Credit Crunch in the Euro Area: A Coopetitive Multi-agent Solution

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Abstract. The aim of this paper is to propose a methodology to attenuate the plague of the credit crunch, which is very common in this period: despite the banking world having available a huge amount of money, there is no available money in the real economy. Consequently, we want to find a way to allow a global economic recovery by adopting a new mathematical model of "Coopetitive Game." Specifically, we will focus on two economic operators: a real economic subject and a financial institute (a bank, for example) with a big economic availability. For this purpose, we examine an interaction between the above economic subjects: the Enterprise, our first player, and the Financial Institute, our second player. The solution that allows both players to win the maximum possible collective profit, and therefore the one desirable for both players, is represented by a coopetitive agreement between the two subjects. So the Enterprise artificially causes (also thanks to the money loaned by the Financial Institute that receives them by the ECB) an inconsistency between spot and futures markets, and the Financial Institute takes the opportunity to win the maximum possible collective gain of the coopetitive game (the two players even arrive to the maximum of the game). We propose hereunder two possible transferable utility solutions, in order to avoid that the envy of the Enterprise, which gains a much less advantage from the adoption of a coopetitive strategy, may compromise the success of the interaction.

Keywords: Credit Crunch, Financial Markets, Financing Policy, Risk, Financial Crisis, Games, Arbitrages, Coopetition.

1 Introduction

In the last years, despite the banking world having available a huge amount of money (on Dec. 2011 and on Feb. 2012 the ECB loaned money to banks at the rate of 1%, respectively 490 and 530 billion euros), there is no available money in the real economy. This phenomenon has begun to show its first sign of life from the second half of 2008, and it reached its peak in Dec. 2011. The credit crunch

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is a wide phenomenon: Europe shows a decrease of 1.6% in loans to households and businesses. In Italy, this phenomenon is particularly pronounced, because the decline in loans was even of 5.1% from 2008. *Where's the money loaded by ECB?* Badly, the money remained caged in the world of finance: with some of the money from the ECB, banks bought government bonds (so the spread went down); another part of the money is used by the banks to rectify their assets in accordance with EBA requirements (European Banking Authority); the rest of the money was deposited at ECB, at the rate of 0.5% (lower than the rate at which they received it). Moreover, from the second half of 2008, the deposits of European banks at the ECB have quadrupled. In view of this, our model takes a different dimension and different expectations: in our model, the bank (the speculator) put money in the real economy by lending to the Enterprise; it eliminates the risk of losing money for the economic crisis and obtains a gain by an agreement with the Enterprise (which gains something too). The credit crunch, by our model, should be gradually attenuated until it disappears.

In this paper, by using game theory (for the complete study of a game see also [4,5,10]) we propose a method aiming to attenuate the phenomenon of the credit crunch and, consequently, a way to allow a global economic recovery. For the achievement of our aim, we propose the introduction of a tax on speculative financial transactions, in order to stabilize the financial markets (see also[23,8,9]). Moreover, we propose a method of using money (that were provided to banks by the ECB) that allows the money to get into the real economy without getting stuck in the world of finance. Our aim is attained without inhibiting the possibilities of profits and, for this purpose, we present and study an advantageous coopetitive model and two different compromise solutions.

2 Description of the Initial No-coopetitive Game

2.1 Methodologies

The Carfì and Musolino's model ([7]) is based on a construction on 3 times.

0) At time 0 the Enterprise can choose whether to buy futures contracts to hedge the market risk of the underlying asset, which (the Enterprise knows) should be bought at time 1, in order to conduct its business activities.
1) The Financial Institute, on the other hand, acts—with speculative purposes—on spot market (buying or short-selling the asset at time 0) and futures market (with the action contrary to that on the spot market: if the Financial Institute sells short on spot market, it purchases on the futures market, and vice versa). Thus, the Financial Institute may take advantage of the temporary misalignment of the spot and futures prices that would be created as a result of a hedging strategy by the Enterprise.
2) At time 2, the Financial Institute cashes or pays the sum determined by its behavior in the futures market at time 1.

2.2 Strategies of the Players

In Carfì and Musolino's model ([7]) the first player is an Enterprise that may choose whether to buy futures contracts to hedge by an upwards change in the price of the underlying asset that it (the Enterprise) has to buy at time 1 for the conduct of its business. Therefore, the Enterprise has the possibility to choose a strategy $x \in [0, 1]$, which represents the percentage of the quantity of the underlying M_1 that the Enterprise purchases via futures, depending on its intends:

1. to not hedge ($x = 0$),
2. to hedge partially ($0 < x < 1$),
3. to hedge totally ($x = 1$).

On the other hand, the second player is a Financial Institute operating on the spot market of the same underlying asset. The Financial Institute works in our game also on the futures market:

- taking advantage of possible gain opportunities—given by misalignment between spot prices and futures prices of the asset;
- or accounting for the loss obtained, because it has to close the position of short sales opened on the spot market.

These are just actions to determine the win or the loss of the Financial Institute.

The Financial Institute can therefore choose a strategy $y \in [-1, 1]$, which represents the percentage of the quantity of the underlying M_2 that it can buy (in algebraic sense) with its financial resources, depending on its intends:

1. to purchase the underlying on the spot market ($y > 0$);
2. to short sell the underlying on the spot market ($y < 0$);
3. to not intervene on the market of the underlying ($y = 0$).

3 Coopetitive Approach

For the display of our game (proposed as a remedy to the credit crunch), it is necessary to pass from the Carfì and Musolino's model ([7]) to a game set in a coopetitive context (see [3,2,12,17,27,32,1,13,14,15,24,25,26] about coopetition). In particular, we follow the Carfì's definition of coopetitive game (for some examples see [6,11])

3.1 The Idea

A coopetitive game is a game in which two or more players (participants) can interact *cooperatively and non-cooperatively at the same time*. Even Brandenburger and Nalebuff, creators of coopetition ([1]), did not define, precisely, a *quantitative way to implement coopetition* in the Game Theory context.

The problem to implement the notion of coopetition in Game Theory is summarized in the following question:

– *how do, in normal form games, cooperative and non-cooperative interactions live together simultaneously, in a Brandenburger-Nalebuff sense?*

In order to explain the above question, consider a classic two-player normal-form gain game $G = (f, >)$—such a game is a pair in which f is a vector valued function defined on a Cartesian product $E \times F$ with values in the Euclidean plane $\mathbb{R}^2$ and $>$ is the natural strict sup-order of the Euclidean plane itself (the sup-order is indicating that the game, with payoff function f, is a gain game and not a loss game). Let E and F be the strategy sets of the two players in the game G. The two players can choose the respective strategies $x \in E$ and $y \in F$

– cooperatively (exchanging information and making binding agreements);
– not-cooperatively (not exchanging information or exchanging information but without possibility to make binding agreements).

The above two behavioral ways are mutually exclusive, at least in normal-form games:

– the two ways cannot be adopted simultaneously in the model of normal-form game (without using convex probability mixtures, but this is not the way suggested by Brandenburger and Nalebuff in their approach);
– there is no room, in the classic normal form game model, for a simultaneous (non-probabilistic) employment of the two behavioral extremes *cooperation* and *non-cooperation*.

Towards a Possible Solution. A manner to pass this *impasse*, according to the idea of coopetition in the sense of Brandenburger and Nalebuff is Carfì's coopetitive game model, where

– the players of the game have their respective strategy-sets (in which they can choose cooperatively or not cooperatively);
– there is a common strategy set C containing other strategies (possibly of different type with respect to those in the respective classic strategy sets) that *must be chosen cooperatively*;
– the strategy set C can also be structured as a Cartesian product (similarly to the profile strategy space of normal form games), but in any case the strategies belonging to this new set C must be chosen cooperatively.

3.2 Two Players Coopetitive Games

Definition (of coopetitive game). *Let E, F, and C be three nonempty sets. We define* ***two-player coopetitive gain game carried by the strategic triple*** (E, F, C) *any pair of the form $G = (f, >)$, where f is a function from the Cartesian product $E \times F \times C$ into the real Euclidean plane $\mathbb{R}^2$ and the binary relation $>$ is the usual sup-order of the Cartesian plane (defined component-wise, for every couple of points p and q, by $p > q$ iff $p_i > q_i$, for each index i).*

Remark (coopetitive games and normal form games). The difference between a two-player normal-form (gain) game and a two-player coopetitive (gain) game is the fundamental presence of the third strategy Cartesian-factor C. The presence of this third set C determines a total change of perspective with respect to the usual examination of two-player normal form games, since we now have to consider a normal form game $G(z)$, for every element z of the set C.

3.3 Normal Form Games of a Coopetitive Game

Let G be a coopetitive game in the sense of the above definitions. For any cooperative strategy z selected in the cooperative strategy space C, there is a corresponding normal form gain game

$$G_z = (p(z), >),$$

upon the strategy pair (E, F), where the payoff function $p(z)$ is the section

$$f(., z) : E \times F \to \mathbb{R}^2,$$

of the payoff function f of the coopetitive game—the section is defined, as usual, on the competitive strategy space $E \times F$, by

$$f(., z)(x, y) = f(x, y, z),$$

for every bi-strategy (x, y) in the bi-strategy space $E \times F$.

Let us formalize the concept of game-family associated with a coopetitive game.

Definition (the family associated with a coopetitive game). *Let $G = (f, >)$ be a two-player coopetitive gain game carried by the strategic triple (E, F, C). We naturally can associate with the game G a family $g = (g_z)_{z\in C}$ of normal-form games defined by*

$$g_z := G_z = (f(., z), >),$$

for every z *in C, which we shall call* ***the family of normal-form games associated with the coopetitive game** G.*

Remark. It is clear that with any of the above family of normal form games

$$g = (g_z)_{z\in C},$$

with $g_z = (f(., z), >)$, we can associate:

- a family of payoff spaces

  $$(\mathrm{im} f(., z))_{z\in C},$$

  with members in the payoff universe $\mathbb{R}^2$;
- a family of Pareto maximal boundary

  $$(\partial^* G_z)_{z\in C},$$

  with members contained in the payoff universe $\mathbb{R}^2$;

– a family of suprema

$$(\sup G_z)_{z\in C},$$

with members belonging to the payoff universe $\mathbb{R}^2$;

– a family of Nash zones

$$(\mathcal{N}(G_z))_{z\in C};$$

with members contained in the strategy space $E \times F$;

– a family of conservative bi-values

$$v^{\#} = (v_z^{\#})_{z\in C};$$

in the payoff universe $\mathbb{R}^2$.

And so on, for every meaningful known feature of a normal form game.

Moreover, we can interpret any of the above families as *set-valued paths* in the strategy space $E \times F$ or in the payoff universe $\mathbb{R}^2$.

It is just the study of these induced families which becomes of great interest in the examination of a coopetitive game G and which will enable us to define (or suggest) the various possible solutions of a coopetitive game.

Solutions of a Coopetitive Game. The two players of a coopetitive game G—according to the general economic principles of *monotonicity of preferences* and of non-satiation—should choose the cooperative strategy z in C in order that:

– *fixed a common kind of solution for any game G_z, say $S(z)$ the set of these kind of solutions for the game G_z, we can consider the problem to find all the optimal solutions (in the sense of Pareto) of the set valued path S, defined on the cooperative strategy set C. Then, we should face the problem of* ***selection of reasonable Pareto strategies*** *in the set-valued path S via proper selection methods (Nash-bargaining, Kalai-Smorodinsky, and so on).*

4 The Shared Strategy

We have two players, the Enterprise and the Financial Institute, each of them has a strategy set in which to choose its strategy; moreover, the two players can cooperatively choose a strategy z in a third set C. The two players choose their cooperative strategy z to maximize (in some sense that we specify) the gain function f.

The strategy $z \in [0, 1]$ is a shared strategy, which represents the percentage of the highest possible money M_3 that the European Central Bank lends to the Financial Institute with a very low interest rate (hypothesis highly plausible according to the recent anti-crisis measures adopted by the ECB). By convention, we assume this interest rate equal to 0. The two players use the loan so that the Enterprise can create an even higher misalignment between spot and futures price, misalignment that is exploited by the Financial Institute. In this way, both players can get a greater win than that obtained without a shared strategy z. The two players can then choose a shared strategy depending on what they want:

- to not use the money of the ECB ($z = 0$);
- to use a part of the money of the ECB so that the Enterprise purchases futures ($0 < z < 1$);
- to use totally the money of the ECB so that the Enterprise purchases futures ($z = 1$).

Remark. In the following coopetitive game, we do not introduce the uncertainty (and we do not consider extreme events in our economic world) and so we suppose that attempts of speculative profit (modifying the asset prices) are successful. In fact our interest is to show that a tax on speculative profits can limit speculation, and not to determine if or how much speculators gain. Anyway, even without uncertainty, our model remains likely, plausible and very topical because

- in a period of crisis, behavioral finance suggests ([16,22,31]) the vertical diffusion of a behavior (the so-called herd behavior [19,29]) conforming to that adopted by the great investors;
- just the decrease (or increase) in demand influences the prices of the asset ([21]).

5 The Payoff Function of the Enterprise

In practice, to the payoff function f_1 of the paper [7], that is, the function defined by

$$f_1(x, y) = -nuM_1(1 - x)y,$$

for every (x, y) in the bi-strategy space S, we must add the payoff-consequence $v_1(y, z)$ of the shared action z of the game, consisting in buying futures contracts and selling them at time 1 (action decided by both players and performed by the Enterprise).

In paper [7], we have already chosen $M_1 = 1$ and $nu = 1/2$, and so we have

$$f_1(x, y) = -(1/2)(1 - x)y,$$

for every (x, y) in the bi-strategy space S: this is the first component of the initial game we shall represent in the present paper.

Payoff Consequence of the Shared Strategy. The payoff function addendum $v_1(y, z)$, of the Enterprise, is given by the quantity of futures bought, that is, the term zM_3, multiplied by the difference, $F_1u^{-1} - F_0$, between the futures price at time 1—when the Enterprise sells the futures—and the futures price at time 0 - when the Enterprise buys the futures.

Remark. Similarly to what happened to the Financial Institute in Carfì and Musolino's model ([7]) because of the introduction of a tax on speculative transactions, also the Enterprise has to pay a tax on the sale of the futures contracts (see [28,30,33] for the benefits of the taxation on financial transactions: we follow exactly this lane of thought). We assume that this tax is equal to the impact of the Enterprise on the futures price, in order to avoid speculative acts created by itself.

We have:

$$h_1(x, y, z) = f_1(x, y) + zM_3(F_1(x, y, z)u^{-1} - m(x + z) - F_0) \tag{1}$$

where:

(1) zM_3 is the quantity of futures purchased.
(2) F_0 is the futures price at time 0. It represents the price established at time 0 that has to be paid at time 1 in order to buy the asset. We assume that it is given by

$$F_0 = S_0 u. \tag{2}$$

S_0 is, on the other hand, the spot price of the underlying asset at time 0. S_0 is a constant because our strategies x, y, and z do not influence it, while $u = 1 + i$ is the factor of capitalization of interests. By i we mean risk-free interest rate charged by banks on deposits of other banks, the so-called "LIBOR" rate.
(3) $F_1(x, y, z)$ is the futures price (established) at time 1, after the Enterprise has played its strategy x and the shared strategy z. We assume that the price $F_1(x, y, z)$ is given by

$$F_1(x, y, z) = S_1 u + mu(x + z), \tag{3}$$

where
(a) S_1 is the spot price at time 1. We assume that it is given by

$$S_1(y) = (S_0 + ny)u,$$

where n is the marginal coefficient that measures the impact of y on $S_1(y)$.
(b) m is the marginal coefficient that measures the impact of x and z on $F_1(x, y, z)$.

$F_1(x, y, z)$ depends on x and z because an increase/decrease of futures demand influences upward/downward the futures price ([18]). The value S_1 should be capitalized because it follows the Hull's relationship between futures and spot prices ([20]). The value $m(x + z)$ is also capitalized because the strategies x and z are played at time 0 but have effect on the futures price at time 1.
(4) $m(x + z)$ is the normative tax paid by the Enterprise on the sale of futures, referred to time 1. We assume that the tax is equal to the impact of the strategies x and z (adopted by the Enterprise) on the futures price F_1.

(5) u^{-1} is the discount factor. $F_1(x, y, z)$ must be actualized at time 1 because the money for the sale of futures are cashed at time 2.

Remark. The values m and n depend on the ability (of our players) to influence the spot and futures markets and the behavior of other financial agents (see the remark in Section 4).

The Payoff Function of the Enterprise. Recalling Equations 2 and 3 and substituting them into Eq. 1, we have

$$h_1(x, y, z) = f_1(x, y) + zM_3[((S_0 + ny)u^2 + m(x + z)u)u^{-1} - m(x + z) - S_0u],$$

that is,

$$h_1(x, y, z) = f_1(x, y) + M_3nuyz. \tag{4}$$

From now we assume $M_3 = 1$ for the sake of simplicity.

6 The Payoff Function of the Financial Institute

In our initial no-coopetitive game, the payoff function of the second player (already analyzed in the paper [7]) is

$$f_2(x, y) = yM_2mx,$$

for every (x, y) in the bi-strategy space S. In paper [7] we have already chosen $M_2 = 2$ and $m = 1/2$, so we have

$$f_2(x, y) = yx,$$

for every (x, y) in the bi-strategy space S: this is the second component of the initial game we shall represent in the present paper.

The initial no-coopetitive payoff function of the Financial Institute at time 1 is given by the multiplication of the quantity of asset bought on the spot market, that is yM_2, by the difference among:

1. the futures price $F_1(x, y)$ (it is a price established at time 1 but cashed at time 2) transferred to time 1, that is $F_1(x, y)u^{-1}$;
2. the purchase price—net of the tax introduced by the normative authority on financial transactions ([7])—of asset at time 0, say S_0, capitalized at time 1 (in other words we are accounting for all balances at time 1).

But in our coopetitive game, instead of the futures price $F_1(x, y)$, we have to consider the futures price $F_1(x, y, z)$ that takes into consideration the shared strategy z (in fact at time 0 the Enterprise buys the additional quantity zM_3 of futures contracts than our initial no-coopetitive game, and the futures price F_1 changes consequently).

The Payoff Function of the Financial Institute of our coopetitive game is defined by:

$$f_2(x, y) = yM_2(F_1(x, y, z)u^{-1} - nuy - S_0u), \qquad (5)$$

where:

(1) y is the percentage of asset that the Financial Institute purchases or sells on the spot market of the underlying;
(2) M_2 is the maximum amount of asset that the Financial Institute can buy or sell on the spot market, according to its economic availability;
(3) S_0 is the price paid by the Financial Institute in order to buy the asset on spot market at time 0. S_0 is a constant because our strategies x, y, and z do not influence it.
(4) nuy is the normative tax on the futures price, paid at time 1. We are assuming that the tax is equal to the incidence of the strategy y of the Financial Institute on the spot price at time 1, that is, $S_1(y) = (S_0 + ny)u$ (see also the paper [7]).
(5) $F_1(x, y, z)$ is the futures price (established) at time 1, after the Enterprise has played its strategy x and the shared strategy z. The price $F_1(x, y, z)$ is given by

$$F_1(x, y, z) = S_1(y)u + mu(x + z),$$

where $S_1(y) = (S_0 + ny)u$ is the spot price at time 1, and$u = 1 + i$ is the factor of capitalization of interests. With m we intend the marginal coefficient that measures the impact of x and z on $F_1(x, y, z)$. $F_1(x, y, z)$ depends on x and z because a change of futures demand influences the futures price ([18,20]). The value S_1 should be capitalized because it follows the fundamental relationship between futures and spot prices (see [20]). The value $m(x + z)$ is also capitalized because the strategies x and z are played at time 0 but have effect on the futures price at time 1.
(6) u^{-1} is the discount factor. $F_1(x, y)$ must be translated at time 1, because the money for the sale of futures is cashed at time 2.

The Coopetitive Payoff Function of the Financial Institute. Recalling functions F_1 and f_2, we have

$$h_2(x, y, z) = yM_2m(x + z), \qquad (6)$$

So, we have

$$h(x, y, z) = (f_1(x, y), f_2(x, y)) + yz(nuM_3, M_2m), \qquad (7)$$

for every strategy triple (x, y, z) of our coopetitive game. In this paper, we shall represent the following numerical case:

$$h(x, y, z) = (-(1/2)(1 - x)y, xy) + yz(1/2, 1).$$

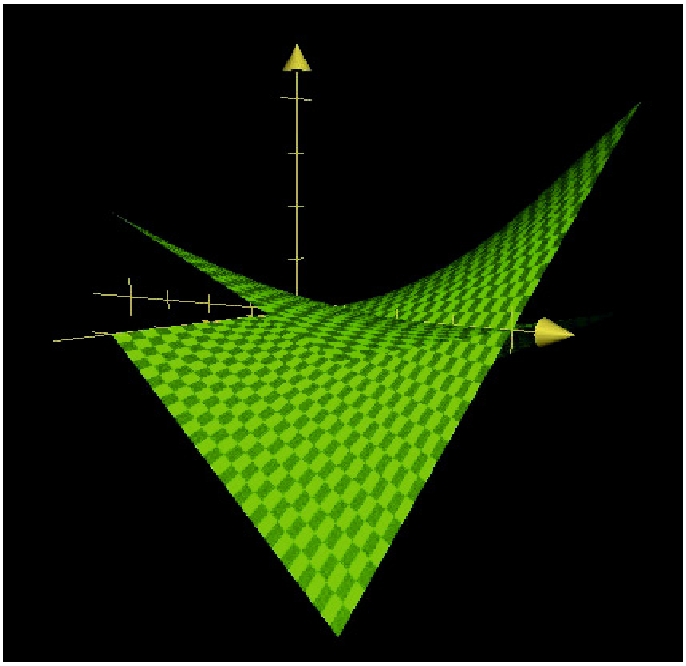

Fig. 1. Initial game $f = h(., 0)$

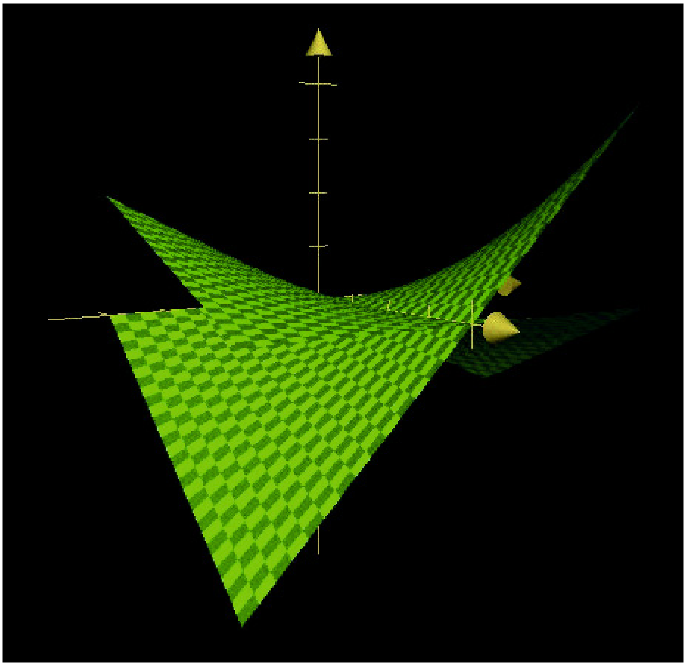

Fig. 2. Initial game $f = h(., 0)$

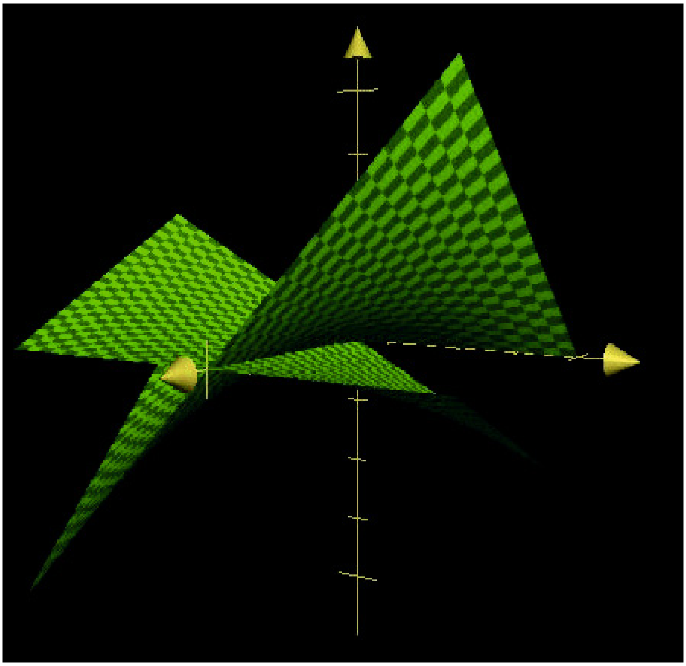

Fig. 3. Initial game $f = h(., 0)$

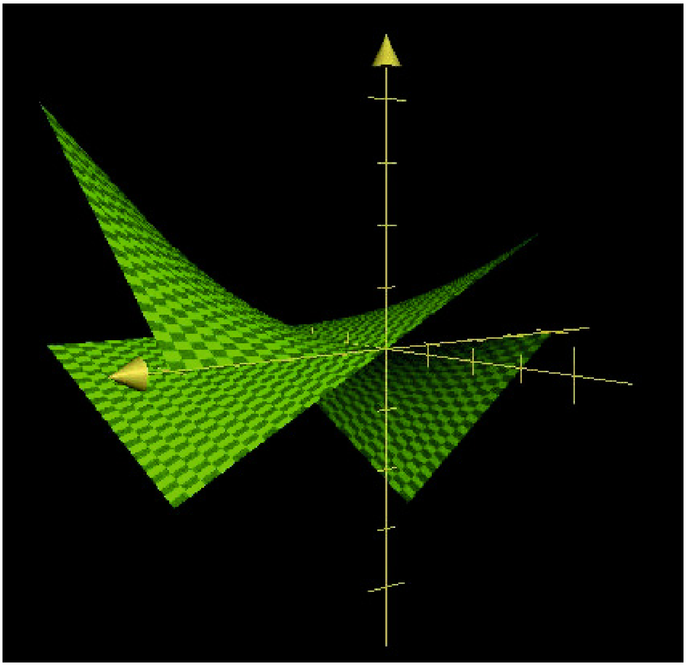

Fig. 4. Initial game $f = h(., 0)$

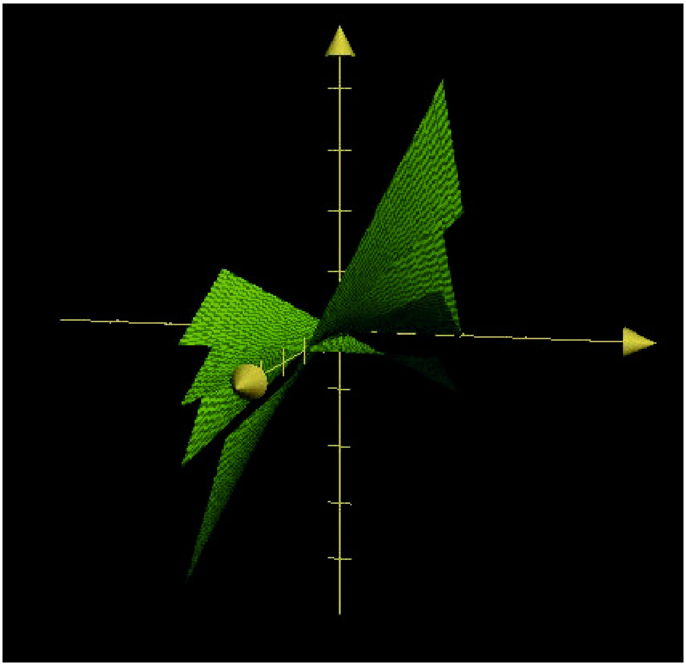

Fig. 5. Games $f = h(.,0)$ and $f = h(.,1)$

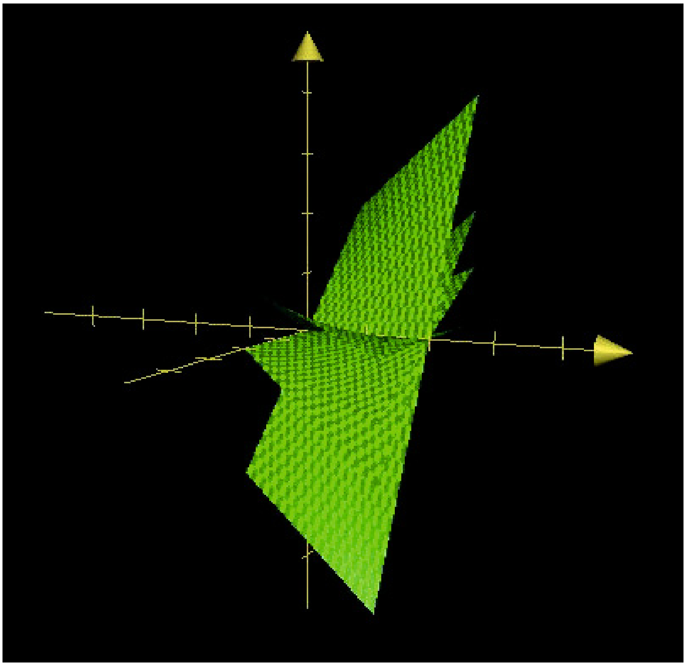

Fig. 6. Games $f = h(.,0)$ and $f = h(.,1)$

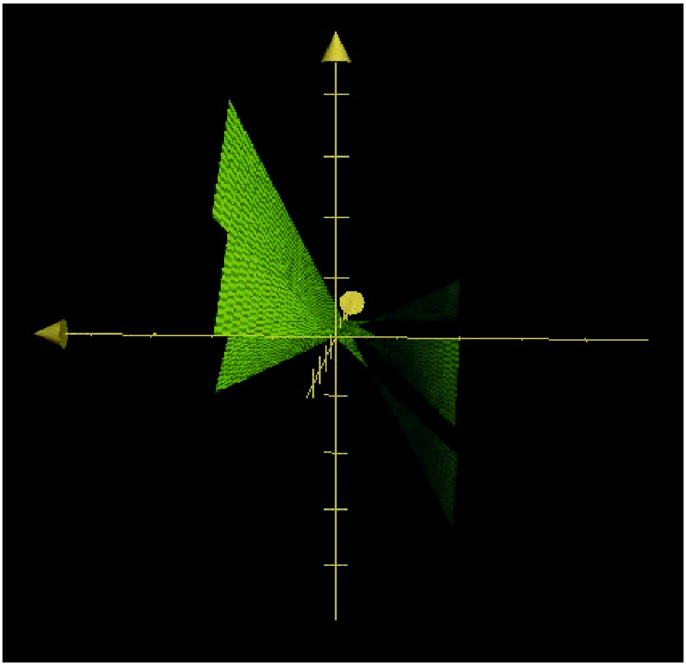

Fig. 7. Games $f = h(., 0)$ and $f = h(., 1)$

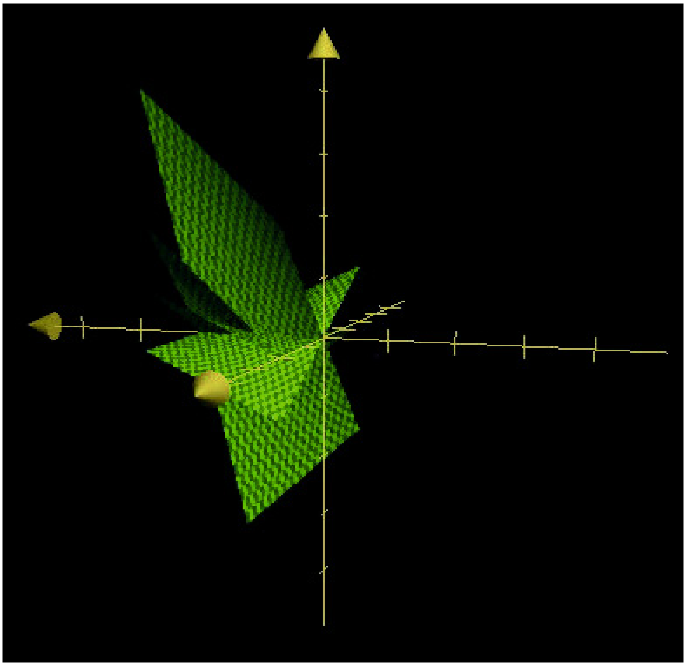

Fig. 8. Games $f = h(., 0)$ and $f = h(., 1)$

7 The Coopetitive Translating Vectors

We note immediately that the new function is the same payoff function f of the first game already studied in paper [7],

$$f(x, y) = (-nuyM_1(1 - x), yM_2mx),$$

translated by the vector function

$$v(y, z) := zy(nuM_3, M_2m).$$

Recalling that $y \in [-1, 1]$ and $z \in [0, 1]$, we see that the vector $v(y, z)$ belongs to the 2-range $[-1, 1](nuM_3, M_2m)$.

7.1 Coopetitive Payoff Space

Concerning the payoff space of our coopetitive game $(h, >)$, we note a meaningful result. We observe that (since any shared strategy z is positive):

1. the part of the initial payoff space $f(S_{\geq})$ (where $S_{\geq}$ is the part of S such that the second projection pr_2 is greater than 0) is translated upwards, when we consider the transformation by the coopetitive extension h of f and the shared variable z is increasing;
2. the part of the initial payoff space $f(S_{\leq})$ (where $S_{\leq}$ is the part of S such that the second projection pr_2 is less than 0) is translated downwards, when we consider the transformation by the coopetitive extension h of f and the shared variable z is increasing.

Proposition. *Let $S := E \times F = [0, 1] \times [-1, 1]$ and $Q := S \times [0, 1]$. Then, the payoff space $h(Q)$ is the union of $h(., 0)(S)$ and $h(., 1)(S)$.*

Proof. The strategy space S is the union of $S_{\geq} := [0, 1] \times [0, 1]$ and $S_{\leq} := [0, 1] \times [-1, 0]$. We shall split the proof into two parts.

Part 1. We will show that the shared strategy that maximizes the wins when $y \geq 0$ is always $z = 1$, that is, we'll show that $h(x, y, z) \leq h(x, y, 1)$, for every $y \geq 0$ and every x in E, i.e., $(x, y) \in S_{\geq}$. Recalling the definition of h, we have to show that

$$(-nuyM_1(1-x), yM_2mx) + yz(nuM_3, M_2m) \leq (-nuyM_1(1-x), yM_2mx) + y(nuM_3, M_2m),$$

that is,

$$yz(nuM_3, M_2m) \leq y(nuM_3, M_2m)$$

and therefore we have to prove that $yz \leq y$, which is indeed verified for any $y \geq 0$. We can show also that $h(x, y, z) \geq h(x, y, 0)$, for every $y \geq 0$ and every x in E. Indeed, we have to show that

$$(-nuyM_1(1 - x), yM_2mx) + yz(nuM_3, M_2m) \geq (-nuyM_1(1 - x), yM_2mx),$$

that is,

$$yz(nuM_3, M_2m) \geq 0$$

and therefore $yz \geq 0$, which is indeed verified for any $y \geq 0$. Since, with $x \in [0,1]$ and $y \in [0,1]$, we have

$$h(x,y,0) \leq h(x,y,z) \leq h(x,y,1),$$

we obtain that the payoff part $h([0,1]^3)$ is included in the union of the images $h(.,0)(S_{\geq})$ and $h(.,1)(S_{\geq})$.

Part 2. We will show that the shared strategy that maximizes the losses when $y \leq 0$ is always $z = 1$, that is, we will show that

$$h(x,y,z) \geq h(x,y,1),$$

for every $y \leq 0$ and every x in E, i.e., for every $(x,y) \in S_{\leq}$. Recalling the definition of h, we have to show that

$$(-nuyM_1(1-x), yM_2mx) + yz(nuM_3, M_2m) \geq (-nuyM_1(1-x), yM_2mx) + y(nuM_3, M_2m),$$

that is to say

$$yz(nuM_3, M_2m) \geq y(nuM_3, M_2m)$$

and therefore we have to prove $yz \geq y$, which is indeed verified for any $y \leq 0$. We can also show that

$$h(x,y,z) \leq h(x,y,0),$$

for every $y \leq 0$ and every x in E. Indeed, we have to show that

$$(-nuyM_1(1-x), yM_2mx) + yz(nuM_3, M_2m) \leq (-nuyM_1(1-x), yM_2mx),$$

that is,

$$yz(nuM_3, M_2m) \leq 0,$$

that is equivalent to $yz \leq 0$, which is indeed verified for any $y \leq 0$. Since, when $x \in [0,1]$ and $y \in [0,-1]$, we have

$$h(x,y,1) \leq h(x,y,z) \leq h(x,y,0),$$

we obtain that the payoff part $h(S_{\leq} \times [0,1])$ is included in the union of the images of $h(.,1)(S_{\leq})$ and $h(.,0)(S_{\leq})$. This completes the proof. ■

Hence, transforming our bi-strategic space S by $h(.,0)$ (in dark green) and $h(.,1)$ (in light green), in Fig.9 we have the whole payoff space of the our coopetitive game $(h,>)$. If the Enterprise and the Financial Institute play the bi-strategy $(1,1)$, and the shared strategy 1, they arrive at the point $B'(1)$, which is the maximum of the coopetitive game G, so the Enterprise wins $1/2$ (amount greater than $1/3$ obtained in the cooperative phase of the no-coopetitive game in the paper [7]) while the Financial Institute wins even 2 (an amount much greater than $2/3$, value obtained in the cooperative phase of the no-coopetitive game in paper [7]).

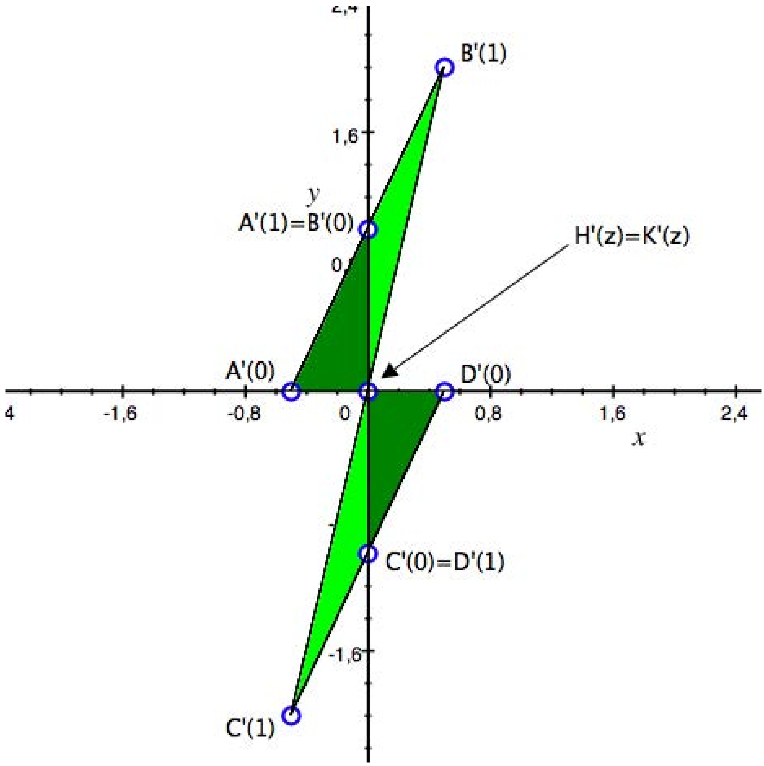

Fig. 9. The payoff space of the coopetitive game, $h(Q)$

8 Kalai-Smorodisky Solution

Why a Kalai-Smorodinsky Solution? The point $B'(1)$ is the maximum payoff of the game (with respect to the usual component-wise order of the payoff plane). But the Enterprise could be not satisfied by the gain 1/2, value that is much less than the win 2 of the Financial Institute. In addition, playing the shared strategy 1, the Enterprise increases only slightly the win obtained in the no-coopetitive game, on the contrary our Financial Institute gains more than double. For this reason, precisely to avoid that the envy of the Enterprise can affect the game, the Financial Institute might be willing to cede part of its win to the Enterprise by contract, in order to balance fairly the distribution of money.

Maximum Collective Gain. One way would be to distribute the *maximum collective profit* of the coopetitive game, that is, the maximum value of the *collective gain function*

$$g : \mathbb{R}^2 \to \mathbb{R} : g(X, Y) = X + Y$$

on the (compact) payoffs space of the game G, say $W = \max_{h(Q)} g$. The maximum collective profit is attained (evidently) at the maximum point point $B'(1)$, which is the only bi-win belonging to the straight line $g = 5/2$ and to the payoff space. Hence the Enterprise and the Financial Institute play the coopetitive 3-strategy $(1, 1, 1)$, in order to arrive at the payoff $B'(1)$ and then split the wins obtained by contract. From a practical point of view: the Enterprise buys futures to create artificially (also thanks to the money borrowed from the European Central Bank) a significant misalignment between futures and spot prices, misalignment which is exploited by the Financial Institute getting the maximum win $W = 5/2$.

First Possible Division of Maximum Collective Gain. For a possible **fair division** of the win $W = 5/2$, we propose a *transferable utility Kalai-Smorodinsky method.* The bargaining problem we face is the pair (Γ, α), where:

1. our decision constraint Γ is the transferable utility Pareto boundary of the game (straight line $X + Y = 1$);
2. we take the supremum of the game $\alpha = (1/2, 1)$ as threat point of our bargaining problem.

Solution. For what concerns the solution: we join $\alpha = (1/2, 1)$ with the supremum

$$\sup(\Gamma \cap [\alpha, \rightarrow [),$$

according to the classic Kalai-Smorodinsky method, supremum which is given by $(3/2, 2)$.

The coordinates of the intersection point P', between the straight line of maximum collective gain (i.e., $X + Y = 2.5$) and the segment joining α and the considered supremum (the segment is part of the line $\alpha + \mathbb{R}(1, 1)$), give us the desirable division of the maximum collective win $W = 5/2$, between the two players.

Second Possible Division. For another possible quantitative division of the maximum win $W = 5/2$, between the Financial Institute and the Enterprise, we propose a *transferable utility Kalai-Smorodinsky method.* The bargaining problem we face is the pair $(\Gamma, B'(0))$, where:

1. our decision constraint Γ is the transferable utility Pareto boundary of the coopetitive game (straight line $g = 5/2$);
2. we take, in our initial no-coopetitive game, the payoff with maximum possible collective profit, which is the point $B'(0) = (0, 1)$, as threat point of our bargaining problem (the payoff $B'(0)$ corresponds to the most likely Nash equilibrium of the initial no-coopetitive game—see paper [7]).

Solution. For what concerns the solution: we join $B'(0)$ with the supremum

$$\sup(\Gamma \cap [B'(0), \rightarrow [),$$

according to the classic Kalai-Smorodinsky method, supremum which is given by $(5/2, 3/2)$. The coordinates of the intersection point P, between the straight line of maximum collective gain (i.e. $g = 2.5$) and the segment joining $B'(0)$ and the considered supremum (the segment is part of the line $(0,1) + \mathbb{R}(1,1)$) give us the desirable division of the maximum collective win $W = 2.5$, between the two players. In Fig. 10 is shown the situation.

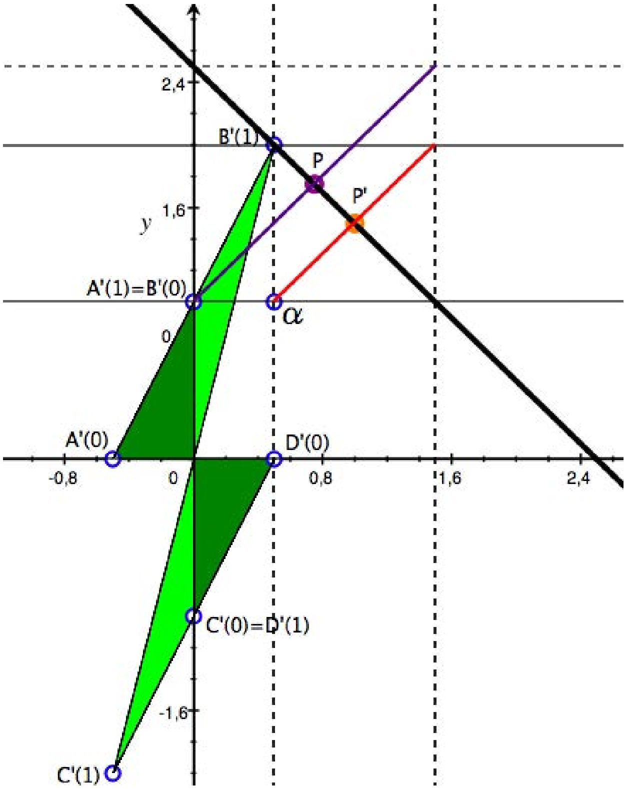

Fig. 10. Transferable utility solutions in the coopetitive game: cooperative solutions

Thus $P = (3/4, 7/4)$ and $P' = (1, 3/2)$ suggest as solution that the Enterprise receives respectively $3/4$ or 1 by contract by the Financial Institute, while at the Financial Institute remains the win $7/4$ or $3/2$.

Why Are There Differences between the Two Possible Division of Collective Profit? The difference between the points P and P' are due to the different method used.

About the point P', we consider as threat point the sup $\alpha = (1/2, 1)$ of the no-coopetitive game. Therefore, the division is more profitable for the Financial Institute because it can obtain in the no-coopetitive game a higher maximum profit (that is 1) than the Enterprise (that can obtain 1/2).

About the point P, we consider as threat point the retro-image of the most likely Nash equilibrium B of the no-coopetitive game. Therefore, the division is even more profitable for the Financial Institute because, according to most likely Nash equilibrium, it should obtain its higher maximum profit (that is 1) while the Enterprise does not win anything.

9 Conclusions

The games just studied suggest a possible regulatory model by which the phenomenon of the credit crunch (which in recent years has put in crisis small and medium enterprises in Europe) should be greatly attenuated. Moreover, the financial markets are stabilized through the introduction of a tax on financial transactions. In fact, in this way it could be possible to avoid speculations, which constantly affect modern economy. The Financial Institute could equally gain without burdening the financial system by unilateral manipulations of traded asset prices and, especially, the Financial Institute invests the money received by the ECB in the real economy lending money to the Enterprise (which also gains something).

No-coopetitive Game. The unique optimal solution is the cooperative one (exposed in paper [7]), otherwise the game appears like a sort of "your death, my life." This type of situation happens often in the economic competition and leaves no escapes if either player decides to work alone, without a mutual collaboration. In fact, all no-cooperative solutions lead dramatically to mediocre results for at least one of the two players.

Coopetitive Game. We can see that the game becomes much easier to solve in a satisfactory manner for both players. Moreover, the money received by the ECB is put into real economy by the Financial Institute: in fact the bank (our second player) issues a loan for the Enterprise (our first player), which uses the money in order to buy assets for its business activities. Both the Enterprise and the Financial Institute reduce their chances of losing than the no-coopetitive game, and they can even easily reach to the maximum of the game: so the Enterprise wins 1/2 and the Financial Institute wins 2. If they instead take the tranfer utility solutions with the Kalai-Smorodisky method, the Enterprise increases up to three times the payout obtained in the cooperative phase of the no-coopetitive game (3/4 or 1 instead of 1/3), while the Financial Institute wins twice more than it did before (7/4 or 3/2 instead of 2/3). We have moved from an initial competitive situation that was not so profitable to a coopetitive highly profitable situation for both the Enterprise and the Financial Institute.

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