Modelling the Intertemporal Choice through the Dynamic Time-Perception

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Abstract. The process of intertemporal choice is intimately linked with the concept of discounting function. Usually the benchmark in this important financial tool is the instant 0. This is an actual constraint for economic agent decision-making; indeed, in many situations, individuals have to decide at instants different from 0. Obviously, this introduces a multicriteria decision making framework in which a group of agents can (or cannot) cooperate in order to obtain greater profitabilities in function of the time variable. In this financial context, it is necessary to choose between transitive and non-transitive choice, giving rise to additive and non-additive (which includes subadditive and superadditive) discounting, respectively. Finally, another classification distinguishes between discounting with increasing or decreasing impatience.

Keywords: Discounting function, instantaneous discount rate, impatience, stationarity, additivity, multicriteria decision making.

1 Introduction

In many situations, an individual or a firm must decide what amount is equivalent to \$1 available t periods after a benchmark. This issue is a well-known problem in actuarial framework and *intertemporal choice* (Cruz and Muñoz, 2005 and 2006; Cruz and Ventre, 2011a), that is our present perspective. Indeed the benchmark is the instant at which we have the information necessary to replace a future with a present amount. If we take into account the criterion available at instant 0, the intertemporal choice will be named *static*, while if the criterion is available at variable time d, the intertemporal choice is said to be *dynamic*. Obviously, this last case provides us a

A.G.S. Ventre et al. (Eds.): *Multicriteria & Multiagent Decision Making*, STUDFUZZ 305, pp. 109–114.
DOI: 10.1007/978-3-642-35635-3_9

variable criterion, depending on the benchmark (time *d*), which gives us a multicriteria decision making process in time.

This approach is equivalent to know a capitalization or discounting function:

- if $t \leq 0$, we will be using a *capitalization function*,
- if $t \geq 0$, we will be using a *discounting function*.

We will deal only with discounting functions, that are functions $F(t)$ of one time variable *t* (if the benchmark is 0) or functions $F_2(d,t)$ of two variables *d*, *t* (if the benchmark is the instant *d*). In this way, we will say that a discounting function is the mathematical expression of the intertemporal choice of a subject or a company, or, equivalently, the intertemporal choice is quantified by a discounting function. This shows the coincidence of the main topic both in "discounting function" and in "intertemporal choice".

Nevertheless, there is an intermediate way to obtain the equivalent of $1 at instant *d* with the criterion available at time 0. In this case, we will be using the *discounting factor*:

$$f(0,d,t) = \frac{F(d+t)}{F(d)}.$$

In other words, despite this factor discounts from $d+t$ to *t*, the employed criterion is the current one at time 0. Finally, the relationship between the two former notations is stationarity. Indeed, a discounting function is said to be *stationary* if $F_2(d,t)$ is independent of *d*, that is, the criterion of intertemporal choice does not change in time (Harvey, 1986 and 1994), in which case it will be simply denoted by $F(t)$.

On the other hand, when dealing with intertemporal choice, we can use an objective or a subjective discounting function. An *objective* discounting function is a given criterion of choice (linear, hyperbolic (Azfar, 1999) or exponential discounting), that is known by the two subjects involved in a financial transaction. A *subjective* discounting function is a criterion of choice deduced from the particular preferences of an individual or a group of individuals. In this paper we will focus on the issue of subjective intertemporal choice.

Finally, in intertemporal choice, the monetary unit can be replaced with a reward (for instance, an apple) with a given utility (Benzion *et al.*, 1999). The organization of the present paper is as follows. In Section 2, we will introduce the general notations and the concept of stationary intertemporal choice by exhibiting the different forms of definition. Thus, in a natural way, the concepts of transitivity and (im)patience arise, giving rise to increasing and decreasing (im)patience, and to subadditive and superadditive intertemporal choice. These issues will be developed in sections 3 and 4. Finally, Section 5 summarizes and concludes.

2 Stationary and Dynamic Intertemporal Choice

In this Section, we will start with some notation. As indicated in Section 1, given $1 available at time *t*, the value $F(t) = F_2(0,t)$ represents the amount subjectively equivalent at time 0. Said in other words, an economic subject is indifferent between $1, available at time *t*, and $F(t)$, available at time 0. $F(t)$ is said to be a *spot discounting function*. In this context, we would like to highlight the noteworthy relationship between the discounting function and the *instantaneous discount rate* (Maravall, 1970; Gil, 1993):

$$F(t) = e^{-\int_0^t \delta(x)\mathrm{d}x}, \tag{1}$$

where $\delta(t) = -\left.\frac{\mathrm{d}\ln F(x)}{\mathrm{d}x}\right|_{x=t}$. We are now interested in continuity in time of the defined criterion of choice, that is, transitivity. In order to deal with this problem, we have at our disposal two alternatives for describing future choices:

- $F_3(0,d,t)$ which denotes the amount equivalent at time *d* to $1 available at time $d+t$, where the benchmark is instant 0, that is, the choice involves futures dates with present criteria. As a particular worthwhile case, we can cite the discounting factor $f(0,d,t) = \frac{F(d+t)}{F(d)}$, which incorporates a condition of transitivity in intertemporal choice. Indeed, we have:

$$f(0,d,t)f(0,d+t,s) = f(0,d,t+s).$$

  The expression form $F_3(0,d,t)$ without involving stationarity has been studied by several authors (see, for example, Mulazzani (1993)).
- $F_2(d,t)$, which denotes the amount equivalent at time *d* to $1 available at time $d+t$. Observe that here the benchmark is instant *d*.

In general, $F_3(0,d,t)$ is said to be a *forward discounting function* and, in particular, $f(0,d,t)$ is said to be the *discouting factor* associated to the spot discounting function $F(t)$. Finally, $F_2(d,t)$ is said to be a *dynamic discounting function*.

On the other hand, *d* is the delay, that is, a date later than today, and *t* is the interval, that is, a period of time after date *d*. The action of the interval *t* over the delay *d* gives rise to another delay $d+t$. Observe that, in the discounting function $F(t)$, the delay coincides with the interval.

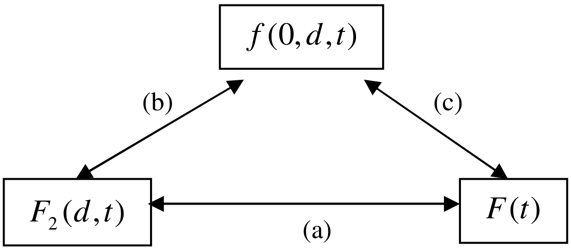

Chart 1. Some different possibilities to define stationarity

Next, in order to define stationarity as an invariance of the discounting function through time, we are going to develop each of the three arrows in chart 1:

a) Following this arrow, we can define stationarity by the following equation:

$$F(t) = F_2(d,t). \tag{2}$$

A discounting function satisfying equation (2) is said to be *stationary* (Harvey, 1986 and 1994).

b) Following this arrow, we can define stationarity by the following equation:

$$F_2(d,t) = f(0,d,t), \tag{3}$$

or, equivalently,

$$F(d)F_2(d,t) = F(d+t). \tag{4}$$

A discounting function satisfying equation (4) is said to be *additive*.

c) Finally, following this arrow, we can define stationarity by the following equation:

$$F(t) = f(0,d,t), \tag{5}$$

or, equivalently,

$$F(d)F(t) = F(d+t), \tag{6}$$

that is a functional equation whose solution (Aczél, 1987) is the well-known exponential discounting $F(t) = e^{-kt}$. Observe that condition (5) is stronger than condition (3), because condition (5) does not consider variable benchmark and moreover incorporates condition (2), that is, (5) = (3) + (2).

Strictly speaking, we will refer to stationarity by means of equation (2).

3 Impatience in Intertemporal Choice

A noteworthy characteristic of exponential discounting $F(t) = e^{-kt}$ is that its instantaneous discount rate is constant through time, $\delta(t) = k$. Intuitively, the instantaneous discount rate represents the degree of (im)patience of the intertemporal choice quantified by the corresponding discounting function (Prelec and Loewenstein, 1991; Thaler, 1981). This property allows us a (non-dichotomous) transversal classification of both stationary and dynamic discounting functions, in the following way:

- Discounting functions with *increasing impatience*, whose instantaneous discount rate is increasing (also called with *decreasing patience*).
- Discounting functions with *decreasing impatience*, whose instantaneous discount rate is decreasing (also called with *increasing patience*).

Table 1 summarizes the concepts and gives some examples:

Table 1. A classification of discounting functions according to their impatience

A classification of discounting functions			
	Constant impatience	**Variable impatience**	
		Increasing impatience	**Decreasing impatience**
Expression of *F*(*t*)	$F(t)=e^{-kt}$	$F(t)=1-dt$	$F(t)=\frac{1}{1+it}$
Instantaneous discount rate	$\delta(t)=k$	$\delta(t)=\frac{d}{1-dt}$	$\delta(t)=\frac{i}{1+it}$

4 Subadditive and Superadditive Intertemporal Choice

In Section 2 (eq. (4)), we have defined additive discounting functions. Observe that, despite additivity is a kind of stationarity, its financial interpretation is the following. An investor is indifferent between the following behaviors:

- placing an initial amount during the interval $[0,d]$, then disinvesting and immediately placing the resulting amount during the interval $[d,d+t]$, or
- placing the initial amount during the whole interval $[0,d+t]$ without splitting it.

This condition allows us to present a (non-dichotomous) classification of non-additive discounting functions:

- *Subadditive* discounting functions: $F(d)F_2(d,t)<F(d+t)$.
- *Superadditive* discounting functions $F(d)F_2(d,t)>F(d+t)$.

Table 2 summarizes the concepts and gives some examples:

Table 2. A classification of discounting functions according to their additivity/non-additivity

Another classification of discounting functions			
	Additive	**Non-additive**	
		Subadditive	**Superadditive**
Expression of *F*(*t*)	$F(t)=e^{-kt}$	$F(t)=\frac{1}{1+it}$	$F(t)=1-dt$

Observe that the concepts of subadditive and superadditive discounting functions involve a certain degree of increasing and decreasing impatience, respectively. Nevertheless, despite their importance in intertemporal choice, they have been presented as independent of stationarity. Looking for the relationships among these features is useful in order to detect possible inconsistencies in individual choices (Cruz and Ventre, 2011b).

5 Conclusion

We have exhibited some features of intertemporal choice from the point of view of stationarity. Namely, starting from the spot and forward discounting factors, we can deduce the concepts of pure stationarity and additivity, and their respective violations: on the one hand, increasing and decreasing impatience, and, on the other hand, subadditivity and superadditivity. Despite these concepts are presented in the financial literature as independent as each other, in this paper we demonstrate their common origin and their relationships.

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