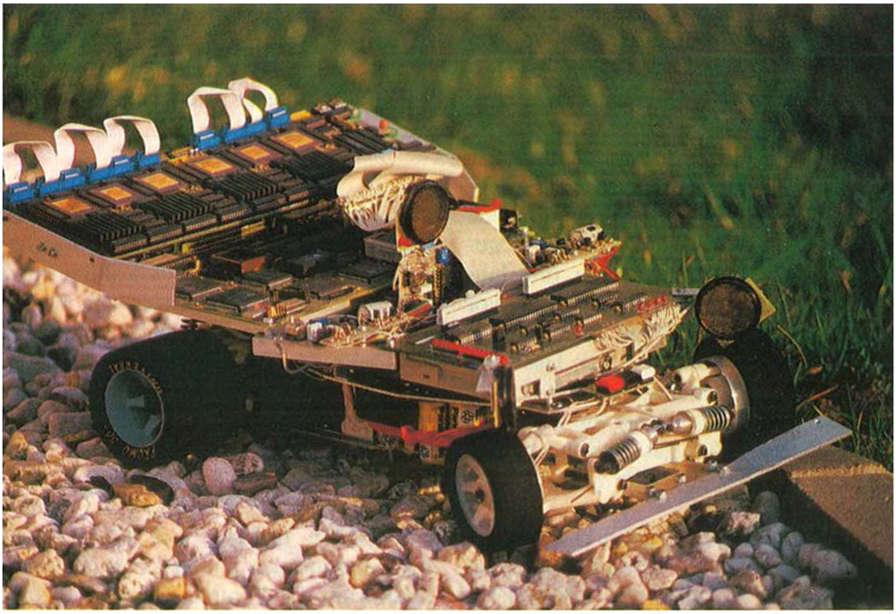

Fig. 110.4. "Fuzzy Car" with 250 rules that could go autonomously from *A* to *B* without hitting an obstacle

110.7 Modeling Different Types of Uncertainty

Until 1971 to model uncertainty was easy for me: I knew probability theories and statistics , worked on stochastic programming and these were the only tools I could use. This became much more difficult after the inception of Fuzzy Set Theory by Professor Zadeh, because now I had to distinguish between different kinds of uncertainty and try to choose the correct theory to model the type of uncertainty under consideration. In the meantime there exist, to my knowledge, about 25 different uncertainty theories and the choice of the "correct" or applicable theory in a certain situation is often not easy. It is my experience, that most people act in such a situation according to the "hammer principle" (If I have a hammer in my hand, everything starts looking like a nail!). Since most still know only probabilities or statistics they will use these as a modeling tool. What, however, if one is aware of other uncertainty theories? Which one should one apply?

Fig. 110.5. Lotfi Zadeh with our fuzzy car at our booth

I assume that most of the uncertainty theories are "formal theories", developed on the basis of some stated or not stated axioms or assumptions a consistent theory. Of course, one can combine theories (such as probabilistic sets or fuzzy event). But generally they remain formal theories. For a few (primarily probabilistic) theories there exists already empirical evidence that can facilitate the choice.

But how about the others, when one wants to apply them to real situations or problems? To my mind [50] each of the formal theories would have to state clearly the assumptions and axioms on which it is built. This is important not only for the mathematics used, but also for the types of inputs assumed (i.e. linguistic or numerical, if numerical on which scale level etc.) and for the types of outputs required. Each theory could then be characterized by a vector of characteristics, which can be compared with the characteristics of the phenomenon to be modeled. In applications in decision making the time horizon also plays an important part. While for strategic planning probability theory may be appropriate, for control problems fuzzy set theory may be the correct choice.

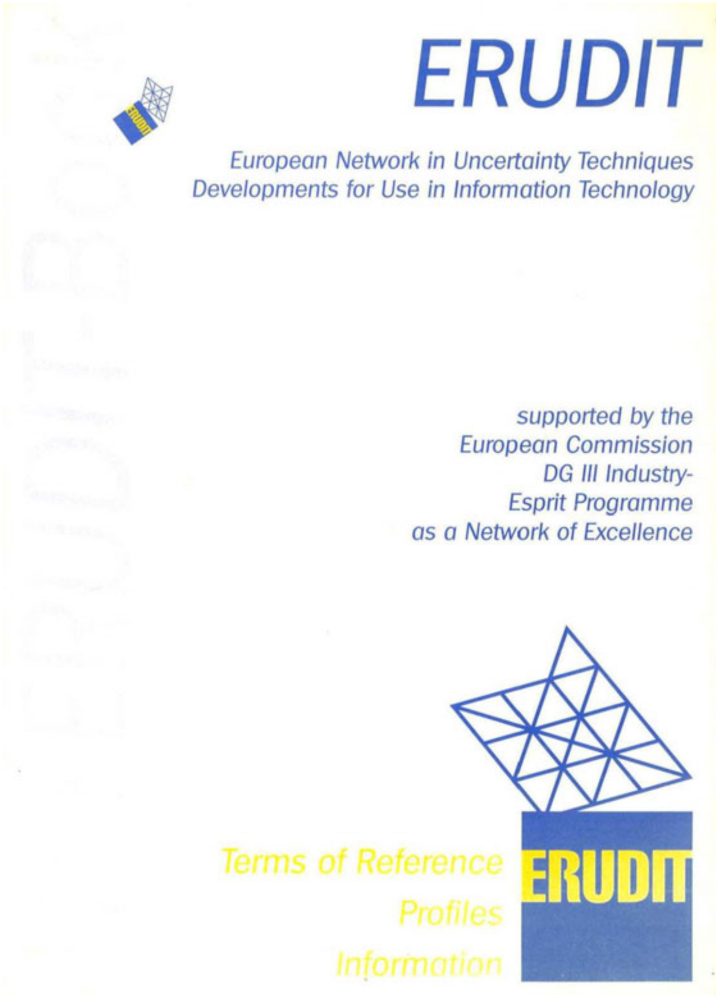

Fig. 110.6. ERUDIT, the "European Network in Uncertainty Techniques"

110.8 Conclusions

The inception of fuzzy set theory by Lotfi Zadeh has been a centennial breakthrough for the entire area of uncertainty modeling. It is not only the theory itself and all the additional concepts that Lotfi has invented and worked out, that have changed all our views, but he has also triggered very many useful other theoretical developments and he is still doing that! I cannot congratulate and thank him enough for what he has done for scientific advancements. The challenges he has formulated will still take a long time in the future to satisfy. For me personally Lotfi has been much more: Not only a brilliant scientist but also a very good friend, who has often in my life helped

with professional and personal advices (which always turned out to be correct). For all that I want to thank him from all my heart and hope that we can enjoy his company still for a long time.

References

1. Adlassnig, K.-P.: A Fuzzy Logical Model of Computer-assisted Medical Diagnosis. Methods of Information in Medicine 19, 141–148 (1980)
2. Baas, S.M., Kwakernaak, H.: Rating and Ranking of Multiple-aspect Alternatives Using Fuzzy Sets. Automatica 13, 47–58 (1977)
3. Babuska, R.: Fuzzy Modeling for Control. Kluwer, Dordrecht (1998)
4. Baldwin, J.F., Guild, N.C.F.: Feasible Algorithms for Approximate Reasoning Using Fuzzy Logic. Fuzzy Sets and Systems 3, 225–251 (1980)
5. Bandemer, H., Nöther, W.: Fuzzy Data Analysis. Kluwer, Dodrecht (1992)
6. Bellman, R.E., Zadeh, L.A.: Decision-making in a Fuzzy Environment. Management Science 17 B, 141–164 (1970)
7. Bezdek, J.C.: Pattern Recognition with Fuzzy Objective Function Algorithms. Kluwer, New York (1981)
8. Blockley, D.I.: The Role of Fuzzy Sets in Civil Engineering. Fuzzy Sets and Systems 2, 267–278 (1979)
9. Buckles, B.P., Petry, F.E.: A Fuzzy Model for Relational Databases. Fuzzy Sets and Systems 7, 213–226 (1982)
10. Buckley, J.J.: Possibility and Necessity in Optimization. Fuzzy Sets and Systems 25, 1–13 (1988)
11. Goguen, J.A.: L-fuzzy sets. Journal of Mathematical Analysis and Applications 18, 145–174 (1967)
12. Goodman, I.R., Nguyen, H.T.: Uncertainty Models for Knowledge-based Systems. North Holland, Amsterdam (1985)
13. Gupta, M.M., Yamakawa, T. (eds.): Fuzzy Logic in Knowledge-Based Systems, Decision and Control. North-Holland, Amsterdam (1988)
14. Hirota, K.: Industrial Applications of Fuzzy Technology. Springer, Berlin (1993)
15. INFORM, N.N. FuzzyTech.1 Reference Manual; Users Manual. Inform Aachen (1995)
16. Kacprzyk, J.: Multistage Decision Making and Fuzziness. TÜV Rheinland, Köln (1983)
17. Kaufmann, A.: Introduction to the Theory of Fuzzy Subsets. Academic Press, New York (1975)
18. Kickert, W.J.M.: Fuzzy Theories on Decision Making. Kluwer, Leiden (1978)
19. Klir, G.J., Folger, T.A.: Fuzzy Sets, Uncertainty, and Information. Prentice Hall, Englewood Cliffs (1988)
20. Kosko, B.: Neural Networks and Fuzzy Systems. Prentice Hall, Englewood Cliffs (1992)
21. Lee, S.C.: Fuzzy Neural Networks. Mathematical Biosciences 23, 151–177 (1975)
22. Mamdani, E.H., Assilian, S.: An Experiment in Linguistic Synthesis with a Fuzzy Controller. International Journal of Man-Machine Studies 7, 1–13 (1975)
23. Meyer, A., Zimmermann, H.-J.: Applications of Fuzzy Technology in Business Intelligence. International Journal of Computers Communications & Control VI, 40–423 (2011)
24. Mizumoto, M., Zimmermann, H.-J.: Comparison of Fuzzy Reasoning Methods. Fuzzy Sets and Systems 8, 253–283 (1982)
25. Popper, K.R.: The Logic of Scientific Discovery. Routledge, London (1959)

26. Rommelfanger, H., Hanuschek, R., Wolf, J.: Linear Programming with Fuzzy Objectives. Fuzzy Sets and Systems 29, 31–48 (1989)
27. Roubens, M.: Pattern Classification Problems and Fuzzy Sets. Fuzzy Sets and Systems 1, 239–253 (1975)
28. Ruspini, E.A.: A New Approach to Fuzzy Clustering. Information and Control 15, 22–32 (1969)
29. Shafer, G.A.: A Mathematical Theory of Evidence. Princeton University Press (1976)
30. Smets, P.: Probability of a Fuzzy Event: An Axiomatic Approach. Fuzzy Sets and Systems 7, 153–164 (1982)
31. Smithson, M.: Fuzzy Set Analysis for Behavioral and Social Sciences. Springer, New York (1987)
32. Sugeno, M.: Industrial Applications of Fuzzy Control. Elseveir, Amsterdam (1985)
33. Tanaka, H., Asai, K.: Fuzzy Linear Programming Problems with Fuzzy Numbers. Fuzzy Sets and Systems 13, 1–10 (1984)
34. Thole, U., Zimmermann, H.-J., Zysno, P.: On the Suitability of Minimum and Product Operators for the Intersection of Fuzzy Sets. Fuzzy Sets and Systems 2, 167–180 (1979)
35. Tong, R.M., Bonissone, P.P.: Linguistic Decision Analysis Using Fuzzy Sets. In: Zimmermann, H.-J., Zadeh, L.A., Gaines, B.R. (eds.) Fuzzy Sets and Decision Analysis. Elsevier, Amsterdam (1984)
36. Verdegay, J.-L., Delgado, M. (eds.): The Interface Between Artificial Intelligence and Operations Research in Fuzzy Environments. TÜV Rheinland, Köln (1989)
37. Werners, B.: Interactive Multiple Objective Programming Subject to Flexible Constraints. European Journal of Operational Research 31, 342–349 (1987)
38. Yager, R.R.: On a General Class of Fuzzy Connectives. Fuzzy Sets and Systems 4, 235–242 (1980)
39. Yager, R.R.: Families of OWA Operators. Fuzzy Sets and Systems 59, 125–148 (1993)
40. Zadeh, L.A.: Fuzzy Sets. Information and Control 8, 338–353 (1965)
41. Zadeh, L.A.: Fuzzy Algorithms. Information and Control 19, 94–102 (1969)
42. Zadeh, L.A.: Outline of a New Approach to the Analysis of Complex Systems and Decision Processes. IEEE Transactions on Systems, Man and Cybernetics 3, 28–44 (1973)
43. Zadeh, L.A.: The Role of Fuzzy Logic in the Management of Uncertainty in Expert Systems. Fuzzy Sets and Systems 11, 199–227 (1983)
44. Zimmermann, H.-J.: Fuzzy Programming and Linear Programming with Several Objective Functions. Fuzzy Sets and Systems 1, 45–55 (1978)
45. Zimmermann, H.-J.: Testability and Meaning of Mathematical Models in Social Sciences. Mathematical Modelling 1, 123–139 (1980)
46. Zimmermann, H.-J.: Fuzzy Sets, Decision Making and Expert Systems. Kluwer, Boston (1987)
47. Zimmermann, H.-J., Zysno, P.: Latent Connectives in Human Decision Making. Fuzzy Sets and Systems 4, 37–51 (1980)
48. Zimmermann, H.-J., Zysno, P.: Decisions and Evaluations by Hierarchical Aggregation of information. Fuzzy Sets and Systems 10, 243–266 (1983)
49. Zimmermann, H.-J. (ed.): Practical Applications of Fuzzy Technologies. Kluwer, Boston (1999)
50. Zimmermann, H.-J.: An Application-oriented View of Modeling Uncertainty. European Journal of Operational Research 122(2), 190–199 (2000)
51. Zimmermann, H.-J.: Fuzzy Set Theory and its Applications, 4th edn. Kluwer, Boston (2001)

Part III

Postscriptum

111

How We Got Fuzzy (1976 - 1980)

Didier Dubois and Henri Prade

Abstract. This short note reports on our beginnings in fuzzy set theory and possibility theory, indicating our interests and concerns at that time, also acknowledging the support of many persons who in some way or other have helped us develop our research work.

111.1 Introduction

This paper is unusual in our writings. It does not not contain any scientific results or proposals, nor does it offer a survey of some topic. It is not even just a piece of testimony on the development of research in fuzzy set theory in the late seventies. It rather intends to illustrate how research is a matter of personal taste and interest, but also of good or bad luck, of perseverance through the hazards of life, of persons encountered who were sympathetic, critical or just indifferent to your enterprise. It also makes it clear that research is not an activity whose development can be fully planned and evaluated in advance, although more and more people in charge of its organization would like to make us believe to the contrary, in these days of unreasonable love of money and short-term profit.

We first explain in which circumstances we heard of fuzzy sets for the first time, why we decided to work on this topic, and at which point in time we finally started to better understand what they may be useful for. We highlight the opportunity offered to us of enjoying a one-year post-doctoral position in American universities with a lot of freedom for writing our first book. Finally we show how, back to France, we continued to develop our ideas, while experiencing how badly considered and poorly understood were fuzzy sets in that time, but also encountering various supports and encouragements from key people.

111.2 Encountering Fuzzy Sets

After getting our engineering degrees in aeronautics that we both obtained in 1975 from the Ecole Nationale Supérieure de l'Aéronautique et de l'Espace, a French "Grande Ecole" (usually abbreviated as "Sup'Aéro"), we prepared a Doctor-Engineer thesis for two years (the usual duration at that time for a French PhD thesis) at the Department of Automatic Control in the "Centre d'Etude de Recherche de Toulouse" (CERT-DERA) in France. Our respective thesis topics were the

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optimization of bus transportation networks (DD[1]) and the real time management of scheduling problems (HP). Nine months after starting our thesis research works, in June 1976, at lunch time, a friend[2] mentioned the arrival, at the Sup'Aéro library, of several volumes dealing with a strange thing called "sous-ensemble flou"[3]. He explained us that it had something to do with a generalized set theory with graded membership. Actually, these were the three first volumes of an introductory treatise in French ("for the benefit of engineers", as said in the complete title) by Arnold Kaufmann (1911-1994) [39, 40, 41], a series also including [42, 44], only the first volume of which was translated in English [43].

In fact, we had heard of fuzzy sets for the very first time some weeks before, in a prospective working paper about future lines of research in production engineering by a French professor, Lucas Pun, from Bordeaux. Indeed, in this report, he advocated the relevance of the general idea of using fuzzy sets in this area (see, e.g. [62] – another paper we saw later on). However, the working paper we had in hands gave absolutely no detail about fuzzy sets. So, when our friend reported us about the arrival of Kaufmann's books and a bit about their contents, our curiosity was immediately aroused, because we soon realized that it might be connected to multiple-valued logic, a topic for which one of us (HP) had an older personal interest[4]. This interest was connected to a general concern for logic in general, that both of us shared, since, during the last year of our engineering school, we had attended an optional course on propositional and first-order logics, given for the first time by Hervé Gallaire [36], a professor of computer sciences at Sup'Aéro, and a renowned database specialist. So, in the afternoon of the same day, we borrowed the three volumes from the library, and started to look at their contents. Very rapidly, we got convinced of the close relation between fuzzy sets and min/max-based multiple-valued logic, and were impressed by the large range of potential applications advocated by the author, including tools for linguistics [40] and decision modeling [41]. We got excited by this new idea, and we asked our respective PhD advisors[5] the permission to devote one month of our PhD time to a bibliographical study in order to see if, as suggested by L. Pun, fuzzy sets had any potential for the respective topics of our theses. We got their green light immediately without any problem (as we expected) since they

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[1] In the following we indicate by 'DD' (for 'Didier') and 'HP' (for 'Henri') to whom a particular piece of information refers, when necessary.

[2] Georges Aicardi, also from "Sup'Aéro" and preparing a thesis in another field.

[3] The French translation of fuzzy sets.

[4] This interest for logic as a tool for describing the world had prompted him four years before to read an introductory book in logic [11]. This excellent treatise also presented the Piaget group of transformations of propositional sentences, and non-classical logics were mentioned within half a page. This triggered a desire to understand how a multiple-valued logic works in terms of truth tables, and led to reinvent the min-based conjunction and the max-based disjunction, before discovering two years later that such things were already known for a long time, in another more advanced introductory book [8] including a whole chapter on non-classical calculi.

[5] Jean-François Le Maître, a specialist of urban systems (DD), and Jacques Delmas, a specialist of automatic control and production systems (HP).

were open-minded, and Kaufmann was at the time a highly regarded name [31] as the author of many books introducing new topics in engineering such as matrix calculus and operations research in the two previous decades. He was famous at least in the engineering circles to which our advisors belonged[6].

111.3 First Writings

The result of this first (fuzzy) month of bibliographical search was a (handwritten !) CERT-DERA technical report [19] with an unorthodox title[7]. Following the advice of our supervisors, we were bold enough to send this report to professor Kaufmann himself. To our surprise, he quickly replied in a very encouraging letter (See Fig. 111.1). This report was a synthesis of the main basic notions of fuzzy set theory. It also emphasized the potential interest of fuzzy constraints and fuzzy algorithms in areas such as the ones we were dealing with in our theses. At this stage, we had mainly identified the capability of fuzzy logic for expressing trade-offs between constraints and goals, and more generally its possible use for modeling linguistically described procedures (in that respect the paper by Zadeh [73] where he outlined his "linguistic" rule-based approach made a great impression on us, when we discovered it a bit later). Still, we felt that the impact of fuzzy sets as a tool for solving the problems to be addressed in our theses remained limited. However, we still found the idea attractive and tried to keep up with the publication of new results in the fuzzy set area until the beginning of 1977, when we finally discovered an article by Ramesh Jain [46] advocating the interest of computations with fuzzy numbers based on Zadeh's extension principle [74]. We were immediately convinced that fuzzy numbers were the kind of notion that would be very useful for modeling ill-known task duration times or transportation times in our problems. Yet at that time, no practical computation method with fuzzy intervals had been published, even for particular cases. The pioneering investigations of Mizumoto and Tanaka [47] mainly dealt with algebraic properties of fuzzy arithmetic operations. After some joint research, we were lucky enough to discover a parametric representation of fuzzy numbers (the so-called L-R representation, now quite popular). We could then perform arithmetic operations on fuzzy numbers, as well as extended max and min operations between intervals, by

[6] It might have been quite a different situation, had we prepared our theses directly in the university world: For instance, we later heard that at about the same time some young colleague working at the university lab that we joined later on (and whom we still know), was strongly advised by older colleagues not to pursue the research line on fuzzy sets that she had just started. In fact, we later on received several testimonies of such states of fact in different places: Toulouse, Lyon, Paris, etc... Fuzzy sets were really a controversial topic at that time.

[7] "Le flou, kouackseksa ?, meaning "Fuzzy, what is this?", where "kouackseksa" is an onomatopoeia for the French "quoi que c'est que çà", a young child phonetic approximation of the standard French query "qu'est-ce que c'est que çà".

Corenc-Montfleury, le 30 novembre 1976

A. KAUFMANN
2, allée du Chêne
Corenc-Montfleury
38700 - LA TRONCHE

Messieurs Didier DUBOIS & Henri PRADE
Ingénieur ENSAE
C.E.R.T.- D.E.R.A.
2, avenue Edouard Belin
31055 - TOULOUSE CEDEX

Cher Monsieur DUBOIS et cher Monsieur PRADE,

J'ai bien reçu votre aimable lettre et le rapport que vous avez rédigé sur le Flou pour le CERT-ONERA.

Vous avez fait là un excellent travail de synthèse que je vais parcourir plus en détail mais, ce que j'ai déjà lu mérite ce compliment. Si je trouve quelques points conduisant à des remarques, je vous les transmettrai.

En ce qui concerne les applications du flou à la R.O., il y a de plus en plus d'applications. D'abord, comme vous le soulignez vous-même, en ce qui concerne l'emploi des heuristiques, des problèmes multi-critères et de divers problèmes de programmation dans l'incertain (ce qui est assez général même si cela semble paradoxal); mais aussi dans de nombreux problèmes où le comportement d'un (ou plusieurs) opérateurs humain doit être pris en compte (diagnostic, décisions, apprentissage, créativité). Vous trouverez dans le tome IV de ma série de livres diverses approches sur ces sujets (ce tome IV doit paraître chez MASSON le mois prochain). Dans le tome V qui est presque fini et qui paraîtra en juin 7 7, une suite d'autres applications sont données.

La bibliographie sur le flou contient maintenant plus de 800 titres dont je possède une liste approchant 700. Plus d'une centaine de thèses ont été passées sur des sujets se situant dans ce domaine des mathématiques appliquées (Ph. D., et équivalentes).

Je vous signale que le meilleur spécialiste japonais, le Professeur SUGENO, est actuellement au L.A.A.S. à TOULOUSE; je vous conseille d'aller le voir de ma part.

Bien qu'étant maintenant à la retraite je me déplace quand même beaucoup à l'étranger et en France. Je ne sais pas si j'aurai une opportunité pour aller à TOULOUSE; si cela est possible je viendrai vous voir.

Une question importante pour vous, en ce qui concerne les applications du flou à la R.O., il faudrait que vous preniez contact avec le Docteur en Médecine Roland SAMBUC - Laboratoire de Physique Médicale - Faculté de Médecine de Marseille - 27, boulevard Jean-Moulin.- 13385 - MARSEILLE. Il vous adressera sa thése dans laquelle il a introduit le concept de sous-ensemble Phi-flou (cas particulier des sous-ensembles φ- flous, particulièrement bien adapté pour mieux cerné l'imprécis dans des marges). Prenez aussi contact avec le Professeur Claude PONSARD - Directeur de l'Institut de Mathématiques Economiques - Faculté de Science Economique et de Gestion - 4, boulevard Gabriel- 21000 - DIJON.- Le groupe du Professeur PONSARD travaille entièrement avec les nouveaux concepts flous sur des problèmes économiques.

Je vous écrirai plus tard, avec des commentaires sur votre travail quand je l'aurai lu en détail. Je vous renouvelle mes compliments.

Très cordialement à vous.

Fig. 111.1. 1st letter by A. Kaufmann, after he received "Le flou, kouackseksa ?" in 1976. Note his generosity, the broadness of his view, and his care to put people in relation: the letter encouraged us to contact M. Sugeno, R. Sambuc, and C. Ponsard.

simple computations on the parameters. These results first appeared in [20][8]; they were later published in [24, 23] and soon applied to shortest path problems [25]. In the meantime, we had become aware of the work of Nahmias [48] and realized that his findings on the addition of fuzzy intervals were particular cases of ours, restricted to triangular membership functions. We had also realized that fuzzy arithmetics generalized interval arithmetics through the use of cuts. But it was only later on (in 1978, before writing our first book) that we read Nguyen's fundamental paper on the extension principle [51], where its cut-worthiness was studied in depth.

We were also lucky enough to meet several key people that in some way or other influenced our future works in Toulouse before the 1977 summer vacations. First, in december 1976, we heard that a fuzzy set researcher, Michio Sugeno, was a visiting scholar in another neighboring laboratory on the same campus, the L.A.A.S.,[9] for several months (it was pointed out in Kaufmann's letter on Fig. 111.1). He was there thanks to the support of Georges Giralt, the future father of robotics research in France. After a recent sabbatical in Berkeley, Giralt had become a sympathizer of fuzzy sets. Thus, we had the privilege to discuss very early with Michio Sugeno, who gave us a copy of his landmark PhD dissertation [69]. It was also the opportunity to meet a young CNRS researcher from the same laboratory, Gérald Banon, interested in fuzzy measures and Shafer's belief functions [68], whose work [5] would be the departure point of our chapter on this topic in our 1980 book [28]. A bit later we also had the chance to meet Elie Sanchez, back from Berkeley, who also gave us a copy of his remarkable PhD thesis on fuzzy relation equations and their applications to medicine. He was the first scholar to reveal the existence of possibility theory [75] to us[10]. This was a brand new topic at that time, to which he had just contributed [64]. These lucky encounters clearly contributed to enrich our view of the field and led to new developments [21], while we were completing our PhD dissertations [13, 14, 55, 57] that we finally defended in October 1977. We had successfully applied for post-doctoral fellowships so as to pursue our works in the US. Just before our departure, Kaufmann strongly suggested us to take this opportunity and write a book on fuzzy sets. It was an unexpected advice given by a very unusual, generous and experienced man to 25 year old researchers! In fact, we decided to take this advice seriously.

[8] The title of this report "Le flou, mécédonksa !, meaning "Fuzzy ? this is it!", where "mécédonksa" is an onomatopoeia for the French "mais c'est donc çà". It was echoing, in the same style as in the title of our first opus, our feeling to have finally identified a reason for advocating the usefulness of fuzzy sets.

[9] L.A.A.S. stands for "Laboratoire d'Automatique et d'Analyse des Systèmes". It was already at that time a very important French CNRS laboratory.

[10] Thanks to Shafer's book [68], we became aware almost at the same time that an English economist, George Lennox Sharman Shackle (1903-1992) [66] had already felt the need for a similar calculus [18], but on the basis of quite different motivations. This is a good example of the fact that the emergence of new theories may be the result of multiple attempts. A bit later HP had the chance, at a PhD committee, to meet Shackle, a delightful old-fashioned English professor, who was glad to discover that his ideas were starting to have a revival [67], to which we later contributed when providing a decision-theoretic axiomatization of possibility theory.

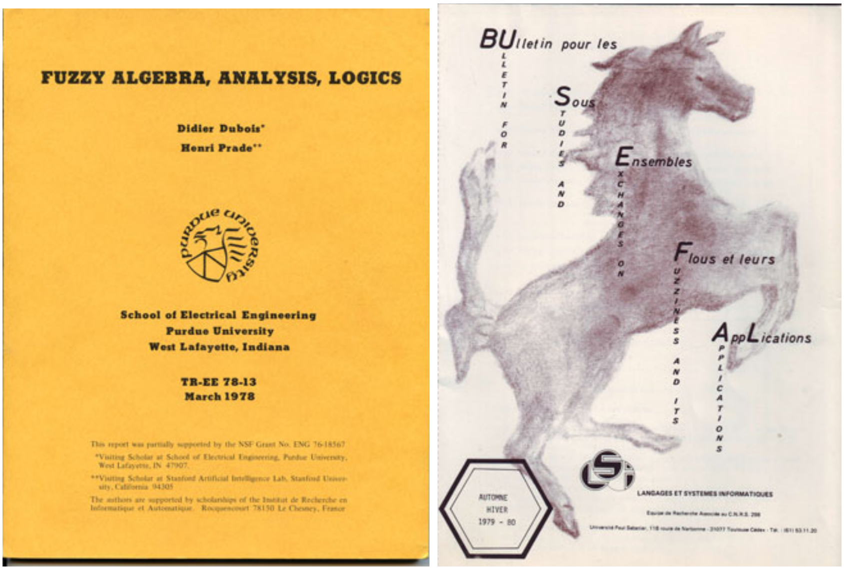
111.4 Discovering North-American Research

In November 1977, we left for Purdue University (DD) and Stanford University (HP) respectively, supported by one-year IRIA[11] scholarships, with one idea in mind: to write that book. The choice of these universities was differently motivated. On the one hand, Prof. King-Sun Fu (1930-1985), a leading figure in pattern recognition in that time, had already done some remarkable work on fuzzy automata, but also on the axiomatics of fuzzy set connectives in relation to decision analysis [35]. DD sent a letter to him expressing his high interest for Fu's paper on connectives, and the latter was kind enough to welcome the visit in his group of a young researcher interested in fuzzy sets. On the other hand, the Stanford AI Lab. was one of the very few leading research places in artificial intelligence in those days. Thanks to the support of Georges Giralt, HP was accepted in the group of Tom Binford in order to learn AI and robotics, and more particularly, planning. At that time, nobody was interested in fuzzy sets[12] at the Stanford AI Lab. On the other hand, Stanford was only one hour by car from Berkeley University and the Electronics Research Laboratory at Evans Hall, where Prof. Zadeh's seminar was taking place.

American university libraries were a paradise for two young French researchers willing to write a research monograph: they were generally open all day long (even late in the evening), the whole week, and they allowed you to have a direct access to books and journals. Moreover they contained almost everything you may need. It was for us an enormous difference with the French system, even if we were very privileged at the time of our thesis since our laboratory had access to the French Army library "CEDOCAR" (Centre de Documentation de l'Armement) where it was at least possible to order copies of articles. In order to work together on our project, we decided to spend one month in Albuquerque, New Mexico around Christmas vacations, since it was sort of mid-way between LaFayette, Indiana (where Purdue University is) and San Franciso: it took each of us about 36 hours by bus to reach the place! Apart from visiting Santa Fé, we spent days of intensive work, trying to build an organized view of our readings, and to develop our own ideas: we wrote there the first versions of 5 papers which later were published in journals, and a long analysis of Zadeh's paper on the PRUF representation language. It resulted in a thick Purdue University technical report [22] (see Fig. 111.2.a). Later, in April we met again for several weeks in Menlo Park (near Stanford) for preparing the tentative table of contents of the future book, that we then presented to Prof. Zadeh. As he wrote it later in the foreword to the book, he "was rather skeptical" on the possibility of

[11] IRIA, now INRIA (Institut National de Recherche en Informatique et Automatique), is a French organization for research in applied mathematics, computer sciences and automatic control, which in that time was offering some scholarships every year for post-doctoral staying in foreign research laboratories.

[12] It was not just indifference, since HP was then encouraged to write a note [56] in order to make it clear that robotics and fuzzy set had nothing to do with each other. This rare piece should have appeared in an annual report, but, fortunately was finally never published. However, due to his broadmindedness, Tom Binford left the freedom of their research lines to members of his group.



(a) (b)

Fig. 111.2. (a): Purdue Univ. TR-EE 78-13 [22] (b): 1st issue of BUSEFAL, Jan. 1980

"writing an up-to-date research monograph on fuzzy sets and systems". Nevertheless, we decided to go on, and we spent almost three months (from the end of June to mid-September) writing the book, during a hot summer in Purdue. It was handwritten, since at that time, text processing tools were in their infancy, and not in current use. Prof. King-Sun Fu encouraged us continuously during this period, even reading the manuscript and making some suggestions. Once the writing was over, we had the text type-written by a professional typist (at our own expense). Prof. Fu was then instrumental for recommending our work and having it accepted in the prestigious "Mathematics in Science and Engineering" series edited by Richard Bellman (1920-1984) at Academic Press. After receiving the galley proofs and making a substantial update during the fall of 1979 (we were then back in France), the book was finally published the next year [28] and proposed at an affordable price for interested researchers, while paying significant royalties (this situation fully contrasts with the one that became usual about 20 years later, when books became much more expensive, while publishers were just printing ready-to-process files in low cost countries, while significantly reducing royalties).

Our year in the US was clearly a very rich experience for each of us, not only because of the success of the project and the publication of the book, but also because of the new style of life and research we experienced, and all the people, colleagues, friends we encountered. It is clearly not possible to mention them here. Let us just report that one of us (DD), just before leaving back to France, attended his first conference in Philadelphia, where he presented results from his thesis work [14], and had thus the opportunity to meet Ronald Yager [71] for the first time. Ron presented a family of new connectives for fuzzy sets (his now widely acknowledged subclass of t-norms and co-t-norms), a work quite unorthodox at that time where operators other than min and max were not really considered.

111.5 Back to France

When we came back to France in the last term of 1978, our professional situation was not the same. HP had just got a CNRS “attaché de recherche" position in an artificial intelligence research group [10] in a Toulouse university laboratory, later to become part of our present IRIT laboratory at the very beginning of the nineties. DD still had to find a position; during one year he worked first in Paris then at Grenoble IMAG laboratory as an engineer (where he could often visit Arnold Kaufmann, now retired, but still active), and finally got a permanent research engineer position at CERT-DERA laboratory in Toulouse in March 1980, in the very lab where he had worked on his Ph. D. thesis. Finally, we became both CNRS “chargés de recherche" in the same group at Toulouse university in 1984.

At that time, there were not so many people in France interested in fuzzy sets, apart from Kaufmann. The main others were Elie Sanchez in Marseille working in computer-assisted medical diagnosis (as well as Roland Sambuc [63], the first to propose the use of interval-valued fuzzy sets), Claude Ponsard (1928-1990), a professor of economics in Dijon [54], Robert Féron [32, 33, 4] in econometrics, Daniel Ponasse [53] (with Nicole Blanchard [6] who died early, Achille Achache, and Josette and Jean-Louis Coulon [12]) in pure mathematics in Lyon, Noël Malvache (1943-2007) and Didier Willaeys [70] in automatic control in Valenciennes, and Bernadette Bouchon [9] a young CNRS researcher in Paris, working in Claude-François Picard group. Picard was the father of questionnaire theory [52], one of the very rare influent persons in the academic world to be interested in fuzzy sets; unfortunately he died very early from a heart attack by the end of 1979. We should also mention the early work of Jean-Pierre Aubin [3] introducing the idea of fuzzy coalition in game theory. As can be seen, the interest for fuzzy sets had quite different motivations. Besides, fuzzy sets at that time remained controversial in most academic circles, even if it was becoming possible to publicize them in large audience journals or newspapers, e.g. [59].

In order to foster international communication between researchers in fuzzy sets, who, at that time, were topically and geographically scattered (remember Europe was cut in two blocks, and Internet was still in infancy, operating in a few American universities only), we had the idea by the end of 1979 to launch a quarterly bulletin BUSEFAL (a double acronym in English and French as can be read on the cover (Fig. 111.2.b). Each issue of this international bulletin reached about 100 pages from the beginning, and later went beyond 300 pages, publishing short contributions on new research trends, as well as many news on recently published papers. It published announcements and programs of scientific manifestations. It has been edited and published in our laboratory in Toulouse for 19 years since 1980 (issues 1 to 76), until the research assistant of our group, Yves Luvisutto, who took care of the assembling, printing and mailing, retired (and was not replaced)[13]. BUSEFAL played an important role for scientific communication between the West and the East, and China as well; many now renowned scholars in Fuzzy Sets from Eastern European countries (Krassimir Atanassov, Slavka Bodjanova, Arkady N. Borisov, Ernest Czogała (1941-1998), Józef Drewniak, Janos Fodor, Robert Fuller, Siegfried Gottwald, Janusz Kacprzyk, Leonid Kitainik, Lazlo Koczy, V. B. Kuz'min, Jiri Mockor, Wolfgang Näther, Constantin Negoita, Vilém Novák, Maria Nowakowska (1928-1989), Walenty Ostasiewicz, Witold Pedrycz, Radko Mesiar, Jaroslav Ramik, Beloslav Riecan, H.-N. Teodorescu, Maciej Wygralak to name a few), and from China (Cao Zhi-Qiang, Li Hongxing, Liu Yingming, Wang Peizhuang, Wang Zhenyuan, Zhang Jinwen (1930-1993)), published short notes in BUSEFAL in the eighties and nineties.

Fig. 111.3. Abraham Kandel, Henri Prade, Masao Mukaidono, in Evanston, IL, 3-5 June, 1980, at the 10th IEEE International Symposium on Multiple-Valued Logic

In other respects, the years 1979-1980 for us were rich in events of different kinds which durably influenced our future work. First, 1979 is the year of the "arrival" of triangular norms and co-norms in the fuzzy set world. It happened almost simultaneously in two different places. On June 28, 1979, in Duke University at Durham, one of us (HP) was presenting our joint work [26] on different fuzzy set theoretic

[13] The bulletin continued until issue 92, at LISTIC laboratory in Annecy, where the contents of issues 15 to 92 are available on line `http://www.listic.univ-savoie.org/modules.php?name=Busefal`, thanks to the efforts of Laurent Foulloy and the help of Patrick Bosc.

operators, when Ulrich Höhle came to him after the talk and told him "Do you know that the operators you just presented are triangular norms and have been studied for a long time"? It was the first encounter with Ulrich who was also using the binary operation $\max(0, a+b-1)$ in his presentation, but in the setting of much more elaborated mathematics [37]. The conference in Durham was also the opportunity to meet Peter Klement [45] for the first time. Thanks to Ulrich, we rapidly learnt about the solutions to the functional equation of associativity and the work of Berthold Schweizer (1929-2010) and Abe Sklar on triangular norms [65] after Karl Menger (1902-1985), and even one of us (HP) had the chance to receive a full collection of reprints on the topic from the hands of Abe Sklar, taking advantage of a conference at Northwestern University in June 1980 (see Fig. 111.3). We rapidly realized the interest of triangular for fuzzy set theory both as fuzzy set connectives (see the final version of [26] and [28, 15, 17, 60]), but also for defining decomposable fuzzy measures [61, 29]. But, triangular norms and co-norms were independently known in another "fuzzy circle". Indeed, Claudi Alsina and Enric Trillas had been for several years studying probabilistic metric spaces [1] and functional equations, before starting to work on fuzzy sets in the late seventies [2].

1979 was also the year of the first International Seminar on Theory of Fuzzy Sets in Linz (Austria) organized by Peter Klement at J. Kepler Universität, in Linz (Austria). We attended the seminar from the beginning: in 1979, one of us (HP) presented the nomenclature of fuzzy measures [58] that was going to appear in our book [28], while the second year (see Fig. 111.4) the other (DD) emphasized the interest of triangular norms for fuzzy sets [17]. This yearly seminar, that is still going on to-day, was bound to play a major role in the development of fuzzy set mathematics, and we were again lucky enough to be among the few (less than 10) early participants, that included Ron Yager and the pioneer of fuzzy topology Robert Lowen. After attending the 1st Linz Seminar, HP continued from Linz towards Bucharest and visited Constantin Negoita[14] [49], whose book written with Dan Ralescu [50] we regarded highly. In 1979, DD presented the first works in interactive and constrained fuzzy arithmetics (t-norm-based additions, and fuzzy expectations [30]) at the IEEE conf. on Decision and Control (Fig. 111.5).

In 1980, in Lyon, Robert Féron [15] (the inventor of fuzzy random variables, also a follower of Maurice Fréchet (1878-1973)) took the initiative to organize a CNRS Round Table: "Quelques applications concrètes utilisant les derniers perfectionnements de la théorie du flou" ("Some concrete applications using the most recent

[14] Quite naively, especially if we consider that Rumania was under the law of a communist regime, the travel to Bucharest was rather unprepared, and the visit was done without preliminary announcement. It had funny aspects: when arriving at Negoita's address as given in Fuzzy Sets and Systems, i.e. Str. Traian 204, HP discovered an orthodox church. It turns out that Negoita's father was a pope! Fortunately, his mother was outside hanging out washed clothes, and she called his son who arrived half an hour later fully amazed to meet an absolutely unexpected visitor. In spite of it, an impromptu scientific visit of his laboratory was organized.

[15] He also came to the Acapulco Inter. Cong. on Applied Systems Research & Cybernetics; see Fig. 111.6

Fig. 111.4. H. W. Martin, Didier Dubois, Robert Lowen, Ronald R. Yager, Ulrich Höhle, Erich Peter Klement. Photo by W. Schwyla. 2nd International Seminar on Fuzzy Set Theory, Linz, Sept. 1980.

advances in fuzzy theory") on June 23-25. Interestingly enough, the organizing committee (in Lyon, on January 25, 1980, to which one of us (HP) took part thanks to Negoita's support), included highly reputed mathematicians, such as Joseph Kampé de Feriet (1893-1982), Robert Fortet (1912-1998) [34], and Gustave Choquet (1915-2006) (at a time where Choquet integral was not yet considered by fuzzy set researchers!). However, only Kampé de Feriet, who was the first to point out the interpretation of a fuzzy set membership function as the contour function in a Shafer belief structure [38], came and participated to the meeting in June.

We were fortunate enough to take part in this meeting with two presentations each, including preliminary versions of our works on links between probability and

Fig. 111.5. Masaharu Mizumoto, Elie Sanchez, Didier Dubois, Ronald R. Yager, J. Baldwin, Lotfi A. Zadeh. *18th IEEE Conference on Decision & Control, Fort Lauderdale*, Dec. 12-14, 1979.

Fig. 111.6. G. Jumarie, Henri Prade, Masao Mukaidono, Ronald R. Yager, Robert Féron, Lotfi A. Zadeh, Erich P. Klement, Dan Ralescu, W. H. Benson, in Acapulco, Dec. 12-15, 1980

TABLE RONDE : QUELQUES APPLICATIONS CONCRETES
UTILISANT LES DERNIERS PERFECTIONNEMENTS DE LA
THEORIE DU FLOU

A - ORGANISATION GENERALE :

Le Comité d'organisation de la table ronde sur le flou s'est réuni le vendredi 25 janvier et a décidé ce qui suit.

1°) La table ronde sur le flou aura lieu à l'Université de LYON I, Mathématiques, du lundi 23 juin 1980 à 9 heures au mardi 24 juin à 19 heures, et sera composée de 4 demi-journées consacrées respectivement :

- au calcul des possibilités
- à la classification automatique
- aux applications du flou à la programmation et son adéquation aux sciences économiques et humaines
- a l'axiomatisation mathématique des méthodes employées dans la théorie du flou.

B - PROGRAMME :

1°) Programme de la table ronde : (Département de Mathématiques) Université LYON 1, 43 Bd. du 11 Novembre 1918 Villeurbanne :

Thème 1 : Lundi 23 juin - 9H 12 H
Titre : Calcul des possibilités
Exposé introductif L. A. ZADEH (durée 1 H)
Intervenants : H.T. NGUYEN, H. PRADE, D. DUBOIS (durée 30 minutes)
débat : (durée 1H 30)
Présidence de la session J. KAMPE DE FERIET

Thème 2 : lundi 23 juin - 15H 18 H
Titre : Classification automatique et aide à la décision
Exposé introductif : M. RUSPINI
Intervenants : E. SANCHEZ, S. OPPENHEIM ET B. DUBUISSON, Mlle B. BOUCHON (durée 30 minutes)
Débat : (durée 1H 30)
Présidence de la session : A. DUSSAUCHOY.

Thème 3 : mardi 24 juin - 9H 12H
Titre : Applications du flou à la programmation linéaire, l'analyse des systèmes et à l'économie.
Exposé introductif : HJ. ZIMMERMANN (1 heure)
Intervenants : C. NEGOITA, D. WILLAEYS ET M. MALVACHE, C. CARLSSON (30 minutes au total)
Débat : (durée 1H 30)
Présidence de la session D. RALESCU.

Thème 4 : mardi 24 juin - 15H 18H
Titre : fondements d'une mathématique floue
Exposé introductif D. RALESCU (1Heure)
Intervenants : P. KLEMENT, M. PREVOT, S. GOTTWALD (durée 30 minutes)
Débat : (durée 1H 30)

Fig. 111.7. Preliminary program of the first 2 days of the meeting organized by R. Féron in Lyon on June 23-25, 1980. Excerpt of an announcement BUSEFAL n° 2, April 1980.

possibility [16] and triangular-based decomposable measures[16] [61] (we discovered only later that Kampé de Feriet's theory of information measures used the same structure). Participants to the Lyon meeting included many other scholars who were going to be involved in fuzzy set research in a way or another in the following years. Here is an incomplete list: G. Banon, E. Backer, J. Baldwin, B. Bouchon, N. Blanchard, C. Carlsson, A. Di Nola, B. Dubuisson, C. Dujet, H. Emptoz, M. Gupta, S. Gottwald, E. Hisdal, K. Hirota, U. Höhle, L. Itturioz, A. Kaufmann, A. Kandel, E. P. Klement, R. Lopez de Mantaras, R. Lowen, N. Malvache, C. Negoita, H. Nguyen, S. Oppenchaim, C. Ponsard, D. Ponasse, M. Prévot, D. Ralescu, E. Ruspini, E. Sanchez, P. Smets, R. Vallée, A. Ventre, D. Willaeys, R. Yager, L. Zadeh, H. Zimmermann. See Fig. 111.7 for the program of the two first days. It is also at this event that we had the chance to meet Philippe Smets (1938-2005) [7] for the first time, who became our friend and with whom we were going to share many happy days in joint European projects and works.

Fig. 111.8. Ronald Yager in Acapulco, Dec. 12-15, 1980, at the *International Congress on Applied Systems Research & Cybernetics*; photo H. Prade

[16] A funny experience, a bit later the same year, was to present this idea in a seminar in Berkeley in front of Dennis Lindley, a very gentle man, and a leading advocate of Bayesian statistics who was visiting Zadeh at that time and to see how puzzled he was by the claim that probabilities were (also) characterized by the postulate $\forall A, B$ s.t. $A \cup B = X, g(A \cap B) = \max(0, g(A) + g(B) - 1)$. This small story is just to illustrate how *any of us* may be confined in mental habits and have difficulties to grasp a new view, even for an already known object.

Many more scattered facts or events that contributed to our formation in these years are omitted here (for instance, the Inter. Cong. on Applied Systems Research & Cybernetics, where Ron Yager organised an important session track on fuzzy sets and possibility theory, see Fig. 111.8). This was the beginning of several years of efforts for having fuzzy set theory and possibility theory more largely accepted. In that respect, the first important misunderstandings we had to face were about their relations with probability theory (and a decade later with formal logic). Thanks to supports and circumstances, we were lucky enough to approach two renowned researchers in probability theory, Michel Métivier (1931-1988) and then Alain Bensoussan, to show them the potentials of fuzzy sets and possibility theory. They were part of our Doctorat d'Etat or Habilitation committees a few years later.

Fig. 111.9. Didier Dubois and Henri Prade, in Marseille, July. 19-21, 1983, *IFAC Symposium on Fuzzy Information, Knowledge Representation and Decision Analysis*; photo by L. A. Zadeh

111.6 To Conclude

In these concluding remarks, we would like first to recall mottos that we often heard from by L. A. Zadeh (but also A. Kaufmann) as pieces of advice in those years: "Be thick-skinned", "Whatever is said to you, take it as a compliment". The latter guiding rule is to be understood as an injunction not to give up in face of criticisms, especially partisan ones, when your own ideas and intuitions are the result of serious thinking. However, this should go together with a form of humility, since we should always remember that often comments or remarks made by others may bring us references or ideas that we have ignored until now. In that sense, research is a collective venture. Practicing it regularly in a joint manner, as we have done for more than three decades, is certainly a good way of coping with criticisms, and more importantly to cross-fertilize ideas. It is also important to keep in mind that what may appear later simple, straightforward, or even obvious has not always been so, that apparently easy steps may take time as soon as they are devoted to new directions, and that the path towards new conceptual and methodological advances is a long chaotic route with difficulties, but also rich in joys and encounters. This specificity of research makes it distinct from teaching and engineering tasks, which are respectively aiming at organizing and transmitting what is already known and at looking for practical solutions immediately applicable in particular areas. This is poorly understood by state agencies that highly privilege application-oriented research those days, forgetting that ideas and tools that are really new are only discovered thanks to a mixture of dedicated work and chance, which takes time.

References

1. Alsina, C., Trillas, E.: Sobre las t-normas cont'nuas por la izquierda. Actas XII R. A. M. E., Málaga (1976, Abstract), pp. 223–225 (1983)
2. Alsina, C., Trillas, E., Valverde, L.: On Non-distributive Logical Connectives for Fuzzy Sets Theory. In: BUSEFAL, n°3, L.S.I., Univ. Paul Sabatier, Toulouse, pp. 18–29 (1980)
3. Aubin, J.-P.: Fuzzy Core and Equilibrium of Games Defined in Strategic Form. In: Ho, Y.-C., Mitter, S.K. (eds.) Directions in Large-Scale Systems, pp. 371–388. Plenum, New York (1976)
4. Auray, J.-P., Prade, H., Féron, R.: A Pioneer in Soft Methods for Probability and Statistics. In: Dubois, D., Lubiano, M.A., Prade, H., Gil, M.Á., Grzegorzewski, P., Hryniewicz, O. (eds.) Proceedings of the 4th International Conference on Soft Methods in Probability and Statistics (SMPS 2008), Toulouse, September 8-10, pp. 27–32. Springer (2008)
5. Banon, G.J.F.: Distinction entre plusieurs sous-ensembles de mesures floues. Note interne n°78.1.11 L.A.A.S.-A.S., Toulouse, France. Also in: Proceedings Colloque International sur la Théorie et les Applications des Sous-Ensembles Flous, Marseille, pp. 20–24 (September 1978)
6. Blanchard, N.: Injections, surjections, bijections floues, cardinaux flous. In: Busefal, n° 2, L.S.I., Univ. Paul Sabatier, Toulouse, pp. 8–18 (1980)
7. Bersini, H., Denoeux, T., Dubois, D., Prade, H.: In Memoriam Philippe Smets (1938-2005). Fuzzy Sets and Systems 157, 3–7 (2006)
8. Blanché, R.: Introduction à la Logique Contemporaine. Collection U2, Armand Colin (1968)
9. Bouchon, B.: Fuzzy Questionnaires. Fuzzy Sets and Systems 6, 1–10 (1981)
10. Cayrol, M., Farreny, H., Prade, H.: Possibility and Necessity in a Pattern-matching Process. In: Proceedings of the IXth International Congress on Cybernetics, Namur, Belgium, September 8-13, pp. 53–65 (1980)
11. Chauvineau, J.: La Logique Moderne. Que Sais-Je? n° 745. Presses Universitaires de France, Paris (1969)

12. Coulon, J., Coulon, J-L.: Engendrement des sous-espaces vectoriels flous et fermetures de Moore floues. In: Busefal, n° 3, L.S.I., Université Paul Sabatier, Toulouse, pp. 10–17 (1980)
13. Dubois, D.: Quelques outils méthodologiques pour la synthèse de réseaux de transport. Thèse de Docteur-Ingénieur, ENSAE Toulouse (1977)
14. Dubois, D.: An Application of Fuzzy Sets Theory to Bus Transportation Network Modification. In: Proceedings of the Joint Automatic Control Conference "Control Theory Meets the Real World of Application", Philadelphia, October 15-20, vol. III, pp. 53–60
15. Dubois, D.: Quelques classes d'opérateurs remarquables pour combiner des ensembles flous. In: BUSEFAL, n° 1, L.S.I., Université Paul Sabatier, Toulouse, pp. 29–35 (1980)
16. Dubois, D.: Sur les liens entre les notions de probabilité et de possibilité. Table ronde C.N.R.S., "Quelques applications concrêtes utilisant les derniers perfectionnements de la théorie du flou", Dept. of Mathématiques, Université de Lyon-1, Lyon, June 23-24 (1980)
17. Dubois, D.: Triangular Norms for Fuzzy Sets. In: Proceedings of the 2nd International Seminar on Fuzzy Set Theory, Johannes Kepler Universität, Linz, Austria, September 15-20, pp. 39–68 (1980)
18. Dubois, D.: Un précurseur de la théorie des possibilités: G. L. S. Shackle. In: BUSEFAL, n° 2, L.S.I., Université Paul Sabatier, Toulouse, pp. 70–73 (1980)
19. Dubois, D., Prade, H.: Le flou, kouacksexa? (170 p.), Note interne D.E.R.A., Toulouse (October 1976)
20. Dubois, D., Prade, H.: Le flou, mécédonksa! (243 p.), Note interne D.E.R.A., Toulouse (April 1977)
21. Dubois, D., Prade, H.: Flou et catastrophes dans les systèmes (Le flou, cétenkorsa!) (224 p.), Rapport Scientifique – DERA, Toulouse (October 1977)
22. Dubois, D., Prade, H.: Fuzzy Algebra, Analysis, Logics. Technical Reports, Purdue University, n°TR-EE 78-13 (March 1978). Contents: A. Fuzzy Real Algebra: Some Results (37 p.); B. Systems of Linear Fuzzy Constraints (21 p.); C. Towards Fuzzy Analysis: Integration and Differentiation of Fuzzy Functions (51 p.); D. Operations in a Fuzzy-valued Logic (21 p.); E. A Summary Comment on 'Theory of Approximate Reasoning' & 'PRUF-A Meaning Representation Language for Natural Languages' by Zadeh, L.A. (20 p.); F. An Alternative Fuzzy Logic (12 p.)
23. Dubois, D., Prade, H.: Comment on 'Tolerance Analysis Using Fuzzy Sets' and 'A Procedure for Multiple Aspect Decision-making'. International Journal of Systems Science 9(3), 357–360 (1978)
24. Dubois, D., Prade, H.: Operations on Fuzzy Numbers. International Journal of Systems Science 9(6), 613–626 (1978)
25. Dubois, D., Prade, H.: Algorithmes de plus courts chemins pour traiter des donnŽes floues. R.A.I.R.O. – Operations Research 12(2), 213–227 (1978)
26. Dubois, D., Prade, H.: New Results about Properties and Semantics of Fuzzy-set-theoretic Operators. In: Proceedings of the 1st Internatuional Symposium on Policy Analysis & Information Systems, Durham, N.C., June 28-30, pp. 167–174 (1979); Revised version in: Wang, P.P., Chang, S.-K. (eds.): Fuzzy Sets: Theory and Applications to Policy Analysis and Information Systems, pp. 59–75. Plenum Publ. (1980)
27. Dubois, D., Prade, H.: Various Kinds of Interactive Addition of Fuzzy Numbers. Application to Decision Analysis in Presence of Linguistic Probabilities. In: Proceedings of the 18th IEEE Conference on Decision & Control, Fort Lauderdale, FL, pp. 783–787 (December 1979)
28. Dubois, D., Prade, H.: Fuzzy Sets and Systems: Theory and Applications. Mathematics in Science and Engineering Series, vol. 144. Academic Press, New York (1980)

29. Dubois, D., Prade, H.: A Class of Fuzzy Measures Based on Triangular Norms. A General Framework for the Combination of Uncertain Information. International Journal of General Systems 8(1), 43–61 (1982)
30. Dubois, D., Prade, H.: Unfair Coins and Necessity Measures: A Possibilistic Interpretation of Histograms. Fuzzy Sets and Systems 10(1), 15–20 (1983); Preliminary version in: Klement, E.P. (ed.): Proceedings of the 3rd International Seminar on Fuzzy Set Theory, Johannes Kepler Universität, Linz, Austria, September 7-11, pp. 223–231 (1981)
31. Dubois, D., Prade, H., Sanchez, E.: Professor Arnold Kaufmann (18 August, 1911–15 June, 1994). Fuzzy Sets and Systems 84(2), 121–123 (1996)
32. Féron, R.: Ensembles aléatoires flous. Comptes Rendus de l'Académie des Science de París, Série A 282, 903–906 (1976)
33. Féron, R.: Economie d'échange aléatoire floue. Comptes Rendus de l'Académie des Science, Série A 282, 1379–1382 (1976)
34. Fortet, R., Kambouzia, M.: Ensembles aléatoires et ensembles flous. Publications Econométriques 9(1) (1976)
35. Fung, L.-W., Fu, K.S.: An Axiomatic Approach to Rational Decision-making in a Fuzzy Environment. In: Zadeh, L.A., Fu, K.S., Tanaka, K., Shimura, M. (eds.) Fuzzy Sets and Their Application to Cognitive and Decision Processes, pp. 227–256. Academic Press, New York (1975)
36. Gallaire, H.: Eléments d'informatique théorique – Théorie des automates. Théorie des langages. Calculabilité. Logique. Cours Sup'aéro (1974)
37. Höhle, U.: Minkowski Functionals of L-fuzzy Sets. In: Proceedings of the 1st International Symposium on Policy Analysis & Information Systems, Durham, N.C., June 28-30, pp. 178–186 (1979)
38. Kampé de Fériet, J.: Une interprŽtation des mesures de plausibilitŽ et de crŽdibilitŽ au sens de G. Shafer et de la fonction d'appartenance dŽfinissant un ensemble flou de L. Zadeh. Publ. I.R.M.A. de Lille I 2(6), II.01–II.20 (1980). Translated into "Interpretation of Membership Functions of Fuzzy Sets in Terms of Plausibility and Belief". In: Gupta, M.M., Sanchez, E. (eds.) Fuzzy Information and Decision Processes, pp. 93–98. North-Holland, Amsterdam (1982)
39. Kaufmann, A.: Introduction à la Théorie des Sous-Ensembles Flous à l'usage des ingénieurs. 1 Eléments Théoriques de Base, Masson et Cie., Paris (1973)
40. Kaufmann, A.: Introduction à la Théorie des Sous-Ensembles Flous à l'usage des ingénieurs. 2. Applications à la Linguistique et à la Sémantique, Masson et Cie., Paris (1975)
41. Kaufmann, A.: Introduction à la Théorie des Sous-Ensembles Flous à l'usage des ingénieurs. 3. Applications à la Classification, et la Reconnaissance des Formes, aux Automates et aux Systèmes, aux Choix des Critères, Masson et Cie., Paris (1975)
42. Kaufmann, A.: Introduction à la Théorie des Sous-Ensembles Flous à l'usage des ingénieurs. 4. Compléments et Nouvelles Applications. Masson et Cie., Paris (1976)
43. Kaufmann, A.: Introduction to the Theory of Fuzzy Subsets. Fundamental Theoretical Elements, vol. 1. Academic Press, New York (1975)
44. Kaufmann, A., Dubois, T., Cools, M.: Exercices avec solutions sur la théorie des sous-ensembles flous, Masson (1975)
45. Klement, E.P.: Characterization of Fuzzy Measures by Classical Measures. In: Proceedings of the 1st International Symposium on Policy Analysis & Information Systems, Durham, N.C, June 28-30, pp. 187–194 (1979)
46. Jain, R.: Tolerance Analysis Using Fuzzy Sets. International Journal of Systems Science 7, 1393–1401 (1976)

47. Mizumoto, M., Tanaka, K.: The Four Operations of Arithmetic on Fuzzy Numbers. Systems, Computers, Controls 7(5), 73–81 (1976)
48. Nahmias, S.: Fuzzy Variables. Fuzzy Sets and Systems 1, 97–110 (1978)
49. Negoita, C.V.: Fuzzy Sets. New Falcon Publications, Tempe (2001)
50. Negoita, C.V., Ralescu, D.A.: Applications of Fuzzy Sets to Systems Analysis. Interdisciplinary Systems Research Series, vol. 11. Birkhaüser, Halsted Press, Basel, Stuttgart (1975)
51. Nguyen, H.T.: A Note on the Extension Principle for Fuzzy Sets. Journal of Mathematical Analysis and Applications 64, 369–380 (1978)
52. Picard, C.: Théorie des Questionnaires. Gauthier-Villars (1965)
53. Ponasse, D.: Algèbres floues et algèbres de Łukasiewicz. Revue Roumaine de Mathématiques Pures et Appliquées 23(1), 103–113 (1978)
54. Ponsard, C.: Contribution à une théorie des espaces économiques imprécis. Publications Econométriques 8(2), 1–43 (1975)
55. Prade, H.: Ordonnancement et temps réel. Thèse de Docteur-Ingenieur, C.E.R.T./ E.N.S.A.E., Toulouse, vol. 1 (125 p.), vol. 2 (101 p.) (October 1977)
56. Prade, H.: Why Fuzzy Set Theory Does Not Seem Very Useful for Industrial Robotic Systems ... But May Be Relevant for a Lot of Other Applications. Unpublished research note (7 p.), Stanford A.I. Lab. (1978)
57. Prade, H.: Using Fuzzy Set Theory in a Scheduling Problem: A Case Study. Fuzzy Sets and Systems 2(2), 153–165 (1979)
58. Prade, H.: Nomenclature of Fuzzy Measures. In: Proceedings of the 1st International Seminar on Theory of Fuzzy Sets, Johannes Kepler Universität, Linz, Austria, September 24-29, pp. 9–25 (1979)
59. Prade, H.: Le flou devient mathématique. Le Monde-Dimanche, XV (Mai 11, 1980)
60. Prade, H.: Unions et intersections d'ensembles flous. In: BUSEFAL, n° 3, L.S.I., Univ. Paul Sabatier, Toulouse, pp. 58–62 (1980)
61. Prade, H.: Une approche des 'mesures floues' basée sur les normes triangulaires. Table Ronde CNRS sur le flou, Lyon, Juin 23-25 (1980)
62. Pun, L.: Use of Fuzzy Formalism in Problems With Various Degrees of Subjectivity. In: Gupta, M.M., Saridis, G.N., Gaines, B.R. (eds.) Fuzzy Automata and Decision Processes, pp. 357–378. North-Holland (1977)
63. Sambuc, R.: Fonctions ϕ-floues. Application à l'aide au diagnostic en pathologie thyroidienne. Thèse, Université de Marseille
64. Sanchez, E.: On Possibility Qualification in Natural Languages. Electronics Research Laboratory Memorandum M77/28. University of California, Berkeley, 1977. Information Sciences 15, 45–76 (1978)
65. Schweizer, B., Sklar, A.: Probabilistic Metric Spaces. North Holland, Amsterdam (1983)
66. Shackle, G.L.S.: Decision, Order & Time: In Human Affairs. Cambridge University Press, Cambridge (1961) (2nd edn., 1969)
67. Shackle, G.L.S.: Foreword to "Espérance Mathématique de l'Utilité FIoue (by B. Mathieu-Nicot)", Collection de I'I.M.E. No. 29, Dijon, Sirey (1985)
68. Shafer, G.: A Mathematical Theory of Evidence. Princeton University Press (1976)
69. Sugeno, M.: Theory of Fuzzy Integrals and its Applications. Doctor of Engineering dissertation. Tokyo Institute of Technology, Tokyo (1974)
70. Willaeys, D., Malvache, N., Hammad, P.: Utilization of Fuzzy Sets for Systems Modelling and Control. In: Proceedings of the IEEE Conference on Decision and Control, Including the 16th Symposium on Adaptive Processes Fairmont Hotel, New Orleans, LA, December 7-9, pp. 1435–1439 (1977)

71. Yager, R.R.: On a General Class of Fuzzy Connectives. Fuzzy Sets and Systems 4, 235–242 (1980)
72. Zadeh, L.A.: Fuzzy Sets. Information and Control 8, 338–353 (1965)
73. Zadeh, L.A.: Outline of a New Approach to the Analysis of Complex Systems and Decision Processes. IEEE Transactions on Systems, Man and Cybernetics 3, 28–44 (1973)
74. Zadeh, L.A.: The Concept of a Linguistic Variable and its Application to Approximate Reasoning. Information Sciences, Part 1: 8, 199–249, Part 2: 8, 301–357, Part 3: 9, 43–80 (1975)
75. Zadeh, L.A.: Fuzzy Sets as a Basis for a Theory of Possibility. Fuzzy Sets and Systems 1, 3–28 (1978)

112

Living in an Uncertain Universe

Brian R. Gaines

112.1 Introduction

It has been interesting to revisit the era when my path crossed with those of John Andreae, Joe Goguen, George Klir, Ladislav Kohout, Abe Mamdani, Gordon Pask, Ted Poppelbaum and Lotfi Zadeh, all precious friends and colleagues, some of whom are, unfortunately, no longer with us; their ideas live on and flourish as a continuing inspiration. I was tempted to entitle this article after Gurdieff, *Meetings with Remarkable People*, and that will be its focus, particularly the background to the initial development of fuzzy controllers. There have been studies analyzing their history and the basis of their success—this article provides some details from my personal experience.

112.2 My Background

My path began in my schooldays when I was an electronics, mathematics and philosophy hobbiest, having been entranced by Wiener's *I am a Mathematician*, Hardy and Littlewood's *Pure Mathematics*, Bertrand Russell's *Principles of Mathematics*, Kant's *Critique of Pure Reason* and Hegel's *Science of Logic*. Logic and mathematics were ways of modelling and understanding the world, and electronics a way of influencing and controlling it. I saw everything from cosmology and society to the world of ideas as a cybernetic system with hierarchies of feedback loops evolving through temporary equilibria punctuated by periods of chaos; I still do.

The mathematics I valued was algebra and logic. I saw arithmetic as an over-specified algebraic structure, notions of continua and infinity as useful fictions for modelling indefinitely extensible algebraic structures, and the major mathematical modelling task as being the search for underlying generative systems that were logical, concise and complete; I still do.

I spent a year at ITT's semiconductor research laboratories at Footscray before going up to study mathematics at Trinity, Cambridge in 1959. I saw myself as an electronics engineer, quantum physics as the foundation of electronics and mathematics as the foundation of physics. I took with me some of the semiconductor devices I had fabricated at ITT and investigated who might find them useful; this earned me a place in Richard Gregory's cognitive psychology laboratory as his electronics technician.

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112.3 Learning Machines at ITT

I saw an advertisement in *Nature* for positions in a new *Learning Machines* project at *Standard Telecommunication Laboratory* (STL), ITT's Research Laboratories at Harlow, specifying qualifications in topology, cybernetics, neural networks, and other topics that interested me, and contacted the Project Leader, John Andreae. During my years at Cambridge I spent my vacations working with John and his team, David Hill, Owen Morgan and Peter Joyce, eventually taking over his position part-time when he left to take up a Chair at the University of Canterbury in Christchurch, New Zealand.

Fig. 112.1. Three pioneers of computational intelligence: from left to right, John Andreae (learning machines), David Hill (speech recognition), Ted Poppelbaum (stochastic computing in pattern recognition), IFIP 1968, Edinburgh

I first came across the work of Lotfi Zadeh whilst at ITT. Andreae's STelLA, the STL learning automaton, digitally encoded the state space of the system to be controlled and and learned to generate inputs to keep the system state in a specified region of that space; Zadeh and Desoer's book, *Linear System Theory: The State Space Approach* [53] was the bible for that approach to system design. In a 1963 symposium on general system theory Lotfi had presented a general formulation of *The notion of state in system theory* [49] that encompassed linear systems and automata. Although he had specifically excluded stochastic and anticipative systems it was clear how to extend his abstract framework to such systems and apply them to learning controllers.

I envisioned general learning components as black boxes in system design where the designer no direct control or knowledge of their internal states, customizing them for particular tasks through external techniques: *coding* input stimuli to make learning easier; *training* them through task sequences designed to facilitate learning; and *priming* them through linguistic stimuli designed to provide an initial problem solution to be refined by experiential learning. My state spaces were those of the learning components' capabilities, my design techniques were behavioural, and I developed a theoretical framework based on Ross Ashby's [3] algebraic formulation of Gerard Sommerhoff's model of adaptivity in living systems in *Analytical Biology* [46] and Lotfi's formulation of adaptivity in control systems in *On the definition of adaptivity* [48].

I kept track of Lotfi's research and saw his 1965 paper on *Fuzzy sets* [50] as an interesting engineering application of set theory based on Łukasiewiz infinite-valued logic [30]. I was interested in that logic as Moh Shaw-Kwei [45] had speculated the axiom of comprehension might not be subject to Russell's paradox in the corresponding set theory (still open in 1965 [6], proved in 1979 by Richard White [47]), a paradox that had fascinated me since I read *Principles of Mathematics*; it seemed to present a pitfall for any axiomatic system theory.

I knew that Łukasiewiz's axioms could be subsumed under axiomatic probability logics where I saw Carnap's [5] logical interpretation, Savage's [42] subjective one, and Shackle's [43] partial order over possibilities, as better models of uncertainty than frequentist ones, but found nothing I could use immediately in the fuzzy sets paper. Lotfi's footnote that "the membership function can be taken to be a suitable partially ordered set" was more appealing to my algebraic frame of mind than a mapping to the numeric range [0,1]; it still is.

112.4 Experimental Psychology at Cambridge

When I graduated from the Mathematics Tripos, Richard suggested I study for a PhD with him. Oliver Zangwill, the Chair of Experimental Psychology, accepted me but said I must get a psychology degree also. I took Part II of the Psychology Tripos after a year, preparing by writing past exam papers for my tutor, Alan Watson, acquiring the necessary background from journal papers, writing an essay on a methodology for animal experiments based on my model of adaptivity, and attending lectures on topology, probability, logic and algebra to extend my mathematical proficiency.

My doctoral research was funded through a contract with the Ministry of Defence to study adaptive training of perceptual-motor skills, and I built an analog computer in order to emulate a flight simulator and collect data on the learning of pilots from RAF Oakington under different training regimes.

My studies of human operators at Cambridge were synergistic with my studies of learning machines at STL. My paper with John for the 1966 IFAC Congress, *A learning machine in the context of the general control problem* [23], emphasized the same techniques, of *coding*, *training* and *priming*, that I was using in my human operator studies. We *coded* the inputs and outputs of the learning systems to ensure they provided a natural topology for the problem space; *trained* the systems through a task progression dynamically generated through feedback from their performances; and *primed* them by linguistically specifying behaviours that seemed, *prima facie*, to be initially useful.

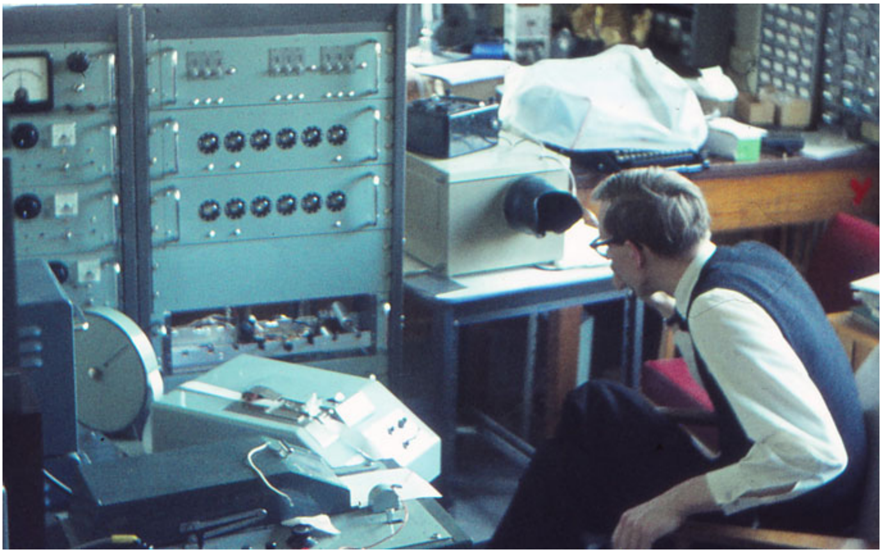

Fig. 112.2. A young Brian Gaines and analog computer in his laboratory in the Department of Experimental Psychology, Cambridge, 1964

The paper ends with a remark that seems prophetic for later developments in fuzzy control:

> "It is customary to think of controllers in terms of optimality but, when the plant is indeterminate or time-varying and the controller itself is required to be widely applicable, such a concept loses much of its force. When fabrication and storage costs have also to be taken into account, one can only ask whether satisfactory control is possible and, if so, how much it will cost."

That captures the ethos in which John Andreae, Igor Aleksander, Abe Mamdani and I worked on learning controllers.

112.5 Stochastic and Possibilistic Computing

In Richard's laboratory I investigated stereoscopic vision with an oscilloscope I had built with two small cathode ray tubes sending separate stimuli to each eye; enabling subjects to rotate and move simulated images of 3- and 4-D skeleton cubes and hypercubes. I speculated on the neural mechanism through which disparity was used to perceive depth; it seemed to necessitate cross-correlation between the stimuli from the two eyes. I thought it might be modelled as a neural process whereby stimulus intensities encoded as the probability of a neuron firing would be multiplied and cross-correlated by a neuron firing when it received pulses from corresponding neurons in both eyes.

I implemented this notion in one of my projects for the ITT learning machine, a front-end neural net to learn more useful encodings of the input stimuli regardless of their source and nature; John named it *Gadafter*, the 'Gaines adaptive filter.' I developed an adaptive filter based on discrete logic gates with stimuli encoded as the generating probabilities of random pulse streams, and a stochastic version of the adaptive digital elements (ADDIEs) for making the weight changes in STelLA's learning protocol, leaky stochastic integrators computing running averages.

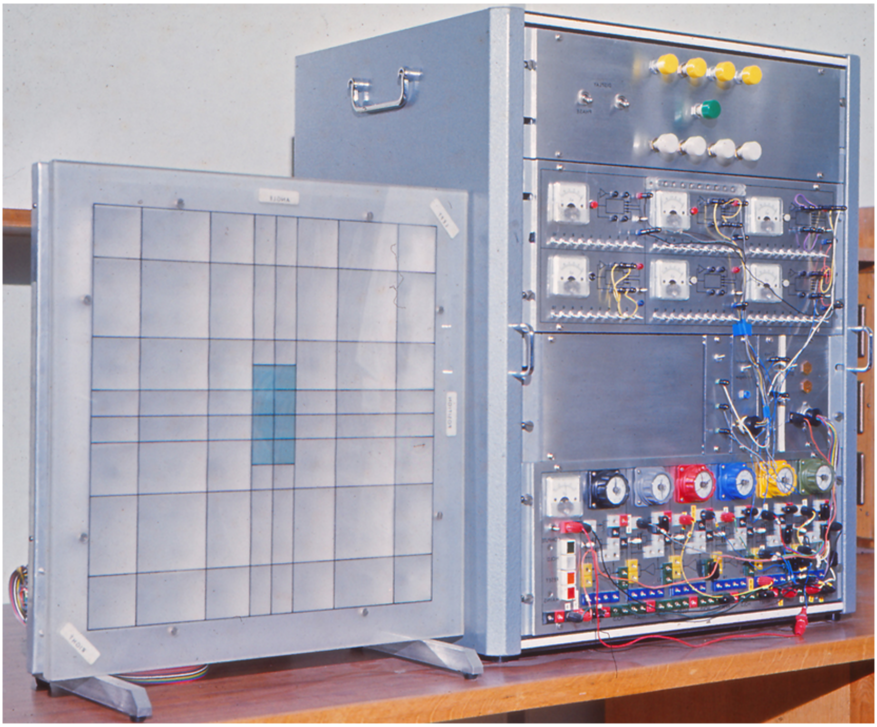

Fig. 112.3. On right, stochastic analog computer built by Brian Gaines & Peter Joyce; on left, visual display of STelLA learning controller's trajectory to the specified region of state space—Standard Telecommunication Laboratories, 1965

By the time I returned to the ITT Laboratories for the Summer of 1965 I had a fully developed scheme for a *stochastic analog computer* that resulted in a massive patent application [8] with 54 claims and papers in *Electronics* [9] and at the 1967 Spring Joint Computer Conference [10]. Ted Poppelbaum at the University of Illinois contacted me as he was an independent co-inventor of similar techniques for pattern recognition systems [40]. I visited him at Champaign, Urbana in 1967 and we became close friends, exchanging papers and research reports until he died in 1994.

I proved that a Rosenblatt *Perceptron* with digital weights did not converge because it could not implement steepest descent and went into limit cycles, but that a variant of the Novikoff convergence proof did apply to stochastic perceptrons built with stochastic integrators [11]. I saw the asynchrony between random sequences as generating a requisite variety of behaviours that led to the *possibility* of convergence, and the existence of optimal trapping states as leading to its *necessity*, a result consistent with Ross Ashby's notion of inherent adaptivity in a system with many states of equilibrium [2].

In 1968 when Ladislav Kohout came to be my graduate student at the University of Essex he referred me to Rescher's [41] proof of equivalence between probabilistic logic and the modal logic S5, which eventually led to reports and papers on a *calculus of possibility, probability and eventuality* [14] that encompassed fuzzy logic, and *possibilistic automata theory* [24] based on the calculus.

112.6 Linguistic Priming of Controllers

The community of those researching machine learning, neural networks and artificial intelligence in the UK in the 1960s was fairly small and the members well-known to one another. I met with Igor Aleksander, Mike Brady, Jim Doran, Pat Hayes, Abe Mamdani, Donald Michie, Ted Newman, Gordon Pask, Pete Uttley, Yorick Wilks, and others, at various meetings within a cybernetics, artificial intelligence, machine learning, and pattern recognition ethos. I knew Abe as Igor's student working on the *SLAM* deterministic digital adaptive modules applied to pattern recognition [1], and Abe knew me as working on stochastic digital adaptive modules applied to learning machines and to modelling human adaptive control of unstable systems. We used the new IEE journal, *Electronics Letters*, as a vehicle for rapid publication as it guaranteed publication (or rejection) within 6 weeks, and I refereed some of Igor and Abe's paper and suspect they refereed some of mine.

Alexander Luria from the USSR was a friend of Oliver Zangwill's and a frequent visitor to the Experimental Psychology Department at Cambridge. I was fascinated by his research on the positive impact of verbal behaviour on performance of perceptual-motor skills [33] and built this into the experimental design for my studies of training human operators, investigating the trade-off between *priming* my subjects with helpful control strategies and non-verbal training techniques, and collecting their verbalizations in the very difficult control task I had set them.

My results demonstrated a strong effect of such priming and, when I attempted to show that the success of my training techniques had little or no dependence on human psychology but were systemic and would apply to any learning automaton capable of carrying out the task, I wanted to be able to prime my stochastic neural networks with the same verbal input as I had provided my subjects.

I did so by having the stochastic Perceptron *imagine* itself with the input specified, taking the action specified, and *rewarding* itself for so doing [13]. This enabled me to replicate the positive effect of verbal instructions on my human subjects with identical phenomena in artificial adaptive controllers, demonstrating that the effects of coding, priming and training were all *cybernetic* phenomena in Wiener and Ashby's terms, a major thrust of my doctoral thesis which was very positivistic and behaviourist in keeping with the ethos of experimental psychology at that time.

112.7 The Genesis of Fuzzy Control

Abe was at presentations I made on this research at IEE Control System Colloquia, had a copy of my thesis, and in 1971, when he completed his doctoral research and was appointed a lecturer at Queen Mary College, set his graduate student, Sedrak Assillian, the task of replicating the results using a realistic engineering situation, the control of a small steam engine, a task known to involve non-linearities and time-varying behaviour that was not amenable to linear modelling and optimal control approaches.

By that time I was Reader in the Department of Electrical Engineering Science, Essex University, Technical Director of two companies I had founded in 1968, one offering timeshared computer services, the other a minicomputer I had designed, and executive editor of the International Journal of Man-Machine Studies (IJMMS) that John Gedye and I founded in 1968. I was very busy and cannot recollect whether I even knew of the research or met Sedrak before Abe asked me to be his doctoral examiner in 1974.

Sedrak's thesis [4] is a model of scholarship. He considers the relative merits of Perceptron-like adaptive threshold logic elements (ATLE) and Bayesian learning elements as adaptive controllers, analyses the impact of different input and output encodings on their potential learning performance, and studies empirically both types of learning controller, finding little difference between them. For his first study he used human operators as targets for the learning controllers to emulate, and got poor results which he attributes to erratic control strategies that were difficult to emulate, possibly resulting from operator fatigue as the training took hours at a time over several days.

For the second study he used a digital controller as the role model and again got poor results which he attributes to the controller again being difficult to emulate but this time because it was using analog inputs and making finer distinctions than were available to the learning controller using quantized inputs. His third study was retrospective in that he used the "fuzzy logic controller" described in the second half of his thesis as a trainer to be emulated by his learning controllers and found that the ATLE could emulate it perfectly indicating that the results of his first two experiments were not defects of his learning algorithms.

The second half of the thesis investigated priming the learning system with a linguistic description of a suitable control policy. He notes that I translated linguistic hedges into precise inputs whereas Waterman translated them to a range of values, and proposes to use fuzzy logic to do so automatically using Lotfi's techniques for representing conditional statements in linguistic variables described his 1973 paper,

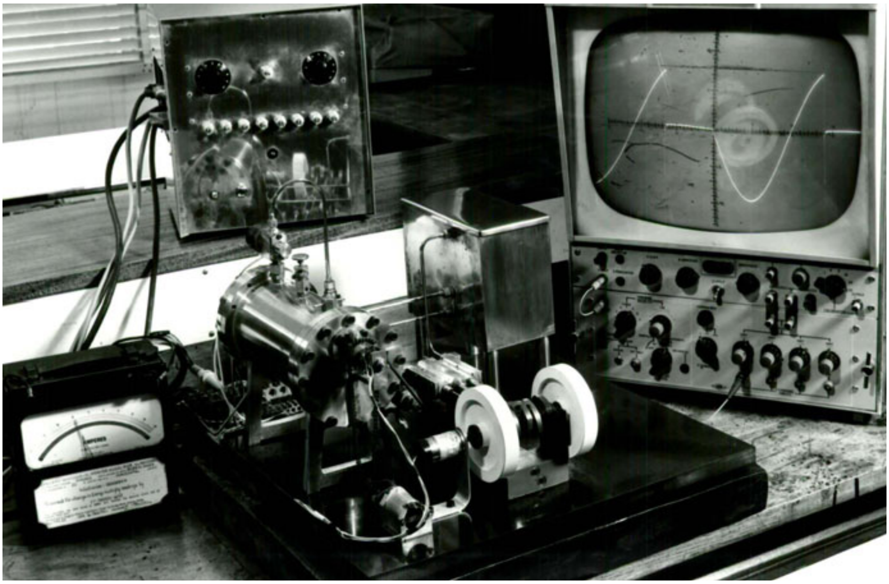

Fig. 112.4. Abe Mamdani and Sedrak Assilian's steam engine, QMC, 1974

Outline of a new approach to the analysis of complex systems and decision processes [51]. He defined 8 linguistic variables (positive big, etc) for his two sets of inputs and outputs and represented them as fuzzy distributions over his quantized input and output variables. He developed heuristic linguistic rules based on his own experience for controlling the steam engine, tested them, identified where control was ineffective and adjusted the rules (but not the fuzzy distributions) for better control.

One can summarize Sedrak's achievement to be that of providing the human trainer with a means to communicate linguistically with a control system and to monitor the impact of specific instructions on the performance in order to improve it. He remarks that I had also found in my experiments that it was not possible to choose a good set of instructions in advance and that feedback as to the effect of different instructions was necessary to develop the best set.

We both had emulated the way in which human trainers of perceptual-motor skills in athletes provide verbal instruction and modify it in the light of its effect on the trainee. Where Sedrak's made a major advance was in using Lotfi's algorithms to provide a principled translation from linguistic rules to a control policy. It was also interesting that his tuning of the linguistic statements resulted in a control policy that was very effective and primed the controller so well that no further learning improved it.

112.8 Evolution of Fuzzy Control

My reaction to Sedrak's thesis was three-fold: first, to request a paper for IJMMS as I saw it as a major advance in linguistic interaction between people and computers;

second, to see if fuzzy logic was necessary or whether I could replicate the results with stochastic elements; third; to investigate fuzzy logic further. Abe and Sedrak published *An experiment in linguistic synthesis with a fuzzy logic controller* [36] in IJMMS in 1975 and it generated widespread interest.

I published *Stochastic and fuzzy logics* [15] in *Electronics Letters* in 1975 showing that one could substitute probability logic for fuzzy logic in interpreting Sedrak's linguistic specification and obtain the same control policy. Ladislav and I subsumed all forms of uncertainty in dynamical systems within the framework of *possibilistic automata*, unifying fuzzy and probabilistic sequential systems within an algebraic framework, and published *The logic of automa* [24] in Gerge Klir's *International Journal of General Systems* in 1975.

Abe took these results in his stride citing them in his papers [36] [35] and noting that there were other ways of implementing linguistic heuristics that gave the same results as fuzzy logic. Neither of us saw the details of the implementation as important; it was the use of linguistic statements to communicate with a learning controller and lead it to develop an effective policy that was the breakthrough. In 2009 he still emphasized in his correspondence with Enric Trillas that "Fuzzy control should not be seen as an experimental proof of the correctness of fuzzy logic." [34]

Abe had interested many in industrial control world-wide in the potential applications of linguistically specified heuristics in designing controllers for difficult plants that required human operators because conventional automatic control techniques were inapplicable. The instructions for the human operators in Peray and Waddell's 1972 book on *The Rotary Cement Kiln* [39] were so similar in form to those Sedrak had used as to represent an independent confirmation of the industrial value of the approach.

Abe and I organized a series of workshops on *Discrete Systems and Fuzzy Reasoning* [37], the first of which took place at Queen Mary College in January 1976 with a 300 page proceedings published by University of Essex. By 1982 the popular science press also found the potential applications of fuzzy reasoning intriguing [31].

I had already published eight papers citing fuzzy sets in IJMMS prior to 1974. In the first volume in 1969 I had solicited one from Bill Kilmer and Warren McCulloch where Bill cites his paper on *Biological applications of the theory of fuzzy sets and fuzzy algorithms* [32]. I asked Joe Goguen to submit his report on *Concept representation in natural and artificial languages: Axioms, extensions and applications for fuzzy sets* [29] and published it in 1974. Gordon Pask [38] in 1973 cites three papers by Lotfi on fuzzy algorithms in the context of Manna's non-deterministic algorithms and suggests that my work on *Axioms for adaptive behavior* [12] provides a bridge between the two; my paper cites Lotfi heavily for his work on adaptivity but not his later work on fuzzy sets—Gordon somehow made a link.

112.9 Visiting Lotfi at Berkeley

In 1975 I was appointed Professor of Computer Systems at Essex University and Head of the Department of Electrical Engineering Science. I went to visit Ted

The original idea of fuzzy logic came from Lotfi Zadeh (left centre), professor at the University of California at Berkeley. He and other theorists have since developed some sophisticated mathematics to accompany the basic ideas. It is not yet clear, though, how the theory relates to other, more conventional, approaches to non-rigorous concepts and arguments. It is slightly worrying to the proponents of fuzzy logic that people working on "expert systems", for example for chemical analysis and medical diagnosis, have not needed to invoke Zadeh's work directly.

Abe Mamdani (left) lecturer in the Department of Electrical Engineering at Queen Mary College, London, worked with one of his students on a fuzzy-logic controller for a steam engine. Brian Gaines (far left), is one of the computer scientists whose ideas led to the use of fuzzy logic in process controllers. Gaines, formerly a professor of computer science at Essex University, is now a consultant based in London. □

Fig. 112.5. From *Computing with a human Face, New Scientist*, May 1982

Poppelbaum at Champaign, Illinois, and drove to Bloomington, Indiana, to present a paper by Ladislav and myself on *Possible automata* [25] at the International Symposium on Multivalued Logic, where Ryszard Michalski suggested recasting the results within his *variable-valued logic* framework.

I went to Los Angles to visit Joe and Lotfi, gave a seminar at UCLA on my research on the identification of stochastic systems, and stayed at Joe's apartment. We went into UCLA's Moog Synthesizer Laboratory at 2am, and listened to the resulting 'music' while discussing whether my modelling process could be cast as a category-theoretic adjunction in a a similar way to his unification of the identification problems for linear systems and automata in his 1972 paper *Realization is universal* [28].

I visited Lotfi at Berkely and gave a presentation on the QMC controller studies. I think this was the first time we had met but I knew his work on system theory and he knew mine on stochastic computing, and we quickly engaged in discussions of fuzzy sets, their relation to multi-valued logics, and their applications to heuristic control. His office shelves were stacked with large plastic laundry baskets containing reprints of papers from all over the world sent to him by those working on fuzzy sets and their applications, and he gave me permission to look through them, make notes and copy some.

I asked Lotfi for a paper for IJJMS and published *A fuzzy-algorithmic approach to the definition of complex or imprecise concepts* [52] in 1976. I published notes on my meeting with Lotfi, *Research notes on fuzzy reasoning* [17], in the 1976 workshop proceedings.

112.10 Fuzzy Logic Bibliography, Survey and Synthesis

When I returned to the UK I wrote to over 200 authors requesting copies of papers relating to fuzzy sets. Ladislav and I read them, classified them, developed an annotated bibliography, sent it to the authors for comment and published in 1976 as *The fuzzy decade: a bibliography of fuzzy systems and closely related topics* [26] comprising some 1,100 items.

In the same year I published a paper, *Foundations of fuzzy reasoning* [16], setting fuzzy reasoning within the framework of the relevant literature on multi-valued logics, noting that fuzzification of logic left the definition of implication open, and suggesting a range of possible fuzzy implication functions. It was interesting as I reviewed the current literature in writing this paper to find a number of these now associated with my name and used in a wide range of applications.

I published my synthesis of probability and fuzzy logics as a *Fuzzy and probability uncertainty logics* [18] in *Information and Control* in 1978, suggesting that there was a *standard uncertainty logic* (SUL) subsuming probability and fuzziness, each of which could be derived from it by additional axioms: *excluded middle* and *truth functionality*, respectively. However, Sedrak's linguistic controller algorithm could be derived directly within a SUL [19] and did not need commitment to either additional axiom.

112.11 Monotype and SGSR

In 1978 I left Essex University, having been head-hunted by the UK National Enterprise Board to be Technical Director and Deputy Chief Executive of the Monotype

Corporation, a nineteenth century British printing equipment company that was in financial trouble but had the possibility of recovery through new computer technology. Monotype had some 4,000 employees, 26 subsidiaries around the world and was on the verge of bankruptcy; the job required a major time commitment and continuous travel. In 1978 also George Klir invited me to become President of the Society for General Systems Research which provided some balance between my industry and academic lives.

While at Monotype I continued to edit IJMMS, and managed to fit in the banquet speech at the First North American Fuzzy Information Processing Group Workshop at Utah State University, in May 1982, between my industry trips in North America—later published as *Precise past—fuzzy future* [19]. After that trip I collapsed on return to England and was harangued by a doctor who told me to buy some medical textbooks and treat my body with the same care as I did my computers.

112.12 Moving to North America

My wife, Mildred Shaw, and I decided that it might be time to slow down and emigrated to Canada in 1982 with funding from the Canadian government for me to form a research company developing handwriting recognition tablets, and a Professorship in Computer Science at York University for Mildred.

In 1985 David Hill, by then at the University of Calgary, nominated me for the Killam Research Chair there, and we remained at UofC for 15 years until retiring out to the West coast of Vancouver Island at the end of 1999. At Calgary we became heavily involved in knowledge acquisition for knowledge-based systems research, collaborated with John Boose to launch a series of three annual knowledge acquisition workshops in Banff, Europe and South East Asia, and an associated journal and book series. By that time fuzzy reasoning had become well-established and figured in many of the KAW papers.

112.13 Reflections in Retirement

In the summer of 2004, after 36 years, I retired as editor of IJHCS (an updated title for IJMMS, also incorporating the KA journal) and stopped travelling. My research interests include: the role of knowledge in civilization, from the origins of *homo erectus* to the present day and beyond [22]; knowledge acquisition, representation and inference methods and tools [20] [27]; the nature of human rationality in everyday reasoning and the sciences [21]; and in minimalist, defeasible substructural logics that are adequate to model much human reasoning, solve many AI problems, and could have been developed by Aristotle within the framework of syllogistics.

For me the most significant issue in understanding human knowledge processes over the millennia is how we coordinate our activities by communicating using concepts that are open and ill-defined, often with no common agreement on their meaning and usage. Even in close-knit scientific communities there can be commonly used concepts that are interpreted in very different ways [44].

Human civilization 'muddles through' [7] very effectively in a way that seems foreign to logic and rationality. However, one can also view the phenomena as resulting from our intrinsic uncertainties about the world being accurately communicated through our use of appropriately vague terms in our languages. We convey not only the properties of the world but also our uncertainties about them; this is a rational process of the social brain that we need to model in our theories and associated *soft computing* tools.

112.14 Concluding Remarks

I will finish as I started by remarking on how privileged I have been to have friendships and intellectual dialogue with so many remarkable people. Abe and I had a lot of fun together and it was fascinating to see his ideas propagate into a control industry that had achieved so much based on linear systems theory but had a substantial residue of processes that were not tractable within that framework. I have often remarked that there is no such thing as 'nonlinear systems theory'—it is the heterogenous mess that is left when the linearization paradigm fails—Abe made a significant dent in that mess.

I see papers attempting to reconstruct the basis of those achievements and will echo what Abe has said—you need to go back to Sedrak's thesis for such reconstruction; it is readily available on the web from QMC [4].

Lotfi and I continue to communicate although our paths no longer cross. It is a pleasure to have this opportunity to say how much I respect and admire all he has achieved from his seminal analyses of the notions of *state* and *adaptivity* to his development of computational algorithms for *computing with words* that have inspired so many theoretical innovations and industrial applications.

I have enjoyed re-reading much of the early literature and catching up with the recent literature in writing this article, and look forward to seeing the other contributions in this book. One privilege of having been involved in this community is the insights it provides into how new constellations of knowledge form in our society.

References

1. Aleksander, I., Mamdani, E.H.: Microcircuit Learning Nets: Improved Pattern Recognition by Means of Pattern Feedback. Electronics Letters 4(20), 425–426 (1968)
2. Ashby, W.R.: Design for a Brain. Chapman Hall, London (1952)
3. Ashby, W.R.: The Set Theory of Mechanism and Homeostasis, Biological Computer Laboratory, Technical report 4.7, University of Illinois (1962)
4. Assilian, S.: Artificial intelligence in the Control of Real Dynamic Systems, Ph.D. thesis (1974), http://qmro.qmul.ac.uk/jspui/handle/123456789/1450
5. Carnap, R.: Logical Foundations of Probability. University of Chicago Press, Chicago (1950)
6. Chang, C.C.: Infinite Valued Logic as a Basis for Set Theory, Logic, Methodology and Philosophy of Science. In: Bar-Hillel, Y. (ed.) Proceedings of the 1964 International Congress, pp. 93–100. North-Holland, Amsterdam (1965)

7. Fortun, M., Bernstein, H.J.: Muddling Through: Pursuing Science and Truths in the 21st Century. Counterpoint, Washington (1998)
8. Gaines, B.R.: Stochastic Computing Arrangement, UK Patent 1, pp. 184, 652 (1966)
9. Gaines, B.R.: Stochastic omputer Thrives on Noise. Electronics, 72–79 (1967)
10. Gaines, B.R.: Stochastic Computing. In: Spring Joint Computer Conference, vol. 30, pp. 149–156. AFIPS, Atlantic City (1967)
11. Gaines, B.R.: Techniques of Identification with the Stochastic Computer. In: Proceedings IFAC Symposium on the Problems of Identification in Automatic Control Systems, pp. 1–10 (1967)
12. Gaines, B.R.: Axioms for Adaptive Behaviour. International Journal of Man-Machine Studies 4, 169–199 (1972)
13. Gaines, B.R.: The Learning of Perceptual-motor Skills by Men and Machines and its Relationship to Training. Instructional Science 1(3), 263–312 (1972)
14. Gaines, B.R.: A Calculus of Possibility, Eventuality and Probability, Technical Report EES-MMS-FUZI-75, Department of Electrical Engineering Science, University of Essex (1975)
15. Gaines, B.R.: Stochastic and fuzzy logics. Electronics Letters 11(9), 188–189 (1975)
16. Gaines, B.R.: Foundations of Fuzzy Reasoning. International Journal of Man-Machine Studies 8(6), 623–668 (1976)
17. Gaines, B.R.: Research Notes on Fuzzy Reasoning. In: Mamdani, E.H., Gaines, B.R. (eds.) Discrete Systems and Fuzzy Reasoning, pp. 45–49. University of Essex, Colchester (1976)
18. Gaines, B.R.: Fuzzy and Probability Uncertainty Logics. Information and Control 38(2), 154–169 (1978)
19. Gaines, B.R.: Precise Past—Fuzzy Future. International Journal of Man-Machine Studies 19(1), 117–134 (1983)
20. Gaines, B.R.: Designing Visual Languages for Description Logics. Journal of Logic, Language and Information 18(2), 217–250 (2009)
21. Gaines, B.R.: Human Rationality Challenges Universal Logic. Logica Universalis 4(2), 163–205 (2010)
22. Gaines, B.R.: Knowledge Capture Through the Millennia: From Cuneiform to the Semantic Web. In: Proceedings of the Sixth International Conference Knowledge Capture, K-CAP 2011. ACM, New York (2011)
23. Gaines, B.R., Andreae, J.H.: A Learning Machine in the Context of the General Control Problem. In: Proceedings of the 3rd Congress of the International Federation for Automatic Control. Butterworths, London (1966)
24. Gaines, B.R., Kohout, L.J.: The Logic of Automata. International Journal of General Systems 2(1), 191–208 (1975)
25. Gaines, B.R., Kohout, L.J.: Possible Automata. In: Proceedings of the International Symposium Multiple – Valued Logic, pp. 183–196. IEEE, Long Beach (1975)
26. Gaines, B.R., Kohout, L.J.: The Fuzzy Decade: A Bibliography of Fuzzy Systems and Closely Related Topics. International Journal of Man-Machine Studies 9, 1–68 (1977)
27. Gaines, B.R., Shaw, M.L.G.: Computer Aided Constructivism. In: Caputi, P., Viney, L.L., Walker, B.M., Nadia, C. (eds.) Constructivist Methods, pp. 183–222. Wiley, New York (2012)
28. Goguen, J.A.: Realization is Universal. Mathematical System Theory 6(2), 359–374 (1972)
29. Goguen, J.A.: Concept Representation in Natural and Artificial Languages: Axioms, Extensions and Applications for Fuzzy Sets. International Journal of Man-Machine Studies 6, 513–561 (1974)

30. Hay, L.S.: Axiomatization of the Infinite-valued Predicate Calculus. Journal of Symbolic Logic 28(1), 77–86 (1963)
31. Herman, R.: Computing with a Human Face. New Scientist 94(1304), 368–371 (1982)
32. Kilmer, W.L.: Biological Applications of the Theory of Fuzzy Sets and Fuzzy Algorithms. In: Procter, L.D. (ed.) Biocybernetics of the Central Nervous System. Little Brown, Boston (1969)
33. Luria, A.R.: The Role of Speech in the Regulation of Normal and Abnormal Behavior. Pergamon Press, Oxford (1961)
34. Mamdani, E.H., Trillas, E.: Correspondence between an Experimentalist and a Theoretician. In: Trillas, E., Bonissone, P.P., Magdalena, L., Kacprzyk, J. (eds.) Combining Experimentation and Theory. STUDFUZZ, vol. 271, pp. 1–18. Springer, Heidelberg (2011)
35. Mamdani, E.H.: Advances in the Linguistic Synthesis of Fuzzy Controllers. International Journal of Man-Machine Studies 8, 669–678 (1976)
36. Mamdani, E.H., Sedrak, A.: An Experiment in Linguistic Synthesis with a Fuzzy Logic controller. International Journal of Man-Machine Studies 7(1), 1–13 (1975)
37. Mamdani, E.H., Gaines, B.R.: Discrete Systems and Fuzzy Reasoning. University of Essex, Colchester (1976)
38. Pask, G., Scott, B.C.E., Kallikourdis, D.: A Theory of Conversations and Individuals (Exemplified by the Learning Process on CASTE). International Journal of Man-Machine Studies 5, 443–566 (1973)
39. Peray, K.E., Waddell, J.J.: The Rotary Cement Kiln. Chemical Publishing, New York (1972)
40. Poppelbaum, W.J., Afuso, C., Esch, J.W.: Stochastic computing elements and systems. In: Proceedings of the American Federation of Information Processing Societies, Fall Joint Computer Conference, vol. 31, pp. 635–644. Books, Inc., New York (1967)
41. Rescher, N.: A Probabilistic Approach to Modal Logic. Acta Philosophica Fennica 16, 215–226 (1963)
42. Savage, L.J.: The Foundations of Statistics. Wiley, New York (1954)
43. Shackle, G.L.S.: Decision, Order and Time in Human Affairs. Cambridge University Press, Cambridge (1961)
44. Shaw, M.L.G., Gaines, B.R.: Comparing Conceptual Structures: Consensus, Conflict, Correspondence and Contrast. Knowledge Acquisition 1(4), 341–363 (1989)
45. Moh, S.-K.: Logical Paradoxes for Many-valued Systems. Journal of Symbolic Logic 19(1), 37–40 (1954)
46. Sommerhoff, G.: Analytical Biology. Oxford University Press, London (1950)
47. White, R.B.: The onsistency of the Axiom of Comprehension in the Infinite-valued Predicate Logic of Łukasiewicz. Journal of Philosophical Logic 8(1), 509–534 (1979)
48. Zadeh, L.A.: On the Definition of Adaptivity. Proceedings IEEE 51(3), 469–470 (1963)
49. Zadeh, L.A.: The Concept of State in System Theory. In: Mesarovic, M.D. (ed.) Views on General Systems Theory, pp. 39–50. Wiley, New York (1964)
50. Zadeh, L.A.: Fuzzy Sets. Information and Control 8, 338–353 (1965)
51. Zadeh, L.A.: Outline of a New Approach to the Analysis of Complex Systems and Decision Processes. IEEE Transactions Systems, Man & Cybernetics SMC-3(1), 28–44 (1973)
52. Zadeh, L.A.: A Fuzzy-algorithmic Approach to the Definition of Complex or Imprecise Concepts. International Journal Man-Machine Studies 8, 249–291 (1976)
53. Zadeh, L.A., Desoer, C.A.: Linear system theory: The state space approach. McGraw-Hill, New York (1963)

113

Dialogue on Scientific Theories and Fuzziness –Fuzzy-Philosophical Investigations

Rudolf Seising

113.1 Preliminary Remark

The historiography of modern science comprises some famous dialogues, starting with thirtysix Socratic dialogues ascribed to Plato. Then there is Galileo's famous *Dialogo sopra i due massimi sistemi del mondo* that is the model for the fictitious debate on scientific theories and their relationships, that is presented in this chapter, and this subject of *Philosophical Investigations* reminds the reader of the posthumous edited book by Ludwig Wittgenstein.

Galileo Galilei (1564-1642) published his *Dialogo – Dialogue Concerning the Two Chief World Systems* in 1632 [8]. It was the first book on science written in Italian (instead of Latin) and therein the author compared the new world system of Nicolaus Copernicus (1473-1643) with the traditional one of Claudius Ptolemy (90-168).

Three protagonists debate on this subject, named Sagredo, Salviati, and Simplicio. While Simplicio argues for the Ptolomaic world system that was also the view of Aristotle, Salviati argues for the Copernican position, and Sagredo is an intelligent disputant who starts with a neutral opinion. The debate takes place at the palace of Sagredo at the Canale Grande in Venice (see Figure 113.1, left).

About three and a half centuries later the three savants appeared again in the book *Are Quanta Real?* [11] published by the Swiss physicist Josef Maria Jauch (1914-1974) and translated into English in 1989.[1] In this book the three fight on scientific knowledge and nature perception against the background of the controversy on the foundations of quantum mechanics.

Today, almost half a century later, in the times of coexistent real and virtual worlds, avatars for three scholars enter into a dialogue. These scholars are well-known for their work in the 20th century philosophy of science, and their avatars meet to debate the historical development of scientific theories and their fuzziness! Their Latin names are: Carolus, Thomas and Ludovicus.[2]

[1] The German-written original *Die Wirklichkeit der Quanten* appeared already in 1973, [10].

[2] The word avatar means in Hinduism, a "manifestation" of a deity to earth. However, today "avatar" denotes a computer-graphical representation in games or virtual worlds.

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(a) (b)

Fig. 113.1. (a): Frontpage of Galilei's *Dialogo* with copper engraving by Stefano della Bella (1610-1664); from left to right: Aristotle, Ptolemy and Copernicus; (b) The Campanile on the campus of the University of California at Berkeley, CA

113.2 Introduction

Last year, Lotfi A. Zadeh gave me the permission to search, scan and photocopy the materials in his office at the University of California at Berkeley and in his home to continue my historical research on the genesis and development of the theory of Fuzzy Sets and Systems. One late evening, when I came home to the appartment that I have rented for my stay in Berkeley, I was terribly tired from work and I laid down on the sofa. On TV there was an episode of the series *Star Trek – The Next Generation*. I remember watching the English actor Patrick Stewart as the starship Enterprise's Captain Jean-Luc Picard. I know exactly that I gave some thought to the similarity of Steward and Lotfi Zadeh – may be because they both have bold heads. It was one of the Star Trek sequels where the actors used the so-called "holodeck". Star Trek's holodeck is a closed room that can replicate a wide variety of environments in which various people, objects, and places of past, present and future time can be simulated as a virtual reality. Usually the protagonists of the series use the holodeck technology for their research and training, but also for recreation and entertaining. The crew members like to interact with the program and its characters. When I was young, I was very interested in the episodes where the Star Trek actors met avatars of historical notabilities. However, as all too often, this evening I fell asleep and the following dialogue happened in my dream.

113.3 Dialogue

Carolus: Why are we here?

Ludovicus: Damned! – We agreed on not discussing such metaphysical questions: Why are we here? – What is there? – What is it like? – What is the meaning of life? – Why is there something? – Why isn't there nothing? – Does God exist? – It was my deep hope that we conform with never debating senseless questions! – Whereof one cannot speak, thereof one must be silent.[3]

Carolus: Calm down! You are too sensitive to the sound of my words. I thought that after the many years you passed away you would not behave that Viennese! I just wanted to know: What place is this?

Thomas: This is the campus of Berkeley and we are sitting in front of the campanile, also called Sather Tower, UC Berkeley's most recognizable symbol. It's a bell and clock tower, completed in 1914. We are also close to the buildings of the philosophy department and the history department. Here, I became professor of the history of science in 1961. However, before that I studied in Harvard and later I got a professorship of philosophy and history of science in Princeton. Did I ever told you that I was teaching in both of the departments ...

Ludovicus: Yes Thomas, you told us this a good many times! – Carolus, for a moment I was thinking that we have arrived in Venice.

Carolus: In Venice is the Campanile di San Marco that is much older and this one here has resemblance. Also when I was in England I saw the Joseph Chamberlain Memorial Clock Tower at the University of Birmingham, and there is also the Torre del Mangia in Siena, Italy. ... You'd think that all unversity campus have a tower but if you find one without tower, this hypothesis would be falsified ....

Ludovicus: Stop it, Carolus! You madden me. I was already looking for a poker! It is not only the problem to know whether there is a tower or not. It is also a problem to decide what is a tower! – There exist so many buildings that could be considered as towers in and we can imagine so many more towers ... *I mean the Tower of Babel, the Tower of London, church spires, watch towers, siege towers, television towers, the Eiffel Tower, the Tower(s) of Hanoi and so on. What is common to them all? Don't say: "There must be something common, or they would not be called 'towers' " but look and see whether there is anything common to all. For if you look at them you will not see something that is common to all, but similarities, relationships, and a whole series of them at that. To repeat: don't think, but look! Look for example at look-outs, with their multifarious relationships. Now pass to church spires; here you find many correspondences with the first group, but many common features drop*

[3] This is a quotation by Wittgenstein; it is the last sentence in his *Tractatus* [16].

out, and others appear. When we pass next to castle-towers, much that is common is retained, but much is lost. Are they all "stonelike"? Compare television towers with siege towers. Or is there always a viewing balcony, or a restaurant deck? Think of the Tower of Pisa. In watch towers there is a ladder; but for television towers this feature has disappeared. Look at the wooden siege towers and a water tower made from concrete or stone. Think now of mine head towers; here is the element of height, but how many other characteristic features have disappeared! sometimes similarities of detail. And we can go through the many, many other groups of towers in the same way; can see how similarities crop up and disappear. And the result of this examination is: we see a complicated network of similarities overlapping and crisscrossing: sometimes overall similarities.[4]

Carolus: How times flies, Ludovicus! I remember the days of neo-positivistic ideas in Vienna, in Berlin and later in Cambridge and Chicago. For me it is still one of the most remarkable developments in history of philosophy that you turned diametrically opposed your views on language. Nowadays it is obvious for us that not all phenomena could be expressed in terms of language. By the way, this was the reason for the break down of the Empiricism à la Carnap. Instead of analyis of the language of science philosophers turned to the analysis of its theories and methodologies.

Ludovicus: Carolus, even your *Logic of Scientific Discovery* was published already in 1934 in German it became not influential before the English edition appeared in 1959.[5] Because I passed away in 1951, I may be missed some information. What was the big success of your book?

Carolus: My work is one of the milestones in History of Philosophy of Science and I became old enough to enjoy public famousness. The *Logic of Scientific Discovery* heralded a shift in differentiating between science and non-science, metaphysics or pseudo-science. In the "pre-Popper-times" philosophers tried to fix this demarcation in scientific language ...

Thomas: Carolus, I hate it when you maintain this with such an arrogant manner. But in the merits you are right. However, Ludovicus' *Tractatus* played the role of a Bible for this view on philosophy of science, that was called "Logical Empiricism" or "Logical Positivism" or "Neopositivism" that was significantly involved by the members of the Vienna Circle ...

Ludovicus: ... but that is a presentation of my previous philosophical view on language, logic and the world! As you mentioned already, I thought extremely different when I came back to philosophy in the 1930s. Today philosophers distinguish sharply between Wittgenstein I, the author of the *Tractatus* and Wittgenstein II,

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[4] As the reader will notice, this is a slightly variation of parts of §66 in Wittgenstein's [17].

[5] See: [15]. Popper published the German book in 1934. He rewrote it in English, republished in 1959.

the author of the *Philosophical Investigations*, by the way, the latter was published in 1953, after my death, and I would have never published it this way! But that 's life and that's death!

Carolus: In my metatheory named "Critical Rationalism" the decision of what is science and what is not science is related to theories and methods in these fields and not in the precision of the terms of language. I created this alternative concept to that of the Vienna Circle and the other Logical Empiricists who tried to analyze the constitution or the structure of scientific theories by using modern logic, particularly Rudolf Carnap wrote in 1928 *The Logical Structure of the World*.[6] For Carnap and many others theories are sets of propositions and these propositions are built from data via induction.

I say: On the contrary! For Critical Rationalists scientific theories are *not* built from data by induction! There is no logical way from data to theory! Theories are hypotheses or conjectures and scientists test these hypotheses in experiments with intend to refute them. Even a great number of positive test results can not confirm a scientific theory, but if there is only one outcome that is negative, this one counterexample shows that the theory is falsified. Look at this squirrel!

Carolus points to one of the many squirrels on the campus (see Figure 113.2 left side).

There are so many squirrels on this campus and they all look different, they all live on nuts and similar things but I know that some of them also consume meat and perhaps there are even "flying squirrels". Well, that is a nice example for a little theory or hypothesis. I say: *There is no flying squirrel*. It is not possibel to verify this theory because to this end we would have to investigate all existing squirrels and we have to check wether they can fly or not, and may be they can but they don't do during the time that we observe them. However, we can try to falsify this hypothesis and if we find one solely flying squirrel, then the hypothesis is refuted. To cut a long story short: In Critical Rationalism the falsifiability is the criterion of demarcation between what is scientific and what is not.

Thomas: There is another reason why Logical Empirism has nowadays very few supporters. It seems very clear that we can not reduce all our knowledge to sensual data. Therefore, we need so-called theoretical elements in addition to the empirical ones. These additional elements are being understood only in the context of a theory. They are more abstract, they are more distant from our perceptions than observational terms. To factor these elements in Logical Empiricism Carnap and Carl G. Hempel introduced in the 1950s the so-called "double language model".[7] Whereas observational and therefore non-theoretical terms are elements of the observation language, theoretical terms are elements of the theoretical language.

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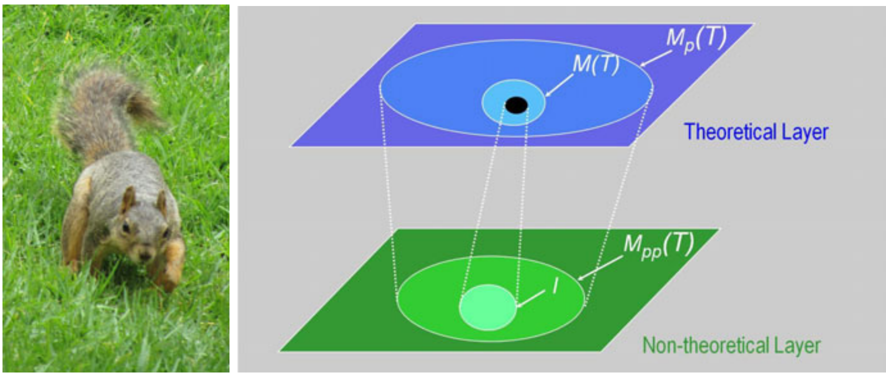
[6] Carnap published [4] in German in 1928, the English translation appeared in 1967.
[7] See [6, 9].

Ludovicus: In former times we would have named this metaphysics!

Thomas: Later, also Willard van Orman Quine – we both were Fellows at Harvard – criticized the empiricist differentiation between "analytical" and "synthetical". In short, the Logical Empiricism collapsed. If you want to name this metaphysics, then I say: Without metaphysics no knowledge is possible!

Ludovicus: Let's take Carolus' example of "flying squirrels": They exist! "Flying squirrels, scientifically known as *Pteromyini* or *Petauristini*, are a tribe of 44 species of squirrels (family Sciuridae)." Of course, the name is a theoretical term because Wikipedia says: "Flying squirrels are not capable of powered flight like birds or bats; instead, they glide between trees. They are capable of obtaining lift within the course of these flights, with flights recorded to 90 meters (295 ft)."[8] Roughly spoken, we can consider the term "flying" as an element of the theoretical language of the theory of squirrels and its meaning is not our usual meaning of "flying".

However, in Biology these flying squirrels belong to the family of squirrels, and there are "tree squirrels, ground squirrels, chipmunks, marmots (including woodchucks), flying squirrels, and prairie dogs." [9] and *I can think of no better expression to characterize these similarities than "family resemblances"; for the various resemblances between members of a family: build, features, colour of eyes, gait, temperament, etc. etc. overlap and crisscross in the same way. And I shall say: "squirrels" form a family.*[10]



(a) (b)

Fig. 113.2. (a): Squirrel on the Berkeley campus; (b) Structures of a theory T in the metastructuralist view of scientific theories

[8] See: http://en.wikipedia.org/wiki/Flying_squirrel
[9] See for more details http://en.wikipedia.org/wiki/Squirrel
[10] As the reader will notice, this is a slightly variation of parts of §67 in Wittgenstein's [17].

Thomas: I am so sorry that we never met in lifetime, Ludovicus. Your philosophy of language games and familiy resemblance is exciting and I wonder why we don't try to combine it with my ideas on *The Structure of Scientific Revolutions*, that I published here at Berkeley, 50 years ago.[11]

In this book I critisized Carolus' view on theory dynamics in science. As I could show in many cases of my historical research work, no replacement of a theory by another happened because of falsification.[12] In my view theory change in science is not a rational process and therefore we need assistance from sociology and psychology to explain the paths of science through history.

My historical research convinced me that there were periods of "prescience" that lack any theory or paradigm, then there were periods of "normal science" with paradigm monopole and finally there were times of crisis that triggered "scientific revolutions". Most scientists in most periods have been "normal scientists". They are involved with puzzle-solving. Only if there were many anomalies in opposition to the current paradigm a crisis appeared and a scientific revolution could happen. However, we have discussed this very often in life, Carolus. Do you remember the conference organized by Imre Lakatos at Bedford College in London, 1965?[13]

Carolus: I conceded already in my lifetime: I did not concentrate my attention on the periods of normal sciences. You can say that this was a failure! However, my interests was the change of theories in the field of science. Yes, normal science exists and there are normal scientists but this is the bad thing! To my mind scientists have to critically analyze their theories at any time and therefore I did not differentiate various periods. To make this clear: I still think that it is a scandalon that there is normal science! In the early times, the adventure of science startet with the method of trial and error and, as I said in my *Logic of Scientific Discovery*, from then until today, the methodology of scientific progress is falsifiability.

To say a word to your "paradigm shifts": I still think that the idea on rival paradigms in times of a scientific crisis has an irrational element.

Thomas: In this point I agree with you, Carolus, it was straightforward my intention to emphasize that these processes can not be explained in terms of pure rationality! In the way I see it, a paradigm shift can be activated by sociological, economical, political or other reasons and there is no logical determinism in theory dynamics. I have to emphasize that the theory in a new paradigm may use concepts that are totaly different from the concept of the former theory in the old paradigm. Even if they have got the same or a similar name, the new concepts may have a very different meaning. It is also not necessary that a concept in the new paradigm has an analogon in the old paradigm!

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[11] See: [13]. Kuhn exemplified later that the idea to this book went back to 1947. In that year as a graduate student at Harvard University he was asked to teach a science class for humanities undergraduates on historical case studies.

[12] See for these examples Kuhn's book [13, 12].

[13] For details see: [7].

Carolus: This means that if two paradigms are incommensurable then we have no chance to compare them directly. Do you really want to join your irrational system with that “Wittgenstein II language philosophy” that is not strong rational as well?

Thomas: Yes, I think that it could be a very fruitful combination in philosophy and history of science!

Ludovicus: I appreciate very much this suggestion to step forward in our discussion instead of backwards movements! May be you can explain what do you have in mind by this “combination”?

Thomas: Shortly after your death a second trend in obtaining systematic rational reconstructions of empirical theories was established, the so-called Suppes approach ...

Carolus: Patrick Suppes is a philosopher and statistician living in Stanford, very close to Berkeley.

Thomas: That’s him. – Well, to get a rational reconstruction of the theory in question the first step consists of an axiomatization that seeks to determine the mathematical structure of the theory. Now, the difference between the old view – we call it Carnap approach – and the new Suppes approach can be found in the manner in which this task is performed. As you can imagine, Carnap was firmly convinced that only formal languages can provide suitable tools to achieve the desired precision. Consequently, the Carnap approach claims that a theory has to be axiomatized within a formal language.

Ludovicus: That’s right! – Could you please explain any more about this other view, the Suppes approach?

Thomas: In this approach one is able to axiomatize physical theories in a precise way without recourse to formal languages.

Ludovicus: Eh? – I see, this turns toward my late philosophy ...

Thomas: No rush, Ludovicus! The Suppes approach – later the fully developed approach was called “Metatheoretical Structuralims” – uses informal logic and set theory. Already in the 1950s Suppes proposed to include the axiomatization of empirical theories of science in the metamathematical program of the French group “Bourbaki” [14] and in the 1970s the physicist Joseph D. Sneed developed informal semantics meant to consider not only mathematical aspects, but also application

[14] “Nicolas Bourbaki” is the pseudonym of a group of mathematicians in France in the 20th-century. Since 1935 the group presented their mathematical research work in a series of volumes. It was the aim of the group to establish a founding of all mathematics on set theory. [3]

subjects of scientific theories in this framework, based on this method[15] Sneed presented this view as stating that all empirical claims of physical theories have the form "x is an S" where "is an S" is a set-theoretical predicate. Every physical system that fulfills this predicate is called a model of the theory. Let's take again our example of towers: "x is a tower " means that every thing that fulfills the predicate "tower" is a model of our little tower theory. Thus, there is the set $M(Towers)$ of all tower models!

Carolus: What's the use of it?

Ludovicus: I think I got the picture! The Carnap approach and also the other approaches of Logical Empiricism regarded a theory as being a system of logical propositions i.e. linguistic formations. In contrast, this metastructuralist view of Sneed considers a theory as an object that comprises mathematical structures.

Thomas: That's it, Ludovicus. To make it clear, let's get back to our little tower example! Wikipedia explains: "A tower is a tall [architectural] structure, usually taller than it is wide, often by a significant margin."[16]

Now, we follow the framework of the metastructuralist approach to reconstruct a theory of towers – however, this will be a very simple theory and we name it "Wiki-Theory of Towers", WTT for short. First we have a set of all architectural structures x that have a significant margin. We call this set $M_p(WTT)$ the *potential* models of our "Wiki-Theory of Towers" WTT.

We recall the condition in Wikipedia's sentence: the architectural structure *has to be taller that it is wide* to be a tower. We can phrase this condition as an axiom of our WTT: For this we have to indicate two magnitudes: the height $H(x)$ and width $W(x)$. To formulate the WTT's axiom we say $H(x) >> W(x)$.

Now, all potential models of WTT (all $x \in M_p(WTT)$) which in addition fulfill this axiom are models of WTT. Therefore, they build the set $M(WTT)$

Carolus: That's a nice set-theoretical finger exercise but can this approach picture all wherewithal to present modern science studies? Can it model the relationships between scientific theories, eg. specialization, generalization, reduction, and of utmost importance, the change and replacement of theories, the dynamics that was our subject in the last half hour?

Thomas: This can be done by set theory. Metastructuralists use thereto the concepts of set theoretical relations. – A specialization of a theory is represented by a more special axiom. That's food for thought: From our Wikipedia theory of towers, WTT, we could obtain a more special theory of towers by a special axiom, e.g. the theory

15 [14].

16 Not to be confused with the structures of the metatheoretical structuralism we completed Wikipedia's definition by the adjective "architectural"; see http://en.wikipedia.org/wiki/Tower

of *wooden* towers, the theory of *university* towers or the theory of towers *that are two times hight as wide*. Hence, we obtain a specialization of a theory by fulfillment of this additional axiom. In my last example that would be $H(x) = 2 \times W(x)$.

Therefore, the set of models of the more special theory, let's name it $WTT_{H=2\times W}$, are a subset of the models of the general theory WTT, i.e. we have the set relation $M(WTT_{H=2\times W}) \subseteq M(WTT)$ for this specialization relationship between the two theories.

Carolus: I wonder – and who'd have thunk that I would take the part of an Empirist – where is the connection to empirical phenomena in this view? Where are the scientists observation terms?

Thomas: You are right, Carolus, besides the layer of theoretical structures there has to be another one that comprises the empirical data and phenomena. Though scientists establish laws and they introduce empirical theories that say that the laws hold for the data, they also observe real systems or phenomena and they measure data. That is to say: To study systems or phenomena in reality, scientists connect them with a theoretical structure. To this end they give the real systems a structure themselves. Woflgang Balzer, one of the today's metastructuralists wrote in this regard: "How to do that is not clear! — This is one of the central problems in the philosophy of science. [...] The problem is that we create a connection between real systems and theoretical structures. We assume that this can be done. Without this assumption it is senseless to talk about empirical science."[17]

In Metastructuralism the connection between non-theoretical and theoretical structures of a theory T, i.e. the potential models in $M_p(T)$ and the models in $M(T)$ are represented by another set theoretical relation, the so-called *Theoretization*. What is important is that in this view theoretical terms are theoretical relative to a theory T, i.e. a concept is not theoretical at all but it is T-theoretical to the respective theory T.

Let me try to explain this in terms of our example, the theory of towers. Let's say *hight* $H(x)$ *and width* $W(x)$ are theoretical concepts relative to our theory WTT. That means that *hight* $H(x)$ *and width* have got their meaning by the theory WTT, they are WTT-theoretical concepts.

If we remove all theoretical terms of a theory T in its potential models $M_p(T)$, then we get structures in a T-non-theoretical layer; we call these structures the "partial potential models" of theory T and we name their set $M_{pp}(T)$. In our example, without the WTT-theoretical concepts of *hight* $H(x)$ and *width* $W(x)$ we obtain $M_{pp}(WTT)$. These are obviously all architectural buildings.

Finally, every empirical theory T has a class I of intended application systems that is a subset of all partial potential models in $M_{pp}(T)$. E.g., for the tower example Wikipedia says that all "architectural structures that have a significant margin". I try to draw the whole metastructuralist conceptualization on the ground. The sets $M_p(T)$ and $M(T)$ and the sets $M_{pp}(T)$ and I are located in different "layers":

[17] The sentence that is quoted here is in the German book [1], the translation into English is by the author.

Thomas draws Figure 113.3 on the ground.

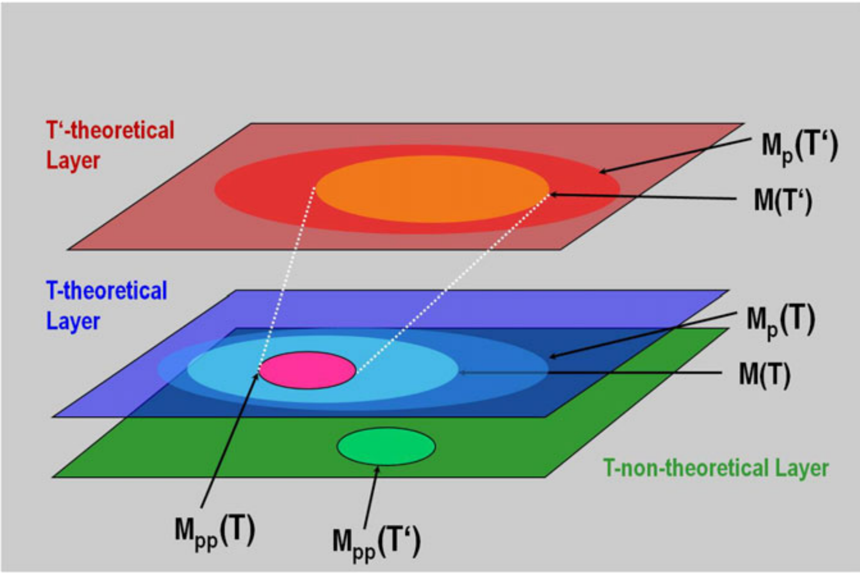

Fig. 113.3. Structures of the intended application systems of theory T, its theoretical structures and of their Theoretization T'

$M_{pp}(T)$ and I are structures in the layer of empirical concepts, whereas $M_p(T)$ and $M(T)$ are structures in a theoretical layer of this schema. And with a spotted lines I indicate the connection between the two layers, the "Theoretization" between the theories T and T'. Indeed, this is a set-theoretical relation for it holds: T' is a theoretization of T if and only if $M_{pp}(T') \subseteq M(T)$.

Carolus: Now, I am very interested in your answer to my next question: Have these metastructuralists also set-theoretical tools to model the change of theories, say, from T^{old} to T^{new}, as the change from Ptolemy's geocentric universe to Copernicus' heliocentric model or from Newtonian Mechanics to Einstein's Special Relativity Theory? Can they express these scientific revolutions by set theory?

Thomas: Well, Carolus, they try to do this. They defined an intertheoretical relation that is called "reduction" to reconstruct these kind of theory change. Let me see whether I can sketch this as well!

Thomas draws Figure 113.4 on the ground.

Thomas: If you have two theories, say T_{old} and T_{new}, they say that T_{old} reduces T_{new} if the following conditions are fulfilled:

1. $\rho \subseteq M_p(T_{old}) \times M_p(T_{new})$
2. For all x and x': if $< x, x' > \in \rho$ and $x' \in M(T_{new})$, then $x' \in M(T_{old})$.

Ludovicus: Stop! Please stop! This discussion became to mathematical to my mind! – Thomas, it was your intention to find a combination of your ideas in history of science and my philosophical system of family resemblances. But what you discussed in the last minutes and what you showed in your drawings has nothing to do with my philosophy. I doubt that this kind of set theoretical relation can represent theory changes like scientific revolutions. As you both agreed some time ago, these scientific revolutions that you, Carolus, called "theory replacement" and that you, Thomas, explained as a paradigm shift, are not pure rational changes. Between the old and the new theory there is no one-to-one-relation, therefore we have to respect some unsharpness in these transformations that can not be represented by hard math!

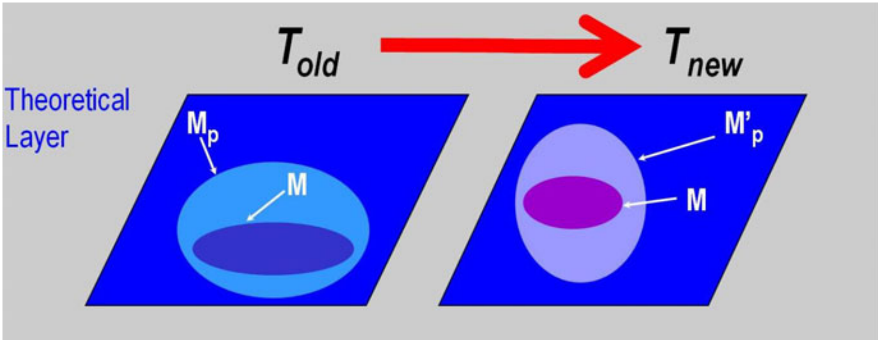

Fig. 113.4. The relation of reduction between theories T^{old} and T^{new}

Thomas: You are right, Ludovicus, and I wanted to add that to represent changings of theories or paradigms instead of this reduction relation very often a so-called approximative reduction is used.

Carolus: Oh, I see. With this "approximative reduction" they can try to reconstruct paradigm shifts as the one from Newtonian Mechanics to Quantum Mechanics, or to the Special Relativity Theory, isn't it? How does this look like?

Thomas: Oh, there are different proposals. Some metastructuralists use the concept of converging series of models of a theory, others favored to establish topological spaces of models ...

Ludovicus: Stop it, please! – Again I have to say that this sounds to high-mathematically in my mind! I doubt that all these approaches that base on our precise system of mathematics will result in a success of our philosophical problems. I am looking for a tool that is appropriate to model language games!

At this point in my dream I got in the middle of the dialogue. Hitherto I was just a kind of a neutral bystander but now I could not keep still. – As is usually the case the holodeck would not run a program without interaction of the user. Interestingly, in that dream I borrowed some phrases from Lotfi Zadeh.

I: Please accept my apologies. Excuse my interrupting. I overheared you interesting debate. 50 years ago, in the year of the publication of your *Structure of Scientific Revolutions*, Thomas, a professor of Electrical Engineerig at this university in Berkeley wrote already on "the gap that reflects the fundamental inadequacy of the conventional mathematics — the mathematics of precisely-defined points, functions, sets, probability measures, etc. -– for coping with the analysis of biological systems." In that paper he said that "we need a radically different kind of mathematics, the mathematics of fuzzy or cloudy quantities which are not describable in terms of probability distributions."[18] Three years later he established a new mathematical theory[19]. He introduced "Fuzzy Sets" — as classes or sets that '"are not classes or sets in the usual sense of these terms, since they do not dichotomize all objects into those that belong to the class and those that do not". In fuzzy sets '"there may be a continuous infinity of grades of membership, with the grade of membership of an object x in a fuzzy set A represented by a number $f_A(x)$ in the interval $[0,1]$.'"[20]. Please do not hesitate to disagree with me: I think that Fuzzy Sets are a suitable formalism to solve your problems.

Thomas: Who is this scientist at this university? It is a pity that I never met him!

I: The name is Zadeh, Lotfi Asker Zadeh, better known as Lotfi A. Zadeh! His office on this campus is very close in Soda Hall. Having graduated with a Bachelor of Science in electrical engineering from the University of Tehran in 1942 and after working for a year as a technical contractor with the United States army forces in Iran, Zadeh came to the US in 1944. He applied to the Massachusetts Institute of Technology (MIT) in Cambridge, Massachusetts, and was accepted to continue his studies. For the thesis he completed with Robert M. Fano, he was awarded the degree of Master of Science in 1946. Then, he did move to New York, where he obtained a position at Columbia University as an instructor. In 1959 he had become a professor at Berkeley and 1963/64 he was chairman of the department of electrical engineering. Prior to the publication of his first paper on fuzzy sets in 1965, he was concerned with systems analysis, decision analysis and information systems. For his scientific work he received among many others the IEEE Richard W. Hamming Medal, the IEEE Medal of Honor, the ASME Rufus Oldenburger Medal, the B. Bolzano Medal, the Kampe de Feriet Medal, the Grigore Moisil Prize, the Honda Prize, the Okawa Prize, the IEEE Millennium Medal, the ACM 2001 Allen Newell

[18] The reader will notice that this is a quotation of the paragraph in that Lotfi A. Zadeh used the word "fuzzy"in a journal article for the first time, see: [18, p. 857].

[19] In 1965 Zadeh's "Fuzzy Sets" appeared: [19].

[20] The reader will notice that these are quotations from [19]

Award, the Nicolaus Copernicus Medal, the Franklin Institute Medal, the High State Award ‘Friendship Order’, and 25 honorary doctorates.

Carolus: Well, that is very, very impressive! I do not know many names of that highly honored scientists, but concerning his new mathematics, this “Fuzzy Set Theory”: the fuzzy set’s “membership values between 0 and 1 sound very much to be probabilities, that is nothing news!

I: Oh, scientists in the Fuzzy community had to listen to that very often in the last 50 years, but Fuzzy Sets and probabilities are totally different! The theory represents linguistic uncertainties, you can use my fuzzy sets to label linguistic terms that are values of linguistic variables. If you would say, e.g., that this squirrel is brown or that this tower is high, there is some impreciseness in these words ....

Ludovicus: That’s what I want to stress! Language is not precise!

I: Therefore you can use fuzzy sets to compute with these imprecise magnitudes and in Fuzzy Logic we established a tool for approximate reasoning. In the 1970s Abe Mamdani and Sedrak Asilian used Zadeh’s concept of Fuzzy Algorithms to write a small program that was able to control a steam engine and today there are so many fuzzy application systems...

Carolus: Well, that sounds to be an example for the pragmatic way of American engineering and I can accept that engineers are comfortable with “good enough solutions”, but we are philosophers and our reasoning has to be exact.

Ludovicus: Come on, Carolus, your question reminds me the Gottlob Frege’s opinion! In my view it’s obvious that we need Fuzzy Logic and Approximate Reasoning in epistemology and philosophy of science! In our example of towers: *One might say that the concept “tower” is a concept with blurred edges. “But is a blurred concept a concept at all?” Is an indistinct photograph [of a tower] a picture of a tower at all? Is it even always an advantage to replace an indistinct picture by a sharp one? Isn’t the indistinct one often exactly what we need? Frege compares a concept to an area and says that an area with vague boundaries cannot be called an area at all. This presumably means that we cannot do anything with it. But is it senseless to say: “Stand roughly there?”*[21]

Carolus: So, Thomas, what do you have in mind? How would you use this theory of fuzzy sets in philosophy and history of science?

Thomas: Why did nobody use fuzzy sets and fuzzy relations instead of usual sets and set relations to reconstruct all the structures in the elaborated Suppes approach? If I understand this man correctly then this “extension priciple” was very successful

[21] As the reader will notice, this is a slightly variation of parts of §71 in Wittgenstein’s [17].

in engineering in the last decades in which there was a literally “Fuzzy Boom” in the 1980s: after the steam engines they used fuzzy control in cameras, washing machines, rice cookers, ...

Ludovicus: There! – Why did nobody try to establish a “Fuzzification” of this Metastructuralism in philosophy and history of science?

I: Oh, I published some papers in conference proceedings and books to start this kind of research program a few years ago and I called it “Fuzzy Structuralism”. With this view on philosophy of science I tried to find an approach to bridge the gap between science and technology on the one hand and humanities and social sciences on the other hand!

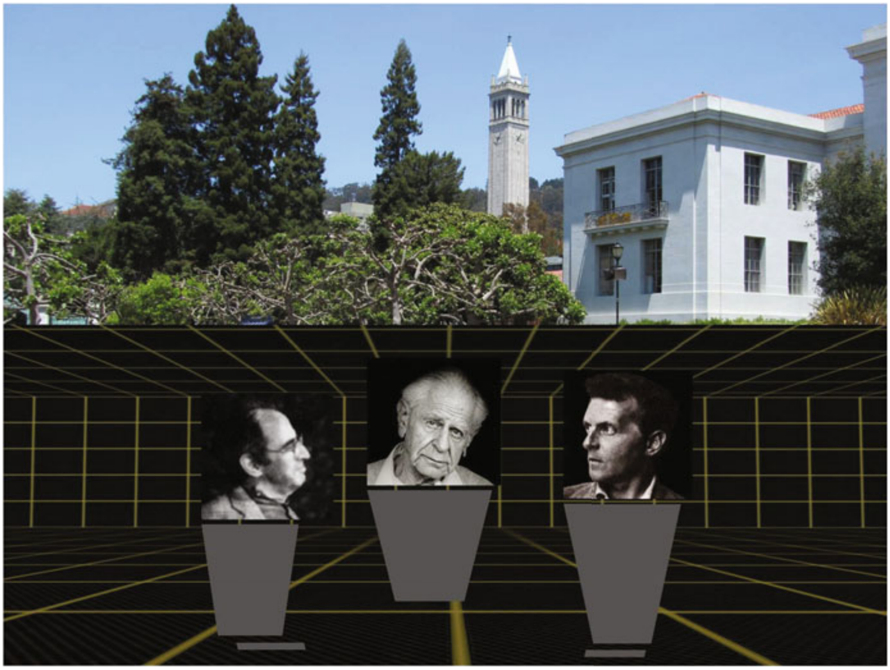

Fig. 113.5. The holodeck-avatars debating at the Berkely campus

Ludovicus: We will get this out! – Carolus, because you said that our reasoning has to be exact, I want to direct your attention once more to Wikipedia’s explanation: “A tower is a tall [architectural] structure, usually taller than it is wide, often by a significant margin.” There are two words that are anything but exact, these words are “usually” and “often”. I would say that all so-called “definitions”, even in science include such imprecise concepts to a greater or lesser extent because they use linguistic terms. I think that Fuzzy Logic is suitable to represent the kind of reasoning we need in philosophy. As I wrote already 50 years ago: *The results of philosophy are the uncovering of one or another piece of plain nonsense and bumps that the understanding has got by running its head up against the limits of language.*[22].

Carolus: The sands are running out. We have to go.

Normally, Star Trek's holodeck-stories close when the user of the holodeck program says 'Computer, end program!" but instead of that, I suddenly woke up very confused.

References

1. Balzer, W.: Empirische Theorien: Modelle –Strukturen – Beispiele. Die Grundzüge der modernen Wissenschaftstheorie. Vieweg, Braunschweig (1982)
2. Balzer, W., Moulines, C.U., Sneed, J.D.: An Architectonic for Science. The Structuralist Program. Reidel, Dordrecht (1987)
3. Bourbaki, N. (pseudo.): Elements of Mathematics: Theory of Sets. Addison-Wesley, Reading (1968)
4. Carnap, R.: Der Logische Aufbau der Welt. Felix Meiner Verlag, Leipzig (1928); English: The Logical Structure of the World. Pseudoproblems in Philosophy (translated by Rolf A. George). University of California Press, Berkeley (1967)
5. Carnap, R.: Logische Syntax der Sprache (= Schriften zur wissenschaftlichen Weltauffassung, Band 8). Julius Springer, Vienna (1934); English translation: The Logical Syntax of Language. Kegan Paul, London (1937)
6. Carnap, R.: The Methodological Character of Theoretical Concepts. In: Feigl, H., Michael, S. (eds.) Minnesota Studies in the Philosophy of Science. The Foundations of Science and the Concepts of Psychology and Psychoanalysis, vol. I, pp. 38–76. University of Minnesota Press, Minneapolis (1956)
7. Fuller, S.: Kuhn vs. Popper: The Struggle for the Soul of Science. Columbia University Press, New York (2005)
8. Galilei, G.: Dialogo sopra i due massimi sistemi del mondo. Landini, Florence (1632); English: Dialogue Concerning the Two Chief World System (translated by Stillman Drake). University of California Press Berkeley (1953)
9. Hempel, C.G.: The Theoretician's Dilemma. In: Feigl, H., Scriven, M., Maxwell, G. (eds.) Minnesota Studies in the Philosophy of Science. Concepts, Theories and the Mind-body Problem, vol. II, pp. 37–98. University of Minnesota Press, Minneapolis (1958)
10. Jauch, J.M.: Die Wirklichkeit der Quanten. Ein zeitgenössischer galileischer Dialog. Hanser, Munich (1973)
11. Jauch, J.M.: Are Quanta Real? A Galilean Dialogue (with a new Foreword by Douglas R. Hofstadter). Indiana University Press, Bloomington (1989)
12. Kuhn, T.S.: The Copernican Revolution. Harvard University Press, Cambridge (1957)
13. Kuhn, T.S.: The Structure of Scientific Revolutions, 1st edn. University of Chicago Press (1962)
14. Sneed, J.D.: The Logical Structure of Mathematical Physics. Reidel, Dordrecht (1971)

[22] As the reader will notice, this is quote of a sentence in [17, §119].

15. Popper, K.R.: Logik der Forschung. Mohr-Siebeck, Tübingen (1934, 1935); English translation: The Logic of Scientific Discovery. Routledge, London (1959)
16. Wittgenstein, L.: Logisch-philosophische Abhandlung, Wilhelm Ostwalds Annalen der Naturphilosophie (1921); also the Edition of the works of Wittgenstein:Werkausgabe, vol. 1, includes Tractatus. Suhrkamp, Frankfurt am Main (1980); English Translation prepared by C.K. Ogden with assistance from G.E. Moore, F.P. Ramsey, L. Wittgenstein: Tractatus Logico-Philosophicus. Routledge & Kegan Paul, London (1922)
17. Wittgenstein, L.: Philosophical Investigations. Blackwell Publishing, Oxford (1953)
18. Zadeh, L.A.: From Circuit Theory to System Theory. Proceedings of the IRE 50, 856–865 (1962)
19. Zadeh, L.A.: Fuzzy Sets. Information and Control 8, 338–353 (1965)

114

Fuzziness, Probability, Uncertainty and the Foundations of Knowledge

Paul J. Werbos

114.1 Introduction

This paper will describe how fuzzy logic, neural networks and other fundamental approaches to the nature of knowledge and epistemology fit together, both at a philosophical level and at the level of practical technology. The views herein are my own, but the bulk of the credit really belongs to Lotfi Zadeh and to the unusual, rich dialogue he has created through the *Berkeley Initiative for Soft Computing* (BISC). Only this very special kind of dialogue can really bring out the many cross-connections which exist in these complex fields of research. Lotfi has done an amazing job of pushing the community just hard enough, through clear but tricky questions, to get ever deeper into a wide range of issues related to fuzzy logic and to soft computing in general. As a role model, he can be somewhat intimidating at times; how many of us could really keep up such an incredible pace past the age of 90, having such a huge impact on a world culture which is often so resistant to new ways of thinking? Like his ideas, his personal example is well worth remembering, no matter how high we build on these foundations.

In the discussions of the BISC list, we can see two major aspects to the study of fuzzy logic. The first major aspect is the use of fuzzy logic as a practical tool in engineering and information technology. Section 114.2 will discuss how fuzzy logic has become more and more popular in engineering in recent years. Section 114.3 will discuss the connection between fuzzy logic and neural networks, and how they can be upgraded to overcome the limitations of today's use of those technologies, for applications which demand the highest possible levels of performance. In discussing methods for applications like prediction and data mining, it will show how there is a kind of ladder of intelligent systems, of ever higher capability. The second main aspect is more a matter of logic and philosophy. The practical success of fuzzy logic has raised more and more questions about its greater significance. How does fuzziness relate to larger questions about probability, uncertainty, artificial intelligence, and the ways that we humans choose to do our own reasoning? Actually, our practical experience in making things work (sections 114.2 and 114.3) has a lot to say of importance to these basic questions. Section 114.4 will briefly state my views about fuzzy logic and logic in the normal situation, the classical universe which most us think we are living in. Section 114.5 will briefly discuss how that picture is changed by modern physics, from quantum theory to the future.

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114.2 Fuzzy Logic in Engineering: Meat and Potatoes Today

Fuzzy logic started a huge spurt of growth doe to Japan's *Fifth Generation Computer System* (FGCS) program of the 1980's.

Leaders of Japan have always understood that machine intelligence is a crucial goal for computer technology, and that hardware without the best software does not provide the most useful computing capability. Thus FGCS contained a major component of artificial intelligence (AI). In those days, the people who led (and still lead) the field of mainstream AI had given up on brain-like learning systems, and adopted a philosophy of "all words, all following instructions." More precisely, expert systems based on binary logic were seen as the route to higher order machine intelligence. This normally meant going to experts, to ask them for the "if then" rules needed to perform some task. The "intelligent system" would simply apply these hard-coded rules to decide what to do, in any application calling for an action or an answer in words.

As a matter of wise policy, Japan decided not to put 100% of their AI resources into just one paradigm. Thus they set aside about 10% for work which would still use if-then rules, but would use fuzzy logic to implement the rules.

For example, consider the rule:

"If the boiler is too hot and pressure is rising, turn it off."

In traditional AI, "boiler is too hot" would be translated into a logical proposition which is always either "true" or false;" in other words, a rule is provided which turns the phrase "boiler is too hot" into a function of available information, a function which is always zero or one. If they are decent engineers, they also pick a critical level for the rate of increase of pressure, and get a function which is one or zero. In the resulting control system, there is a critical threshold for the temperature and for the rate of increase of pressure; as soon as that threshold is reached, the boiler is turned off quite abruptly.

In the fuzzy logic version of the same rule, we would simply develop more continuous functions to represent the two input conditions, and we would perhaps consider dialing down the boiler gradually (if it is the kind of boiler which allows that). We can apply the same general approach, but without the very abrupt transitions, which really have no basis in the physics of the task anyway.

The Japanese reported that the traditional expert systems led to very little real benefit, but the fuzzy logic projects succeeded and supported many practical applications. Much of the industry in Japan then started to produce products using fuzzy logic for appliances and many other areas. The same simple comparison between binary logic and fuzzy logic applies to many engineering applications today, where rules from experts can be useful but binary functions do not fit as well to the physical reality, which involves many continuous variables. Of course, it also helps that fuzzy logic is very easy to use in this way.

From the viewpoint of AI people, already committed to using rules, the benefit of fuzzy logic is that it lets us develop rules which fit physical plants better than binary rules do. From the viewpoint of control engineering, which already uses continuous variables, the main benefit is that fuzzy logic provides an understandable interface to the human. It makes it easier for the human to discuss and create controllers, and to explain the controllers in words to other humans.

An interesting example may help shed light on this second benefit, and on some common mistakes in science. Legally, I cannot name names, but I can tell the story in general terms.

One year, two proposals came to NSF (National Science Foundation), from different people. One proposed to develop a fuzzy logic controller for a certain kind of system. There would be work to improve performance and work to prove stability and so on. Another proposal wanted to develop a "soft switching gain scheduling controller" for the same task, with the same methods used to analyze stability and performance. In fact, almost all of the equations in the two proposals were exactly the same. Both went to similar types of reviewers.

In the end, both proposals received a mix of something like "very good" and "poor," about 50-50. But the kinds of reviewers who rated one as "poor" would rate the other as "very good," and vice-versa. The same work, the same mathematics – but human beings are distracted by words, and do not always penetrate to what is really going on, even when they are very proud of their mathematical skills.

In my case, I would have given a slight edge to the fuzzy proposal, because IN ADDITION to the mathematics, it also would provide a human interface.

Because the key advantage of fuzzy logic here is the natural "white box" interface to human beings, research on that interface with human beings could be important in telling us how to make the best use of that advantage, and how to make the best use of fuzzy logic and related approaches in general.

114.3 More Advanced Possibilities for Fuzzy Logic in Engineering

In principle, fuzzy rules could be applied to any task in the domains of decision, control and management or in prediction, classification, data mining and state estimation. However, in most of those applications, feedforward soft-switching simply does not give the greatest possible performance. Many companies and researchers do not need or cannot afford the highest possible level of performance; in those cases, simple fuzzy rules can often be a superior alternative to other low-cost off-the-shelf tools such as the simple forms of reinforcement learning or machine learning taught in first-year courses in computer science. But in some applications domains (and research), the key challenge is how to get to maximum achievable performance.

This leads into a key challenge for fuzzy logic: how can we design systems across this range of domains, which can still use fuzzy logic and provide a "white box" interface, but also provide the greatest possible level of performance by some kind of quantitative metric?

To narrow down this question, and avoid trying to write a whole textbook here, I will focus on two more specific but still very general domains:

1. How could we use fuzzy logic in "cognitive optimization" – developing learning or data based systems for computing optimal decisions or controls $\boldsymbol{u(t)}$ to manage or assist complex systems, in the general case where there is assumed to be some kind of nonlinearity, random disturbance and complexity?
2. How could we use fuzzy logic in "cognitive prediction" – developing learning or data-based systems for modeling, simulating, classifying or inferring the state of systems which send us a stream of time-series data $\boldsymbol{X}(t)$ which is assumed to be just one "window" into a larger system $\boldsymbol{x}(t)$ governed by

$$\boldsymbol{x}(t+1) = \boldsymbol{f}(\boldsymbol{x}(t), \boldsymbol{u}(t), \boldsymbol{e}(t)) \tag{114.1}$$

   where $\boldsymbol{f}$ is an unknown function, $\boldsymbol{u}$ is a vector of exogenous variables – often just the decisions of a companion control system, and $\boldsymbol{e}$ is a vector of random numbers? (Of course, this also includes inferring causality in data mining.)

Note that equation 114.1 may look simpler than the general "state space" model used in control theory, but it is trivial to convert any stochastic state space model to this form.

Fuzzy Logic and Cognitive Optimization

Twenty years ago, many researchers were interested in developing a new kind of intelligent control system, combining the highest "intelligence" available from AI with control systems based on the more popular forms of classical control theory. NSF and the *Electric Power Research Institute* (EPRI) jointly sponsored a workshop on intelligent control to evaluate the possibilities. That in turn led to a Foreword to [1] authored by five NSF Program Directors, including the Deputy Associate Director of Engineering, containing Figures 114.1 and 114.2 below:

Figure 114.1 basically reminds us that there is a natural, smooth interface between fuzzy logic and neural networks, unlike the awkward cut-and-paste interface many were proposing for binary AI and linear control designs. Since both approaches deal with nonlinear functions, focused mainly on variables ranging from 0 to 1, a better interface should be possible by combining those two approaches.

Figure 114.2 suggests a more specific way to combine the two. We can use fuzzy logic to implement a control design which reflects the domain knowledge of the user, and use the same kind of learning methods developed in the neural network field to adapt that controller further, for maximum performance. And then we can still go back and tell the user what the new adapted fuzzy controller actually looks like.

Translating that vision into practice is not as simple as it may sound. Where do we find the learning or training methods able to tune the fuzzy logic system (or anything else) for maximum expected performance, in an uncertain environment? And how do we get stability results after we do?

Before we can maximize a measure of performance, we need to decide what that measure of performance $U(t) = U(\boldsymbol{X}(t))$ actually is. The problem of maximizing the expected value of $U(t)$ summed over future time, is a dynamic programming

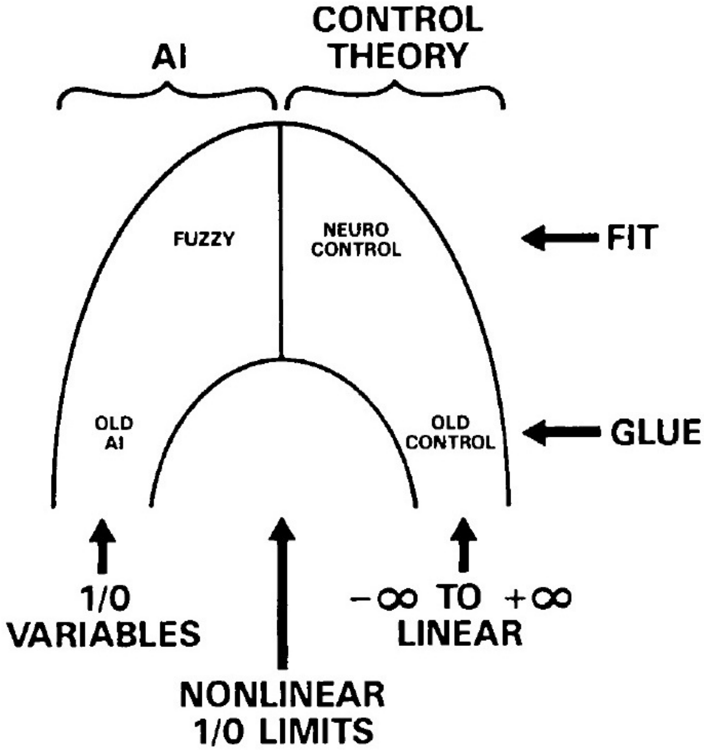

Fig. 114.1. Approaches to Intelligent Control

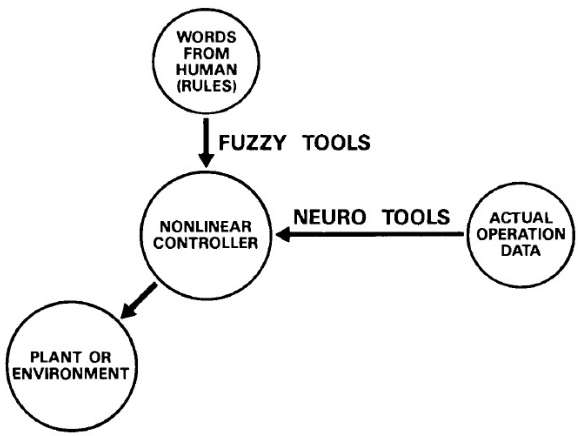

Fig. 114.2. Combining Fuzzy Logic and Neural Networks

problem. Some would say that this is "simply" a Partially Observed Markov Decision Process (POMDP). There is no computationally feasible general method for exactly solving a POMDP, because of the "curse of dimensionality." However, new general methods have been developed for *approximating* and *adapting towards* the optimal solution. There is a substantial body of research and applications in the new field of adaptive or approximate dynamic programming (ADP) summarized in a stream of books based largely on projects funded by NSF and by large-scale industry efforts [35, 20, 12]. Unlike the later simple forms of reinforcement learning, which generally assume binary logic or lookup tables, these methods were initially developed for the case of continuous variables, as in fuzzy logic systems or neural networks or econometric models.

One effective way to address stability issues is formulate a utility function U which penalizes any excursion into undesired states. Many stability theorems have been proven showing that "solving the Hamilton Jacobi Bellman equation" (i.e., maximizing U) gives the most robust possible controller in the general nonlinear case. There is a large literature on many approaches to stability in ADP [35, 20, 12].

Regarding stability – some theoreticians have argued that we will never be able to use anything but classical linear control in systems like aerospace or cars, because those industries depend on theorems for the whole of their business. This is simply not accurate. Most higher-level complex systems are inherently nonlinear, and they require verification and validation methods more like "six nines." As an example – Chuck Jorgenson of NASA Ames Research Center demonstrated years ago how a simple neural network controller could restore stability and land a full MD-11 commercial airplane after all the hydraulic controls were locked up, as part of their Reconfigurable Flight Control program. He reported that many people were initially skeptical about whether stability could be restored at all when an aircraft is so crippled. He said that demonstrating that with a live physical MD-11 was their second greatest achievement of the year. Their greatest achievement was the suite of verification and validation procedures [19] needed to get permission to do this on a live airplane. His work was essentially a spinoff from the seminal initial work by White and Sofge at McDonnell-Douglas, reported in [35].

But how can we apply these methods to get optimal performance out of fuzzy controllers? All the basic methods of ADP – Heuristic Dynamic Programming (HDP), Dual Heuristic Programming (DHP), and their action-dependent versions – are presented in spinoff from the seminal initial work by White and Sofge at McDonnell-Douglas, reported in [35] with pseudocode *for the general case* in which the controller may be a neural network, a set of fuzzy rules, or any other differentiable system. George Lendaris of Portland State (pdx.edu) has published many demonstrations of how to apply ADP to training fuzzy logic systems to solve difficult, serious challenges in nonlinear decision and control (e.g. in [20]).

Lendaris also made many of the tools available on his NWIL website, but there is a crucial need for further research to make these kinds of tools easier to use and more widely available. Because the quality of ADP depends on the quality of one's understanding of the system one is trying to optimize, and because function approximation issues are crucial in ADP, section 114.3 is important to achieving the best performance in cognitive optimization as well.

Fuzzy Logic, Cognitive Prediction and the Quality of Function Approximation

People often say that extreme polarization of political parties in Washington, based on crisp ideological positions, now makes it very difficult to implement a unified synthesis or to make progress on many important issues. The same may be said of the many different disciplines and schools of thought which address different aspects of cognitive prediction. Among those disciplines are statistics, neural networks, signal processing, system identification in control theory, and machine learning in AI.

This section will only present the basics of this challenge, and its connection to fuzzy logic, but even so the story is not simple. I will take a kind of step-by-step historical approach, for the sake of clarity.

Early Foundations: Maximum Likelihood and Bayesian Approaches

How can we identify or estimate the function $\boldsymbol{f}$ in equation 114.1 based on data on the observed data $\boldsymbol{X}(t)$ for a time-series or string of observations, from $t = 1$ to $t = T$, or from a set of such strings?

In 1972, when I began work on cognitive prediction for my PhD thesis at Harvard, the mainstream of statistics was moving towards a powerful consensus about the best way to perform such tasks – "maximum likelihood theory." In classical maximum likelihood or Bayesian statistics, one tries to find the function $\boldsymbol{f}$ out of some set F of possible functions which has the highest probability of being true; one "calculates" the probability of truth by using Bayes' Theorem as follows:

$$Pr(\boldsymbol{f} \mid \text{Data}) = \frac{Pr(\text{Data} \mid \boldsymbol{f})Pr(\boldsymbol{f})}{Pr(\text{Data})} \tag{114.2}$$

where "Data" refers to the entire dataset of $\boldsymbol{X}(1)$ and $\boldsymbol{u}(1)$ through $\boldsymbol{X}(T)$ and $\boldsymbol{u}(T)$. These vectors may be made up of any combination of continuous or binary variables, though we usually emphasize the continuous case. In classical Bayesian statistics, the user would supply the function $Pr(\boldsymbol{f})$, the prior probability function, which represents what he or she already knows prior to considering the data. In maximum likelihood statistics, people would usually assume that all models are equally likely apriori, so that we pick the function $\boldsymbol{f}$ in F which maximizes $Pr(\text{Data} \mid \boldsymbol{f})$, which is called the "likelihood term." The likelihood term can be calculated directly by use of equation 114.1 and basic probability theory. Theoreticians like Carnap and Jeffreys (leaders of the maximum likelihood school in philosophy) and Rao (who originated "information geometry") modified this recipe by recommending functions $Pr(\boldsymbol{f})$ which slightly penalize functions with excessive degrees of freedom; however, these adjustments were normally very small, like using $\sqrt{T-n}$ instead of $\sqrt{T}$ in ordinary statistics [36].

Software packages were developed which would allow the user to insert some kind of prior probability function $Pr(\boldsymbol{f})$ into the computer, along with data, and learn what function $\boldsymbol{f}$ has the highest probability. But those software packages never became very popular, for many reasons. Users felt very uncomfortable about encoding their knowledge into the forms of $Pr(\boldsymbol{f})$ allowed by these programs. They also found it difficult to disentangle their beliefs at that stage from what they learned from data. And most of these users wanted to write reports or journal articles in which the statistical results as such were "objective." Many believers in the Bayesian viewpoint (like myself at that time) felt that they could more easily convolve knowledge from data with knowledge from other sources *after the fact*, in the later stage when they interpret the meaning and significance of what the statistics said. In cognitive prediction, we seek to build universal learning systems, capable of converging to the right model, in an open-minded way, without the need to have "the right" prior probabilities. And so, we gravitated towards the maximum likelihood approach, which appeared to offer a kind of universal direct brute force recipe for tuning and evaluating any well-defined stochastic model.

Notice that maximum likelihood theory still lets us pick any family F of functions that we choose. Usually, when we have prior knowledge about a system we are studying, we will simply choose a family of functions F which incorporates that knowledge. Usually, we do that by specifying F as a *family* of functions defined by $\boldsymbol{f}(\boldsymbol{x},\boldsymbol{u},\boldsymbol{e},W)$, where this function $\boldsymbol{f}$ is specified by the user as a function of weights or parameters W. Maximum likelihood methods are used to find the set of weights W which yields the maximum probability of truth (still calculated by use of equation 114.1). If the user picks a few possible choices for the functional form f, maximum likelihood statistics also let us compare the degree of fit and degree of probability of the alternatives.

Notice that maximum likelihood theory allows us to pick functional forms $\boldsymbol{f}$ *which represent a set of fuzzy rules, or any type of neural network.* There is absolutely no conflict here between the probabilistic principles of maximum likelihood theory and the use of fuzzy logic or neural networks (or both).

Many strange misconceptions have emerged amongst very proud people who do not seem to understand the simple basic principles here. For example, some have asserted that it is "scientific" to require that $\boldsymbol{f}$ must be linear, and "unscientific" to allow nonlinear functional forms $\boldsymbol{f}$ – even when modeling systems which are well-known to be nonlinear, and even when modeling neurons themselves. In fact, when one uses a set of fuzzy rules or a neural network, and one uses maximum likelihood methods to estimate the weights W and assess the quality of fit, this is mathematically a special case of nonlinear regression. It is no less rigorous and no more rigorous than nonlinear regression in general. Years before Sutton's paper on "TD" methods appeared, I presented both ADP methods and neural network methods for cognitive prediction in precisely this way [24]. All the statistics which are familiar from nonlinear regression are available in these cases as well. The appropriate error functions for Gaussian continuous variables and for binary variables are well-known [36], and may be used as easily and rigorously here as in conventional statistics.

Nevertheless, there are some important practical issues which need to be addressed, in getting maximum performance in cognitive prediction, even when we stick to the maximum likelihood approach.

I will discuss these first in the "static" case of supervised learning, and then in the more general case.

Towards Maximum Performance for Models $\boldsymbol{X}(t+1) = \boldsymbol{f}(\boldsymbol{u}(t), \boldsymbol{e}(t), W)$

Many times in fuzzy logic and in neural networks, we are asked to learn some kind of simple static mapping, from a vector of inputs at one time to a vector (or classification) of outputs. In neural networks, we call this a "supervised learning" problem. Usually, the inputs are called "X" and the outputs are called "Y", but here I am using different notation, simply to make it clear that this is just a special case of equation 114.1. This is the special case where we "see everything" (where $\boldsymbol{X}$ is the same as $\boldsymbol{x}$) and where there is no memory of the past embedded in the model. Supervised learning or classification is used very often on datasets without any time dimension.

Maximum likelihood offers us a complete recipe for how to find W and evaluate the degree of fit, in this case. There is a lot of art for the user in finding plausible sets of fuzzy rules $\boldsymbol{f}$ and in thinking about how random disturbances $\boldsymbol{e}$ might affect the system being studied. The art of how to translate one's prior knowledge and values into stochastic fuzzy systems is important [32, 25], but beyond the scope of this chapter. But once you have done that translation, it is straightforward in principle to apply equation 114.1.

In practice, two difficulties arise.

First, finding the weights W which maximize $Pr(\text{Data} \mid \boldsymbol{f})$ is not a trivial computation, in general. People have been known to complain about iterations, local minima and such with simple neural networks; however, the problem is far worse in traditional nonlinear regression packages, where numerical instability actually occurs more often, depending of course on the choice of $\boldsymbol{f}$ by the user. Both with neural network models and with smooth fuzzy logic systems, we can use the original, general form of backpropagation to compute the derivatives more easily [28, 29]. The simple restriction to variables between 0 and 1, or -1 and $+1$ (except in some scaling preprocessors or postprocessors as needed) avoids the most common forms of instability one tends to see in nonlinear regression. There is really no way to provide useful absolute guarantees that one has found a truly global minimum of error, in the general nonlinear case, in less than astronomical time, in any form of nonlinear regression; however, as Widrow has pointed out, it is useful enough if our estimation package always *improves* on the user's initial guess for W, and if it allows us to *compare* the degree of fit of outcomes which emerge from different starting points.

In the 1980's and 1990's, many methods were found to get much faster convergence and learning with backpropagation than naïve steepest descent or naïve use of the simplest methods from operations research. For example, I had some success myself with my adaptive learning rate algorithm [35] and with varieties of extended Kalman filter learning [10]. Others have found it useful to use the signs of derivatives, at times, rather than respond to their magnitudes; for example, many people swear by RPROP. Similar kinds of methods could also be used with systems of fuzzy rules. I am not aware of any really comprehensive review or analysis of the many ways available to speed up learning or convergence.

Second, in order to really get to maximum performance in the general case, we need to choose a functional form $\boldsymbol{f}(W)$ which can do a good job of approximating the true best function $\boldsymbol{f}$ no matter what the true function is. Researchers showed long ago that there are many "universal approximators" which can approximate "any" function $\boldsymbol{f}$ to any desired degree of accuracy, if you add enough terms to the approximator. Among the many universal approximators are fuzzy rules, multilayer perceptrons (MLP, the most popular form of artificial neural network), radial basis functions, Taylor series, spline functions and simple or interpolated lookup tables.

Unfortunately, most of these universal approximators are not very useful in practice when the state vector $\boldsymbol{x}$ consists of more than just three or four continuous variables. They can approximate more complex functions in theory, but only when more and more parameters are added, which raises *both* the cost of computation *and* the amount of data required to learn the approximation. For the standard linear basis function approximators, like Taylor series or Gaussian approximators, the cost grows exponentially with the number of variables [20], just as it does with simple look-up tables. But when the true function $\boldsymbol{f}$ is a smooth function, Barron has proven [20, 3] that the cost of using an MLP rises only as a gentle power function of the number of variables, and that normal learning procedures do well enough.

Many have interpreted Barron's results as follows. For applications which do not demand high performance, or which only involve three or four variables, you have a wide variety of choices, and you can afford to use things which are easy and fast. But if there are more input variables, and you want high performance, the best choice is to use an MLP, even though the learning and convergence sometimes take work.

On the other hand, what if we want to make sure that our predictors or controllers are easy for humans to interpret and initialize?

In order to get maximum flexibility and performance from fuzzy logic systems, I have proposed that we use elastic fuzzy logic [28]. I would conjecture that elastic fuzzy logic (ELF) has the same kind of high quality universal approximation capability that MLPs do – but with an important caveat. For maximum flexibility, it is necessary that the learning system be able to change the definition of words or even learn new words altogether. ELF with a fixed vocabulary is the next best choice.

At the end of the day, humans who do not have a large enough vocabulary may simply not be able to achieve the highest level of performance in some tasks. We cannot escape the problem of choosing between mediocre performance, versus learning new words and new ideas (and new "features" of reality). Learning systems which are not able to learn new words or features simply cannot do as well as those which do, in the general case.

It is tempting to write a great deal more about "white box" versus "black box" modeling, and the issue of learning new words or features, but for reasons of length, I will move directly to another set of fundamental issues.

Toward Maximum Performance in the General Time-Series Case

Using fuzzy logic in the general time-series case (equation 114.1) involves all the same issues discussed in the previous section, but adds important new complications. (See [35, 28].) Actually, the progression from supervised learning to the time-series case is an excellent example of what happens in general when we progress from one level of intelligence, to the next higher level of intelligence. Everything in the lower level still applies to the higher level, but new dimensions are added. Neoplatonists have sometimes used the saying "As below, so above" (or vice-versa) to refer to this kind of connection.

The prediction and estimation of time-series is crucial to engineering, and also to our understanding of intelligence in the brain [31]. However, it has also become a very lucrative cottage industry for many people, making it possible for many to make money simply by grabbing well-known things off the shelf or following fads without regards to performance or serious underlying principles (or what happens to their clients). For this reason, through NSF, I have been funding a number of challenge competitions intended to get people thinking again about what actually works. Isabelle Guyon and Sven Crone have been particularly active in developing competitions which are brought to multiple conferences, not just neural networks but statistics and finance and others, in order to get competition across a very wide range of disciplines and methods. Empirical results alone are not enough to unify our understanding of the best way to do things, but they do play an essential role.

Shortly before the workshop on the time-series competition held at the International Joint Conference on Neural Networks in 2007 (IJCNN2007), Sven Crone told me he had sad news to report to me. There were many dozens of teams from all disciplines working very hard on his competitions. But the teams from machine learning, computer science and neural networks – clever and hard-working as they were – were all doing much worse in performance than those relying on simple basic time-series statistics. To do anything useful with time-series data, we need to have a thorough understanding of what the simple off-the-shelf methods were that let the statisticians perform so well at the beginning of the contest. Any university which graduates students who will work in cognitive prediction should make sure that they have a thorough understanding of the basic, seminal text by Box and Jenkins [4] which expounds those methods, widely used today in exactly the way that Box and Jenkins recommended.

Box and Jenkins developed tools and analysis which help us use simple stochastic models of the form:

$$X(t+1) = a_0 X(t) + \ldots + a_n X(t-n) + e(t) + b_1 e(t-1) + \ldots + b_m e(t-m) \quad (114.3)$$

which is called an "ARMA(n,m) process." (They used different letters, but this is the main model. They also introduced a variation based for nonstationary processes based on integration of this, beyond the scope of this chapter.) Notice that this is just a univariate model, describing a single time-series, not a vector X.

Also notice that there is only one random number, $e(t)$, generated at any moment of time. Of course, it did not take long for theorists to generalize this to the idea of a vector nonlinear ARMA model, NARMA (n,m):

$$\boldsymbol{X}(t+1) = \boldsymbol{f}(\boldsymbol{X}(t),\ldots,\boldsymbol{X}(t-n),\boldsymbol{e}(t),\ldots,\boldsymbol{e}(t-m)) \tag{114.4}$$

However, there are practical problems in building computer tools to address this general case, as I will discuss. Of course, this more general case would include the possibility that f may represent an elastic fuzzy system [28] or a neural network. There has also been discussion of vector NARMAX models, which also contain a vector of eXogenous variables u:

$$\boldsymbol{X}(t+1) = \boldsymbol{f}(\boldsymbol{X}(t),\ldots,\boldsymbol{X}(t-n),\boldsymbol{e}(t),\ldots,\boldsymbol{e}(t-m),\boldsymbol{u}(t),\ldots,\boldsymbol{u}(t-q)) \tag{114.5}$$

Box and Jenkins developed simple tools to estimate ARMA(n,m) models, based on maximum likelihood theory, and to guide the choice of n and m from empirical data. These tools may be viewed as the first truly universal learning machine, for learning from a database (as opposed to learning from one observation at a time). So long as the world is linear, and so long as one variable X is not affected by any other variables, this provides the greatest performance which it is possible to achieve. When this tool does not yield strong predictions, maximum likelihood theory would say that there is still no way to do better, because there is no way to make reliable strong predictions when there is not enough data to justify doing so. These tools have in fact continued to do quite well, compared to most other things. They are used in many practical applications where there is a lot of data but great difficulty in understanding what lies behind the data. Box and Jenkins showed that systems which start out being autoregressive (i.e. following equation 114.3 with $m = 0$) turn into full-fledged ARMA processes whenever there is random error in observing the system variables.

The readers should be warned that many control engineers were confused by reading papers in statistics in which "$\boldsymbol{u}$" was used to refer to random variables (what may of us call $\boldsymbol{e}$), rather than controls or exogenous variables; thus they would call equation 114.4 a "NARX system." I remember once trying to correct a student who said: "In our field, we do it right. We know that "MA" stands for exogenous, and that "X" stands for moving average."

In applications like large-scale econometric modeling, it is more common to use models of the more general form in equation 114.5, with $m = 0$, and with $\boldsymbol{f}$ a function which is linear in the weights or parameters, using functional forms chosen by the user based on great laborious study of the specific variables and domain being predicted. For example, the most accurate predictions of US industrial energy ever achieved (about 1% mean average percentage error for forecasts running ten years into the future) were obtained form a model of that type [26]. Models of this kind directly address, estimate and exploit causal relations, relations in which the change in one variable at time t lead to changes in other variables at time $t+1$. Strictly speaking, f is often defined implicitly in econometrics, by a system of the form:

$$0 = \boldsymbol{g}(\boldsymbol{X}(t+1), \boldsymbol{X}(t), \ldots, \boldsymbol{u}(t-q)) \tag{114.6}$$

which is called a simultaneous equation model. This is all still done by use of maximum likelihood.

For my own PhD thesis, I ended up working for a well-known political scientist, Karl Deutsch, who had written a book which described how neural network kinds of concepts could be used to understand political systems [6]. He had worked through many graduate students using traditional methods, all of which failed to yield good political forecasts based on his model of nationalism and social communications [5]. Looking at this system, I could easily imagine why conventional econometric methods might have failed: there was a lot of measurement noise here, which would convert the system into an ARMA kind of process, for which new tools were needed. But to combine the power of ordinary econometrics, in accounting for causality, together with the power of ARMA modeling, I would need to develop new tools.

The idea here was that linear vector ARMA models offer us the next level of universal learning system, still based on maximum likelihood theory, strictly better and more universal than the univariate version. I discussed this with George Box himself, when he visited Harvard. He was excited about this new approach in general, but the best algorithms he had been able to find anywhere [4] were absurdly expensive; they would rise in cost as the sixth power of the number of variables in the system. I remember vividly going to bed in the dorm one day, with very acute pain in my stomach, based on fear that I would be able to solve Deutsch's problem and not be able to graduate at all. I told myself: "I now know how to adapt a whole BRAIN – a neural network system – in $o(N)$. Why should a little system like this present such a problem?"

At that point, I realized that I could generalize the system I had developed for neural networks, now called backpropagation, so that it would work on any differentiable system, including vector ARMAX estimation (or elastic fuzzy logic). Since Harvard did not want to hear about neural networks at that time, this became the basis for my thesis [23]. More precisely, I proved the general chain rule for ordered derivatives (generalized backpropagation), and developed a tool in the Time Series Processor (TSP) software package at MIT to implement vector ARMAX estimation using backpropagation to estimate such systems. I used it to estimate Deutsch's model successfully. The TSP manager from the Federal Reserve heard about this tool, and ported it there, from whence it proliferated elsewhere. In 2011, the citation for the Nobel Prize in Economics stressed the pioneering work of the recipient in actually using vector ARMAX estimation in large-scale econometric modeling.

But even so, all of this is still just the linear case, and it relies on prior knowledge which is relatively rare even in real-world macroeconomics.

In the 1990's, I did some consulting for a small company called BehavHeuristics, based in College Park Maryland, which had contracts with several major airline companies to try to predict demand for seats and other things. At one point, they, like other people I knew, were locked in a perpetual competition with the Box Jenkins people. One day, their well designed MLPs would be predicting things better than the Box Jenkins models used in the airlines themselves; the next day, it would be the other way around; and it kept changing. The MLPs were much better in capturing causal effects across variables, and in learning the shape of nonlinearities, because of their universal approximation abilities [2,3]. But the simple univariate ARMA models were better in capturing time-series kinds of effects. So one day I said: why not get the best of both worlds? Why not use a time-lagged recurrent network (TLRN), defined by:

$$\boldsymbol{X}(t+1) = \boldsymbol{f}(\boldsymbol{X}(t), \boldsymbol{R}(t), \boldsymbol{u}(t), W)(+\boldsymbol{e}(t)) \tag{114.7}$$

$$\boldsymbol{R}(t+1) = \boldsymbol{g}(\boldsymbol{X}(t), \boldsymbol{R}(t), \boldsymbol{u}(t), W) \qquad ? \tag{114.8}$$

By the way, I had published papers on this general case long before people began "reinventing" many special cases of it. I put the random noise term here in parentheses, because they really were interested in prediction, not in stochastic modeling for its own sake. Nevertheless, in this project, as in many other situations, it made perfect sense simply to train the weight so as to minimize least square error, using standard maximum likelihood methods. Note how this is a generalization of equation 114.5, where square error is also being minimized.

My argument was that this is a kind of third step up, past univariate ARMAX and vector ARMAX. So long as we implement $\boldsymbol{f}$ and $\boldsymbol{g}$ as universal nonlinear function approximators, this becomes a universal NARMAX learning system – and the best we can do for a more general class of dynamic systems. In cases which truly are linear, the vector ARMAX and this system should both end up converging to the same answer, with the same quality of fit, when there is enough data – but when the system to be estimated is nonlinear, this system should converge to the right answer, more than the linear ARMAX could. BehavHeuristics thereafter found themselves beating the Box-Jenkins tools quite consistently after they made the transition. They used MLPs for $\boldsymbol{f}$ and $\boldsymbol{g}$, but if my conjecture of the previous section holds, elastic fuzzy logic should be able to do the same.

In 1992, I also had a chance to meet Lee Feldkamp of Ford, at a conference on fuzzy logic led by Prof. Yamakawa in Iizuka. Feldkamp's group was doing a very systematic in-depth evaluation of all the tools which might help them solve some companies critical to Ford. They were especially concerned about how to meet vey tough accuracy standards for predicting misfires, and other events, required by the tough new Clean Air Act. After experimenting with TLRNs, Ford became the world's practical leader in implementing these methods in an industrial strength way. The President of Ford made a commitment in Business Week in 1998 that all Ford cars in the world would be carrying TLRNs, to meet air quality regulations. Feldkamp's group did numerous important studies published at IJCNN and other conferences.

Given this history, in 2007, when Sven Crone told me how it was going, I quickly informed the Ford group. They did not have a lot of time to put into this, the way the student groups did, but it was easy enough to crank the competition data into their package, and see what came out. At the conference, Sven informed me that Ford had done the best of anyone at the competition.

The Ford package is not widely available, although the publications are. Lee Feldkamp retired relatively recently, and many of the capabilities of his group have since been replicated at Toyota [17], Siemens and elsewhere, with TLRNs systems just as powerful. At the energy/industry meeting at IJCNN07, industry representatives agreed unanimously that capability in using TLRNs was what they wanted most from students they might hire.

Though the Ford package was not widely available, the Neurodimensions package developed by Curt Lefebvre and Jose Principe [16] with NSF support had important TLRN capabilities, and some people also recommended the SNNS package from Stuttgard. Lefebvre set up a company to apply TLRNs to coal-fired power generators, and recently reported in discussion that his systems are now used in about 20% of the coal-fired generation in the US.

Two especially important papers on TLRNs appeared in the workshops in adaptive and learning systems, held by Prof. Narendra of Yale. In one paper [7], Feldkamp and Prokhorov of Ford reported on a comparison of three alternative methods for state estimation involving engines. Of course, they were paid to get the best results by whatever method they could find. They compared particle filters, TLRN and extended Kalman filters (EKF) on the same problem. They found that the well-executed particle filters and the TLRNs both outperformed EKF substantially on the same problem, which one might expense, since EKF is inherently a kind of local approximation, while the others converge to the right answer. However, because of the universal approximation property, the TLRN could achieve these results at much less computational cost and complexity. The fascinating result implicit here is that the recurrent vector, $\boldsymbol{R}$, acts as a condensed representation of the "belief state" of the system, the whole probability distribution for what the unknown variables might be; this makes sense, since minimizing square error in equation 114.7 does imply that. Another important result was by Eduardo Sontag, who showed that his formulation of the TLRN (more or less equivalent) acts as a universal approximator for dynamical systems, not just static mappings as in the work of Barron.

I would like to conclude this section with a few quick further observations.

First, equations 114.7 and 114.8 do not explicitly represent where noise might come from inside a stochastic process. Thus in 1990, I developed a still more general architecture, the Stochastic Envcoder/Decoder/Predictor (SEDP), which does exactly that, in the general nonlinear case for continuous variables [35]. Given the results of Feldkamp and Prokhorov, I wondered for many years just how useful or necessary that would be in a system like the brain, which could afford to use equations 114.7 and 114.8 for its real-time operation. However, it now seems clear that the brain must learn to make decisions and develop partial models to operate over larger time intervals, and cope with spatial complexity, as I will describe later; thus I now believe that some variation of SEDP must indeed be present in an intelligent system as powerful as the brain, and even that it can be mapped into known neural circuitry [31].

Second, the IJCNN competitions are an ongoing process, now large enough that a brief summary is difficult. In a later section, I will discuss spatial complexity and challenges which involve it. In IJCNN11, Ford itself sponsored a forecasting competition involving vehicle safety, for which more than 200 entries were received. They did not enter that contest themselves... but at the conference, when asked, they smiled and said that yes their in-house package continues to outperform everyone else, but it was useful for them to have continuing validation of that, and to identify good university groups to work with.

Third, just as Barron has proved some important theorems about universal properties of MLPs [3,3], there seems to be room to prove that something like TLRN is a universal learning system, in the sense that if the true $\boldsymbol{f}$ is sampled from the set of smooth functions (with higher probability for smoother functions), that a TLRN will always converge to the right answer as fast as anything else or faster, with no more than some bounded need for additional data for the comparison with any specific alternative. There is considerable room both for fundamental mathematical results, and new general purpose software and software guides, to bring this level of performance to more people, and make it the off-the-shelf easy alternative.

Many people believe that universal learning systems like the brain are impossible because of the No Free Lunch Theorem. No universal learning system can do as well as a hard-wired system designed for the specific problem domain. And yet, as Box and Jenkins showed us, we can develop systems which are universal for a certain class of environments; when they have enough time to learn, they can converge to a perfect job. Furthermore, the vector ARMAX systems are a perfect superset of the univariate Box-Jenkins systems; if a system happens to be a set of disconnected univariate time series, the simpler methods may be a bit better in the beginning, when data is limited, but it only takes a finite, bounded amount of learning for the more general system to do just as well, even in the special cases which fit the univariate model. In the same way, the TLRN methods for prediction of a NARMAX system are a superset of the vector ARMA methods, because of the universal function approximation capabilities in the TLRN.

Nevertheless, I doubt that TLRNs trained or tuned by maximum likelihood methods could satisfy this kind of theorem. I doubt that they could give us the best possible performance, even when we make allowances for learning time and assume smooth functions $\boldsymbol{f}$. The reason for this is that the maximum likelihood approach needs to be augmented in order to reflect the two key principles of dynamic robustness (which is very different from robustness as commonly practiced in control theory) and simplicity, to be discussed in the next section.

Well-designed TLRN packages offer us the greatest universal performance now available, but more powerful universal systems for cognitive prediction are possible.

From Maximum Likelihood to Simplicity, Robustness and Vector Intelligence

How could we build a universal learning system for cognitive prediction, which could actually satisfy the kind of universal learning theorem envisioned at the end of the previous section? I would use the term "vector intelligence" to describe such a universal learning system. Of course, the challenge of vector intelligence in systems which learn from one observation at a time is more difficult than the challenge of vector intelligence from a fixed database, where one is allowed to iterate over and over again, as in traditional statistics. This section will discuss the challenge of how to achieve vector intelligence in the usual offline case of statistics, in cognitive prediction. Although this gives more capability and universality than a simple maximum likelihood system, it is still only one step up on the ladder of intelligence. The next section after this will describe how to get higher than that, based on concepts more powerful than vector intelligence itself.

Two key principles need to be injected into a universal learning system in order to get to vector intelligence: (1) simplicity; and (2) statistical robustness. Simplicity comes first; I was already well aware of its importance in 1973, when I started my work using ARMA methods for my Harvard PhD thesis.

Simplicity, or Occam's Razor, may be viewed either as a neat way to cheat in order to prove the theorem, or as a fundamental principle of epistemology, philosophy and the nature of human knowledge. Both views are correct. Simplicity itself is not a simple thing to understand.

For centuries and centuries, philosophers have argued that everything we know about the world we live in is based on what we learn from our stream of experience. When we try to build universal learning systems, we are trying to build systems which do the same. But philosophers learned long ago that there are certain difficulties in this kind of learning. For example, consider the following two theories about the world we live in:

(A) The sun rises every day;
(B) The sun rises every day until ten days after the publication of this book, when a choir of angels will suddenly appear instead of the sun, and everyone who has only five fingers on each hand will be removed from the earth.

Both theories fit our past experience equally well. Thus from a maximum likelihood viewpoint, they are equally likely to be true. But do you believe it? Are you really expecting a choir of six-fingered angels to appear a few days after this book comes out? The obvious solution to this paradox is to penalize theory B somehow for the way it adds additional complexities which do not add to the empirical fit. We really have no choice about this, since learning systems for the nonlinear case cannot work without it. The mammal brain itself could not work without such a simplicity mechanism, as it struggles to survive in a world far more complex than the worlds of univariate linear time series. More concretely, if we want to build a computer simulator, to sample possible functions $\boldsymbol{f}$ to test how well different learning systems perform, we can't really do it unless we bias the sampling to favor functions simple enough that we can actually implement them.

The standard way to address this problem is to assume a prior probability distribution, $Pr(\boldsymbol{f})$ in equation 114.2, of the form:

$$Pr(\boldsymbol{f}) = c \exp\left(-kC(\boldsymbol{f})\right) \tag{114.9}$$

where C is a measure of the complexity of $\boldsymbol{f}$. This kind of prior probability is commonly called an "uninformative prior." The goal here is to assume the weakest possible prior probabilities, so that the learning system is as open-minded as possible, without preventing it from being able to learn what it tries to learn when it has enough experience to learn from. Even for static linear systems, when the number of variables is large, it is now well known [14, 11] that we get better performance from using "ridge regression," which assumes:

$$C(\boldsymbol{f}) = \sum_{i=1}^{n} W_i^2 \tag{114.10}$$

where n is the number of weights in the linear model.

In vector intelligence, we think of the time-series $\boldsymbol{X}(t)$ as a vector, as a set of numbers with no special structure, as in traditional control theory and as in Barron's theorems. We aim to develop learning systems which converge in the situation where functions $\boldsymbol{f}$ come from a probability distribution like that of equation 114.9, and where complexity is measured by something like Barron's measure of smoothness. If we "cheat" by using the same complexity measure (or something closely related to it) in the learning system itself, universal learning theorems should be possible. In practice, we could do this simply by using penalty functions when we train our neural networks or elastic fuzzy logic system; that generally boils down to picking weighs W which minimize something like:

$$P(W) + \sum_{t=1}^{T} E(\boldsymbol{X}(t), \widehat{\boldsymbol{X}}(t)) \tag{114.11}$$

where $\widehat{\boldsymbol{X}}$ is the prediction of our network, where E is a standard measure of error (such as square error for continuous variables or the logistic function [36] for binary variables), and where "P" is some kind of penalty function. We can use the same kind of process to sample possible "true" functions to test the vector intelligence of competing methods. The subject of penalty functions is a large area, important to vector intelligence, but for reasons of space I will not say more here. This subject is also an area where more fundamental well-grounded mathematical research is needed, connecting the concept of uninformative priors and universality with the design of mathematical learning systems.

In the end, we can build a "next generation" universal learning system simply by starting from TLRNs, but modifying them by the use of penalty functions. These penalty functions, like 114.10, must be much stronger than the simple allowances for degrees of freedom already familiar in maximum likelihood statistics [36] and information geometry. In practical econometrics, people can often achieve good results by using good judgment in their choice of simple, elegant functional forms $\boldsymbol{f}$, without having to use ridge regression, but for universal learning systems that is not good enough. This general approach to vector intelligence was where I started, in 1973, when I began the practical statistical work for my PhD thesis [23].

The empirical work with real data forced me very violently to give up the pure probability-oriented approach I have described up until now. At some level, the results were quite good, and followed the traditional script quite well. The vector ARMAX tool basically cut predictions errors in half, compared to traditional AR (regression) versions of the same models. That was basically true across a wide variety of actual and simulated data. The work on simulated data beautifully followed the goals for a more universal learning system. If, like many researchers today, I simply wanted to graduate quickly and avoid making waves, I would have stopped right there. But my conscience as a scientist told me that I should do some additional tests, to see how well these methods would do against some kind of simple "devil's advocate" alternative. I compared all of these forecasts (except for the more detailed modeling of language change in Norway) against the forecasts of a simple exponential growth extrapolation. The extrapolation generally got errors only half as large as those with the ARMA tool.

In later years, I found out that many people who work with data know that simple extrapolation can often work much better than elaborate models, multivariate or univariate. But I wanted to know why. After all, simple exponential growth is a special case of equation 114.3. Why do maximum likelihood methods fail so badly to find the best model in this case?

Since I was testing predictions over multiple time periods, there seems to be an obvious explanation. The extrapolation was tuned to get minimum error multiperiod prediction, where the maximum likelihood methods basically end up minimizing error in predicting just one period of time ahead. Thus I immediately proposed the "pure robust method," in which we tune models to minimize multiperiod error. In recent years, researchers such as followers of Vapnik have sometimes said: "Forget the probability of truth. Just tune the model so that it would have made you the most money in the past, if you had used it. Don't maximize truth, maximize dollars." Earlier researchers sometimes told me: "Don't think of these simple models as something true or false. Think of them as approximations to a more complex systems. Instead of probability of truth, think about the quality of approximation."

More careful logic convinced me that both of these extreme viewpoints – the probability of truth viewpoint and the past-dollars viewpoint – are highly deceptive. I showed that an intermediate kind of approach can do better than both. It is unlikely that brains could do as well as they do if they did not find a way to blend both. In the second half of chapter 10 of [1], I summarized these results, and provided some suggestions for how we could develop a more principled version of the working compromise I had already found.

Notice that these two extremes – the followers of probability of truth and the followers of approximation-only – are similar to the kind of people who debate fuzzy logic (approximate reasoning) versus probability, without asking how best to blend the two essential types of tool.

Unfortunately, today's culture for research in prediction still tends to be polarized between people loyal to one extreme or the other. The crucial work needed to develop an effective, universal synthesis has yet to be done. Just this past year, some important related new work has been reported by Mark Tobenkin under Professor Tedrake of MIT, under funding from the NSF Cognitive Optimization and Prediction (COPN) topic [25].

Once simplicity and robustness are fully incorporated, and cognitive prediction is paired with cognitive optimization, we should be able to develop universal learning machines for full-fledged vector intelligence. In 1990, I felt that we could explain the higher order intelligence of mammal brains as the emergent outcome of vector intelligence operating in the brain. To develop such a universal vector intelligence system is still an important, unfulfilled research challenge, because of the gaps I mentioned above. In my view, the *Handbook of Intelligent Control* [35] still is the closest thing we have to describing how to build such a system.

However, work in the 1990's convinced me that vector intelligence is still only one step on the ladder of general intelligent systems, and that mammal brains have a much more powerful underlying learning capability, to be described very briefly in the next section.

From Vector Intelligence to the Mouse

During the 1990's, I developed a new theory about the mathematics needed to carry us up from vector intelligence to the level of intelligence we see even in the simplest mammal brains, like the brain of the mouse. For reasons of length, I will not get into all the details here, which are summarized in Figure 114.3.

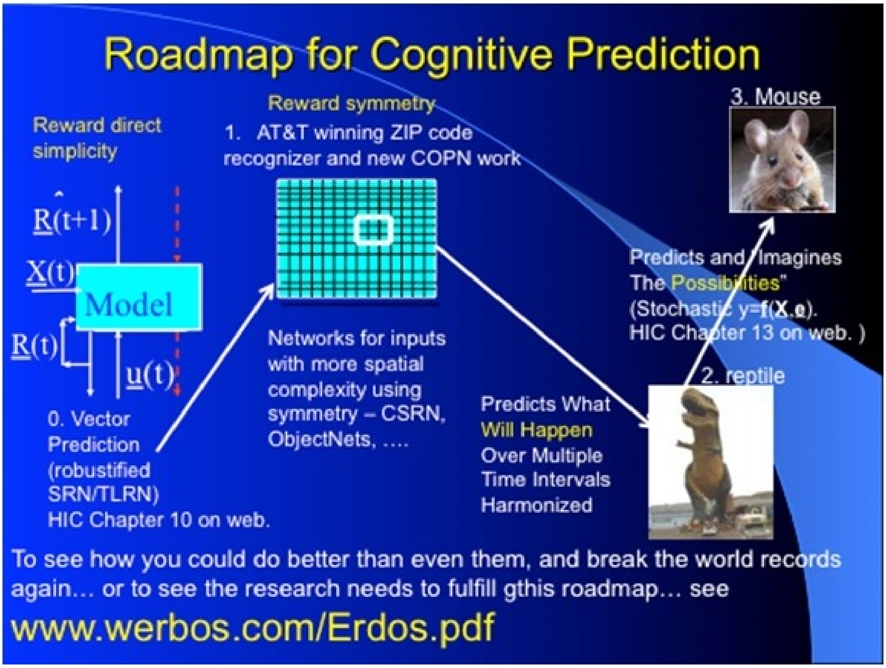

Fig. 114.3. A Roadmap for Cognitive Prediction from Vector Intelligence to the Mouse

For practical purposes, the theory is that we need to take *three* big steps up the ladder of intelligence, to climb as high as the smallest mouse. For today's research community, the most important challenge (aside from consolidating vector intelligence) is to really nail down that first step, rising up from vector intelligence to learning systems which can properly account for spatial complexity. The key to this is to learn and exploit symmetry properties in our environment, and eventually to connect them to what AI people call "world modeling" [1]. This entails developing and using new types of neural network and fuzzy logic systems such as the Cellular Simultaneous Recurrent Network and the ObjectNet [10], which allow us to handle thousands of inputs effectively in a general-purpose learning system, without needed to depend on hand-crafted feature detectors. This past year or two, researchers such as LeCun and Ng under COPN funding, and Schmidhuber under European funding, have broken all previous records on many complex well-known benchmark challenges in object recognition, phoneme recognition and others, by properly using of these. (See [32] and IJCNN07 for examples.) Liang, Venayagamoorthy and Harley [13] have shown how ObjectNets can be used to handle large-scale challenges in managing electric power grids which are crucial to the economics of renewable energy. A few years earlier, Fogel used a simple feedforward variety of Object Net to develop the world's first system able to play chess at a master class level by using only what it learned on its own, without humans inserting detailed rules specific to that particular game [8].

In formal mathematics, intelligence with spatial complexity addresses the problem of addressing functions f which come from more challenging distributions than from the set of smooth functions, but in which smooth functions are well represented as one possibility.

In my view, the challenge of reverse engineering the higher intelligence (cognitive optimization and prediction) seen *in* the mouse brain is the most important challenge to all of basic mathematical science in the new century we live in.

114.4 From the Mouse to Logic in the Classical Universe

Many philosophers have been debating: "What is the proper foundation for us humans to use in reasoning? Is it classical logic, or is it fuzzy logic, or something else? What is the proper position on the meaning of probability? Should probabilities always be thought of in frequentist terms, or is there some basis for the Bayesian notion of subjective probabilities? If so, how do probabilities and fuzziness relate to each other?"

In my view, we should never forget that 99% of the intelligent levels of the human brain are exactly equivalent to similar structures in the brain of the mouse. This refers to the basic learning structures, like the six-layer neocortex, and not to the specific things which individual humans actually learn in a lifetime, which define the usual Broca areas you often see in color pictures of the surface of the brain. The other 1% is important, but we need to understand the 99% to have a basic understanding of what is going on.

Because life demands that we make decisions [21,18], we and mice have no choice but to assign subjective probabilities (subjectively, at a nonverbal level) to questions like: "If I try to cross that field, what is the probability that a fox will catch me and eat me? If I devote my life to superstring research what is the probability that it will all turn out to be a great big ball of nonsense ten years after I graduate?"

What could the basis be of estimating such probabilities, when we cannot use rigorous frequentist methods? As a response – the answer is that we get them by articulating what we know and see at the nonverbal level of our intelligence. More precisely, we humans are engaged in an ongoing process of articulating thoughts form the nonverbal level into words, logic and mathematics; reasoning at the verbal level (with nonverbal inputs); and translating back from symbols to subsymbolic understanding. That process itself is the true foundation for all kinds of logic as practiced by humans [32]. In some sense, traditional logic, fuzzy logic and continuous mathematics, as well as music, are all on an equal footing as languages we learn to use to try to articulate and explain what we learn at the nonverbal level. As in section 114.3, the most accurate articulation often requires some *combination* of approximation or fuzziness and probability.

More formally, as we rely more and more on reasoning with symbols, it becomes more and more important that we learn systems of axioms and procedure which do not get us into trouble [32]. For that, I have proposed that we should take as axiomatic: (1) the concept of objective reality, affirming that we live in a universe of continuous variables; (2) our foundation in what we learn from personal subjective reality and nonverbal experience; and (3) the quest to reconcile the two and get full value from both, as we "look at ourselves in the mirror." In a sense, (2) is really the one invariant, since objective reality is something we learn about from experience, but the concept of objective reality is a very pervasive and powerful concept, which shapes our ability to understand our own minds.

There is a certain paradox here. For objective reality, I propose that we consider the possibility that the state of the universe over all space time corresponds to the state of $\varphi(x_\propto)$, where φ is a set of fields (scalars, vectors, tensors, and maybe spinors) defined over points $(x_\propto)$ in $3+1$-dimensional Minkowski space, governed by classical Lagrangian field equations [30,33,34]. Everything we do, think and are may be simply patterns over those fields. This in turn is meaningful in an axiomatic sense only if such field equations are meaningful. Many have questioned whether that is true, because Gödel has proven the incompleteness of early formulations of logic and arithmetic, and others have questioned the continuum hypothesis. Nevertheless, Gödel himself [9] has acknowledged that more recent formulations of logic based on Russell's theory of types and extensions of that are enough to show us that we are on firm ground here. There is room for research to find the "best" theory of types, but the concept of objective reality in Minkowski space remains quite tenable.

Of course, if we arrive at new understandings of physics, we will have to modify (114.1) somewhat.

Within the classical framework, causality always moves from part to future. The mouse brain itself was clearly designed on the basis of that kind of principle. Causality is basically a property of the usual time-forwards models like equation 114.3 and 114.5. Box and Jenkins [4] clearly display a causality axiom which is asymmetric in time, and is essentially the same as what is assumes in much of advanced physics today [22]. The axiom basically says that the random or unpredictable disturbances may correlate with variables in future time (because they may cause changes in them) but not with previous values of the same variables (because then they would not random relative to the past). Establishing causality is basically a matter of developing models of the world we live in (cognitive prediction), and analyzing properties like sensitivities [29] of those models.

In this view, our uncertainty about the world can always be represented completely, in principle, by the subjective probability distribution $Pr(\varphi(x_\infty))$ of the state of the universe. However, we need a great chain of approximations and simplified representations, which sometimes includes fuzzy logic, in order to reason approximately about that probability distribution, as accurately as we can within the constraints of finite brains and finite files. Many of the debates about types of uncertainty are essentially discussions of alternative ways of trying to give good approximations which capture some of the man types of complicated situations which can exist in our world.

114.5 From the Classical Universe to Quantum Reality

Virtually no serious physicists today would say that we really live in a classical universe. However, there are many different views of what it means to live in a quantum universe, and we certainly do not yet know the true "laws of everything," the ultimate laws of physics. Here I will briefly discuss a few alternatives.

Quantum mechanics actually began with efforts by Heisenberg to promote an alternative form of multivalued logic, much further away from traditional binary logic than fuzzy logic is. Instead of interpreting "propositions" as binary functions, true or false, or as continuous functions, he proposed interpreting them as matrices over an infinite valued space, whose values would essentially be more matrices. Many people thought of this as a very fringe kind of idea – until it turned out to explain the only known way to correctly predict the spectrum of helium. But people mainly segued from this original idea to a simplified view which is even now called "the Copenhagen version of quantum theory" (much to the distress of Heisenberg's old collaborators). A few of us have at times worked with the idea of representing quantum theory as an outcome of logic over the complex plane or the unit circle, but it really doesn't work out so well and I do not see much empirical basis for it.

Many people believe that the Copenhagen version of quantum theory and the modern many-worlds theories and a dozen other varieties are all equivalent for all practical purposes. That is very convenient for many people, politically. However, it is not true [30].

It turns out that classical Lagrangian field theory over Minkowski space is still logically tenable and consistent with experiment, so long as we get rid of the grafted-on assumption of universal time-forwards causality [30, 33, 34]. Thus the notion of objective reality as discussed above is still tenable, since I carefully did not include time-forwards causality in the list of proposed axioms.

Some physicists have argued that we simply cannot entertain the concept of causality running both forwards and backwards at times, because of how our brains are constructed. However, the universal function approximation abilities in the mouse brain clearly include an ability to learn recurrent relations and a family of nonsmooth functions, which would encompass this kind of physics. Mathematicians have learned to work with mixed forwards-backwards stochastic differential equations. Hans Georg Zimmermann of Siemens has even shown how we can get better industrially useful predictions of economic variables by training TLRNs in a way which involves mixed forwards-time and backwards-time relations.

In my view, the highest level of reasoning which we are capable of builds on the concepts of section 114.4, by accounting for such backwards-time quantum effects, and also accounting for some kind of collective intelligence effects across different people, which provide additional nonverbal as well as verbal inputs.

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References

1. Albus, J.S.: Outline for a Theory of Intelligence. IEEE Transactions on Systems, Man and Cybernetics 21(2), 473–509 (1991)
2. Barron, A.R.: Universal Approximation Bounds for Superpositions of a Sigmoidal Function. IEEE Transactions on Information Theory 39(3), 930–945 (1993)
3. Barron, A.R.: Approximation and Estimation Bounds for Artificial Neural Networks. Machine Learning 14(1), 113–143 (1994)
4. Box, G.E.P., Jenkins, G.M.: Time-Series Analysis: Forecasting and Control. Holden-Day, San Francisco (1970)
5. Deutsch, K.W.: Nationalism and Social Communications, 2nd revised edn. MIT Press, Cambridge (1966)
6. Deutsch, K.W.: The Nerves of Government. Glencoe, New York (1967)
7. Feldkamp, L.A., Prokhorov, D.V.: Recurrent Neural Networks for State Estimation. In: Proceedings of the Workshop on Adaptive and Learning Systems, Narendra edn. Yale University (2003), Posted with permission at `http://www.werbos.com/FeldkampProkhorov2003.pdf`
8. Fogel, D.B., Hays, T.J., Han, S.L., Quon, J.: A Self-learning Evolutionary Chess Program. Proceedings of the IEEE 92(12), 1947–1954 (2004)
9. Gödel 1944:126 footnote 17: “Russell’s mathematical logic” appearing in Kurt Gödel: Collected Works: vol. II Publications 1938-1974. Oxford University Press, New York (1944)
10. Ilin, R., Kozma, R., Werbos, P.J.: Beyond Backpropagation and Feedforward Models: A Practical Raining Tool for more Efficient Universal Approximator. IEEE Transactions on Neural Networks 19(3), 929–937 (2008)
11. Johnstone, I.M.: Statistical Challenges of High-dimensional Data. Philosophical Transactions of the Royal Society, Mathematical, Physical and Engineering Sciences 367(1906), 4237 (2009)
12. Lewis, F.L., Liu, D.: Reinforcement Learning and Approximate Dynamic Programming for Feedback Control. Wiley (2012)
13. Liang, J., Venayagamoorthy, G.K., Harley, R.G.: Wide-Area Measurement based Dynamic Stochastic Optimal Power Flow Control for Smart Grids with High Variability and Uncertainty. IEEE Transactions on Smart Grid 3(1), 59–69 (2012)

14. Morris, C.N.: Parametric Empirical Bayes Inference: Theory and Applications. Journal of the American Statistical Association 78(381), 47–55 (1983), Article Stable `http://www.jstor.org/stable/2287098`
15. National Science Foundation, Emerging Frontiers in Research and Innovation (2008), `http://www.nsf.gov/pubs/2007/nsf07579/nsf07579.pdf`
16. Príncipe, J.C., Euliano, N.R., Curt Lefebvre, W.: Neural and Adaptive Systems: Fundamentals through Simulations. Wiley, New York (2000)
17. Prokhorov, D.V.: Prius HEV Neurocontrol and Diagnostics. Neural Networks 21(2-3), 458–465 (2008)
18. Raiffa, H.: Decision Analysis. Addison-Wesley, Reading (1968)
19. Schumann, J.M., Yan, L. (eds.): Applications of Neural networks in High Assurance Systems. Springer, Berlin (2010)
20. Si, J., Barto, A.G., Buckler Powell, W., Wunsch, D. (eds.): Handbook of Learning and Approximate Dynamic Programming. IEEE Press Series on Computational Intelligence. Wiley-IEEE Press (2004)
21. von Neumann, J., Morgenstern, O.: The Theory of Games and Economic Behavior. Princeton U. Press, Princeton (1953)
22. Walls, D.F., Milburn, G.J.: Quantum Optics. Springer, New York (1994)
23. Werbos, P.J.: Beyond Regression: New Tools for Prediction and Analysis in the Behavioral Sciences, Ph.D. Thesis, Committee on Applied Mathematics, Harvard U. (1974); reprinted in its entirety in [27]
24. Werbos, P.J.: Building and understanding adaptive systems: A Statistical/Numerical Approach to Factory Automation and Brain Research. IEEE Transactions on Systems, Man and Cybernetics 17(1), 7–20 (1987)
25. Werbos, P.J.: Rational Approaches to Identifying Policy Objectives. Energy: The International Journal 15(3/4), 171–185 (1990)
26. Werbos, P.J.: Econometric Techniques: Theory Versus Practice. Energy: The International Journal 15(3/4), 213–236 (1990)
27. Werbos, P.J.: The Roots of Backpropagation: From Ordered Derivatives to Neural Networks and Political Forecasting. Wiley (1994)
28. Werbos, P.J.: Elastic Fuzzy Logic: A Better Fit to Neurocontrol and True Intelligence. Journal of Intelligent and Fuzzy Systems 1, 365–377 (1993); reprinted and updated in Gupta, M. (ed.): Intelligent Control. IEEE Press, New York (1995)
29. Werbos, P.J.: Backwards Differentiation in AD and Neural Nets: Past Links and New Opportunities. In: Bucker, H.M., Corliss, G., Hovland, P., Naumann, U., Norris, B. (eds.) Automatic Differentiation: Applications, Theory and Implementations, Springer, New York (2005), Posted at `http://www.werbos.com/AD2004.pdf`
30. Werbos, P.J.: Bell's Theorem, Many Worlds and Backwards-Time Physics: Not Just a Matter of Interpretation. International Journal of Theoretical Physics 47(11), 2862–2874 (2008)
31. Werbos, P.J.: Intelligence in the Brain: A Theory of How it Works and How to Build it. Neural Networks 22(3), 200–212 (2009)
32. Werbos, P.J.: Neural Networks and the Experience and Cultivation of Mind. In: Neural Networks (special issue based on IJCNN 2011, published electronically, hard copy in press) (2012)
33. Werbos, P.J.: A Three Step Program For Return To Reality. Problems of Nonlinear Analysis in Engineering Systems, an International IFNA-ANS Journal (in press, 2012), a preliminary draft is posted at `http://www.scribd.com/doc/80026749/ThreeStep-v5`

34. Werbos, P.J.: Solitons for Describing 3-D Physical Reality: The Current Frontier. In: Chua, Adamatzky (eds.) Chaos, CNN, Memristors and Beyond. World Scientific (forthcoming)
35. White, D.A., Sofge, D.A. (eds.): Handbook of Intelligent Control. Van Nostrand Reinhold, New York (1992)
36. Wonnacott, T.H., Wonnacott, R.J.: Introductory Statistics for Business and Economics, 4th edn. Wiley (1990)

Errata: On Fuzziness

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In the original version, error has occurred in the following:

1. In page XXIV, the title is given in Table of content (Vol. I) is Fuzzy Clouds. It should read as "Fuzzy Cloud": Sfumato versus Chiaroscuro in Music

2. The following List of Contributors have to be included in "Volume I" (Studfuzz 298):

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