Studies in Fuzziness and Soft Computing

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Multicriteria and Multiagent Decision Making with Applications to Economics and Social Sciences

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Preface

The book provides a comprehensive and timely report on the topic of decision making and decision analysis in economics and the social sciences. The various contributions included in the book, selected using a peer review process, present important studies and research conducted in various countries around the globe. The majority of these studies are concerned with the analysis, modeling and formalization of the behavior of groups or committees that are in charge of making decisions of social and economic importance. Decisions in these contexts have to meet precise coherence standards and achieve a significant degree of sharing, consensus and acceptance, even in uncertain and fuzzy environments. This necessitates the confluence of several research fields, such as foundations of social choice and decision making, mathematics, complexity, psychology, sociology and economics. The main topics being investigated are:

- scientific and philosophical foundations of social choice and decision together with relevant aspects of uncertainty in complexity;
- planning, control and use of land and cities
- social, economic and financial systems

These topics are comprehensive discussed in the various contributions included in this book.

The foundational aspect is the main focus of the work by Patrik Eklund, Mario Fedrizzi, and Robert Helgesson (*Monadic Social Choice*)**.** Here the authors show how monads and substitutions allow for a separation between social choice, as value, and social 'choosing' as operation. The key reference lies in a seminal concept of the theory of fuzzy sets, namely Goguen category Set(L).

Silvia Bortot and Ricardo Alberto Marques Pereira's work (*The Generalized Gini Welfare Function in the Framework of Symmetric Choquet Integration*) deals with Social Welfare and Choquet integration. It shows that any 2-additive symmetric Choquet integral can be written as the difference between the arithmetic mean and a multiple of the classical Gini inequality index, with a given constraint. In the special case of positive parameter values this result corresponds to the well-known Ben Porath and Gilboa formula for Weymark generalized Gini welfare functions.

The paper by Enrico Ciavolino and Giovanni Indiveri (*Entropy Based Estimators in the Presence of Multicollinearity and Outliers*) describes two estimators inspired by the concept of entropy that allow the authors to robustly cope with multicollinearity, in one case, and with outliers, in the other. The Generalized Maximum Entropy estimator optimizes the Shannon's entropy function subject to consistency and normality constraints. The Least Entropy-Like estimator is a novel prediction error model coefficient identification algorithm that minimizes a nonlinear cost function of the fitting residuals.

The role of fuzzy regression in dealing with the causal complexity occurring in social phenomena is studied by Antonio Maturo, Fabrizio Maturo (*Research in Social Sciences: Fuzzy Regression and Causal Complexity*). The authors analyze in details some aspects of the fuzzy regression, and define suitable operations between fuzzy numbers. Finally, they defines some critical remarks about the causal complexity and logical limits of the assumption of linear relationship between variables.

Antonio Maturo and Aldo G.S. Ventre (*Multiobjective Decision Making, de Finetti Prevision and Fuzzy Prevision*) introduce an approach to multiobjective decision making in the context of finite de Finetti random numbers. The objectives are events, the action of an alternative with respect to an objective is seen as a finite de Finetti conditional random number. The global score of an alternative with respect to an objective, is the de Finetti prevision. Coherence conditions are investigated and criteria for aggregating scores are defined.

The research conducted by Pietro D'Amico, Ferdinando Di Martino, and Salvatore Sessa (*A GIS as a Decision Support System for Planning Sustainable Mobility in a Case-Study*) is about territorial analysis. It deals with the finding, selection, valuation, weighting and synthesis of a set of indicators to monitor the Coordination Plan of the District of Napoli. The method, implemented with a GIS, is intended to enhance sustainable mobility, one of the main goals of the above-mentioned coordination plan.

Šárka Hošková-Mayerová, Václav Talhofer, and Alois Hofmann (*Decision-Making Process with respect to the Reliability of Geo-Database*) developed a system which focuses on data and spatial information precision, and on reliability evaluation. They describes the intervention of a fire rescue unit as case scenario and show how the proposed system can be used in practice.

Rational use of energy is the central topic of the work by Antonella Violano and Francesca Verde (*Protocol ITACA: a Decision Tool for an Energetically Efficient Building Management*) Their paper describes the Italian Protocol ITACA, a tool which evaluates the interrelations between building and surrounding environment.

The Evaluation of Interventions in Urban Areas: Methodological Orientations in the Programming of Structural Funds for the period 2007–2013 by Barbara Ferri starts from an analysis of innovation in urban and territorial policies and investigates the changes taking place during the evaluation of interventions in urban areas, also in the context of local development. The paper underlines the central importance of impact evaluation together with models based on targets and discusses the limits of quantitative evaluation methods.

The paper *Assessing Plans and Programs for Historic Centers Requalification: an Interactive Multicriteria Approach* by Salvatore Ercolano, Fabiana Monacciani, and Pietro Rostirolla proposes a decision support system for the definition and implementation of complex policies aiming at the global preservation and enhancement of cultural heritage, in a context of poor funding. The proposed methodology is applied to a decision problem derived from the "Great Program for the Historic Center of Napoli", enrolled in the UNESCO World Heritage List since 1995.

Fabio De Felice and Antonella Petrillo (*Decision Making Analysis to Improve Public Participation in Strategic Energy Production Management*) propose a multicriteria methodological approach based on the Analytic Hierarchy Process methodology (AHP) to examine the scope and feasibility of the AHP integrated with public participation approach. The main goal is to incorporate the prioritization criteria for the assessment of various energy policies for power alternatives, and evaluate these policies against these criteria.

The paper *Development Policies in China: an Analysis of the Territorial Imbalances* by Roberta Arbolino verifies the efficacy of the rebalancing policies adopted in China, in order to establish their role in the convergence process. The main contribution of this work consists in evaluating the rebalancing policies implemented locally as a whole through a disaggregated analysis. This method not only produces useful information on the *in progress* convergence process between the most developed provinces and the inland areas, but also gives specific insights on the suitable strategies that need to be implemented by the different provinces.

Carmen Costea and Diana Tâmpu (*The MAS Models Use - an Imperative Approach to Build a New Economic Paradigm*) discuss the benefits of multiagent models in economy. They claim that a scientific revolution is needed now more than ever in economy, in order to get out of the endless recession and to enable economic growth during the crisis. They show how multicriteria or multiagent models may promote this revolution.

Starting from the point of view that legal policies are not neutral in terms of social impact and that law can be analyzed using complex system tools, Noemi L. Olivera, Araceli N. Proto Claudia M. Sarris (*Quantum Decision Making, Legal Complexity and Social Behaviour*), discuss why individuals opt for some legal instruments, which are not of compulsory application, or do not choose them. Using the available data and a quantum decision making model, the authors describe, for the Argentinean case, why among the available typical joint venture regimes, the Temporary Union of Firms is preferred by users rather than the Group of Collaborating or the Consortium of Cooperating Firms.

The paper *Analysis of the Italian Banking System Efficiency; a Stochastic Frontier Approach* by Cesare Imbriani, Luca Giordano, and Antonio Lopes focuses on the efficiency of Italian banks, in terms of parametric cost and profit functions, taking into account the dualistic structure which characterizes the Italian economy, the bank size and the juridical form. An analysis of some features of the Italian banking system during the decade 1998–2008 leads to confirm, in particular, the ability of local small Mutual Banks to effectively and successfully compete in the markets characterized by global operators.

David Carfì and Francesco Musolino (*Credit Crunch in the Euro Area: a Coopetitive Solution*) propose a methodology to attenuate the plague of the credit crunch in the Euro area, which is very common today: despite the banking world possesses a huge amount of money, there is no available money in the real economy. The authors claim that a way to allow a global economic recovery is to adopt the new mathematical model of *Coopetitive Game* with two economic interacting operators: a real economic subject and a financial institute with a big economic availability.

The paper *Modelling the Intertemporal Choice through the Dynamic Time-Perception* by Salvador Cruz Rambaud and Viviana Ventre deals with the process of choice over time. Intertemporal choice is intimately related to the concept of discounting function. A multicriteria framework is introduced in which a group of agents can (or cannot) co-operate in order to obtain a greater profitability. In this financial context, it is necessary to choose between transitive and non-transitive choice, giving rise to subadditive and non-additive discounting.

The process of choice over time is also the main subject of the paper by Salvador Cruz Rambaud, María José Muñoz Torrecillas (*An Analysis of Inconsistency in Intertemporal Choice).* Here the authors stress the issue of non-constant discounting. According to some empirical studies, economic agents do not always use constant discount rates over time. One of the most important problems of non-constant discounting is inconsistency in intertemporal choice. This work shows that one of the main sources of inconsistency is subadditivity.

The paper *Intuitionistic fuzzy preference relations and hypergroups* by Irina Cristea shows a connection between intuitionistic fuzzy relations and hypergroups. Here, the author constructs a hypergroup associated with a binary relation, that has been naturally induced by an intuitionistic fuzzy relation. The author investigates in which conditions the hypergroup is a join space or a reduced hypergroup, in the framework of the intuitionistic fuzzy preference relations.

The work from Janusz Kacprzyk, Slawomir Zadrozny, Hannu Nurmi and Mario Fedrizzi (*On some voting paradoxes: a fuzzy preference and a fuzzy majority perspective*) deals with group decision making. A group of individuals (decision makers) provide their individual preference relations concerning an issue over some set of options. The problem is to find a solution, i.e. an alternative or a set of alternatives which reflects in the best possible way the preferences of the group of individuals as a whole. This work shows how fuzzy preferences may help alleviate some known voting paradoxes.

Ronald R. Yager (*Using Agent Importance to Combat Preference Manipulation in Group Decision Making*) considers a problem that can arise in group decision-making when the selection process is based upon a group preference function obtained by an aggregation of the individual preference functions of the group members. The author describes and formalizes the possible degeneracy of a group decision making process when a strategic manipulation is triggered by individual agents. This strategic behavior could lead to a form of impossibility. Some ways of modifying the formulation of the group decision functions to discourage manipulations are suggested.

Bice Cavallo, Livia D'Apuzzo, and Massimo Squillante (*Pairwise Comparison Matrices over Abelian Linearly Ordered Groups: a Consistency Measure and Weights for the Alternatives*) provide a survey of results related to pairwise comparison matrices over a real divisible and continuous abelian linearly ordered group. This approach allows the author to unify different approaches (e.g. multiplicative, additive, fuzzy). In this way, the consistency condition is expressed in terms of the group operation.

Moreover, under the assumption of divisibility, a consistency measure, expressed in terms of distances, is provided.

Leandro Pecchia and Paolo Melillo (Analytic Hierarchy Process for Health Technology Assessment. A case study for selecting a maintenance service contract) use the Analytic Hierarchic Process (AHP) to improve Health Technology Assessment. They discuss a method that is able to track decision processes and make stakeholders understand the work done by decision-makers (DMs); to properly weight the most appropriate DM for each dimension of the problem, to extend decision processes to DMs who are not skilled in complex mathematical methods.

A large spectrum of problems that may be encountered during decision making and decision analysis in the areas of economics and the social sciences, together with a broad range of tools and techniques that may be used to solve those problems, are presented in detail in this book, making it an ideal reference work for all those interested in analyzing and implementing mathematical tools for application to relevant issues involving the economy and society.

Contents

Development Policies in China: An Analysis of the Territorial Imbalances

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Abstract. The aim of this paper is to verify the efficacy of the rebalancing policies adopted in China, in order to establish their role in the convergence process.

The main contribution consists in evaluating the rebalancing policies implemented locally as a whole through a disaggregated analysis, obtaining in addition to useful information for the decision maker on the *in progress* convergence process between the most developed provinces and the inland areas, specific insights on the suitable strategies to be implemented by each province.

The economic expansion started in December 1978, as a consequence of the decision taken by the Chinese Communist Party, led to the country economic openness, but also entitled regional disparities.

Three rebalancing policies — China Western Development, Revitalize Northeast China, Rise of China Central Plan — have been established since 1999 by the Chinese Government. They differ essentially in the targets and the destination areas.

Based on data referred to the years 1995, 2000 and 2010 and taken from the "National Bureau of Statistics" in China, this paper follows a five steps - based approach:

1. Survey of the policies;
2. Recognition of the objectives associated with each policy;
3. Building up of the indicator for measuring policies impacts;
4. Pre and post testing policies;
5. Check of the role played by policies in the convergence process.

Results show that the rebalanced policies have been crucial in stimulating and effectively allowing a strong development in China as a whole, specifically in all those provinces for which initially the Chinese Government did not make up any efforts, believing that their development would have been followed consequently to that of the coastal provinces.

However, the final goal of the convergence has not been totally met and the level of development of inland areas is certainly lower than the coastal ones.

Keywords: Policies, Development, evaluation, territorial imbalances.

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1 Introduction

The traditional analysis of regional imbalances highlighted the ability of market mechanisms in determining a path of convergence of less developed areas to production levels of the advanced ones (Solow, 1956; Swan, 1956). Conversely, analyses based on Keynesian models (Harrod, 1939; Domar, 1946) showed an unbalanced view of the market economy functioning. In Hirschman (1958), imbalances acquired a positive role, with their distribution of profits and losses, as a stimulus to the economic transformation.

This vision of development requires interventions in less developed areas, focusing on their ability to solicit additional investments, concentrating them in space.

The main idea was not to bring about development for all territories, but to stimulate active forces (economies of agglomeration, marshal external economies) so as to favor "polarization effects" (attraction of investment, capital, qualified labor) and possible "ripple effects" (purchase of intermediate goods, raw materials, absorption of the unemployed concealed by developed areas). Along the same line of research, other concepts have been proposed like the "vicious and virtuous circles" (Nurkse, 1958) or "cumulative causation" (Myrdal, 1957), for which the market mechanisms let the interregional disparities increase. In contrast Williamson (1965), recognizing however the polarization of development in the central areas of a country, set out an optimistic vision for which, after a certain level of income, the strong area would carry-over effects on the weak, reducing regional disparities of growth following an inverted U-shaped path.

China is an emblematic case of imbalances between coastal and inland areas.

In 1978, the Chinese government introduced the so-called open door policy, allowing the economic openness of the country and its rapid rise on the global scene.

However, economic growth has not occurred evenly across the country: economic openness, aimed at attracting foreign capitals and technologies, has been implemented for long only to the so-called “Special Economic Zones”, overlooking the development of other regions.

The main problem was that the effect of towing, promoted by conventional economics theories, consisting in leading development from strong to weak areas, was not achievable, as the disparities in terms of income per capita and the territorial disparities between Coast and Hinterland demonstrate.

Only from the 1990s a change of policy occurred, so as to take into account, at least in theory, the established dual system and the redistribution of resources to poorer provinces (Dumerger, 2003).

2 China Embalanced Policies

By creating the Special Economic Zones, Deng Xiaoping wanted mainly to attract foreign capitals and favor the entering of the country on the international market. The government then put in place complex development programs aimed precisely to make these areas attractive to foreign investors[1].

Results of the open door policy were, therefore, positive: the strong economic growth (the growth of GDP was about 9% from 1978 to 1990), the improved average living conditions of the population, the increase of consumptions and international integration (Centre for the Study of enterprise, 1997) were some evidences.

In view of a major expansion, China's transition towards a market economy brought about new issues of unbalanced growth[2] among the coastal and inland provinces : as pointed out by Weber (2005) " were only large metropolitan coastal regions to reap the benefits of a development model essentiality export-oriented the reform of the banking system, the entry of large foreign capital, the restructuring of SOEs (however still ongoing) and the emergence of numerous domestic firms, which is characterized by a real private enterprice".

Then, the Chinese government ratified, since 1999, three *ad hoc* reforms.

a) China Western Development.

The Primary purpose of this policy was the realization of economic growth in the western provinces through capital investments and development of natural resources.

Most of the projects proposed by the government focused on the development of reliable infrastructures, creating favorable environment for investments and skilled workforce, and tended also to maintain the ecological balance of the western region, which in the long term would have contributed to the improving of local life standard. However, in order to ameliorate the urban infrastructure of inner regions, most of the funds were assigned to transportation, energy and communication.

Particularly important was the commitment of the State in encouraging and improving the region's industries, focusing on the most competitive sectors, namely: nuclear industry, electronics, solar energy.

b) Revitalize Northeast China.

The policy implemented for the North-East region covered the three provinces of Heilongjiang, Jilin and Liaoning and the five Eastern prefectures of Inner Mongolia: it was primarily aimed at revitalizing the area, transforming the existing industrial system in national and international basis for the manufacture equipment and crucial raw materials.

In fact, although it was always considered the cradle of the industrialized China, the adaptation process to structural reforms and economic transition of the country

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[1] For an exhaustive review see Yingqi Wei, Bo Liu and Xiaming Liu (2005) and Arbolino (2008)

[2] However, this opening process also caused the negative effects, such as:

- the increase in unemployment following the increase of labor force entry into the market;
- the high inflation (21.7% in 1994);
- a legal system out of step with economic development;
- the uneven distribution of income in the population (particularly among workers of state-owned and private, urban and agricultural areas);
- unbalanced development to the detriment of the inner regions, disadvantaged by the strategy of opening coastal and Special Economic Zones.

was found to be very hard. The business and the market suffered a big loss, while the support offered by central administration turned out to be ineffective because of fundamental structural and institutional problems.

In 2003, guidelines were established for encouraging in the region the building up of institutional and structural reforms, primarily through private sector growth and for the increasing of investments[3] abroad.

To this purpose, major efforts were voted in particular to the reform of enterprises already operating in the territory, to the manufacture of efficient and coordinated use of existing resources and to the increase of production efficiency. In addition, it was necessary to coordinate properly the relationship between the actors of the reform through the direct involvement of both the central administration, as the local government, central government departments and enterprises, but also as a clear division of spheres of competence.

c) Rise of Central China Plan.

After the interventions delivered both in the western and eastern regions, the need to focus on the central region, as the connecting area between the growth of the east and west of the country, rose, accelerating the development, so far inadequate.

Exploiting the geographical advantage of being located between a region with an advanced level of development as the east, and a less developed region like the West area however very rich of resources , the target chosen for this area was to turn it into the largest production base of grain in China, as base of raw materials and energy production, introducing advanced technologies in industry and exploiting foreign investment, in line with the idea of making a balanced interregional development .

Coherently with this goal, the need to adjust the structure of the agricultural sector was first established. Then, investments in agricultural infrastructures and for the construction of an ecological environment increased, while the realization of high quality agricultural production and the development cycle of all agricultural activities: forestry, animal husbandry and fishing were promoted.

Furthermore, for both the traditional and the high-tech industry it was necessary to extend the industrial production from an economic, qualitative and quantitative standpoint, in order to improve the industrial competitiveness of the region.

3 About the Effect Analyses

In order to consider the benefits of a given policy it is necessary to identify changes that would have occurred even without interventions (Heckman, J., and J. Hotz, 1989).

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[3] It was immediately carried out an important reform of business, by reducing the share of government involvement in business and offering incentives for development of private enterprises in order to inject new energy economy, also in tax for enterprises, has been a reduction in VAT collection, so as to attract foreign investments and encourage the restructuring and technological upgrading in existing businesses, revitalization and improvement of old industrial structure of the region.

This means that to determine the effect of public policy, experimental or quasi-experimental methods need to be employed to reconstruct what would have happened to those involved by that policy if they had not been involved, through a situation called " counterfactual "(Farrington, 2003).

However, the inability in some cases to manipulate the selection process is the major limit of this method: basically, in many cases the characteristics of the intervention do not allow to selectively apply a policy to some and not to other stakeholders, without altering the operation.

The inability to directly observe the counterfactual situation creates a dilemma: if the effect of a policy is defined as the difference between what happened after the intervention and what would have happened without the intervention, it can never be determined with absolute certainty, as this would imply to make a comparison (with respect to the variables on which public policy will affect) between an observable value and a hypothetical one, which is unobservable.

From the non-observability of counterfactual follows, as a logical consequence, the non-observability of the effect. Strictly speaking, an effect can never be observed (or so "measured") directly, because it is not possible to observe simultaneously the same subjects in the status of the beneficiaries of an intervention and that of non-beneficiaries.

The fact that an effect is never directly observable, however, does not exclude the possibility of arguing something plausible about such an effect.

To the extent that the counterfactual can be plausibly reconstructed with other information, it is still possible to estimate the effect as the difference between the situation observed post-intervention and the (likely) reconstruction of the counterfactual situation (A Martini, M. Sisti, 2007 - A Martini, M. Sisti, 2009).

Statistical -economic evaluators define these methods as non-experimental, while the sociological or psychological ones define them as quasi-experimental (Blundell, R., Costa Dias, M., 2000, Campbell DT, Stanley, JC 1963).

Through these methods the policy maker observes what happens: the counterfactual will be approximated by observing what happens to other people and / or in other periods of time.

Among the different strategies of analysis[4] to arrive at a quantitative estimate of the effects in our work we preferred to simultaneously use the "pre-post comparison for treated units " and "with-without comparison" (Heckman, J., 2001).

The pre–post comparison for the covered units is the leader of what in literature evaluation is sometimes defined as "one group pre-post design." The main characteristic of this family of approaches is not to use any of the non-treated group of units, but to refer only to treated units using the available information on the subject exposed to the policy before the policy had been adopted.

4 Methodology

In order to evaluate the rebalancing policies' efficacy adopted in China, the net contribution of each analyzed intervention on the provinces has been considered.

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[4] For a review see V. Moffit R, 199. Card D., Krueger A., 1995 ,Card D. 1990.

To date, some authors ((Hongyi Harry Lai, 2002, Demurger S., JD Sachs, WT Woo, S. Bao, G. Chang, A. Mellinger, 2002, Yeung, YM 2004, Fan Jie. 2004) have already considered the problem, however, without analyzing individual policies. This, in our view allow, instead at capturing the different impacts in different regions obtained as a result of policies put in place.

Our analyses have been carried out at large-scale (in terms of policies), but also at a greater level of detail, so as to put in evidence the differences between the two approaches.

To verify the existence of a convergence process, the country has been divided joining the provinces/municipalities under subjugation of several politics, overcoming the traditional political–geographical divisions .

Table 1 summarizes for each policy the area of action, the strategy, the objectives and the implementation period. It was possible to monitor ex post only the western China development, while the other type of policy monitoring is ongoing.

Table 1. Policies for reducing regional imbalances

	PROVINCES	OBJECTIVES	YEARS
China Western Development	GUANGXI INNER MONGOLIA NINGXIA TIBET XINJIANG GANSU GUIZHOU QINGHAI SHAANXI SICHUAN YUNNAN CHONGQING	- INFRASTRUCTURE IMPROVEMENT - ECOLOGICAL ENVIRONMENT PROTECTION - INDUSTRIAL RESTRUCTURING	1999-2010
Revitalize Northeast China	HEILONGJIANG JILIN LIAONIG	-INSTITUTIONAL AND STRUCTURAL RENOVATION - INDUSTRIAL BASE REJUVENATION - INCREASE IN FOREIGN INVESTMENT	2003- 2015
Rise of China Central Plan	SHANXI HENAN ANHUI HUBEI HUNAN JIANGXI	-INDUSTRIAL MODERNIZATION -OPTIMIZATION OF FOREIGN INVESTMENT - REGIONAL COOPERATION	2009-2014

To read the impacts of individual policies, some indicators, expressing the most important aspects and related to the phenomenon under examination, have been selected according to the general objectives, or to areas of intervention of policies.

The study area covers 28 Chinese provinces and three municipalities. Data have been taken from "the National Bureau of Statistics of China" for the period 1995-2000-2010.

To measure the effectiveness of policies, we proceeded comparing the economic results, expressed in percentages, found in the provinces in which policies aimed at reducing inequalities have been applied and that occurred in the rest of China.

In particular it has been verified:

1) the effectiveness of policies on growth :

- through a spatial comparison (in terms of average annual growth rate over the previous period, 1995-2000, which is taken as a proxy of the trend in the absence of interventions) and territorial comparison (both to the rest of China and to the coastal areas);

2) the effectiveness of policies in narrowing the gap with more advantaged areas, using:

- The percentage variation of the absolute values of the Year 2010 with respect to the year 2000, which represents the reduction or absence of remote inland areas to coastal areas.

To calculate this indicator and compare the areas of different sizes it was necessary to normalize the data relatively to the GDP or the surface or the population.

In the next three tables the impacts on the Territory Realized by Each policy are presented.

The success or the failure of the declared objectives has been totally evaluated through a qualitative judgment, considering all the examined indicators. It has been summarized in the last column using a color scale where darker colors represent the greatest impacts, while the lighter colors express the least impacts[5].

Among the analyzed policies the China Western Development (now CWD) results the policy that obtained the best benefits, both in comparison with growth rates that would be obtained in the absence of intervention and compared to other areas investigated.

In particular, the highest values of growth rates in the considered period are read with reference to infrastructure provision (e.g. 35.84% on highways), but with highly variability rates — the highest values are obtained in the province of Tibet .

However, this type of intervention affected not only the inland areas, but China as a whole (the value for the examination indicators is about 30%).

Even with regard to corporate restructuring, good results especially in the secondary and tertiary have been achieved. Indeed, in line with the promoted objectives the average annual growth in both areas was around 20%, while the lowest values affected the primary sector.

Good results have been achieved also by the objectives of "environment protection" and "science emphasis" : data on the environment protection are primarily evident in the western, instead with reference to implemented expenditure levels , the coastal areas have the highest rates of growth followed by the rest of China.

Finally, with reference to reducing the gap, also the distance of the western from the coastal areas among the period 2000 to 2010 appears to be small.

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[5] The dark blue represents the achievement of a very rilevant impact, the light blue corresponds with a rilevant impact, the green is equivalent to negligible impact and finally the orange coincides to ineffective impacts.

Table 2. Efficacy In China western development policy

China western development		Spatial Index (1995-1999)	Territorial index (2000-2010)			Narrowed gap (2000-2010)			Efficacy
Objective	**Indicator**	**No policy**	**A) China western development**	**B) Other part of the China**	**C) Coastal Provinces**	**D) C/A 2000**	**E) C/A 2010**	**E/F**	
Improving infrastructures	Length of Railways in Operation (KM)	3.02%	4.98%	4.21%	6.06%	90%	56%	0,62	
	Length of Navigable Inland Waterways(KM)	2.41%	10.97%	11.14%	9.61%	-85%	-92%	1,09	
	Length of Highways(KM)	1.95%	35.84%	30.57%	30.06%	84%	48%	0,57	
	Total Investment in Fixed Assets in the Whole Country (ml yuan)	11.83%	26.05%	23.30%	20.82%	774%	495%	0,64	
Corporate restructuring	Primary Industry	4.87%	11.35%	10.52%	9.83%	54%	43%	0,80	
	Secondary Industry	8.84%	20.51%	17.94%	17.86%	-69%	-51%	0,74	
	Tertiary Industry	10.92%	18.70%	18.47%	19.33%	63%	-21%	-0,33	
Environment protection	Investment Completed in Treatment of Industrial Pollution	N.D.	12.08%	4.30%	2.47%	41%	40%	0,98	
	Investment Completed in the Treatment of Industrial Waste Water(10,000 Yuan)	N.D.	6.79%	3.15%	2.08%	503%	77%	0,15	
	Treatment of Industrial Waste Gas(10,000 Yuan)	N.D.	15.77%	5.01%	0.67%	374%	118%	0,32	
	Treatment of Solid Waste(10,000 Yuan)	N.D.	6.80%	-3.40%	-8.53%	561%	18%	0,03	
	Treatment of Noise Pollution(10,000 Yuan)	N.D.	10.35%	0.88%	0.35%	650%	15%	0,02	
	Treatment of Other Pollution(10,000 Yuan)	N.D.	22.93%	7.56%	11.62%	393%	37%	0,10	
Science emphasis	R&D Expenditure by Region (100ml yuan	N.D.	20.03%	23.41%	23.53%	1220%	262%	0,21	

If we compare the Revitalize Northeast China policy (RNC) RNC to the previous, the effectiveness of the policy appears lower.

With respect to all objectives — wealth, employment, Industrial Revitalization — the rates are significantly increased, although still lower when compared with the rest of China or the coastal areas.

This is true especially with reference to the secondary sector, where despite the growth rates being high, they are lower in comparison with other areas. Analyzing in particular the policies we observe that Jilin obtains for almost all considered indicators the highest rates of growth (GRP 15.70%, secondary industry 20.97%), with the exception of rates on employed, that are higher in the province of Liaoning.

About the gap between the area it appears that the difference between GRP is decreased, sign of growth in the Northeast, together with the employed and the revenue from principal business, while no other relevant effects are readable on the remaining indicators.

Table 3. Efficacy In Revitalize Northeast China policy

Revitalize Northeast China		Spatial Index (1995-1999)	Territorial index (2000-2010)			Narrowed gap (2000-2010)			Efficacy
Objective	**Indicator**	No Policy	**A) Revitalize Northeast China**	**B) Other part of the China**	**C) Provinces no policy**	**D) A/C 2000**	**E) A/B 2010**	**E/F**	
Wealth	Gross Regional Product (100 ml yuan)	7.98%	14.4%	16.2%	16.0%	435%	394%	0,91	
	Per Capita GRP (yuan)	7.56%	14.2%	14.7%	13.4%	868%	590%	0,68	
Employment	Number of Employed Persons (10000 person)	0.00%	1.5%	2.1%	3.0%	279%	293%	1,05	
	Urban Employed Persons (10000 person)	0.00%	1.4%	5.0%	6.8%	309 %	206%	0,66	
	Rural Employed Persons (10000 person)	0.00%	1.5%	1.0%	1.1%	336%	280%	0,84	
Industrial revitalization	Ratio of Value-added to Gross Industrial Output Value (2007)	M.D	0.3%	0.3%	-0.7%	212%	163%	0,77	
	Secondary Industry	7%	16.5%	18.6%	17.9%	202%	193%	0,96	
	Number of Enterprises(Ge)	-29%	11.6%	10.8%	11.9%	403%	337%	0,84	
	Total Assets(100 Ml Yuan)	M.D	13.7%	18.1%	18.2%	200%	270%	1,35	
	Original Value of Fixed Assets(100 Million Yuan)	10%	13.9%	17.0%	17.2%	154%	163%	1,06	
	Revenue from Principal Business(100 Ml Yuan)	4%	24.8%	26.0%	25.3%	324%	276%	0,85	
	Ratio of Total Assets to Industrial Output Value	M.D	3.9%	6.7%	5.6%	132%	224%	1,70	

The third policy maximizes the optimization of foreign investment, achieving very high percentage rates, while the impact on the industrial Modernization is smaller if we compare the data with the previous years and with the other areas.

Analyzing the provinces individually we observe that the province that achieves growth rates considerably higher than the others is Jianxi (with rates around 40% on the values of optimization of foreign investment), while the target in manufacturing of high quality is achieved primarily by the province of Shanxi.

Changes in growth of the transaction value in the technical Market are significant in the provinces of Anhui (10%), Jiangxi (9.50%) and Hubei (7.40%).

Evaluating the effectiveness in terms of relative distance between the areas, in the analyzed period, the most considered indicators remain unchanged or increased. Relevant exceptions are: transaction value in technical market, the rate of products with excellent quality, Total Imports of Foreigh-funded Enterprises. However, these rates are high in terms of decreased distance between the central provinces and the coastal area.

Table 4. Efficacy In Rise of Central China Plan policy

Rise of Central China Plan		Spatial Index (1995-1999)	Territorial index (2000-2010)			Narrowed gap (2000-2010)			Efficacy
Objective	**Indicator**	**No policy**	**A) Rise of Central China Plan**	**B) Other part of the China**	**C) Provinces no policy**	**D) A/C 2000**	**E) A/B 2010**	**E/F**	
Optimizing foreign investment	Total Exports of Foreigh-funded Enterprises(10000dollar)	4.88%	27.57%	21.76%	21.30%	901%	2001%	2,22	
	Total Imports of Foreigh-funded Enterprises(10000dollar)	7.28%	26.33%	20.06%	19.31%	1578%	987%	0,63	
Industrial modernization	Transaction Value in Technical Market by Region (100 MlYuan)	M.D.	5.24%	7.80%	8.94%	128%	51%	0,40	
	Rate of Products with Excellent Quality(%)	4.12%	8.48%	11.57%	9.97%	515%	101%	0,20	
	Rate of Products with First Grade Quality(%)	7.21%	-2.07%	-1.03%	-2.19%	506%	525%	1,04	
	Rate of Products with Quality(%)	-5.37%	-6.96%	-12.68%	-13.12%	47%	63%	1,33	
	Three Kinds of Application for Patents Accepted(Jian)	9.05%	23.98%	23.77%	25.81%	62%	117%	1,90	
	Three Kinds of Application for Patents Granted(Jian)	19.67%	20.77%	24.13%	25.44%	44%	47%	1,06	

5 The Factorial Analysis

To assess the benefits and to observe changes in the structure of the provinces involved in the examined policies, we synthesized the indicators previously considered by a factor analysis. The analysis of a large number of indicators, with highly differentiated rates between municipalities, is dealt with by methods of multidimensional analysis of data (AMD), that study the association between variables or the relationships between the units through the organization of information structures in multiple dimensions, each of them possibly with multiple levels (Hotelling, 1933; Benzécri, 1973) as the factor analysis.

Factor analysis was developed to satisfy the need to simultaneously examine the interrelationships between multiple aspects of a single system. In order to provide an overview, we used Factor Analysis, so as to look at a multitude of variables related to each other in various combinations and summarized in a single structure easier to visualize and interpret.

The most important structural variable, Asia-Pacific, represented by the axis 1 horizontal to the plan explains about 63% of the total information.

It describes on the right side the provinces characterized by a balanced and developed economic structure while the vertical axis, which explains 17% of the total information, describes the economy of the provinces mainly driven by foreign investment. The graph shows the projection of the clusters of provinces grouped by policy on the factorial axes.

It is possible to observe that the economy of coastal areas is changed, the CWD and RCCP have undertaken in course of development based on domestic demand, while the contribution made by the RNC is less effective.

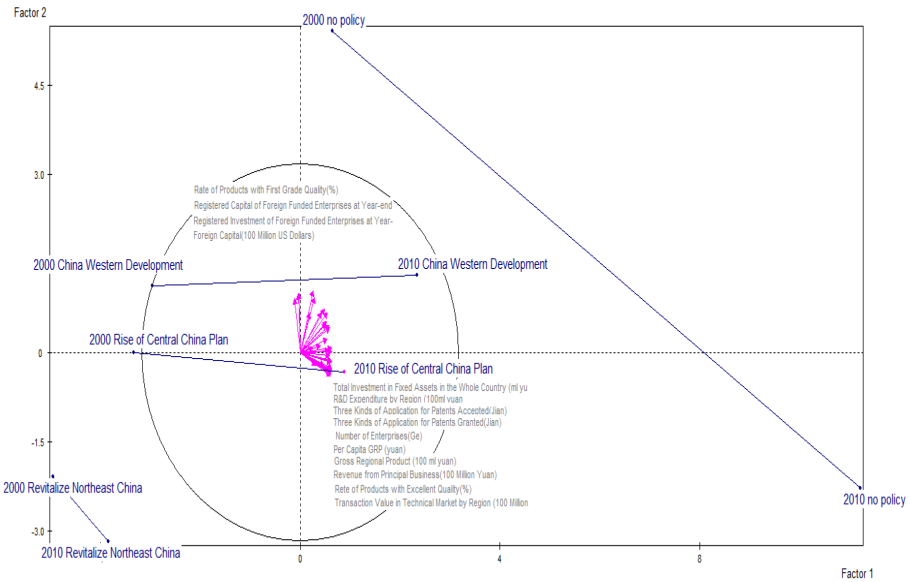

Fig. 1. Representation of the cluster on the factorial plane relative to the first two principal components

6 Conclusion

The policies aimed at rebalancing have had the merit to stimulate and to actually allow a strong development in China, or rather in all those provinces that initially were neglected by the Chinese government , because of the idea that focusing primarily on the development of the provinces coastal growth would have followed for the others.

So, for now, while it is true that the Western, Central and Northeast provinces have been characterized by massive government interventions, targeted industrial policies, investment plans in view of specific objectives that have been totally achieved, it is also true that the ultimate goal of the balance has not been completely met.

In practice, the growth generated at the same time inequalities and the gap between regions has not yet been filled, making these actions appear as "bad".

However, some considerations can be drawn:

First, it should be recognized that China is still in transition and there are many strategies for rebalancing in progress, for which there are long intervals.For instance, the policy implemented in favor of the central provinces, Rise of Central China Plan, which was introduced just two years ago, includes targets whose completion is expected by 2014.

Moreover, in evaluating the results achieved by China, it cannot be denied the recent global economic turmoil and, in particular, the events in 2008, that influenced in a decisive way the Chinese economy from both domestic and foreign relations points of view.

Obviously this has negatively affected the Chinese economy: given the international situation, in the first 10 months of 2008 the export of Chinese goods on a large scale has been hit hard, and economic growth has declined by about 10%. Moreover, the decline in trade with foreign countries has caused huge disruption of livelihood, while a majority of small enterprises with low competitiveness have even failed. However, the Chinese government, faced with this serious economic situation in the second half of 2008, has had the ability to implement a flexible and timely restructuring of macroeconomic policy, financial policy of transforming from "stable" to "active" and monetary policy from "restricted" to "appropriately relaxed", earning the positive consideration of other states. The year 2008 was also the year in which the Chinese economy had to grapple in other ordeals such as the extraordinary natural disasters.

In practice, despite an increasingly unstable external environment and the many calamities that have struck China as never before, the interest of the whole world for this state, characterized by a timely reassessment of economic policies, the successful organization of the Olympics and the reaction to the financial crisis, is always increasing; while from the point of view of the internal growth the needs of rebalancing are still strong.

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The Generalized Gini Welfare Function in the Framework of Symmetric Choquet Integration

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Abstract. In the context of Social Welfare and Choquet integration, we briefly review, on the one hand, the classical Gini inequality index for populations of $n \geq 2$ individuals, including the associated Lorenz area formula, and on the other hand, the k-additivity framework for Choquet integration introduced by Grabisch, particularly in the additive and 2-additive symmetric cases. We then show that any 2-additive symmetric Choquet integral can be written as the difference between the arithmetic mean and a multiple of the classical Gini inequality index, with a given interval constraint on the multiplicity parameter. In the special case of positive parameter values this result corresponds to the well-known Ben Porath and Gilboa's formula for Weymark's generalized Gini welfare functions, with linearly decreasing (inequality averse) weight distributions.

Keywords: Social Welfare, Gini Inequality Index, Symmetric Capacities and Choquet Integrals, OWA Functions, 2-Additivity and Equidistant Weights.

1 Introduction

The Gini inequality index [24,25,21,15] plays a crucial role in Social Welfare Theory and the measurement of economic inequality [2,45]. In the literature several extensions of the Gini index have been proposed [14,47,48,49,16,9,4], in particular the generalized Gini inequality index and the associated welfare function introduced by Weymark [47] on the basis of Blackorby and Donaldson's correspondence formula [5,6],

$$A_G(\mathbf{x}) = \bar{x} - G_A(\mathbf{x})$$

where $G_A(\mathbf{x})$ denotes the (absolute) generalized Gini inequality index, $A_G(\mathbf{x})$ is the associated generalized Gini welfare function, and $\mathbf{x} = (x_1, \ldots, x_n)$ represents the income distribution of a population of $n \geq 2$ individuals. Recently, the extended interpretation of this formula in terms of the dual decomposition [19] of aggregation functions has been discussed in [20,1].

The generalized Gini welfare functions introduced by Weymark have the form

$$A(\mathbf{x}) = \sum_{i=1}^{n} w_i x_{(i)}$$

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where $x_{(1)} \leq x_{(2)} \leq \ldots \leq x_{(n)}$ and $w_i \in [0,1]$ for $i = 1,\ldots,n$, with $\sum_{i=1}^{n} w_i = 1$. These welfare functions correspond to the ordered weighted averaging (OWA) functions introduced by Yager [50], which in turn correspond (see [17]) to the symmetric Choquet integrals. Moreover, the principle of inequality aversion for welfare functions requires non-increasing weights, $1 \geq w_1 \geq w_2 \geq \ldots \geq w_n \geq 0$, with $\sum_{i=1}^{n} w_i = 1$.

The use of non-additivity and Choquet integration [12] in Social Welfare and Decision Theory dates back to the seminal work of Schmeidler [43,44], Ben Porath and Gilboa [3], and Gilboa and Schmeidler [22,23]. In the discrete case, Choquet integration [41,10,13,26,27,36] corresponds to a generalization of both weighted averaging and ordered weighted averaging, which remain as special cases. For recent reviews of Choquet integration see [32,35,33,34].

The complex structure of Choquet capacities can be described in the k-additivity framework introduced by Grabisch [28,30,29,7,8,40]. The 2-additive case, in particular, has been examined in [40,37,38]. Due to its low complexity and versatility it is relevant in a variety of modeling contexts.

The characterization of symmetric Choquet integrals (OWA functions) has been studied in [7,8,18,40]. It is shown that in the k-additive case the generating function of the OWA weights is polynomial of degree $k-1$. In the symmetric 2-additive case, in particular, the generating function is linear and thus the weights are equidistant, in analogy with the classical Gini welfare function.

In this paper we examine explicitly the family of symmetric Choquet integrals (OWA functions) of the 2-additive type and show that any 2-additive OWA function can be written as the difference between the arithmetic mean and a multiple of the classical Gini inequality index, with a given interval constraint on the multiplicity parameter. In the special case of positive parameter values, this result corresponds to the well-known Ben Porath and Gilboa's formula [3] for Weymark's generalized Gini welfare functions with linearly decreasing (inequality averse) weight distributions.

The paper is organized as follows. In Section 2 we review the classical Gini index for populations of $n \geq 2$ individuals, including the Lorenz area formula in the discrete case. In Section 3 we present the basic definitions and results on capacities and Choquet integrals, particularly in the additive and 2-additive cases. In Sections 4 and 5 we consider symmetric Choquet integration and we present the main result of the paper, concerning the parametric expression of the 2-additive OWA functions in terms of the arithmetic mean and a multiple of the classical Gini inequality index.

2 Gini Inequality Index and Welfare Function

Consider a population of $n \geq 2$ individuals whose income distribution is represented by $\mathbf{x} = (x_1,\ldots,x_n)$. Typically the range of the income values is taken to be $[0,\infty)$ but in this paper, apart from the derivation of the Lorenz area formula below, it could be the whole real line.

We define the *(absolute) classical Gini inequality index* as

$$G_A^c(\mathbf{x}) = -\sum_{i=1}^{n} \frac{n-2i+1}{n^2} x_{(i)} \tag{1}$$

where $x_{(1)} \le x_{(2)} \le \ldots \le x_{(n)}$. This expression shows explicitly the coefficients of the ordered income variables and is the most convenient in our presentation. In what follows we will omit "classical" and refer only to "Gini inequality index."

The traditional form of the Gini inequality index $G_A^c(\mathbf{x})$ is given by

$$G_A^c(\mathbf{x}) = \frac{1}{2n^2} \sum_{i,j=1}^{n} |x_i - x_j| \tag{2}$$

which can be easily shown to be equivalent to (1). In fact, the double summation expression for $n^2 G_A^c(\mathbf{x})$ as in (2) corresponds to

$$\begin{array}{rcccccccl} (x_{(n)} - x_{(n-1)}) & + & (x_{(n)} - x_{(n-2)}) & + \ldots + & (x_{(n)} - x_{(2)}) & + & (x_{(n)} - x_{(1)}) \\ & + & (x_{(n-1)} - x_{(n-2)}) & + \ldots + & (x_{(n-1)} - x_{(2)}) & + & (x_{(n-1)} - x_{(1)}) \\ & & & \vdots & & & \\ & & & + & (x_{(3)} - x_{(2)}) & + & (x_{(3)} - x_{(1)}) \\ & & & & & + & (x_{(2)} - x_{(1)}) \end{array} \tag{3}$$

which can be rewritten as

$$(n-1)x_{(n)} + ((n-2)-1)x_{(n-1)} + \ldots + (1-(n-2))x_{(2)} + (-(n-1))x_{(1)}\,. \tag{4}$$

It follows that

$$n^2 G_A^c(\mathbf{x}) = \frac{1}{2} \sum_{i,j=1}^{n} |x_i - x_j| = -\sum_{i=1}^{n} (n-2i+1)x_{(i)}\,. \tag{5}$$

In the discrete case, the Lorenz area formula can be derived as follows. Consider

$$V(\mathbf{x}) = \sum_{i=1}^{n} (x_{(1)} + \ldots + x_{(i)}) = nx_{(1)} + (n-1)x_{(2)} + \ldots + x_{(n)} \tag{6}$$

$$U(\mathbf{x}) = \sum_{i=1}^{n} (x_{(i)} + \ldots + x_{(n)}) = x_{(1)} + 2x_{(2)} + \ldots + nx_{(n)}\,. \tag{7}$$

We can easily express $U(\mathbf{x})$ in terms of $V(\mathbf{x})$,

$$\begin{aligned} U(\mathbf{x}) &= \sum_{i=1}^{n} (x_{(i)} + \ldots + x_{(n)}) \\ &= \sum_{i=1}^{n} [(x_{(1)} + \ldots + x_{(n)}) - (x_{(1)} + \ldots + x_{(i)}) + x_{(i)}] \\ &= n^2 \bar{x} - V(\mathbf{x}) + n\bar{x} = n(n+1)\bar{x} - V(\mathbf{x}) \end{aligned} \tag{8}$$

where $\bar{x} = (x_{(1)} + \ldots + x_{(n)})/n$. Since

$$\begin{aligned} n^2 G_A^c(\mathbf{x}) &= -\sum_{i=1}^{n} (n-2i+1)x_{(i)} \\ &= -((n-1)x_{(1)} + (n-3)x_{(2)} + \ldots + (-n+1)x_{(n)}) \end{aligned} \tag{9}$$

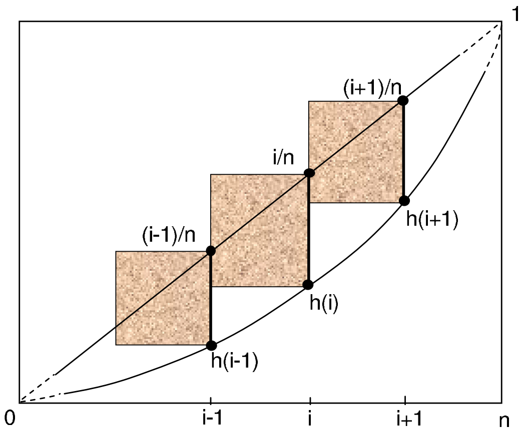

Fig. 1. Lorenz area in the discrete case

we can write $G_A^c(\mathbf{x})$ in terms of $\bar{x}$ and $V(\mathbf{x})$,

$$n^2 G_A^c(\mathbf{x}) = -(V(\mathbf{x}) - U(\mathbf{x})) = n(n+1)\bar{x} - 2V(\mathbf{x})\,. \tag{10}$$

Consider now the area illustrated in Fig. 1. The diagonal line and the Lorenz "curve" are hypothetical and are indicated only to suggest the analogy with the continuous case. In the discrete case we have just the vertical differences between the diagonal i/n values, associated with uniform cumulative income distribution, and the actual cumulative income distribution expressed by the $h(i)$ values,

$$h(i) = \frac{x_{(1)} + \ldots + x_{(i)}}{x_{(1)} + \ldots + x_{(n)}} \tag{11}$$

where we assume $x_{(i)} \geq 0$ for $i = 1, \ldots, n$ and $x_{(n)} > 0$, so that $\bar{x} > 0$.

The total area H in Fig. 1 is therefore given by

$$\begin{aligned} H = & \sum_{i=1}^{n} \left(\frac{i}{n} - h(i) \right) = \sum_{i=1}^{n} \left(\frac{i}{n} - \frac{x_{(1)} + \ldots + x_{(i)}}{x_{(1)} + \ldots + x_{(n)}} \right) \\ = & \frac{1}{n\bar{x}} \left[\sum_{i=1}^{n} \left(i\bar{x} - (x_{(1)} + \ldots + x_{(i)}) \right) \right] \\ = & \frac{1}{n\bar{x}} \left[\frac{n(n+1)}{2}\, \bar{x} - V(\mathbf{x}) \right] \\ = & \frac{1}{n\bar{x}} \left[\frac{n^2}{2}\, G_A^c(\mathbf{x}) \right] = \frac{n}{2\bar{x}}\, G_A^c(\mathbf{x})\,. \end{aligned} \tag{12}$$

Finally, we obtain

$$G_A^c(\mathbf{x}) = \frac{H}{n/2}\, \bar{x} \tag{13}$$

where the Lorenz area $H/(n/2)$ corresponds to the *relative Gini inequality index*.

The *welfare function* associated with the classical Gini inequality index is

$$A^c_G(\mathbf{x}) = \bar{x} - G^c_A(\mathbf{x}) \tag{14}$$

and it can be written as

$$A^c_G(\mathbf{x}) = \sum_{i=1}^{n} \frac{2(n-i)+1}{n^2} x_{(i)} = \sum_{i=1}^{n} \frac{1}{n} x_{(i)} + \sum_{i=1}^{n} \frac{n-2i+1}{n^2} x_{(i)} \tag{15}$$

where the coefficients of the Gini index sum up to zero, $\sum_{i=1}^{n}(n-2i+1) = 0$.

3 Capacities and Choquet Integrals

In this section we present a brief review of the basic facts on Choquet integration, focusing on the additive and 2-additive cases as described by their Möbius representations. For recent reviews on Choquet integration see [32,35,33,34] for the general case, and [40,37,38] for the 2-additive case.

Consider a finite set of interacting individuals $N = \{1, 2, \dots, n\}$. The subsets $S, T \subseteq N$ with cardinalities $0 \leq s, t \leq n$ are usually called coalitions.

The concepts of capacity and Choquet integral in the definitions below are due to [12,46,13,26,27].

Definition 1. *A* capacity *on the set N is a set function* $\mu : 2^N \longrightarrow [0,1]$ *satisfying*

(i) $\mu(\emptyset) = 0$, $\mu(N) = 1$ *(boundary conditions)*
(ii) $S \subseteq T \subseteq N \quad \Rightarrow \quad \mu(S) \leq \mu(T)$ *(monotonicity).*

Capacities are also known as *fuzzy measures* [46] or *non-additive measures* [13]. Given two coalitions $S, T \subseteq N$, with $S \cap T = \emptyset$, the capacity μ is said to be

- additive for S, T if $\mu(S \cup T) = \mu(S) + \mu(T)$,
- subadditive for S, T if $\mu(S \cup T) < \mu(S) + \mu(T)$,
- superadditive for S, T if $\mu(S \cup T) > \mu(S) + \mu(T)$.

In general the capacity μ is additive over N if $\mu(S \cup T) = \mu(S) + \mu(T)$ for all coalitions $S, T \subseteq N$, with $S \cap T = \emptyset$. Otherwise, the capacity μ is subadditive over N if $\mu(S \cup T) \leq \mu(S) + \mu(T)$ for all coalitions $S, T \subseteq N$ with $S \cap T = \emptyset$, with at least two such coalitions for which μ is subadditive in the strict sense. Analogously, the capacity μ is superadditive over N if $\mu(S \cup T) \geq \mu(S) + \mu(T)$ for all coalitions $S, T \subseteq N$ with $S \cap T = \emptyset$, with at least two such coalitions for which μ is superadditive in the strict sense. In the additive case, $\sum_{i=1}^{n} \mu(i) = 1$.

Definition 2. *Let* μ *be a capacity on* N*. The* Choquet integral *of a point* $\mathbf{x} = (x_1, \dots, x_n) \in [0,1]^n$ *with respect to* μ *is defined as*

$$\mathscr{C}_\mu(\mathbf{x}) = \sum_{i=1}^{n} [\mu(A_{(i)}) - \mu(A_{(i+1)})] x_{(i)} \tag{16}$$

where $(\cdot)$ *indicates a permutation on N such that* $x_{(1)} \leq x_{(2)} \leq \dots \leq x_{(n)}$*. Moreover,* $A_{(i)} = \{(i), \dots, (n)\}$ *and* $A_{(n+1)} = \emptyset$*.*

In the additive case, since

$$\mu(A_{(i)}) = \mu(\{(i)\}) + \mu(\{(i+1)\}) + \ldots + \mu(\{(n)\}) = \mu(\{(i)\}) + \mu(A_{(i+1)}) \qquad (17)$$

the Choquet integral reduces to a weighted mean,

$$\mathscr{C}_\mu(\mathbf{x}) = \sum_{i=1}^{n} [\mu(A_{(i)}) - \mu(A_{(i+1)})]\, x_{(i)} = \sum_{i=1}^{n} \mu(\{(i)\}) x_{(i)} = \sum_{i=1}^{n} \mu(\{i\}) x_i \qquad (18)$$

where the weights are given by $w_i = \mu(\{i\})$, for $i = 1, \ldots, n$.

A capacity μ can be equivalently represented by its Möbius transform m_μ [42,29].

Definition 3. *Let μ be a capacity on N. The* Möbius transform *associated with the capacity μ is defined as*

$$m_\mu(T) = \sum_{S \subseteq T} (-1)^{t-s} \mu(S) \qquad T \subseteq N \qquad (19)$$

where s and t denote the cardinality of the coalitions S and T, respectively.

Conversely, given the Möbius transform m_μ, the associated capacity μ is obtained as

$$\mu(T) = \sum_{S \subseteq T} m_\mu(S) \qquad T \subseteq N. \qquad (20)$$

In the Möbius representation, the boundary conditions take the form

$$m_\mu(\emptyset) = 0 \qquad \sum_{T \subseteq N} m_\mu(T) = 1 \qquad (21)$$

and the monotonicity condition is expressed as follows [39,11]:

$$\sum_{S \subseteq T} m_\mu(S \cup i) \geq 0 \qquad i = 1, \ldots, n \qquad T \subseteq N \setminus i. \qquad (22)$$

This form of monotonicity condition derives from the original monotonicity condition in Definition 1, expressed as $\mu(T \cup i) - \mu(T) \geq 0$ for all $i \in N$ and $T \subseteq N \setminus i$.

The Choquet integral in Definition 2 can be expressed in terms of the Möbius transform in the following way [36,29]

$$\mathscr{C}_\mu(\mathbf{x}) = \sum_{T \subseteq N} m_\mu(T) \min_{i \in T}(x_i). \qquad (23)$$

Defining a capacity μ on a set N of n elements requires $2^n - 2$ real coefficients, corresponding to the capacity values $\mu(T)$ for $T \subseteq N$. In order to control exponential complexity, Grabisch [28] introduced the concept of k-additive capacities.

A capacity μ is said to be *k-additive* [28] if its Möbius transform satisfies $m_\mu(T) = 0$ for all $T \subseteq N$ with $t > k$, and there exists at least one coalition $T \subseteq N$ with $t = k$ such that $m_\mu(T) \neq 0$.

We consider now, in particular, the 1-additive (or simply additive) case and the 2-additive case, and we revisit formulas (20) - (23).

- In the additive case, the decomposition formula (20) takes the simple form

$$\mu(T) = \sum_{i \in T} m_\mu(\{i\}) \qquad T \subseteq N, \tag{24}$$

the boundary conditions (21) reduce to

$$m_\mu(\emptyset) = 0 \qquad \sum_{i \in N} m_\mu(\{i\}) = 1 \tag{25}$$

and the monotonicity condition (22) reduces to

$$m_\mu(\{i\}) \geq 0 \qquad i = 1, \ldots, n. \tag{26}$$

Moreover, for additive capacities, the Choquet integral in (23) reduces to

$$\mathscr{C}_\mu(x_1, \ldots, x_n) = \sum_{i \in N} m_\mu(\{i\}) x_i. \tag{27}$$

- In the 2-additive case, the decomposition formula (20) takes the form

$$\mu(T) = \sum_{\{i\} \subseteq T} m_\mu(\{i\}) + \sum_{\{i,j\} \subseteq T} m_\mu(\{ij\}) \qquad T \subseteq N, \tag{28}$$

the boundary conditions (21) reduce to

$$m_\mu(\emptyset) = 0 \qquad \sum_{\{i\} \subseteq N} m_\mu(\{i\}) + \sum_{\{i,j\} \subseteq N} m_\mu(\{ij\}) = 1 \tag{29}$$

and the monotonicity condition (22) reduces to

$$m_\mu(\{i\}) \geq 0 \qquad m_\mu(\{i\}) + \sum_{j \in T} m_\mu(\{ij\}) \geq 0 \quad i = 1, \ldots, n \quad T \subseteq N \setminus i. \tag{30}$$

Moreover, for 2-additive capacities, the Choquet integral in (23) reduces to

$$\mathscr{C}_\mu(\mathbf{x}) = \sum_{\{i\} \subseteq N} m_\mu(\{i\}) x_i + \sum_{\{i,j\} \subseteq N} m_\mu(\{ij\}) \min(x_i, x_j)\,. \tag{31}$$

4 Symmetric Capacities and Choquet Integrals

We examine the basic definitions and results presented in the previous section in the particular case of symmetric capacities and Choquet integrals.

Definition 4. *A capacity μ is said to be* symmetric *if it depends only on the cardinality of the coalition considered*

$$\mu(T) = \mu(t) \qquad \textit{where} \qquad t = |T|\,. \tag{32}$$

Accordingly, for the Möbius transform m_μ associated with a symmetric capacity μ we use the notation

$$m_\mu(T) = m_\mu(t) \qquad \text{where} \qquad t = |T|\,. \tag{33}$$

Consider a Choquet integral with respect to a symmetric capacity μ. Then the Choquet integral reduces to an Ordered Weighted Averaging (OWA) function [50],

$$\mathscr{C}_\mu(\mathbf{x}) = \sum_{i\in N}[\mu(n-i+1) - \mu(n-i)]x_{(i)} = \sum_{i\in N} w_i x_{(i)} = A(\mathbf{x}) \tag{34}$$

where

$$w_i = \mu(n-i+1) - \mu(n-i) \tag{35}$$

correspond to the OWA weights. The traditional form of OWA functions as introduced by Yager [50] (OWA operators) is as follows:

$$A(\mathbf{x}) = \sum_{i\in N} \tilde{w}_i x_{[i]} \tag{36}$$

where $\tilde{w}_i = w_{n-i+1}$ and $x_{[1]} \geq x_{[2]} \geq \ldots \geq x_{[n]}$.

For a symmetric capacity μ, (25) - (26) and (29) - (30) take the following form:

- In the additive case the boundary conditions (25) reduce to

  $$m_\mu(0) = 0 \qquad\qquad n\,m_\mu(1) = 1 \tag{37}$$

  and the monotonicity condition (26) reduces to

  $$m_\mu(1) \geq 0\,. \tag{38}$$

  From the boundary conditions (37) we have $m_\mu(1) \geq 1/n$ and the OWA function is simply the arithmetic mean.
- In the 2-additive case the boundary conditions (29) reduce to

  $$m_\mu(0) = 0 \qquad\qquad n\,m_\mu(1) + \frac{n(n-1)}{2}\,m_\mu(2) = 1 \tag{39}$$

  and the monotonicity condition (30) reduces to

  $$m_\mu(1) \geq 0 \qquad m_\mu(1) + t\,m_\mu(2) \geq 0 \qquad\qquad 1 \leq t \leq n-1\,. \tag{40}$$

In the next section we present a detailed treatment of the 2-additive symmetric case.

5 Symmetric Capacities and Choquet Integrals: The 2-Additive Case and the Gini Inequality Index

Consider now the 2-additive symmetric case as discussed in the previus section. Let

$$\alpha = m_\mu(1) \qquad\qquad \beta = m_\mu(2)\,. \tag{41}$$

From the boundary conditions (39) we have

$$n\alpha + \frac{n(n-1)}{2}\beta = 1 \qquad \alpha = \frac{1}{n} - \frac{n-1}{2}\beta\,. \tag{42}$$

From the monotonicity condition (40) it follows that

$$\alpha \geq 0 \qquad \alpha + (n-1)\beta \geq 0 \tag{43}$$

where the second constraint corresponds to the dominating worst case $t = n-1$ in (40). Substituting α as in (42) in the two conditions (43) we obtain

$$-\frac{2}{n(n-1)} \leq \beta \leq \frac{2}{n(n-1)}\,. \tag{44}$$

Consider now the OWA operator as in (34) and (35),

$$A(\mathbf{x}) = \sum_{i=1}^{n} w_i x_{(i)} \qquad w_i = \mu(n-i+1) - \mu(n-i)\,. \tag{45}$$

In the 2-additive case we have that

$$\mu(n-i+1) = (n-i+1)\,\alpha + \frac{(n-i+1)(n-i)}{2}\beta \tag{46}$$

$$\mu(n-i) = (n-i)\,\alpha + \frac{(n-i)(n-i-1)}{2}\beta \tag{47}$$

and therefore we obtain

$$w_i = \alpha + (n-i)\beta = \frac{1}{n} + \frac{n-2i+1}{2}\beta \tag{48}$$

where β is subject to the constraints (44).

Introducing the notation $u_i = (n-2i+1)/2,\ i = 1,\ldots,n$, notice that $\sum_{i=1}^{n} u_i = 0$ and the coefficients $u_i,\ i = 1,\ldots,n$ are linearly decreasing $u_1 > u_2 > \ldots > u_n$ with $u_1 = (n-1)/2$ and $u_n = -(n-1)/2$.

The main result of the paper is then the following.

Proposition 1. *Any 2-additive OWA function can be written as*

$$A(\mathbf{x}) = \bar{x} - \frac{1}{2}\beta n^2 G_A^c(\mathbf{x}) \tag{49}$$

where β is a free parameter subject to the constraints $-\dfrac{2}{n(n-1)} \leq \beta \leq \dfrac{2}{n(n-1)}$.

The proof follows straightforwardly from (45) - (48), associated to the constraints (44), and the definition of the classical Gini inequality index (1).

Given that

$$A(\mathbf{x}) = \sum_{i=1}^{n} w_i x_{(i)} = \sum_{i=1}^{n} \frac{2+\beta(n^2-2in+n)}{2n} x_{(i)} \tag{50}$$

we must have $\beta \geq 0$ in order to have non-increasing weights. In Proposition 1, the strict case $\beta > 0$ corresponds to the well-known Ben Porath and Gilboa's formula [3] for Weymark's generalized Gini welfare functions with linearly decreasing (inequality averse) weight distributions, see also [31].

In particular, with $\beta = 2/n^2$ we obtain the classical Gini welfare function

$$A(\mathbf{x}) = A_G^c(\mathbf{x}) \qquad \alpha = \frac{1}{n^2} \quad \beta = \frac{2}{n^2}. \tag{51}$$

Regarding the choice of the parameter values α and β, we introduce

$$a = n\alpha \qquad\qquad b = \frac{n(n-1)}{2}\beta \tag{52}$$

and then the boundary and monotonicity constraints (42) - (43) take the simple form

$$a + b = 1 \qquad\qquad a \geq 0 \qquad a + 2b \geq 0 \tag{53}$$

from which we obtain $a = 1 - b$ and $-1 \leq b \leq 1$. In this notation the general form (49) of a 2-additive OWA function is as follows:

$$A(\mathbf{x}) = \bar{x} - \frac{n}{n-1}\, b\, G_A^c(\mathbf{x}) \tag{54}$$

where $-1 \leq b \leq 1$. The classical Gini case $\alpha = 1/n^2$ and $\beta = 2/n^2$ corresponds to $a = 1/n$ and $b = (n-1)/n$.

Other interesting parameter choices for a, b could be $a = k/n$ and $b = (n-k)/n$ with $k = 0, \ldots, n$. In the case $k = 0$ the whole Choquet capacity structure lies in the edges, whereas the case $k = 1$ corresponds to the classical Gini inequality index; the remaining cases correspond to increasingly weak structure being associated to the edges, towards the additive case $k = n$.

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Credit Crunch in the Euro Area: A Coopetitive Multi-agent Solution

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Abstract. The aim of this paper is to propose a methodology to attenuate the plague of the credit crunch, which is very common in this period: despite the banking world having available a huge amount of money, there is no available money in the real economy. Consequently, we want to find a way to allow a global economic recovery by adopting a new mathematical model of "Coopetitive Game." Specifically, we will focus on two economic operators: a real economic subject and a financial institute (a bank, for example) with a big economic availability. For this purpose, we examine an interaction between the above economic subjects: the Enterprise, our first player, and the Financial Institute, our second player. The solution that allows both players to win the maximum possible collective profit, and therefore the one desirable for both players, is represented by a coopetitive agreement between the two subjects. So the Enterprise artificially causes (also thanks to the money loaned by the Financial Institute that receives them by the ECB) an inconsistency between spot and futures markets, and the Financial Institute takes the opportunity to win the maximum possible collective gain of the coopetitive game (the two players even arrive to the maximum of the game). We propose hereunder two possible transferable utility solutions, in order to avoid that the envy of the Enterprise, which gains a much less advantage from the adoption of a coopetitive strategy, may compromise the success of the interaction.

Keywords: Credit Crunch, Financial Markets, Financing Policy, Risk, Financial Crisis, Games, Arbitrages, Coopetition.

1 Introduction

In the last years, despite the banking world having available a huge amount of money (on Dec. 2011 and on Feb. 2012 the ECB loaned money to banks at the rate of 1%, respectively 490 and 530 billion euros), there is no available money in the real economy. This phenomenon has begun to show its first sign of life from the second half of 2008, and it reached its peak in Dec. 2011. The credit crunch

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is a wide phenomenon: Europe shows a decrease of 1.6% in loans to households and businesses. In Italy, this phenomenon is particularly pronounced, because the decline in loans was even of 5.1% from 2008. *Where's the money loaded by ECB?* Badly, the money remained caged in the world of finance: with some of the money from the ECB, banks bought government bonds (so the spread went down); another part of the money is used by the banks to rectify their assets in accordance with EBA requirements (European Banking Authority); the rest of the money was deposited at ECB, at the rate of 0.5% (lower than the rate at which they received it). Moreover, from the second half of 2008, the deposits of European banks at the ECB have quadrupled. In view of this, our model takes a different dimension and different expectations: in our model, the bank (the speculator) put money in the real economy by lending to the Enterprise; it eliminates the risk of losing money for the economic crisis and obtains a gain by an agreement with the Enterprise (which gains something too). The credit crunch, by our model, should be gradually attenuated until it disappears.

In this paper, by using game theory (for the complete study of a game see also [4,5,10]) we propose a method aiming to attenuate the phenomenon of the credit crunch and, consequently, a way to allow a global economic recovery. For the achievement of our aim, we propose the introduction of a tax on speculative financial transactions, in order to stabilize the financial markets (see also[23,8,9]). Moreover, we propose a method of using money (that were provided to banks by the ECB) that allows the money to get into the real economy without getting stuck in the world of finance. Our aim is attained without inhibiting the possibilities of profits and, for this purpose, we present and study an advantageous coopetitive model and two different compromise solutions.

2 Description of the Initial No-coopetitive Game

2.1 Methodologies

The Carfì and Musolino's model ([7]) is based on a construction on 3 times.

0) At time 0 the Enterprise can choose whether to buy futures contracts to hedge the market risk of the underlying asset, which (the Enterprise knows) should be bought at time 1, in order to conduct its business activities.
1) The Financial Institute, on the other hand, acts—with speculative purposes—on spot market (buying or short-selling the asset at time 0) and futures market (with the action contrary to that on the spot market: if the Financial Institute sells short on spot market, it purchases on the futures market, and vice versa). Thus, the Financial Institute may take advantage of the temporary misalignment of the spot and futures prices that would be created as a result of a hedging strategy by the Enterprise.
2) At time 2, the Financial Institute cashes or pays the sum determined by its behavior in the futures market at time 1.

2.2 Strategies of the Players

In Carfì and Musolino's model ([7]) the first player is an Enterprise that may choose whether to buy futures contracts to hedge by an upwards change in the price of the underlying asset that it (the Enterprise) has to buy at time 1 for the conduct of its business. Therefore, the Enterprise has the possibility to choose a strategy $x \in [0, 1]$, which represents the percentage of the quantity of the underlying M_1 that the Enterprise purchases via futures, depending on its intends:

1. to not hedge ($x = 0$),
2. to hedge partially ($0 < x < 1$),
3. to hedge totally ($x = 1$).

On the other hand, the second player is a Financial Institute operating on the spot market of the same underlying asset. The Financial Institute works in our game also on the futures market:

- taking advantage of possible gain opportunities—given by misalignment between spot prices and futures prices of the asset;
- or accounting for the loss obtained, because it has to close the position of short sales opened on the spot market.

These are just actions to determine the win or the loss of the Financial Institute.

The Financial Institute can therefore choose a strategy $y \in [-1, 1]$, which represents the percentage of the quantity of the underlying M_2 that it can buy (in algebraic sense) with its financial resources, depending on its intends:

1. to purchase the underlying on the spot market ($y > 0$);
2. to short sell the underlying on the spot market ($y < 0$);
3. to not intervene on the market of the underlying ($y = 0$).

3 Coopetitive Approach

For the display of our game (proposed as a remedy to the credit crunch), it is necessary to pass from the Carfì and Musolino's model ([7]) to a game set in a coopetitive context (see [3,2,12,17,27,32,1,13,14,15,24,25,26] about coopetition). In particular, we follow the Carfì's definition of coopetitive game (for some examples see [6,11])

3.1 The Idea

A coopetitive game is a game in which two or more players (participants) can interact *cooperatively and non-cooperatively at the same time*. Even Brandenburger and Nalebuff, creators of coopetition ([1]), did not define, precisely, a *quantitative way to implement coopetition* in the Game Theory context.

The problem to implement the notion of coopetition in Game Theory is summarized in the following question:

- *how do, in normal form games, cooperative and non-cooperative interactions live together simultaneously, in a Brandenburger-Nalebuff sense?*

In order to explain the above question, consider a classic two-player normal-form gain game $G = (f, >)$—such a game is a pair in which f is a vector valued function defined on a Cartesian product $E \times F$ with values in the Euclidean plane $\mathbb{R}^2$ and $>$ is the natural strict sup-order of the Euclidean plane itself (the sup-order is indicating that the game, with payoff function f, is a gain game and not a loss game). Let E and F be the strategy sets of the two players in the game G. The two players can choose the respective strategies $x \in E$ and $y \in F$

- cooperatively (exchanging information and making binding agreements);
- not-cooperatively (not exchanging information or exchanging information but without possibility to make binding agreements).

The above two behavioral ways are mutually exclusive, at least in normal-form games:

- the two ways cannot be adopted simultaneously in the model of normal-form game (without using convex probability mixtures, but this is not the way suggested by Brandenburger and Nalebuff in their approach);
- there is no room, in the classic normal form game model, for a simultaneous (non-probabilistic) employment of the two behavioral extremes *cooperation* and *non-cooperation*.

Towards a Possible Solution. A manner to pass this *impasse*, according to the idea of coopetition in the sense of Brandenburger and Nalebuff is Carfì's coopetitive game model, where

- the players of the game have their respective strategy-sets (in which they can choose cooperatively or not cooperatively);
- there is a common strategy set C containing other strategies (possibly of different type with respect to those in the respective classic strategy sets) that *must be chosen cooperatively*;
- the strategy set C can also be structured as a Cartesian product (similarly to the profile strategy space of normal form games), but in any case the strategies belonging to this new set C must be chosen cooperatively.

3.2 Two Players Coopetitive Games

Definition (of coopetitive game). *Let E, F, and C be three nonempty sets. We define* ***two-player coopetitive gain game carried by the strategic triple*** (E, F, C) *any pair of the form $G = (f, >)$, where f is a function from the Cartesian product $E \times F \times C$ into the real Euclidean plane $\mathbb{R}^2$ and the binary relation $>$ is the usual sup-order of the Cartesian plane (defined component-wise, for every couple of points p and q, by $p > q$ iff $p_i > q_i$, for each index i).*

Remark (coopetitive games and normal form games). The difference between a two-player normal-form (gain) game and a two-player coopetitive (gain) game is the fundamental presence of the third strategy Cartesian-factor C. The presence of this third set C determines a total change of perspective with respect to the usual examination of two-player normal form games, since we now have to consider a normal form game $G(z)$, for every element z of the set C.

3.3 Normal Form Games of a Coopetitive Game

Let G be a coopetitive game in the sense of the above definitions. For any cooperative strategy z selected in the cooperative strategy space C, there is a corresponding normal form gain game

$$G_z = (p(z), >),$$

upon the strategy pair (E, F), where the payoff function $p(z)$ is the section

$$f(., z) : E \times F \to \mathbb{R}^2,$$

of the payoff function f of the coopetitive game—the section is defined, as usual, on the competitive strategy space $E \times F$, by

$$f(., z)(x, y) = f(x, y, z),$$

for every bi-strategy (x, y) in the bi-strategy space $E \times F$.

Let us formalize the concept of game-family associated with a coopetitive game.

Definition (the family associated with a coopetitive game). *Let $G = (f, >)$ be a two-player coopetitive gain game carried by the strategic triple (E, F, C). We naturally can associate with the game G a family $g = (g_z)_{z\in C}$ of normal-form games defined by*

$$g_z := G_z = (f(., z), >),$$

for every *z in C, which we shall call **the family of normal-form games associated with the coopetitive game** G.*

Remark. It is clear that with any of the above family of normal form games

$$g = (g_z)_{z\in C},$$

with $g_z = (f(., z), >)$, we can associate:

- a family of payoff spaces

  $$(\mathrm{im} f(., z))_{z\in C},$$

  with members in the payoff universe $\mathbb{R}^2$;
- a family of Pareto maximal boundary

  $$(\partial^* G_z)_{z\in C},$$

  with members contained in the payoff universe $\mathbb{R}^2$;

- a family of suprema
$$(\sup G_z)_{z\in C},$$
with members belonging to the payoff universe $\mathbb{R}^2$;
- a family of Nash zones
$$(\mathcal{N}(G_z))_{z\in C};$$
with members contained in the strategy space $E\times F$;
- a family of conservative bi-values
$$v^{\#} = (v_z^{\#})_{z\in C};$$
in the payoff universe $\mathbb{R}^2$.

And so on, for every meaningful known feature of a normal form game.

Moreover, we can interpret any of the above families as *set-valued paths* in the strategy space $E\times F$ or in the payoff universe $\mathbb{R}^2$.

It is just the study of these induced families which becomes of great interest in the examination of a coopetitive game G and which will enable us to define (or suggest) the various possible solutions of a coopetitive game.

Solutions of a Coopetitive Game. The two players of a coopetitive game G—according to the general economic principles of *monotonicity of preferences* and of non-satiation—should choose the cooperative strategy z in C in order that:

- *fixed a common kind of solution for any game G_z, say $S(z)$ the set of these kind of solutions for the game G_z, we can consider the problem to find all the optimal solutions (in the sense of Pareto) of the set valued path S, defined on the cooperative strategy set C. Then, we should face the problem of* ***selection of reasonable Pareto strategies*** *in the set-valued path S via proper selection methods (Nash-bargaining, Kalai-Smorodinsky, and so on).*

4 The Shared Strategy

We have two players, the Enterprise and the Financial Institute, each of them has a strategy set in which to choose its strategy; moreover, the two players can cooperatively choose a strategy z in a third set C. The two players choose their cooperative strategy z to maximize (in some sense that we specify) the gain function f.

The strategy $z\in[0,1]$ is a shared strategy, which represents the percentage of the highest possible money M_3 that the European Central Bank lends to the Financial Institute with a very low interest rate (hypothesis highly plausible according to the recent anti-crisis measures adopted by the ECB). By convention, we assume this interest rate equal to 0. The two players use the loan so that the Enterprise can create an even higher misalignment between spot and futures price, misalignment that is exploited by the Financial Institute. In this way, both players can get a greater win than that obtained without a shared strategy z. The two players can then choose a shared strategy depending on what they want:

- to not use the money of the ECB ($z = 0$);
- to use a part of the money of the ECB so that the Enterprise purchases futures ($0 < z < 1$);
- to use totally the money of the ECB so that the Enterprise purchases futures ($z = 1$).

Remark. In the following coopetitive game, we do not introduce the uncertainty (and we do not consider extreme events in our economic world) and so we suppose that attempts of speculative profit (modifying the asset prices) are successful. In fact our interest is to show that a tax on speculative profits can limit speculation, and not to determine if or how much speculators gain. Anyway, even without uncertainty, our model remains likely, plausible and very topical because

- in a period of crisis, behavioral finance suggests ([16,22,31]) the vertical diffusion of a behavior (the so-called herd behavior [19,29]) conforming to that adopted by the great investors;
- just the decrease (or increase) in demand influences the prices of the asset ([21]).

5 The Payoff Function of the Enterprise

In practice, to the payoff function f_1 of the paper [7], that is, the function defined by

$$f_1(x, y) = -nuM_1(1 - x)y,$$

for every (x, y) in the bi-strategy space S, we must add the payoff-consequence $v_1(y, z)$ of the shared action z of the game, consisting in buying futures contracts and selling them at time 1 (action decided by both players and performed by the Enterprise).

In paper [7], we have already chosen $M_1 = 1$ and $nu = 1/2$, and so we have

$$f_1(x, y) = -(1/2)(1 - x)y,$$

for every (x, y) in the bi-strategy space S: this is the first component of the initial game we shall represent in the present paper.

Payoff Consequence of the Shared Strategy. The payoff function addendum $v_1(y, z)$, of the Enterprise, is given by the quantity of futures bought, that is, the term zM_3, multiplied by the difference, $F_1u^{-1} - F_0$, between the futures price at time 1—when the Enterprise sells the futures—and the futures price at time 0 - when the Enterprise buys the futures.

Remark. Similarly to what happened to the Financial Institute in Carfì and Musolino's model ([7]) because of the introduction of a tax on speculative transactions, also the Enterprise has to pay a tax on the sale of the futures contracts (see [28,30,33] for the benefits of the taxation on financial transactions: we follow exactly this lane of thought). We assume that this tax is equal to the impact of the Enterprise on the futures price, in order to avoid speculative acts created by itself.

We have:

$$h_1(x, y, z) = f_1(x, y) + zM_3(F_1(x, y, z)u^{-1} - m(x + z) - F_0) \tag{1}$$

where:

(1) zM_3 is the quantity of futures purchased.
(2) F_0 is the futures price at time 0. It represents the price established at time 0 that has to be paid at time 1 in order to buy the asset. We assume that it is given by
$$F_0 = S_0 u. \tag{2}$$
S_0 is, on the other hand, the spot price of the underlying asset at time 0. S_0 is a constant because our strategies x, y, and z do not influence it, while $u = 1 + i$ is the factor of capitalization of interests. By i we mean risk-free interest rate charged by banks on deposits of other banks, the so-called "LIBOR" rate.
(3) $F_1(x, y, z)$ is the futures price (established) at time 1, after the Enterprise has played its strategy x and the shared strategy z. We assume that the price $F_1(x, y, z)$ is given by
$$F_1(x, y, z) = S_1 u + mu(x + z), \tag{3}$$
where
(a) S_1 is the spot price at time 1. We assume that it is given by
$$S_1(y) = (S_0 + ny)u,$$
where n is the marginal coefficient that measures the impact of y on $S_1(y)$.
(b) m is the marginal coefficient that measures the impact of x and z on $F_1(x, y, z)$.

$F_1(x, y, z)$ depends on x and z because an increase/decrease of futures demand influences upward/downward the futures price ([18]). The value S_1 should be capitalized because it follows the Hull's relationship between futures and spot prices ([20]). The value $m(x + z)$ is also capitalized because the strategies x and z are played at time 0 but have effect on the futures price at time 1.
(4) $m(x + z)$ is the normative tax paid by the Enterprise on the sale of futures, referred to time 1. We assume that the tax is equal to the impact of the strategies x and z (adopted by the Enterprise) on the futures price F_1.

(5) u^{-1} is the discount factor. $F_1(x, y, z)$ must be actualized at time 1 because the money for the sale of futures are cashed at time 2.

Remark. The values m and n depend on the ability (of our players) to influence the spot and futures markets and the behavior of other financial agents (see the remark in Section 4).

The Payoff Function of the Enterprise. Recalling Equations 2 and 3 and substituting them into Eq. 1, we have

$$h_1(x, y, z) = f_1(x, y) + zM_3[((S_0 + ny)u^2 + m(x + z)u)u^{-1} - m(x + z) - S_0u],$$

that is,

$$h_1(x, y, z) = f_1(x, y) + M_3nuyz. \quad (4)$$

From now we assume $M_3 = 1$ for the sake of simplicity.

6 The Payoff Function of the Financial Institute

In our initial no-coopetitive game, the payoff function of the second player (already analyzed in the paper [7]) is

$$f_2(x, y) = yM_2mx,$$

for every (x, y) in the bi-strategy space S. In paper [7] we have already chosen $M_2 = 2$ and $m = 1/2$, so we have

$$f_2(x, y) = yx,$$

for every (x, y) in the bi-strategy space S: this is the second component of the initial game we shall represent in the present paper.

The initial no-coopetitive payoff function of the Financial Institute at time 1 is given by the multiplication of the quantity of asset bought on the spot market, that is yM_2, by the difference among:

1. the futures price $F_1(x, y)$ (it is a price established at time 1 but cashed at time 2) transferred to time 1, that is $F_1(x, y)u^{-1}$;
2. the purchase price—net of the tax introduced by the normative authority on financial transactions ([7])—of asset at time 0, say S_0, capitalized at time 1 (in other words we are accounting for all balances at time 1).

But in our coopetitive game, instead of the futures price $F_1(x, y)$, we have to consider the futures price $F_1(x, y, z)$ that takes into consideration the shared strategy z (in fact at time 0 the Enterprise buys the additional quantity zM_3 of futures contracts than our initial no-coopetitive game, and the futures price F_1 changes consequently).

The Payoff Function of the Financial Institute of our coopetitive game is defined by:

$$f_2(x,y) = yM_2(F_1(x,y,z)u^{-1} - nuy - S_0u), \tag{5}$$

where:

(1) y is the percentage of asset that the Financial Institute purchases or sells on the spot market of the underlying;
(2) M_2 is the maximum amount of asset that the Financial Institute can buy or sell on the spot market, according to its economic availability;
(3) S_0 is the price paid by the Financial Institute in order to buy the asset on spot market at time 0. S_0 is a constant because our strategies x, y, and z do not influence it.
(4) nuy is the normative tax on the futures price, paid at time 1. We are assuming that the tax is equal to the incidence of the strategy y of the Financial Institute on the spot price at time 1, that is, $S_1(y) = (S_0 + ny)u$ (see also the paper [7]).
(5) $F_1(x,y,z)$ is the futures price (established) at time 1, after the Enterprise has played its strategy x and the shared strategy z. The price $F_1(x,y,z)$ is given by

$$F_1(x,y,z) = S_1(y)u + mu(x+z),$$

where $S_1(y) = (S_0 + ny)u$ is the spot price at time 1, and$u = 1 + i$ is the factor of capitalization of interests. With m we intend the marginal coefficient that measures the impact of x and z on $F_1(x,y,z)$. $F_1(x,y,z)$ depends on x and z because a change of futures demand influences the futures price ([18,20]). The value S_1 should be capitalized because it follows the fundamental relationship between futures and spot prices (see [20]). The value $m(x+z)$ is also capitalized because the strategies x and z are played at time 0 but have effect on the futures price at time 1.
(6) u^{-1} is the discount factor. $F_1(x,y)$ must be translated at time 1, because the money for the sale of futures is cashed at time 2.

The Coopetitive Payoff Function of the Financial Institute. Recalling functions F_1 and f_2, we have

$$h_2(x,y,z) = yM_2m(x+z), \tag{6}$$

So, we have

$$h(x,y,z) = (f_1(x,y), f_2(x,y)) + yz(nuM_3, M_2m), \tag{7}$$

for every strategy triple (x,y,z) of our coopetitive game. In this paper, we shall represent the following numerical case:

$$h(x,y,z) = (-(1/2)(1-x)y, xy) + yz(1/2, 1).$$

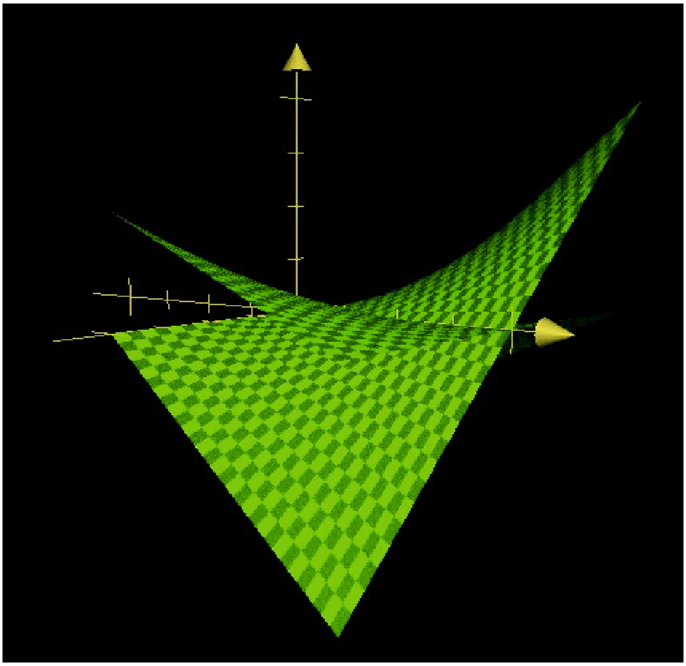

Fig. 1. Initial game $f = h(., 0)$

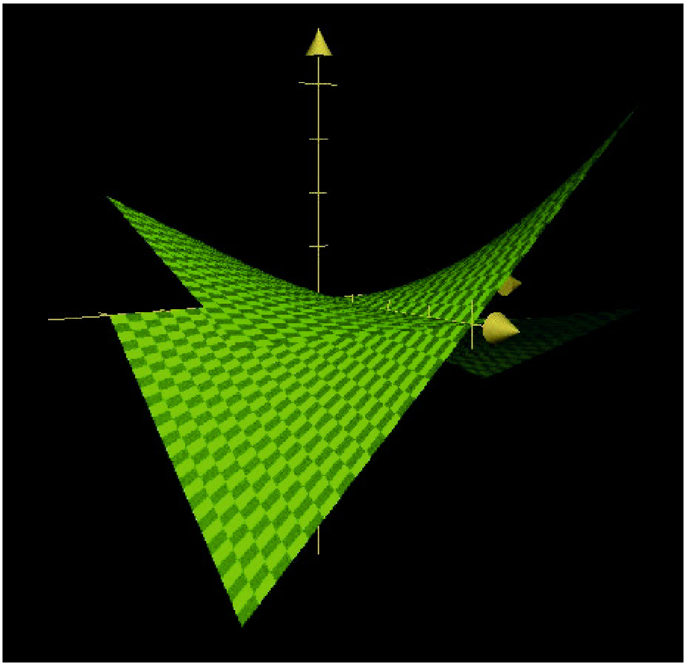

Fig. 2. Initial game $f = h(., 0)$

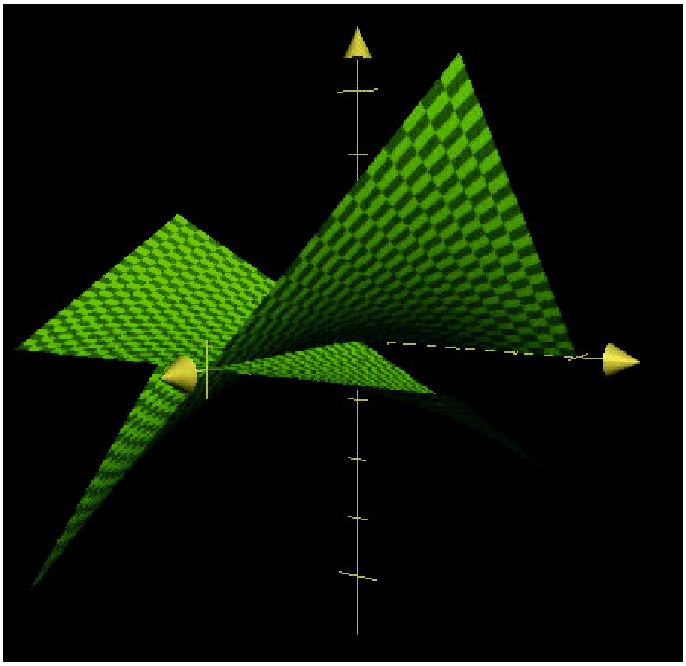

Fig. 3. Initial game $f = h(., 0)$

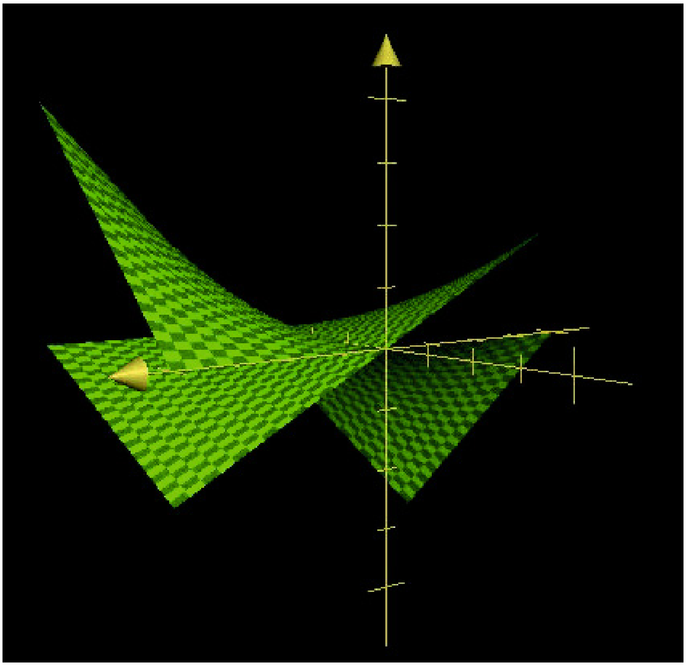

Fig. 4. Initial game $f = h(., 0)$

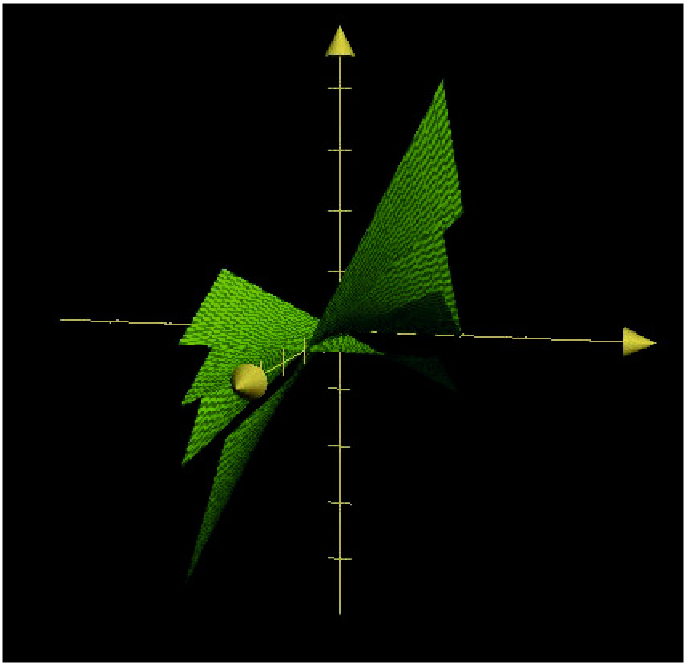

Fig. 5. Games $f = h(.,0)$ and $f = h(.,1)$

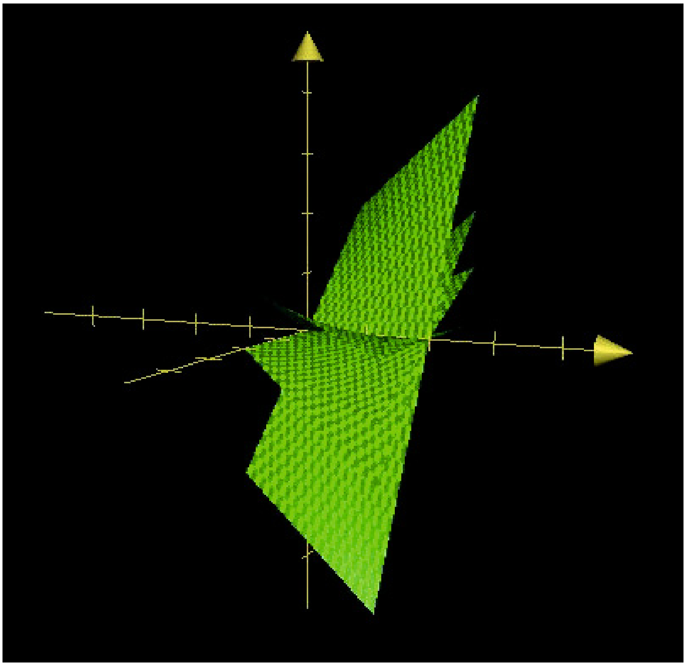

Fig. 6. Games $f = h(.,0)$ and $f = h(.,1)$

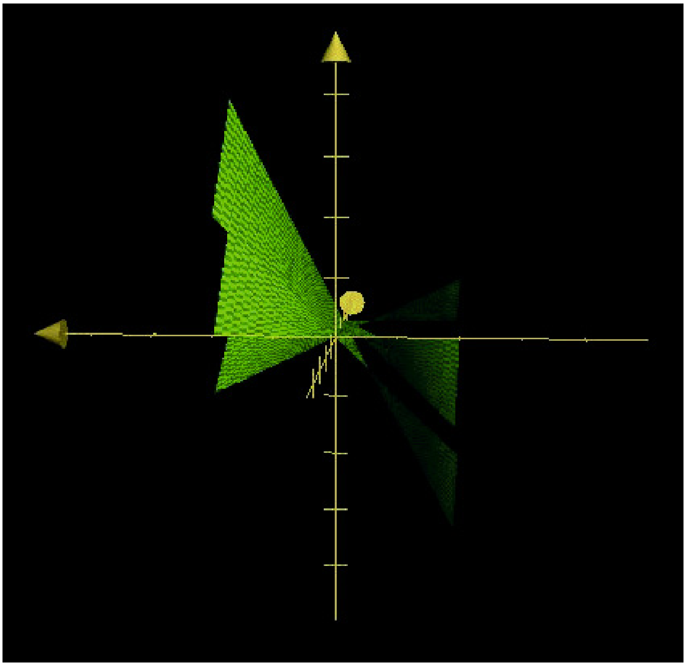

Fig. 7. Games $f = h(.,0)$ and $f = h(.,1)$

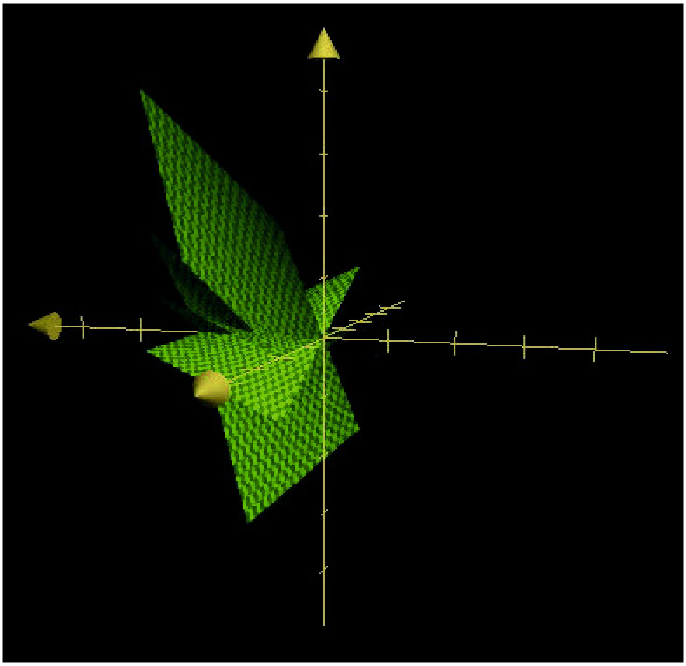

Fig. 8. Games $f = h(.,0)$ and $f = h(.,1)$

7 The Coopetitive Translating Vectors

We note immediately that the new function is the same payoff function f of the first game already studied in paper [7],

$$f(x, y) = (-nuyM_1(1 - x), yM_2mx),$$

translated by the vector function

$$v(y, z) := zy(nuM_3, M_2m).$$

Recalling that $y \in [-1, 1]$ and $z \in [0, 1]$, we see that the vector $v(y, z)$ belongs to the 2-range $[-1, 1](nuM_3, M_2m)$.

7.1 Coopetitive Payoff Space

Concerning the payoff space of our coopetitive game $(h, >)$, we note a meaningful result. We observe that (since any shared strategy z is positive):

1. the part of the initial payoff space $f(S_{\geq})$ (where $S_{\geq}$ is the part of S such that the second projection pr_2 is greater than 0) is translated upwards, when we consider the transformation by the coopetitive extension h of f and the shared variable z is increasing;
2. the part of the initial payoff space $f(S_{\leq})$ (where $S_{\leq}$ is the part of S such that the second projection pr_2 is less than 0) is translated downwards, when we consider the transformation by the coopetitive extension h of f and the shared variable z is increasing.

Proposition. *Let $S := E \times F = [0, 1] \times [-1, 1]$ and $Q := S \times [0, 1]$. Then, the payoff space $h(Q)$ is the union of $h(., 0)(S)$ and $h(., 1)(S)$.*

Proof. The strategy space S is the union of $S_{\geq} := [0, 1] \times [0, 1]$ and $S_{\leq} := [0, 1] \times [-1, 0]$. We shall split the proof into two parts.

Part 1. We will show that the shared strategy that maximizes the wins when $y \geq 0$ is always $z = 1$, that is, we'll show that $h(x, y, z) \leq h(x, y, 1)$, for every $y \geq 0$ and every x in E, i.e., $(x, y) \in S_{\geq}$. Recalling the definition of h, we have to show that

$$(-nuyM_1(1-x), yM_2mx) + yz(nuM_3, M_2m) \leq (-nuyM_1(1-x), yM_2mx) + y(nuM_3, M_2m),$$

that is,

$$yz(nuM_3, M_2m) \leq y(nuM_3, M_2m)$$

and therefore we have to prove that $yz \leq y$, which is indeed verified for any $y \geq 0$. We can show also that $h(x, y, z) \geq h(x, y, 0)$, for every $y \geq 0$ and every x in E. Indeed, we have to show that

$$(-nuyM_1(1 - x), yM_2mx) + yz(nuM_3, M_2m) \geq (-nuyM_1(1 - x), yM_2mx),$$

that is,

$$yz(nuM_3, M_2m) \geq 0$$

and therefore $yz \geq 0$, which is indeed verified for any $y \geq 0$. Since, with $x \in [0, 1]$ and $y \in [0, 1]$, we have

$$h(x, y, 0) \leq h(x, y, z) \leq h(x, y, 1),$$

we obtain that the payoff part $h([0, 1]^3)$ is included in the union of the images $h(., 0)(S_{\geq})$ and $h(., 1)(S_{\geq})$.

Part 2. We will show that the shared strategy that maximizes the losses when $y \leq 0$ is always $z = 1$, that is, we will show that

$$h(x, y, z) \geq h(x, y, 1),$$

for every $y \leq 0$ and every x in E, i.e., for every $(x, y) \in S_{\leq}$. Recalling the definition of h, we have to show that

$$(-nuyM_1(1-x), yM_2mx) + yz(nuM_3, M_2m) \geq (-nuyM_1(1-x), yM_2mx) + y(nuM_3, M_2m),$$

that is to say

$$yz(nuM_3, M_2m) \geq y(nuM_3, M_2m)$$

and therefore we have to prove $yz \geq y$, which is indeed verified for any $y \leq 0$. We can also show that

$$h(x, y, z) \leq h(x, y, 0),$$

for every $y \leq 0$ and every x in E. Indeed, we have to show that

$$(-nuyM_1(1-x), yM_2mx) + yz(nuM_3, M_2m) \leq (-nuyM_1(1-x), yM_2mx),$$

that is,

$$yz(nuM_3, M_2m) \leq 0,$$

that is equivalent to $yz \leq 0$, which is indeed verified for any $y \leq 0$. Since, when $x \in [0, 1]$ and $y \in [0, -1]$, we have

$$h(x, y, 1) \leq h(x, y, z) \leq h(x, y, 0),$$

we obtain that the payoff part $h(S_{\leq} \times [0, 1])$ is included in the union of the images of $h(., 1)(S_{\leq})$ and $h(., 0)(S_{\leq})$. This completes the proof. ■

Hence, transforming our bi-strategic space S by $h(., 0)$ (in dark green) and $h(., 1)$ (in light green), in Fig.9 we have the whole payoff space of the our coopetitive game $(h, >)$. If the Enterprise and the Financial Institute play the bi-strategy $(1, 1)$, and the shared strategy 1, they arrive at the point $B'(1)$, which is the maximum of the coopetitive game G, so the Enterprise wins $1/2$ (amount greater than $1/3$ obtained in the cooperative phase of the no-coopetitive game in the paper [7]) while the Financial Institute wins even 2 (an amount much greater than $2/3$, value obtained in the cooperative phase of the no-coopetitive game in paper [7]).

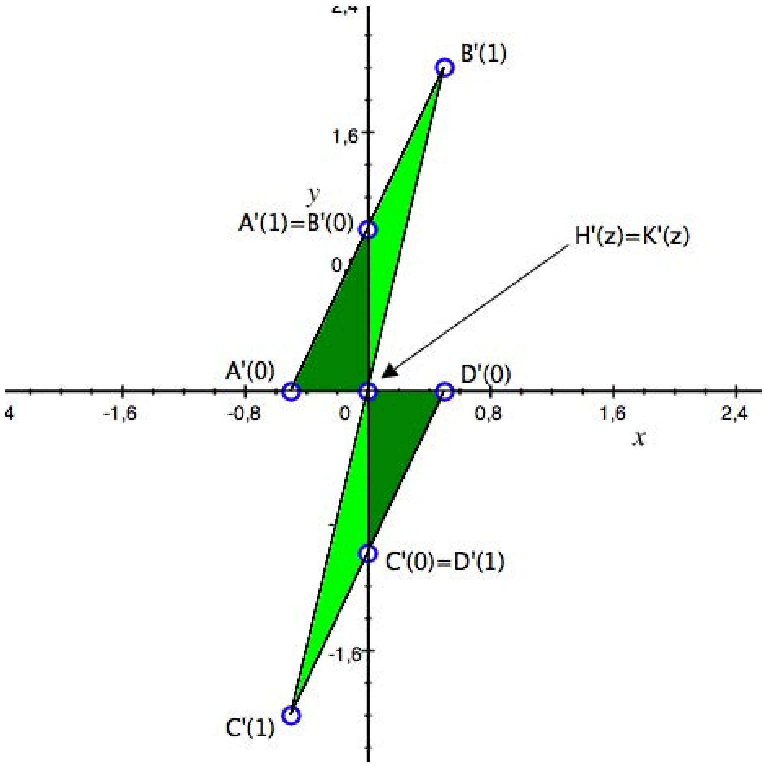

Fig. 9. The payoff space of the coopetitive game, $h(Q)$

8 Kalai-Smorodisky Solution

Why a Kalai-Smorodinsky Solution? The point $B'(1)$ is the maximum payoff of the game (with respect to the usual component-wise order of the payoff plane). But the Enterprise could be not satisfied by the gain 1/2, value that is much less than the win 2 of the Financial Institute. In addition, playing the shared strategy 1, the Enterprise increases only slightly the win obtained in the no-coopetitive game, on the contrary our Financial Institute gains more than double. For this reason, precisely to avoid that the envy of the Enterprise can affect the game, the Financial Institute might be willing to cede part of its win to the Enterprise by contract, in order to balance fairly the distribution of money.

Maximum Collective Gain. One way would be to distribute the *maximum collective profit* of the coopetitive game, that is, the maximum value of the *collective gain function*

$$g : \mathbb{R}^2 \to \mathbb{R} : g(X, Y) = X + Y$$

on the (compact) payoffs space of the game G, say $W = \max_{h(Q)} g$. The maximum collective profit is attained (evidently) at the maximum point point $B'(1)$, which is the only bi-win belonging to the straight line $g = 5/2$ and to the payoff space. Hence the Enterprise and the Financial Institute play the coopetitive 3-strategy $(1, 1, 1)$, in order to arrive at the payoff $B'(1)$ and then split the wins obtained by contract. From a practical point of view: the Enterprise buys futures to create artificially (also thanks to the money borrowed from the European Central Bank) a significant misalignment between futures and spot prices, misalignment which is exploited by the Financial Institute getting the maximum win $W = 5/2$.

First Possible Division of Maximum Collective Gain. For a possible **fair division** of the win $W = 5/2$, we propose a *transferable utility Kalai-Smorodinsky method.* The bargaining problem we face is the pair (Γ, α), where:

1. our decision constraint Γ is the transferable utility Pareto boundary of the game (straight line $X + Y = 1$);
2. we take the supremum of the game $\alpha = (1/2, 1)$ as threat point of our bargaining problem.

Solution. For what concerns the solution: we join $\alpha = (1/2, 1)$ with the supremum

$$\sup(\Gamma \cap [\alpha, \rightarrow [),$$

according to the classic Kalai-Smorodinsky method, supremum which is given by $(3/2, 2)$.

The coordinates of the intersection point P', between the straight line of maximum collective gain (i.e., $X + Y = 2.5$) and the segment joining α and the considered supremum (the segment is part of the line $\alpha + \mathbb{R}(1, 1)$), give us the desirable division of the maximum collective win $W = 5/2$, between the two players.

Second Possible Division. For another possible quantitative division of the maximum win $W = 5/2$, between the Financial Institute and the Enterprise, we propose a *transferable utility Kalai-Smorodinsky method.* The bargaining problem we face is the pair $(\Gamma, B'(0))$, where:

1. our decision constraint Γ is the transferable utility Pareto boundary of the coopetitive game (straight line $g = 5/2$);
2. we take, in our initial no-coopetitive game, the payoff with maximum possible collective profit, which is the point $B'(0) = (0, 1)$, as threat point of our bargaining problem (the payoff $B'(0)$ corresponds to the most likely Nash equilibrium of the initial no-coopetitive game—see paper [7]).

Solution. For what concerns the solution: we join $B'(0)$ with the supremum

$$\sup(\Gamma \cap [B'(0), \rightarrow [),$$

according to the classic Kalai-Smorodinsky method, supremum which is given by $(5/2, 3/2)$. The coordinates of the intersection point P, between the straight line of maximum collective gain (i.e. $g = 2.5$) and the segment joining $B'(0)$ and the considered supremum (the segment is part of the line $(0,1) + \mathbb{R}(1,1)$) give us the desirable division of the maximum collective win $W = 2.5$, between the two players. In Fig. 10 is shown the situation.

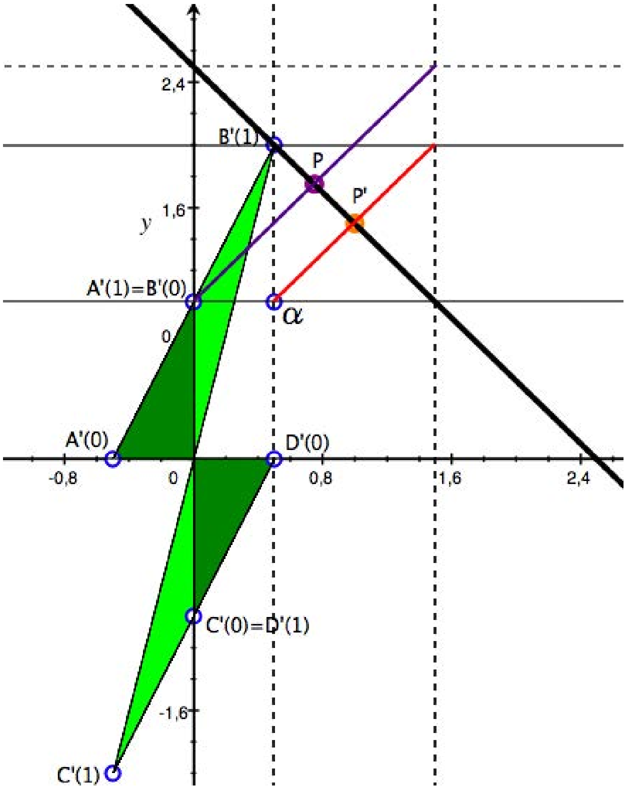

Fig. 10. Transferable utility solutions in the coopetitive game: cooperative solutions

Thus $P = (3/4, 7/4)$ and $P' = (1, 3/2)$ suggest as solution that the Enterprise receives respectively 3/4 or 1 by contract by the Financial Institute, while at the Financial Institute remains the win 7/4 or 3/2.

Why Are There Differences between the Two Possible Division of Collective Profit? The difference between the points P and P' are due to the different method used.

About the point P', we consider as threat point the sup $\alpha = (1/2, 1)$ of the no-coopetitive game. Therefore, the division is more profitable for the Financial Institute because it can obtain in the no-coopetitive game a higher maximum profit (that is 1) than the Enterprise (that can obtain 1/2).

About the point P, we consider as threat point the retro-image of the most likely Nash equilibrium B of the no-coopetitive game. Therefore, the division is even more profitable for the Financial Institute because, according to most likely Nash equilibrium, it should obtain its higher maximum profit (that is 1) while the Enterprise does not win anything.

9 Conclusions

The games just studied suggest a possible regulatory model by which the phenomenon of the credit crunch (which in recent years has put in crisis small and medium enterprises in Europe) should be greatly attenuated. Moreover, the financial markets are stabilized through the introduction of a tax on financial transactions. In fact, in this way it could be possible to avoid speculations, which constantly affect modern economy. The Financial Institute could equally gain without burdening the financial system by unilateral manipulations of traded asset prices and, especially, the Financial Institute invests the money received by the ECB in the real economy lending money to the Enterprise (which also gains something).

No-coopetitive Game. The unique optimal solution is the cooperative one (exposed in paper [7]), otherwise the game appears like a sort of "your death, my life." This type of situation happens often in the economic competition and leaves no escapes if either player decides to work alone, without a mutual collaboration. In fact, all no-cooperative solutions lead dramatically to mediocre results for at least one of the two players.

Coopetitive Game. We can see that the game becomes much easier to solve in a satisfactory manner for both players. Moreover, the money received by the ECB is put into real economy by the Financial Institute: in fact the bank (our second player) issues a loan for the Enterprise (our first player), which uses the money in order to buy assets for its business activities. Both the Enterprise and the Financial Institute reduce their chances of losing than the no-coopetitive game, and they can even easily reach to the maximum of the game: so the Enterprise wins 1/2 and the Financial Institute wins 2. If they instead take the tranfer utility solutions with the Kalai-Smorodisky method, the Enterprise increases up to three times the payout obtained in the cooperative phase of the no-coopetitive game (3/4 or 1 instead of 1/3), while the Financial Institute wins twice more than it did before (7/4 or 3/2 instead of 2/3). We have moved from an initial competitive situation that was not so profitable to a coopetitive highly profitable situation for both the Enterprise and the Financial Institute.

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Pairwise Comparison Matrices over Abelian Linearly Ordered Groups: A Consistency Measure and Weights for the Alternatives

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Abstract. The pairwise comparison matrices play a basic role in multicriteria decision making methods such as the Analytic Hierarchy Process (AHP).

We provide a survey of results related to pairwise comparison matrices over a real divisible and continuous abelian linearly ordered group $\mathcal{G} = (G, \odot, \leq)$, focusing on a $\odot$-consistency measure and a weighting vector for the alternatives.

Keywords: Pairwise comparison matrices, consistency index, abelian linearly ordered group, weighting vector.

1 Introduction

Let $X = \{x_1, x_2, ..., x_n\}$ be a set of alternatives or criteria. A *Pairwise Comparison Matrix* (PCM)

$$A = \begin{pmatrix} a_{11} & a_{12} & ... & a_{1n} \\ a_{21} & a_{22} & ... & a_{2n} \\ ... & ... & ... & ... \\ a_{n1} & a_{n2} & ... & a_{nn} \end{pmatrix}, \tag{1}$$

with a_{ij} encoding the preference intensity of x_i over x_j, is a useful tool for determining a weighted ranking for the alternatives.

In the literature, several kinds of PCMs are proposed, as the entry a_{ij} may assume different meanings: in multiplicative PCMs it represents a preference ratio; in additive PCMs it is a preference difference; in fuzzy PCMs it is a preference degree in [0,1].

In an ideal situation, the PCM satisfies the consistency property, which, in the multiplicative case, is expressed as follows:

$$a_{ik} = a_{ij} \cdot a_{jk} \quad \forall\, i, j, k = 1, \ldots, n. \tag{2}$$

Under condition of consistency, the preference value a_{ij} can be expressed by means of the components of a suitable vector, called consistent vector for $A =$

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(a_{ij}); for a multiplicative PCM, it is a positive vector $\underline{w} = (w_1, w_2, ..., w_n)$ verifying the condition

$$\frac{w_i}{w_j} = a_{ij} \quad \forall\, i, j = 1, \ldots, n.$$

Thus, if $A = (a_{ij})$ is a consistent PCM, then it is reasonable to choose a weighting vector in the set of consistent vectors, while, if $A = (a_{ij})$ is an inconsistent PCM, to look for a vector that is close to be a consistent vector. As an example, for the multiplicative case, we look for a vector such that:

$$\frac{w_i}{w_j} \approx a_{ij} \quad \forall\, i, j = 1, \ldots, n.$$

The multiplicative PCMs play a basic role in the well-known Analytic Hierarchy Process (AHP), the procedure developed by T.L. Saaty at the end of the 1970s [14], [15], [16]. In [2], [3], [4], [5] and [12], properties of multiplicative PCMs are analyzed in order to determine a qualitative ranking on the set of alternatives and find vectors representing this ranking. Additive and fuzzy PCMs are investigated for instance by [1] and [13].

The AHP provides a comprehensive and rational framework for structuring a decision problem, for representing and quantifying its elements, for relating those elements to overall goals, and for evaluating alternative solutions.
"*To make a decision in an organised way to generate priorities we need to decompose the decision into the following steps.*

1. *Define the problem and determine the kind of knowledge sought.*
2. *Structure the decision hierarchy from the top with the goal of the decision, then the objectives from a broad perspective, through the intermediate levels (criteria on which subsequent elements depend) to the lowest level (which usually is a set of the alternatives).*
3. *Construct a set of pairwise comparison matrices. Each element in an upper level is used to compare the elements in the level immediately below with respect to it.*
4. *Use the priorities obtained from the comparisons to weigh the priorities in the level immediately below. Do this for every element. Then for each element in the level below add its weighed values and obtain its overall or global priority. Continue this process of weighing and adding until the final priorities of the alternatives in the bottom most level are obtained.*" [17]

In order to unify the different approaches to the way of building the PCMs, in [7] the authors introduce PCMs whose entries belong to an abelian linearly ordered group (*alo-group*) $\mathcal{G} = (G, \odot, \leq)$. In this way, the consistency condition is expressed in terms of the group operation $\odot$. Under the assumption of divisibility of $\mathcal{G}$, for each $A = (a_{ij})$, a $\odot$-consistency measure $I_{\mathcal{G}}(A)$, expressed in terms of $\odot$-mean of $\mathcal{G}$-distances, is provided; furthermore a $\odot$-mean vector $\underline{w}_m(A)$, satisfying the independence of scale-inversion condition, is chosen as a weighting vector for the alternatives.

In this paper, we provide a survey of results related to PCMs on alo-groups, by focusing on properties of $I_{\mathcal{G}}(A)$ and $\underline{w}_m(A)$.

The paper is organized as follows: Section 2 focuses on alo-groups; Section 3 introduces PCMs on real divisible alo-groups; Section 4 provides concluding remarks and directions for future work.

2 Alo-groups

From now on, $\mathbb{R}$ will denote the set of real numbers, $\mathbb{Q}$ the subset of rational numbers, $\mathbb{Z}$ the subset of relative integers, $\mathbb{N}$ the subset of positive integers and $\mathbb{N}_0$ the set $\mathbb{N} \cup \{0\}$. $\mathcal{G} = (G, \odot, \leq)$ denotes an abelian linearly ordered group (alo-group), e its *identity*, $a^{(-1)}$ the *inverse* of $a \in G$ with respect to $\odot$, $\div$ the *inverse operation* of $\odot$, defined by:

$$a \div b = a \odot b^{(-1)} \quad \forall a, b \in G, \tag{3}$$

and $G^n = \{\underline{w} = (w_1, \ldots, w_n) | w_i \in G, \forall i \in \{1, \ldots, n\}\}$.

Definition 1. *[11] A vector $\underline{w} \in G^n$ is called a $\odot$-*normal *vector if and only if*

$$w_1 \odot w_2 \ldots \odot w_n = e.$$

Definition 2. *[8] The vectors $\underline{w} = (w_1, \ldots, w_n)$ and $\underline{v} = (v_1, \ldots, v_n)$ are $\odot$-proportional if and only if there exists $c \in G$ such that $\underline{w} = c \odot \underline{v} = (c \odot v_1, \ldots, c \odot v_n)$.*

Proposition 1. *[7] If $\mathcal{G} = (G, \odot, \leq)$ is a non-trivial alo-group then it has neither a greatest element nor a least element.*

Proposition 2. *[7] The operation*

$$\begin{aligned} d_{\mathcal{G}} &: G \times G \to G \\ &(a, b) \mapsto d_{\mathcal{G}}(a, b) = (a \div b) \vee (b \div a) \end{aligned} \tag{4}$$

is a $\mathcal{G}$-distance that satisfies the following properties:

1. $d(a, b) \geq e$;
2. $d(a, b) = e \Leftrightarrow a = b$;
3. $d(a, b) = d(b, a)$;
4. $d(a, b) \leq d(a, c) \odot d(b, c)$.

Definition 3. *[7] Let $n \in \mathbb{N}_0$. The (n)-*power *$a^{(n)}$ of $a \in G$ is defined as follows:*

$$a^{(n)} = \begin{cases} e, & if \quad n = 0 \\ a^{(n-1)} \odot a, & if \quad n \geq 1. \end{cases}$$

Definition 4. *[10] Let $z \in \mathbb{Z}$. The (z)-*power *$a^{(z)}$ of $a \in G$ is defined as follows:*

$$a^{(z)} = \begin{cases} a^{(n)}, & if \quad z = n \in \mathbb{N}_0 \\ (a^{(n)})^{(-1)} & if \quad z = -n, \quad n \in \mathbb{N}. \end{cases}$$

2.1 Divisible Alo-groups

Definition 5. *$\mathcal{G} = (G, \odot, \leq)$ is divisible if and only if $(G, \odot)$ is divisible, that is, for each $n \in \mathbb{N}$ and each $a \in G$, the equation $x^{(n)} = a$ has at least a solution.*

If $\mathcal{G} = (G, \odot, \leq)$ is divisible, then the equation $x^{(n)} = a$ has a unique solution. Thus, we give the following definition:

Definition 6. *[7] Let $\mathcal{G} = (G, \odot, \leq)$ be divisible, $n \in \mathbb{N}$ and $a \in G$. Then, the (n)-*root *of a, denoted by $a^{(\frac{1}{n})}$, is the unique solution of the equation $x^{(n)} = a$, that is:*

$$(a^{(\frac{1}{n})})^{(n)} = a. \tag{5}$$

Definition 7. *[7] Let $\mathcal{G} = (G, \odot, \leq)$ be divisible.* $\odot$-mean *$m_\odot(a_1, a_2, ..., a_n)$ of the n elements $a_1, a_2, ..., a_n$ of G is the element $a \in G$ verifying the equality $a \odot a \odot ... \odot a = a_1 \odot a_2 \odot ... \odot a_n$; that is,*

$$m_\odot(a_1, a_2, ..., a_n) = \begin{cases} a_1 & if\ n = 1, \\ (\bigodot_{i=1}^n a_i)^{(\frac{1}{n})} & if\ n \geq 2. \end{cases}$$

Definition 8. *[10] Let $(G, \odot, \leq)$ be divisible. For each $q = \frac{m}{n}$, with $m \in \mathbb{Z}$ and $n \in \mathbb{N}$, and for each $a \in G$, the (q)-power $a^{(q)}$ is defined as follows:*

$$a^{(q)} = (a^{(m)})^{(\frac{1}{n})}.$$

2.2 Real Divisible Alo-groups

An alo-group $\mathcal{G} = (G, \odot, \leq)$ is a *real* alo-group if and only if G is a subset of the real line $\mathbb{R}$ and $\leq$ is the total order on G inherited from the usual order on $\mathbb{R}$. If G is an interval of $\mathbb{R}$ then, by Proposition 1, it has to be an open interval. Examples of real divisible continuous alo-groups are the following:

Multiplicative Alo-group. $]0,+\infty[$ $= (]0, +\infty[, \cdot, \leq)$, where $\cdot$ is the usual multiplication on $\mathbb{R}$. Then, $e = 1$ and for $a, b \in]0, +\infty[$ and $q \in \mathbb{Q}$:

$$a^{(-1)} = 1/a, \quad a \div b = \frac{a}{b}, \quad a^{(q)} = a^q,$$

$$d_{]0,+\infty[}(a, b) = \frac{a}{b} \vee \frac{b}{a};$$

moreover, for $a_i \in]0, +\infty[$, $i \in \{1, \ldots n\}$, $m_\cdot(a_1, ..., a_n)$ is the geometric mean:

$$m_\cdot(a_1, ..., a_n) = \left(\prod_{i=1}^n a_i\right)^{\frac{1}{n}}.$$

Additive Alo-group. $\mathcal{R} = (\mathbb{R}, +, \leq)$, where $+$ is the usual addition on $\mathbb{R}$. Then, $e = 0$ and for $a, b \in \mathbb{R}$ and $q \in \mathbb{Q}$:

$$a^{(-1)} = -a, \quad a \div b = a - b, \quad a^{(q)} = qa,$$

$$d_{\mathcal{R}}(a,b) = |a-b| = (a-b) \vee (b-a);$$

moreover, for $a_i \in \mathbb{R},\ i \in \{1, \ldots n\}$, $m_+(a_1, ..., a_n)$ is the arithmetic mean:

$$m_+(a_1, ..., a_n) = \frac{\sum_i a_i}{n}.$$

Fuzzy group. $]\mathbf{0,1}[=(]0,1[, \otimes, \leq)$, where $\otimes :]0,1[^2 \rightarrow]0,1[$ is the operation defined by

$$x \otimes y = \frac{xy}{xy + (1-x)(1-y)}.$$

Then, $e = 0.5$ and for $a, b \in]0,1[$ and $q \in \mathbb{Q}$:

$$a^{(-1)} = 1 - a, \quad a \div b = \frac{a(1-b)}{a(1-b) + (1-a)b}, \quad a^{(q)} = \frac{a^q}{a^q + (1-a)^q},$$

$$d_{]\mathbf{0,1}[}(a,b) = \frac{a(1-b)}{a(1-b)+(1-a)b} \vee \frac{b(1-a)}{b(1-a)+(1-b)a};$$

moreover, for $a_i \in]0,1[, i \in \{1, \ldots n\}$,

$$m_{\otimes}(a_1, ..., a_n) = \frac{\sqrt[n]{\prod_{i=1}^n a_i}}{\sqrt[n]{\prod_{i=1}^n a_i} + \sqrt[n]{\prod_{i=1}^n (1-a_i)}}. \tag{6}$$

Two divisible continuous real alo-groups are isomorphic with respect to the group operations and the order relation; in particular for each real divisible continuous alo-group $\mathcal{G} = (G, \odot, \leq)$, there exists an isomorphism h between $]\mathbf{0,+\infty}[$ and $\mathcal{G}$. For instance:

$$l : x \in]0, +\infty[\mapsto \log x \in \mathbb{R} \tag{7}$$

is an isomorphism between $]\mathbf{0,+\infty}[$ and $\mathcal{R}$ and

$$\psi : x \in]0, +\infty[\mapsto \frac{x}{x+1} \in]0,1[\tag{8}$$

is an isomorphism between $]\mathbf{0,+\infty}[$ and $]\mathbf{0,1}[$.

Let $\mathcal{G} = (G, \odot, \leq)$ be a real divisible continuous alo-group. For each $a \in G$ and $r \in \mathbb{R}$, we set:

$$I_{a,r} = \{a^{(q)} : q \in Q \quad and \quad q < r\}, \quad S_{a,r} = \{a^{(q)} : q \in Q \quad and \quad q > r\}. \tag{9}$$

In [9], the authors extend the notion of (q)-power, with $q \in \mathbb{Q}$, in Definition 8, to the notion of (r)-power, with $r \in \mathbb{R}$, as follows:

Definition 9. *[9] Let $\mathcal{G} = (G, \odot, \leq)$ be a real divisible continuous alo-group. For each $a \in G$ and $r \in \mathbb{R}$, $a^{(r)}$ is the separation point of sets in (9), thus the following holds:*

$$a^{(r)} = h((h^{-1}(a))^r),$$

with h an isomorphism between $]\mathbf{0},+\infty[$ and $\mathcal{G}$.

Proposition 3. *[9] Let $\mathcal{G} = (G, \odot, \leq)$ be a real divisible continuous alo-group a. For each $a, b \in G$ and $r, r_1, r_2 \in \mathbb{R}$, we have:*

1. $a^{(-r)} = (a^{(r)})^{(-1)} = (a^{(-1)})^{(r)}$;
2. $a^{(r_1)} \odot a^{(r_2)} = a^{(r_1+r_2)}$;
3. $(a^{(r_1)})^{(r_2)} = a^{(r_1 r_2)} = (a^{(r_2)})^{(r_1)}$;
4. $(a \odot b)^{(r)} = a^{(r)} \odot b^{(r)}$;
5. $e^{(r)} = e$.

Proposition 4. *[9] Let $\mathcal{G} = (G, \odot, \leq)$ be a real divisible continuous alo-group and $r \in \mathbb{R}$. Then, (r)-power function:*

$$f_{(r)} : a \in G \to a^{(r)} \in G$$

is strictly increasing if $r > 0$, strictly decreasing if $r < 0$ and is the constant function $f_{(0)} = e$ if $r = 0$.

Proposition 5. *[9] Let $\mathcal{G} = (G, \odot, \leq)$ be a real divisible continuous alo-group and $a \in G$, with $a \neq e$. Then, (r)-exponential function*

$$g : r \in \mathbb{R} \to a^{(r)} \in G$$

is strictly increasing if $a > e$ and strictly decreasing if $a < e$.

3 PCMs on Real Divisible Alo-groups

Let $X = \{x_1, x_2, ...x_n\}$ be a set of alternatives and $A = (a_{ij})$ in (1) the related PCM. We assume that $A = (a_{ij})$ is a PCM over a real continuous divisible alo-group $\mathcal{G} = (G, \odot, \leq)$, that is, $a_{ij} \in G$, $\forall i, j \in \{1, \ldots, n\}$ [7]. We assume that:

1. $\underline{a}_1, \underline{a}_2, \ldots, \underline{a}_n$ are the rows of A;
2. $\underline{a}^1, \underline{a}^2, \ldots, \underline{a}^n$ are the columns of A;
3. $A_{(ijk)}$ is the sub-matrix $\begin{pmatrix} a_{ii} & a_{ij} & a_{ik} \\ a_{ji} & a_{jj} & a_{jk} \\ a_{ki} & a_{kj} & a_{kk} \end{pmatrix}$;
4. A_{ijk} denotes $A_{(ijk)}$ if $i < j < k$ (see [7]);
5. $A^{(r)} = (a_{ij}^{(r)})$.

Definition 10. *For each $A = (a_{ij})$ over $\mathcal{G} = (G, \odot, \leq)$, the* $\odot$-mean vector *associated to A is:*

$$\underline{w}_{m_\odot}(A) = (m_\odot(\underline{a}_1), m_\odot(\underline{a}_2), \cdots, m_\odot(\underline{a}_n)), \qquad (10)$$

where $m_\odot(\underline{a}_i) = m_\odot(a_{i1}, a_{i2}, \ldots, a_{in})$.

Definition 11. *$A = (a_{ij})$ is a $\odot$-*reciprocal *PCM if and only if verifies the condition:*

$$a_{ji} = a_{ij}^{(-1)} \quad \forall\, i, j \in \{1, \ldots, n\}, \qquad (\odot - reciprocity)$$

so $a_{ii} = e\ \ \forall i \in \{1, \ldots, n\}$.

$RM(n)$ will denote the set of $\odot$-reciprocal PCMs of order n. Let us assume $A \in RM(n)$, then we set:

$$x_i \succ x_j \Leftrightarrow a_{ij} > e, \qquad x_i \sim x_j \Leftrightarrow a_{ij} = e, \tag{11}$$

where $x_i \succ x_j$ and $x_i \sim x_j$ stand for "x_i is strictly preferred to x_j"and "x_i and x_j are indifferent,"respectively; the strict preference of x_i over x_j is expressed also by the equivalence:

$$x_i \succ x_j \Leftrightarrow a_{ji} < e. \tag{12}$$

Example 1. The matrix

$$A = \begin{pmatrix} 1 & 2 & \frac{1}{10} \\ \frac{1}{2} & 1 & 3 \\ 10 & \frac{1}{3} & 1 \end{pmatrix}$$

is a $\cdot$-reciprocal PCM on the multiplicative alo-group $(]0, +\infty[, \cdot, \leq)$.

Example 2. The matrix

$$B = \begin{pmatrix} 0 & 2 & -5 \\ -2 & 0 & 3 \\ 5 & -3 & 0 \end{pmatrix}$$

is a $+$-reciprocal PCM on the additive alo-group $(\mathbb{R}, +, \leq)$.

Example 3. The matrix

$$C = \begin{pmatrix} 0.5 & 0.6 & 0.2 \\ 0.4 & 0.5 & 0.7 \\ 0.8 & 0.3 & 0.5 \end{pmatrix}$$

is a $\otimes$-reciprocal PCM on the fuzzy alo-group $(]0, 1[, \otimes, \leq)$.

3.1 $\odot$-consistency

Definition 12. *A is a $\odot$-consistent PCM if and only if*

$$a_{ik} = a_{ij} \odot a_{jk} \quad \forall i, j, k \in \{1, \ldots n\}\ .$$

$CM(n)$ will denote the set of $\odot$-consistent PCMs of order n.

Example 4. The matrix

$$A = \begin{pmatrix} 1 & 2 & 6 \\ \frac{1}{2} & 1 & 3 \\ \frac{1}{6} & \frac{1}{3} & 1 \end{pmatrix}$$

is a $\cdot$-consistent PCM on the multiplicative alo-group $(]0, +\infty[, \cdot, \leq)$.

Example 5. The matrix

$$B = \begin{pmatrix} 0 & 2 & 5 \\ -2 & 0 & 3 \\ -5 & -3 & 0 \end{pmatrix}$$

is a $+$-consistent PCM on the additive alo-group $(\mathbb{R}, +, \leq)$.

Example 6. The matrix

$$C = \begin{pmatrix} 0.5 & 0.6 & 0.\overline{7} \\ 0.4 & 0.5 & 0.7 \\ 1 - 0.\overline{7} & 0.3 & 0.5 \end{pmatrix}$$

is a $\otimes$-consistent PCM on the fuzzy alo-group $(]0, 1[, \otimes, \leq)$.

Definition 13. *Let $A = (a_{ij}) \in CM(n)$. A vector $\underline{w} = (w_1, \ldots, w_n) \in G^n$, is a* $\odot$-consistent *vector for $A = (a_{ij})$ if and only if:*

$$w_i \div w_j = a_{ij} \quad \forall\, i, j \in \{1, \ldots, n\}.$$

Proposition 6. *[7] The following assertions related to $A = (a_{ij})$ are equivalent:*

1. *$A = (a_{ij}) \in CM(n)$;*
2. *there exists a $\odot$-consistent vector $\underline{w}$ for A;*
3. *each column $\underline{a}^k$ is a $\odot$-consistent vector;*
4. *the $\odot$-mean vector $\underline{w}_{m_\odot}(A)$ is a $\odot$-consistent vector.*

Proposition 7. *[8] Let $A \in RM(n)$. The following assertions are equivalent:*

1. *$A \in CM(n)$;*
2. *$a_{ik} = a_{ij} \odot a_{jk} \quad \forall\, i, j, k \in \{1, \ldots, n\} : i < j < k$;*
3. *$\underline{a}_i$ and $\underline{a}_{i+1}$ are $\odot$-proportional vectors ($\underline{a}_{i+1} = a_{i\,i+1}^{(-1)} \odot \underline{a}_i \quad \forall i < n$);*
4. *$\underline{a}^i$ and $\underline{a}^{i+1}$ are $\odot$-proportional vectors ($\underline{a}^{i+1} = a_{i+1\,i}^{(-1)} \odot \underline{a}^i \quad \forall i < n$);*
5. *$a_{ik} = a_{i\,i+1} \odot a_{i+1\,k} \quad \forall i, k : i < k$;*
6. *$a_{ik} = a_{i\,i+1} \odot a_{i+1\,i+2} \odot \ldots \odot a_{k-1\,k} \quad \forall i, k : i < k.$*

3.2 A $\odot$-consistency Measure

In order to measure how much a PCM is far from a consistent one, in [7], the following $\odot$-consistency index is provided:

Definition 14. *[7] Let $A \in RM(n)$ with $n \geq 3$, then the $\odot$-consistency index $I_{\mathcal{G}}(A)$ is defined as follows:*

$$I_{\mathcal{G}}(A) = \Big(\bigodot_{i<j<k} d_{\mathcal{G}}(a_{ik}, a_{ij} \odot a_{jk})\Big)^{(\frac{1}{n_T})}$$

with $T = \{(i,j,k) : i < j < k\}$ and $n_T = |T| = \frac{n(n-2)(n-1)}{6}$.
Thus:

$$I_{\mathcal{G}}(A) = \begin{cases} d_{\mathcal{G}}(a_{13}, a_{12} \odot a_{23}) & if\ n = 3, \\ \big(\bigodot_{i<j<k} I_{\mathcal{G}}(A_{ijk})\big)^{(\frac{1}{n_T})} & if\ n > 3. \end{cases} \tag{13}$$

$I_{\mathcal{G}}(A)$ has an intuitive meaning, because is a $\odot$-mean of $\mathcal{G}$-distances, and is suitable for several kinds of PCMs (e.g. multiplicative, additive and fuzzy).

Proposition 8. *[7] Let $A \in RM(n)$, then:*

$$I_{\mathcal{G}}(A) \geq e, \quad I_{\mathcal{G}}(A) = e \Leftrightarrow A \in CM(n).$$

Proposition 8 proves that there is a unique value of $I_{\mathcal{G}}(A)$ representing the $\odot$-consistency, that is, the identity element e (property that a consistency index must satisfy as required in [6]).

Example 7. Let

$$A = \begin{pmatrix} 1 & \frac{1}{7} & \frac{1}{7} & \frac{1}{5} \\ 7 & 1 & \frac{1}{2} & \frac{1}{3} \\ 7 & 2 & 1 & \frac{1}{9} \\ 5 & 3 & 9 & 1 \end{pmatrix}$$

be a PCM on the multiplicative alo-group $(]0, +\infty[, \cdot, \leq)$, then:

$$\begin{aligned} I_{]0,+\infty[}(A) &= \sqrt[4]{I_{]0,+\infty[}(A_{123}) \cdot I_{]0,+\infty[}(A_{124}) \cdot I_{]0,+\infty[}(A_{134}) \cdot I_{]0,+\infty[}(A_{234})} \\ &= \sqrt[4]{2 \cdot \frac{21}{5} \cdot \frac{63}{5} \cdot 6} = 5.02. \end{aligned}$$

Example 8. Let

$$B = \begin{pmatrix} 0 & -\log 7 & -\log 7 & -\log 5 \\ \log 7 & 0 & -\log 2 & -\log 3 \\ \log 7 & \log 2 & 0 & -\log 9 \\ \log 5 & \log 3 & \log 9 & 0 \end{pmatrix}$$

be a PCM on the additive alo-group $(\mathbb{R}, +, \leq)$, then:

$$\begin{aligned} I_{\mathcal{R}}(B) &= \frac{I_{\mathcal{R}}(B_{123}) + I_{\mathcal{R}}(B_{124}) + I_{\mathcal{R}}(B_{134}) + I_{\mathcal{R}}(B_{234})}{4} \\ &= \frac{0.6931 + 1.4350 + 2.5336 + 1.7917}{4} = 1.6134. \end{aligned}$$

Example 9. Let

$$C = \begin{pmatrix} 0.5\ 0.3\ 0.4\ 0.4 \\ 0.7\ 0.5\ 0.1\ 0.2 \\ 0.6\ 0.9\ 0.5\ 0.8 \\ 0.6\ 0.8\ 0.2\ 0.5 \end{pmatrix}$$

be a PCM on the fuzzy alo-group $(]0,1[, \otimes, \leq)$, then:

$$I_{]0,1[}(C) = \frac{\sqrt[4]{\prod_{i<j<k} I_{]0,1[}(C_{ijk})}}{\sqrt[4]{\prod_{i<j<k} I_{]0,1[}(C_{ijk})} + \sqrt[4]{\prod_{i<j<k}(1 - I_{]0,1[}(C_{ijk}))}} = 0.833.$$

Invariance under Permutation of Alternatives

Let $\pi : \{1,\ldots,n\} \to \{1,\ldots,n\}$ be a bijection, then $(\pi(1),\ldots,\pi(n))$ denotes the corresponding permutation of the n-tuple $(1,\ldots,n)$ and $\Pi : RM(n) \to RM(n)$ the function:

$$\Pi : A = \begin{pmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ a_{n1} & a_{n2} & \ldots & a_{nn} \end{pmatrix} \mapsto \Pi(A) = \begin{pmatrix} a_{\pi(1)\pi(1)} & a_{\pi(1)\pi(2)} & \ldots & a_{\pi(1)\pi(n)} \\ a_{\pi(2)\pi(1)} & a_{\pi(2)\pi(2)} & \ldots & a_{\pi(2)\pi(n)} \\ \ldots & \ldots & \ldots & \ldots \\ a_{\pi(n)\pi(1)} & a_{\pi(n)\pi(2)} & \ldots & a_{\pi(n)\pi(n)} \end{pmatrix}. \tag{14}$$

Proposition 9. *[9] Let $A \in RM(n)$ and Π the function in* (14), *then the following equality holds:*

$$I_{\mathcal{G}}(\Pi(A)) = I_{\mathcal{G}}(A).$$

By Proposition 9, the $\odot$-consistency index $I_{\mathcal{G}}(A)$ is independent from the order in which the alternatives are presented.

Monotonicity under Reciprocity Preserving Mapping

Let $A \in RM(n)$ and $r \in \mathbb{R}$, by Proposition 5, we have:

$$r > 1 \Rightarrow \begin{cases} a_{ij} > e \Rightarrow & e < a_{ij} < a_{ij}^{(r)}, \\ a_{ij} < e \Rightarrow & a_{ij}^{(r)} < a_{ij} < e; \end{cases}$$

$$0 < r < 1 \Rightarrow \begin{cases} a_{ij} > e \Rightarrow & e < a_{ij}^{(r)} < a_{ij}, \\ a_{ij} < e \Rightarrow & a_{ij} < a_{ij}^{(r)} < e; \end{cases} \tag{15}$$

$$r < 0 \Rightarrow \begin{cases} a_{ij} > e \Rightarrow & a_{ij}^{(r)} < e < a_{ij}, \\ a_{ij} < e \Rightarrow & a_{ij} < e < a_{ij}^{(r)}. \end{cases}$$

Thus, if $r > 1$ then $a_{ij}^{(r)}$ represents an intensification of the preference a_{ij}, if $0 < r < 1$ a weakening of the preference and if $r < 0$ a preference reversal.

Proposition 10. *[9] Let $r \in \mathbb{R}$, then the function:*

$$F_{(r)} : A \in RM(n) \mapsto A^{(r)} \in RM(n) \tag{16}$$

is $\odot$-consistency preserving and, if $r \in \mathbb{R}\backslash\{0\}$, it is a bijection.

We study how $I_{\mathcal{G}}(A)$ changes its value, when the function $F_{(r)}$ is applied to A.

Proposition 11. *[9] Let $A \in RM(n)$ and $r \in \mathbb{R}$, then:*

$$I_{\mathcal{G}}(A^{(r)}) = (I_{\mathcal{G}}(A))^{(|r|)} = \begin{cases} (I_{\mathcal{G}}(A))^{(r)} & if\ r \geq 0, \\ (I_{\mathcal{G}}(A))^{(-r)} & if\ r < 0. \end{cases}$$

Corollary 1. *[9] Let $A \in RM(n)\backslash CM(n)$. Then:*

$$I_{\mathcal{G}}(A^{(r)}) \begin{cases} > (I_{\mathcal{G}}(A)) & if\ |r| > 1, \\ < (I_{\mathcal{G}}(A)) & if\ |r| < 1. \end{cases}$$

By Corollary 1, Proposition 10 and Proposition 8, if $A \in RM(n)$, then the following inequality holds:

$$I_{\mathcal{G}}(A^{(r)}) \geq I_{\mathcal{G}}(A) \quad \forall r > 1. \tag{17}$$

Inequality (17) corresponds to the third characterizing property in [6].

Proposition 12. *[9] Let $A \in RM(n)$. Then the function:*

$$m : r \in \mathbb{R} \to I_{\mathcal{G}}(A^{(r)}) \in G$$

satisfies the following properties:

– *if $A \in CM(n)$ then m is the constant function $m : r \in \mathbb{R} \to e \in G$;*
– *if $A \notin CM(n)$ then m is strictly increasing in $[0, +\infty[$ and strictly decreasing in $] - \infty, 0]$.*

For multiplicative, additive and fuzzy cases, for some value of $I_{\mathcal{G}}(A)$, the graphics of $I_{\mathcal{G}}(A^{(r)})$ are shown in Figs 1, 2 and 3.

Strict Monotonicity on Single Entries

Let us consider $A = (a_{ij})$, a $\odot$-consistent PCM, and choose one of its non-diagonal entries a_{pq}. If we change a_{pq} in b_{pq}, by increasing or decreasing its value, and modify its reciprocal a_{qp} accordingly, while all the other entries remain unchanged, then the resulting PCM, $B = (b_{ij})$, is not anymore $\odot$-consistent and, by Proposition 8, $I_{\mathcal{G}}(B) > e$.

Proposition 13 proves that the more b_{pq} is far from a_{pq}, the more $B = (b_{ij})$ is $\odot$-inconsistent. This expresses a sort of monotonicity of the $\odot$-inconsistency with respect to a single entry of the PCM.

Proposition 13. *Let $A \in CM(n)$, $p, q \in \{1, \ldots, n\}$, with $p \neq q$, and $B = (b_{ij})$, $C = (c_{ij}) \in RM(n)$ such that $a_{ij} = b_{ij} = c_{ij}$ for $i \neq p,\ j \neq q$ and for $i \neq q,\ j \neq p$. Then:*

$$\big(e < d_{\mathcal{G}}(b_{pq}, a_{pq}) < d_{\mathcal{G}}(c_{pq}, a_{pq})\big) \Rightarrow (I_{\mathcal{G}}(A) < I_{\mathcal{G}}(B) < I_{\mathcal{G}}(C)).$$

By Proposition 13, $\odot$-consistency index $I_{\mathcal{G}}(A)$ satisfies the fourth property provided in [6].

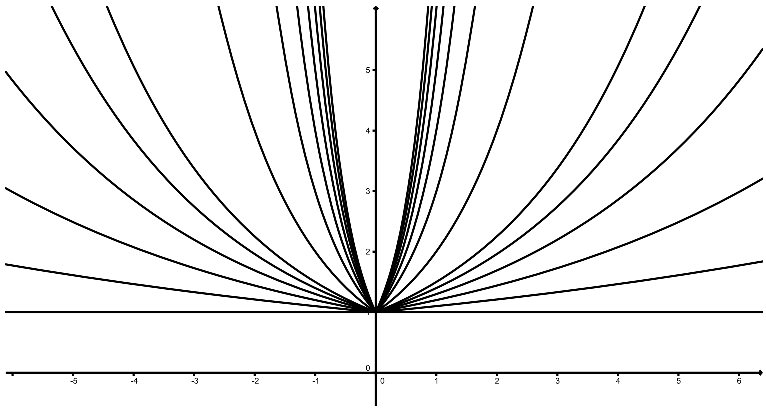

Fig. 1. Multiplicative case: $m : r \in \mathbb{R} \to I_{\mathcal{G}}(A^{(r)}) = I_{\mathcal{G}}((a_{ij}^{r})) = (I_{\mathcal{G}}(A))^{|r|} \in [1, +\infty[$

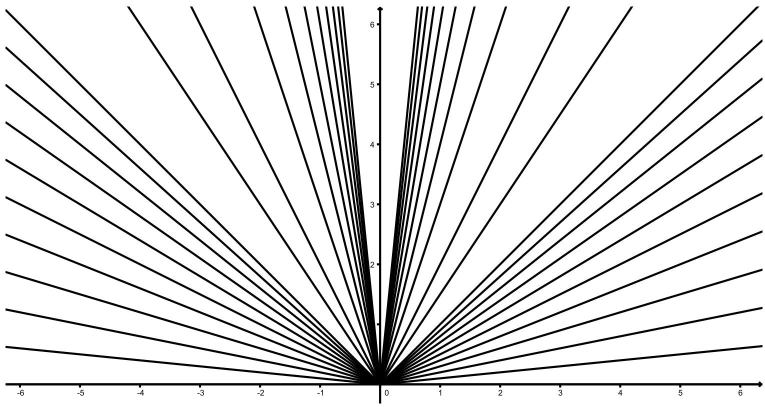

Fig. 2. Additive case: $m : r \in \mathbb{R} \to I_{\mathcal{G}}(A^{(r)}) = I_{\mathcal{G}}((r \cdot a_{ij})) = |r| \cdot I_{\mathcal{G}}(A) \in [0, +\infty[$

3.3 A Weighting Vector for the Alternatives

Proposition 14. *[11] The relation* $\succ$ *in* (11) *is asymmetric, the relation* $\sim$ *in* (11) *is reflexive and symmetric and, for each pair* (x_i, x_j)*, one and only one of the following conditions hold:*

$$x_i \succ x_j, \quad x_i \sim x_j, \quad x_j \succ x_i. \tag{18}$$

Let $\succeq$ denote the relation on X defined by

$$x_i \succeq x_j \Leftrightarrow x_i \succ x_j \textit{ or } x_i \sim x_j. \tag{19}$$

Then, by Proposition 14:

$$x_i \sim x_j \Leftrightarrow (x_i \succeq x_j \textit{ and } x_j \succeq x_i), \quad x_i \succ x_j \Leftrightarrow (x_i \succeq x_j \textit{ and } x_j \succeq x_i). \tag{20}$$

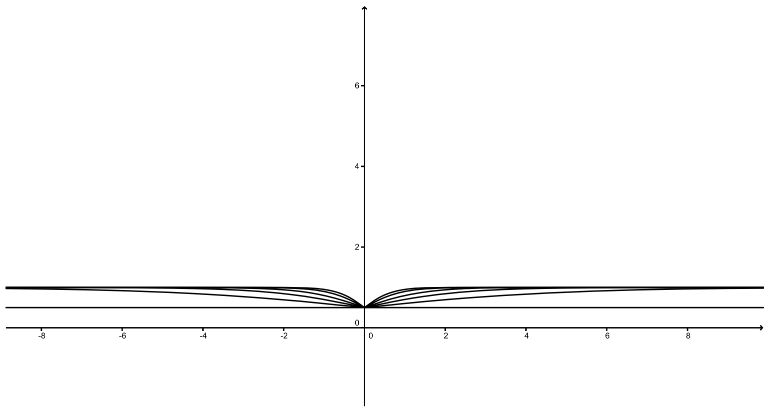

Fig. 3. Fuzzy case: $m : r \in \mathbb{R} \to I_{\mathcal{G}}(A^{(r)}) = I_{\mathcal{G}}((\frac{(a_{ij}^r)}{(a_{ij}^r)+(1-a_{ij}^r)})) = \frac{(I_{\mathcal{G}}(A))^r}{(I_{\mathcal{G}}(A))^r+(1-I_{\mathcal{G}}(A))^r} \in$ $[0.5, 1[$

Proposition 15. *Let $A = (a_{ij}) \in CM(n)$. Then, the relations $\succ$ and $\sim$ are transitive, that is:*

1. $x_i \succ x_j$ *and* $x_j \succ x_k \Rightarrow x_i \succ x_k$,
2. $x_i \sim x_j$ *and* $x_j \sim x_k \Leftrightarrow x_i \sim x_k$.
   Moreover, $\succ$ *and* $\sim$ *verify the following joint transitivity conditions:*
3. $x_i \succ x_j$ *and* $x_j \sim x_k \Rightarrow x_i \succ x_k$,
4. $x_i \sim x_j$ *and* $x_j \succ x_k \Rightarrow x_i \succ x_k$.

Corollary 2. *[11] Let $A = (a_{ij}) \in CM(n)$. Then $\succ$ is a strict order, $\sim$ is an equivalence relation and $\succeq$ is a total weak order on X.*

By Corollary 2, if $A \in CM(n)$ then X is totally ordered by the relation $\succeq$. Hence, there is a permutation $(i_1, i_2, \cdots, i_n)$ of $(1, 2, \cdots, n)$ such that:

$$x_{i_1} \succeq x_{i_2} \succeq ..... \succeq x_{i_n}. \tag{21}$$

We say that the ranking in (21) is the *actual ranking* on X derived from A by means of the equivalences (19) and (11). Then, an *ordinal evaluation vector* for the actual ranking is a vector $\underline{w} = (w_1, w_2, ..., w_n) \in G^n$ verifying the equivalences:

$$x_i \succ x_j \Leftrightarrow w_i > w_j, \qquad x_i \sim x_j \Leftrightarrow w_i = w_j. \tag{22}$$

Proposition 16. *[11] Let $A = (a_{ij}) \in CM(n)$. Each $\odot$-consistent vector $\underline{w} = (w_1, w_2, ..., w_n)$ is an ordinal evaluation vector.*

In [11], the authors focus on the problem of deriving weights for the alternatives from a PCM over a divisible alo-group $(G, \odot, \leq)$, and deal with the following research questions:

RQ1 Let $A = (a_{ij})$ be a $\odot$-consistent PCM. Which vector can be chosen as a weighting vector?

RQ2 Let $A = (a_{ij})$ be a $\odot$-inconsistent PCM. Which vector can be chosen as a weighting vector?

By Proposition 16, $\odot$-consistent vectors are ordinal evaluation vectors for the actual ranking (21) and, by Definition 13 of $\odot$-consistent vector, they are the only ones such that the composition of its components w_i and w_j, by means of $\div$, returns the preference value a_{ij}. Hence it is reasonable to claim that the weighting vector has to be a $\odot$-consistent vector. Thus, the research question **RQ1** changes into:

RQ1'. Let $A = (a_{ij})$ be a $\odot$-consistent PCM. Which $\odot$-consistent vector can be chosen as a weighting vector?

In [11], the $\odot$-mean vector $\underline{w}_{m_\odot}(A) = (m_\odot(\underline{a}_1), m_\odot(\underline{a}_2), \cdots, m_\odot(\underline{a}_n))$ is chosen as weighting vector for the alternatives for the following reasons:

1. $m_\odot(\underline{a}_i)$ represents the $\odot$-mean of the preference intensities of x_i over all the elements x_j;
2. $\underline{w}_{m_\odot}(A)$ is the unique $\odot$-normal vector in the set of $\odot$-consistent vectors (see Definition 1);
3. each $\odot$-consistent vector $\underline{w}$ is $\odot$-proportional to $\underline{w}_{m_\odot}(A)$ (see Definition 2).

Example 10. Let

$$A = \begin{pmatrix} 1 & 2 & 6 \\ \frac{1}{2} & 1 & 3 \\ \frac{1}{6} & \frac{1}{3} & 1 \end{pmatrix}$$

be a multiplicative PCM on $(]0, +\infty[, \cdot, \leq)$, then $\underline{w}_{m.}(A) = (\sqrt[3]{12}, \sqrt[3]{\frac{3}{2}}, \sqrt[3]{\frac{1}{18}})$.

Example 11. Let

$$B = \begin{pmatrix} 0 & 2 & 5 \\ -2 & 0 & 3 \\ -5 & -3 & 0 \end{pmatrix}$$

be an additive PCM on $(\mathbb{R}, +, \leq)$, then $\underline{w}_{m_+}(B) = (\frac{7}{3}, \frac{1}{3}, -\frac{8}{3})$.

Example 12. Let

$$C = \begin{pmatrix} 0.5 & 0.6 & 0.\overline{7} \\ 0.4 & 0.5 & 0.7 \\ 1 - 0.\overline{7} & 0.3 & 0.5 \end{pmatrix}$$

be a fuzzy PCM on $(]0, 1[, \otimes, \leq)$, then $\underline{w}_{m_\otimes}(C) = (0.63, 0.54, 0.33)$.

If A is $\odot$-inconsistent, then the relation $\succeq$ defined in (19) may not provide a ranking on the set X of alternatives and, even if (19) provides a ranking, there is no $\odot$-consistent vector $\underline{w}$ such that $w_i \div w_j = a_{ij}$. Thus, in order to answer **RQ2**, in [10], [11], the authors look for a condition ensuring the existence of a vector $\underline{w} = (w_1, w_2, ..., w_n)$ such that $d_{\mathcal{G}}(w_i \div w_j, a_{ij}) \approx e$, for each $i, j = 1, 2, \ldots, n$, that is, $w_i \div w_j$ is very close to a_{ij}; they provide the following:

$$d_{\mathcal{G}}((m_{\odot}(\underline{a}_i) \div m_{\odot}(\underline{a}_j)), a_{ij}) \begin{cases} = I_{\mathcal{G}}(A)^{(\frac{1}{3})}, & n = 3 \\ \leq I_{\mathcal{G}}(A)^{(\frac{(n-2)(n-1)}{6})}, & n > 3 \; . \end{cases} \tag{23}$$

Formula (23) gives more validity to $I_{\mathcal{G}}(A)$ as $\odot$-consistency measure and more meaning to $\underline{w}_m(A)$; in fact, it ensures that if $I_{\mathcal{G}}(A)$ is close to the identity element then, from one side A is close to be a $\odot$-consistent PCM and from the other side $\underline{w}_m(A)$ is close to be a $\odot$-consistent vector.
Finally, $\underline{w}_m(A)$ satisfies the independence of scale inversion condition [11], that is, $\underline{w}_{m_{\odot}}(A^T)$ and $\underline{w}_{m_{\odot}}(A)$ provide the same ranking for the alternatives.

For these reasons, $\underline{w}_{m_{\odot}}(A)$ is the answer both to **RQ1** and **RQ2**; that is, we choose it as a weighting vector for the alternatives.

4 Final Remark

We consider PCMs on real divisible alo-groups; this approach allows us to unify several approaches proposed in the literature. We focus on properties of the $\odot$-consistency index $I_{\mathcal{G}}(A)$ and the weighting vector $\underline{w}_m(A)$.

In the future, we will investigate, among other things, conditions weaker than $\odot$-consistency that allow us to identify the actual qualitative ranking on X from A. Hence, the problem will be to find vectors agreeing with this ranking.

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Entropy-Based Estimators in the Presence of Multicollinearity and Outliers

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Abstract. The concept and the mathematical properties of entropy play an important role in statistics, cybernetics, and information sciences. Indeed, many algorithms and statistical data processing tools, with a wide range of targets and scopes, have been designed based on entropy. The paper describes two estimators inspired by the concept of entropy that allow to robustly cope with multicollinearity, in one case, and outliers, in the other. The Generalized Maximum Entropy (GME) estimator optimizes the Shannon's entropy function subject to consistency and normality constraints. In regression applications GME allows, for example, to estimate model coefficients in the presence of multicollinearity. The Least Entropy-Like (LEL) estimator is a novel prediction error model coefficient identification algorithm that minimizes a nonlinear cost function of the fitting residuals. As the cost function that is minimized shares the same mathematical properties of entropy, it allows to compute an estimate of the model coefficients corresponding to a positively skewed distribution of the residuals. The resulting estimator exhibits higher robustness to outliers with respect to standard, as ordinary least squares (OLS) model coefficient approaches. Both the GME and LEL estimation methods are applied to a common case study to illustrate their respective properties.

1 Introduction

When talking to Claude Shannon about what name to use for the measure of uncertainty (or information) that he had introduced [18], John von Neumann is quoted for having suggested [22]: *You should call it entropy, for two reasons. In the first place your uncertainty function has been used in statistical mechanics under that name, so it already has a name. In the second place, and more important, nobody knows what entropy really is, so in a debate you will always have the advantage.*

Indeed, entropy is a rather general concept: since it was first acknowledged that it could be used well beyond thermodynamics and statistical mechanics [12], the number of different areas where it has been successfully exploited has grown dramatically.

This paper describes two specific data processing algorithms inspired by the mathematical definition of Shannon's (and Gibb's) entropy. Interestingly, neither of the two

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algorithms are strictly related to probability or classical statistical signal processing theory, but rather they are built exploiting the mathematical properties of entropy. The first of the two algorithms (LEL - Least Entropy-Like estimator) is a model coefficient estimation filter designed to yield robustness to outliers with respect to Ordinary Least Squares (OLS) or similar approaches. The need for robust parameter identification solutions is very strong in the area of control systems and signal processing [2][3][8][16]. The second algorithm is known as GME — Generalized Maximum Entropy [9] and is particularly useful when dealing with multicollinearity. This may occur, by example, in parameter estimation problems with fewer data than parameters or in the presence of rank deficient regression matrices when dealing with linear in the parameters models.

The rationale of the paper is to illustrate two examples of estimation methods designed by exploiting the properties of entropy. In particular, the estimators illustrated in this paper exhibit a noticeable robustness to multicollinearity and outliers. Giving a general overview of entropy-related methods for signal processing or even only parameter estimation goes well beyond the scope of this paper. The discussion will be limited to the LEL and GME approaches. Standard estimators, as OLS, become numerically ill conditioned as the condition number of the regression matrix grows. The OLS solution, in particular, is not defined in the presence of a regression matrix with infinite condition number. On the contrary, the GME approach is not ill conditioned for rank deficient regression matrices and it can robustly tackle the case of multicollinearity (large, but finite, regression matrix condition number). As for the LEL method, this was introduced in [11] and it consists in a model coefficient estimator based on the minimization of an entropy-like cost function. The cost function to be minimized in the LEL method is a nonlinear prediction error function exploiting the mathematical properties of entropy. In particular, this method exhibits an enhanced robustness to outliers as compared with OLS due to the very structure of the cost function to be minimized. Moreover, the proposed solution is computationally much less demanding with respect to alternative outlier robust approaches as the Least Median of Squares [17]. Indeed, thanks to these properties it has also been successfully employed in computer vision applications with stringent computational time requirements [7].

The paper is organized as follows: Sections 2 and 3 focus on briefly summarizing the LEL and GME algorithms, respectively. In Sect.4, we report validation results of the two methods as applied to a classical data set while concluding remarks are reported in Sect. 5.

2 Entropy-Like Estimator

Consider the model

$$y_i = f(x_{i1}, x_{i2}, \dots, x_{im}, \boldsymbol{\theta}_r) + \varepsilon_i \;:\; i = 1, 2, \dots, n. \tag{1}$$

where $\boldsymbol{\theta}_r \in \mathbb{R}^{m\times 1}$ is the unknown parameter vector, $y_i \in \mathbb{R}$ is the response variable, $x_{i1}, x_{i2}, \dots, x_{im}$ are the explanatory variables and ε_i the error term. Index i runs on the number of observations n that is assumed to be strictly larger than m (notice that this might not be the case for the GME method described in the next sections). The error term ε_i is assumed to be a random variable with zero mean. Denoting with

$Z_n = \{(y_i, x_{i1}, x_{i2}, \ldots, x_{im}) : i = 1, 2, \ldots, n\}$ the set of the available observations, a regression estimator T is an algorithm associating to Z_n an estimate $\hat{\boldsymbol{\theta}}$ of $\boldsymbol{\theta}_r$, namely $T(Z_n) = \hat{\boldsymbol{\theta}}$. Prediction error estimators T are designed based on the properties of the regression residuals

$$r_i := y_i - \hat{y}_i \tag{2}$$

being $\hat{y}_i$ the predicted responses $\hat{y}_i = f(x_{i1}, x_{i2}, \ldots, x_{im}, \hat{\boldsymbol{\theta}})$. The most popular prediction error estimators are the Least Squares (LS) and weighted LS (WLS) estimators defined respectively as

$$\hat{\boldsymbol{\theta}}_{LS} = \arg\min_{\boldsymbol{\theta}} \sum_{i=1}^{n} r_i^2 \tag{3}$$

$$\hat{\boldsymbol{\theta}}_{WLS} = \arg\min_{\boldsymbol{\theta}} \left(\mathbf{r}^T \boldsymbol{\Gamma} \mathbf{r}\right) \tag{4}$$

being $\mathbf{r} \in \mathbb{R}^{n\times 1}$ the residual vector $\mathbf{r} = (r_1, r_2, \ldots, r_n)^T$ and $\boldsymbol{\Gamma} \in \mathbb{R}^{n\times n}$ a symmetric positive definite (or eventually semidefinite) matrix of weights. Many other estimators have been proposed in the literature as, by example, M-estimators [10] or Least Median of Squares (LMS) estimators [17] that are defined through the minimization of a properly defined cost function. M-estimators, as LS or WLS estimators, share a common structure related to the additive nature of the corresponding cost function to be minimized: in particular such estimators can be all modeled as

$$\hat{\boldsymbol{\theta}} = \arg\min_{\boldsymbol{\theta}} \sum_{i=1}^{n} \rho(r_i) \tag{5}$$

for some scalar function $\rho : \mathbb{R} \longrightarrow [0, +\infty)$ of the residuals that depend from $\boldsymbol{\theta}$. For the LS estimator ρ is $\rho(r_i) = r_i^2$ while for M-estimators there are many possible different choices [10]. The design of the ρ function for M estimators is usually performed aiming at achieving robustness to outliers resulting in cost functions that, in general, do not admit a closed form solution to the minimization problem as for the LS case.

The LMS estimator [17], instead, is defined through a cost function that differs in nature from the structure reported in Eq. (5); namely the LMS estimator results in

$$\hat{\boldsymbol{\theta}}_{LMS} = \arg\min_{\boldsymbol{\theta}} \text{med}_i \left\{r_i^2\right\} \tag{6}$$

where $\text{med}_i \left\{r_i^2\right\}$ is the median of the squared residuals. This estimator has been shown to exhibit very strong robustness to outliers [17] having, by example, the maximum possible breakdown point (50%). Nevertheless, the LMS estimator cannot be computed in closed form [19] [20] and is thus of limited applicability especially in real-time scenarios. For a qualitative understanding of the robustness of the LMS estimator notice that in case of linear regression problems of the form

$$y_i = \theta_1 x_i + \theta_2, \tag{7}$$

the LMS line corresponds to the center of the stripe in the x, y plane containing half plus one of the data points. Intuitively, it appears that minimizing a cost function as the one in Eq. (6) achieves higher robustness to outliers then cost functions as in (5), because the median of the squared residuals gives a "global" measure of the scatter of the residuals. M or LS cost functions, to the contrary, are not directly related to the distribution of the resulting residuals. Indeed, the by now classical example of F. J. Anscombe [1] explicitly shows how the same LS parameter estimates and residuals can be obtained for very different data sets. Other outlier robust model coefficient estimation techniques as Random Sample and Consensus (RANSAC, [8]) can be casted among the so-called "voting" techniques where a sufficiently large consensus set of data points in agreement with a specific value of the model coefficients is sought for. These methods may be indeed effective, but tend to be computationally demanding.

The proposed Least Entropy-Like (LEL) estimator is designed with the twofold objective of obtaining an estimator that directly relates to the distribution of residuals (in order to achieve high robustness to outliers) while also being quickly computable from a numerical view point. The LEL estimator was first presented in [11]: given the residual r_i in Eq. (2), define:

$$D = \sum_{j=1}^{n} r_j^2, \tag{8}$$

namely the LS estimation cost. Then define the *relative squared residuals* q_i as

$$\text{if } D \neq 0 \implies q_i := \frac{r_i^2}{\sum_{j=1}^{n} r_j^2} \quad : \quad q_i \in [0,1] \text{ and } \sum_{i=1}^{n} q_i = 1, \tag{9}$$

and finally

$$H = \begin{cases} 0 & \text{if } D = 0 \\ -\frac{1}{\log n} \sum_{i=1}^{n} q_i \log q_i & \text{otherwise.} \end{cases} \tag{10}$$

The function H in equation (10) enjoys all the mathematical properties of a normalized *entropy* [6] associated to the sequence of "probability" - like $q_i \ : \ i = 1, 2, \ldots, n$. In particular:

$$H \in [0,1] \tag{11}$$

$$H = 0 \quad \text{if and only if} \quad \begin{cases} r_i = 0 \ \ \forall \ \ i \in [1,n] \\ \text{or} \\ \exists \ ! \ \ i^* : r_{i^*} \neq 0 \text{ and } r_i = 0 \ \ \forall \ \ i \neq i^* \end{cases} \tag{12}$$

$$H = 1 \quad \text{if and only if} \quad r_i^2 = r_j^2 \neq 0 \ \ \forall \ \ i, j \in [1,n]. \tag{13}$$

Indeed, the above is formally equivalent to the Entropy of Information as introduced by Shannon in the 1948 [18] in analogy with the concept that was already known in thermodynamic and mechanical statistics, where Clausius and Boltzamann gave the first functional expression of the entropy, as a measure of the degree of disorder in a thermodynamic system.

Shannon, in particular, defined the entropy of information as a propriety associated to any probability distribution, while, the so-called *experimental entropy* used in thermodynamic is a property of real physic measurements.

Letting X be a random variable with possible outcome x_i (i=1,...,n), its mass probability p_i such that $\sum_{i=1}^{n} p_i = 1$ identifies a global uncertainty measure [18] through the function:

$$H(P) = -\sum_{i=1}^{n} p_i \ln p_i \tag{14}$$

exhibiting the following properties:

- *H(P)* is concave.
- It is equal to zero (perfect certainty) when one of the probabilities is exactly 1.
- It reaches a maximum for uniform probabilities (complete ignorance): $p_1 = p_2 = \ldots = p_n = 1/n$.
- The entropy *H(P)* is a function of the probability distribution and not a function of the actual values taken by the random variable.

With respect to the above properties of entropy, the only possible difference of the entropy-like cost function defined in Eq. (10) is related to the possible singularity $D = 0$. Notice that this would correspond to a perfect LS fit that is quite unlikely. Indeed, there is no practical limitation as prior to computing H in (10) one can always check if the LS fit is perfect. In such case, there is of course no need to compute any other estimate of the parameters. Also notice that for null values of q_i the terms $0 \log 0 = \log 0^0$ in equation (10) are zero.

In words, it can be stated that when the relative squared residuals q_i are properly defined (i.e. $D \neq 0$), the H function is a measure of their spread. When they are not properly defined, it is simply because the residuals are all identically null which corresponds to a null value of H exactly as in the case when all the residuals are zero except one. In Physics, the (Gibbs) entropy of a system admitting n discrete states with probabilities $p_1, p_2, \ldots, p_n$ is computed as $-\sum_{i=1}^{n} p_i \log p_i$. It is a very well-known fact that such function is a very sensitive measure of the distribution of the probabilities. Configurations with only a fraction of highly probable states have a much lower entropy of configurations where most states are approximately equally probable. Motivated by this fact, the function H is defined with the aim of computing a robust estimate of the model parameter vector θ. In particular, given that the entropy-like function H as defined by Eq. (8) depends on $\boldsymbol{\theta}$ through the residuals r_i (equation (2)), the following estimator is proposed:

$$\hat{\boldsymbol{\theta}}_{LEL} := \arg\min_{\boldsymbol{\theta}} H \tag{15}$$

where LEL stands for Least Entropy-Like. Such name was chosen with the twofold objective (i) of underlining that the H function is not properly an entropy and (ii) of avoiding confusion with the Minimum Entropy estimation approach described, by example, in [21][23]. The idea behind the $\hat{\boldsymbol{\theta}}_{LEL}$ estimator defined in (15) is that such estimate will correspond either to making all the residuals null, or to making the relative squared residuals as little equally distributed as possible according to the entropy-like function H, the available data and the model structure. Notice that due to the normalization of the relative squared residuals q_i in (9), forcing them to be "as little equally distributed as possible" means that "most" residual r_i will need to be "small" (with respect to the normalization constant, namely the Least Squares cost D) and "a few" of the residuals

r_i will need to be "large". Data points corresponding to these "large" residuals are candidate outliers. Stated differently, the key to robustness with respect to outliers is related to the fact that the devised penalty function does not directly measure the (weighted) mean square error (that as known tends to level out or "low pass" residuals), but rather the distribution of the relative squared errors. In particular, the devised LEL method tends to enforce a positive skewness to the distribution of the squared relative errors according to a metric give by the entropy-like function H in Eq. (10).

Notice that, in general, there is no guarantee for the H function to have a unique minima with respect to $\boldsymbol{\theta}$. Indeed, the entropy-like penalty function H is highly nonlinear and may have many local minima. The minimization of H needs to be carried out numerically paying attention to the initialization of $\boldsymbol{\theta}$: indeed the proposed estimator should be regarded as a local in nature. The gradient and Hessian matrix of the LEL cost functions can be analytically computed in closed form to aid numerical minimization routines. In case of models that are linear in the parameters as:

$$\mathbf{y} = \mathbf{X}\boldsymbol{\theta} + \boldsymbol{\varepsilon} \tag{16}$$

being $\mathbf{y} \in \mathbb{R}^{n\times 1}$ the $n-$dimensional measurement vector, $\mathbf{X} \in \mathbb{R}^{n\times m}$ the regression matrix, $\boldsymbol{\theta} \in \mathbb{R}^{m\times 1}$ the parameter vector and $\boldsymbol{\varepsilon} \in \mathbb{R}^{n\times 1}$ the measurement noise vector, it can be shown that the gradient of the H cost is always well defined and the elements of the Hessian matrix result eventually ill posed (i.e. infinite) if and only if a residual $r_{i^*} = 0$ for some i^*. This is a highly unlikely situation in practice and even if it should occur an approximation of the Hessian can be computed through a regularization technique based on replacing $r_{i^*} = 0$ with $r_{i^*} = \delta$ for a sufficiently small δ. The gradient and Hessian values of H for the linear in the parameter model (16) have been explicitly computed, but are not here reported for the sake of brevity. Their closed form expressions are used to numerically compute the (local) minimum of H in the case studies described in the paper. For a deeper discussion about the properties of the LEL estimator refer to [11].

For a qualitative and intuitive understanding of the proposed method, the LEL estimates of the of Anscombe data sets are reported in Fig. (1). Anscombe's data sets (or Anscombe's quartet) are four artificial data sets proposed in 1973 by Francis Anscombe [1] to illustrate the importance of graphs and plots in the interpretation of statistical analysis. Each data set is made of 11 (x,y) points in a plane: the plot of the four data sets immediately and intuitively reveals the different structure of the four sets. Yet, if a line $y = \theta_1 x + \theta_0$ is fitted to the data through OLS, the same model coefficients $\hat{\theta}_{LS_1} = 0.5$ and $\hat{\theta}_{LS_0} = 3$ are found for all four data sets. Moreover, the y residual sum of squares of the four data sets is the same revealing the difficulty in analyzing the fitting results in the presence of outliers or of a mismatching model. The LEL estimate $\hat{\theta}_{LEL_1}$ and $\hat{\theta}_{LEL_0}$ of the line $y = \theta_1 x + \theta_0$ coefficients of Anscombe's data sets are expected to generate residuals that are smaller than the OLS residuals for the majority of the points. As illustrated in Fig. (1) and Fig. (2) this is indeed the case for the first three data sets. In the third case where 10 out of the 11 data points fit the model, the LEL method perfectly succeeds in fitting the 10 inliers, whereas the OLS estimated is biased. The fourth is a singular case, as the "real" θ_1 for these points should be infinite and both OLS and LEL, of course, fail. The results plotted in in Fig. (1) and Fig. (2) were obtained by numerically minimizing the LEL cost function starting from the OLS solution.

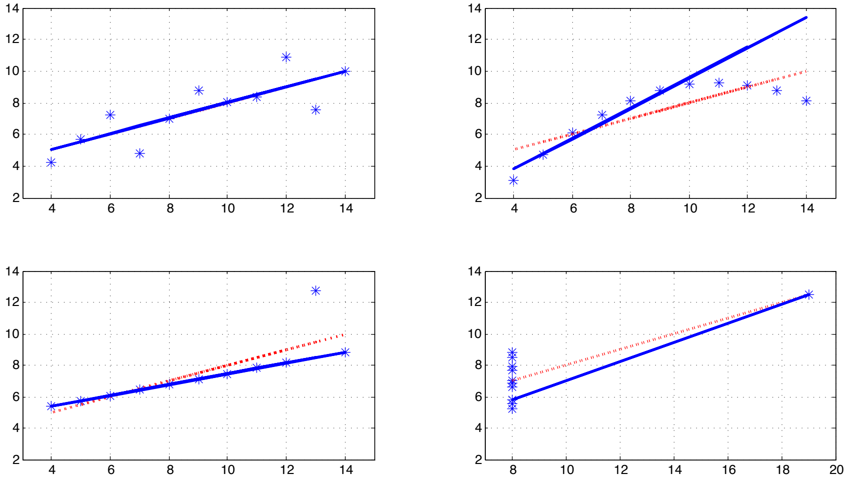

Fig. 1. OLS (red dotted lines) and LEL estimates (blue solid line) for Anscombe's Quarted [1] data sets (blue $*$ points, starting from the top left plot in clockwise direction: sets I, II, III and IV). Notice that in the lower left case the LEL estimator yields the perfect solution, namely the line of the 10 inliers. Also notice that in the bottom right case the regression model is singular hence both the LEL and OLS methods fail.

3 Generalized Maximum Entropy

Building on the concept of information entropy introduced by Claude Shannon [18], the Maximum Entropy Principle (MEP) was firstly proposed by Edwin T. Jaynes [12] [13] defining an objective method for estimating probability distributions in case of limited data. A generalization of the MEP is given to the contribution of Amos Golan *et al.* [9], that proposed an alternative method for parameters estimation called Generalized Maximum Entropy (GME), as an extension of the MEP.

3.1 Shannon's Entropy Measure and the Maximum Entropy Principle

Edwin Jaynes, building on the Shannon's Entropy function in Eq. (14), proposed the *Maximum Entropy Principle* (MEP) [12], [13] to estimate the probability distributions in presence of constraints generated from the data, and given in the form of expectations. Under MEP, the probability distribution is chosen among those distributions consistent with known information (the constraints), that maximizes the entropy. The MEP can be used to solve pure inverse problems defined as follows:

$$\mathbf{y} = \mathbf{X}\mathbf{p} \tag{17}$$

where $\mathbf{y} \in \mathbb{R}^{n\times 1}$, $\mathbf{X} \in \mathbb{R}^{n\times m}$, and $\mathbf{p} \in \mathbb{R}^{m\times 1}$ even for $n < m$. To recover the unknown probability $\mathbf{p}$ vector, the MEP suggests to maximize the *H(P)* function (14) subjected

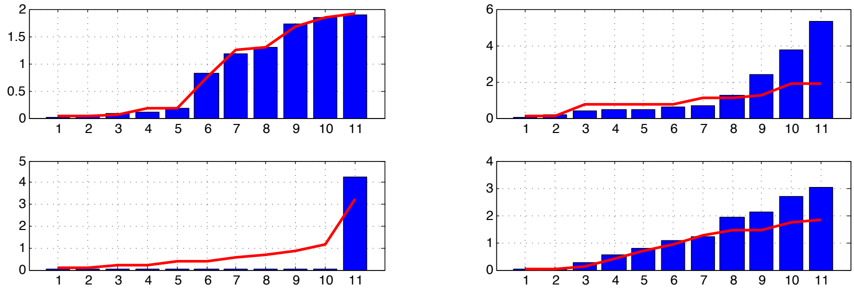

Fig. 2. Sorted absolute value of the residuals for the LEL estimate (bar plot in blue) and the OLS estimate (solid red line) for Anscombe's quartet in figure (1). Cases II and III, in particular, reveal the enhanced outlier robustness of the LEL approach as compared to OLS.

to data consistency and normalization constraints. The data consistency constraint is defined by the Eq. (17). The normalization constraints imply that: $\mathbf{p}^T\mathbf{1} = 1$ being $\mathbf{1}$ an m-dimensional column vector having all components equal to one.

Using Lagranges method we can carry out the analytical solution to the entropy maximization problem as follows:

$$L = -\mathbf{p}^T \ln \mathbf{p} + \boldsymbol{\lambda}^T(\mathbf{y} - \mathbf{X}\mathbf{p}) + \mu(1 - \mathbf{p}^T\mathbf{1}) \tag{18}$$

and the corresponding first-order conditions are:

$$\partial L/\partial \mathbf{p} = -\ln \mathbf{p} - 1 - \mathbf{X}^T\boldsymbol{\lambda} - \mu = 0$$

$$\partial L/\partial \boldsymbol{\lambda} = \mathbf{y} - \mathbf{X}\mathbf{p} = 0$$

$$\partial L/\partial \mu = 1 - \mathbf{p}^T\mathbf{1} = 0$$

The solution of the above conditions will lead to the following estimated value of $\mathbf{p}$:

$$\hat{\mathbf{p}} = \exp(-\mathbf{X}^T\hat{\boldsymbol{\lambda}}) / \sum_j \exp(-\mathbf{X}^T\hat{\boldsymbol{\lambda}}) \tag{19}$$

where $\Omega(\hat{\boldsymbol{\lambda}}) = \sum_j \exp(-\mathbf{X}^T\hat{\boldsymbol{\lambda}})$ is the normalization factor, known also as the partition function, that transforms the relative probabilities into absolute probabilities. The solution (19) can be applied to solve ill-posed problems, as the classical example of the Jaynes's dice experiment: in this case the unknowns are represented by the six unknown probabilities of the dice faces, the term $\mathbf{X}$ in Eq. (17) is determined by the dice model (i.e. $\mathbf{X} = (1,2,3,4,5,6)$) and the a priori knowledge is represented by the average of the outcome, namely 3.5 in case of fair dice. This is an ill-posed problem with one observation and six unknowns, which can be solved by maximizing the constrained *H(P)* function (14).

In case the observed moments are noisy, for instance coming from a sampling experiment, the consistency constraint became stochastic and model (17) can be modified by adding an error term:

$$\mathbf{y} = \mathbf{X}\mathbf{p} + \boldsymbol{\varepsilon} \tag{20}$$

The idea behind Eq. (20) is that the term $\boldsymbol{\varepsilon}$ (in general not equal to zero as in equation (17)) allows to model stochastic moments in $\mathbf{y}$. Consequently, the samples moments are allowed (but not forced) to be different from the underlying population moments, a flexibility that seems natural for finite data sets. Further details are illustrated in the following section adressing the regression model.

3.2 GME Regression Model

The GME estimator is consistent and asymptotically normal under some regularity conditions. The idea underlying the GME estimator consists in viewing the parameters and the error vectors as convex linear combinations of some known discrete support values and unknown proportions to be interpreted as probabilities. Considering a regression model with n observations and m variables:

$$\mathbf{y} = \mathbf{X}\boldsymbol{\theta} + \boldsymbol{\varepsilon} \tag{21}$$

in order to use the MEP method to estimate the regression parameters, the coefficients and the error terms are re-parameterized as a convex combination of expected values of discrete random variables.

Given a parameter $\boldsymbol{\theta}_j$, it is always possible to write it [15] as a convex combination of a support variable as: $\boldsymbol{\theta}_j = \mathbf{z}_j^T \mathbf{p}_j$, where $\mathbf{z}_j^T = [z_{j1}, ..., 0, ..., z_{jM}]$ defines the lower and upper bounds of the j^{th} parameter, with M usually [9] in the interval $2 \leq M \leq 7$. The vector $\mathbf{p}_j^T = [p_{j1}, ..., p_{jM}]$ contains positive probabilities that sum to one.

Similarly, each error term is treated as a discrete random variable: $\boldsymbol{\varepsilon}_i = \mathbf{v}_i^T \mathbf{w}_i$, where $\mathbf{v}_i^T = [v_{i1}, ..., 0, ..., v_{iN}]$ defines the error bound, with M usually [9] in the interval $2 \leq N \leq 7$. The vector $\mathbf{w}_i^T = [w_{i1}, ..., w_{iN}]$ contains positive probabilities that sum to one.

The GME method, therefore, estimates the regression coefficients and the error terms, by recovering the probability distribution of a discrete random variables set. The model (21) can be rewritten as follows:

$$\mathbf{y} = \mathbf{X}\left(\mathbf{I}_{m\times m} \bigotimes \mathbf{z}^T\right)\mathbf{p} + \left(\mathbf{I}_{n\times n} \bigotimes \mathbf{v}^T\right)\mathbf{w} \tag{22}$$

where $\mathbf{I}_{m\times m}$ and $\mathbf{I}_{n\times n}$ are the identity matrices for the parameters and the error terms and the symbol $\bigotimes$ is the Kronecker product.

The supervectors $\mathbf{p}$ and $\mathbf{w}$ contain respectively m and n probabilities vectors, related at each support variable $\mathbf{z}_j$ and $\mathbf{v}_i$. The aim is to estimate the probabilities vectors $\mathbf{p}_j\{j = 1, \ldots, m\}$ and $\mathbf{w}_i\{i = 1, \ldots, n\}$, associated respectively to the $\boldsymbol{\theta}$ and $\boldsymbol{\varepsilon}$ parameters. The estimation is made by the maximization of the Shannon's entropy function:

$$H(\mathbf{p}, \mathbf{w}) = -\mathbf{p}^T \ln(\mathbf{p}) - \mathbf{w}^T \ln(\mathbf{w}) \tag{23}$$

subjected to the *consistency constraints*, that represent a part of the regression model (22), and *normalization constraints*, that means, the element of the probabilities vectors $\mathbf{p}$ and $\mathbf{w}$, have to satisfy respectively the conditions of containing positive probabilities that sum to one.

The optimization problem is obtained via definition of the Lagrangian function, which can be easily solved in the same fashion we reported for the MEP case. The GME has advantages to address some circumstances, as for instance ill-behaved data or no distributional error assumptions ([4], [5], [9]).

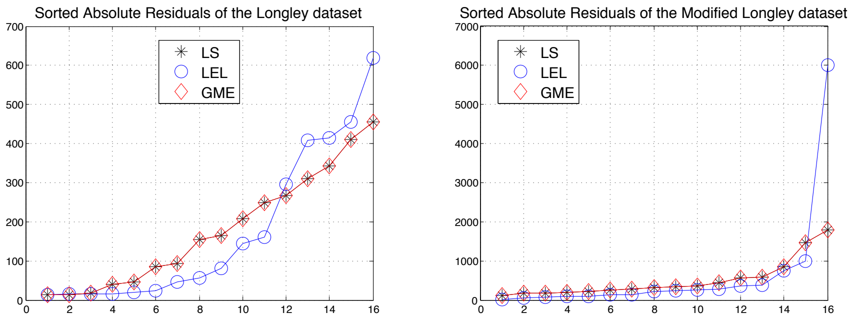

Fig. 3. Sorted Absolute Residuals of the Longley dataset (left) and of the Modified (outlier case) Longley dataset (right). Refer to the text for details.

4 Validation Study: The Longley Data Set Case

In order to compare the proposed entropy, based methods, we have considered the Longley Data Set [14] available through the Internet on the NIST - National Institute of Standards and Technology's website. According to NIST the Linear Least Squares Regression problem on this data set has a "Higher Level of Difficulty". Moreover it is reported that *this classic dataset of labor statistics was one of the first used to test the accuracy of least squares computations. The response variable (y) is the Total Derived Employment and the predictor variables are GNP Implicit Price Deflator with Year 1954 = 100 (x1), Gross National Product (x2), Unemployment (x3), Size of Armed Forces (x4), Non-Institutional Population Age 14 & Over (x5), and Year (x6).*

The difficulty in processing with OLS this data set is basically related to the large condition number ($\kappa = 4.8593E+09$) of the associated regression matrix. This is a typical situation (close to perfect multicollinearity) where the GME approach is particularly useful. The sorted absolute values of the residuals obtained by the GME, LEL, and OLS estimation approaches are depicted in the left plot of Fig. 3. Notice that the GME and OLS solutions are in perfect agreement (in spite of the fact that the OLS estimate is computed by inverting a matrix that is very close to being singular). To the contrary, the LEL estimate yields a different solution. In particular, the LEL Score (i.e. the percentage of LEL residuals in absolute value being smaller than the OLS residuals in absolute value) is of 56.25%. In order to illustrate the effectiveness of the LEL approach to compute robust estimates in the presence of outliers, the Longley data set has been modified by replacing the last y (explanatory variable) value with another y value, in particular imposing $y(16) = y(12)$. All the other data values are left identical. This new data set will be referred to as the "Modified Longley Dataset". The sorted absolute values of the residuals obtained by the GME, LEL and OLS estimation approaches on the Modified Longley Data set are depicted in the right plot of Fig. 3: remarkably, the LEL Score in this case results in 93.75% confirming the robustness of the LEL solution to outliers. Moreover, notice that the largest residual (in absolute value) in this case

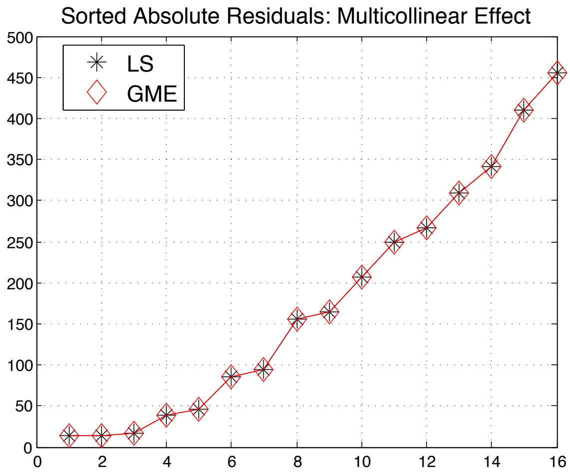

Fig. 4. Multicollinear Effect. Sorted Absolute Residuals of the GME and OLS approaches on a multicollinear modified data set and on the original data set respectively.

corresponds precisely to $y(16)$. Also notice that, once again, the GME and OLS solutions for the Modified Longley Data set are in perfect agreement.

At last, in order to confirm the ability of the GME approach to cope with multicollinearity, the regression matrix of the (original) Longley data set is modified by repeating one of its columns. This yields a singular regression matrix on which both the OLS and LEL algorithms cannot be applied. To the contrary, the GME solution can be computed without numerical issues and the obtained residuals are perfectly equivalent to the OLS residuals obtained with the original (non modified) data set. This is confirmed in the plot of Fig. 4.

5 Conclusion and Discussion

The objective of this paper was to describe two instances of entropy-based estimators illustrating their properties and their performances as compared with more standard methods as the OLS. The described methods consist in the Least Entropy-like LEL and the Generalized Maximum Entropy estimators. The first is particularly useful for model coefficient estimation in the presence of outliers, whereas the second is robust to multicollinearity. Moreover, the LEL solution, although local in nature and hence potentially sensitive to its initialization, is computationally much less demanding than alternative outlier robust approaches as LMS [17] or RANSAC [8]. In order to illustrate the specific characteristics of the two approaches, both methods have been applied to the Longley Data Set [14]. The results confirm the expected outlier robustness properties of LEL and the multicollinearity robustness of GME. The integration of the potentialities of both estimators is one future objective of our research plan.

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The MAS Models Use – An Imperative Approach to Build a New Economic Paradigm

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Abstract. Comparing with other sciences, the quantitative success of economic science is disappointing. People fly to the moon, the touch of a simple button can destroy more than half of the world and energy is extracted with the speed of light. During all these developments of science the economic achievements are somewhere near zero. The economists can praise however with the recurrent inability to predict and revaluate the crises and with the ability to create financial innovations. This study aims to present the benefits of the multi agent (MAS) models in the economy. In order to do that this paper intends to demonstrate the need of such models today. Furthermore are presented some attempts of using multi agent models and concluding with the idea that the economy needs a scientific revolution, and this can be done using multi criteria or multi agent models.

Keywords: multi agent model, heterogeneous agent model, financial crises, Pareto equilibrium.

1 Introduction

Quoting Isaac Newton *"I can calculate the motion of heavenly bodies but not the madness of people"* it is obvious that economic agents aren't rational and the market isn't as efficient as is thought to be. And nevertheless the above concepts are rooted in people's conception worse than axioms, and successfully replace several empirical demonstrations. The perfect efficacy of a free market dates from 1950s, 1960s. Now, the reality is different, and governments, economists, agents, retailers, traders, scientists should take into consideration that market isn't efficient, people think their business in the short term and become blind or totally disinterested in the long term, errors are amplified day after day and the collective irrationality conduct to panic, crashes, and crises.

Financial or economic crises don't occur unpremeditated — or "exogenously," as governments want us to believe. The economists feel when crises will be made present. Like a tsunami, crises are predicted and in many cases the same factors that led to it, also helped finding solutions of it. And even if they are old as the world is, crises are still raging and take by surprise the government and some economists.

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2 The Urgent Need of Multi Agent Models

Today, when the entire world is ruled by the high technology, everyone will assume that those who govern the world economy, such as IMF, WB, and its international counterparts are using sophisticated multi agent quantities models to guide us out of the current economic crisis or even better to predict it. Well, they aren't. Most of the times they based on what history tells, using in this way two types of models which as we can be seen have fatal flows. The first type of model is an econometric one using statistic data from the past, in order to predict the future. But even if the crises are repeating, the world is different from The 1907 Banker's Panic, The Great Depression, The oil crisis, The Asian, or Argentinean crises. We learn more or less from the past, but we fail if we rely only on it in the face of great change. The Black–Scholes model [1] invented in 1973 for evaluating financial options is still used with indicating than there are possible but extreme price movements. More than 25 years ago the use of this formula has been known as October 1987 crash, when Dow Jones index dropped 23% in a single day.

The second type of model is a "dynamic stochastic" one which asserts a very competitive equilibrium of a perfect world. Besides the fact that reality is far from being perfect, the nature rule of crises is to throw off an economy, in order to reposition it on a healthy level. In time of crises all regulations from before are no longer valid.

A long period of time it was thought that financial crises are best to be avoided. F. Allen [2] instead has denied this wisdom, by indicating that under particular conditions a laisser-faire financial system achieves the incentive-efficient or constrained-efficient allocation. Starting from this, the efficiency of a market may require financial crises in equilibrium, which is a utopia. How it is possible to define economic efficiency through tradeoffs? That is, at equilibrium, one side person or criterion can gain only if the other loses. Such zero sum thinking lies at the core of financial crises. The assumption involves Pareto efficiency of Walrasian equilibrium. But like J. Stiglitz said in a Pareto efficient competitive equilibrium, no one can gain from reallocation of resources without someone else to lose, or in time of crises we must think the economy like everyone to survive, and well informed. [3] On the contrary, economies with defective information are in general, not Pareto efficient and from this point of view we can argue that the principle known in the specialist literature as Pareto's Law isn't valid in time of crises. Pareto's Law is valid but only in the period before crises. [4]

The commonly known form of the Pareto law or principle is the 80-20 ratio which states that: 20% of clients account for 80% of sales; 20% of components represent 80% of costs; 20% of the developed nations' control 80% of global wealth, etc. In other words it is demonstrated, by empirical studies, the fact that 80% of effects that crises have on the entire glob are due to 20% of causes. [5]

To illustrate why the global economy needs multi agent-based modelling in order to get out of crises we will take for example only one cause which induced the recession: the inability to evaluate the risk of the adjustable-rate subprime and other mortgages, which were packed into mortgage-backed securities of great complexity.

The rating agencies didn't have effective models to estimate this risk either because of a lack of professionalism, or poor accountability but most probably an intrinsic difficulty in evaluating risk because of the complexity of the problem. This risk is present in the balance sheets of financial institutions that represented the roots of the financial crisis. What a complex model can do is to identify what banks were holding them 5 or 10 years ago, and what banks are holding them now. Now is hard to believe that economists from Wall Street aren't using fancy mathematical models. They do use these models, mainly to model the potential profit and risk of individual trades. Nevertheless their interest in systemic aggregate economic analyze is low.

Emphasizing the idea that the crises must be avoided or prevented by inventing new models is out of this question but they can do contribute at a better understanding of cumulative situations, impacts, and consequences. An agent-based model is a computer simulation of more companies and institutions, which interact with each other through certain, predetermined rules. [6] The decision makers or the agents can be as diverse as they need from individual consumers to policy makers or economist from Wall Street. Such models aren't based on the past trends of the economy or on the supposition that the economy must move toward a predetermined equilibrium state. In return, each agent of those previously mentioned acts according to its own and current situation, the state of the economy, rules that govern its behaviour or his current optimism about the future. The role of the multi agent model is to keep track of interactions of all agents in the economy over time, in order to analyze what will happen and what is their nonlinear behaviour – fact that equilibrium models can't. The multi agent model can simulate an artificial economy considering the way of working of each agent and can simulate various scenarios quantitative and qualitative. The notion of complex economy is available since Adam Smith in the late of 1700s.

Nevertheless, to heal the economy after an economic crisis of such proportion can be harder than the disease itself and even if rational expectations are a reasonable model of human behaviour, the mathematical machinery is overloaded and needs sharp simplifications in order to achieve good results.

An agent-based model instead analyzes the financial economy as a complex system which takes into account human adaptation and learning. It will be created in this way a virtual complex system of acting, in order to help policy makers.

From the few agent-based models of the economy in this article we will present the agent-based financial market model built by Blake LeBaron, [7] the dynamic model that simulates how companies increase and decrease of Robert Axtell, the interactive agent models for understanding monetary economies Mauro Gallegati's and Joseph E. Stiglitz, [8] and the monetary model developed by Robert Clower and Peter Howitt. These models analyze only small portions of the economy but their build is a first step to a global multi agent model.

3 The Agent-Based Financial Market of Blake LeBaron

The agent-based financial market is a new agent based market of Blake LeBaron [7] similar somehow with the Santa Fe artificial stock market but with a lot of differences regarding the plasticity of the construction and its use in the real world. In LeBaron market [7], rules and agents are evolved and compete with each other in their trading activities. The richest agent will survive. Decision making in the various processes is so designed to reproduce the real world heterogeneity, using data from 25 years ago.[9] This transpose of information from the past is strongly correlated to the constant gain learning algorithms. [10]

The market is a partial equilibrium model with two securities, a risk-free asset in infinite supply paying a constant interest rate r_f and a risky security paying investors a random dividend each period.

$$I = \sum_{i=1}^{I} s_i$$

Where I is the number of agents and s_i is the share holding period of agent *i*. The log dividend follows a random walk, with an annualized growth rate of 2% and an annual standard deviation of 6%. [7]

Agents are defined by:

$$u_{i,t} = \mathrm{E}_t \sum_{s=0}^{\infty} (\beta\text{^}\delta \log c_{i,t+s})$$

The time rate of discount β is set to $(0, 95)^{1/12}$ which corresponds to 0.95 annual rate.

The strategy recommends a fraction of savings $\beta w_{i,t}$ to invest in the risky asset as a function of current information z_t. The objective is to:

$$\max_{\alpha j} E_t \ \log[\,1 + \ \alpha_j \, r_{t+1} + \ (1 - \alpha_j\,) r_f]$$

Where α_j is the set of available trading rules. All the agents are using in the current period rules based on their performance from the past using:

$$\max_j E(r_p\,) = \frac{1}{\mathrm{Ti}} \sum_{\mathrm{k}=1}^{\mathrm{Ti}} \log[\,1 + \ \alpha\,(\,\mathrm{z}_{t-k}; w_j) r_{t-k+1} + \left(1 - \alpha(\,\mathrm{z}_{t-k}; w_j)\right) r_f].$$

Where the only heterogeneity across them is their memory *Ti.* Trading is conducted by finding the aggregate demand *(D (pt))* for shares and setting it at the same value with the fixed aggregate of everyone part at a specific time *t:*

$$s_{i,t}(p_t) = \frac{\alpha_i\,(p_t; I_t)\beta w_{i,t}}{p_t} \quad with \;\; w_{i,t} = (p_t + d_t) s_{i,t-1} + (1 + r_f) b_{i,t-1}$$

Where $w_{i,t}$ is the total wealth of the agent *i* and $b_{i,t-1}$ are the bond holding from the previous period. The amount of all these demands gives us the final function of the aggregate demand:

$$D(p_t) = \sum_{i-1}^{I} s_{i,t}(p_t)$$

When $D(p_t) = 1$, the price p_t will be at the equilibrium value. It is important that once $p_t$is revealed the agent i to change his trading rule to a different one and the equilibrium to become temporary.

The result presented by LeBaron[7] after verifying the model on computer experiments shows that an agent-based model is capable of quantitatively replicating many features of actual financial market. The tests are presenting favourable results and generate cross-correlations between volume and volatility along with the leverage asymmetry that matched features of the real data. It was also noted that in places where market seems to be weak and with a high consumption this give a high volatility of the price of the assets. On the other hand, the model matches a lot of characteristics with relative ease that more traditional models don't even consider.[11]

4 The Heterogeneous Agent Model of Stiglitz and Galleti

The heterogeneous agent model of Stiglitz and Galleti [8] offers an alternative in helping economic agents to understand the interconnections that can provide economic growth in time of crisis. The model is able to present endogenous large-scale economic fluctuations by means that make possible the interaction of money demand in the economy and the heterogeneous agents. The model begins with the banal assumption that in the economy there can't be unemployment or liquidity crises. The economic theory based on representative agent framework has in short nothing to say about financial crises and bankruptcies. Any complaint that may be done with regard to the market efficiency is suspected since the beginning, it is the result of extreme hypotheses on which is based the model. [12]

MAS models' elements can be representations of any actors of the market [14]. This model will consider heterogeneous only households, while banks are considered homogeneous. The whole economy is made from *N* companies. Each company produces the same good, with the same production function.[13]

$$Y_{i,t} = K_{i,t}^{\alpha} N_{i,t}^{\beta} \qquad\qquad i = 1, \dots, N$$

Where $Y_{i,t}$ is the output at time *t*, $K_{i,t}$ is the stock of capital employed, and $N_{i,t}$ represents the employment at the particular time *t*. $\alpha, \beta \in (0,1)$.

The production decision of each company at the particular time *t* aims to maximize the profit defined as follows:

$$\prod_{i,t} = p_t K_{i,t}^{\alpha} N_{i,t}^{\beta} - w_t N_{i,t} - p_t\,(r_t - \pi_{i,t}) K_{i,t} \quad [13]$$

Where p is the price for each produced good, r_t is the instantaneous rate of interest on debt securities, and $\pi_{i,t}$ is the rate of increase in the price of capital. $r_t - \pi_{i,t}$ can be analyzed as the real interest rate perceived by firm i.

With the capital stock K_i known, the firm's unemployment N_i is described by the first-order condition for maximizing the equation above:

$$\frac{\partial \pi}{\partial N_i} = 0$$

Which give us the expression for the employment$N_{i,t}$:

$$N_{i,t} = (\beta \frac{p_t}{w_t})^{1/(1-\beta)} K_{i,t}^{\alpha/(1-\beta)}$$

The performance of each company at time *t* will be:

$$Y_{i,t} = (\beta \frac{p_t}{w_t})^{\beta/(1-\beta)} K_{i,t}^{\alpha/(1-\beta)}$$

The investment decision of each firm will be the described by the condition of maximization of the economic profit regarding both $N_i$and K_i:

$$\left.\begin{array}{l} \frac{\partial \pi}{\partial N_i} = 0 \\ \\ \frac{\partial \pi}{\partial K_i} = 0 \end{array}\right\} \longrightarrow K_{i,t}^* = (\beta \frac{p_t}{w_t})^{-\beta(\alpha+\beta-1)} (\frac{r_t - \pi_{i,t}}{\alpha})^{(1-\beta)/(\alpha+\beta-1)}$$

If $K_{i,t}^* > K_{i,t}$, than the company *i* is planning an investment $I_{i,t}^*$. Because usually the investments of companies are funded through liabilities, the aggregate effective investment $Z_{I,t} = \sum_i I_{i,t}$ depends on the size of loan L_t that the bank wants to offer at a specific time *t* as follows:

$$Z_{i,t} = min(Z_{i,t}^*; \frac{L_t}{p_t}) <=> Z_{i,t} = min(\sum_i I_{i,t}^*; \frac{L_t}{p_t})$$

The conclusion of the model is that this model comprehends the possibility of analyzing of complex interactions between representative agents. The model doesn't intend to present that the agents act specifically in an economy. It presents an alternative which enable economic growth during the crisis [15].

5 Conclusions

The economic models based on multi agent approach are able to provide an alternative way of how the government policies affect the economic performance. These models are able to make useful forecasts of the real whole economy, not only certain parts of it.

Innovations in financial products must be analyzed in extreme scenarios and approved only when it is known from certain what effects do they have and how they can be countered.

Even if it is necessary a multidisciplinary collaboration between economists and scientists and a great amount of money use of the multi agent model will be a start of a new successful era, and a first step to the new scientific revolution. The state of mind of those who work in financial and economic domains needs to change.

Long-term stability and development should represent the pragmatic and realistic objectives for any economy. Equations and aesthetic axioms must be replaced with multi-criteria analysis of dates. Population cannot be fed only with economic indicators that provide fake growth and bankruptcies.

Now more than ever the economy needs a scientific revolution to get it out of the endless recession of which effects are surrounding us with every act of business, with any decision making, with any day passing.

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Intuitionistic Fuzzy Preference Relations and Hypergroups

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Abstract. In this paper we present a connection between intuitionistic fuzzy relations and hypergroups. In particular, we construct a hypergroup associated with a binary relation naturally induced by an intuitionistic fuzzy relation. We present some of its properties, investigating when it is a join space or a reduced hypergroup, in the framework of the intuitionistic fuzzy preference relations.

Keywords: Hypergroup, Join space, Intuitionistic fuzzy set, Intuitionistic fuzzy preference relation.

MSC2010: 20N20, 03E72, 90B50.

1 Introduction

In the real life, a lot of problems takes place in an environment in which the goals, the costraints, and the consequences of possible actions are not precisely known (Bellman and Zadeh [3]). The concept of intuitionistic fuzzy set (IFS) introduced by Atanassov [1,2] is one of the mathematical tools higly used to deal with imprecision, vagueness, uncertainty in diverse areas as Computer Science, Social Science, Decision Making, Management Science, Neural Networks, Medicine, Engineering, etc. The application of IFS theory in Decision Making, for example, is very useful to overcome and model the ambiguity generated by diverse factors: a decision maker may not posses a precise or sufficient level of knowledge of the problem, or is unable to discriminate the degree to which one alternative is better than others; it could also happen that the decision maker provides the degree of preference for alternatives, without being sure about it [21,32]. An updated review of the role of IFS theory in decision-making problems, supplemented with a rich bibliography, is presented in [21].

Correspondences between objects are suitable described by relations, that can be crisp or fuzzy. Remaining in the decision-making area, the most frequently used and thus investigated type of relation is that of preference relation, for the first time generalized from the fuzzy case to the intuitionistic fuzzy one by Szimdt and Kacprzyk [32]. A preference relation P on a discrete finite set X of alternatives is characterized by a function $\mu_P : X \times X \longrightarrow D$, where D is the

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domain of representation of preference degrees, and therefore can be expressed by meaning of a square matrix. The preference relations can be mainly classified into the following categories: the multiplicative preference relations [20], fuzzy preference relations [25], intuitionistic fuzzy preference relations [32,33], and interval-valued intuitionistic fuzzy preference relations [34].

On the other hand, fuzzy set theory has interesting applications also in algebra, in particular in algebraic hyperstructure theory, where the connections between the classical structures and fuzzy sets (or their generalizations) determined new crisp hyperstructures, fuzzy subhyperstructures, or fuzzy hyperstructures. A well-known method to obtain new algebraic hyperstructures is to define hyperproducts generated by relations. The most studied such constructions are those of Rosenberg [28] and Corsini [8], investigated later by Corsini and Leoreanu [10], Spartalis et al. [29,30,31], Cristea et al. [11,12,13,14], De Salvo and Lo Faro [17,18], etc. This connection has been extended to n-ary hyperstructures by Davvaz and Leoreanu-Fotea [15,27]. Another way to obtain hyperstructures is given by Chvalina [6] and called "Ends Lemma", used in [22]. Feng [19] obtained fuzzy hypergroups from fuzzy relations, while Jančić-Rašović in [23] constructed hyperrings from fuzzy relations defined on a semigroup. In this article, we continue in the same direction, proposing a method for defining hyperoperations from intuitionistic fuzzy relations.

The rest of the paper is organized as follows. After a short description of the main properties of the intuitionistic fuzzy relations (IFRs), emphasizing those of the intuitionistic fuzzy preference relations (IFPRs), covered in Preliminaries, a brief introduction to the theory of hypergroups associated with binary relations follows in Section 3. We recall the Rosenberg's method and the notion of reduced hypergroup introduced by Jantosciak [24]. Section 4 is dedicated to the construction of hypergroups associated with IFRs, giving examples of IFPRs and discussing their properties connected with join spaces and reduced hypergroups. We end this article with some concluding remarks and possible new lines of research.

2 Preliminaries Concerning Intuitionistic Fuzzy Relations

We recall some definitions concerning intuitionistic fuzzy relation theory and we fix the notations used in this paper.

Diferent generalizations of fuzzy sets have been developed for a better modelling of ambigous problems. The concept of intuitionistic fuzzy set (called also Atanassov's intuitionistic fuzzy set) can be viewed as an alternative approach to define a fuzzy set whenever available information is not sufficient to describe an imprecise, vague concept by means of ordinary fuzzy sets [21].

Definition 2.1. [1,2] An *intuitionistic fuzzy set* (shortly IFS) on a universe X is an object having the form $A = \{(x, \mu_A(x), \nu_A(x)) \mid x \in X\}$, where $\mu_A(x) \in [0,1]$, called the *degree of membership* of x in A, $\nu_A(x) \in [0,1]$, called the *degree of non-membership* of x in A, verify, for any $x \in X$, the relation $0 \leq \mu_A(x) + \nu_A(x) \leq 1$. The class of IFSs on a universe X will be denoted by $\mathcal{IFS}(X)$.

It is clear that an IFS can be considered as a fuzzy set whenever $\nu_A(x) = 1 - \mu_A(x)$, for any $x \in X$, but conversely not.

Definition 2.2. [1,2] An *intuitionistic fuzzy relation* R (shortly IFR) from a universe X to a universe Y is an IFS in $X \times Y$, i.e. a set by the form $R = \{((x,y); \mu_R(x,y), \nu_R(x,y)) \mid (x,y) \in X \times Y\}$, where $\mu_R(x,y) + \nu_R(x,y) \leq 1$, for any $(x,y) \in X \times Y$.

Furthermore, the number $\pi_R(x,y) = 1 - \mu_R(x,y) - \nu_R(x,y)$, for $(x,y) \in X \times Y$, is called the *index* of the element (x,y) in IFR R and it is described as a degree of hesitation whether x and y are in the relation R on not.

The class of IFRs from X to Y will be denoted by $\mathcal{IFR}(X \times Y)$ and the class of IFRs on X will be denoted by $\mathcal{IFR}(X)$.

The *domain* of an IFR $R \in \mathcal{IFR}(X \times Y)$ is the IFS in X defined by $dom(R) = \{(x, \bigvee_{y \in Y} \mu_R(x,y), \bigwedge_{y \in Y} \nu_R(x,y)) \mid x \in X\}$ and the *range* of R is the IFS in Y defined by $rng(R) = \{(x, \bigvee_{x \in X} \mu_R(x,y), \bigwedge_{x \in X} \nu_R(x,y)) \mid y \in Y\}$.

In the following, we mention some basic operations between IFRs. For more details see [5,16].

Definition 2.3. *i*) Let R and S be in $\mathcal{IFR}(X \times Y)$. For every $(x,y) \in X \times Y$, we define

1. $R \subseteq S \Longleftrightarrow \mu_R(x,y) \leq \mu_S(x,y)$ and $\nu_R(x,y) \geq \nu_S(x,y)$
2. $R \preceq S \Longleftrightarrow \mu_R(x,y) \leq \mu_S(x,y)$ and $\nu_R(x,y) \leq \nu_S(x,y)$
3. $R \cup S = \{((x,y), \mu_R(x,y) \vee \mu_S(x,y), \nu_R(x,y) \wedge \nu_S(x,y))\}$
4. $R \cap S = \{((x,y), \mu_R(x,y) \wedge \mu_S(x,y), \nu_R(x,y) \vee \nu_S(x,y))\}$
5. $R^c = \{((x,y), \nu_R(x,y), \mu_R(x,y))\}$.

The family $(\mathcal{IFR}(X \times Y), \cup, \cap)$ is a complete, distributive lattice, with respect to the partially ordering $\preceq$.

ii) Let R in $\mathcal{IFR}(X \times Y)$ and S in $\mathcal{IFR}(Y \times Z)$. Then the composition between R and S is an IFR on $X \times Z$ defined as

$$R \circ S = \{((x,z), \bigvee_{y \in Y} (\mu_R(x,y) \wedge \mu_S(y,z)), \bigwedge_{y \in Y} (\nu_R(x,y) \vee \nu_S(y,z)))\}$$

whenever $0 \leq \bigvee_{y \in Y}(\mu_R(x,y) \wedge \mu_S(y,z)) + \bigwedge_{y \in Y}(\nu_R(x,y) \vee \nu_S(y,z)) \leq 1$.

Now we consider the IFRs defined on a set X.

Definition 2.4. An IFR R on a set X is

1. *reflexive* if $\mu_R(x,x) = 1$ (and consequently $\nu_R(x,x) = 0$), for any $x \in X$;
2. *symmetric* if $\mu_R(x,y) = \mu_R(y,x)$ and $\nu_R(x,y) = \nu_R(y,x)$, for any $x, y \in X$; in the opposite way we will say that it is *asymmetric*;
3. *transitive* if $R^2 = R \circ R \subseteq R$;
4. *antisymmetrical intuitionistic* if, for any $(x,y) \in X \times X$, $x \neq y$, then $\mu_R(x,y) \neq \mu_R(y,x)$, and $\nu_R(x,y) \neq \nu_R(y,x)$, but $\pi_R(x,y) = \pi_R(y,x)$.

5. *perfect antisymmetrical intuitionistic* if, for any $(x, y) \in X \times X$, $x \neq y$ and $\mu_R(x, y) > 0$ or ($\mu_R(x, y) = 0$ and $\nu_R(x, y) < 1$), then $\mu_R(y, x) = 0$ and $\nu_R(y, x) = 1$;
6. an *equivalence* if it is reflexive, symmetric and transitive.

Throughout this paper we focus on the intuitionistic fuzzy preference relations (shortly IFPRs), which are widely applied in decision-making theory, where we deal with the finite set of alternatives $X = \{x_1, x_2, \ldots, x_n\}$ and a decision maker who needs to express his/her preferences over the alternatives, constructing thus a *preference relation* P on the set X. It is characterized by a function $\mu_P : X \times X \longrightarrow D$, where D is the domain of representation of preference degrees. If we pass to the fuzzy case, the definition changes; a *fuzzy preference relation* P on the set X is represented by a membership function $\mu_P : X \times X \longrightarrow [0, 1]$ satisfying several properties: taking $\mu_P(x_i, x_j) = \mu_{ij}$, then $\mu_{ij} + \mu_{ji} = 1$, $\mu_{ii} = 0.5$, for all $i, j = 1, 2, \ldots, n$, where μ_{ij} denotes the preference degree of the alternative x_i over x_j. Generalizing now to the intuitionistic fuzzy case, the definition is given as follows.

Definition 2.5. [32,33] An *intuitionistic fuzzy preference relation* R on the finite set X of cardinality n is represented by a matrix $R = (r_{ij})_{n\times n}$, with $r_{ij} = (\mu_{ij}, \nu_{ij}, \pi_{ij})$, for $i, j = 1, 2, \ldots, n$, where $\mu_{ij} = \mu_R(x_i, x_j)$ is the certainty degree to which x_i is preferred to x_j, $\nu_{ij} = \nu_R(x_i, x_j)$ is the certainty degree to which x_i is non-preferred to x_j, and $\pi_{ij} = \pi_R(x_i, x_j) = 1 - \mu_{ij} - \nu_{ij}$ describes the uncertainty degree (or the hesitation) to which x_i is preferred to x_j. Furthermore, μ_{ij} and ν_{ij} satisfy the following relations: $0 \leq \mu_{ij} + \nu_{ij} \leq 1$, $\mu_{ij} = \nu_{ji}$, $\mu_{ji} = \nu_{ij}$, $\mu_{ii} = \nu_{ii} = 0.5$, for all $i, j = 1, 2, \ldots, n$. It is clear that $\pi_{ij} = \pi_{ji}$, for all $i, j = 1, 2, \ldots, n$.

3 A Brief Introduction to Hypergroups Associated with Binary Relations

Several hyperproducts have been obtained by meaning of binary relations. We recall here that introduced by Rosenberg [28], which is the first one of this type and the most explored one. For a comprehensive overview of hypergroup theory, the reader is refereed to the fundamental books [7,9].

3.1 Rosenberg's Method

Let ρ be a binary relation defined on a nonempty set H. For any pair of elements $(a, b) \in \rho$, we call a a *predecessor* of b and b a *successor* of a.

We addopt the following notations: $L_a = \{b \in H \mid (a, b) \in \rho\}$ and $R_a = \{b \in H \mid (b, a) \in \rho\}$ for the *afterset* and, respectively, *foreset*, of the element a.

For any two elements $x, y \in H$, we define the following hyperproduct

$$x \circ_\rho y = \{z \in H \mid (x, z) \in \rho \ \text{ or } \ (y, z) \in \rho\}.$$

We denote by $\mathbb{H}_\rho$ the hypergroupoid $(H, \circ_\rho)$.

An element $x \in \rho$ is called *outer element* of ρ if there exists $h \in H$ such that $(h, x) \notin \rho^2$. The following theorem states necessary and sufficient conditions for a binary relation ρ to generate a hypergroup $\mathbb{H}_\rho$ in the sense of Rosenberg.

Theorem 3.1. [28] *$\mathbb{H}_\rho$ is a hypergroup if and only if*

1. *ρ has full domain and full range.*
2. *$\rho \subset \rho^2$.*
3. *If $(a, x) \in \rho^2$ then $(a, x) \in \rho$, whenever x is an outer element of ρ.*

It follows immediately the following remark: If ρ is a preordering (reflexive and transitive), then $\mathbb{H}_\rho$ is a hypergroup.

Besides, Rosenberg gave a characterization of the hypergroup $\mathbb{H}_\rho$ in order to be a join space. Let us recall here this result.

Theorem 3.2. [28] *Let ρ be a binary relation with full domain. Then $\mathbb{H}_\rho$ is a join space if and only if*

1. *ρ has full range.*
2. *$\rho \subset \rho^2$.*
3. *If $(a, x) \in \rho^2$ then $(a, x) \in \rho$, whenever x is an outer element of ρ.*
4. *Every pair of elements of H with a common predecessor has a common successor.*
5. *For all $b, c, d \in H$, $a \in L_b$, $\{b, c\} \times L_a \subseteq \rho^2 \setminus \rho$, $L_b \cap L_a = \emptyset$ implies that $L_c \cap L_d \neq \emptyset$.*

We conclude this subsection with the following consequence.

Corollary 3.3. [28] *Let ρ be a binary relation on H with full domain and full range and such that either*

1. *$\rho = \rho^2$ or*
2. *ρ is reflexive and $(a, b) \in \rho^2 \Longrightarrow (a, b) \in \rho$, whenever b is an outer element of ρ.*

Then $\mathbb{H}_\rho$ is a join space if and only if every pair of elements of H with a common predecessor has a common successor.

3.2 Reduced Hypergroups

It may happen that a hyperproduct on a given set H does not discriminate between a pair of elements of H, when the elements play interchangeable roles with respect to the hyperoperation. Thus, a certain equivalence relation can be defined in order to identify the elements with the same properties. In order to explain better this situation, Jantosciak [24] defined on a hypergroup $(H, \circ)$ three equivalences, called *fundamental relations*: two elements x, y in H are called:

1. *operationally equivalent*, and write $x \sim_o y$, if $x \circ a = y \circ a$, and $a \circ x = a \circ y$, for any $a \in H$.

2. *inseparable*, and write $x \sim_i y$, if, for all $a, b \in H$, $x \in a \circ b \Longleftrightarrow y \in a \circ b$.
3. *essentially indistinguishable* if they are operationally equivalent and inseparable.

A *reduced hypergroup* has the equivalence class of each element with respect to the essentially indistinguishable relation a singleton. Therefore, the study of the hypergroups may be divided in the study of the reduced hypergroups and in that of the hypergroups with the same reduced form, as Jantosciak proved in [24]. Necessary and sufficient conditions such that a hypergroup associated with a binary relation is a reduced hypergroup have been presented in the papers [11,12].

A characterization of the fundamental relations for the Rosenberg hypergroup $\mathbb{H}_\rho$ is given in the following result.

Proposition 3.4. [12] *Let $\mathbb{H}_\rho$ be the Rosenberg hypergroup associated with the binary relation ρ defined on H. For any $x, y \in H$, the following implications hold:*

1. $x \sim_o y \Longleftrightarrow L_x = L_y$.
2. $x \sim_i y \Longleftrightarrow R_x = R_y$.
3. $\mathbb{H}_\rho$ is reduced if and only if, for any $x, y \in H$, $x \neq y$, either $L_x \neq L_y$ or $R_x \neq R_y$.

4 Hypergroups Associated with IFRs

4.1 Main Construction

In this section, we present a method to construct a new hypergroupoid starting from an IFR. We will find connections with Rosenberg hypergroup, and thus we will investigate when the obtained hypergroupoid is a hypergroup, or a join space, or a reduced hypergroup. We will focuss more on the case of intuitionistic fuzzy preference relations.

IFRs can induce different binary relations in a universe X. We deal here with that introduced by Burillo and Bustince [4], in order to justify the definition given for an intuitionistic antisymmetrical relation on X.

Let H be an arbitrary finite nonempty set, endowed with an IFR $R = (\mu_R, \nu_R)$. It induces on H the crisp binary relation ρ, defined by

$$x \rho y \Longleftrightarrow \mu_R(y, x) \leq \mu_R(x, y) \wedge \nu_R(y, x) \geq \nu_R(x, y).$$

It is known that, if R is an intuitionistic order on H, then ρ is an ordinary ordering on H [4]. The definition of intuitionistic antisymmetry is fundamental for the proof of this implication. Moreover, if we replace the definition of intuitionistic antisymmetry given by Burillo and Bustince [4] by the one given by Kaufmann [26] for the fuzzy relations, we don't obtain this implication any more in the case of fuzzy relations.

Now we associate with ρ the hyperproduct defined on H in the sense of Rosenberg [28]

$$x \circ_\rho y = L_x \cup L_y,$$

where $L_x = \{z \in H \mid (x,z) \in \rho\}$ is the *afterset* of x, denoted also with $\rho(x)$, or with $x\rho$. As in the previous sections, $\mathbb{H}_\rho$ denotes the associated hypergroupoid $(H, \circ_\rho)$, called Rosenberg hypergroupoid.

Our primary aim is to determine conditions on the IFR R such that the induced crisp relation ρ satisfies the conditions from Rosenberg's theorem (Theorem 3.2). It is not difficult to notice that, for any IFR R, the induced crisp relation ρ has full domain and full range, it is always reflexive, so $\rho \subset \rho^2$, and has no outer element. Indeed, x is an outer element of ρ if there exists $h \in H$ such that $(h,x) \notin \rho^2$; this means that there exists $h \in H$ such that, for any $z \in H$, it holds $(\mu_R(z,h) > \mu_R(h,z)$ and $\nu_R(z,h) < \nu_R(h,z))$ or $(\mu_R(x,z) > \mu_R(z,x)$ and $\nu_R(x,z) < \nu_R(z,x))$, which is impossible for $z = h$, in the first case, and for $z = x$ in the second one. Concluding, it is clear that ρ always satisfies the conditions of Theorem 3.2, so $\mathbb{H}_\rho$ is always a hypergroup.

Two natural questions arrise:

1. *When* $\mathbb{H}_\rho$ *is a join space?*
2. *When* $\mathbb{H}_\rho$ *is a reduced hypergroup?*

Proposition 4.1. *For every IFR* $R = (\mu_R, \nu_R)$ *defined on a nonempty finite set* H*, the associated Rosenberg hypergroup* $\mathbb{H}_\rho$ *is a join space.*

Proof. This is an immediate consequence of Corollary 3.3, since the crisp relation ρ associated with R is reflexive, has no outer element and every pair of elements of H with a common predecessor has a common successor, because, for any $a, b \in H$, we have $(a,b) \in \rho$ or $(b,a) \in \rho$. Thus, if $(x,a),(x,b) \in \rho$, then there exists $y \in \{a,b\}$ such that $(a,y),(b,y) \in \rho$. □

Proposition 4.2. *If the IFR* R *on* H *is symmetric, then the associated crisp relation* ρ *is the total relation on* H*, thus* $\mathbb{H}_\rho$*is the total hypergroup, so it isn't reduced.*

In the following we consider only asymmetric IFRs on H.

Proposition 4.3. *If the IFR* R *is perfect antisymmetrical intuitionistic (and asymmetric), then* $\mathbb{H}_\rho$ *is a reduced hypergroup.*

Proof. We will prove that, for any $x \neq y$, we have that $L_x \neq L_y$, and then, by Proposition 3.4, it follows that $\mathbb{H}_\rho$ is reduced. In order to prove this, it is enough to note that, for $x \neq y$, $x \in L_y$ is equivalent with $y \notin L_x$, and then it is clear that $L_x \neq L_y$.

Let $x \in L_y$ and suppose that $y \in L_x$. Then we obtain that $\mu_R(x,y) \leq \mu_R(y,x) \leq \mu_R(x,y)$ and $\nu_R(x,y) \leq \nu_R(y,x) \leq \nu_R(x,y)$, that is R is symmetric, which is a contradiction of the hypothesis. Thus, it follows that $y \notin L_x$.

Conversely, let $y \notin L_x$, that is $\mu_R(y,x) > \mu_R(x,y)$ or $\nu_R(y,x) < \nu_R(x,y)$. Consider the first case. Since $\mu_R(y,x) > \mu_R(x,y)$, it follows that $\mu_R(y,x){>}0$, and since R is perfect antisymmetrical intuitionistic, we get $\mu_R(x,y)=0$ and $\nu_R(x,y)=1\geq\nu_R(y,x)$. Thereby, $x\in L_y$.

Similarly, suppose that $\nu_R(y,x)<\nu_R(x,y)$. If $\mu_R(x,y)>\mu_R(y,x)$, then $\mu_R(x,y)>0$ and, by the perfect antisymmetry property, it follows that $\mu_R(y,x)=0$ and $\nu_R(y,x)=1$, which is a contradiction with the inequality $\nu_R(y,x)<\nu_R(x,y)$. Therefore, $\mu_R(x,y)\leq\mu_R(y,x)$, which means that $x\in L_y$.

Now the equivalence $y\notin L_x \Longleftrightarrow x\in L_y$ is completely proved and we can conclude that $L_x\neq L_y$, for any $x\neq y$. So $\mathbb{H}_\rho$ is a reduced hypergroup. □

Remark 4.4. It is worth to notice that there exist IFRs on a set H such that they are symmetric and perfect antisymmetrical intuitionistic. The identity Δ defined by

$$\mu_\Delta(x,y)=\begin{cases}1 & \text{if } x=y,\\ 0 & \text{if } x\neq y\end{cases}\quad \nu_\Delta(x,y)=\begin{cases}0 & \text{if } x=y,\\ 1 & \text{if } x\neq y\end{cases}$$

is a such relation. Moreover, all the relations of this type satisfy $\mu_R(x,y)=0$ and $\nu_R(x,y)=0$, for any $x\neq y$.

Proposition 4.5. *If the IFR R is antisymmetrical intuitionistic, then $\mathbb{H}_\rho$ is the total hypergroup or it is a reduced hypergroup.*

Proof. Let R be an antisymmetrical intuitionistic relation such that, for any $x\neq y$, $\mu_R(y,x)<\mu_R(x,y)$. Since $\pi_R(x,y)=\pi_R(y,x)$, it follows that $\nu_R(y,x)>\nu_R(x,y)$ and then $x\rho y$, for any $x\neq y$. Moreover, the associated crisp relation ρ is always reflexive. Thus, we conclude that $L_x=H$, for any $x\in H$, which means that $\mathbb{H}_\rho$ is the total hypergroup.

Let us suppose now that R is an antisymmetrical intuitionistic relation such that there exist $x\neq y$ with $\mu_R(x,y)<\mu_R(y,x)$. Since $\pi_R(x,y)=\pi_R(y,x)$, it follows that $\nu_R(x,y)>\nu_R(y,x)$. We obtain that $x\in L_y$, but $y\notin L_x$, so $L_x\neq L_y$ and thus $\mathbb{H}_\rho$ is a reduced hypergroup, accordingly with Proposition 3.4. □

4.2 The Case of IFPRs

This section is dedicated to the study of the hypergroup $\mathbb{H}_\rho$ associated with the IFPR R, insisting on the meaning of the related hyperoperation.

Let R be an IFPR on the set $H=\{x_1,x_2,\ldots,x_n\}$, represented by the matrix $R=(r_{ij})_{n\times n}$, with $r_{ij}=(\mu_{ij},\nu_{ij},\pi_{ij})$, for $i,j=1,2,\ldots,n$. Then the induced crisp binary relation ρ (in the sense of [4]) is defined by the rule

$$x_i\rho x_j \Longleftrightarrow \mu_R(x_j,x_i)\leq\mu_R(x_i,x_j) \Longleftrightarrow \mu_{ji}\leq\mu_{ij}.$$

Since $\mu_{ji}=\nu_{ij}$, we can also write that

$$x_i\rho x_j \Longleftrightarrow \nu_{ij}\leq\mu_{ij},$$

that is the degree of non-preference of the alternative x_i to the alternatives x_j is less than or equal to the degree of preference of the alternative x_i to x_j, and thus we can simply say that the alternative x_j is preferred less or equal with respect to x_i.

Let us see now which is the afterset L_{x_i} of the alternative x_i. By definition, $L_{x_i} = \{z \in H \mid (x_i, z) \in \rho\}$, i.e. it is the set of all alternatives $z \in H$ that the decision maker prefers less than or equal to the alternative x_i. And therefore the hyperproduct $x_i \circ_\rho x_j$ between two alternatives x_i and x_j is the set of all alternatives z that the decision maker prefers less than or equal to x_i or x_j.

Now we introduce three properties of the alternatives concerning the fundamental relations defined on a hypergroup.

Definition 4.6. We say that two alternatives x_i and x_j are

1. *operationally equivalent* if the elements x_i, x_j are operationally equivalent in the hypergroup $\mathbb{H}_\rho$, that is, for any alternative $a \in H$, the set of all alternatives that the decision maker prefers less than or equal to x_i or a coincide with the set of all alternatives that the decision maker prefers less than or equal to x_j or a.
2. *inseparable* if the elements x_i, x_j are inseparable in the hypergroup $\mathbb{H}_\rho$, that is, for any two alternatives $a, b \in H$, the decision maker prefers x_i less than or equal to a or b if and only if he/she prefers x_j less than or equal to alternatives a or b.
3. *essentially indistinguishable* if they are operationally equivalent and inseparable.

Proposition 4.7. *In a decision-making process, if two alternatives x_i and x_j are operationally equivalent or inseparable, then they are indifferent (one to respect to another) for the decision maker, that is $\mu_{ij} = \nu_{ij}$ (the degree of preference coincides with the degree of non-preference).*

Proof. Let us suppose that the alternatives x_i and x_j are operationally equivalent. A similar discussion can be done in the case they are inseparable. Since $x_i \sim_o x_j$ in the associated hypergroup $\mathbb{H}_\rho$, by Proposition 3.4, it follows that $L_{x_i} = L_{x_j}$. Thererfore $x_i \rho x_j$ and $x_j \rho x_i$, which is equivalent with $\mu_{ji} \leq \mu_{ij}$ and $\mu_{ij} \leq \mu_{ji}$, that is $\mu_{ij} = \mu_{ji} = \nu_{ij}$. □

The converse implication is not true, as we can notice from the following example.

Example 4.8. *Consider $H = \{x_1, x_2, x_3, x_4\}$ the set of four alternatives. Construct on H the IFPRs represented by the following matrices:*

$$R^{(1)} = \begin{pmatrix} (0.5, 0.5, 0) & (0.3, 0.4, 0.3) & (0.4, 0.5, 0.1) & (0.6, 0.3, 0.1) \\ (0.4, 0.3, 0.3) & (0.5, 0.5, 0) & (0.4, 0.4, 0.2) & (0.5, 0.3, 0.2) \\ (0.5, 0.4, 0.1) & (0.4, 0.4, 0.2) & (0.5, 0.5, 0) & (0.7, 0.2, 0.1) \\ (0.3, 0.6, 0.1) & (0.3, 0.5, 0.2) & (0.2, 0.7, 0.1) & (0.5, 0.5, 0) \end{pmatrix}$$

and

$$R^{(2)} = \begin{pmatrix} (0.5, 0.5, 0) & (0.3, 0.4, 0.3) & (0.4, 0.5, 0.1) & (0.6, 0.3, 0.1) \\ (0.4, 0.3, 0.3) & (0.5, 0.5, 0) & (0.4, 0.4, 0.2) & (0.3, 0.4, 0.3) \\ (0.5, 0.4, 0.1) & (0.4, 0.4, 0.2) & (0.5, 0.5, 0) & (0.7, 0.2, 0.1) \\ (0.3, 0.6, 0.1) & (0.4, 0.3, 0.3) & (0.2, 0.7, 0.1) & (0.5, 0.5, 0) \end{pmatrix}.$$

We notice that for both relations we have $\mu_{23} = \nu_{23}$*, so the alternatives* x_2 *and* x_3 *are indifferent (one with respect to another) for the decision maker.*

On the other hand, the induced crisp binary relations are

$$\rho^{(1)} = \Delta \cup \{(x_1, x_4), (x_2, x_1), (x_2, x_3), (x_2, x_4), (x_3, x_1), (x_3, x_2), (x_3, x_4)\},$$

$$\rho^{(2)} = \Delta \cup \{(x_1, x_4), (x_2, x_1), (x_2, x_3), (x_3, x_1), (x_3, x_2), (x_3, x_4), (x_4, x_2)\},$$

where $\Delta = \{(x_1, x_1), (x_2, x_2), (x_3, x_3), (x_4, x_4)\}$ *is the diagonal relation.*

For the first IFPR we obtain that the aftersets and foresets of the elements x_i, $i = \{1, 2, 3, 4\}$*, are:* $L^{(1)}_{x_1} = \{x_1, x_4\}$, $R^{(1)}_{x_1} = \{x_1, x_2, x_3\}$; $L^{(1)}_{x_2} = \{x_1, x_2, x_3, x_4\}$, $R^{(1)}_{x_2} = \{x_2, x_3\}$; $L^{(1)}_{x_3} = \{x_1, x_2, x_3, x_4\}$, $R^{(1)}_{x_3} = \{x_2, x_3\}$; $L^{(1)}_{x_4} = \{x_4\}$, $R^{(1)}_{x_4} = \{x_1, x_2, x_3, x_4\}$.

Because $L^{(1)}_{x_2} = L^{(1)}_{x_3}$ *and* $R^{(1)}_{x_2} = R^{(1)}_{x_3}$*, it follows, accordingly by Proposition 3.4, that* $x_2 \sim_e x_3$*, so the associated hypergroup* $\mathbb{H}_\rho$ *is not reduced.*

Regarding the second IFPR, the aftersets and foresets of the elements x_i, $i = \{1, 2, 3, 4\}$*, are:* $L^{(2)}_{x_1} = \{x_1, x_4\}$, $R^{(2)}_{x_1} = \{x_1, x_2, x_3\}$; $L^{(2)}_{x_2} = \{x_1, x_2, x_3\}$, $R^{(2)}_{x_2} = \{x_2, x_3, x_4\}$; $L^{(2)}_{x_3} = \{x_1, x_2, x_3, x_4\}$, $R^{(2)}_{x_3} = \{x_2, x_3\}$; $L^{(2)}_{x_4} = \{x_2, x_4\}$, $R^{(2)}_{x_4} = \{x_1, x_3, x_4\}$.

In this case, $x_2 \nsim_e x_3$*, and moreover* $x_i \nsim_e x_j$*, for any* $i \neq j, i, j \in \{1, 2, 3, 4\}$*. Thus the associated hypergroup* $\mathbb{H}_\rho$ *is reduced.*

As an immediate consequence of Proposition 4.7, we obtain the following algebraic property.

Corollary 4.9. *If a decision maker doesn't have any sort of indifference between any two distinct alternatives, then the associated hypergroup* $\mathbb{H}_\rho$ *is reduced.*

5 Conclusions and Future Work

In this paper we have started the study of the hypergroups associated with IFRs, considering the particular case of IFPRs. Any IFR induces several crisp binary relations. Here we have considered that one introduced by Burillo and Bustince [4]. Then, a hypergroupoid, in the sense of Rosenberg [28], is associated with the binary relation, and it is proved that it is always a join space. We have extended the fundamental equivalences of Jantosciak [24] to a decision-making process, investigating when the associated Rosenberg hypergroup is reduced.

In a future work, we will analyze this association in the general case of IFRs, considering other types of induced crisp binary relations, or associated hypergroupoids, making a comparison between these cases.

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An Analysis of Inconsistency in Intertemporal Choice*

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Abstract. Some empirical studies have shown that economic agents do not always use constant discount rates through time. That is to say, they do not always use exponential discounting which is a particular case of consistent intertemporal choice, since it predicts there will not be preference reversal when choosing a reward as more valued, independently of the moment of decision making. On the other hand, one of the most important problems of non-constant discounting is its inconsistency in intertemporal choice. In this way, we show that one of the main sources of inconsistency is subadditivity.

JEL Classification: C9, D8, D9.

Keywords: Decision-making, hyperbolic discounting, hazard rate, multicriteria, agent, dynamic inconsistency, subadditivity.

1 Introduction

In the analysis of individual decisions on investment there are several proposed mathematical models trying to capture the underlying mechanisms in these processes that imply a discount transaction. The standard economic model is the exponential discounting (Samuelson, 1937). More recently, some empirical studies (early empirical studies based on hypothetical rewards) point to a hyperbolic or hyperbola-like function, in the way of equation (1), as a better description of choice implying delayed probabilistic rewards (Green *et al.*, 1999; Herrnstein, 1981; Kirby and Marakovic, 1995; Mazur, 1987; Myerson and Green, 1995; Myerson *et al.*, 2001; Rachlin *et al.*, 1991):

$$Y = \frac{A}{(1+bt)^s} \quad , \tag{1}$$

where Y is the subjective value of a reward of amount A, b is a parameter that indicates the degree of discounting, t is the time and s reflects the non-linear scale

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between amount and time or probability. Note that equation (1) becomes a simple hyperbola when s equals one.

This evidence was criticized by other authors (Mulligan, 1996; Rubinstein, 2003; Harrison and Lau 2005) who question the methodology employed in these studies. Furthermore, Harrison *et al.* (2002) find empirical evidence supporting constant discount rates. Even so, we are not going to focus on the methodology of empirical studies. Instead, we are going to focus on a theoretical approach.

In the above mentioned studies, the discount rates used by individuals were higher for short periods of time than for long periods. (Thaler, 1981; Benzion *et al.*, 1989; Cropper *et al.*, 1992). This behavior has been labeled as an anomaly or "paradox of rational choice theory" (Loewenstein and Thaler, 1989). Similarly, Azfar (1999) suggests that "if people are uncertain about their discount factors, it may be perfectly rational for them to discount the distant future at lower rates than the near future". He also shows that if individuals have continuous probability distributions over their constant hazard rates that are not bounded (are unlimited) from 0, then their discount factors should decrease hyperbolically and not exponentially with time, which is consistent with the experimental evidence.

This behavior has been empirically labeled as "hyperbolic discounting". It is consistent with a set of very general results from the natural sciences that find human responses to changes in a stimulus are non-linear and inversely proportional to the existing level of the stimulus. Strotz (1955) adduced no reason whereby the individual discounting function should be logarithmically linear with respect to the distance of the object being viewed.

Let $F(t)$ be the discounting function applied to benefits or costs at delay t, being the instantaneous discount rate $r(t) = -\frac{F'(t)}{F(t)}$. Then, we can formalize the idea that a given increase in the number of years has an impact in the weighting given to this event that is inversely proportional to the initial distance in the future:

$$r(t) = -\frac{F'(t)}{F(t)} = \frac{k}{t} \text{ or equivalently } F(t) = e^{-k\log t} = t^{-k}, \tag{2}$$

being k a positive constant and $t > 1$.

A discounting function $F(t) = e^{-k\log t}$ has the following interesting interpretation: the substitution of t by its logarithm implies that we react to proportional increases more than to absolute increases of distance in time (Heal, 1997). For this reason, this discounting function can be labeled "logarithmic discounting".

We can observe that the logarithmic discounting that we have just described is a particular case of the hyperbolic discounting defined by equation (1). In effect, if we write equation (1) taking a unitary reward, A = 1, we obtain a discount expression similar to equation (2), expressing a logarithmic discount.

Another way of conceptualizing the choice between immediate and delayed rewards is a choice between alternatives that differ with respect to the involved risk.

An immediate reward may be thought as a "sure thing", whereas waiting involves some degree of risk that the delayed reward will not be forthcoming (Myerson *et al.*, 2001)[1]. From this perspective, the exponential model supposes that every additional unit of delay implies a constant marginal increase in degree of risk, i.e., supposes a constant hazard rate (Green and Myerson, 1996). The hyperbolic model, in contrast, supposes that "a choice between an immediate and a delayed reward is a choice between two reinforcement rates, and each additional unit of delay decreases the ratio of amount to delay, resulting in a decrease in the subjective value of delayed reward" (Myerson *et al.*, 2001). So the idea that the hyperbolic function describes temporal discounting better than the exponential one (e.g. Kirby, 1997; Myerson and Green, 1995; Rachlin *et al.*, 1991) arises against the idea that the value decreases with delay because of a constant probability that something could happen, avoiding the delivery of the reward (as is supposed in standard economic models). For further analysis of the difference between exponential and hyperbolic models based on hazard rates, see Green and Myerson (1996).

We can make the hazard rate of a random variable equal to the instantaneous rate of a discounting function (Cruz and Muñoz, 2005). Then, following the hazard rate approach of Green and Myerson, we could have constant hazard rates and, therefore, constant discount rates (for the case of exponential discounting) and variable hazard rates and also variable – increasing or decreasing – discount rates (for the case of hyperbolic discounting). The last case will lead to the dynamic inconsistency that will be studied in this paper.

This paper is organized as follows: after this introduction, in section 2, two equivalent definitions of inconsistent discount functions are presented and it is shown that subadditivity is a source of inconsistency. Finally, Section 3 summarizes and concludes.

2 Non-constant Discounting and Dynamic Inconsistency

Before starting the development of Section 2, we are going to recall some concepts of Finance that are required to understand what follows (see Scholten and Read, 2006). In intertemporal choice, individuals are asked about the preferences between amounts or rewards (i.e., x_1 and x_2) corresponding to different instants of time (i.e., d_1 and d_2). Thus, if an economic agent is indifferent about two pairs (x_1, d_1) and (x_2, d_2), the subjective discounting function, *F*, applied to these amounts has to coincide:

$$x_1 F(d_1) = x_2 F(d_2).$$

Observe that $F(d)$ (resp. $xF(d)$) indicates the value today of \$1 (resp. \$$x$)[2] available at time *d*. But, in experiments, even though the time of valuation is 0, people

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1 For this reason some authors use a FED (front end delay) in the design of their experiments. "The FED design was introduced into discount rate experiments to address concerns about differential credibility", Harrison and Lau (2005).

2 Here we assume risk neutrality over income, that is, $u(x) = x$.

must decide between two choices directly without considering time 0. For example, if they have to decide between a reward available at time *d* and another available at time *d* + *t*, they have to take into account that the interval between the rewards is *t* years. In this case, we will use the discounting factor $F(d+t)/F(d)$.

Finally, take into account that in this last case it is different to compare two rewards available at instants *d* and $d+t$, the time of valuation being *d*, instead of 0. In this case, the time of valuation coincides with the delay and the corresponding notation is $F(d,t)$, an expression that does not have to be a quotient. For example, see the following expression (adapted to our notation):

$$F(d,t)=\frac{1}{1+b\left[(d+t)^a-d^a\right]^c},$$

introduced by Scholten and Read (2006). Observe that this dynamic expression of a discounting function provides us a multicriteria tool for the decision-making process.

Azfar shows that if the source of uncertainty is the hazard rate and individuals learn from their hazard rates, by the sole fact of not having died (or the experiment not having finished) in the present period, then the non-constant discount does not necessarily imply inconsistent behavior[3]. Nevertheless, if individuals have uncertainty about their true hazard rates and they do not learn more about them in the course of time, the non-constant discount would imply a dynamically inconsistent behavior.

Let $F(d,t)$ be a consistent discounting function. If two rewards (x_1,d_1) and (x_2,d_2) are equivalent according to this function, where the time of evaluation is *d*, they will continue being equivalent at another time, d'. More specifically,

Definition 1. An intertemporal choice is said to be *consistent* if:

$$(x_1,d_1)\sim_d (x_2,d_2) \text{ implies } (x_1,d_1)\sim_{d'} (x_2,d_2),$$

for every values *d* and d' both less than or equal to d_1 and d_2 ($d_1<d_2$).

A necessary and sufficient condition for consistent intertemporal choice is that the underlying discounting function is additive. In this case, the financial indifference lines do not depend on the benchmark *d*. Note that in figure 1 it is verified that:

- $t_1'=d_1-d'$ and $t_2'=d_2-d'$
- $t_1=d_1-d$ and $t_2=d_2-d$

[3] Dasgupta and Maskin (2005) also show some choices in which there is no dynamic inconsistency, despite the reversals. They develop an "evolutionary learning model" to explain it. In the same way, Newell and Pizer (2000) state that future beliefs about the appropriate discount rate evolves as the market evolves, meaning that any desire to revise a choice made in the past reflects the process of learning, rather than time-inconsistent behavior.

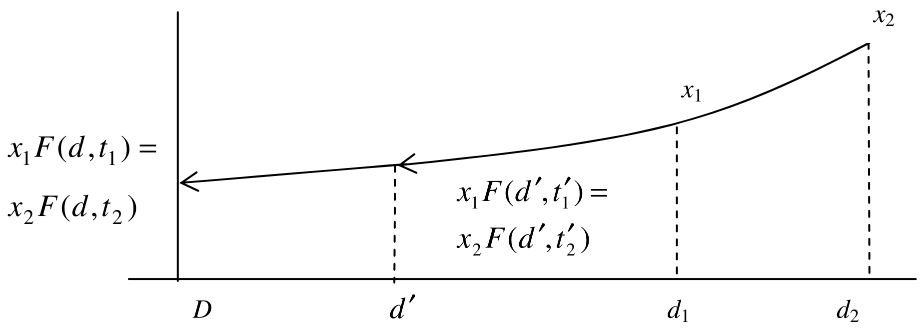

Fig. 1. Equivalence of rewards according to a consistent intertemporal choice

A particular case of consistent intertemporal choice is exponential discounting, since it predicts that there will not be preference reversal, such that “the smaller and sooner reward will have the bigger subjective value, independently of the moment in which the choice will be made” (Green and Myerson, 1996) (see figure 2). In contrast, the hyperbolic discount is not consistent, so that “when the delays are relatively short, the subjective value of the smaller reward is higher than the value of the bigger reward; nevertheless, when both delays are relatively long, the bigger reward (the more distant in time) has the higher subjective value” (Green and Myerson, 1996) (see figure 3). Also Read (2003) states that “exponential discounting is the only way to avoid time inconsistency” and that “hyperbolic discount function can produce time inconsistency”. It is true that the exponential discounting function implies consistency of the intertemporal choice, but it is not the only way to obtain consistency.

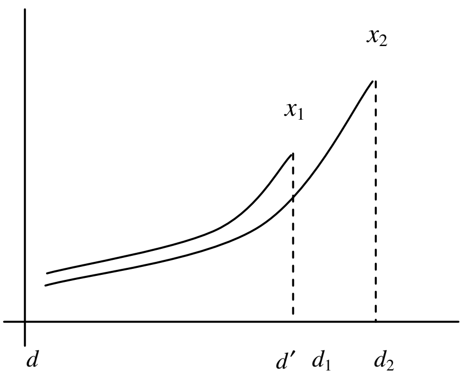

Fig. 2. Exponential model

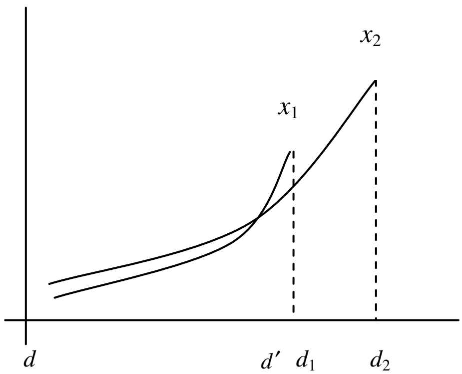

Fig. 3. Hyperbolic model

Definition 2. An intertemporal choice is said to be *inconsistent* if it is not consistent, that is to say, two amounts (or prospects) x_1 and x_2 that mature at d_1 and d_2 are equivalent at the moment d:

$$(x_1, d_1) \sim_d (x_2, d_2),$$

but if we change the benchmark to another instant d', later than d, the equivalence is not verified. That is to say, the following equivalence is not verified

$$(x_1, d_1) \sim_{d'} (x_2, d_2),$$

for some prospects (x_1, d_1) and (x_2, d_2) and for some values d and d' both less than or equal to d_1 and d_2. This situation is represented in figure 4.

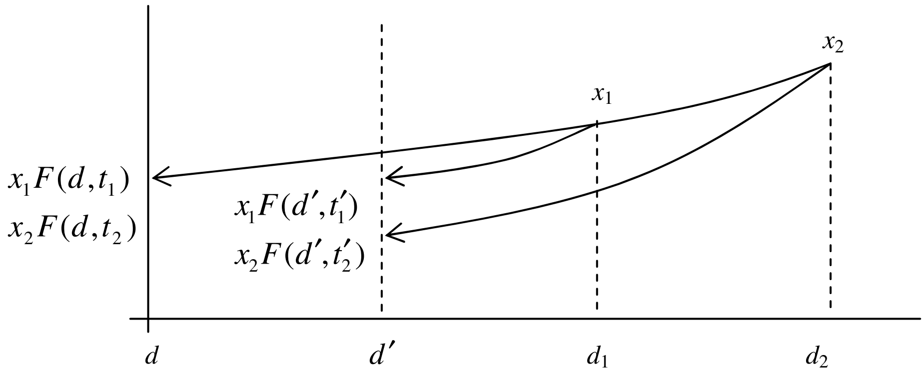

Fig. 4. Indifference curves according to an inconsistent intertemporal choice

Note that the definition of inconsistency given by Green and Myerson (1996) is slightly different:

Definition 3. An intertemporal choice is said to be *inconsistent* if the amount x_1 that matures at d_1 is less preferred than x_2 that matures at d_2, if the appraisal is made at d:

$$(x_1, d_1) \prec_d (x_2, d_2).$$

But, if we change the instant of appraisal to d' the preferences are reversed:

$$(x_1, d_1) \succ_{d'} (x_2, d_2),$$

for some prospects (x_1, d_1) and (x_2, d_2) and for some values of d and d' both less than or equal to d_1 and d_2. This situation is represented in figure 5.

A source of inconsistency can be given by the subadditivity of the discounting function used or, in the case of date-independent functions, by those whose discount factors are increasing with time. Both situations are exemplified in the hyperbolic

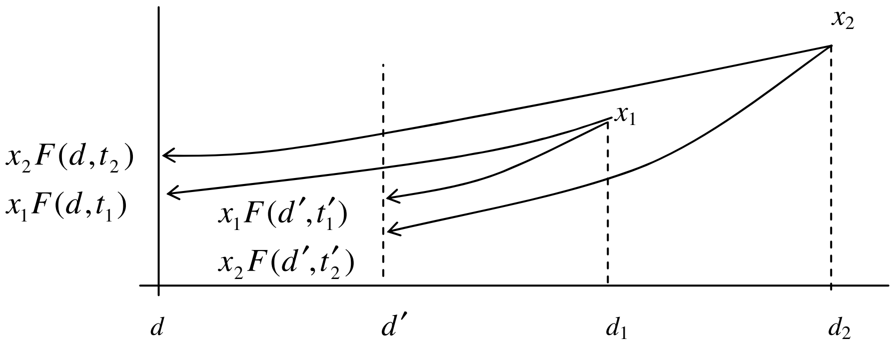

Fig. 5. Indifference curves for rewards (x_1, d_1) and (x_2, d_2) according to an inconsistent intertemporal choice

discount[4]. In effect, it is shown (Cruz and Muñoz, 2004) that a sufficient condition for a discounting function $F(d,t)$ being subadditive is that the respective discounting factor is increasing (or, what is the same, the respective discount rate is decreasing) and the delay effect[5] (Read, 2003) holds.

Definition 4. A discounting function $F(d,t)$ is said to be *subadditive* if the discounting function for the whole interval $(d, t_1 + t_2)$ is greater than the product of the discounting functions for intervals (d, t_1) and $(d + t_1, t_2)$:

$$F(d, t_1 + t_2) > F(d, t_1) \cdot F(d + t_1, t_2),$$

for all d, t_1 and t_2, such that $d < t_1 < t_2$.

That is to say, there is more discounting if we discount in two steps: from t_2 to $d + t_1$ and then from $d + t_1$ to d than if we discount from $t_1 + t_2$ to d, directly.

Proposition 1. Definitions 2 and 3 are equivalent.

Proof

i) $\Rightarrow$. Suppose that definition 2 is verified and that $(x_1, d_1) \sim_d (x_2, d_2)$, but $(x_1, d_1) \prec_{d'} (x_2, d_2)$, as represented in figure 6. Then

$$x_1F(d,t_1) - x_2F(d,t_2) = 0 \text{ and } x_1F(d',t_1') - x_2F(d',t_2') < 0.$$

[4] Following several psychologists, we will consider hyperbolic discounting in opposition to exponential discounting as a non-exponential function in the same sense as the hyperbolic function, that is to say, a decreasing function whose discount rate is also decreasing. So, here the concept of hyperbolic discounting is wider than a simple hyperbola.

[5] The *delay effect* consists of the decrease of the discount rate as waiting time increases, that is, the discount rates tend to be higher in short intervals than in longer ones.

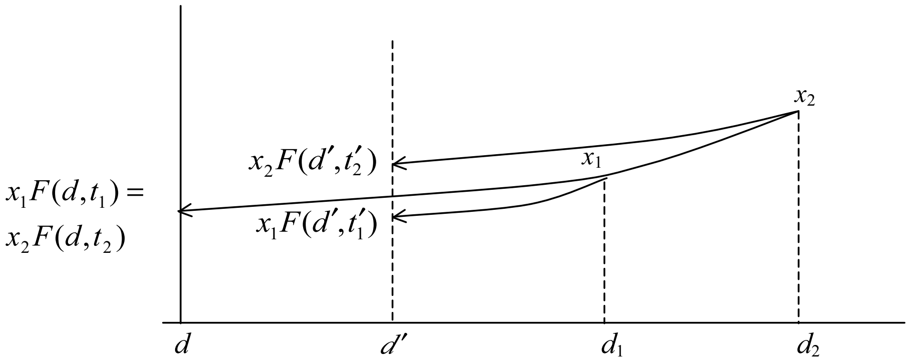

Fig. 6. Equivalence at d but not at d'

Consider the average of $x_1F(d',t_1')$ and $x_2F(d',t_2')$:

$$x = \frac{x_1F(d',t_1') + x_2F(d',t_2')}{2}.$$

Let be $x_1' = \dfrac{x}{F(d',t_1')}$. Then

$$x_1'F(d',t_1') = x < x_2F(d',t_2'),$$

from which

$$(x_1',d_1) \prec_{d'} (x_2,d_2).$$

On the other hand,

$$x > x_1F(d',t_1'),$$

from where

$$\frac{x}{F(d',t_1')} > x_1.$$

So

$$x_1' > x_1,$$

$$x_1'F(d,t_1) > x_1F(d,t_1),$$

and given that

$$(x_1,d_1) \sim_d (x_2,d_2),$$

therefore

$$x_1' F(d,t_1) > x_1 F(d,t_1) = x_2 F(d,t_2)$$

and, thus

$$(x_1', d_1) \succ_d (x_2, d_2).$$

Thus, definition 3 is proved.

ii) $\Leftarrow$. Suppose that definition 3 is verified and that $(x_1, d_1) \prec_d (x_2, d_2)$, but $(x_1, d_1) \succ_{d'} (x_2, d_2)$. Then

$$x_1 F(d,t_1) - x_2 F(d,t_2) < 0 \text{ and } x_1 F(d',t_1) - x_2 F(d',t_2) > 0.$$

Let us consider the function:

$$\phi : (0, d_1] \to \Re$$

defined by:

$$x \to \phi(x) = x_1 F(x, d_1 - x) - x_2 F(x, d_2 - x).$$

It is verified that:

- ϕ is continuous in the interval $(0, d_1]$.
- $\phi(d) = x_1 F(d,t_1) - x_2 F(d,t_2) < 0$.
- $\phi(d') = x_1 F(d',t_1') - x_2 F(d',t_2') > 0$.

Therefore, by Bolzano's theorem, there is a $q \in (0, d_1]$ such that

$$x_1 F(q, d_1 - q) = x_2 F(q, d_2 - q);$$

then $(x_1, d_1) \sim_q (x_2, d_2)$ and, therefore, definition 2 is verified.

Proposition 2 (Existence of inconsistent discounting functions). If a discounting function is subadditive, it will verify definitions 2 and 3, that is to say, it will be inconsistent.

Proof

Consider two rewards (x_1, d_1) and (x_2, d_2), which are equivalent according to the discounting function $F(d,t)$:

$$(x_1, d_1) \sim_d (x_2, d_2),$$

as represented in figure 7.

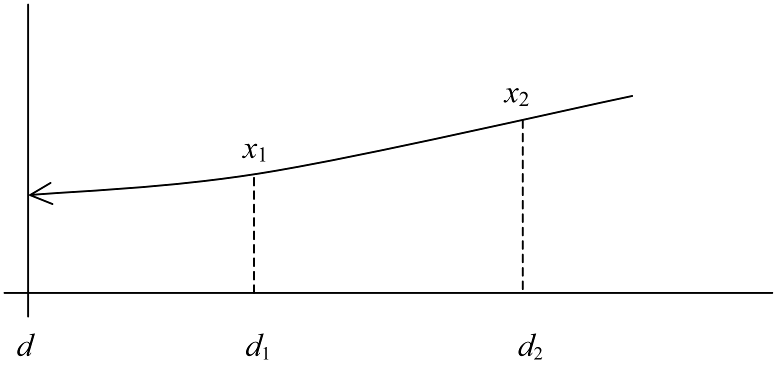

Fig. 7. Equivalence of rewards (x_1, d_1) and (x_2, d_2) according to $F(d,t)$

Therefore, it is verified that:

$$x_1 F(d,t_1) = x_2 F(d,t_2),$$

from which:

$$\frac{x_1}{x_2} = \frac{F(d,t_2)}{F(d,t_1)}.$$

As the discounting function is subadditive, it will verify that:

$$F(d,t_2) > F(d,t_1) \cdot F(d+t_1, t_2 - t_1) = F(d,t_1) \cdot F(d_1, t_2 - t_1).$$

We now have

$$\frac{F(d,t_2)}{F(d,t_1)} > F(d_1, t_2 - t_1),$$

from which we get:

$$\frac{x_1}{x_2} > F(d_1, t_2 - t_1),$$

which implies that:

$$x_1 > x_2 \cdot F(d_1, t_2 - t_1),$$

that is to say,

$$(x_1, d_1) \succ_{d_1} (x_2, d_2).$$

The foregoing proof shows that the discounting function verifies definition 2. Then, given our proof of proposition 1, it will also verify definition 3. □

3 Conclusion

From a hazard rate approach, the exponential model supposes a constant hazard rate (every additional unit of delay implies a constant marginal increase in degree of risk). Being the hazard rate of a random variable equal to the instantaneous rate of a discounting function, the exponential model also implies constant discount rates. Exponential discounting is a particular case of consistent intertemporal choice, since it predicts that there will not be preference reversal. So, agents will prefer the smaller and sooner reward, independently of the moment of decision making.

The hyperbolic model, in contrast, supposes a variable hazard rate (every additional unit of delay implies a decreasing marginal increase in degree of risk) and therefore, non-constant discount rates. It is important to realize that hyperbolic discounting has an important disadvantage: it describes an inconsistent financial choice. For this reason, Section 3 presents two equivalent definitions of inconsistency and proves that subadditivity is a source of inconsistency. To conclude, we can say that, in general, subadditivity is not a necessary condition for inconsistency, since subadditivity implies inconsistency, but inconsistency does not necessarily imply subadditivity. That is to say, there are other sources of inconsistency.

Finally, we would like to make a remark: we have assumed risk neutrality over income, $u(x) = x$. Things may be different if we allow for discounting over utility of income, the fact that people may be risk averse, and that relative risk aversion might not be constant.

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Modelling the Intertemporal Choice through the Dynamic Time-Perception

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Abstract. The process of intertemporal choice is intimately linked with the concept of discounting function. Usually the benchmark in this important financial tool is the instant 0. This is an actual constraint for economic agent decision-making; indeed, in many situations, individuals have to decide at instants different from 0. Obviously, this introduces a multicriteria decision making framework in which a group of agents can (or cannot) cooperate in order to obtain greater profitabilities in function of the time variable. In this financial context, it is necessary to choose between transitive and non-transitive choice, giving rise to additive and non-additive (which includes subadditive and superadditive) discounting, respectively. Finally, another classification distinguishes between discounting with increasing or decreasing impatience.

Keywords: Discounting function, instantaneous discount rate, impatience, stationarity, additivity, multicriteria decision making.

1 Introduction

In many situations, an individual or a firm must decide what amount is equivalent to $1 available *t* periods after a benchmark. This issue is a well-known problem in actuarial framework and *intertemporal choice* (Cruz and Muñoz, 2005 and 2006; Cruz and Ventre, 2011a), that is our present perspective. Indeed the benchmark is the instant at which we have the information necessary to replace a future with a present amount. If we take into account the criterion available at instant 0, the intertemporal choice will be named *static*, while if the criterion is available at variable time *d*, the intertemporal choice is said to be *dynamic*. Obviously, this last case provides us a

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variable criterion, depending on the benchmark (time *d*), which gives us a multicriteria decision making process in time.

This approach is equivalent to know a capitalization or discounting function:

- if $t \leq 0$, we will be using a *capitalization function*,
- if $t \geq 0$, we will be using a *discounting function.*

We will deal only with discounting functions, that are functions $F(t)$ of one time variable *t* (if the benchmark is 0) or functions $F_2(d,t)$ of two variables *d*, *t* (if the benchmark is the instant *d*). In this way, we will say that a discounting function is the mathematical expression of the intertemporal choice of a subject or a company, or, equivalently, the intertemporal choice is quantified by a discounting function. This shows the coincidence of the main topic both in "discounting function" and in "intertemporal choice".

Nevertheless, there is an intermediate way to obtain the equivalent of $1 at instant *d* with the criterion available at time 0. In this case, we will be using the *discounting factor*:

$$f(0,d,t) = \frac{F(d+t)}{F(d)}.$$

In other words, despite this factor discounts from $d+t$ to *t*, the employed criterion is the current one at time 0. Finally, the relationship between the two former notations is stationarity. Indeed, a discounting function is said to be *stationary* if $F_2(d,t)$ is independent of *d*, that is, the criterion of intertemporal choice does not change in time (Harvey, 1986 and 1994), in which case it will be simply denoted by $F(t)$.

On the other hand, when dealing with intertemporal choice, we can use an objective or a subjective discounting function. An *objective* discounting function is a given criterion of choice (linear, hyperbolic (Azfar, 1999) or exponential discounting), that is known by the two subjects involved in a financial transaction. A *subjective* discounting function is a criterion of choice deduced from the particular preferences of an individual or a group of individuals. In this paper we will focus on the issue of subjective intertemporal choice.

Finally, in intertemporal choice, the monetary unit can be replaced with a reward (for instance, an apple) with a given utility (Benzion *et al.*, 1999). The organization of the present paper is as follows. In Section 2, we will introduce the general notations and the concept of stationary intertemporal choice by exhibiting the different forms of definition. Thus, in a natural way, the concepts of transitivity and (im)patience arise, giving rise to increasing and decreasing (im)patience, and to subadditive and superadditive intertemporal choice. These issues will be developed in sections 3 and 4. Finally, Section 5 summarizes and concludes.

2 Stationary and Dynamic Intertemporal Choice

In this Section, we will start with some notation. As indicated in Section 1, given $1 available at time *t*, the value $F(t) = F_2(0,t)$ represents the amount subjectively equivalent at time 0. Said in other words, an economic subject is indifferent between $1, available at time *t*, and $F(t)$, available at time 0. $F(t)$ is said to be a *spot discounting function*. In this context, we would like to highlight the noteworthy relationship between the discounting function and the *instantaneous discount rate* (Maravall, 1970; Gil, 1993):

$$F(t) = e^{-\int_0^t \delta(x)\mathrm{d}x}, \tag{1}$$

where $\delta(t) = -\left.\frac{\mathrm{d}\ln F(x)}{\mathrm{d}x}\right|_{x=t}$. We are now interested in continuity in time of the defined criterion of choice, that is, transitivity. In order to deal with this problem, we have at our disposal two alternatives for describing future choices:

- $F_3(0,d,t)$ which denotes the amount equivalent at time *d* to $1 available at time $d+t$, where the benchmark is instant 0, that is, the choice involves futures dates with present criteria. As a particular worthwhile case, we can cite the discounting factor $f(0,d,t) = \frac{F(d+t)}{F(d)}$, which incorporates a condition of transitivity in intertemporal choice. Indeed, we have:

  $$f(0,d,t)f(0,d+t,s) = f(0,d,t+s).$$

  The expression form $F_3(0,d,t)$ without involving stationarity has been studied by several authors (see, for example, Mulazzani (1993)).
- $F_2(d,t)$, which denotes the amount equivalent at time *d* to $1 available at time $d+t$. Observe that here the benchmark is instant *d*.

In general, $F_3(0,d,t)$ is said to be a *forward discounting function* and, in particular, $f(0,d,t)$ is said to be the *discouting factor* associated to the spot discounting function $F(t)$. Finally, $F_2(d,t)$ is said to be a *dynamic discounting function*.

On the other hand, *d* is the delay, that is, a date later than today, and *t* is the interval, that is, a period of time after date *d*. The action of the interval *t* over the delay *d* gives rise to another delay $d+t$. Observe that, in the discounting function $F(t)$, the delay coincides with the interval.

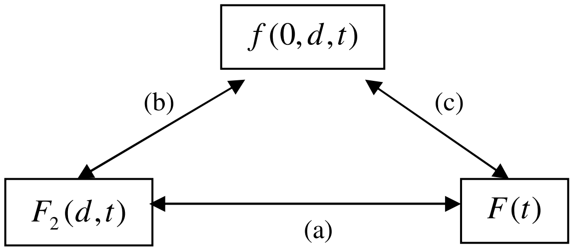

Chart 1. Some different possibilities to define stationarity

Next, in order to define stationarity as an invariance of the discounting function through time, we are going to develop each of the three arrows in chart 1:

a) Following this arrow, we can define stationarity by the following equation:

$$F(t) = F_2(d,t). \tag{2}$$

A discounting function satisfying equation (2) is said to be *stationary* (Harvey, 1986 and 1994).

b) Following this arrow, we can define stationarity by the following equation:

$$F_2(d,t) = f(0,d,t), \tag{3}$$

or, equivalently,

$$F(d)F_2(d,t) = F(d+t). \tag{4}$$

A discounting function satisfying equation (4) is said to be *additive*.

c) Finally, following this arrow, we can define stationarity by the following equation:

$$F(t) = f(0,d,t), \tag{5}$$

or, equivalently,

$$F(d)F(t) = F(d+t), \tag{6}$$

that is a functional equation whose solution (Aczél, 1987) is the well-known exponential discounting $F(t) = e^{-kt}$. Observe that condition (5) is stronger than condition (3), because condition (5) does not consider variable benchmark and moreover incorporates condition (2), that is, (5) = (3) + (2).

Strictly speaking, we will refer to stationarity by means of equation (2).

3 Impatience in Intertemporal Choice

A noteworthy characteristic of exponential discounting $F(t) = e^{-kt}$ is that its instantaneous discount rate is constant through time, $\delta(t) = k$. Intuitively, the instantaneous discount rate represents the degree of (im)patience of the intertemporal choice quantified by the corresponding discounting function (Prelec and Loewenstein, 1991; Thaler, 1981). This property allows us a (non-dichotomous) transversal classification of both stationary and dynamic discounting functions, in the following way:

- Discounting functions with *increasing impatience*, whose instantaneous discount rate is increasing (also called with *decreasing patience*).
- Discounting functions with *decreasing impatience*, whose instantaneous discount rate is decreasing (also called with *increasing patience*).

Table 1 summarizes the concepts and gives some examples:

Table 1. A classification of discounting functions according to their impatience

A classification of discounting functions			
	Constant impatience	**Variable impatience**	
		Increasing impatience	**Decreasing impatience**
Expression of $F(t)$	$F(t)=e^{-kt}$	$F(t)=1-dt$	$F(t)=\dfrac{1}{1+it}$
Instantaneous discount rate	$\delta(t)=k$	$\delta(t)=\dfrac{d}{1-dt}$	$\delta(t)=\dfrac{i}{1+it}$

4 Subadditive and Superadditive Intertemporal Choice

In Section 2 (eq. (4)), we have defined additive discounting functions. Observe that, despite additivity is a kind of stationarity, its financial interpretation is the following. An investor is indifferent between the following behaviors:

- placing an initial amount during the interval $[0,d]$, then disinvesting and immediately placing the resulting amount during the interval $[d,d+t]$, or
- placing the initial amount during the whole interval $[0,d+t]$ without splitting it.

This condition allows us to present a (non-dichotomous) classification of non-additive discounting functions:

- *Subadditive* discounting functions: $F(d)F_2(d,t)<F(d+t)$.
- *Superadditive* discounting functions $F(d)F_2(d,t)>F(d+t)$.

Table 2 summarizes the concepts and gives some examples:

Table 2. A classification of discounting functions according to their additivity/non-additivity

Another classification of discounting functions			
	Additive	**Non-additive**	
		Subadditive	**Superadditive**
Expression of $F(t)$	$F(t)=e^{-kt}$	$F(t)=\dfrac{1}{1+it}$	$F(t)=1-dt$

Observe that the concepts of subadditive and superadditive discounting functions involve a certain degree of increasing and decreasing impatience, respectively. Nevertheless, despite their importance in intertemporal choice, they have been presented as independent of stationarity. Looking for the relationships among these features is useful in order to detect possible inconsistencies in individual choices (Cruz and Ventre, 2011b).

5 Conclusion

We have exhibited some features of intertemporal choice from the point of view of stationarity. Namely, starting from the spot and forward discounting factors, we can deduce the concepts of pure stationarity and additivity, and their respective violations: on the one hand, increasing and decreasing impatience, and, on the other hand, subadditivity and superadditivity. Despite these concepts are presented in the financial literature as independent as each other, in this paper we demonstrate their common origin and their relationships.

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A GIS as a Decision Support System for Planning Sustainable Mobility in a Case-Study

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Abstract. We show the process of research, selection, valuation, weighting and synthesis of a set of indicators to monitor the Coordination Plan of the District of Naples (Provincia di Napoli), during the realization process. The GIS Office and the Planning Office are responsible for choosing the set of indicators and their application, to evaluate if the goals of the Coordination Plan are achievable and if corrective actions should be undertaken as well. The process is specifically applied to enhance sustainable mobility, which is one of the most important goals of the Coordination Plan. Two urban areas lacking infrastructure connections are considered: North Naples and Giuglianese areas. The research is implemented via a GIS, that allows to combine a variety of data and information. It can also be applied in every step of SEA (Strategic Environmental Assessment) and EIA (Environmental Impact Assessment).

Keywords: GIS, sustainable mobility, indicator.

1 Introduction

This study is the result of a team project between the Ph.D. Program in "Evaluation methods for integrated preservation of architectural, urban and environmental heritage of the school of architecture" of the University of Naples Federico II and the GIS office of the District of Naples. The appraisal of projects, plans and programmes has been an increasingly important element in the quest for sustainable development. In the last two decades, in response to the limitations of a project-based approach, practises such as strategic environmental assessment (SEA) and sustainability appraisal have been promoted. The adoption of the European "SEA Directive", Directive2001/12/EC, is one of the highest profile policy development that has accompanied this shift.

Mobility is a major component in ensuring freedom of movement and good quality of life. It is strictly tied to the concept of sustainability, considering that more than 70% of European citizens live in urban areas. Enhancing sustainability is the attempt to relieve the pressures of current environmental challenges and mobility-related problems that most European urban areas are facing. Traffic volumes and congestion,

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air quality, noise pollution, consumption of non renewable resources, a high level of greenhouse gas emissions, social exclusion and urban sprawl are significant challenges to achieve sustainable urban development. Road safety is also an important challenge because of the social and economic costs of road accidents. Therefore it is an essential component of a sustainable mobility.

Local authorities are primarily responsible for urban policies according to the principle of subsidiarity. Nonetheless, the European Union has played a key role since 2001, with the adoption of the "White paper on transport policy". In order to offer specific help for promoting a new culture of urban mobility to local authorities, EU adopted a combination of policy intervention and guidance support. At a policy level, the Green paper "Towards a new culture of urban mobility" in 2007 and the "Action plan on urban mobility" in 2009 represent a milestone. With these two documents, the EC acknowledges the differences that exist between European cities, recognize that they all face similar challenges and stresses the need to implement an approach that should be as integrated as possible. Based on existing policy developments, the EC has also promoted several guidelines and instructions. CIVITAS (CIty-VITAlity-Sustainability Initiative) is probably one of the best known tool for helping European cities in implementing better integrated sustainable urban transport strategies. But there have been many other guidance initiatives, such as ELTIS, TERM, PROPOLIS, some of them dealing with the research of indicators for monitoring sustainable mobility plans.

In this study, we support the SEA process of the Coordination Plan of the District of Naples through the selection of an indicators set to monitor and evaluate the planned actions towards a sustainable mobility in the District. The method applied highlights the chance of transparency and social learning through the appraisal process.

2 The Coordination Plan of the District of Naples

The Coordination Plan of the District of Naples Area, so-called PTCP – Piano Territoriale di Coordinamento Provinciale—outlines the main features of the territorial development in its 92 municipalities. The related Planning Code prescribes that the offices in charge of the Plan and of the Geographical Information System should evaluate the performance of the plan during its realization, in order to verify if the goals are achievable and if corrective actions should be taken into account. The Environmental Report attached to the Plan prescribes to link each topic of the plan to a unique index resulting from a set of specific indicators [1]. Considering that the plan has not been approved yet and that major changes could still take place according to the new political guidance of the District, we did not develop a complete set of indices for all the topics of the Plan. We only examined one theme, i.e. sustainable mobility, and we applied it to 2 out of 11 areas which the district has been divided into. The main aim of this research is highlighting the method to build indices. We selected an indicators set suitable to the area of the District of Naples, in order to obtain a unique index for sustainable mobility, through the valuation and the weighting of the selected indicators. We applied this method to the North Naples Area and Giuglianese Area because they lack mobility infrastructures according to the plan itself. The goals of the Plan [1] linked to sustainable mobility are summarised in fig. 1.

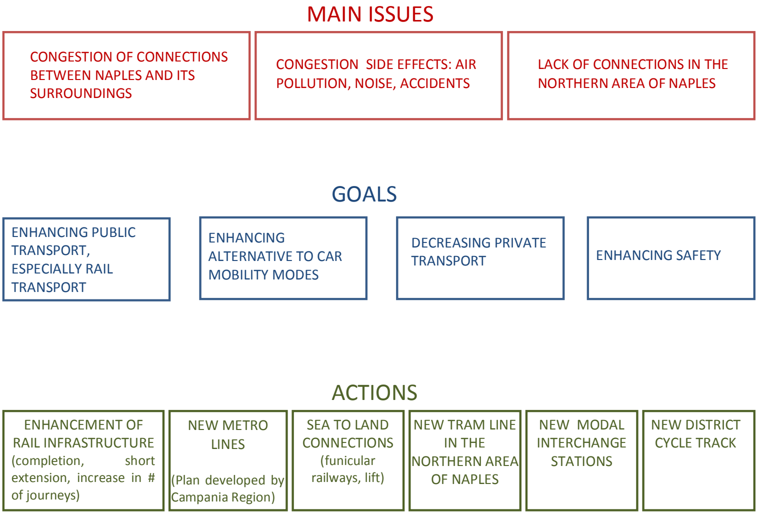

Fig. 1. The Coordination plan goals and actions

3 The Selection of Indicators

The selection of 22 indicators for sustainable mobility is the result of the intersection between the lists available in scientific literature and the databases available for the District of Naples. According to the PTCP Environmental Report, the use of existing databases is highly recommended to avoid double checking, which would result in a waste of public funds [1].

Lists of indicators applicable to sustainable mobility can be found in studies applied to urban development and to transportation system as well. Indeed, there are only few applications for sustainable urban mobility [2].

Indicators lists area available in several EU initiative. We have selected and analyzed in detail the following ones:

– European Common Indicators: the initiative was promoted in 1999, and resulted in the selection of ten indicators related to sustainable urban planning; some of them, such as local mobility and passenger transportation, are directly tied to sustainable mobility;
– TISSUE, Trends and Indicators for monitoring the EU thematic Strategy on Sustainable development of Urban Environment: it was developed from 2004 to 2007; it analyses the indicators developed within the V Programme in the DPSIR (driving forces, pressure, state, impact, response) framework; the selected indicators are divided in five areas, one of them is dedicated to mobility;
– TERM, Transport and Environment Reporting Mechanism: developed since 1999, the main goal of the initiative is defining an indicators list related to transport field and applicable to EU members; the list of indicators is updated and monitored on a yearly base;

– PROPOLIS, Planning and Research of Policies for Land use and transport for Increasing urban Sustainability: developed from 2000 to 2004, it researched and tested integrated land use and transport policies, tools and comprehensive assessment methodologies to define sustainable long run urban strategies and to demonstrate their effects in European cities.

List of indicators can be found in literature as well. Litman [3] lists 39 indicators divided into three groups (fundamental, optional and expert). Jeon and Amekudzi [4] analyse 16 studies related to transport indicators and get a list of 16 elements. Costa et al. [5] identify a list of indicators through a Internet research and then select a final list of 24 items resulting from a weighting process via analytical hierarchy process with an expert team. Barker [6] analyses the San Antonio (Texas) transport system, using per capita miles-vehicle travel as a key indicator. All the phenomena connected to congestion are linked to this indicator and strategies for increasing sustainability are suggested.

Moreover, all national and local databases available for the District of Naples have been analysed, in order to verify if they could contribute to the definition of the indicators. At a national level, ISTAT, the institute of statistics, ISPRA, the institute of environmental protection, and ACI, the Automobile Club of Italy, have been very helpful to define some of the selected indicators, such as the percentage of people either walking or biking to school or to work and the percentage of cars respecting emission thresholds. At a local level, we analyzed the database of ARPAC, the Regional Agency for Environmental Protection. It monitors a wide set of pollutants that are directly tied to mobility. The pollutant detection units net is not widespread in all the Region. In fact it is designed to monitor pollutants in the five main towns. However it has been helpful to define trends and thresholds for each indicator.

The District of Naples has a complementary pollutant detection net: it has not been fully developed yet but it is going to be increased to cover most of the area. The results of the monitoring activities are published by the Environment Section. The District is also undertaking several actions in the field of road safety: a database of roads accidents, seriously injured and deaths has already been developed. However it should be updated. Finally, the District of Naples is responsible for the local public transport companies through its Transport Section. Its database has been very helpful to define several indicators, such as the percentage of people living within 300 meters from a bus stop and the yearly public funding for local public transports.

We can say that one of the strength of this study has been the integration of all the databases available in the District of Naples in a planning perspective via the GIS.

To select indicators, two approaches can be used [7]: bottom up and top down. In the former, indicators are selected by citizenship and stakeholders. This approach enhances the transparency of the process and the social learning during the appraisal process. In the latter, indicators are selected by technicians: this is the approach that has been adopted in this study, according to the PTCP Planning Code.

The first result of the research is the list of 22 indicators (table 1) for monitoring sustainable mobility in the District of Naples.

4 Getting the Indicators

In order to get the indicators, it is necessary to process the available data in a geographical framework through GIS. Hereby, it is reported the procedure to get one of the selected indicators: the percentage of people living in a walking distance from a metro/railway station. The European Environmental Agency defines walking distance as a length that can be covered in fifteen minutes, that means more or less 300/500 meters. The data available to process this indicator (Fig. 2) are the shape files of the District railways system and of census islands, and the data base of national census.

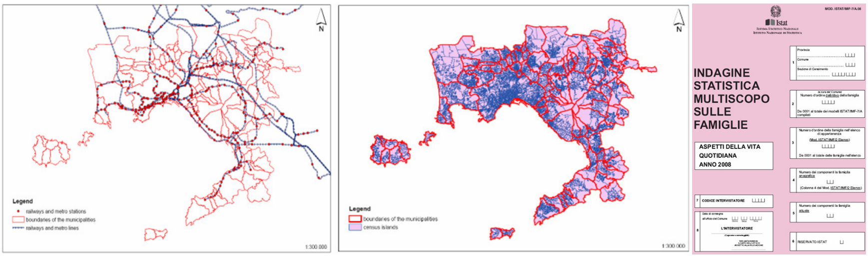

Fig. 2. Shape file of rail tracks and stations, shape file of census islands in the District of Naples, cover sheet of Italian census questionnaire

Firstly, the census data base was joined to the shapefile of census islands on the base of the islands Id. Then the average population has been calculated in every island (Fig. 3).

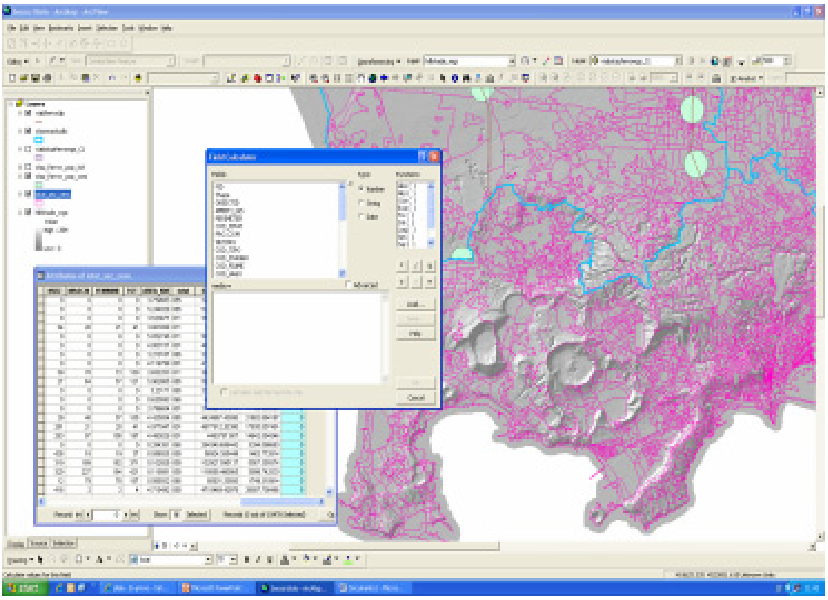

Fig. 3. Join of census data base to the shape file of the census

Secondly, a new feature class containing the geographic subset of our case study has been created by clipping the District data set on the boundaries of the analyzed area (Fig. 4).

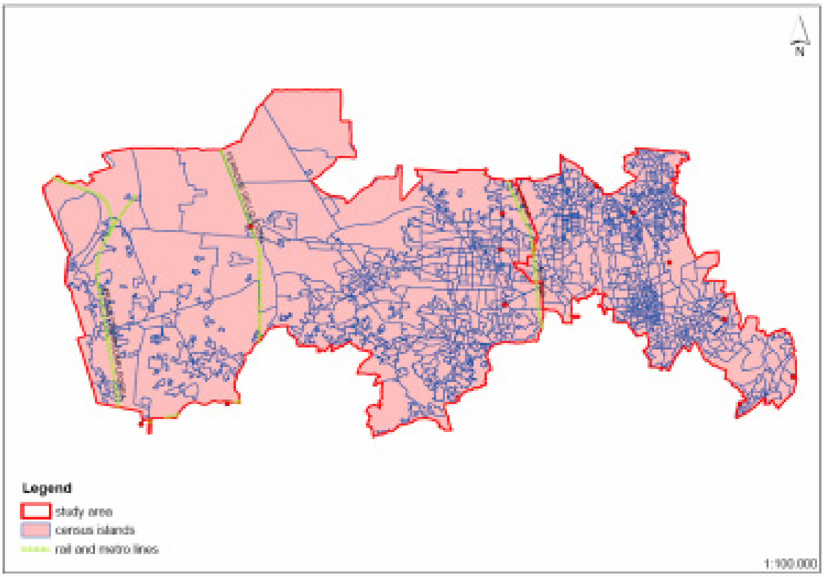

Fig. 4. Clip of the data set on the study area

Thirdly, buffer polygons have been created at 500 meters distance around the stations feature. The optional dissolve has been performed to remove overlapping buffers (Fig. 5).

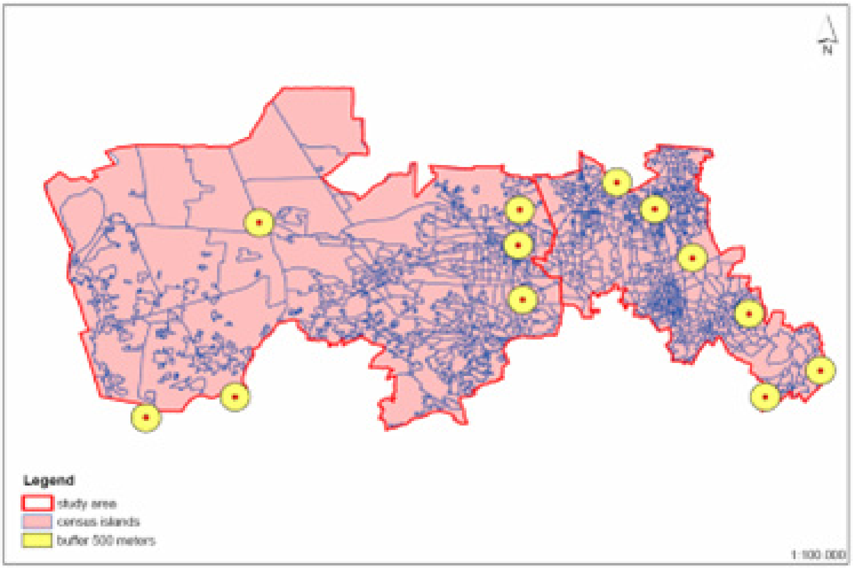

Fig. 5. Buffer of the metro/railways stations

Then the buffer shape file has been used as clip feature of the census islands. As a result we obtained the areas of census islands contained within buffers. Finally, we calculated the number of inhabitants within a walking distance from stations as the product of average population within census islands and the areas of census islands contained in the buffer. The calculus of percentages is the ratio between the number of people in the buffer areas and the total people living in each town multiplied by 100 (Fig. 6).

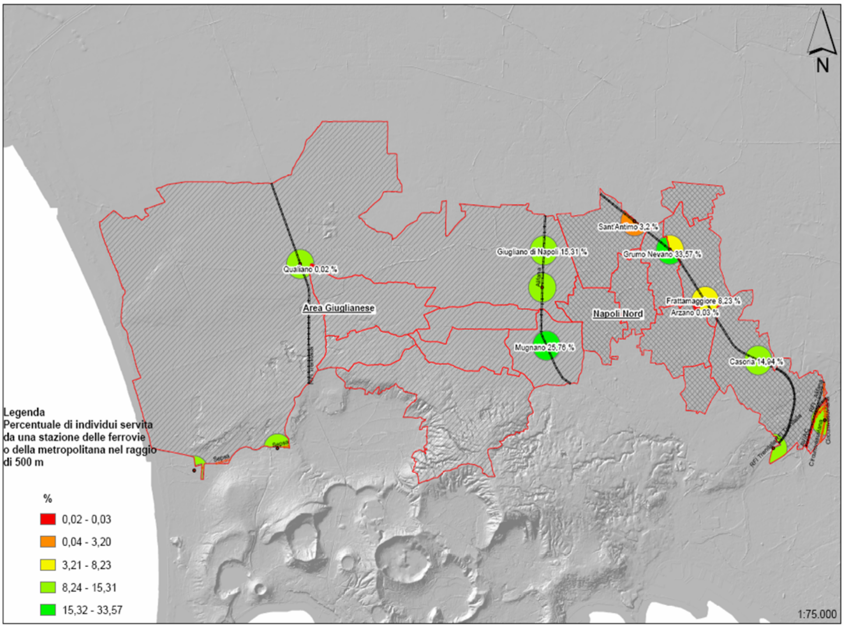

Fig. 6. Intersection of census islands with station buffers and count of % of people within 500 meters from a rail station

5 Getting the Index

In order to get the unique index for sustainable mobility, we evaluated and normalized the indicators first, and then weighted and aggregated them.

A value function aims at linking the values of the function to corresponding 'raw' indicator values x. There are several methods to link the indicator values to a function, such as direct rating and indifference methods. For further details on this topic see von Winterfeldt and Edwards [7, 8, 13, 14]. In this research we applied the direct rating method. First, the worst and best x values are defined and assigned the minimum and maximum values of the function (zero and unity, respectively). The best and worst 'raw' indicator values have been established for each indicators according to either thresholds within laws and regulations, or to best and worst value registered in the area, in the Region and in the State. According to Lautso et al. [9], not only should linearity be the starting point for value functions, but, if alternative scales for x are available, the one leading to the most linear value function should be adopted. In this study we adopted linear value function.

The indicator values do not have any common unit of measure. There would be no need of further processing to assess the plan, but the number of indicators to be taken into account is unpractical. It should be pointed out that we have selected a list of 22 indicators for only one of the topics of the plan. To standardise onto a common scale the indicators and to enable the weighting, we have used value functions with a y domain [0,1].

Thus each indicator must be assigned a weight that determines its importance compared to the other indicators. Doing this, we determine how a change in indicators value affects the index. There are several methods to weight indicators, such as analytical hierarchy process (AHP), trade off, rating and ranking methods. For further details on this topic see Nijkamp et al. [10] and Fusco Girard et al. [11]. This study applies a rating method named allocation of budget. The choice was based on feasibility: the use of the AHP method, that is very common in literature, would result too long. In addition, the high number of indicators could result in a high consistency ratio, that implies the repetition of comparisons. The weighting process started with a focus group with the GIS and Planning Offices. During this focus group, the list of indicators available has been presented and discussed. As a result of the debate, the list of indicators has been divided into four groups: environmental efficiency, economic efficiency, liveability and accessibility, which are part of the social dimension [2]. After a few days, the personnel was asked to fill in a questionnaire: they had to assign 100 scores among the four groups, and then to distribute them among the indicators. The data were processed and submitted to another focus group with the same personnel. They were asked to review the results of the weighting process and given the chance to change the budget allocation. Nobody changed the chosen scheme. The distribution of scores within the four groups is shown in Fig. 7. The weight of each indicator is reported in Table 1.

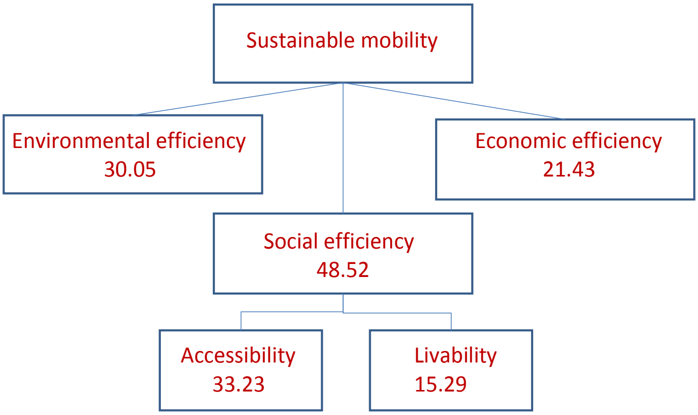

Fig. 7. Distribution of weights within the three dimensions of sustainability

Table 1. Selected indicators list and results of weighting process

INDICATORS			Dimension weights	Indicator weights
Environment	1	Maximum value of CO	30.05	7.39
	2	Maximum value of NO_2		5.50
	3	Maximum value of O_3		4.09
	4	Maximum value of particulate		5.88
	5	Maximum value of SO_2		4.93
	6	% of cars respecting emission thresholds		2.26
Economy	7	Yearly Public Transport funding	21.43	10.06
	8	# of vehicles / square km		3.38
	9	# of inhabitants / # of cars		4.13
	10	# of inhabitants / # of motor bikes		3.85
Society: liveability	11	Yearly # accidents	15.29	4.55
	12	Yearly # of seriously injured and deaths		3.48
	13	# of accidents / area		1.66
	14	# of seriously injured and deaths / area		1.38
	15	% of people exposed to harmful noise		4.21
Society: accessibility	16	% of people walking / biking to school / work	33.23	5.07
	17	% of people living within 300 meters from a bus stop		6.07
	18	% of people living within 500 meters from a metro station		7.86
	19	Bicycle track length / inhabitants		3.13
	20	Bicycle track length / main roads length		2.76
	21	Railways and main roads length / area		3.18
	22	Railways length / area		5.16
		TOTAL	100	100

The last step has been the aggregation of indicators:

$$\text{Mobility index} = \sum_{i=1}^{n} \omega_i v_i(x_i) \Big/ \sum_{i=1}^{n} \omega_i \qquad (1)$$

where:

- n = *# of indicators*
- ω_i = *weights of indicators*
- v_i = *value function*
- x_i = *raw indicators values.*

Fig. 8 shows the map of the mobility index resulting from the aggregation of data for each municipality of the North Naples and Giuglianese Area. Green colour represents a high performance of the plan actions. In contrast, red colour represents a bad performance. It must be pointed out that the map is the result of a hypothesis of a set of data during the realisation process.

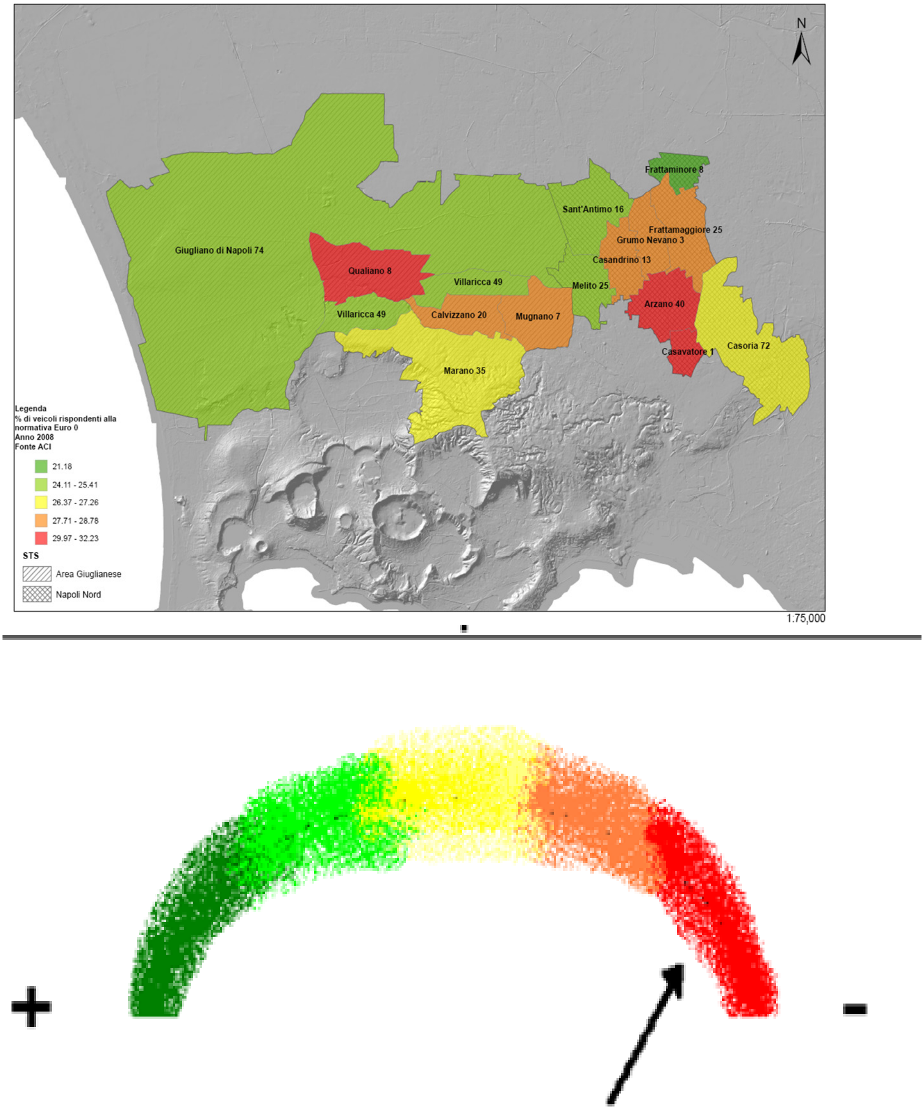

Fig. 8. Map showing the mobility index in the municipalities of Napoli Nord and Giuglianese areas. Green colour represents a high performance of the plan actions. In contrast, red colour represents a bad performance.

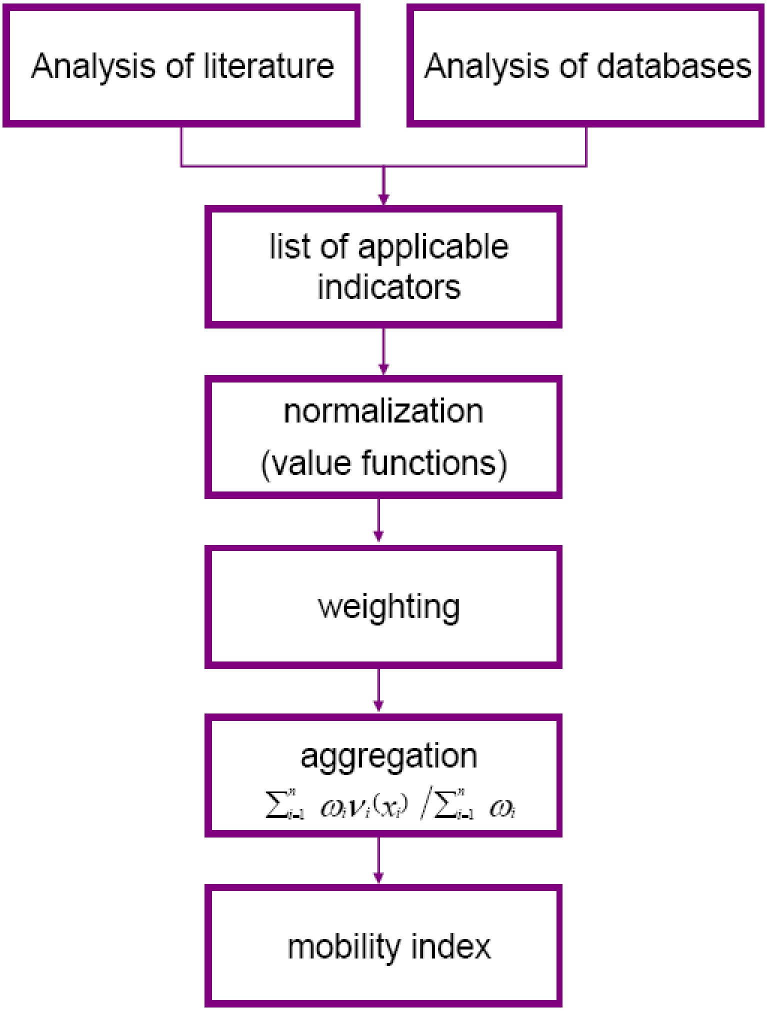

Fig. 9. Flow chart showing the process to get complex indices

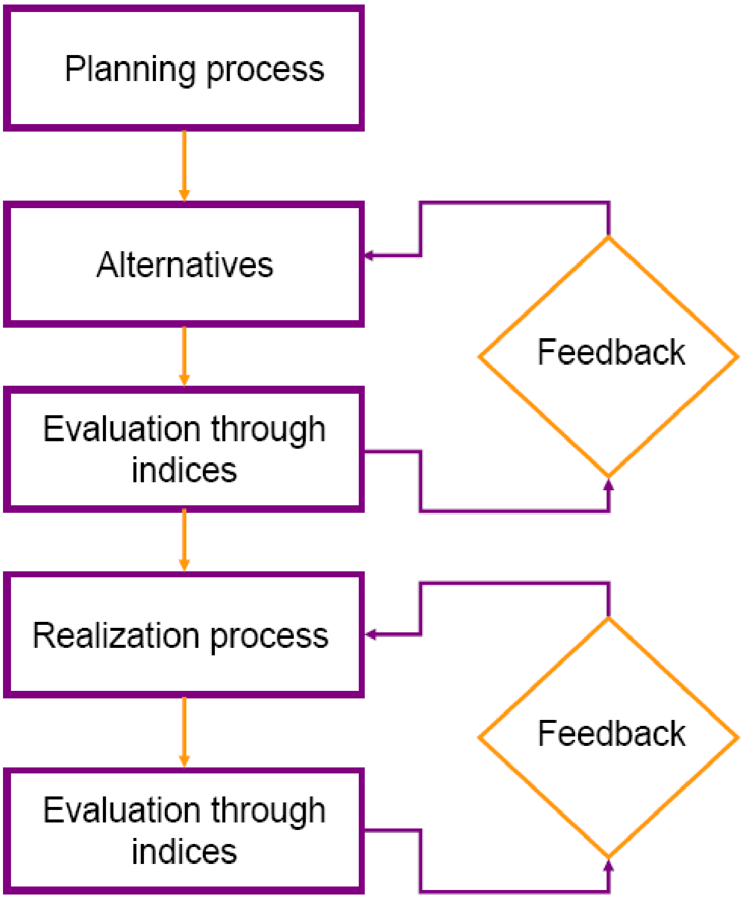

Fig. 10. Flow chart showing the role of indices in both planning and realization process

6 Conclusions

The flow chart in Fig. 9 shows the process that has been followed in this paper. The research designed and implemented a database via a GIS (see, e.g., [15, 16, 17]). Not only could this GIS be updated, but it can also convert the huge quantity of information within each indicator in one complex index. It can give immediately the trend towards sustainable mobility. The database is useful to compare either a zone of the district in different times or different areas of the district at the same time. We strongly believe that this is a really powerful tool to support decisions: it can help assessing the plan in its realization process and defining feedback actions as well (Fig. 10).

A similar experience has been done by the District of Milan, where the list of indices was also used to define alternative plans and to assess the best options during the planning process [12]. Naples has not adopted this procedure and uses indices only to evaluate the performance of the plan.

The process of building indices can help transparency in planning procedures and social learning through the appraisal process, if citizenship and stakeholders were involved. The combined use of participatory techniques and multiple criteria analysis takes conflicting interest into account and is the only way to solve them in a common vision. The method implemented in this research should be also applied to planning process (Fig. 10) and should involve most of the society.

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Decision-Making Analysis to Improve Public Participation in Strategic Energy Production Management

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Abstract. Environmental challenges decisions are often characterized by complexity, irreversibility, and uncertainty. Much of the complexity arises from the multiple-use nature of goods and services, difficulty in monetary valuation of ecological services and the involvement of numerous stakeholders. From this point of view, the objective of this paper is to propose a multicriteria methodological approach based on the Analytic Hierarchy Process methodology (AHP) in order to examine the scope and feasibility of AHP integrated with public participation approach. The main goal is to incorporate the prioritization criteria for the assessment of various energy policies for power alternatives, and evaluate these policies against these criteria. The three types of energy selected are: electricity production from wind farms, thermal power plants, and nuclear power plants. The results show that our model can help in the decision-making process and increase the transparency and the credibility of the process including tangibles and intangibles attributes.

Keywords: Environmental, Analytic Hierarchy Process, Key performance indicators, Public Participation.

1 Introduction

Environmental decisions are often characterized by complexity, irreversibility, and uncertainty. It is essential to use mathematical models, in order to evaluate vagueness and uncertainty (Hošková-Mayerová *et. al.* 2013). Under these circumstances, conventional methods such as cost-benefit analysis are not adapted to evaluate environmental decisions (Ananda, 2003). In this context, public participation and environmental impact assessment are recent developments in all countries; however, considerable advances have been made in their development. From this point of view multicriteria techniques are considered as a promising framework for evaluation since they have the potential to take into account conflictual, multidimensional, incommensurable, and uncertain effects of decisions explicitly (Carbone et al., 2000; Munda, 2000; Omann, 2000). The most widely used multicriteria methods include the Analytic Hierarchy Process (AHP), multiattribute utility theory, outranking theory, and goal programming. In this study we focused our attention on AHP because it proves useful

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when many interests are involved and a number of people participate in the judgment process (Saaty, 2005). So this method can be used in environmental challenges planning as it can accommodate conflictual, multidimensional, incommensurable, and incomparable sets of objectives. On the other hand, principles and practice of public participation can serve to promote environmental equity for disadvantaged social groups. The literature on participation and participatory processes stems broadly from two major areas: political sciences with discussions on democracy and citizenship especially within the context of regional and local planning (Pateman, 1970; Munro-Clark, 1990; Davis, 1996); and development theory (Wignaraja et al., 1991; Vettivel, 1992; Rahman, 1993; Nelson and Wright, 1995; Chambers, 1997). According to Creighton (2005), public participation, in principle, involves every person, although it may not be possible to *"reach"* all the individuals and some may not be interested in being involved. However, it is necessary to ensure that the participants involved represent those who are directly, or indirectly, affected by the proposed project and those who can positively or negatively influence the project outcomes (Lizarralde, 2011). These include (i) government/project initiators; (ii) lay public who are affected, or have interests in, the proposed project; (iii) private organizations, such as design institutes and construction companies; (iv) professional organizations and educational institutions; and (v) pressure groups such as the NGOs and mass media. By involving the public effectively in the decision-making process, project success may increase due to (i) a reduction in project time and cost (Creighton, 2005); (ii) the development of more innovative plans and solutions through the incorporation of the community's collective wisdom (CCSG, 2007); (iii) the accomplishment of needs or concerns of a cross-section of society without sacrificing the project goals (Woltjer, 2009); (iv) community acceptance, which can increase the legitimacy of government decisions (Moore & Warren, 2006); (v) an opportunity to promote mutual learning (Manowong & Ogunlana, 2008); (vi) a desire to protect individual and minority rights (Plummer & Taylor, 2004); (vii) an achievement of sustainable project lifecycle management (Varol, Ercoskun, & Gurer, 2011); and (viii) the promotion of collaborative governance (Enserink & Koppenjan, 2007). However, the success of public participation does not depend only on the genuine attitude of the project organizers in soliciting public opinion, but also requires the careful planning and organization of every participatory activity. The effectiveness of this practice in preventing or reducing environmental inequity definitely depends upon the use of participation methodology catering to the cultural and social needs of such groups. These methods need to provide appropriate forms of information, suitable venues for participation, and access to expertise and education which enable the public to understand policy issues and formulate preferences. The extent to which public preferences are incorporated in policy decisions determines the worth of public participation programs in promoting environmental equity (Hampton, 1999). From this point of view we noted that some of the participatory methods developed so far have often been criticized as lacking efficacy because of poor rigor and need of better structuring and analytical capabilities. In spite of this criticism, several studies applying the AHP to incorporate public participation have concluded that the AHP method is worth pursuing (Kangas, 1994, 1999; Ananda and Herath, 2003, Mau-Crimminsa *et al.* 2005). Thus, the objective of this paper is to propose a multi criteria methodological approach based on the AHP in order to examine the scope and feasibility of AHP integrated with public participation and stakeholder preferences in environmental challenges planning (De Felice *et al.* 2010). Our project is based on the assumption that the barriers to effective decision-making that exist between local communities and other stakeholders cannot be broken down by one party acting alone. The study is applied in a real case study concerning three different energy production processes: electricity production from wind farms, thermal power plants, and nuclear power plants because fossil fuels, renewable energy and nuclear (Entzinger and Ruan, 2006) are known as the three major energy sources of the world. Forsberg (2009) emphasized that these energy sources are treated as competing energy resources and economics and environmental constraints determine which energy source will be selected. In all projections, the world energy consumption is expected to increase depending on various demographic, technological and economic growth assumptions particularly in developing countries (Nakicenovic and Swart, 2000; Duffey, 2005; Fiore, 2006). The paper is organized as follows: the Analytic Hierarchy Process approach is described in section 2, the research approach and methodology is analyzed in section 3, the model and case study are proposed in section 4. Lastly, in the Conclusions the results are analyzed.

2 The Analytic Hierarchy Process: Theory Approach

The AHP was developed by Thomas Saaty (Saaty, 1980) in the early 1970s. The strength of the AHP approach lies in its ability to structure a complex, multiattribute, multiperson, and multiperiod problem hierarchically. In addition, it can also handle both qualitative (through representing qualitative attributes in terms of quantitative values) and quantitative attributes. The general approach followed in AHP is to decompose the problem and make pairwise comparisons of all the elements (attributes, alternatives) at a given level with respect to the related elements in the level above. AHP usually involves three stages of problem solving: the principles of decomposition, comparative judgments, and synthesis of priority. Some key and basic steps involved in this methodology are:

1. State the problem.
2. Broaden the objectives of the problem or consider all actors, objectives, and the outcome.
3. Identify the criteria influencing the behavior.
4. Structure the problem in a hierarchy of different levels constituting goal, criteria, sub-criteria, and alternatives.
5. Compare each element in the corresponding level and calibrate them on the numerical scale. This requires n(n-1)/2 comparisons, where n is the number of elements with the considerations that diagonal elements are equal or 1 and the other elements will simply be the reciprocals of the earlier comparisons.
6. Perform calculations to find the maximum eigenvalue and consistency index CI.
7. If the maximum eigenvalue and CI are satisfactory then decision is taken based on the normalized values; otherwise the procedure is repeated till these values lie in a desired range.

We note that pairwise comparisons of the elements in each level are conducted with respect to their relative importance toward their control criterion based on the principle of AHP. Saaty suggested a scale of 1-9 when comparing two components. The score of a_{ij} in the pairwise comparison matrix represents the relative importance of the component in row (i) over the component in column (j), i.e., $a_{ij}=w_i/w_j$. The score of 1 represents equal importance of two components and 9 represents extreme importance of the component i over the component j. The reciprocal value of the expression ($1/a_{ij}$) is used when the component j is more important than the component i. If there are n components to be compared, the matrix A is defined as in (1):

$$A = \begin{bmatrix} 1 & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ \frac{1}{a_{1n}} & \cdots & 1 \end{bmatrix} \tag{1}$$

After all the pairwise comparison is completed the priority weight vector (w) is computed as the unique solution of: $Aw= \lambda_{max}w$ where λ_{max} is the largest eigenvalue of matrix A.

As we said, in addition to final preference weights, the AHP permits to calculate the consistency index (Anderson et al., 1994; Saaty, 2000). This index measures preference transitivity for the person doing the pairwise comparisons. To illustrate the meaning of preference transitivity, if a person prefers choice A over B, and B over C, then do they prefer A over C? This index provides a useful check because the AHP method does not inherently prevent the expression of preference intransitivity when ratings are being performed. The AHP consistency index compares a person's informed preference ratings to those generated by a random preference expression process. The consistency index (CI) of the derived weights could then be calculated by Equation (2):

$$CI = \frac{\lambda_{max-n}}{n-1} \tag{2}$$

An arbitrary but generally accepted tolerable level of inconsistent preference scoring with the AHP is less than or equal to 10% of the total number of judgments. Finally, there is an issue of aggregation of individual decisions to form a group consensus decision. Saaty (2000) suggests that there are two possible types of group decision situations: (1) a small group of individuals working closely together with homogeneous preferences or (2) a larger number of individuals, possibly geographically scattered, with non-homogeneous preferences. The former requires a deterministic approach while the latter requires a statistical approach to group synthesis (Saaty, 2000). Definitely, the AHP facilitates multiple criteria weighting in complex choice situations. An advantage of the AHP is that it is capable of providing numerical weights to options where subjective judgments of either quantitative or qualitative alternatives constitute an important part of the decision process. This is often the case with natural resources planning on public lands. Thus, the purpose of this study was to test the AHP as a means of improving public participation in an energy production process.

3 Research Approach and Methodology

The main objective of our work is to develop a participatory decision-making model to use when dealing with key environmental decisions together with local communities and other important stakeholders. To achieve this participatory decision-making model, the following objectives are envisaged:

- Balance the starting knowledge level of all partners.
- Analyze and parameterize conflicts of interest in natural resource management.
- Identify the reference cases.
- Model the decision-making process helping the local communities.
- Model the participation process.
- Improve decision-making procedures.
- Develop proper support for participation, discussion, learning, evaluation, prioritization, communication, traceability, etc.
- Improve the capability of local communities to become a partner when defining natural resource management policies.
- Develop procedures for collective working on line.
- Construct an Analytic Network Model to enhance participatory approaches.

To structure the decision problem we identified and structured objectives which required careful empirical and literature investigations (De Felice and Petrillo, 2010). They provide the basis for quantitative modeling. According to Keeney (1992) we can classify objectives in two types: fundamental objectives and means objectives. The fundamental objectives are the issues or attributes that stakeholders genuinely care about, and means objectives are ways to accomplish the fundamental objectives. Objective hierarchies can be constructed using this classification. For example, ecologically sustainable development could be the fundamental objective and economic, social and environmental objectives could be the means objectives in case of forest decisions. According to these consideration we identified attributes to measure these objectives. Research framework is illustrated in Figure 1.

Here below is the description of the methodological steps:

- **STEP 1. Definition of the problem.** The aim of this step is to identify the environmental problem with the local community.
- **STEP 2. Constructing the AHP model.** The decision-making process will be structured by AHP techniques in respect of social, environmental and economic principles. Problem components as well as tangible/intangible decision variables will be defined and clustered. Relations among components will be defined as well as the definition of the scale of preferences. Problem structuring will be carried out by considering scientific literature as well as judgments of experts and public decision makers.
- **STEP 3: Evaluation of priorities**. The aim of this step is to evaluate priorities among different alternatives.

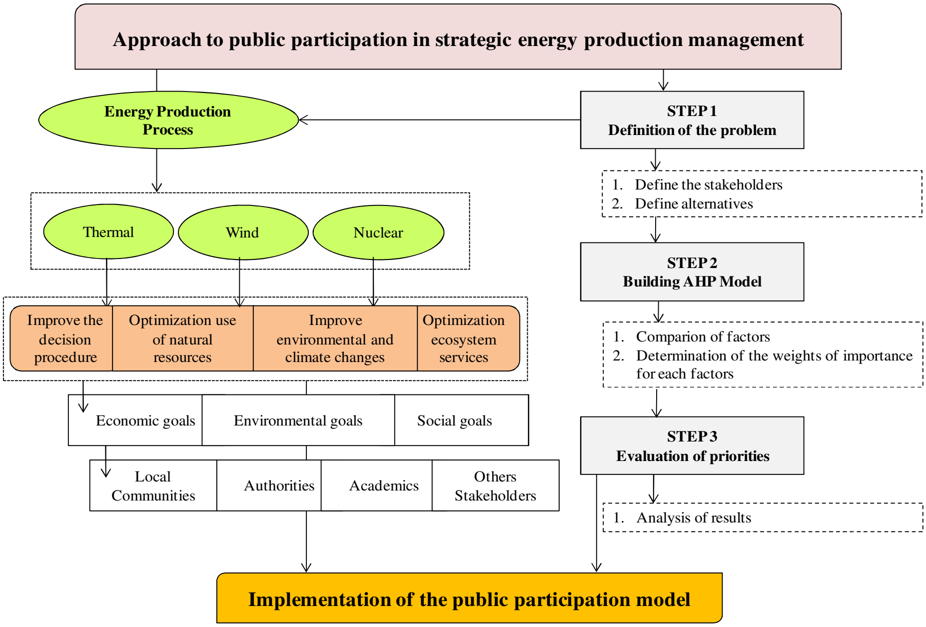

Fig. 1. Research framework

4 Case Study

The objective of this paragraph is to examine through a simple case study the scope and feasibility of our methodology in incorporating stakeholder preferences into energy environmental policies.

4.1 STEP 1. Definition of the Problem

The main objective is to identify a priority schedule within the framework of the global environment and energy policies to assist decision makers in the selection of energy production options. To achieve this aim, an approach based on comparisons of three basic energy production processes: nuclear, renewable energy (wind) and thermal power have been implemented.

4.1.1 Selection of Stakeholders

Identifying or rather selecting stakeholder groups (policy makers, planners, and administrators in government and other organizations is a difficult task. The process of selection has to be open and transparent. We chose a group composed of Industrials, Citizens, Environmentalists, Agriculturists, and Tourism Operators.

4.1.2 Selection of Alternatives

As alternatives we chose three types of energy: electricity production from wind farms, thermal power plants, and nuclear power plants. In Table 1 the alternatives are described.

Table 1. Characterization of Alternatives

Alternatives	**Features**	**Capacity factor**	**Investment costs (average value)**	**Operating and maintenance costs/capital %**
Nuclear Plants	Public acceptance does not exist due to some uncertainties related to nuclear energy such as economic performance, proliferation of dangerous material, the threat of terrorism, operation safety, and radioactive waste disposal.	60–100%	€3000/kW	50
Thermal Plants	Coal is an essential energy source to generate electricity for thermal power plants. The poor quality of this lignite is responsible for a considerable amount of air pollution.	70–90%	€1300/kW	97
Wind Plants	Wind power as a practical electric power generation is now becoming more prominent among renewable and the other energy options and all researches focused on improving wind energy generation. Wind energy is accepted by public, industries, and politics as a clean, practical, economical, and eco-friendly option.	20–40%,	€1100/kW	25

Other points to be considered include:

- Environmental risks, impacts, and waste-emissions of wind energy production systems can be neglected compared to others, and depend on regional characteristics.
- Nuclear energy is able to compete with other energy sources when the operating cost is less than 210$/kWh year or 2.4cent/kWh (Yildirim and Erkan, 2007).

- Energy policies are restricted by global and international long-term objectives of environmental policies.
- The capacity factor is defined as the ratio of the actual energy produced in a given period to the hypothetical maximum possible capacity (i.e. running full time at rated power).

4.2 STEP 2. Constructing the AHP Model

Developing effective energy policy requires that policy-makers take into account the multiple objectives of multiple stakeholders and their conflicting interests. From this point of view the structure of the proposed AHP Participatory Model is shown in Figure 2.

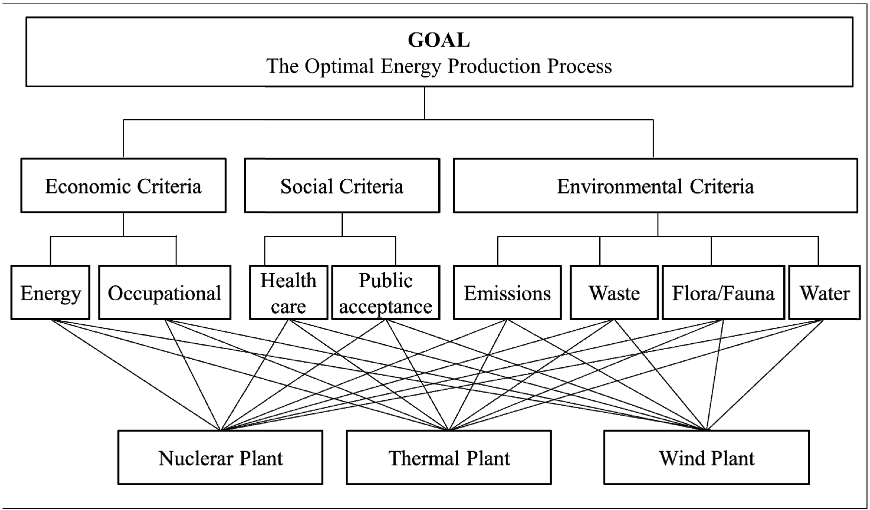

Fig. 2. Proposed AHP Participatory Model

The use of *multicriteria approach* involves developing a decision model comprising decision attributes (criteria), sub-criteria, and alternatives. Criteria and sub-criteria are described in Table 2.

Table 2. Criteria and Sub-criteria

Criteria	Sub-criteria	
Social Goals *Social benefits achieved from the development of sustainability level.*	Health Care	Activities that not ensure safeguard of population
	Public Acceptance	Acceptance of qualitative and quantitative consequences on the environment.
Economic Goals *Processes associated with planning, scheduling, and coordinating activities. The effectiveness in managing assets to support environmental demand satisfaction.*	Energy	Evaluation of total energy production.
	Occupational	Activities can build value through new jobs.
Environmental Goals *Activities can build value through sustainable methods.*	Emissions	The evaluation and implementation of actions to reduce environmental impacts.
	Waste	The needs to reduce waste due to energy production.
	Flora/Fauna	Evaluation of actions helps to maintain biodiversity and reduce environmental damage.
	Water	The need to satisfy the requirements for water preservation.

In Table 3 a scenario with key relationships of AHP factors is shown.

Table 3. Relation between Sub-criteria and Alternatives

Sub Criteria	Nuclear	Wind	Thermal
Health Care	High risks	No risks	Low risks
Public Acceptance	High resistance	Low resistance	Medium resistance
Energy	High production	Low production	Medium production
Occupational	High value	Medium value	Medium value
Emissions	Radiation	Noise	CO2, NO_x, SO_2, HM, HW, and fly ash
Waste	High. Radioactive waste. Difficult and expensive disposal and storage	No waste	Medium
Flora/Fauna	Bad preservation	High preservation	Bad preservation
Water	Bad	Good	Bad

4.2.1 Comparison of Factors

Since the problem has been structured as a hierarchy, the relations between elements in succeeding levels are obtained by making pairwise comparisons.

4.2.2 Determination of the Weights of Importance for Each Factor

The weights of the decision objectives from the stakeholder group's point of view and that represent the results of our model are presented in Table 4.

Table 4. Weights of decision objectives

Criteria	Sub Criteria	Nuclear	Wind	Thermal	Consistency Index (CI)
Social	Health Care	0,157	0,593	0,249	0,051
	Public Acceptance	0,155	0,519	0,326	0
Economic	Energy	0,686	0,126	0,186	0,09
	Occupational	0,593	0,157	0,249	0,051
Environmetal	Emissions	0,117	0,614	0,268	0,07
	Waste	0,09	0,279	0,626	0,082
	Flora/Fauna	0,121	0,558	0,319	0,0175
	Water	0,131	0,66	0,208	0,051

4.3 STEP 3: Evaluation of Priorities

The global priorities can be calculated on the basis of: 1) the weighting scheme for the stakeholder groups; 2) the importance of objectives from the point of view of the stakeholder groups; 3) the relative priorities of decision alternatives with respect to the objectives. Final results, the following score; 1) Nuclear 0.190; 2) Wind 0.271; 3) Thermal 0.537.

4.3.1 Analysis of Results

We can note that nuclear power has a higher priority of economic goal because of its high capacity factor, efficiency and ever ready generating electricity. Social and environmental goals have lower priority numbers than the thermal plant and wind plant. The thermal plant has the lowest priority from social goals because of huge amounts of waste and high air pollution profiles related with climate changes and global warming. Environmental and social factors make wind power the leader in the public eye as it has negligible negative impacts on the environment and on human health. Public acceptance of social factors, which is the reason for the lowest priority number for nuclear power, is the main indicator on decision-making. Figure 3 shows the results of sensitivity analysis of economic goal. The vertical dotted line is initially set at 0.5 on the X-Axis for the priority of the Goal (left figure). The respective priorities of the alternatives is indicated by the Y-Axis values where their lines intersect the vertical line: Nuclear is 0.393, Thermal is 0.242, and Wind is 0.365. Grab the vertical line with and move it to the right to see that there is a crossing point around 80% (0.80) after which the Nuclear is again the best choice. Sensitivity can be done for the other criteria. The sensitivity analysis shows how the alternatives were prioritized relative to other alternatives with respect to each objective as well as overall objective.

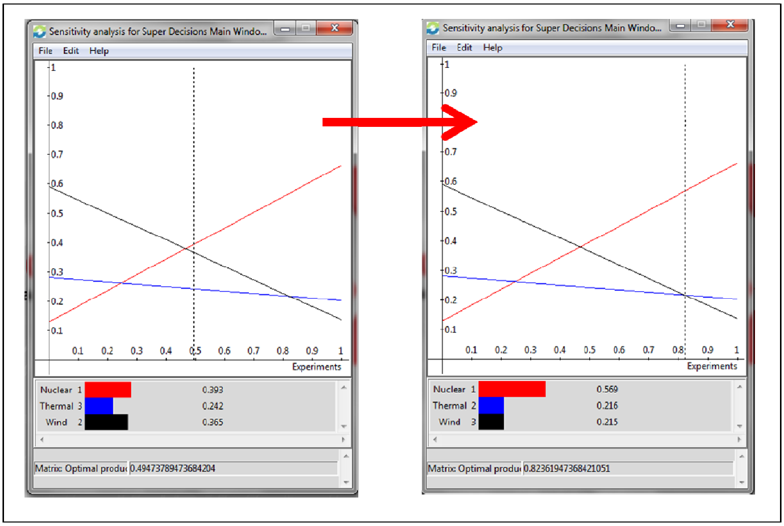

Fig. 3. Sensitivity analysis

5 Conclusions

Quantifying stakeholder preferences in environmental management is a complex task. From this point of view the methodologies of public participation can be judiciously selected and modified to promote equity. The most critical aspect of promoting equity through participation is the extent to which public preferences are incorporated in policy decisions which govern environmental quality. Limited incorporation reduces participation programs to an inconsequential democratic drama. In this context, the objective of this paper was to propose a multicriteria methodological approach based on the Analytic Hierarchy Process methodology (AHP) in order to examine the scope and feasibility of a modeling process integrated with public participation for environmental assessment. AHP allows for participation of more than one person as a decision maker, which is important in dealing with several stakeholder groups. Another advantage of the AHP is the ability to include many decision makers in an electronic meeting environment. Therefore, we decided to use the AHP in this study for the following reasons: (1) the AHP is a structured decision quantitative process which can be documented and replicated, (2) it is applicable to decision situations involving multi-criteria, (3) it is applicable to decision situations involving subjective judgment, (4) it uses both qualitative and quantitative data, (5) it provides measures of consistency of preference, (6) there is ample documentation of AHP applications in academic literature, (7) the AHP is suitable for group decision-making.

The results of this study could:

- Provide valuable information regarding decision-making tools for strategic environmental management.
- Facilitate discussions on the environmental matter;
- Increase public awareness of environmental/social/economic effects of alternatives;
- Spread environmental information;
- Increase e-participation (e-Democracy) of people in the decision-making process to achieve public awareness consensus;
- Point out decision makers and procedures of decision processes.

The end result of the model is a measure of the decision maker's relative preference of one attribute over another attribute. It is concluded that the model is an effective way to improve participatory decision-making in complex decision situations and to clarify public preferences more rigorously. The application presented here has some limitations therefore future research should focus on: (1) integrating AHP model with benefits, opportunities, costs, and risks analysis; (2) improving cooperation between the respondent and the analyst; (3) designing innovative and user-friendly questioning protocols; (4) developing full-scale case studies.

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Monadic Social Choice

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Abstract. In this paper, we show how monads and substitutions allows for a separation between social choice and social 'choosing'. Choice as value and choosing as operation is modeled using underlying signatures and related term monads. These monads are arranged over Goguen's category $\mathtt{Set}(L)$, which provides the internalization of uncertainty both in choice as well as choosing.

Keywords: Choice function, monad, Kleisli category, substitution.

1 Introduction

The discipline of social choice originates from objective probability used in justice and as pioneered by French mathematicians Borda [4] and de Condorcet [5]. The balance between individual liberty and societal authority was related to the risk of innocent citizens being wrongly convicted and punished for crime. Social justice as well as social order required that particular risk to be minimized. Condorcet argued that judicial tribunals could manage probabilities and errors, taking into account also some minimum required plurality to guarantee the probability. Uncertainty based voting schemas then are just behind the corner, and is the historical prerequisite also for choice theory.

Objective probability eventually turns subjective, and probabilists believe they have keys to inference mechanisms as well. Some modern time improvements can be seen in these directions, but generally speaking, probability is not logical.

The subject of social choice was revived in the 20th century by Arrow [1] who, facing the inconsistencies of group decisions, put the discipline of social choice in a structured axiomatic framework leading to the birth of social choice theory in its modern form. As Sen [14] pointed out "Arrow's impossibility theorem is a result of breathtaking elegance and power, which showed that even some very mild conditions of reasonableness could not be simultaneously satisfied by any social choice procedure, within a very wide family". Accordingly, impossibility results in social choice theory have been seldom considered as being destructive of the possibility of social choice and welfare economics. Sen [14] argued against that view, claiming that formal reasoning about postulated axioms, as well as informal understanding of values and norms, both point in the productive direction of overcoming social choice pessimism and of avoiding impossibilities.

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Arrow's focused on individual values and ranking, together with impossibility theorems, and dealt with individual preferences and choice processes. Probabilities are not in the ingredients, but rather operators and functions, and properties about them. There are no counterparts in probability theory for these concepts. From a logical point of view, Arrow uses implicitly underlying signatures, even if they are never formalized, and since they are not formalized, it is never seen that these choice functions indeed could have been integrated into a logical framework. Arrow follows von Neumann and Morgenstern's "mathematical tradition" [13] in his success stories of economic and social sciences, but also without ending up in any logical framework.

In [6] we assumed that making a distinction between choice and mechanism for choice could advantageously enrich the theoretical framework of social choice theory opening the way to categorical approaches. The idea of generalizing the Arrow's paradigm through a new architecture of social choice procedure was introduced, e.g., by Bandyopadhyay [2] and then extended in [3] where a social choice procedure is proposed which depends both on the way a set of alternatives is broken up into the subsets and the sequence in which each of these subsets is taken up for consideration.

Our standpoint in this paper is that social choice functions must identify the difference between 'we choose' and 'our choice', the former being the operation of choosing, the latter being the result of that operation. We view this from a signature point of view, i.e., using formalism involving signatures and their algebras. Classically, and without consideration of underlying categories, a signature $\Sigma = (S, \Omega)$ consists of sorts, or types, in a set S, and operators in a set Ω. More precisely, Ω is a family of sets $(\Omega_n)_{n \leq k}$, where n is the arity of the operators in Ω_n. An operator $\omega \in \Omega_n$ is syntactically written as $\omega : s_1 \times \cdots \times s_n \longrightarrow s$, where $s_1, \ldots, s_n, s \in S$. Operators in Ω_0 are constants. Given a set of variables we may construct the set of all terms over the signature. This set is usually denoted $\mathsf{T}_\Omega X$, and its elements are denoted $(n, \omega, (t_i)_{i \leq n})$, $\omega \in \Omega_n$, $t_i \in \mathsf{T}_\Omega X, i = 1, \ldots, n$, or $\omega(t_1, \ldots, t_n)$.

In this algebraic formalism, ω corresponds to the operation of choosing, and $\omega(t_1, \ldots, t_n)$ is a result of choosing, i.e., a choice. Note that both the operator ω as well as the term $\omega(t_1, \ldots, t_n)$ are syntactic representations of mechanisms for choosing and choices. The semantics of ω is a mapping $A(\omega) : A(s_1) \times \cdots \times A(s_n) \longrightarrow A(s)$.

Social choice is basically seen as a mapping

$$f : X_1 \times \cdots \times X_n \longrightarrow X$$

where agents $i \in \{1, \ldots, n\}$ are choosing or arranging elements in sets X_i. The aggregated social choice related to $x_i \in X_i, i = 1, \ldots, n$ is then represented by $f(x_1, \ldots, x_n)$. In most cases $X_1 = \cdots = X_n = X$, and the social choice function is then

$$f : X \times \cdots \times X \longrightarrow X. \tag{1}$$

This can be seen either as a semantic representation which has an underlying choice operator in its signature, or it is syntactic and elements in X are basically constant operators, i.e., $X = \Omega_0$ in some operator domain.

In the view of 'choosing' we would replace X with the set of substitutions. More precisely, let C be the Kleisli category $\mathtt{Set}_{\mathbf{T}_\Omega}$, where $\mathbf{T}_\Omega$ is the term monad over $\mathtt{Set}$. Elements σ in $\mathrm{Hom}_{\mathsf{C}}(X, X)$ are then substitutions $\sigma : X \longrightarrow \mathsf{T}_\Omega X$, and $\mathcal{X} = \mathrm{Hom}_{\mathsf{C}}(X, X)$ is the corresponding set of substitutions capturing the notion of individual choice and choosing. The choice function

$$\varphi : \mathcal{X} \times \cdots \times \mathcal{X} \longrightarrow \mathcal{X} \tag{2}$$

therefore may consider and compute with not just the output, the choice, but also with all the operators, i.e., the whole mechanism of choosing, leading to that particular term.

We will expand these ideas to cover uncertainty modeling, and we will show how representation of uncertainty can be seen as related to an appropriate choice of an underlying category. Furthermore, we will see how all this can be embedded into a many-sorted framework.

In the literature there are some previous categorical approaches Keifing's [11] objective is similar to ours, namely a unification of framework, and indeed unification of concept, results, and theorem framework based on more or less formal methods. Keiding involves categories and Hom functors, but the categorical framework remains rather poor, as there is no use of operators. The set $\mathrm{Hom}(A, \mathsf{P}X)$ indeed comes with no structure. It is simply a set of mappings. In the end, we will have a $\mathrm{Hom}_{\mathsf{C}}$ functor, where C also can carry uncertainty once (many-sorted) term monads are constructed over Goguen's category $\mathtt{Set}(L)$. It then integrates both operators and uncertainties, and even more so, operators working internally over uncertainties. Eliaz [10], making no reference to [11], does not add any new formalism or formal methodology.

2 Monads and Underlying Categories

A monad (or triple, or algebraic theory) over a category C is denoted $\mathbf{F} = (\mathsf{F}, \eta, \mu)$, where $\mathsf{F} : \mathsf{C} \longrightarrow \mathsf{C}$ is a covariant functor, and $\eta \colon \mathsf{id} \longrightarrow \mathsf{F}$ and $\mu \colon \mathsf{F} \circ \mathsf{F} \longrightarrow \mathsf{F}$ are natural transformations satisfying $\mu \circ \mathsf{F}\mu = \mu \circ \mu\mathsf{F}$ and $\mu \circ \mathsf{F}\eta = \mu \circ \eta\mathsf{F} = \mathsf{id}_{\mathsf{F}}$. Any monad $\mathbf{F}$ over a category C, gives rise to a Kleisli category $\mathsf{C}_{\mathbf{F}}$ whose objects are $\mathrm{Ob}(\mathsf{C}_{\mathbf{F}}) = \mathrm{Ob}(\mathsf{C})$, and morphisms are $\mathrm{Hom}_{\mathsf{C}_{\mathbf{F}}}(X, Y) = \mathrm{Hom}_{\mathsf{C}}(X, \mathsf{F}Y)$. Morphisms $f : X \longrightarrow Y$ in $\mathsf{C}_{\mathbf{F}}$ are morphisms $f : X \longrightarrow \mathsf{F}Y$ in C, with $\eta_X : X \longrightarrow \mathsf{F}X$ being the identity morphism. Composition of morphisms in $\mathsf{C}_{\mathbf{F}}$ is defined as

$$(X \xrightarrow{f} Y) \diamond (Y \xrightarrow{g} Z) = X \xrightarrow{\mu_Z \circ \mathsf{F}g \circ f} \mathsf{F}Z.$$

Let L is a completely distributive lattice, and let $\mathtt{Set}(L)$ be the (Goguen) category where objects are pairs (A, α) with $\alpha \colon A \longrightarrow L$, and morphisms $(A, \alpha) \xrightarrow{f} (B, \beta)$ are mappings $f \colon A \longrightarrow B$ such that $\beta(f(a)) \geq \alpha(a)$ for all $a \in A$. The category $\mathtt{Set}$ is not isomorphic to $\mathtt{Set}(2)$, where $2 = \{0, 1\}$.

For a set of sorts S, the many-sorted category of sets $\mathtt{Set}_S$ has objects $\{X_\mathtt{s}\}_{\mathtt{s}\in S}$, where $X_\mathtt{s}$, $\mathtt{s} \in S$, are objects in $\mathtt{Set}$. Morphisms $f_\mathtt{s} : X_\mathtt{s} \longrightarrow Y_\mathtt{s}, \mathtt{s} \in S$, in $\mathtt{Set}$, produce morphisms $\{f_\mathtt{s}\}_{\mathtt{s}\in S} : \{X_\mathtt{s}\}_{\mathtt{s}\in S} \longrightarrow \{Y_\mathtt{s}\}_{\mathtt{s}\in S}$ in $\mathtt{Set}_S$. For a morphisms $\{g_\mathtt{s}\}_{\mathtt{s}\in S} : \{Y_\mathtt{s}\}_{\mathtt{s}\in S} \longrightarrow \{Z_\mathtt{s}\}_{\mathtt{s}\in S}$, composition with $\{f_\mathtt{s}\}_{\mathtt{s}\in S}$ is sort-wise, i.e., $\{g_\mathtt{s}\}_{\mathtt{s}\in S} \circ \{f_\mathtt{s}\}_{\mathtt{s}\in S} = \{g_\mathtt{s} \circ f_\mathtt{s}\}_{\mathtt{s}\in S}$. For objects in $\mathtt{Set}_S$, set operations are also defined sort-wise.

Functors $\mathsf{F}_\mathtt{s}, \mathsf{G}_\mathtt{s} : \mathtt{Set} \longrightarrow \mathtt{Set}$ can be lifted to functors $\mathsf{F}_S = \{\mathsf{F}_\mathtt{s}\}_{\mathtt{s}\in S}$ and $\mathsf{G}_S = \{\mathsf{G}_\mathtt{s}\}_{\mathtt{s}\in S}$ from $\mathtt{Set}_S$ to $\mathtt{Set}_S$, and composition is again sort-wise, i.e., $\mathsf{F}_S \circ \mathsf{G}_S = \{\mathsf{F}_\mathtt{s} \circ \mathsf{G}_\mathtt{s}\}_{\mathtt{s}\in S}$.

The product $\prod_{i\in I} \mathsf{F}_i$ and coproduct $\coprod_{i\in I} \mathsf{F}_i$ of covariant functors F_i over $\mathtt{Set}_S$ is defined as

$$(\prod_{i\in I} \mathsf{F}_i)\{X_\mathtt{s}\}_{\mathtt{s}\in S} = \prod_{i\in I} \mathsf{F}_i\{X_\mathtt{s}\}_{\mathtt{s}\in S}$$

and

$$(\coprod_{i\in I} \mathsf{F}_i)\{X_\mathtt{s}\}_{\mathtt{s}\in S} = \coprod_{i\in I} \mathsf{F}_i\{X_\mathtt{s}\}_{\mathtt{s}\in S}$$

with morphisms being handled accordingly.

The many-sorted underlying category $\mathtt{Set}_S(\{L\}_{\mathtt{s}\in S})$ is defined sort-wise with respect to L. That is, objects are indexed sets of pairs $\{(A_\mathtt{s}, \alpha_\mathtt{s})\}_{\mathtt{s}\in S}$ with $\alpha_\mathtt{s} : A_\mathtt{s} \longrightarrow L_\mathtt{s}$ and morphisms $\{f_\mathtt{s}\}_{\mathtt{s}\in S}\colon \{(A_\mathtt{s}, \alpha_\mathtt{s})\}_{\mathtt{s}\in S} \longrightarrow \{(B_\mathtt{s}, \beta_\mathtt{s})\}_{\mathtt{s}\in S}$ are such that $\beta_\mathtt{s}(f_\mathtt{s}(a)) \geq_\mathtt{s} \alpha_\mathtt{s}(a)$ for all $\mathtt{s} \in S$ and $a \in A_\mathtt{s}$.

3 The Term Monad over $\mathtt{Set}_S(\{L\}_{\mathtt{s}\in S})$

A many-sorted signature $\Sigma = (S, \Omega)$ over $\mathtt{Set}_S$ consists of a set S of sorts considered as a set in ZF, and a set Ω of operators as an object in $\mathtt{Set}$. Operators in Ω are indexed by sorts and syntactically denoted $\omega : \mathtt{s}_1 \times \cdots \times \mathtt{s}_n \longrightarrow \mathtt{s}$, where n is the arity of the operation. We may write Ω_n for the set (as an object of $\mathtt{Set}$) of n-ary operations. Clearly $\Omega = \coprod_{n\leq k} \Omega_n$, where k is a cardinal number representing the 'upper bound of arities'.

Let now $\{(\Omega_n, \vartheta_n) \mid n \leq k\}$ be a family of objects in $\mathtt{Set}(L)$. Further, let $(\Omega, \vartheta) = \coprod_{n\leq k}(\Omega_n, \vartheta_n)$ be a fuzzy operator domain, i.e., $\vartheta_n : \Omega_n \longrightarrow L$. Note, we write $\Omega^{\mathtt{s}_1\times\cdots\times\mathtt{s}_n \longrightarrow \mathtt{s}}$ for the set of operations $\omega : \mathtt{s}_1 \times \cdots \times \mathtt{s}_n \longrightarrow \mathtt{s}$.

A many-sorted signature $\Sigma = (S, (\Omega, \vartheta))$ over $\mathtt{Set}(L)$ consists again of a set S of sorts considered as a set in ZF, and a pair (Ω, ϑ) (of operators) as an object in $\mathtt{Set}(L)$.

Let

$$\mathsf{T}^0_\Sigma = \mathrm{id}_{\mathtt{Set}_S(\{L\}_{\mathtt{s}\in S})}$$

and

$$\mathsf{T}^0_{\Sigma,\mathtt{s}}\{(X_\mathtt{s}, \xi_\mathtt{s})\}_{\mathtt{s}\in S} = (X_\mathtt{s}, \xi_\mathtt{s}).$$

For convenience, given an object A in a category $\mathtt{C}$, we will make use of the constant functor $A_\mathtt{D} : \mathtt{D} \longrightarrow \mathtt{C}$ which assigns any object in $\mathtt{D}$ to A, and morphisms in $\mathtt{D}$ to the identity morphism id_A in $\mathtt{C}$. Further, for $\mathtt{s}_1, \ldots, \mathtt{s}_n \in S$ we define a functor $\arg^{\mathtt{s}_1\times\cdots\times\mathtt{s}_n} : \mathtt{Set}_S(\{L\}_{\mathtt{s}\in S}) \longrightarrow \mathtt{Set}(L)$ by $\arg^{\varnothing}(\{(A_\mathtt{s},\alpha_\mathtt{s})\}_{\mathtt{s}\in S}) = (\{\varnothing\},\top)$ and

$$\arg^{\mathtt{s}_1\times\cdots\times\mathtt{s}_n}(\{(A_\mathtt{s},\alpha_\mathtt{s})\}_{\mathtt{s}\in S}) = (\arg^{\mathtt{s}_1\times\cdots\times\mathtt{s}_n}(\{A_\mathtt{s}\}_{\mathtt{s}\in S}), \arg^{\mathtt{s}_1\times\cdots\times\mathtt{s}_n}(\{\alpha_\mathtt{s}\}_{\mathtt{s}\in S}))$$

where

$$\arg^{s_1\times\cdots\times s_n}(\{A_\mathtt{s}\}_{\mathtt{s}\in S}) = \prod_{i=1,\ldots,n} A_{\mathtt{s}_i} \text{ and}$$
$$\arg^{s_1\times\cdots\times s_n}(\{\alpha_\mathtt{s}\}_{\mathtt{s}\in S})(a_1,\ldots,a_n) = \bigwedge_{i=1,\ldots,n} \alpha_{s_i}(a_i).$$

The functor

$$(\Omega^{\mathtt{s}_1\times\cdots\times\mathtt{s}_m\longrightarrow\mathtt{s}},\vartheta_m)_{\mathtt{Set}_S(\{L\}_{\mathtt{s}\in S})} \times \mathsf{arg}^{\mathtt{s}_1\times\cdots\times\mathtt{s}_m} : \mathtt{Set}_S(\{L\}_{\mathtt{s}\in S}) \longrightarrow \mathtt{Set}(L)$$

now allows to define

$$\begin{aligned}\mathsf{T}^1_{\Sigma,\mathtt{s}}\{(X_\mathtt{s},\xi_\mathtt{s})\}_{\mathtt{s}\in S} &= \coprod_{\substack{\mathtt{s}_1,\ldots,\mathtt{s}_m\\ 0\le m\le k}} ((\Omega^{\mathtt{s}_1\times\cdots\times\mathtt{s}_m\longrightarrow\mathtt{s}},\vartheta_m)_{\mathtt{Set}_S(\{L\}_{\mathtt{s}\in S})} \\ &\qquad \times \mathsf{arg}^{\mathtt{s}_1\times\cdots\times\mathtt{s}_m}\{(X_\mathtt{s},\xi_\mathtt{s})\}_{\mathtt{s}\in S}) \\ &= \coprod_{\substack{\mathtt{s}_1,\ldots,\mathtt{s}_m\\ 0\le m\le k}} ((\Omega^{\mathtt{s}_1\times\cdots\times\mathtt{s}_m\longrightarrow\mathtt{s}},\vartheta_m) \\ &\qquad \times (\prod_{i=1,\ldots,m} X_{\mathtt{s}_i}, \bigwedge_{i=1,\ldots,m} \xi_{s_i})) \\ &= (T^1_{\Sigma,\mathtt{s}}\{X_\mathtt{s}\}_{\mathtt{s}\in S},\beta_\mathtt{s})\end{aligned}$$

where

$$\beta_\mathtt{s}(\omega : \mathtt{s}_1\times\cdots\times\mathtt{s}_m\longrightarrow\mathtt{s},(x_i)_{i\le m}) = \vartheta_m(\omega)\wedge\arg^{\mathtt{s}_1\times\cdots\times\mathtt{s}_m}(\{\xi_\mathtt{s}\}_{\mathtt{s}\in S})((x_i)_{i\le m}),$$

and $(x_i)_{i\le m}\in\prod_{i=1,\ldots,m} X_{\mathtt{s}_i}$. We then have

$$\mathsf{T}^1_{\Sigma}\{(X_\mathtt{s},\xi_\mathtt{s})\}_{\mathtt{s}\in S} = \{\mathsf{T}^1_{\Sigma,\mathtt{s}}\{(X_\mathtt{s},\xi_\mathtt{s})\}_{\mathtt{s}\in S}\}_{\mathtt{s}\in S}.$$

Further,

$$\begin{aligned}\mathsf{T}^\iota_{\Sigma,\mathtt{s}}\{(X_\mathtt{s},\xi_\mathtt{s})\}_{\mathtt{s}\in S} &= \coprod_{\mathtt{s}_1,\ldots,\mathtt{s}_m} ((\Omega^{\mathtt{s}_1\times\cdots\times\mathtt{s}_m\longrightarrow\mathtt{s}},\vartheta_m)_{\mathtt{Set}_S(\{L_s\}_{s\in S})} \\ &\qquad \times \mathsf{arg}^{\mathtt{s}_1\times\cdots\times\mathtt{s}_m}\circ\bigcup_{\kappa<\iota}\mathsf{T}^\kappa_\Sigma\{(X_\mathtt{s},\xi_\mathtt{s})\}_{\mathtt{s}\in S})\end{aligned}$$

and

$$\mathsf{T}^\iota_{\Sigma}\{(X_\mathtt{s},\xi_\mathtt{s})\}_{\mathtt{s}\in S} = \{\mathsf{T}^\iota_{\Sigma,\mathtt{s}}\{(X_\mathtt{s},\xi_\mathtt{s})\}_{\mathtt{s}\in S}\}_{\mathtt{s}\in S},$$

for each positive ordinal ι. Finally, let $\mathsf{T}_\Sigma = \bigvee_{\iota<\bar{k}} \mathsf{T}^\iota_\Sigma$ where $\bar{k}$ is the least cardinal greater than k and $\aleph_0$. Terms of sort $\mathtt{s}$ are denoted $\mathsf{T}_{\Sigma,\mathtt{s}} = \mathsf{arg}^{\mathtt{s}} \circ \mathsf{T}_\Sigma$.

Clearly, each $\mathsf{T}_{\Sigma,\mathtt{s}} : \mathtt{Set}_S(\{L_{\mathtt{s}}\}_{\mathtt{s}\in S}) \longrightarrow \mathtt{Set}(L)$ is a functor and, by extension, so is

$$\mathsf{T}_\Sigma\{(X_{\mathtt{s}}, \xi_{\mathtt{s}})\}_{\mathtt{s}\in S} = \{\mathsf{T}_{\Sigma,\mathtt{s}}\{(X_{\mathtt{s}}, \xi_{\mathtt{s}})\}_{\mathtt{s}\in S}\}_{\mathtt{s}\in S}.$$

Note, it is easy to verify that

$$\mathsf{T}_{\Sigma,\mathtt{s}}\mathsf{T}_\Sigma\{X_{\mathtt{s}}\}_{\mathtt{s}\in S} = \mathsf{arg}^{\mathtt{s}}\ \mathsf{T}_\Sigma\{X_{\mathtt{s}}\}_{\mathtt{s}\in S}$$

and T_Σ is therefore idempotent.

The extension of T_Σ to a monad is enabled by the natural transformations

$$(\eta_s^{\mathsf{T}_\Sigma})_{(X_{\mathtt{s}},\xi_{\mathtt{s}})} : (X_{\mathtt{s}}, \xi_{\mathtt{s}}) \longrightarrow \mathsf{T}_{\Sigma,\mathtt{s}}\{(X_{\mathtt{s}}, \xi_{\mathtt{s}})\}_{\mathtt{s}\in S}, \text{ and}$$
$$(\mu_s^{\mathsf{T}_\Sigma})_{(X_{\mathtt{s}},\xi_{\mathtt{s}})} : \mathsf{T}_{\Sigma,\mathtt{s}}\mathsf{T}_\Sigma\{(X_{\mathtt{s}}, \xi_{\mathtt{s}})\}_{\mathtt{s}\in S} \longrightarrow \mathsf{T}_{\Sigma,\mathtt{s}}\{(X_{\mathtt{s}}, \xi_{\mathtt{s}})\}_{\mathtt{s}\in S}$$

that are simply defined, with the help of idempotency of T_Σ, by

$$(\eta_s^{\mathsf{T}_\Sigma})_{(X_{\mathtt{s}},\xi_{\mathtt{s}})}(x_{\mathtt{s}}, \alpha_{\mathtt{s}}) = (x_{\mathtt{s}}, \alpha_{\mathtt{s}}), \text{ and}$$
$$(\mu_s^{\mathsf{T}_\Sigma})_{(X_{\mathtt{s}},\xi_{\mathtt{s}})}(x_{\mathtt{s}}, \alpha_{\mathtt{s}}) = (x_{\mathtt{s}}, \alpha_{\mathtt{s}}).$$

We write $\eta^{\mathsf{T}_\Sigma} = \{\eta_s^{\mathsf{T}_\Sigma}\}_{\mathtt{s}\in S}$ and $\mu^{\mathsf{T}_\Sigma} = \{\mu_s^{\mathsf{T}_\Sigma}\}_{\mathtt{s}\in S}$.

Proposition 1. $\mathbf{T}_\Sigma = (\mathsf{T}_\Sigma, \eta^{\mathsf{T}_\Sigma}, \mu^{\mathsf{T}_\Sigma})$ *is a monad over* $\mathtt{Set}_S(\{L_s\}_{\mathtt{s}\in S})$.

Remark 1. The many-sorted, many-valued, term monad specialized to a one-pointed set of sorts $S = \{\mathtt{s}\}$ collapses to the classical many-valued term monad.

Remark 2. Morphisms

$$\{f_{\mathtt{s}}\}_{\mathtt{s}\in S} : \{(X_{\mathtt{s}}, \alpha_{\mathtt{s}})\}_{\mathtt{s}\in S} \longrightarrow \{(Y_{\mathtt{s}}, \beta_{\mathtt{s}})\}_{\mathtt{s}\in S}$$

in $\mathtt{Set}_S(\{L_{\mathtt{s}}\}_{\mathtt{s}\in S})_{\mathbf{T}_\Sigma}$, the Kleisli category of $\mathbf{T}_\Sigma$, capture the notion of many-sorted and many-valued variables being substituted by many-sorted terms over many-sorted and many-valued variables.

4 Preference Relations

Arrow [1] studied social welfare functions, the arguments of which are named components of social states. These functions map n-tuples of individual preferences (orderings [1]) into a collective preference:

$$f : (X^m)^n \to X^m$$

Here the assumption is that X is an ordering $(X, \preceq)$ with suitable properties. The preference value in this case is an ordinal value and not a scale value. Clearly, choice functions can also involve scale values, so that

$$f : (\mathbb{R}^m)^n \to \mathbb{R}^m$$

i.e. using the real line, or some suitable closed interval within the real line, for the preference (scale) values. Note how the underlying signature handles this situation internally for $\mathcal{X} = \mathrm{Hom}_{\mathtt{C}}(X, X)$, where $\mathtt{C}$ is the Kleisli category $\mathtt{Set}(L)_{\mathbf{T}_\Sigma}$.

Computing with preferences is less transparent with orderings built into the set X of alternatives [6]. Also in this case there is a corresponding underlying signature capturing this situation.

5 Conclusion and Future Work

In the presentation above we still use only terms. Sentences, satisfaction $\models$ (based on the algebraic models of the signature), and entailment $\vdash$ are not yet included. Axioms of the logic and inference rules for entailment are then also missing, so we no 'logic of choice' at this point, and this has fallen outside the scope of this paper. See [7] for a treatment of generalized general logic.

Going beyond the distinction between choosing and choice, and entering rationality of choice, Mill [12] said that behavior is based on custom more than rationality. Custom is clearly based on particular algebras acting as models and used in $\models$, whereas rationality is based on representable sentences interrelated by $\vdash$. These aspects are investigated in future work.

In consensus reaching [8,9] we have a dynamic situation of aggregated choice, where individual preferences change within a consensus reaching mechanism. This opens up interesting perspectives as consensus reaching in our substitution model for social choice now also reaches the level of 'choosing', i.e., consensus is reached either on 'choice' level including dynamics for the 'choosing' level, or can even be a stronger consensus on 'choosing' levels as well. Similar situations appear in negotiation.

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Assessing Plans and Programs for Historic Centers Regeneration: An Interactive Multicriteria Approach

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Abstract. The paper proposes a decision support system (DSS) for definition and implementation of complex policies for globally preserving and valorizing the cultural heritage, in a context of limited financing.

The proposed methodology is inspired by the fundamentals of Goal Programming. The multiobjective decision problem is articulated into two phases: the first, aimed at allocating the public financial resources among different kinds of homogeneous actions; the second, aimed at selecting the punctual investments to be financed with the optimal resources assigned to each action during the first stage.

In order to find the best compromise solutions, the original multiobjective problem is transformed into a monobjective constrained problem. For this purpose, phase 1 is supported by a linear programming model, while phase 2 by a binary one. The first and the latter are interactive DSS allowing to identify, through a series of iterative steps, the best compromise solution, if it exists.

The proposed methodology is applied to a decision problem derived from the "Great Program for the Historic Center of Naples", launched by the Municipality in year 2007 with the aim of triggering a requalification process involving the whole historic center, enrolled in the UNESCO World Heritage List since 1995.

Keywords: financial allocation, investments selection, mathematical programming, decision support system

1 Introduction

The task of preserving and valorizing towns' historic centers is implied by the wider duty of cultural heritage protection; such task is defined both by the Constitutions of modern Countries and in many international documents such as the Athens and Venice Charts, the UNESCO Agreement and the European Chart on Architectural Heritage.

In particular, the last two documents state the concept of cultural heritage "global" protection, and stress the need of not considering historic centers as a simple sum of buildings or assets, but as a whole, whose preservation must be integrated into city planning and economic programming.

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Following this, in the last years, various local communities have started devising new policies aimed at preserving and valorizing their cultural heritage, even in consideration that, in many different cases, such strategies have been proved to play a key role in inducing the start up of virtuous transformation processes of degraded areas, also thanks to the joint action of public and private efforts.

Unfortunately, though, the aim of "global" preservation has to face strict public budget constraints avoiding the implementation of wide restoration programs; therefore, accurate financial programming and coherent investments selection is strongly required in order to successfully preserve and valorize towns' historic centers.

The key questions to be faced, therefore, are the following: "how to define the best financial allocation to better contribute in restoration program’s targets reaching?", "how to choose among the various possible restoration projects?", "how to verify the coherence between projects selection and program’s targets attainment?"

Answering to such questions is very complex, and the current evaluative procedures, unfortunately, are still unable to effectively support decision making in this field: the first problem to be faced, in fact, is due to the difficulty in identifying the causal relations existing between the possible typologies of actions and their impacts on social relevant targets; the second problem, instead, is related to the fact that many of the available information are either fuzzy or qualitative, with the consequence that they often remain unused.

In other words, current evaluative procedures for supporting the design of restoration programs are undermined by the difficulty of obtaining reliable quantitative data and of using and synthesizing all the available "weak" information.

As a result, the design of complex restoration policies for historic centers is currently based only on technical and political judgment, while economic assessment is often limited to the evaluation of monetary impacts and confined at the end of the decision making process for a mere validation of the choices already set by the technical and political staff.

In order to overcome the current limits characterizing the evaluation procedures linked with programs design and implementation, therefore, a robust methodology for effectively supporting final decision making is strongly needed.

Next sections, therefore, describe one of the possible solutions to be adopted for this specific purpose, based on the use of the mathematical programming methods.

The usefulness of such methodology will be shown through its application to a specific case study related to the sector of cultural heritage and inspired to the real problems that bubbled up when designing “the Great Program for the Historic Center of Naples”, an ambitious Program whose aim is that of triggering a huge requalification and development process involving the whole Naples historic center, enrolled in the UNESCO World Heritage List since 1995. Such Program, launched by the Municipality in year 2007, has been temporary stopped due to the new rules set by Stability Pact; anyway, the evaluative problems put in place by such an experience are still very updated and, for this reason, such case study will be further used as a mere exemplificative starting point for showing the features of the proposed DSS.

2 The Great Program for the Historic Center of Naples

The Great Program for the Historic Center of Naples" (from now on GP) has been conceived by the Municipality as a very ambitious Program implying both the restoration of historic monuments and buildings, and the implementation of various other "physical" and "un-material" interventions in the Historic Center enrolled in the UNESCO World Heritage List.

Such Program is articulated into two main documents: the Strategic Orientation Document (S.O.D.), and the Urban Integrated Program (U.I.P.). The S.O.D. is aimed at defining the general strategy (actors, synergies, instruments, roles) to be adopted in the entire UNESCO area; the U.I.P., instead, is the operating document, which relies, at least before the stop set by the Stability Pact, on a total financing of about 240 million Euros out of the POR-FESR 2007-2013 (objective 6.2), plus additional 110 Euros out of other funding specifically addressed to some sectors such as tourism, welfare, security, transports, entrepreneurship, for a total budget available of more than 350 million Euros.

The GP pursues 15 different targets, all of them converging toward the general aim of global requalification and development of the Historic Center of Naples. In particular, three groups of targets can be identified as relevant: a) targets directly related to the GP general strategies, declined by the S.O.D.; b) targets indirectly derived from considering the GP as an instrument for fostering local economic development; c) targets specifically traceable from the UNESCO directories regarding the sites enrolled in the World Heritage List.

To reach such targets, 9 different kinds of actions have been identified; the actions represent homogeneous expenditure categories and they are implemented by several punctual investments. We can split the actions into 4 groups:

a. *interventions on monumental heritage/buildings.* This group comprises all the actions regarding interventions on the external facades and for the internal requalification of public – private monumental heritage/buildings;
b. *interventions on ordinary goods/buildings*. This group comprises all the actions regarding interventions on buildings with no artistic value, whose requalification and refunctionalization contribute to valorize the historic center;
c. *Requalification of open spaces and urban areas*. This group comprises the actions regarding the re-making of urban furniture;
d. *Interventions of urban archeology*. This group comprises the actions regarding the archeological excavations and all the interventions for requalification, safeguard, and valorization of urban areas.

3 The Proposed Methodology for Supporting the Design of the Great Program for the Historic Center of Naples

As evident, the Great Program for the Historic Center of Naples is a very ambitious program, whose global and efficient implementation would require huge financing and high planning capacities by the decision maker.

Given the context of strict budget constraints, however, the only way to successfully succeed in pursuing the wide set of GP goals is to make an accurate programming for the scarce resources available, and a coherent selection of the investment projects to be included into the Plan.

Following this, the decisional process should be articulated into two consecutive steps: the first aimed at supporting the “Programming Phase” or, in other words, at defining the amount of resources to be devoted to each GP action in order to get the best compromise impacts on all the GP targets; the second, aimed at supporting the “Budgeting Phase” or, in other words, at defining, with respect to the budget constraints identified for each action, the investments set that better contributes to the targets pursued by the action they belong to.

In order to operatively fulfill the GP targets, a wide set of 247 investments projects has been proposed since the year 2007 by citizens, institutions, and economic operators to be included into the Program; starting from such wide and overlapping list of projects, then, the final decisional stage to operatively implement the GP should be the selection of the projects to be actually included in the plan, in order to reach, at best, the ultimate goals it has been conceived for.

Next sections will describe, more in detail, the features of the two evaluative steps, and the techniques to be used for supporting the related decisions implied by each.

3.1 Step 1 – Programming Phase

Aim of the first step of the model is to define the best budget structure of the Program in order to get the best compromise impacts on the GP most relevant objectives.

Step one could be run out by considering that the definition of the GP expenditure Program is a typical multi-objective problem: the preliminary S.O.D. and U.I.P., in fact, state that heterogeneous objectives should be pursued through the use of the public financing available.

Given the trade-offs among objectives and in consideration of the general budget constraint, however, no financial allocation (among the different GP actions) capable of maximizing all the GP objectives, exists.

As a result, the hypothesis of identifying an optimal solution to the problem should be rejected, while a solution of "best compromise" should be pursued; such solution is the one implying a financial allocation whose impacts are considered "acceptable" for all the decision makers involved in the process; the level of satisfaction attributed to a solution, therefore, is not an absolute concept, but it is linked to the structure of the decision maker preferences.

In consistence with the principles of Goal Programming, the research of the "best compromise" solution could be operated by transforming the "original" multi-objective problem into a "new" mono-objective constrained problem with continuous variables, where one of the GP targets is set as the objective function, and the remaining ones are treated as constraints, whose minimum (maximum) value must be respected.

In such a model, the control variable is the amount of financial resources devoted to the GP and the basic hypothesis is that each objective is linearly linked to the GP financing assigned to the various actions impacting on them. In particular, in consideration of the low level of knowledge regarding the relation existing between the action and the objectives, the impacts that the first have on the second can be quantified by using a Delphi approach, where experts may be asked either to use a scoring system (e.g., -5/ 5 scale) where negative/positive scores must be attributed in case of negative/positive impact on the target, or to give quantitative evaluations based on the activation coefficient linked to a unit of expense observed in other similar interventions.

The solution to such mono-objective constrained problem leads to the identification of the best budget allocation for a given structure of the model (according to the objective function chosen and the constraints set), and to the evaluation of the economic impacts related to such financial plan.

By modifying the model structure (by choosing a different objective function and/or modifying the constraints value), alternative budget allocations could be easily generated, and their relative impacts easily compared with those related to other scenarios previously found.

The approach used for the generation of the various scenarios is interactive, because it is based on a dialogue with the decision maker: at each step, the model provides a new solution to be proposed to the decision maker, described in terms of impacts on the GP targets; after the creation of each scenario, then, the decision maker is asked about its degree of approval with the impacts and, in case of low satisfaction, he is asked to provide additional information (e.g. specification of new constraints, changing of the objective function, etc.) for generating a new solution.

The process ends when the decision maker identifies the "best compromise" budget structure, that is the financial allocation bringing to an acceptable level of all the GP objectives.

In this way, therefore, the first step of the proposed approach allows both to generate the budget structure of the GP and to make an ex-ante evaluation of the impacts associated to it: the result is the identification of both *effective* (able to reach a solution, if it exist, where all the objectives are at an acceptable level) and *feasible* (in terms of capacity in respecting the financial and other existing constraints) expense Program to be adopted.

3.2 Step 2 – Budgeting Phase

Once defined the amount of resources to be devoted to each GP action, the second step of the approach consists in identifying, for each of them, the investments set to be realized with that resources.

Like in step one, the selection phase is a typical multi-objective problem. Each investments Plan, in fact, pursues a "best compromise" solution among a set of conflicting objectives among which: (a) some are related to the contribution that each investment project gives to the reaching of the GP targets; (b) others are related to projects' micro-economic performance, strictly due to their technical and economic specificities (e.g., minimize project investment cost, maximize project Financial Net Present Value, maximize project Economic Net Present Value, etc.).

Again, like in step 1, the research of the "best compromise" solution could be operated by transforming the original multi-objective problem into a "new" mono-objective constrained problem, where the decision variables are binary variables linked with the investments projects.

The solution to such new problem leads to the identification of the best investments set to be included into the Program, for a given structure of the model, and a prevision of the impacts associated to its implementation.

By interacting with the decision maker, it is possible to modify the model structure and, consequently, calculate a new solution, characterized by a different Plan configuration bringing to a different set of impacts.

The second step process ends when the decision maker is satisfied by all the impacts of a given Plan configuration.

In this way, therefore, the second step of the model allows, at the same time, to generate and evaluate the investments Plan to be adopted, by ensuring both its *effectiveness* and its *feasibility*.

4 Description and Formalization of the Model

This section describes more in detail the mathematical models to be used in order to implement the proposed evaluative approach.

In particular, paragraph 4.1 describes the model used for supporting the programming phase (step 1); paragraph 4.2, instead, is focused on the model supporting the budgeting phase (step 2).

4.1 Step 1 – Defining the "Best Compromise" Budget Structure for the GP

Let's indicate with:

- O_j = the set of J objectives ($j=1, ..., J$) pursued by the GP;
- X_n = the decisional continuous variable, that is to say the amount of resources to be assigned to each n-action ($n= 1, ..., N$);
- C_{nj} = the average unitary impact of the expense X_n on the *j-th* objective.

The search of the "best compromise" solution must be operated by respecting a set of exogenous constraints, defined before starting the interaction phase and considered as un-modifiable; such constraints regard the minimum and maximum value within which the decisional variables may range:

$$X_n^{min} \leq X_n \leq X_n^{max} \quad [1]$$

In addition to the above, the global budget constraint must be considered, as well: the total amount of resources assigned to the various actions, in fact, cannot exceed the total resources *(K)* assigned to the Program:

$$\sum_{n=1}^{N} X_n \leq K \quad [2]$$

Assumed that the value drawn by the generic target j depends on how the financial resources are distributed among the actions, such general relation could be written as $f_j(X)$ (with $j = 1,\dots, J$); more in detail, the value of each j objective may be represented as a linear combination of the decisional variables according to the performance coefficients C_{nj}

$$f_j(X) = \sum_{n=1}^{N} C_{nj} X_n \quad [3]$$
$$\forall\, j = 1,\dots,J$$

As a result, the following multi-objective linear programming model is obtained:

$$\max f(X) = [f_1(X); \dots\ f_j(X)] \quad [4]$$
$$\text{S.T.}$$
$$X_n^{min} \le X_n \le X_n^{max}$$
$$\sum_{n=1}^{N} X_n \le K$$

Given that the j-targets are different from each other, and often conflicting among them, no repartition of financial resources allowing to optimize all the targets actually exists.

As said before, in fact, in multi-criteria problems the concept of "optimality" is substituted by that of "acceptable compromise", which implies the research of "satisfactory" results.

In order to find the "best compromise" solution, the first step is a technical one and consists in identifying the "ideal solution" vector: for each target $f_j(X)$, the ideal value $f_j^*(X)$ is calculated within the model by hypothesizing to optimize only one target, without caring of the level of the remaining ones, and considering only the exogenous constraints.

This technical phase is developed before starting the interaction with the decision maker.

Obviously, the ideal solution is external to the region of the feasible solutions; in other words, no real solution could ever generate such optimal values, otherwise this would mean that the targets are not conflicting. In general, in fact, a generic solution $\boldsymbol{f}(X)$, at least in one of its $f_j(X)$ elements, will present a lower value then the corresponding $f_j^*(X)$ belonging to the "ideal" vector $\boldsymbol{f}^*(X)$.

Anyway, the identification of the ideal vector is very useful because it represents a benchmark allowing to compare the "real" results obtained by the elaboration of the various scenarios while searching for the "best compromise" solution.

Once identified the "ideal vector", next step consists in identifying the best compromise solution: to do this, the DSS model should start interacting with the decision maker, in order to identify (within the entire set of targets to be pursued) both the objective function to be maximized (or minimized) and the set of discretional constraints describing decision maker's preferences.

The DSS generates a Pareto-efficient solution, described both in terms of impacts on the several relevant targets and in terms of value associated to each decision variables; such solution, then, is proposed to the decision maker. Furthermore the DSS provides information about the distance between the selected solution and the ideal one (ideal vector); such distance represents the "regret" with reference to the value that the target achieves in the ideal solution. Starting from these information, the decision maker is able to identify the target on which he wants to intervene, in consideration of both its specific level of attainment (compared with the ideal value) and of that of the other targets.

The decision maker, then, may define on such target a new constraint on the minimum accepted level of satisfaction; the DSS, then, generates a new solution that respects the new discretional constraint, and describes the new result achieved. This procedure goes on until a "best compromise" solution is accepted by the decision maker. The new solution to be proposed to the decision maker may be generated by using different approaches. The simplest one is to transform the original multiobjective problem into a monobjective problem, where one of the *J* objective functions is optimized subject to the several constraints, both exogenous (the availability of resources, the technologies etc) both endogenous, set in by the decision maker as minimum acceptable level in achieving the objectives. If the decision maker does not change his previous choices, he will arrive at the best compromise solution, if any, in a number of steps equal to *J-1*.

In the GP specific application, the target chosen for maximization is the - "*Recovery of monumental heritage*" (from now on *target 1*), given the high stress toward this objective highlighted in all the GP strategic documentation; therefore the objective function of model [4] could be substituted with the following:

MAX

$$\sum_{n=1}^{N} X_n C_{n1} \qquad [5]$$

ST

$$X_n^{min} \le X_n \le X_n^{max}$$

$$\sum_{n=1}^{N} X_n \le K$$

$$\sum_{n=1}^{N} C_{nj} X_n \ge B_j$$

$$\forall\ j = 2,\ldots,J$$

where B_j is the minimum acceptable level of *j-th* objective, discretionally set by the decision maker.

In this way, the problem of finding the best financial allocation of GP funds is represented through a simple linear programming model, to be solved by the iterative use of the interactive DSS.

In particular, the “best compromise” solution found for our case study (the one considered as the most "satisfactory" according to the DM preferences) has been identified after 7 iterations of the model, and resulted in the following funding distribution:

- interventions on monumental heritage/buildings, 51,8 %;
- interventions on ordinary goods/buildings, 13,3%;
- requalification of open spaces and urban areas, 27,9%;
- Interventions on urban archeology, 7%.

4.2 Step 2 - Selecting the Best Investments Set to Be Included in the GP

Once defined the optimal allocation of GP financial resources among the N actions, next task consist in identifying, within each action, the investments set to finance with those X_n resources.

In order to better understand features of the proposed model to be used during step 2, this paragraph shows an exemplification of the selection phase, limited to a subset of 27 investment projects, proposed by different stakeholders, and belonging to the action "*restoration and reconstruction for social purposes"* (from now on *action a*), which has been assigned (X_a^* in the application described in previous paragraph) of the 15,4% of the GP total resources.

Let's indicate with:

- aO_s = the set of S objectives *(s=1, ..., S)* considered as relevant for *action a* ($S = 16$ in our simulation);
- Y_i = the binary variable associated to the generic investment project i-th *(i=1, ..., I)* belonging to *action a* (I = 27 in our simulation); Y_i is binary because its value will be 1/0 if the related project is selected/not selected in the optimum;
- Z_{is} = the impact generated by the generic project i on the generic objective s.

The search of the "best compromise" solution must be operated by respecting the constraint regarding the maximum amount of resources available for projects implementation. In other words, the costs of the selected investments to be financed by the GP resources, therefore, must be lower or equal than the financial constraint defined for *action a*:

$$\sum_{i=1}^{I} K_i^* Y_i \le X_a^* \qquad [6]$$

where:

- $X_{a.2}^*$ is the optimal amount of resources attributed to *action a* in Step 1;
- K_i^* is the investment cost of project i belonging to *action a*.

In addition to constraint [6], the model must also take into consideration the relations of "mutual exclusion" and "complementarity" among the investment projects.

Given that the *S* objectives are different from each other, and conflicting among them, no projects selection allowing to optimize all the targets actually exists, therefore an "acceptable compromise" should be searched for.

To do this, the first step consists, again, in identifying the "ideal solution" vector.

Once identified it, we need to identify both the objective function to be maximized (or minimized) and the set of discretional constraints describing decision maker's preferences.

In our specific application, the maximization of ENPV has been chosen as the objective function, given that such index summarizes, in itself, many information regarding the whole economic performance of the projects.

In the new model, therefore, the objective function becomes the following:

MAX

$$\sum_{i=1}^{I} ENPV_i^{*} Y_i \qquad [7]$$

ST

$$\sum_{i=1}^{I} Z_{is}^{*} Y_i \geq D_s$$

where:

$ENPV_i$ is the value of the Economic Net Present Value associated to project *i*;

D_s is the minimum level to be reached by objective s, discretionally set by the decision maker.

In this way, the problem of finding the best investment set to be included in *action a* of the GP, to be financed with the resources X_a^{*} devoted to *action a,* is represented by a binary programming model: by defining new D_s values, the decision maker may easily determine different solutions to be compared, in order to find the "best compromise" one. The process goes on until a "satisfactory" solution is achieved.

With reference to our simulation, the "best compromise" solution for the budgeting phase (the one allowing to obtain an acceptable deviation from the ideal solution for all the targets) has been identified after 6 iterations of the model. Such solution implies the inclusion in the GP of 18 projects from *action a* (of which 13 "material" and 5 "un-material"), for a total costs of € 35.737.500 (of which the 43% co-financed by the GP and the remaining 57% by private resources).

5 Conclusions

In conclusion, beyond the specific results obtained with reference to the GP case study, the main strengths of the proposed approach are the following:

- It allows to implement a relation map to understand the links between the instruments and the objectives;
- It allows to make the decision maker's choices more "explicit", by reducing the "hidden" space that could be seldom found in many DSS;
- It offers a coherent and flexible analytical framework for quickly representing the effects of different alternative choices;
- It helps determining efficient, effective, and feasible solutions, given the existing constraints and the decision maker's preferences, therefore allowing to easily exclude all the dominated solutions.

In addition to the above, another important feature of the approach is that the same methodology is used, with little changes, to support both the programming and budgeting phases, thus allowing to simplify model understanding and results reading by the decision maker.

Finally, the structure of the model allows a very simplified interaction between the Analyst and the decision maker: the latter's task, in fact, is not that of determining plausible trade-off values among the impacts, but only that of being oriented towards the determination of acceptable solution in terms of impacts obtainable; in other words, the decision maker should only express its judgment on the attainable impacts; if he is satisfied with them, this implicitly means that he is accepting the underlying budget structure, on the contrary, he must simply communicate where his dissatisfaction comes from, and a new scenario will be generated by taking into consideration new constraints.

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The Evaluation of Interventions in Urban Areas: Methodological Orientations in the Programming of Structural Funds for the Period 2007–2013

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Abstract. The most recent interpretations of the concept about *development* have profoundly changed the purpose of the *evaluation* for public investment, requiring not only the analysis of the efficiency and effectiveness of programs, but also the ability to manage the interests and demands of *social actors* and various *institutional levels* involved in implementing policies. Starting from an analysis of the innovations in *urban* and *territorial policies*, the paper investigates the changes taking place in the evaluation of interventions in urban areas, extending the considerations to the more general question about *local development*. In particular, we find some remarks arising from recent documents issued by the European Commission for the period 2007 – 2013, concerning the evaluation of projects and policies implemented through the EU cohesion policy. The paper also underlines the centrality of the *impact evaluation* together with *models based on targets*; it resumes some observations about limits attributed to the quantitative evaluation methods, which are traditionally favored, but sometimes deemed as unfit to understand the complex dynamics of the development processes.

Keywords: urban and regional policies, program evaluations, decision making models in assessing impacts.

1 The New Urban and Territorial Policies: From Innovating Actions Supporting Sustainable Development to Strategic Planning

The European territorial policies assign to cities a new role aimed at strengthening the competitiveness of their respective regional areas. The 2007 – 2013 Community Strategic Guidelines emphasize that urban areas are essential in achieving the sustainability goals in the EU as all cities are the center of business and economic lever; moreover social, cultural, and environmental aspects of development are concentrated and interconnected in the cities.

Since the 1990s, the European Commission has promoted a growing interest in policies having impact on the cities, reserving part of the European Regional

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Development Fund (ERDF) to the *regeneration of the suburbs*. In 1994, the European Commission established the *Urban Program* for the economic and social regeneration of cities and urban areas in crisis.

In the 1990s, the question of *urban sustainability* has been stressed in many international and Community documents such as Agenda 21 in 1992, Aalborg Charter in 1994, Action Plan of Lisbon in 1996, Habitat Agenda in 1996. The document "*Structural Funds and Sustainable Development in Urban Areas*" [7] is useful for a comprehensive review of the main EU documents addressing the issue of the urban environment. In 1998, the *Framework for Action for the Sustainable Urban Development* [5] by European Commission underlined that the actions on cities can contribute to the achievement of three important objectives: *cohesion*, through the intervention on areas of exclusion and poverty, which focus largely in the cities; *competitiveness*, improving the efficiency of cities, their accessibility and attractiveness for investors (especially outside the EU); *sustainable development*, to be achieved through the energy efficiency and a better use of few resources [21].

Subsequently urban policies become part of the objectives of the Structural Funds, the main instrument of the EU action on the urban dimension. The European Union has provided for specific initiatives for the 2000 – 2006 programming; in particular, the *Urban II* was more directed to the purposes of urban environment redevelopment, continuing the positive experience of Urban in 1994. The Urban II Initiative was characterized by the promotion of a large partnership at the local level through a *bottom up* approach; other features relate to innovative strategies for social rehabilitation and economic development of urban centres, and exchange of experiences about sustainable urban development in the EU. Urban areas characterized by high social and environmental degradation are eligible for funding; recovery strategies on the social sphere and employment are proposed, improving the environment and public transport [7]. In the 1990s the European Commission had also provided the *Urban Pilot Projects*, as additional Innovative Actions aimed at financing interventions of urban regeneration, with specific reference to historic centers, industrial areas, and neighborhoods characterized by socio-economic degradation.

In the Italian context, since the early 1990s the tools for urban planning have been formulated in a large variety of detailed plans aimed at encouraging the redevelopment of urban areas. The reference is to *Integrated Intervention Programs, renewal and regeneration Programs, neighborhood Contracts, Programs for Territorial Sustainable Development*, which have stressed a new concept of urban regeneration as "strategy" aimed both to the rehabilitation of degraded parts of the city, both to the reequipment of the city as a whole, combining social interventions, and encouraging the participation of the numerous actors involved.

The results obtained are still in depth examination through an ongoing work of critical analysis; however, the plurality of tools and kinds of intervention has been the consequence of the need to overcome the strictness of the traditional urban development plans.

Nowadays the main changes in urban areas are *dispersion and sprawling trends*, *growing networks of regional systems,* the *presence of rural areas* having inadequate external relationships, and accessibility. These aspects require new forms of urban governance, both in terms of *planning* and in terms of *decision making*. Relatively to planning issue, it is necessary a renewed co-operation between different planning authorities, working at different spatial scales; about decision making aspects, new forms of private/ public partnerships, community participation, and strategic planning procedures are needed [21]. Complexity of territorial planning requires an *integrated approach* [7]. The *Europe 2020 Strategy* underlines that the new challenges for European cities are *competitiveness, social inclusion, sustainability, multilevel governance, smartness*. Strengthening the competitiveness in the European territories requires a *strategic vision of development* to identify opportunities and potential of cities and their territories, aiming to the system of local resources, planning, and promoting networks with other cities, in the national and European context [8]. Cooperation and networks of cities are essential to improve governance of urban interventions in a context of increasing decision making complexity.

S*trategic planning* is an innovative governance tool to support public institutions in promoting supra-local networks of decision-makers, so to achieve a *shared vision* for development strategies. Significant experiences of Strategic Planning for a middle-longterm regeneration of cities have been implemented in some European areas, in order to deal with the main challenges of the contemporary city. The main peculiarity of the Strategic Plans is the *cooperation* between local governments, knowing that the municipal level is too limited to generate strategies for improving the competitiveness of the involved territories [19].

The Strategic Plan is a voluntary act enabled in Italy since the programming of national resources for underdeveloped areas (FAS funds, CIPE Resolution 20/2004). It is developed through the involvement of the multiplicity of institutional, social, economic, and cultural stakeholders. The relationship between the local government and the system of public and private actors makes it possible for better development strategies and the redefinition of the intervention priorities on the basis of the achieved and achievable results.

Evaluation is essential for the plan success, in terms of effective implementation (*process evaluation*) and goodness of results (*outcome evaluation*). However, the *integrated approach* in development processes—seen as combination of different sectorial interventions on the same geographical area—makes it difficult to evaluate the relationship between actions and their effects on the relevant issues of development. For this reason, in order to make *sustainable* the contents provided by the strategic plans, it would be necessary for a strengthening of the traditional evaluation tools based on the *criterion of efficiency*, seen as the ability of the plan to achieve the objectives. In general, in addition to *exante* indicators of success and *in itinere* and *expost* measurements, it would be useful the analysis of the contribution that the development policies provide to the local economic growth and to the quality of life [14]. This involves the identification of the *causal link* between the *output* of an intervention and the effects on the relevant assets on which the policy will affect (*outcome levels*).

2 Evaluation and Planning of Urban and Territorial Policies Financed by the Structural Funds: Some Experiences in a Regional Context

In the frame of planning for regional development policies *based on results*, the use of *evaluation* has been greatly extended: today city and regional authorities have to fulfill new evaluation aims in relation both to the formulation and implementation of programs, and to the sustainability of projects.

In general, an evaluation process pursues different interconnected aims [11]: *cognitive aim*, to acquire data on the effects, and results of policies; *decision making support*, to provide information to guide choices; *transparency*, to account to the European Commission and to the Italian state for what has been achieved and learned from the implementation phase; it is important also for the institutional partners and for the other stakeholders and citizenship.

The evaluation procedures implemented during the 1990s in the European context have been directed primarily to explore "*how the money was spent*" under the necessity of monitoring the results achieved with the delivery of Structural Funds. It was often pointed out that the great majority of integrated projects funded in the city through the operational Programmes (OPs) and other Community initiatives have not a strong strategic framework and a shared vision of development [3]. The new interest for cities and urban development in terms of *cohesion, competitiveness, and sustainable growth* of territories requires more appropriate evaluation tools.

The Regulation (EC) 1083/2006 stresses the role of *multi-phase evaluation process* (Section 47 (2)) for programming of actions to promote the development of territories, with a differentiation between:

- *exante evaluations* of Programs, to optimize the allocation of financial resources within operational programs and to improve programming quality, identifying disparities, gaps and potential for development, goals to be achieved, results expected, and the coherence of the strategy proposed for each region. The priorities of the Community and the lessons from previous programming are also considered, as well as the quality of the procedures of implementation, monitoring, assessment, and financial management;
- *intermediate evaluations,* linked to the monitoring of operational programs in particular where their implementation presents a significant departure from the goals initially set or where proposals are made in order to obtain a revision of the programs;
- *expost evaluations* of Programs, relating to the whole operational programs in order to examine the degree of utilization of resources, the effectiveness and efficiency of Fund programming, and the socio-economic impacts. They draw conclusions about the policy on economic and social cohesion, identifying the factors that contribute to the success or failure of the implementation of programs and identifying best practices.

The following table shows the role of *multi-phase evaluation* in the decision making process, outlining the main approaches and tools at each stage.

Table 1. Multi-phase evaluation process of Programs

Phases	***Objectives***	***Approaches and Tools***
Exante evaluation	It provides feedback on the validity of the choices with respect to *social needs*, *consistency* of plans, *sustainability of governance*, definition of criteria for *priority actions* to be implemented for the territorial development. This is an evaluation to support the definition of the program, following a logic of *policy and program design*. The purpose of the ex ante evaluation is to optimize the allocation of resources and to improve the quality of the programming process [13]. The evaluation questions in general are based on judgement criteria which can be grouped into the following main categories [13]: • *relevance* of the program to needs identified • *coherence* (internal and external) • *effectiveness*, to understand whether the objectives of the program are likely to be achieved • *efficiency,* to understand the best relationship between resources employed and results to be achieved • *utility* and *longer term sustainability*, judging the likely impacts against wider social, environmental, and economic needs.	The *relevance* of the strategic general setting requires a SWOT analysis, to examine Strengths, Weaknesses, Opportunities, Threats of the context. *Coherence* analysis needs evaluation of the program in terms of compatibility between general objectives, regional scenarios, European and national guidelines (*external coherence*); evaluation of the program in relation to specific objectives, activities, indicators, and expected results (*internal coherence*) . About *efficiency*, the evaluation may refer to strategic aspects, as different ways of reaching the same socio-economic objectives and achieving the same impact [11]. About *utility and sustainability*, the evaluation focuses on *expected impacts and results*, verifying the appropriateness of the objectives and the indicators identified, as well as the proposed quantification on the basis of past experience and appropriate benchmarks [13]. At this stage the evaluation needs to integrate the *Strategic environmental Assessment* (SEA), considering the effects of program (including alternatives) on environmental objectives.
Intermediate evaluation	This evaluation is both an assessment process, and a monitoring. The assessment process shall keep under review the organization and evolution of a program, following a logic of *compliance*. The monitoring provides data on physical and financial progress of the program, following a logic of *accountability*. This is helpful in understanding the critical points and provide relevant indications to the general trend of the interventions, suggesting changes in strategy or implementation methodology. The intermediate evaluation helps to develop the implementation of the program.	*Efficiency and effectiveness* analysis is further developed: assessment of *outputs, results and impacts* achieved; analysis of quality of the implementation mechanisms, in terms of *time and cost*. Study of the early effects of the program by means of *field research and data collection* focused on the concluded projects [see 2, 5]. *Statistical models* may be appropriate in some cases to estimate impacts on different types of data. These can include *input/output analysis or econometric models* [13].

Table 1. (*continued*)

Expost evaluation	The ex post evaluation analyzes the effects of policies on the territories in which program has been implemented. It also provides some *lessons from past development experiences*, becoming a source of knowledge to refocus the development efforts in the new programs, following a logic of *learning*. The attention is focused on good practices, but also on the worst practices, so to understand errors and avoid repeating failures.	Cross-sectoral integrated assessment of the expected effects, such as analysis of coherence with the general aims of interventions, as well as analysis of impacts on many issues (environmental, territorial, economic, social). Causal analysis for *impact evaluation*, experimental or quasi-experimental designs, counterfactual analysis, participatory approaches [20].

The following parts 2.1 and 2.2 of the paper are mostly referred to concepts and arguments presented at the Congress held in Naples in 2010 on the evaluation of public funding for Structural Policies (see [13]). In the subsections we find a description of some questions about the evaluation of urban interventions implemented by the Structural Funds during the 2000 – 2006 period in a specific regional context; the main trends for the analysis of the regional development programs for the 2007 – 2013 period are also outlined.

2.1 Evaluation of Urban and Territorial Policies in Abruzzo during the 2000 – 2006 Programming

The experience of the Abruzzo Region in the frame of urban and territorial policies has mainly covered the *Integrated Territorial Projects* (TIPs) and the *Framework Programme Agreements* (FPAs). The first ones refer to projects formulated according to a *bottom-up* approach and funded in the South Italy during the Community program 2000 – 2006. The TIPs should be viewed in a research phase for new methods of interventions, focused on the importance of the *local development*. The TIPs focus on the sharing of local needs in a specific local context and on the concentration of Structural Funds spending toward a single goal of development. These tools for local growth are also characterized by the aspects of strengthening the role of the municipalities, wide participation in public decision-making, and integration of different interventions within a single project aimed at resolving a relevant territorial issue [1]. The FPAs concern agreements between different levels of government for the identification of infrastructures to be financed by the National Fund for Underdeveloped Areas for the development policies for the South.

The analysis of the Territorial Integrated Programmes (TIPs) implemented in Abruzzo during the period 2000 – 2006 was essentially an *evaluation process* to assess the completeness and consistency of programs in respect to the strategies and guidelines of the development Regional programmes (RPD). It affected mainly criteria related to *validity and consistency of the motivations* of the TIP, in order to analyse whether the different sectorial interventions with a single management were able to capture specific opportunities of the territory. The identification of the *basic strategy* of the project is considered essential for the evaluation of public policies. The identification of the objectives for each program is necessary if you adopt a valuation method "*centred on the objectives*", comparing the target set and the results pursued [1].

Other elements of analysis are: *internal consistency of the project* in relation to needs and objectives of the local context and in relation to the technical-economic-management aspects of each project; *cost-effective* of coordination and management; methods of *implementation* of the TIP and *expected results*; *external coheren*ce of the project, in relation to priorities and strategies of the European policies, and also to the objectives of the Regional Development Programme (RDP).

About the formulation of the TIPs, it was pointed out in particular that the *technical-economic* specifications of the interventions would require a clearer identification of projects. In addition, it would be appropriate to analyse the *consistency* of the program not only with respect to the *higher level* tools (Programming Document and RPD, in the specific case of the PIT), but also with the *local planning* tools adopted or being adopted, in order to frame the project/program in the planning and programming of regional and local level. Therefore, it is considered appropriate an estimate of the economic, financial, and environmental feasibility.

The evaluation of the interventions included in the FPAs for urban areas in the *intermediate* stage was mainly based on the analysis of the *relevance of the strategy*, *effectiveness analysis,* and *ability to implement*. In particular, the last one is evaluated on the basis of *financial efficiency*, in terms of capacity for commitment and expenditure, and *procedural efficiency*, as implementation and success, for each FPA examined, considering the activation of single interventions, the delay in implementation and the probability to complete works before the deadline [13].

A critical analysis of the main signs of development connected with each program (TIP and FPA) in different contexts is still in the process of defining.

2.2 Evaluation of Urban and Territorial Policies in Abruzzo for the 2007-2013 Programming

The current phase of programming has introduced the urban strategies in the regional cohesion policy, to analyze the various opportunities of the territories and to reduce disparities in regional development. Regions identify in their programming documents the *priorities* of their urban policy, the financial *resources* and the *cities* for specific interventions.

At national level, the priorities of the National Strategic Framework (NSF) 2007-2013 on spatial and urban planning (Priority 8) reaffirm the importance of ensuring an *integrated vision* of development, to include the issues of urban-territorial planning, cultural heritage, landscape, environmental, social, and economic development, with particular reference to the importance of the integration of investments for urban areas with sectorial policies.

In Abruzzo, the priorities of the regional urban policy for the 2007-2013 period can be recognized in some operational documents relating to the Structural and national Funds. In particular, the European Regional Development Fund (ERDF) Operational

Programme for 2007-2013, under Axis IV "Territorial development", has the following objectives for urban areas: improving the image of the city, strengthening contribution to innovation, entrepreneurship, and knowledge-based economy, creating more and better quality jobs, reducing social inequalities. The program underlines the importance of shared planning strategies to be implemented through *Strategic Planning* and *Integrated Planning*. Latest regional planning documents emphasize the purpose of a *balanced growth* in the local contexts, considering their assets and encouraging higher services and infrastructures. The documents underline the strategic position of Abruzzo in the international and national level, as a region of central importance in the Euro-Mediterranean area. At present, policies for the regeneration of urban areas in Abruzzo have made use of a large variety of operational programs for about 2.200 projects and investments of more than 6 billion euros. However, it has proved *the lack of a logical network* of cities and the consequent absence of integration among *territorial systems*. For these reasons, it is hoped the definition of an *Operational Programme for the cities* to ensure coherence to *strategic planning* and *ordinary urban planning*, defining *priorities* for urban policy on the basis of the new strategic regional trends [13].

An appropriate evaluation of the results of urban policies implemented in different territorial contexts appears to be essential in this regard.

The main innovation introduced by the EU cohesion policy for the period 2007-2013 is the introduction of *Regional Plans of Evaluation*. The Regulation (EC) 1083/2006 provides indicative guidance for assessments, distinguishing between *evaluations with a strategic nature*, aimed at providing relevant information to ensure the *consistency* of the program in relation to the Community and national priorities, and *evaluations of operational support* to the supervision of programs. Based on these guidelines, the *Evaluation Plan* of the Abruzzo Region for 2007-2013 unitary programming offers *strategic and operational evaluations*.

Strategic evaluations are intended primarily to investigate the *process of territorial rebalancing*, examining the relationship between development policies and territory, considering also the most effective interventions in reducing the economic and social imbalance in the internal areas.

Operational evaluations concern the following aspects: the *self-assessment* activities, to analyze strengths and weaknesses of the program; the *process evaluation,* to identify the difficulties that could undermine the implementation of the program, including any corrective actions on administrative procedures and participatory governance; the *product evaluation*, aimed at providing the expected and/or unexpected results of programs, in reference to consistency and effectiveness of the actions to achieve the planned objectives. In particular, the *process evaluations* will examine "the added value of *Integrated planning* compared to the more traditional planning and implementation of interventions, identifying the territorial and institutional contexts in which the integrated planning works better". The analysis aims to highlight the *good management practices*, in order to strengthen synergies among the relevant actors in the territory.

The following table is an explanatory scheme of the differences between strategic and operational evaluations, according to the indicative guidelines of the European Commission [11].

Table 2. Strategic and Operational evaluations

Evaluation Plans	***Contents***
Strategic evaluation	It provides analyses to: - examine the development of a program in relation to *Community and national priorities;* - assess the macro-economic impact of Structural Funds, taking into account the *impact indicators*, which are more suitable for strategic considerations; - propose *adjustments* in line with changes in the socio-economic environment; - focus on *specific themes* which are of strategic importance for the program; - identify *good practices*.
Operational evaluation	It provides analyses to measure and analyze progress in implementation of program, considering: - relevance of the quantified objectives, expressed as *output and result indicators*; - data on *financial and physical progress*, so to provide recommendations on how to improve the performance of the program, in terms of Efficiency and Effectiveness; - functioning of administrative structures and quality of implementation mechanism.

Some authors highlight that techniques to be used in the Evaluation of programs are evolving in the scientific debate, considering that this kind of evaluation is very different from the ex-ante evaluation and selection of projects.

3 Methodological Perspectives in the Evaluation of Urban and Territorial Policies

As already noted, the analysis of the regional development policies and their ability to generate effects on the local contexts is still in progress. Indeed, the potential economic, social, and institutional effects are captured in a longer time than it is necessary for the implementation of the interventions; sometimes the impact is in areas which are different from the contexts regarding the projects under examination [2].

With specific reference to urban policies formulated in the Abruzzo context, the transition from the period 2000-2006 to the 2007-2013 one is characterized by a greater focus on shared strategies, to be achieved through the formulation of *Strategic Plans* of the city. These tools bring unity to the numerous tools for urban regeneration tested to date, investing rural and mountain areas, and overcoming the traditional dualistic view of "coast-inland areas" in the region. The experience evaluation developed in relation to the urban interventions have focused mainly on “operational” aspects. The lack of an integrated vision of the development strategies implemented in different urban contexts requires a reminding of the evaluation models, replacing the "*target-result*" logic with a more "*strategic*" level of the evaluation.

The *Cost-benefit analysis* and the *Effectiveness evaluation* are the traditional tools to estimate the collective well-being resulting from the implementation of an intervention (see [9] as regards the latest methodological guidelines of the European Commission). However, the concept of territorial competitiveness requires not only the consideration of the effectiveness of projects, but also the analysis of the territory as a whole, in its specificity and complexity: the only quantitative analysis are not enough to understand why some policies have certain effects in a given context.

The monitoring system of EU policy is aimed at providing information on the progress made in the Operational Programmes with regard to the indicators, in order to intervene where problems arise. It is also important to show which objectives have been achieved with public resources [5]. In Italy several territorial indicators were built under the Community Support Framework (CSF) for 2000-2006 to assess the *quality of policies*. The CSF proposed a "soft" use for the *context indicators* and "hard" use for the *performance indicators*.

The *context indicators* describe the multidimensionality of territorial development with reference to *well-being*, *quality of services*, *quality of housing*, *availability of infrastructure*, *employment*. These indicators concern the main characteristics of the areas, supporting the policy making processes and allowing to assess their effectiveness.

The *performance indicators* are primarily related to the strengthening of *institutional capacity* (institution building), that is necessary for the success of policies. In accordance with this, it is now widely accepted that the mechanism of the Community and national *Performance Reserve* has introduced some important innovations, rewarding the National and Regional Operational Programmes based on effectiveness, management and financial implementation, highlighting the institutional progress toward the reform and simplification of the public administration, integration of measures in the territorial planning, concentration of interventions in a reduced number of objectives.

Experience has shown the need for further reminding on physical indicators and in particular on *core indicators*, in order to account the effects of EU policies. The Working Document No. 2 of the European Commission (2006) [11] provided a guide to the identification of indicators for monitoring and evaluation, offering 41 core indicators to be used for the European Fund for Regional Development (ERDF) and the Cohesion Fund within the *Convergence* and *Regional Competitiveness and Employment* objectives (Tab. 4).

These indicators, subject to revision and subsequent simplifications, are described in the most recent Working Document 7 of the EC (2009) [5], so that all European countries can develop a standard accounting of the progress of cohesion policy. With regard Urban Development, the indicators are: *number of projects ensuring sustainability and increasing attractiveness of towns and cities; number of projects promoting business, entrepreneurship and new technologies; number of projects providing services for the promotion of equal opportunities and social inclusion of minorities and young people*.

Table 3. Indicators for monitoring and evaluating of CSF 2000-06

Quantitative measurable variables	***Objectives***
In order to evaluate the *overall effectiveness* of the program: - **Context indicators (soft use)**	Contex indicators describe the socio-economic features of the context. They represent an essential tool both in the programming phase, both in the construction of projects, both in the verification phase. By way of instance, some context indicators for the "Cities" Axis of CSF are: - % of families who declare very or fairly difficult to reach supermarkets - equipment of air quality monitoring stations (values per 100.000 inhabitants) - users of public transport on the total number of people who have moved for work and have used means of transportation (%).
- **Breaking variables**	Breaking variables are indentified during the definition of the strategy, to describe the potential for development of the area. In particular, they represent the sectors through which an acceleration of public investments increases productivity and growth.
In order to strengthen *institution building*: - **Performance indicators (hard use)**	Performance Indicators describe the process objectives. The allocation of resources to reward Regions depends on the achievement of specific targets. The *performance* concerns: - *Institutional progress* (modernization of pa, innovations for the greater effectiveness of Structural Funds spending, sectoral reforms) - *Quality of programming* (project integration and concentration of financial resources on a limited number of objectives).

The following table shows the main indicators to measure and observe the development policies of the National Strategic Framework (NSF) 2007-13.

The table has been structured on the basis of information from the Department for Development Policy and Economic Cohesion of the Italian Ministry for Economic Development.

The main evaluation issues emerged from the recent documents of the Commission for the current programming promote the *learning process* of the evaluation, with the definition and application of methods to understand *what has* (*or has not*) *worked* and then the *real impact* of policies [23]. The *fieldwork* is preferred to understand the overall improvements of the development policies. It is suggested a *realistic approach* to the evaluation, with particular attention to the comparative analyses to study the effects of a policy in different contexts. The aim is to overcome the "goal-result" logic based on *statistical indicators* which concern the outcomes of a single

Table 4. Indicators of NSF 2007-2013

Types of Indicators	*Objectives*
Indicators to observe the NSF	To be used as a support to the orientation of policy actions and assessing the effectiveness of policies. The reference is to the *Database of indicators for territorial development policies* (approximately 200 indicators, grouped by thematic priority of the NSRF, on www.istat.it).
Indicators of Programs:	To be used for the measurement of phenomena induced by development programs in implementation of the NSF
- Output	Relating to each objective of program , these indicators are required at the national level and apply to all projects of programs funded by NSF. The indicators are identified and managed by the Unified Monitoring system 2007-2013. They consist of: - *Physical Indicator* , in order to capture the concrete manifestations of the projects; - *Employment Indicator* , which is temporary and /or additional, for each intervention.
- Result and Impact	Starting from a conceptual distinction between *results* (effect of interventions on the direct beneficiaries) and *impact* (effect of interventions on a wider context, which also includes the indirect beneficiaries), quantified targets (target) distinguished for areas were provided, to be achieved during the 2007-2013 programming period. Indicators of *Essential Services* are here included, referring to sectors (education, care services for children and the elderly, urban waste management, integrated water services) considered fundamental for the improvement of the living conditions of citizens and for economic activities. They set binding targets on final goals, explicitly expressed in terms of *service to citizens*, on the achievement of which a financial reward for the eight Regions of the South and the Ministry of Education depends.
Core indicators	They are a limited number of data to be used for the measurement of phenomena induced by the implementation of programs supported by the ERDF. These indicators are requested by the European Commission for the purpose of reporting to the European Parliament what has been achieved (see [5]).

intervention in a specific geographical area [17]. But The measurement of the *added value* of territorial development policies is still considered a complex system under definition, since characterized by *incomplete information* and *uncertainty* of the causal links between objectives and actions [18].

In 2002, the European Commission introduced the *Impact Assessment* (IA) (CEC, 2002) to improve policies in achieving Sustainable Development objectives. In the "*Socio Economic Tools for Sustainability Impact Assessment*" of the European Commission (DG Research, 2002) we read that the aim of this evaluation approach *is to provide a set of quantitative and qualitative decision variables that will guide and support policy-makers in taking decisions*.

The document describes the *tools* to assess impacts in *qualitative*, *quantitative*, and in *monetary terms* with reference to the approaches developed in Europe: methods commonly used to evaluate trade-offs and support decision making, such as cost-effectiveness, cost-benefit and multi-criteria analysis, and methods of *participated evaluation*. We find also the treatment of *risks* and *uncertainties* arising from incomplete knowledge, with reference to the Monte Carlo Analysis, Sensitivity Analysis, Delphi Methods and Meta-Analysis.

The IA was later reviewed as a series of logical steps for the definition of policies (CEC, 2005, p. 4); the procedure integrates together different sectors and dimensions (economic, environmental and social), replacing all existing assessments with mono-sectorial nature (environmental assessments, equal opportunities, economy, health) (CEC, 2004b). The "*Impact Assessment Guidelines*" (CEC 2009/92) of the European Commission provides the analytical steps to follow in the IA work: i*dentification of economic, social and environmental impacts; qualitative assessment of the more significant impacts,* assigning likelihoods (e.g., low, medium or high probability) that the impact will occur (or conversely the risk that the impact will not occur) and estimating the magnitude of each impact (providing reasonable ranges)*; in-depth qualitative and quantitative analysis of the most significant impacts*. These methodological indications suggest the need for further investigation of the complex issues treated, which we can find developed in many Commission documents providing a *Toolkit* on specific aspects about economic, social, and environmental impacts (see annexes 8, 9, 10, 11 of CEC 2009/92).

One of the methodological approaches developed by the international scientific community to understand the effects of policies is the *counterfactual evaluation*. This approach aims to study the "effects of causes" on a well-defined group of beneficiaries, so to understand whether a given output produces a desired or unexpected effect. The counterfactual approach is rarely used today in the Italian context, but widely developed in the social field in the United States and Canada. Given the impossibility to observe the counterfactual elements, the ability of the evaluator is to remind the situation "without intervention" in a plausible manner [17]. In this regard, some recent theories are oriented to the use of Regression Analysis for evaluating causal effects of interventions, with particular reference to *Regression-discontinuity* (RD) methods and *fuzzy regression discontinuity* (FRD) designs, as suggested by Lee e Lemieux (2009) and Lemieux et al (2011).

The classification of methods and techniques to be used at the *conclusion/result* stage of the policy cycle is copious, however, the aim of "*knowledge production*" appears to overlap with the aim of "*accountability*". This last is to be achieved through the use of indicators, useful "to demonstrate results", but ineffective in the pursuit of the most complex cognitive purpose on the effects of policies. For this reason, in order to improve the programming capabilities and give an answer to questions such as "*what works, for whom, as and under what circumstances*", it seems appropriate to dedicate specific attention to the different evaluation approaches which have different aims of knowledge.

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Decision-Making Process with Respect to the Reliability of Geo-Database

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Abstract. The aim of this paper is to propose a method that would focus on data and spatial information precision and reliability evaluation. The resulting characteristics of data reliability can be applied in various command and control systems. The method to contribute to increase the quality of decision-making process is proposed. Finally, the case scenario is focused on an intervention of a fire rescue unit and presents the proposal for the use of the system in practice.

Keywords: reliability, decision-making process, mathematical modeling, spatial data, GIS, quality assessment, utility value.

1 Introduction

Command and control systems used in various branches of rescue systems use digital geographic data and information increasingly more often. Geographic data are collected from various sources with the use of various technologies. It results into position and thematic properties inhomogeneity. In spite of this situation, data are stored and used in a common spatial database or they are used for various kinds of spatial analyses. Obtained information can be applied in a decision-making process. Precision and reliability of such information can significantly influence the final solution.

The aim of exploiting geographical data is to accelerate decision-making processes and to optimize deployment of forces and resources. In case of fire and rescue service, it is the elimination of consequences of technical and environmental disasters.

However for correct decision-making it is necessary to know the complex characteristics of geographical data in relation to the task that is being solved so that the reliability of background analyses for decision-making was clear.

By implementing the methods of value analysis and mathematical modeling it is possible to create an assessment system of spatial data complex usability. Based on

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input characteristics of the used spatial data and databases, quality characteristics and their changes can be calculated with the help of analytical methods. By comparing costs necessary for different variants of enhancement or for an adjustment of database quality it is possible to optimize both the total usability and the costs securing the required data quality.

2 Possibilities of Assessment of Digital Geoinformation Quality

Each product including DGI has to be made for the specific user and only his satisfaction with the product is the final criterion for quality of this product evaluation. Usability as an expression for a product's potential to accomplish the goal of the user is often mentioned term. Usability can be described as results of technical quality parameters evaluation added with textual remarks related to customer requirements. The other approach is to apply some system which enables to combine different possibilities of expression of quality parameters. The application of the Value Analysis Theory (VAT) (Miles, 1989) is one of possibilities.

Five essential criteria imply from DGI quality review. Their assessment gives the baseline for relatively reliable determination of each product utility value (Talhofer & Hofmann, 2009). Each of the criteria is mathematically assessable through independent tests and can be described as a quality parameter. In the next table (Table 1) there is a list of main used criteria. The whole criteria are in (Talhofer, Hoskova-Mayerova, & Hofmann, 2012).

Table 1. List of the main criteria for the spatial geodatabase utility value evaluation

Main criteria	Main criteria characteristics
Data model content – k_1	Complexity of conceptual landscape model and compliance of required resolution of geometric and thematic data
Technical functionality – k_2	Transparency of data sources and methods for secondary data derivation, position accuracy, thematic accuracy, logical consistency, and data completeness
Database timeliness – k_3	Degree of adherence geographic data to the time changes in the landscape
Landscape importance – k_4	Value of inverse distance to objects of interest
Techniques of application and safety – k_5	Data standardization, independency on application software, data protection

2.1 Assessment Function

The product or a part of the product resultant function utility degree may be assessed based on the above mentioned criteria using a suitable aggregation function (Talhofer, Hoskova-Mayerova, & Hofmann, 2012), (Talhofer, Hoskova, Hofmann, & Kratochvil, 2009).

$$F = p_3 k_3 p_4 k_4 (p_1 k_1 + p_2 k_2 + p_5 k_5) \tag{1}$$

The chosen form of the aggregation function concerns also the case the user gets data on an area beyond his interest or data obsolete so that their use could seriously affect or even disable the DGI functions. The weight of each criterion is marked as p_i, where $i = 1, \ldots, 5$. The mentioned aggregation function proves the product status at the questioned instant and its utility rate. It is applicable also to experiments to find the ways of how to increase product utility at minimum cost increment.

2.2 Individual Benefit Cost Assessment

Organizations, such as the Geographic Service of the Army of the Czech Republic or the Czech Office for Surveying, Mapping, and Cadastre, are usually responsible for DGI databases development continuously covering all the Czech Republic area or some parts of the World. Digital Landscape Model (DLM25 or DMU25 in the Czech language), Multinational Geospatial Co-Production Program (MGCP) or Vector Map Level 1 (VMap1) can be mentioned as examples from military branch (for more see (Talhofer, Hoskova-Mayerova, & Hofmann, 2012), (Talhofer & Hofmann, 2009)).

The DGI are usually developed and maintained by individual partial components of the complete database, such as save units, measurement units, map sheets, etc. Therefore, it is quite a good idea to assess their utility value in the above-described system within the established storing units introducing *individual benefit value*. Similarly, the individual benefit value can be applied for the selected part of master databases from given *area of interest* which is used for certain task.

When assessing database utility, it is useful to define *ideal quality level* at first. The ideal level is used as a *comparison standard* to express each criterion compliance level. Using the comparison standard the individual criteria compliance level and consequently aggregate utility may be assessed.

The compliance level of each individual criterion $u_{n,s}$ is given as $u_{n,s} = k_s / k_s^*$, where k_s is for the value of s^{th} criterion compliance and k_s^* is for the level of compliance of s^{th} criterion or its group criterion of the comparison standard.

Thus the aggregate individual benefit value (*individual functionality* – U_n) of the n^{th} save unit is defined by the aggregation function of the some type as (1). Therefore:

$$U_n = p_3 u_{n,3} p_4 u_{n,4} (p_1 u_{n,1} + p_2 u_{n,2} + p_5 u_{n,5}) \tag{2}$$

The individual criteria weights are identical with the weights in database utility value calculation.

Particular criteria usually consist of several sub criteria (see Table 1). The authors took 20 criteria into their consideration; hence the equation for calculation the aggregate individual utility value is therefore a function of 20 variables that characterize the levels of compliance for each individual criterion.

Any modification of selected criterion has an impact on the value of U^n. Individual variables are independent one to another, so the derivation of the function can model the changed utility values or individual utility values.

$$dU = \frac{dU_n}{du_{n,i}} \tag{3}$$

where $i = 1,\ldots, 5$, $n = 1,\ldots, N$, and N is number of all saved units in the database.

Determination of dU value is thus feasible in two ways regarding the desired information structure. When assessing *individual variables effects* on the individual functionality value, while the other variables keep constant values, it is necessary to differentiate U function as follows:

$$dU = \frac{dU_n}{du_{n,i}} \frac{du_{n,i}}{dx} \tag{4}$$

where x is one of the 20 mentioned variables.

In practice, however, such situations may arise that multiple factors may change at the same time. For example the technical quality of database changes in all its parameters—the secondary data derivation methods will improve location and attribute accuracy and the data integrity will increase, and moreover, the data stored in a geodatabase accessible to all authorized users. In this database the data are maintained properly with respect to all topologic, thematic, and time relations. In such a case it is suitable to define dU value as a total differential of all variables describing the modified factors. For more details see (,).

Database functionality degree is comparable to the cost necessary for provisions—direct used material, wages, other expenses (HW, SW, cost of amortization, tax and social payments, etc.), research and development cost, overhead cost, and others. One example of the whole expenses for up-dating of one map sheet (area of it is 64 km^2) is in the next table (Table 2). Listed expenses are valid for operations doing in the conditions of the Military Geographic and Meteorological Institute of the Czech Armed Forces (MGMI).

Functionality and cost imply *the relative cost efficiency (RCE)* calculated as follows:

$$RCE = \frac{F}{\sum_{i=1}^{n} E_i}, \qquad i = 1, \ldots, N \tag{5}$$

In this formula F is the aggregation function calculated by (1). Similarly to individual utility value, it is possible to consider the impact of particular variables of expenses E_i on final RCE. The goal is to find such solution as the functionality will be maximized and the expenses will be minimized.

Table 2. Expenses for up-dating one sheet of digital geodatabase DMU25

Stage	Working operation	Norm in hours	Direct wage in CZK	Direct mater i-al in CZK	Social found in CZK	Total direct costs in CZK	Produc tion overhea ds in CZK	Admini strative expense s in CZK	Total expense s in CZK
Preliminary works	Documentary establishment	0,5	60	-	22	81	146	159	408
	Sources collection and preparing	16,0	1919	-	706	2611	4698	5099	13114
	Basic editorial preparation	12,0	1 432	-	530	1962	3524	3825	9841
	Data structure preparation	1,0	152	-	56	208	374	406	1044
Database up-dating	Topographic evaluation	178,8	25081	1335	9280	35696	61700	66967	173643
	Revision	71,3	10001	1	3700	13703	24604	26704	68711
	Photogrammetri c evaluation	13,8	1784	-	660	2444	4390	4765	12259
	Revision	0,6	77	-	28	106	190	207	531
	Check-in in the field preparing	2,5	350	-	129	480	862	936	2407
	Check-in in the field	12,5	1753	-	648	2402	4313	4681	12044
	Data completion	20,0	2805	1	1038	3844	6901	7490	19273
	Final revision of up-dating	25,0	3506	-	1297	4804	8626	9363	24090
	Control drawing	1,0	140	2	51	194	345	374	964
	General revision	2,0	280	-	103	384	690	749	1926
	Export of data	0,5	76	-	28	104	187	203	522
Database management	Data storage in the database	1,0	152	-	56	208	374	406	1044
Total		358,5							341821

3 Usage of Value Analyses in the Decision-Making Process

The DGI benefit cost assessment including individual benefit cost is a task for a data manager or a geographer-analyst who is responsible to provide a demanding project. The system enables him to consider which quality parameters are possible to improve in given time, with given technological conditions, with given sources, with given co-workers, etc.

The system of spatial data quality evaluation and application of the VAT should help to answer most of the previous questions.

Frequent task in decision-making processes is based on judging various options of solutions of the given problem. The options are calculated with the help of spatial analyses, in which technical and technological factors are taken into account, for instance the performance of technical resources, their usability in terrain, questions of co-operation of the individual units, etc. Spatial analyses work with available data, yet it is always necessary to take into account also the quality of the data. This can significantly influence the whole decision-making process.

Geographical information systems used in management systems usually work with a certain version of spatial database that has a certain level of quality which is projected into the quality of the solution of the resulted analysis. It is not possible to approach the solution of analysis as a final result but it is necessary to take into consideration also a certain level of vagueness (Hoskova & Cristea, 2010). The vagueness level of the result is then given to the manager either verbally or with the help of visualization (Kubíček & Šašinka, 2011). In any case, it is essential to take into account the vagueness level in the final phase of decision-making.

Decreasing the vagueness level of the resulted analysis is possible, e.g., by improvement of quality of the database which is, however, a time-consuming process.

The complete system of relations between a geographical database and a decision-making process is described in a case study in the following chapter.

4 Case Study

In order to verify the VAT methodology the task of *Cross Country Movement* (CCM) was chosen as an example. CCM can be solved as a common problem or with consideration of certain types of vehicles. The detailed theory of CCM is explained in (Rybansky & Vala, Relief impact on transport, 2010) and (Rybansky, 2009). This publication is actually related to the questions of military technology, however, with regard to the fact that it is a study describing the interaction of chassis of vehicles and terrain; the conclusions are valid also for the rescue technology used by the fire and rescue service.

The case study was solved in two phases. In the first phase, the relation between database quality and reliability of the decision-making process was judged. In the second phase, a database of higher quality was used and its qualitative characteristics in relation to more types of emergency vehicles were examined.

The solution can offer to the officer in duty not only one possibility, but the variants from which he/she can choose according to his/her intentions and the current situation in the given area.

4.1 Cross Country Movement

The main goal of CCM is to evaluate the impact of geographic conditions on a movement of vehicles in terrain. For the purpose of classification and qualification of geographic factors of CCM, it is necessary to determine:

- particular degrees of CCM
- typology of terrain practicability by kind of military (civilian) vehicles
- geographic factors and features with significant impact on CCM

As a result of the geographic factors impact evaluation we get three degrees of CCM: passable terrain, passable terrain with restrictions, or impassable terrain.

The impact of geographic factor can be evaluated as a *coefficient of deceleration* 'C_i' from the scale of 0 to 1. The coefficient of deceleration shows the real (simulated) speed of vehicle v in the landscape in the confrontation with the maximum speed of given vehicle v_{max}. The impact of the whole n geographic factors can be expressed by the formula:

$$v = v_{\max} \prod_{i=1}^{n} C_i, \qquad n = 1, \ldots, N. \tag{6}$$

The main coefficients of deceleration are listed in the next table.

Table 3. Main coefficients of deceleration

Basic coefficient	**Geographic signification and impact**
C_1	Terrain relief (gradient of terrain relief and micro relief shapes)
C_2	Vegetation cover
C_3	Soils and soil cover
C_4	Weather and climate
C_5	Hydrology
C_6	Build-up area
C_7	Road network

Each coefficient consists of several subcoefficients. For example decisive effect on the coefficient of deceleration of vehicle movement by effect of soil type C_3 have such factors as the sort of soil (depends on soil granulation); a type of soil at factual weather conditions, which affects above all the adhesive force and rolling friction of vehicle wheels/tracks; the vegetation cover of soil; the roughness of terrain surface. These factors have decisive effect, which is given by relation:

$$C_3 = \prod_{i=1}^{n} C_{3i}, i = 1,2,3$$

where:

- C_{31} is coefficient of deceleration by effect of soil type (sort) factor,
- C_{32} is coefficient of deceleration by effect of factor of vegetation cover,
- C_{33} is coefficient of deceleration by effect of surface roughness factor.

For given vehicle (its technical properties) the values of deceleration coefficients are counted from ascertained properties of geographic objects stored in the spatial geodatabase. Using formula (6) it is possible to create a cost map in which the value of each pixel is the final (modeled) speed. The *cost map* can be used as a source for calculation of the fastest path, the most reliable path etc.

4.2 First Phase of the Case Study

In the first phase of the case study, dependence of the results of analyses on the quality of used data and the way of interpretation of the results of analyses in decision-making processes were examined. The aim was to find an optimized path for a terrain vehicle TATRA 815 in free terrain (Tatra, 2010). The complete procedure is described in (Talhofer, Hofmann, Hošková-Mayerová, & Kubíček, 2011). Only results and main conclusions are stated in the following text.

The master DGI database is usually used as a base for spatial data analyses. The national or international databases as DMU25, VMAP1, or MGCP are very detailed, carefully maintained, and used in many applications. But nobody can suppose that those databases contain all information he could need.

The task of CCM solution could require more information that is available in the master database. Geographer-analyst has to consider which information and in what quality can he/she obtain from the master database. E.g., all forests in the area of interest are necessary to be selected for mentioned C_{2i} coefficients. Further he has to find out all their properties and their accuracy or count how many characteristics are missing. The next step is the individual functionality value of the given part of master database evaluation. In the next picture (Fig. 1) there is a basic scheme of the database for CCM creation.

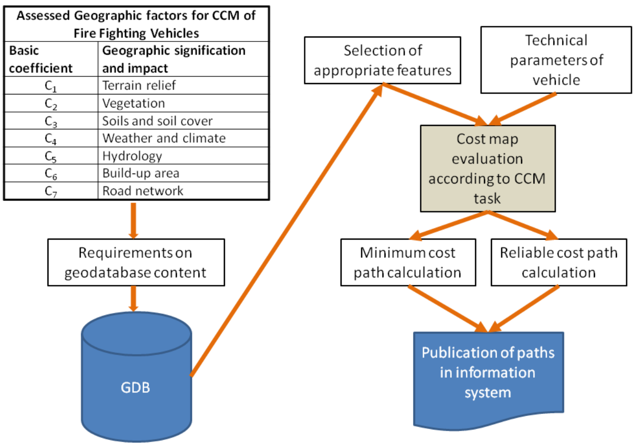

Fig. 1. Spatial analyses without database quality evaluation

Not all attributes are available within the used thematic spatial databases. So far the incompleteness of attributes has been omitted. Thus, the real state-of-the-art has not been taken into account and the resulting CCM path has been considered as 'certain'. One of the possibilities to make the resulting path closer to reality is to take the data attribute incompleteness into account and inform the decision maker (commander) about the uncertain parts of the path.

Two variants of the DMU25 database were utilized for the case study. The feature properties were defined according to the *Feature Attribute Coding Catalogue* (FACC) adapted as *Catalogue of the Topographic Objects* (CTO) (MTI, 2005) in the first variant updated in 2005. The 4^{th} edition of CTO was transformed in accordance with the *DGIWG Feature Data Dictionary* (DGIWG-500, 2010) in 2010 and a transformed edition (updated in 2010) was used in the second variant (MoD-GeoS, 2010).

The smaller personal database was created in the area of Brno of the size approximately 400 km^2 and all objects necessary for CCM evaluating were selected from DMU25 databases of both variants. The individual utility value was counted for both variants. On the base of statistical analysis 12.65% objects have problems mainly due to incomplete attributes in the first variant of DMU25 while 3.45% objects have similar problems in the second one. The time difference is 5 years between both variants. Hence, the individual utility value was calculated using the formula (3) as 0.6887 for the 2005 variant and 0.8825 for the 2010 variant. The ideal quality level is 1.0068. Both variants were used for CCM of TATRA 815 evaluation.

ArcGIS 9.3 was used for all calculations and analyses. In the next figures there are the main results – cost maps. The cost of each pixel, which is the simulated speed of vehicle, is symbolized in the gray scale where a darker tone signifies higher costs, higher speed in this case.

The minimum cost paths were evaluated using both cost maps and the same process created in ModelBuilder were applied. The results are shown in the next figures.

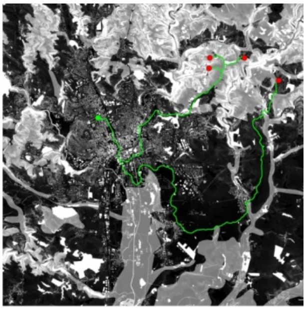

Fig. 2. The minimum cost paths in CM of 2005 version. The initial point is green, the destinations are red

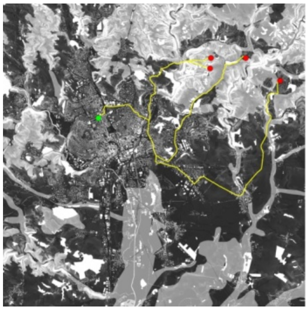

Fig. 3. The minimum cost paths in CM of 2010 version. The initial point is green, the destinations are red

The comparison of both results presented over the topographic situation is shown in the next picture (Fig. 4).

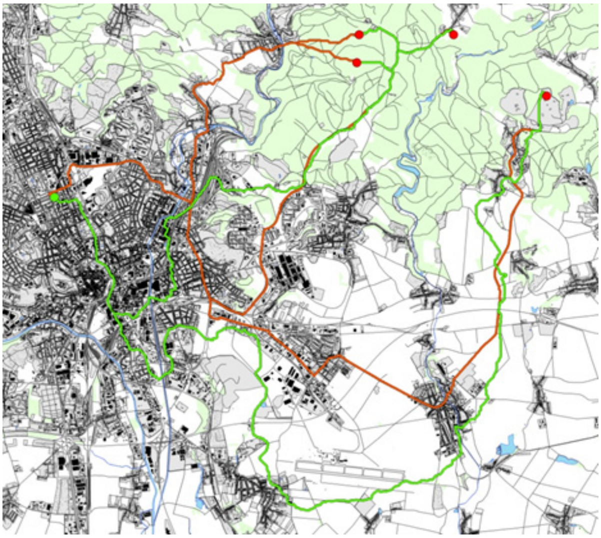

Fig. 4. Comparison of two variants of minimum cost paths. Red ones correspond to 2005 version and the green ones to 2010 version.

The obtained results proved that the results of spatial analyses are highly dependent on overall quality of digital spatial data. Relatively small changes in the database can cause significant differences in received results.

4.3 Model Conditions of the Second Phase of the Case STUDY

Conditions for the solution of the second phase of the case study came from the established operational procedures that are used in actions of fire and rescue service units. The aim of this phase was to show possibilities of using geographical data to optimize decision-making, i.e., reduction of time before the action is commenced, optimization of engaging of equipment and human resources, and last but not least cost-saving of financial sources.

For the solution itself the following model task was chosen:

Let's have an averagely equipped fire and rescue service station, in which these vehicles are available:

- one road fire fighting vehicle with the tank capacity of 9,000 liters,
- one terrain fire fighting vehicle with the tank capacity of 9,000 liters,
- one terrain fire fighting vehicle with the tank capacity of 3,000 liters,
- one road transport vehicle (transport car, hoses, and pump)

At this station a fire in relatively inaccessible terrain is reported. The intensity of the fire is so high that it is presumed that at least 20,000 liters of water will be needed to extinguish it. There is no hydrant available in that locality; what is more, weather conditions make the use of helicopter impossible. Operational officer must command the vehicles to go in a way so that he/she optimized time and exploitation of the available fire fighting vehicles. The task is to decide to engage specific Technical parameters of Fire Fighting Vehicles

After consultation with officers in Fire Brigade in Brno, the following vehicles were chosen for the case study (see Table 4). Technical details were obtained from websites of producers (THT, 2012) and specified also in Brno Fire Brigade.

Table 4. The technical characteristics of available Fire Fighting Vehicles

Vehicle number	1	2	3	4	5
Parameter	Tatra T815-731R32 6x6	Mercedes-Benz Unimog 4x4	Mercedes-Benz Atego 1329 AF 4x4	Mercedes-Benz Actros 3355 A 45 6x6	Renault Midlum
Length (m)	9,19	6,70	7,60	10,07	6,42
With (m)	2,55	2,34	2,55	2,55	2,37
High (m)	2,85	3,20	3,33	3,76	3,05
Maximum climbing capability	36°	45°	30°	20°	15°
Maximum climbing capability up to rigid step (m)	0,5	0,5	0,5	0,5	0,3
Maximum width of trench (m)	0,9	0,5	0,5	0,9	0,3
Maximum road speed (kph)	100	90	105	100	90
Maximum depth of wade (m)	1,2	1,2	0,5	0,5	0,5
Water tank capacity (liters)	9000	3200	1200	9500	0

4.4 The Procedure of Solution

As the base for the solution, database of the same locality as in the first phase of the case study was used, only in version of 2010. According to the parameters of the solved CCM task and technical characteristics of the fire fighting vehicles, requirements of its contents were derived, which were later used to calculate traffic ability in the terrain.

Based on the stated requirements, appropriate data of the geo-database were taken and a cost map was calculated from the given space. With the help of the cost map for individual vehicles, possible variants of path from the fire station to the action point were calculated. Such a solution is possible to be made for various emergency vehicles, or more precisely for vehicles which are available and suitable for the given kind of action. The procedure of solution was once again processed in the environment of ArcGIS 10 using Model Builder tool, in which the input parameters of vehicles may be changed.

Results can be visualized on the operational officer's screen who—according to the calculated paths and their parameters (distance, travel time, possibility to come directly to the place of fire, or more precisely the necessity to include time to spread hoses from place where the given vehicles can go to in case the traffic ability is limited) and with inclusion of water tanks capacity of vehicle—has to decide which vehicle or vehicles will be sent to an action.

4.5 Procedure of Calculation

According to CCM theory (Rybansky & Vala, Relief impact on transport, 2010), individual coefficients of deceleration C_1 to C_7 were calculated, based on which complete cost maps for individual types of vehicles were calculated. The simulated speed of the given vehicle in the given pixel was once again taken as a pixel cost in the cost map. In the individual cost maps the cheapest—in this case the fastest paths—from the fire brigade station to the place of fire were calculated. The place of fire was purposely set away from settlements in free terrain and away from communications. Also, time of arrival to the place of fire for individual vehicles was calculated and the possibility for vehicles to go through the free terrain to the place of fire was checked.

The complete calculation was once again—based on the mathematical model—programmed in the environment ModelBuilder of ArcGIS system. In the picture (Fig. 5) model of calculation of coefficient of deceleration of maximum speed C_3 is shown and there is also stated a formal mathematical description of the calculation.

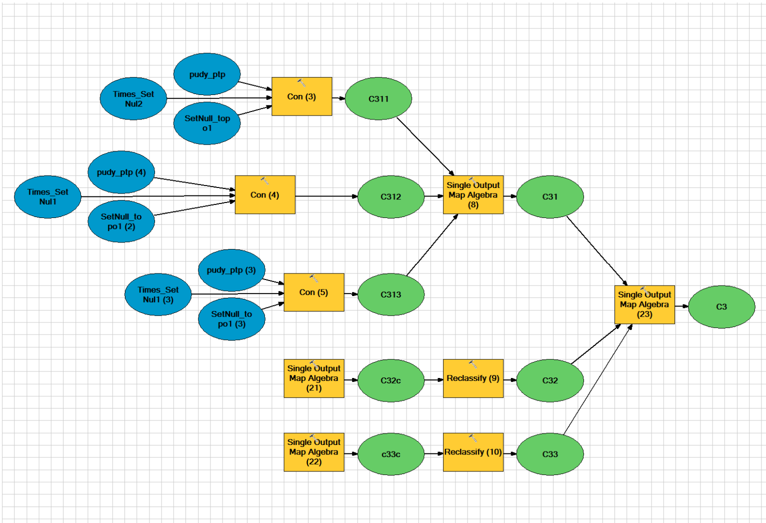

Fig. 5. Example of data procedure model in ArcGIS ModelBuilder

4.6 Reached Results

Reached results of calculation of the individual routes are collectively stated in the chart (Table 5). Although the place of action was purposely chosen to be away from other communications, the differences of routes with the individual vehicles were smaller than we had expected. Apparently, the reason is the fact that the studied location was chosen in an urbanized area with a sufficient number of hard communications and technical parameters of the chosen emergency vehicles enable reasonable movement also in the free terrain unless they get into extreme conditions. The authors suggest continuing in this case study also with phase three, in which less urbanized area will be chosen. This will enable to compare the reached results.

Using our results, the optimization of decisions of the operational officer would concern only the composition of the sent vehicles. Based on the calculations, the operational officer gets information that all vehicles that are available are able to arrive to the place of fire practically at the same time. He/she can then choose the variant that he/she sends simultaneously vehicles 1, 2, and 4; or 1, 3, and 4.

Also average speeds of all vehicles were calculated. Basically, they correspond to the experience of members of fire and rescue brigade from real action.

Table 5. Results of calculation of the individual routes

Vehicle number	**1**	**2**	**3**	**4**	**5**
Parameter	Tatra T815-731R32 6x6	Mercedes-Benz Unimog 4x4	Mercedes-Benz Atego 1329 AF 4x4	Mercedes-Benz Actros 3355 A 45 6x6	Renault Midlum
Total distance (km)			11. 2		
Time (min)	12.0	13.6	12.0	11.4	13.6
Calculated average speed (kph)	56	51	56	61	51
Water tank capacity (liters)	9000	3200	1200	9500	0
Number of ways needed	3	7	17	3	1*

* Transport lorry - for transportation of hoses and portable motorized fire engine

Examples of cost maps and calculated routes are shown in the following pictures.

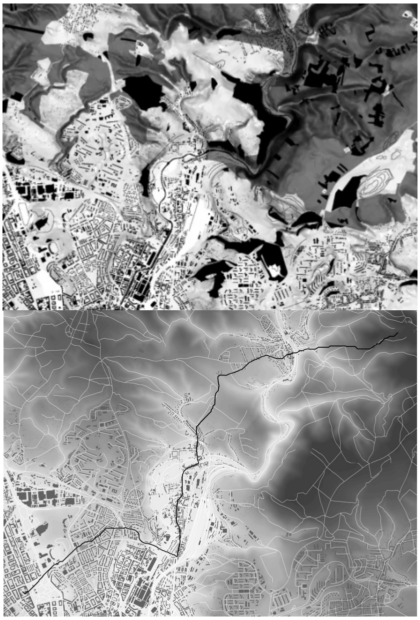

Fig. 6. Cost map and path example

Fig. 7. Cost map calculation example

5 Discussion and Conclusion

The first phase of the case study has demonstrated a strong relationship between quality data and the results of spatial analysis. Likewise, it has pointed to the problem of defining quality.

In the second phase of the study, the resulted paths gained in the described spatial analysis are basically the same for all tested vehicles. This case frequently happens in reality, especially in an urbanized locality. Then the results of the geographical analyses are used in the decision-making process as one of the bases for optimization of engaging forces and sources. Nevertheless, it is not possible to underestimate also these procedures as there can happen a situation when it is necessary to send vehicles to a complicated terrain, and the operational officer must be able to correctly interpret occasional different routes of the individual emergency vehicles which are given not only by their technical parameters but also by the quality of used data, and to optimally decide accordingly.

Values of technical parameters of vehicles under consideration were used for all calculations. From the experience of members of fire departments, however, it is clear that the real reached average speed of vehicles especially in urbanized areas is usually half with regard to their technical specifications. We could not take this thought into our consideration so far. However, in the further development we will take into account level I of traffic density related to the time of day when the routes will be calculated. The level of traffic density will be taken as another coefficient of deceleration.

For both cases were developed formalized procedures, the samples are in the preceding text. This procedure is applicable to the whole territory of the Czech Republic.

Models for support of decision-making processes are generally always multi-criterial and their success depends both on the correct setting of count and structures of used criteria as well as on the reliability of real values that are being worked with at the particular moment. As the results of the procedure described in this article as well as scientific studies and procedures (e.g., (Condorelli & Mussumeci, 2010), (Linlin, Lijun, Jianming, Chao, & Xiangpei, 2012)) prove, models can be very useful in the complete decision-making process. However, it is necessary to approach critically their results with respect to the data reliability and to the level of the complete model complexity.

In the future a real-time risk analysis approach which dynamically evaluates the risk at discrete time points during the whole transportation process will be considered.

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Analysis of the Italian Banking System Efficiency: A Stochastic Frontier Approach

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Abstract. The paper provides an analysis of some features of the Italian banking system during the decade 1998 – 2008. In particular, it focuses on the efficiency of Italian banks—in terms of parametric cost and profit functions—taking into account the dualistic structure which characterizes the Italian economy, the bank size, and the juridical form. During this period the Italian banking system has experienced a higher level of competition and significant ownership changes; these phenomena had a relevant impact on the performance of all banks. In particular, we found a reduction of differences in the efficiency between Northern and Southern banks. In addition, small banks exhibit a higher level of efficiency compared with the large ones. Finally, we observe that Mutual Banks improved in a significant way their performance compared with the banks organized as limited companies and cooperative. These results confirm the ability of local small Mutual Banks to effectively and successfully compete in the markets characterized by global operators. The reason for the continuing vitality of local banks is due to the fact that they offer a different product from large global banks and attract customers, specially small local firms, which external global banks would find difficult to serve.

Keywords: Italian Banking System, Stochastic Frontiers, Cost and Profit Efficiency JEL: D2, G21.

1 Introduction

The 1990s was a particularly intense decade for the Italian banking system, in which a reform of the credit market was launched aimed to promote competition among intermediaries through a substantial review of the old 1936 banking law and a deep reorganization of the banking system in terms of both ownership and legal structures of credit companies.

The privatization of the banking system and the liberalization of the credit market have increased the competition which individual intermediaries are subjected to, facilitating this by rationalizing the use of resources and by a thorough review of banking management. Moreover, the Italian banking industry is characterized by another dimension of territorial nature, which has no equivalent in the other European countries. It cannot be ignored that the restructuring process of the banking system

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has been far from uniform in terms of territorial structure of intermediaries' activity and in terms of financing of productive activity in the weak areas of the country, with relatively less satisfying results concerning operational efficiency.

These problematic data sum up the economic crisis of Southern Italy which, throughout the first half of the 1990s, has led to the disappearance of a genuine local banking system in the South, which starting from the second half of the decade has progressively been absorbed by Northern banks. If the outcome of these processes is a strengthening of the banking system as a whole and improved performance in terms of productive and allocative efficiency, it is natural to ask after almost 20 years, if these goals have been achieved.

The aim of this paper concerns, thus, the analysis of the Italian banking system efficiency. The analysis of the proposed efficiency relies on an estimated stochastic frontier of cost and profit, taking into account the dimensional profile, the legal-organizational structure, and the territorial implications. The work is structured as follows. Section 2 will focus on the most relevant aspects of the reorganization process of the Italian banking system throughout the past 20 years, considering also into account how the banks have reacted to the global financial crisis in 2009 – 2010. Section 3 will examine some aspects of the methodological nature related to the estimate of the stochastic frontier. Section 4 provides some comments on the results of the econometric analysis on a representative sample of Italian banks between 1998 and 2008. Some final considerations, in Section 5, will conclude the paper.

2 The Italian Credit System Restructuring in the Last Decades

At the end of the 1980s the Italian banking system was highly segmented, predominantly public controlled[1], and essentially impermeable to the competition of foreign intermediaries.

The bank was seen more as an institution with a social function rather than an entrepreneurial activity; the establishment of new institutions was limited by the supervisory authorities and the banks operated in a kind of quasi-monopolistic market.

It was with the entry of the new Banking Act in 1993 that the new regulatory framework was organized into a system. The system described by this law reverses the principles that have long characterized the credit industry (specialization time, institutional pluralism, separation between bank, and industry). The banks authorized by the Bank of Italy today are all similar on a legal level and can operate across the board, without limitations in terms of operations and services offered to the customers.

The 1993 Banking Act favors the creation of a competitive environment in the banking system, designing a system based on entrepreneurship, and a free market. As a consequence, the objectives of the management exerted by the Bank of Italy have changed: efficiency and competitiveness of the financial system are added to the former objectives of stability, compliance with the rules, and a sound management.

[1] Consider that in the eighties the activity of the public controlled banks touched upon 70% of all the intermediated funds of the banking system.

The changes in the legal framework favored the reorganization of the banking system and, in particular, have reproposed the problem of the operative dimension of the Italian banks compared to those of the main OECD countries.

The importance of scale economies in the banking industry has constituted an important strand of empirical literature throughout the 1980s and 1990s, but it is far from reaching unequivocal conclusions. This is even more evident in the Italian context, in which the presence of scale economies, especially for larger companies, is far from predictable; in fact, the relevance of economies of scope seems much more significant in terms of financial services diversification (Giannola and Lopes, 1996; Imbriani and Lopes, 1999).

Caution is due to the difficulties that may arise in the managing of large intermediaries that involve high quality leaders and management and appropriate corporate governance rules. In the absence of such conditions, the concentration process could exacerbate the effects of a possible corporate crisis. Moreover, the incorporation of small banks in larger bodies could lead to a lack of funding for small local firms (Avery and Samolyk, 2004).

Indeed, during the period from 1999 to 2003, Bank of Italy data show that the quota of deposits of the smaller banks has increased from 26% to 31% that the share of the medium-sized banks remains at 18%, while for the larger banks there has been a decrease from 56% to 51%. Regarding the credit, the market share of the smaller banks from 1999 to 2003 increased from 25% to 31%; the medium sized banks maintain a constant share of approximately 20%, while the larger banks have experienced a decrease from 55% to 49%; this trend tends to grow stronger during the decade.

In addition, some empirical studies (Ferri and Inzerillo, 2002) show the persistence of credit rationing phenomena regarding small- and medium-sized companies; there is reason to believe that the large universal banks have not been able to meet the demand of financial services coming from small companies; the growth prospects of the banks with strong territorial roots would be enhanced. In fact, many retain that local Mutual banks are better equipped than larger national banks to assist small and medium enterprises. In the Italian case, Mutual banks (BCC) and Cooperative banks (PB) begin and grow with a vocation to support small businesses in their local area, even more so than other local banks organized as limited companies (LC) (De Bruyn and Ferri, 2005).

It has often been observed that the widespread presence on the territory of local banks has allowed a continuous stream of finances aimed at small and medium firms, that otherwise would have suffered a severe rationing as a result of the contraction of the volume of credit supplied by large intermediaries resulting from mergers.[2]

The opening of a bank deposit implies an immediate knowledge of the entrusted client, which precedes any loan concession. This advantage of possessing information becomes increasingly important if it establishes a long-term contract with the client. Indeed, a continuing relationship for the bank becomes an exclusive and long lasting

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[2] Bonaccorsi di Patti and Gobbi (2001) found that acquisition reduces the supply of credit to small companies; in addition, Sapienza (2002) showed that acquisition increases the probability that the bank will terminate credit reports, particularly with small enterprises which were previously entrusted with the acquired bank. See also Berger et al. (1998).

asset (Petersen and Rajan, 1995). If on the one hand, the exclusivity of the relationship with one bank exposes the firm to the risk of expropriation of part of its profits, on the other hand it creates the conditions for offering an implicit insurance service: the bank is ready to provide emergency credit lines when the company is facing temporary liquidity crisis or to isolate it from sudden increases in interest rates (interest rate smoothing) due, for example, to a tightening of monetary policy (Berlin and Mester, 1999).

Some of these aspects may be amplified, nevertheless, if the bank and the customer interact in the same area and if the bank has a mutual structure. This category of intermediaries tends to supply most of the credit to their members, on which there should be increased information available to the bank compared to those related to other cases. The admission of a member into the *club* of a mutual bank is based on the *liking* or *satisfaction* of the other members; they accept a new member that is considered *reliable* (Cesarini et al., 1997; Angelini et al., 1998; Cornes and Sandler, 1996; Dowd, 1994).

The mutual structure of a bank provides incentives that make entrusted members active participants in the bank life. The objective of being a successful bank is shared by the members (Varian, 1990). Such a system leads to a form of reciprocal checks—*peer monitoring*—creating the necessary incentives to encourage the members to behave in the interests of the financing bank. The problems between the bank and the members can be solved more easily in the case of the Mutual banks (Berger and Udell, 2002). Peer monitoring makes screening and monitoring of the Mutual banks more efficient, contributing positively to the reduction of constraints to which they are normally subjected (Fonteyne, 2007).

In Italy, the problems outlined above take on a particular meaning when the dualistic character (Imbriani, 2003) of the production system is considered.[3] The increasing competition and the consequent removal of the constraints on the location of branches, has been particularly intense in the South.

The Southern banks in fact have been characterized from the outset for financial coefficients which are lower than those of the rest of Italy (Giannola, 2007). Due to both this aspect and the difficult environment in which they operate, relatively less satisfying results are produced in terms of operative efficiency. These problematic data added to the Southern economic crisis throughout the first half of the 1990s, has led to the disappearance of the national dimension of the Southern banks and the dissolution of a local banking system, which, starting from the second half of the decade, has been gradually absorbed by Northern banks.

With the aim of providing a quantitative indication of the property restructuring processes, where Southern local banks systematically enter into the sphere of northern external banks, it can be said that the credit system, still independently managed, is unable to control less than 30% of the Southern credit market (Butzbach and Lopes, 2006).

If the processes of reorganization and merging of the credit market in Italy represent, to a certain extent, a necessary reinforcement for competing in larger

[3] There is a pronounced debate surrounding the incidents that have led to a substantial liquidation of an independent banking system in the Southern Italy with reference to Alessandrini (2001), Giannola (2002, 2007) and Bongini and Ferri (2005).

markets, it must once again be reiterated that in a dualistic context, this strategy may have negative consequences on small firms operating in the weakest areas.

The question is whether the weakening of the local banks' system, owned by local people, has increased the difficulties of credit access for Southern businesses. Some studies (Panetta, 2003) come to the conclusion that the property restructuring of the Southern banking system would not have determined these negative consequences. On the contrary, the restructuring would have improved the conditions of the Southern credit market. Moreover, such conclusions do not run parallel with the widespread perception of small Southern firms for which that access to bank credit them, is more problematic.

Various sample surveys carried out in Southern companies come to the conclusion that in the Southern Italy credit rationing is perceived as a serious problem and that, at least in part, is related to the property reorganization of the Southern banks. If it has allowed a partial recovery of operative efficiency of the banking system, it has also made access to credit more difficult (Bongini and Ferri, 2005; Butzbach and Lopes, 2006).

According to Bank of Italy, it can be seen the tendency toward downsizing the supply of credit of the larger banks. In the Centre-North such a percentage has decreased, between 1999 and 2005, from above 50% to slightly more than 45%. In the South the reduction has been more significant and has exceeded seven percentage points. At the other extreme of the scale, it can be seen that also the Southern regions have achieved a substantial alignment in the credit provided by the institutions of smaller and minimum dimensions toward the national value of 30%. Regarding the medium-sized credit companies, the South seems to diverge from the national data. In fact, while in Italy and in the Centre-North this percentage tends to exceed 20%, in the South at the end of 2004 it was approximately 15%. Similar considerations can also be carried out regarding deposits; all these trends continue in subsequent years. This result is in part due to the numerous acquisitions of smaller Southern banks by non-local groups and the substantial downsizing of the larger Southern banks.

The global financial crisis that also hit the Italian economy during the biennium 2008—2009 has affected access to credit, especially small businesses based in the South. According to the Bank of Italy, the annual growth rate of loans to Italian companies was 10.2% while for Southern firms it was 7.9% and this tendency was further strengthened during 2009.

In this regard, the Bank of Italy noted that the slowdown in lending by the major banking groups in 2008, led to the same focus into business less risky and this trend was partly offset by the behavior of small and medium banks through an increase of loans to more financially vulnerable firms. The result was a significant shift in market shares for the benefit of the smaller banks.

The phenomenon can be partially explained by the introduction of strict regulations on capital requirements (Basel II) that, by requiring banks to set aside capital proportionate to the risks undertaken and evaluated based on credit scoring mechanisms, can push the big banks—mostly adopting automated rating systems—to limit the credit toward the most opaque firms; otherwise, the small local banks rooted in the territory, by virtue of the accumulated information about its customers, including small and very small (small companies, craftsmen, traders), are able to arrive at an assessment of the creditworthiness of financial information regardless of the balance sheet information. In conclusion, it is expected that the international financial crisis, through the loss of confidence in the banking system, results in an abrupt tightening of credit rationing for small firms more opaque and localized in the southern regions.

In the light of these issues outlined so far, the question of the recovery of efficiency, experienced by the Italian banking system throughout the last years, will now be further examined by means econometric techniques.

3 The Bank Efficiency Analysis through the Construction of Stochastic Frontiers

3.1 Econometric Technique

According to the economic theory, the degree of technical efficiency of a production unit is evaluated by observing whether a combination of given factors of production has made it possible to achieve the highest level of a product, or if the level of production observed has been achieved with the smallest possible use of productive resources. The analysis of technical efficiency is based on the identification of the production function, or the geometric points that identify the highest product level achievable for each given use of productive factors (Forsund et al., 1980). The measure of the distance of each production unit from this frontier is the most immediate way to assess its efficiency (Farrel, 1957).

The methodologies which are most frequently used in order to identify the production frontier are divided into parametric and non parametric. The former start with a specification of the production function and the parameters are estimated with econometric techniques (Stochastic Frontier Analysis). The non-parametric methodologies do not make any assumptions about the functional form behind the phenomenon to be estimated and make use of linear programming techniques (Data Envelopment Analysis).

For the present work we are limited to use only the former, which despite in some cases of being unfavorably conditioned by the arbitrary aspect of the choice of the functional form that links the production factors to the results of the production process, avoids confusion between statistical errors and real inefficiency using inferential techniques, as they allow us to evaluate how well the model can be adapted to an observed situation, and therefore the adequacy of the chosen explanatory variables, which is not possible with a non-parametric approach.

Literature developments[4] have helped to identify other measures of efficiency that are not only linked to the technology used in production, but which identify the allocation of productive factors and therefore the ability of the firm to minimize the production costs of a determined level of production, given the prices of the factors. In this case, one talks about cost efficiency, which is analyzed by constructing a cost function:

[4] For all the theoretical and methodological aspects of the concepts of efficiency and the measurement techniques, see Coelli et al. (1999) and Kumbhakar and Lovell (2000).

$$C = C\,(y, w, u_c, v_c) \tag{1}$$

Where C are the total production costs, y is the vector of the output quantity, w is the vector of the input prices, u_c is a measure of cost inefficiency and v_c is a random error that could be due to measurement errors and/or a shock suffered by the company and for which it may, temporarily, experience higher or lower costs.

Two operators can attain the same level of efficiency in terms of costs, but one of the two may be more efficient than the other concerning marketing expertise and therefore attaining a higher level of profits.

The ability of the enterprise of efficiently combining the production and the sales factors is evaluated through the specification and the estimation of parameters of the profit frontier, given the output prices:

$$\Pi = \Pi(w, p, u_\Pi, v_\Pi) \tag{2}$$

Where Π are the total profits, w is the vector of the input prices, p is the vector of the output prices, u_Π is a profit inefficiency measure and v_Π is a random error that may be due to measurement errors and/or external shock which the bank has undergone and that due to these, could temporarily experience profits which are higher or lower compared to the minimum or maximum. Regarding the profit function, several considerations in the literature suggest the adoption of alternative versions,[5] in which the price vectors of the output p are not considered and the levels of production y are included; therefore the proposed specifications are as follows:

$$\Pi = \Pi(w, y, u_\Pi, v_\Pi) \tag{3}$$

The usual frontier stochastic models, initially proposed by Aigner et al. (1977) and Meeusen and Van Den Broeck (1977), do not include any explanatory efficiency variable in the phase of the frontier estimation. Generally, the previous type of approach found in the literature was that proposed by Pitt and Lee (1981) and Kalirajan (1981). In those papers a two stage technique is used, which aims to investigate the explanatory factors of efficiency: in the first stage, the stochastic frontier is estimated and the inefficiency component is identified; in the second one the inefficiency values are regressed on a set of variables which are supposed to be able to explain the trend.

As noted by Kumbhakar et al. (1991), Reifschneider and Stevenson (1991), and Huang and Liu (1994), the two stage approach is incorrect because in the specification of the regression model at the second stage, the hypotheses concerning the inefficiency distribution, on which the stochastic frontiers are based, contradict each other.

An alternative approach to the two stages, which does not present the aforementioned limits, is the one originally proposed by Kumbhakar et al. (1991) and then adapted for panel models by Battese and Coelli (1995).

[5] See contributions, reported in the financial sector, of Berger and Mester (1997); Humprey and Pulley (1993 and (1997).

Considering a generic production function for panel models we have:

$$Y_{it} = \exp(x_{it}\beta + V_{it} - U_{it}) \tag{4}$$

where Y_{it} is the output produced by the unit in year t; x_{it} is a dimension vector $(1 \times K)$ referring to the input of the production function; β is a vector of parameters of the production function that must be estimated; V_{it} is the stochastic component that can be distributed as a Normal variable $iid \rightarrow N(0;\sigma_v^{\ 2})$ with average zero and variance $\sigma_v^{\ 2}$, independently distributed by the component of inefficiency U_{it}. U_{it} is a non-negative variable and it measures the real technical inefficiency; it is considered to be independently, but not identically distributed. U_{it} is therefore obtained through the cut off at zero of a normal distribution with average $z_{it}\delta$, and variance $\sigma_u^{\ 2}$; z_{it} is a vector $(1\times m)$ of explanatory variables linked to the levels of inefficiency of the different economical units observed over time, δ is a vector $(m\times 1)$ of coefficients to be estimated. The inefficiency component U_{it}, included in the equation (5), can be specified as:

$$U_{it} = z_{it}\delta + W_{it} \tag{5}$$

where the random variable W_{it} can be obtained by truncation of a normal distribution with zero mean, variance σ^2, and truncation point equal to $-z_{it}\delta$, such that $W_{it} \geq -z_{it}\delta$, This assumption is consistent with the hypothesis that U_{it} is a non-negative variable extracted from a distribution $N^+(-z_{it}\delta,\ \sigma^2)$. We employed a simultaneous ML estimation of the above parameters in equations (4) and (5). The maximum likelihood function, and the partial derivatives with respect to the model parameters have been calculated by Battese and Coelli (1993), the same function is then parameterized following Battese and Corra (1977) and therefore we will have that $\sigma_S^2 \equiv \sigma_V^2 + \sigma^2$ and $\gamma \equiv \dfrac{\sigma^2}{\sigma_S^2}$.

Once we obtained the total residuals from the estimated function (U_{it} + V_{it}), we isolated pure inefficiency (U_{it}) following the approach suggested by Jondrow et al. (1982) and finally calculated the efficiency score using the estimator proposed by Battese and Coelli (1993). The efficiency score of the i-th unit in year t is then equal to :

$$E_{it} = \exp(-U_{it}) = \exp(-z_{it}\delta + W_{it}) \tag{6}$$

3.2 Model Specification

We assume that the bank uses three inputs: 1) collected funds (x_1); 2) deposits (x_2); 3) labor (x_3) and produces three outputs: 1) loans to ordinary customers (y_1); 2) loans to financial institutions (y_2); 3) other financial assets in portfolio (y_3); input prices are 1) cost of collected funds (w_1); 2) cost of deposits (w_2); 3) labor cost (w_3).

Total costs (TC) are calculated considering all costs incurred by the bank including interest. Total profits (Π) are derived from the difference between total revenues and total costs. In the revenues are included all interest incomes and commission incomes (including deposits).

The cost function (and profit) estimated is a Translog type (Caves and Christeensen, 1980); as already said, following Battese and Coelli (1995), in addition to outputs (**y**) and input prices (**w**), we insert the variables (**z**) describing the factors affecting the mean distribution of the inefficiency variable (U_{it}) for each bank.

$$\ln(TC_{it}) = \alpha_0 + \sum_{i=1}^{3} \alpha_i \ln y_{it} \sum_{j=1}^{3} \beta_j \ln w_{jt} + t_1 T + \frac{1}{2}\left[\sum_{i=1}^{3}\sum_{j=1}^{3} \delta_{ij} \ln y_{it} \ln y_{jt} + \sum_{i=1}^{3}\sum_{j=1}^{3} \gamma_{ij} \ln w_{it} \ln w_{jt} + t_{11} T^2\right]$$
$$+ \sum_{i=1}^{3}\sum_{j=1}^{3} \rho_{ij} \ln y_{it} \ln w_{jt} + v_{it} + u_{it} \tag{7}$$

First of all, we inserted a trend variable (T), to capture structural changes causing translation of Hicks neutral type frontier; second we added a scale variable represented by the logarithm of total assets (TA) in order to control the large variability of bank dimension.

According to Hughes and Mester (1994), if banks are not risk neutral, they do not choose the equity level exclusively in terms of cost minimization. On the contrary if banks are more risk adverse, may choose to finance their loans with a higher proportion of equity compared to debt (in other words choosing to use less indebtedness). Since the equity is a source of funding typically more expensive, this may suggest that banks more risk averse produce its output in a less efficient way. As a consequence, the assessment of efficiency would be distorted by the choice of the mix of production factors which is affected by the different risk aversion of banks involved and this diversity must be taken into account (Kwan and Eisenbeis, 1995; Shrieves and Dahl, 1992).

These considerations, concerning the different risk aversion of bank management, seem to be even more important in the Italian situation which is characterized by banks with a different legal structure, and presumably, different risk preferences. As highlighted by Giordano and Lopes (2007), the level of capital used by the Mutual banks is much higher than that used by the Cooperative banks or by limited companies; this difference portrays a higher risk aversion of the Mutual banks, granted that in the latter, the mutualistic aspect blends together the aims of the owners and of the clients (Mayers and Smith, 1988). When such diversities are not considered, a distorted estimate of the efficiency of the intermediaries who are more averse to risk could be possible. This is the reason why, in this paper, we introduce a level of capitalization—capital on total assets (FEC) of the intermediaries—as an efficiency explanatory variable[6].

[6] Similar considerations can be made when analysing the distribution of the same ratio – capital on total assets – according to the bank size. As predicted, it can be noted that the smaller banks report slightly higher levels of capitalisation compared to larger ones because of their increased risk aversion and because they predominantly coincide with the Mutual banks.

Another important issue that we take into account is the relationship between banks efficiency and non-performing loans (NPL). In the following analysis we assume that the different environmental and macroeconomic conditions in which banks are involved may cause a deterioration of the quality of loans and, as a consequence, the performance of banks in terms of efficiency. In other words, given the sharp dualism of the Italian economy and taking account of the historical differences in terms of percentage of bad loans between banks in different regions of the country (see table 2), the higher level of non-performing loans in Southern Italy is due to more adverse economic conditions faced by banks operating in this area. In this context, the bank management may achieve lower levels of efficiency not as a result of poor screening and monitoring activities of customers but because of tighter external constraints. Therefore, the variable which captures the quality of assets (NPL) is inserted into the vector of the explanatory variables of efficiency[7].

Finally, we include a variable measuring the intensity of credit (IC) as a proxy of the role of the traditional credit activity; the credit intensity is calculated by the ratio between customer credits and total earning assets. In addition, we took into account the importance of institutional aspects related to the legal nature of the bank, i.e. if it is a Limited company, a Cooperative bank or a Mutual bank. The hypothesis to be tested is that the different legal structure affects corporate strategies with regard to more traditional activities connected with the supply of loans to firms and on banking efficiency; in order to do so, the (IC) variable was multiplied by dummy variables relating to Cooperative and Mutual banks.

3.3 Data and Variables

The estimates have been made on a sample of 526 banks coming from the *Bilbank* archive for the period 1998 – 2008. The banks for which the budgetary information was available for at least 10 years out of 11 were included in the sample; estimates were made using 5686 observations corresponding approximately to 76% of the total observations relating to the entire Italian banking system.

The sample is broken down to take account of firm size[8], legal structure (Limited company, Cooperative bank, and Mutual bank), and Headquarter location (Northern, Central, and Southern Italy). The inflation has been removed from all the series using the value-added deflator for the banking sector (the base year is 1995). In Tables 1 and 2 are shown respectively, the structure of the sample and sample means of the variables in question.

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[7] In a previous paper the authors tested the exogeneity hypothesis of NPL variable using the empirical Granger Causality applied to a sample of 550 banks for the period 1993-2003, see Giordano and Lopes (2009).

[8] The breakdown according to the dimensional criterion was made following the Bank of Italy criterion for which the dimensions are five groups: "major banks" (with total resources exceed 60 billion euro), "large banks" (26 to 60 billion euro), "medium-sized banks" (9 to 26 billion euro), "small banks" (from 1.3 to 9 billion euro) and "smaller banks" (with lower average total resources to 1.3 billion euro). Here we preferred to merge major banks with large banks and small banks with minor banks.

Table 1. Sample composition

Year	Banks (Total)	Legal Structure: Limited Company (LC)	Cooperative Banks (PB)	Mutual Banks (BCC)	Headquarter Location: Nortern Italy	Central Italy	Southern Italy
1998	498	102	35	361	287	103	108
1999	502	105	31	366	296	102	104
2000	518	110	33	375	301	107	110
2001	522	113	32	377	304	107	111
2002	521	114	30	377	305	106	110
2003	524	117	28	379	304	107	113
2004	524	118	27	379	307	104	113
2005	517	111	27	379	302	103	112
2006	523	119	25	379	304	105	114
2007	526	122	24	380	305	107	114
2008	511	115	24	372	296	104	111

Source: Bilbank (ABI - Italian Banking Association).

Limited company banks collect a greater amount of funds, compared to the Cooperative banks and Mutual banks; we found the same pattern with regard to all the other outputs and inputs. It is evident the dominant role of LC banks in the Italian credit industry. On the other tail of the distribution can be found Mutual banks that, despite their relative abundance, show a very small mean value of the above variables. Cooperative banks face short distance from the LC banks, replicating, in fact, the industrial features of the latter.

In addition, we observe a significant difference in the cost of funds held by banks operating in different areas of the country. Banks headquartered in the North sustain a lower cost of raised funds, compared to Central and Southern banks indicating a tightening of supply of funds raised gradually from North to South. Likewise, Cooperative banks support a lower cost for raised funds with respect to Mutual banks and Limited company banks. The cost of labor does not show substantial differences between the various institutional categories and the three geographical areas. The cost of deposits is virtually identical for the banks operating in the three main areas of the country. The cost of deposits turns out to be rather higher for LC banks compared to other types of intermediaries, due to the mutual nature of Cooperative and Mutual banks.

The variable measuring the intensity of Credit (IC) is a proxy for the productive specialization of intermediaries in traditional credit activity; it might be able to explain the paths of cost and profit efficiency of different banks with different characteristics. The data in table 2 show clearly that the traditional lending activity orientation is significantly lower for banks headquartered in the South than banks headquartered in the rest of Italy. These data provide evidence of a higher risk faced by Southern intermediaries operating in areas with problematic economic conditions. For these reasons, they may prefer to lend to other lending institutions or to invest in financial assets, ultimately weakening the support for the growth of the local productive activities. In addition, LC banks show a higher percentage of assets represented by loans to customers, followed by Cooperative and Mutual banks. This may be due to the greater willingness for traditional lending activity of LC banks; it might also indicate the existence of a wide margin for growth in the lending activities of Mutual banks.

Table 2. Sample mean of variables

Variables	Italian Banking System	Legal Structure			Headquarter Location		
		Limited Company (LC)	Cooperative Banks (PB)	Mutual Banks (BCC)	Northern Italy	Central Italy	Southern Italy
Collected Funds (x_1)	1,13 (mld)	4,19 (mld)	2,67 (mld)	81 (mln)	1,20 (mld)	1,79 (mld)	309 (mln)
Customer Deposits (x_2)	1,09 (mld)	3,86 (mld)	2,82 (mld)	126 (mln)	1,19 (mld)	1,38 (mld)	573 (mln)
Employees	533	1.881	1.398	60	561	695	304
Wages(x_3)	31,9 (mln)	116,9 (mln)	79,6 (mln)	3,6 (mln)	33,9 (mln)	41,3 (mln)	17,7 (mln)
Loans to customers (y_1)	1,5 (mld)	5,74 (mld)	3,96 (mld)	159 (mln)	1,75 (mld)	2,24 (mld)	567 (mln)
Loans to Financial Institutions (y_2)	445 (mln)	1,75 (mld)	871 (mln)	17,4 (mln)	470 (mln)	603 (mln)	227 (mln)
Other Financial Assets (y_3)	306 (mln)	980 (mln)	939 (mln)	54,5 (mln)	303 (mln)	473 (mln)	157 (mln)
Collected Funds cost(w_1)	0,034	0,034	0,031	0,034	0,032	0,035	0,039
Deposits cost (w_2)	0,017	0,02	0,016	0,016	0,017	0,018	0,017
Labour cost (w_3)	59.782	62.157	56.973	59.279	60.434	59.441	58.338
Credit Intensity (IC)	0,63	0,67	0,63	0,62	0,681	0,628	0,5
Total assets (TA)	2,64 (mld)	9,57 (mld)	6,60 (mld)	243 (mln)	2,85 (mld)	3,72 (mld)	1,06 (mld)
Financial equity capital (FEC)	0,108	0,085	0,09	0,117	0,112	0,095	0,111
Non performing loans (NPL)	0,005	0,007	0,008	0,005	0,004	0,006	0,007

Source: Bilbank (ABI - Italian Banking Association).

We included Total assets (TA) as a scale variable affecting mean distribution of cost and profit because the Italian banking system is polarized between LC banks (medium-large size banks) with a total assets at an average of 9.57 billion euros and a multitude of small and very small Mutual banks with total assets amounted on average to 243 million euros.

The variable FEC (financial equity capital), as already mentioned, is a proxy of risk aversion of bank management and is the ratio between equity and total assets of the bank. There is a clear difference between Mutual banks and LC banks, with the latter characterized by a percentage of the total equity equal to 11,7% compared with 8,5% of the former; while Cooperative banks fall in an intermediate position.

Finally, the variable NPL (non-performing loans) measures the asset quality of intermediaries depending on the economic environment in which banks have to operate. Data indicate a percentage of bad loans in the South much higher than in the North, with the banks headquartered Centre much closer to the dynamics of the Southern ones.

4 Econometric Results

Table 3 gives some parameters[9] of the estimated cost and profit functions (Equations 1 and 2) following the Battese and Coelli (1995) approach. With regard to the cost function, all parameters are significant at 1%, except (α_2), which is negative and significant at 10%. The profit function parameters are all significant with the exception of (α_1) and (β_2).

The parameter ($\gamma = \sigma^2/\sigma^2_S$) is 0.96 for the cost frontier and 0.98 for profit frontier, respectively. These values confirm the importance of the inefficiency component in explaining the deviations of the observed economic units from the efficient frontier.

Table 3. Some parameters of *Translog Cost/Profit function*

Functions	α_0	α_1	α_2	α_3	β_1	β_2	β_3
Cost function	10,45	0,73	-0,16	0,95	1,25	2,05	0,104
(t-ratio)	(5,42)***	(7,56)***	(-1,90)*	(9,04)***	(8,24)***	(13,64)***	(28,58)***
Profit function	17,05	-0,067	-0,27	-0,15	0,33	-0,09	-0,009
(t-ratio)	(15,60)***	(-0,90)	(-3,88)***	(-2,24)**	(3,23)***	(-0,87)	(-4,14)***

(***)= 1% significance level, (**)= 5% significance level, (*)= 10% significance level

The Likelihood ratio test (LR) of the correct specification of the model[10] is constructed by testing the null hypothesis that the parameters of the explanatory variables of efficiency are all zero ($\delta_0 = \delta_1 = \delta_2 = \cdots = \delta_n = 0$). In both cases (cost and profit function) we reject it at 1%.

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[9] Note that the estimated parameters of cost or profit functions showed in table 3 are not equivalent to the elasticity of the dependent variable with respect to the quantities and prices, because of the presence of cross-products, not reported in the table. Consequently, the interpretation of the signs of the parameters must be cautious (Berger and Mester, 1987).

[10] The LR test is calculated as: $LR = -2\ln\{[L(H_0)/L(H_1)]\}=2\ln\{[L(H_1)]-\ln[L(H_0)]\}$, degrees of freedom equal to the imposed restrictions; finally the critical values are taken from Kodde and Palm (1986).

Table 4. Stochastic frontiers - Specification tests

Functions	sigma-squared	γ	LR	LR critical value	LR decision test
Cost function	0,463	0,96	3755,07	27,133	Rejected
(t-ratio)	(23,82)***	(438,36)***	-	-	-
Profit function	0,6	0,98	8670,77	27,133	Rejected
(t-ratio)	(33,15)***	(1867,9)***	-	-	-

(***)= 1% significant level, (**)= 5% significant level, (*)= 10% significant level

With regard to the impact of the explanatory variables[11] (table 5), the trend variable (T) tends to reduce the expected value of the inefficiency of cost but not profit; in other words, the learning process allows banks to improve their performance only in terms of cost reduction, whereas, on the contrary, the sign of the parameter indicates a deterioration of the capacity of intermediaries to improve performance in terms of achieving the maximum potential profit.

Similar considerations can be carried out regarding the effects of scale variable (TA). Clearly, the growth in size—which optimizes the use of inputs in the production of output—is accompanied by an excessive product standardization that has negative repercussions in terms of quality. This output deterioration prevents, to some extent, the larger banks to place the various outputs profitably in the market thereby moving away from the profit frontier.

Table 5. Sign and Significance of parameters explaining efficiency

Functions	δ_0	δ_1 (BCC)	δ_2 (PB)	δ_3 (IC)	δ_4 (T	δ_5 (TA)	δ_6 (NPL)	δ_7 (FEC)	δ_8 (BCC*IC)	δ_9 (PB*IC)
Cost function	+	-	-	-	-	-	+	-	-	(*)
Profit function	-	+	(*)	+	+	+	+	+	-	-

(*) Not significant parameter.

With regard to the dummy variables related to the mutual banks (BCC), it is possible to observe a positive effect on cost efficiency and a negative one on profit efficiency. Therefore, BCC enjoy advantages in terms of information and relationship that allows them to offer services to customers at a price relatively cheaper than LC banks. On the contrary, the profit frontier of profits seems to get away for the BCC because their mutual nature could mitigate the management profit maximization objective.

[11] Therefore, the correct interpretation of the signs is as follows: the negative sign means that the variable reduces the average inefficiency (efficiency increasing), the positive sign increases the average inefficiency (efficiency reduction).

Being a cooperative bank (PB) results in a reduction of cost inefficiency relatively to LC banks, but has no effect on the efficiency of profit. Therefore, although this kind of banks do not enjoy special advantages or disadvantages compared to LC banks regarding profits, the mutual nature and their local roots (stronger than the LC banks) allow them to exhibit cost advantages like BCC banks.

These results confirm the ability of local Mutual banks to compete effectively and successfully in markets with global player operators. The enduring vitality of these local banks is due to the fact that they use more intensively intangible information during screening and monitoring activity; in addition, they offer a different product compared to the big global banks and they deal with a clientele that is not served by larger external banks (De Young et al., 2004, Carter et al., 2004, Berger et al. 2004).

As expected, the highest level of capitalization (FEC) has a negative effect on the profit efficiency, noting that a greater risk aversion management determines a sub optimal input combination compared to banks that are larger users of borrowed capital. On the contrary, the (FEC) variable has positive impact on cost efficiency; in other words, the most capitalized banks are favored in terms of cost efficiency. This result may seem counterintuitive because the more intensive use of capital should be inefficient due to the increased cost of equity compared to those of others. By the way, we should also take into account another effect that goes in the opposite direction, namely the lower cost of supply of two important inputs in the production function of banks: i) "stock funds collected" and ii) "customer deposits", because of lower risk premium required by providers of these funds toward the most highly capitalized banks (and therefore with lower risk of default). We must therefore assume that this effect (lower cost of funds raised) prevails on the other one (higher level of capitalization).

Concerning the Credit intensity (IC), we must distinguish between the effect on the entire banking system performance and the effect only on (BCC). The increase in (IC) variable increase cost efficiency and reduces profit efficiency for the system as a whole. However, this phenomenon must be appropriately interpreted in the light of multiplicative variables that attempt to separate the effect of lending to customers depending on whether it refers to different types of banks (BCC*IC). BCC banks can effectively improve their performance, both in terms of cost and profit, in increasing loans to customers. These banks, then, may continue to expand their business in traditional lending activity; in other words, they can count on competitive advantages in this market compared to the large universal LC banks. The competitive advantages of Mutual banks in the traditional banking intermediation sector are derived from the fact that the lending activity is typically based on soft information acquisition and relies on established customer relationships which play a decisive role in determining the quality of products offered and the cost to produce them.

With regard to PB banks, credit intensity appears to have no significant effect on cost inefficiency; in fact, these banks have long ago departed from traditional operational paradigm of mutual bank and have diluted the competitive advantage that still characterizes BCC banks. The credit intensity has positive effect in terms of profit efficiency. Again, the competitive advantage compared to the LC banks is in the lower organizational complexity of cooperative banks (less vertically integrated structures) and their residual ability to offer customized products with higher added value than the standard products offered by large, impersonal LC banks.

Finally, the (NPL) variable captures the effect that the loans quality exerts on cost and profit efficiency. In other words, we are assuming that the quality of assets is primarily an exogenous variable, beyond the management control, depending crucially on the economic environment in which banks operate. The empirical evidence indicates that the increase of bad loans leads to a deterioration in the performance of both costs and profits as intermediaries have to bear higher costs for screening and monitoring activities in an environment characterized by adverse macroeconomic conditions.

We complete the results presentation showing cost and profit efficiency scores as evolved between 1998 and 2008, dividing the banks by size, legal status, and headquarter location.

With regard to the cost efficiency score according to the legal status, it should be noted in Table 6 that Mutual banks have on average a positive cost efficiency differential compared to Limited company banks and Cooperative banks. This gap between BCC and LC banks, after a decrease between 1998 and 2000, remains pretty constant up to 2004 and then falls to some extent in the next two years to return to grow over 2007 and 2008. The differential in favor of the PB Banks remains fairly stable until 2004, then declines over the next two years and grows again in the years 2007-2008.

Regarding profit efficiency score dynamics shown in Table 7, the differential in favor of BCC banks has a tendency to rise continuously until 2007 and then declines in 2008; the gap in favor of the PB Banks, although fluctuating, has remained fairly stable until 2006; growing in 2007 and declining in the following year.

Table 6. Cost Efficiency (mean values) - Legal Structure

Year	All Banking System	Limited Company Banks(LC)	Cooperative Banks (PB)	Mutual Banks (BCC)
1998	0,8238	0,6720	0,7235	0,8764
1999	0,8317	0,7283	0,7983	0,8643
2000	0,8511	0,7581	0,8287	0,8804
2001	0,8717	0,7564	0,8474	0,9083
2002	0,8802	0,7795	0,8504	0,9130
2003	0,8748	0,7674	0,8308	0,9112
2004	0,8813	0,7795	0,8455	0,9156
2005	0,8957	0,8164	0,8614	0,9213
2006	0,8880	0,8477	0,8558	0,9027
2007	0,9106	0,8587	0,8904	0,9286
2008	0,9110	0,8453	0,8954	0,9323

These results confirm that, in the Italian banking system, there is a widespread presence of Mutual banks that stand out positively from other types of banks. In other words, the process of consolidation of the Italian banking system, characterized by the adoption of the common organization of the limited company aimed to pursue higher levels of efficiency, does not seem to find strong support from this empirical evidence.

Table 7. Profit Efficiency (mean values) - Legal Structure

Year	All Banking System	Limited Company Banks(LC)	Cooperative Banks (PB)	Mutual Banks (BCC)
1998	0,9053	0,8988	0,9359	0,9042
1999	0,9279	0,9079	0,9409	0,9326
2000	0,9216	0,9021	0,9325	0,9263
2001	0,9128	0,8916	0,9203	0,9185
2002	0,9107	0,8972	0,9225	0,9138
2003	0,9196	0,8916	0,9258	0,9278
2004	0,9226	0,8944	0,9245	0,9312
2005	0,9129	0,8768	0,8932	0,9249
2006	0,9024	0,8637	0,8859	0,9156
2007	0,8694	0,8175	0,8791	0,8854
2008	0,8271	0,8007	0,8346	0,8348

An examination of Table 8 shows that small banks have a higher average level of cost efficiency than the larger ones; this gap does not seem to decrease during 1999 – 2005 period, in the following two years it becomes negative; finally, it increases again in 2008. The gap between small and medium banks is always positive; it decreases between 2003 and 2008 and increases in the last two years. Time would seem to exert a negative effect on cost efficiency. The persistent problems of costs for larger banks may depend on structural rigidities that impede a rapid decline in the unit cost or the adoption of more efficient production methods.

Regarding profit efficiency, we observe in Table 9 a gradual expansion, although marked by wide fluctuations, the gap between small and large banks until 2007 and then a significant reduction in 2008; the same pattern may be observed in the gap between medium and large banks. Finally, we observe an increasing gap between small and medium banks favorable to the former.

Table 10 shows a widening gap in terms of cost efficiency unfavorable to Southern banks during 1998 – 1999; then, this trend stops and the gap decreases until 2002, but it widens again until 2006. In 2007, there is a reduction in the gap which widens in 2008. The gap in terms of cost efficiency between Southern banks and banks headquartered in the Central Italy fluctuates around zero until 2004 and then becomes increasingly unfavorable to the Southern ones until 2006; in 2007 there is a gap reduction which increased again in 2008. Overall, cost efficiency score dynamics shows a clear and permanent inferiority of the southern banks compared to those with local headquarters in the rest of Italy.

Table 8. Cost Efficiency (mean values) - Bank Size

Year	All Banking System	Large banks	Medium banks	Small banks
1998	0,8238	0,5427	0,6573	0,8336
1999	0,8317	0,8021	0,7957	0,8335
2000	0,8511	0,8236	0,8068	0,8534
2001	0,8717	0,8259	0,8058	0,8756
2002	0,8802	0,7958	0,8072	0,8854
2003	0,8748	0,7777	0,7926	0,8808
2004	0,8813	0,7774	0,8116	0,8869
2005	0,8957	0,8459	0,8320	0,9000
2006	0,8880	0,9160	0,8848	0,8876
2007	0,9106	0,9265	0,8859	0,9119
2008	0,9110	0,8921	0,8697	0,9136

Table 9. Profit Efficiency (Mean values) - Bank Size

Year	All Banking System	Large banks	Medium banks	Small banks
1998	0,9053	0,9028	0,8330	0,9078
1999	0,9279	0,9092	0,8633	0,9306
2000	0,9216	0,9179	0,8804	0,9233
2001	0,9128	0,8615	0,8605	0,9162
2002	0,9107	0,8763	0,8838	0,9126
2003	0,9196	0,7916	0,8809	0,9240
2004	0,9226	0,8861	0,8644	0,9263
2005	0,9129	0,7727	0,8593	0,9186
2006	0,9024	0,7919	0,8336	0,9091
2007	0,8694	0,7144	0,7893	0,8780
2008	0,8271	0,7829	0,7427	0,8327

Table 10. Cost Efficiency (mean values) - Headquarter Location

Year	All Banking System	Northern Italy	Central Italy	Southern Italy
1998	0,8238	0,8300	0,8043	0,8260
1999	0,8317	0,8442	0,8046	0,8230
2000	0,8511	0,8614	0,8325	0,8410
2001	0,8717	0,8761	0,8589	0,8721
2002	0,8802	0,8809	0,8758	0,8824
2003	0,8748	0,8790	0,8622	0,8756
2004	0,8813	0,8881	0,8697	0,8742
2005	0,8957	0,9031	0,8861	0,8842
2006	0,8880	0,9062	0,8699	0,8558
2007	0,9106	0,9191	0,8949	0,9028
2008	0,9110	0,9218	0,8933	0,8986

Table 11. Profit Efficiency (mean values) - Headquarter Location

Year	All Banking System	Northern Italy	Central Italy	Southern Italy
1998	0,9053	0,9089	0,9013	0,8995
1999	0,9279	0,9305	0,9245	0,9240
2000	0,9216	0,9226	0,9148	0,9252
2001	0,9128	0,9151	0,9077	0,9112
2002	0,9107	0,9135	0,9036	0,9095
2003	0,9196	0,9223	0,9158	0,9161
2004	0,9226	0,9249	0,9194	0,9194
2005	0,9129	0,9153	0,9082	0,9108
2006	0,9024	0,9031	0,8966	0,9057
2007	0,8694	0,8724	0,8668	0,8638
2008	0,8271	0,8273	0,8260	0,8276

Regarding profit efficiency scores reported in Table 11, the unfavorable gap between Southern and Northern banks is reduced gradually until 2000, then it tends to worsen until 2003; in the following three years there is a decrease in the difference which increases again in 2007 followed by a new reduction in 2008. What appears clear is that the convergence between the performance of Northern and Southern banks was reached by means of a deterioration of the performance results of the former rather than an improvement of the latter.

The comparison between banks based in the South and those based in the Center is favorable to the first ones, although the gap is characterized by large fluctuations; it is good until 2006, it deteriorates significantly in 2007, and then follows a new improvement in 2008.

We may conclude that the ownership changes occurred since the late 1990s, which "stabilized" and "consolidated" Southern banks, have obtained only a partial gap reduction in terms of cost efficiency; in addition, we found a progressive alignment and convergence performance in terms of profit efficiency even though this process is achieved at lower levels.

5 Concluding Remarks

At the beginning of the 1990s the Italian banking system was conditioned by a predominantly public ownership, a low concentration, an insufficient international projection, a capital inadequacy, as well as a modest income capacity. The last 15 years have seen a significant restructuring process relative to all these aspects, which gradually lifted many structural limitations. Nevertheless, the work toward a modernized system is still far to be completed and problematic elements still occur which need further examination.

The drive toward a rationalization of the use of inputs, aimed at reducing costs, has not occurred in the terms desired by the Bank of Italy and the convergence process toward increased allocative efficiency among the various components of the banking system does not seem to have occurred yet.

One aspect that emerges more clearly is the superiority of the Mutual banks, in terms of cost and profit efficiency, compared to the rest of the system. This type of bank is aligned with the organizational structure of a mutual bank; it has strong territorial roots and it is based on relationship banking. Despite the fact that these banks take up a small share of the market (7%), there may still be prospects for them in terms of profitable expansion in the loans market. This is consistent with the hypothesis of an underlying demand for credit which does not meet with the offer of the larger banks but can be adequately met by smaller banks of decentralized structures (or rather, in the Italian situation, by Mutual banks). The empirical results are in line with a substantial amount of empirical evidence based on other credit systems (United States and Germany), which reported a performance deterioration of major banks organized as limited companies.

These results highlight also a substantial efficiency gap to the detriment of larger banks (they benefit from economies of scale, even if in the Italian case this is doubtful) and a unique process of convergence of the Cooperative banks to the lowest levels of efficiency of the banks organized in the form of limited companies. In this regard, it can be pointed out that the traditional bank has not lost its importance: in particular, smaller banks can expand their market shares and profit opportunities.

If, on one side, Mutual banks invest more in intangible information (soft information), develop more intense customer relationships and adopt a less vertical structure, on the other side, the process of consolidation of the Italian credit market has encouraged the growth of average size intermediaries and the adoption of hierarchical models which are more rigid.

Regarding the Southern banks, the massive ownership changes through their acquisition by the other Italian banks occurred at the end of the nineties, at least in terms of modernization of Southern banking system, have not achieved the expected results; we have observed a persistent gap unfavorable to Southern banks with respect to the rest of the other Italian banks particularly evident until 2005. Since 2006 we note a gap reduction, but this result is mainly due to a sharply reduction in the overall efficiency levels in all the Italian banking system. Moreover, the poor asset quality (due to the external environment of the bank) adversely affects cost and profit efficiency. As a consequence, banks operating in more disadvantaged areas of Southern Italy get lowest levels of cost and profit efficiency.

Several warnings emerge concerning the trends in the Italian banking system and we must ask whether the significant structural changes taking place are enough to increase efficiency or rather if the future scenario, which has become more critical after the global financial crisis, will not impose the problem of availability of credit, or the problem, more generally, of the absence of a virtuous model of bank – enterprise relationship able to operate as a development factor in the Italian economy.

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On Some Voting Paradoxes: A Fuzzy Preference and a Fuzzy Majority Perspective

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Abstract. Group decision making, as meant in this paper, is the following choice problem which proceeds in a multiperson setting. There is a group of individuals (decision makers, experts, ...) who provide their testimonies concerning an issue in question as individual preference relations over some set of option (alternatives, variants, ...). The problem is to find a solution, i.e., an alternative or a set of alternatives which best reflects the preferences of the group of individuals as a whole. First, we survey main developments in group decision making under fuzziness and outline some basic inconsistencies and negative results in group decision making and social choice, and show how they can be alleviated by some plausible modifications of underlying assumptions, mainly by introducing fuzzy preference relations and a fuzzy majority. We concentrate on how to derive solutions under individual fuzzy preference relations, and a fuzzy majority equated with a fuzzy linguistic quantifier (e.g., most, almost all, ...), and discuss a related issue of how to define a "soft" degree of consensus in the group. Finally, we show how fuzzy preferences can help alleviate some known voting paradoxes.

Keywords: fuzzy logic, linguistic quantifier, fuzzy preference relation, fuzzy majority, group decision making, social choice, consensus.

1 Introduction

The purpose of this paper is to briefly discuss an important issue of *voting paradoxes*, meant as some intuitively implausible, surprising, or even difficult to imagine, and generally unwelcome results that occur in voting type situations. In this paper we will deal with the voting paradoxes in the perspective of the fundamental works by Nurmi (1999) (cf. also Nurmi and Meskanen 1999). Basically, in those works one type of the voting paradoxes is formed by the classic, best known paradoxes which are known as those of Condorcet and Borda. The former

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is related to the intransitivity of collective preference relation that are employed in pairwise majority comparisons, and the latter just shows that it is may be possible that an intuitively implausible alternative is elected, the one defeated by all other alternatives in pairwise comparisons by a majority of votes (cf. also [2,5]). There is a rich literature on how to try to avoid those, and many other, voting paradoxes. In this paper we will focus on an approach that is based on the use of individual fuzzy preference relations and the fuzzy majority as proposed and developed by Kacprzyk, Zadrożny, Nurmi and Fedrizzi [27,28,29].

In the next sections, we will first briefly summarize the essence of a fuzzy preference relation and fuzzy majority based approach to group decision making (and voting), then provide a brief overview of main voting paradoxes, and then show some effective and efficient methods of avoiding (or alleviating) those paradoxes by employing elements of the fuzzy preference and fuzzy majority based approach.

2 Group Decision Making: A Fuzzy Preference and Fuzzy Majority Based Approach

We consider the case of multiperson decision making, more specifically group decision making, practically from the perspective of social choice (voting), under some fuzzification of preferences and majority. We assume that there is a set of alternatives and a set of individuals who provide their testimonies as *preferences* over the set of alternatives. The problem is to find a *solution*, i.e., an alternative (or a set of alternatives) which is best acceptable by the group of individuals as a whole.

Since its very beginning group decision making has been plagued by negative results, the essence of which is that no "rational" choice procedure satisfies all "natural", or plausible, requirements; a notable example is Arrow's impossibility theorem (cf. Arrow [1] or Kelly [30]), or some results by Gibbard and Satterthwaite, McKelvey, Schofield, etc. – cf. Nurmi [40].

A main direction is here based on the *individual* and *social fuzzy preference relation*. If we have a set of $n \geq 2$ alternatives, $S = \{s_1, \ldots, s_n\}$, and a set of $m \geq 2$ individuals, $E = \{1, \ldots, m\}$, then an individual's $k \in E$ individual fuzzy preference relation in $S \times S$ assigns a number in $[0, 1]$ to the preference of one alternative over another according to individual k; for some conditions, see Fodor and Roubens' [9].

Another basic element underlying group decision making is the concept of a *majority*. Some of the above-mentioned negative results in group decision making are closely related to too strict a representation of majority (e.g., at least a half, at least $2/3$, ...). A natural way out is clearly to somehow make that strict concept of majority softer, closer to its real human perception.

A Natural manifestations of a soft (fuzzy) majority are the so-called *linguistic quantifiers* as, e.g., most, almost all, much more than a half, etc., which can be dealt with by fuzzy-logic-based calculi of linguistically quantified statements as proposed by Zadeh [52], and some other approaches, notably Yager's [49] ordered weighted averaging (OWA) operators (cf. Yager and Kacprzyk [50] and Yager, Kacprzyk and Beliakov [51]). For simplicity and brevity, we will employ here the classic Zadeh's [52] approach, and refer the reader to, for instance, Kacprzyk, Zadrożny, Fedrizzi and Nurmi [27] for information on the use of other approaches. The fuzzy linguistic quantifiers, which stand for fuzzy majorities, are basically tools for a linguistic quantifier driven aggregation.

We use a standard notation and setting. A fuzzy set A in $X = \{x\}$, is characterized and equated with its membership function $\mu_A : X \longrightarrow [0,1]$ such that $\mu_A(x) \in [0,1]$ is the grade of membership of $x \in X$ in A, from full membership to full nonmembership, through all intermediate values. For a finite $X = \{x_1, \ldots, x_n\}$, we write $A = \mu_A(x_1)/x_1 + \cdots + \mu_A(x_n)/x_n$. Moreover, $a \wedge b = \min(a,b)$ and $a \vee b = \max(a,b)$.

A *linguistically quantified statement*, e.g., "most (Q) experts are convinced (F)", is generally written as

$$Qy\text{'s are } F \tag{1}$$

where Q is a linguistic quantifier (e.g., most), $Y = \{y\}$ is a set of objects (e.g., experts), and F is a property (e.g., convinced).

We can assign to the particular y's (objects) a different importance (relevance, competence, ...), B, which may be added to (1) yielding a *linguistically quantified statement with importance qualification*, e.g., "most (Q) of the important (B) experts (y's) are convinced (F)", written as

$$QBy\text{'s are } F \tag{2}$$

In Zadeh's [52] method, a (proportional, as assumed here) fuzzy linguistic quantifier Q is assumed to be a fuzzy set defined in $[0,1]$. For instance, $Q =$ "most" may be given as

$$\mu_Q(x) = \begin{cases} 1 & \text{for } x \geq 0.8 \\ 2x - 0.6 & \text{for } 0.3 < x < 0.8 \\ 0 & \text{for } x \leq 0.3 \end{cases} \tag{3}$$

which may be meant as that if at least 80% of some elements satisfy a property, then *most* of them certainly (to degree 1) satisfy it, when less than 30% of them satisfy it, then *most* of them certainly do not satisfy it (satisfy to degree 0), and between 30% and 80%—the more of them satisfy it the higher the degree of satisfaction by *most* of the elements.

Property F is defined as a fuzzy set in Y. For instance, if $Y = \{X, W, Z\}$ is the set of experts and F is a property "convinced", then $F =$ "convinced" $= 0.1/X + 0.6/W + 0.8/Z$ which means that expert X is convinced to degree 0.1, W to degree 0.6 and Z to degree 0.8. If now $Y = \{y_1, \ldots, y_p\}$, then it is assumed that truth(y_i is F) $= \mu_F(y_i)$, $i = 1, \ldots, p$.

Then, the truth of (1) is calculated as:

$$r = \frac{1}{p} \sum_{i=1}^{p} \mu_F(y_i) \tag{4}$$

$$\text{truth}(Qy\text{'s are } F) = \mu_Q(r) \tag{5}$$

With importance qualification, B is defined as a fuzzy set in Y, and $\mu_B(y_i) \in [0,1]$ is a degree of importance of y_i: from 1 for definitely important to 0 for definitely unimportant, through all intermediate values. We rewrite first "QBy's are F" as "$Q(B$ and $F)y'$s are B" which leads to the following counterparts of (4) and (5):

$$r' = \frac{\sum_{i=1}^{p}[\mu_B(y_i) \wedge \mu_F(y_i)]}{\sum_{i=1}^{p} \mu_B(y_i)} \tag{6}$$

$$\text{truth}(QBY'\text{s are } F) = \mu_Q(r') \tag{7}$$

The method presented is simple and efficient, and has proven to be useful in a multitude of cases, also in this paper.

Group decision making (equated here with social choice) proceeds as follows. We have a set of $n \geq 2$ alternatives, $S = \{s_1, \ldots, s_n\}$, and a set of $m \geq 2$ individuals, $E = \{1, \ldots, m\}$. Each individual $k \in E$ provides his or her testimony as to the alternatives in S, as individual fuzzy preference relations in $S \times S$.

An *individual fuzzy preference relation* of individual k, R_k, is given by its membership function $\mu_{R_k} : S \times S \longrightarrow [0,1]$ such that

$$\mu_{R_k}(s_i, s_j) = \begin{cases} 1 & \text{if } s_i \text{ is definitely preferred to } s_j \\ c \in (0.5, 1) & \text{if } s_i \text{ is slightly preferred to } s_j \\ 0.5 & \text{in the case of indifference} \\ d \in (0, 0.5) & \text{if } s_j \text{ is slightly preferred to } s_i \\ 0 & \text{if } s_j \text{ is definitely preferred to } s_i \end{cases} \tag{8}$$

Basically, two lines of reasoning may be followed here (cf. Kacprzyk [15]):

- a direct approach: $\{R_1, \ldots, R_m\} \longrightarrow$ solution, that is, a solution is derived directly (without any intermediate steps) just from the set of individual fuzzy preference relations, and
- an indirect approach: $\{R_1, \ldots, R_m\} \longrightarrow R \longrightarrow$ solution, that is, from the set of individual fuzzy preference relations we form first a social fuzzy preference relation, R (to be defined later), which is then used to find a solution.

A solution is here, unfortunately, not clearly understood – cf. Nurmi [36] – [40]. First, in the case of group decision making under fuzzy preferences only, i.e., under a conventional majority, we start with solution concepts that do not require any preference aggregation at all. One of them is that of a *core* or a *set of undominated alternatives*, under a nonfuzzy required majority be r (e.g., at least 50%).

Definition 1. An alternative $x \in S$ belongs to the *core* iff there is no other alternative $y \in S$ that defeats x by the required majority r.

We can extend the notion of a core to cover fuzzy individual preference relations by defining a *fuzzy α-core* as follows (cf. Nurmi [36]):

Definition 2. An alternative $s_i \in S$ belongs to the *fuzzy α-core* S_α iff there exists no other alternative $s_j \in S$ such that $r_{ji} > \alpha$ for at least r individuals.

The intuitive interpretation of the fuzzy α-core is obvious: an alternative is a member of S_α if and only if a sufficient majority of voters does not feel strongly enough against it.

Another nonfuzzy solution concept with much intuitive appeal is a *minimax set* defined in a nonfuzzy setting as:

Definition 3. If, for each $x, y \in S$, we denote the number of individuals preferring x to y by $n(x, y)$, and define $v(x) = \max_y n(y, x)$ and $n* = \min_x v(x)$, then the *minimax set* is

$$Q(n*) = \{x \mid v(x) = n*\}$$

Thus, $Q(n*)$ consists of those alternatives that in pairwise comparison with any other alternative are defeated by no more than $n*$ votes. Obviously, if $n* < m/2$, where m is the number of individuals, then $Q(n*)$ is singleton and $x \in Q(n*)$ is the core if the simple majority rule is being applied.

Analogously, we can define *the minimax degree set* $Q(\beta)$ as follows:

Definition 4. *Given:* $s_i, s_j \in S$ *and, for individuals* $k = 1, \ldots, m$, $v_D^k(x_j) = \max_i r_{ij}$, $v_D(x_j) = \max_k v_D^k(x_j)$, *and let* $\min_j v_D(x_j) = \beta$.

Then

$$Q(\beta) = \{x_j \mid v_D(x_j) = \beta\}$$

A more general solution concept, the α-*minimax set* (cf. Nurmi [36]) denoted $Q^\alpha(v_f^\alpha)$, is defined as follows:

Definition 5. *Let* $n_\alpha(x_i, x_j)$ *be the number of individuals for whom* $r_{ij} \leq \alpha$ *for some value of* $\alpha \in [0, 0.5)$. *We now define* $\forall x_i \in S : v_f^\alpha(x_i) = \max_j n_\alpha(x_i, x_j)$ *and* $\bar{v}_f^\alpha = \min_i v_f^\alpha(x_i)$.

Then the α*-minimax set is defined as*

$$Q^\alpha(v_f^\alpha) = \{x_i \mid v_f^\alpha(x_i) = \bar{v}_f^\alpha\}$$

It can be shown that $Q^\alpha(v_f^\alpha) \subseteq Q(n*)$ (see [36]).

Now, we will show some basic solution concepts based on a social fuzzy preference relation that is obtained by an aggregation of the individual fuzzy preference relations. We will concentrate on those derived along the lines of Nurmi [36].

Definition 6. The set S_α of α-*consensus winners* is defined as: $s_i \in S_\alpha$ iff $\forall s_j \neq s_i : r_{ij} \geq \alpha$, with $0.5 < \alpha \leq 1$

Whenever S_α is nonempty, it is a singleton, but it does not always exist.

Definition 7. Let $\bar{r}_j = \max_i r_{ij}$ and $\bar{r} = \min_j \max_i r_{ij}$. Then $s_i \in S_M$ is the set of *minimax consensus winners* if and only if $\bar{r}_i = \bar{r}$.

Clearly S_M is always nonempty, but not necessarily a singleton.

Now, we will consider some solution concepts of group decision making but when we both have fuzzy preference relations and a fuzzy majority.

We will first employ the direct approach, i.e., $\{R_1, \ldots, R_m\} \longrightarrow$ solution to derive two popular solution concepts: fuzzy cores and minimax sets.

Definition 8. *Conventionally, the core is defined as a set of undominated alternatives, i.e., those not defeated in pairwise comparisons by a required (strict!) majority* $r \leq m$*, i.e.*

$$C = \{s_j \in S : \neg \exists s_i \in S \text{ such that } r^k_{ij} > 0.5 \text{ for at least } r \text{ individuals}\} \quad (9)$$

As the first fuzzification attempt, Nurmi [36] who has extended it as follows:

Definition 9. *The fuzzy* α*-core is defined as*

$$C_\alpha = \{s_j \in S : \neg \exists s_i \in S \text{ such that } r^k_{ij} > \alpha \geq 0.5 \text{ for at least } r \text{ individuals}\} \quad (10)$$

that is, as a set of alternatives not sufficiently (at least to degree α*) defeated by the required (still strict!) majority* $r \leq m$*.*

The concept of a fuzzy majority has been here proposed by Kacprzyk [15] and has turned out to be useful and adequate.

We start by denoting

$$h^k_{ij} = \begin{cases} 1 \text{ if } r^k_{ij} < 0.5 \\ 0 \text{ otherwise} \end{cases} \quad (11)$$

where $i, j = 1, \ldots, n$ and $k = 1, \ldots, m$.

Thus, h^k_{ij} just reflects if alternative s_j defeats (in pairwise comparison) alternative s_i ($h^k_{ij} = 1$) or not ($h^k_{ij} = 0$). Then, we calculate $h^k_j = \frac{1}{n-1} \sum_{i=1, i \neq j}^n h^k_{ij}$ which is clearly the extent, from 0 to 1, to which individual k is not against alternative s_j, where 0 standing for definitely against to 1 standing for definitely not against, through all intermediate values. Next, we calculate $h_j = \frac{1}{m} \sum_{k=1}^m h^k_j$ which it to what extent *all* the individuals are not against alternative s_j. And, finally, we calculate $v^j_Q = \mu_Q(h_j)$ is to what extent, from 0 to 1 as before, Q (say, most) individuals are not against alternative s_j.

Then:

Definition 10. *The fuzzy* Q*-core is defined (cf. Kacprzyk [15]) as a fuzzy set,*

$$C_Q = v^1_Q / s_1 + \cdots + v^n_Q / s_n \quad (12)$$

i.e., as a fuzzy set of alternatives that are not defeated by Q *(say, most) individuals.*

Notice that in the above basic definition of a fuzzy Q-core, we do not take into consideration to what degrees those defeats of one alternative by another are. They can be accounted for in a couple of plausible ways.

First, the degree of defeat in (11) may be replaced by

$$h^k_{ij}(\alpha) = \begin{cases} 1 \text{ if } r^k_{ij} < \alpha \leq 0.5 \\ 0 \text{ otherwise} \end{cases} \quad (13)$$

where, again, $i, j = 1, \ldots, n$ and $k = 1, \ldots, m$. Thus, $h^k_{ij}(\alpha)$ just reflects if alternative s_j sufficiently (i.e., at least to degree $1 - \alpha$) defeats (in pairwise comparison) alternative s_i or not.

We can also explicitly introduce the strength of defeat into (11) via:

$$\hat{h}^k_{ij} = \begin{cases} 2(0.5 - r^k_{ij}) \text{ if } r^k_{ij} < 0.5 \\ 0 \qquad\qquad\quad \text{otherwise} \end{cases} \tag{14}$$

where, again, $i, j = 1, \ldots, n$ and $k = 1, \ldots, m$. Thus, $\hat{h}^k_{ij}$ just reflects how strongly (from 0 to 1) alternative s_j defeats (in pairwise comparison) alternative s_i.

Then, by following the same procedure we can derive an α/Q-fuzzy core and an s/Q-fuzzy core.

Another intuitively justified solution concept may be the minimax (opposition) set which may be defined for our purposes as follows.

Definition 11. *Let $w(s_i, s_j) \in \{1, 2, \ldots, m\}$ be the number of individuals who prefer alternative s_j to alternative s_i, i.e., for whom $r^k_{ij} < 0.5$.*

If now $v(s_i) = \max_{j=1,\ldots,n} w(s_i, s_j)$ and $v^* = \min_{i=1,\ldots,n} v(s_i)$, then the minimax (opposition) set is defined as

$$M(v^*) = \{s_i \in S : v(s_i) = v^*\} \tag{15}$$

i.e., as a (nonfuzzy) set of alternatives which in pairwise comparisons with any other alternative are defeated by no more than v^* individuals, hence by the least number of individuals.

Nurmi [36] extends the minimax set, similarly in spirit to his extension of the core (10), to the *α-minimax set* as follows:

Definition 12. *Let $w_\alpha(s_i, s_j) \in \{1, 2, \ldots, m\}$ be the number of individuals who prefer alternative s_j to alternative s_i at least to degree $1 - \alpha$, i.e., for whom $r^k_{ij} < \alpha \leq 0.5$.*

If now $v_\alpha(s_i) = \max_{j=1,\ldots,n} w_\alpha(s_i, s_j)$ and $v^*_\alpha = \min_{i=1,\ldots,n} v_\alpha(s_i)$, then the α-minimax set is defined as:

$$M_\alpha(v^*_\alpha) = \{s_i \in S : v_\alpha(s_i) = v^*_\alpha\} \tag{16}$$

i.e., as a (nonfuzzy) set of alternatives which in pairwise comparisons with any other alternative are defeated (at least to degree $1 - \alpha$) by no more than v^* individuals, hence by the least number of individuals.

A fuzzy majority was introduced into the above definitions of minimax sets by Kacprzyk [15] as follows.

We start with (11), i.e.,

$$h^k_{ij} = \begin{cases} 1 \text{ if } r^k_{ij} < 0.5 \\ 0 \text{ otherwise} \end{cases} \tag{17}$$

and $h^k_i = \frac{1}{n-1} \sum_{j=1, j \neq i}^{n} h^k_{ij}$ is the extent, between 0 and 1, to which individual k is against alternative s_i. Then $h_i = \frac{1}{m} \sum_{k=1}^{m} h^k_i$ is the extent, between 0 and 1, to which all the individuals are against alternative s_i. Next, $t^Q_i = \mu_Q(h_i)$ is the extent, from 0 to 1, to which Q (say, most) individuals are against alternative s_i, and $t^*_Q = \min_{i=1,\ldots,n} t^Q_i$ is the least defeat of any alternative by Q individuals.

Finally:

Definition 13. *The Q-minimax set is defined as*

$$M_Q(t^*_Q) = \{s_i \in S : t^Q_i = t^*_Q\} \tag{18}$$

And analogously as for the α/Q-core and s/Q-core, we can explicitly introduce the degree of defeat $\alpha < 0.5$ and s into the definition of the Q-minimax set.

In the case of an indirect derivation, we follow the scheme: $\{R_1, \dots, R_m\} \longrightarrow R \longrightarrow$ solution, i.e., from the individual fuzzy preference relations we determine first a social fuzzy preference relation, R, and then find a solution from such a social fuzzy preference relation.

The indirect derivation involves two problems:

- how to find a social fuzzy preference relation from the individual fuzzy preference relations, i.e.,
$$\{R_1, \dots, R_m\} \longrightarrow R$$
- how to find a solution from the social fuzzy preference relation, i.e.,
$$R \longrightarrow \text{ solution}$$

In this paper, we will not deal in more detail with the first step, i.e., $\{R_1, \dots, R_m\} \longrightarrow R$, and assume a (most) straightforward alternative that the social fuzzy preference relation $R = [r_{ij}]$ is given by

$$r_{ij} = \begin{cases} \frac{1}{m}\sum_{k=1}^{m} a^k_{ij} \text{ if } i \neq j \\ 0 \quad\quad\quad\quad\quad \text{otherwise} \end{cases} \tag{19}$$

where $a^k_{ij} = \begin{cases} 1 \text{ if } r^k_{ij} > 0.5 \\ 0 \text{ otherwise} \end{cases}$. Notice that R obtained via (19) need not be reciprocal, i.e., $r_{ij} \neq 1 - r_{ji}$, but it can be shown that $r_{ij} \leq 1 - r_{ji}$, for each $i, j = 1, \dots, n$.

In the second case, i.e., $R \longrightarrow$ solution, a solution concept of much intuitive appeal is here the consensus winner (cf. Nurmi [36]) which will be extended under a social fuzzy preference relation and a fuzzy majority.

We start with

$$g_{ij} = \begin{cases} 1 \text{ if } r_{ij} > 0.5 \\ 0 \text{ otherwise} \end{cases} \tag{20}$$

which expresses if s_i defeats (in the whole group's opinion!) s_j or not.

Next $g_i = \frac{1}{n-1}\sum_{j=1, j\neq i}^{n} g_{ij}$ which is a mean degree to which s_i is preferred, by the whole group, over all the other alternatives. Then, $z^i_Q = \mu_Q(g_i)$ is the extent to which alternative s_i is preferred, by the whole group, over Q (e.g., most) other alternatives.

Finally:

Definition 14. *The fuzzy Q-consensus winner is defined as*

$$W_Q = z^1_Q/s_1 + \cdots + z^n_Q/s_n \tag{21}$$

i.e., as a fuzzy set of alternatives that are preferred, by the whole group, over Q other alternatives.

And analogously as in the case of the core, we can introduce a threshold $\alpha \geq 0.5$ and s into (20) and obtain a *fuzzy* α/Q*-consensus winner* and a *fuzzy* s/Q*-consensus winner*, respectively.

This concludes our brief exposition of how to employ fuzzy linguistic quantifiers to model the fuzzy majority. We did not present some other solution concepts as, e.g., minimax consensus winners (cf. Nurmi [36], Kacprzyk [15]) or those based on fuzzy tournaments which have been proposed by Nurmi and Kacprzyk [45]and are relevant in the voting context.

One should also notice that in a number of recent papers by Kacprzyk and Zadrożny [25] it has been shown that the concept of Kacprzyk's [15] fuzzy Q-core can be a general (prototypical) choice function in group decision making and voting; for instance, those of: a "consensus solution", Borda's rule, the minimax degree set, the plurality voting, the qualified plurality voting, the approval voting-like, the "consensus + approval voting", Condorcet's rule, the Pareto rule, Copeland's rule, Nurmi's minimax set, Kacprzyk's Q-minimax, the Condorcet looser, the Pareto inferior alternatives, etc. This result, though of some relevance to the problem of dealing with voting paradoxes, is however beyond the scope of this paper.

To summarize this section, the fuzzy preferences and a fuzzy majority can be used to derive more flexible and human consistent versions of main solution concepts in group decision making, in a natural connection to voting.

3 Remarks on Some Voting Paradoxes and Their Alleviation

Voting paradoxes are an interesting and very relevant topic that has a considerable theoretical and practical relevance. In this paper we will just give some simple examples of well-known paradoxes and indicate some possibilities of how to alleviate them by using some elements of fuzzy preferences and a fuzzy majority. The paper is based on the works by Nurmi [43], [43], Nurmi and Kacprzyk [46], and Kacprzyk, Zadrożny, Fedrizzi and Nurmi [27,28,29].

In most case, one distinguishes between the so-called classic paradoxes, the so-called Condorcet and Borda paradoxes, and some other ones, which are interesting too but maybe less known. Basically, the former pertains to the non-transitivity of a collective preference relation if it is formed using pairwise majority comparisons, while the latter shows the possibility that an intuitively implausible alternative is elected even if it is defeated by all other alternatives in pairwise comparisons by a majority of voters (cf. Black [2], DeGrazia [5], Nurmi [43]).

3.1 Condorcet's Paradox

An example of Condorcet's paradox is shown in Table 1. There are 3 voter groups of equal size whose preferences over alternatives A, B and C are represented by the rank order indicated below each group. The equal size of the groups is not necessary but any two of them should constitute a majority. A collective preference relation formed by pairwise comparisons of alternatives using the majority rule results in a cycle: A is preferred to B, B is preferred to C and C is preferred to A.

Table 1. Condorcet's paradox

Group I	*Group II*	*Group III*
A	B	C
B	C	A
C	A	B

3.2 Borda's Paradox

An instance of Borda's paradox is given in Table 2 in which alternative A wins clearly by a plurality of votes and, yet, both B and C beat A under pairwise comparisons using the majority rule.

Table 2. Borda's paradox

voters 1-4	*voters 5-7*	*voters 8,9*
A	B	C
B	C	B
C	A	A

One can easily notice that a common characteristic in these classic paradoxes is the violation of intuitively plausible requirements. In the case of Condorcet's paradox, the result obtained by using the majority rule on a set of complete and transitive preferences is intransitive. In the case of Borda's paradox, the winner in the plurality sense is different from the winner in the sense that the winner is to beat all the other ones in pairwise comparisons.

3.3 Some other Paradoxes

Among other, less known paradoxes, presumably the most important in this class is the so-called *additional support paradox* which occurs whenever some additional support makes a winning alternative a non-winning one. Unfortunately, many commonly used voting procedures are plagued by this.

An example of an additional support paradox is shown in Table 3. Suppose that the voters vote according to their preferences, and in the first round alternative A gets 22 votes, B – 21 votes and C – 20 votes. Since no alternative gets more than 50% of the 63 votes, there will be a second round between A and B. Suppose that A wins as the 20 voters who have favored C will presumably vote for A rather than their lowest ranked B. Hence A is the winner. Suppose now that A obtains some more support, say, 2 out of those 21 voters with the preference ranking BCA. We now have 24 voters with the preference ranking ABC, 19 voters with the ranking BCA and 20 voters with the ranking CAB. A runoff is again needed, now between A and C. But now C wins by 39 votes against 22. Thus, an additional support would be disastrous for A.

Table 3. Additional support paradox

22 voters	*21 voters*	*20 voters*
A	B	C
B	C	A
C	A	B

Another type (maybe even a class) is the *choice set variance paradoxes* the crucial feature of which is a counter-intuitive variation in choice sets under certain types of modifications in the set of alternatives or the preference profile.

An example of a choice set variance paradox is given in Table 4 which shows the preference profile of a 100-voter electorate divided into two equal parts. Suppose that the plurality runoff system is used and that the votes are counted separately in each part of the electorate. In both parts A is the winner. In the left half there is a runoff between A and B yielding A as the winner. In the right half A is the winner with more than 50% of the votes. Suppose now that the whole electorate is taken as a whole. This implies C to be the winner. Thus, in spite of being the winner in both halves of the electorate, A is not the winner in the entire set of voters.

Table 4. Consistency paradox

20	*20*	*10*	*26*	*4*	*20*
A	B	C	A	B	C
B	C	A	B	C	B
C	A	B	C	A	A

Two specific choice set variance paradoxes, the so-called *Ostrogorski's paradox* and the *referendum paradox* are related to the majority rule that is a foundation of all democratic systems. They occur also in contexts in which the choice sets, obtained by using the majority rule, are combined. Hence, they are often called the compound majority paradoxes.

Table 5 shows the essence of the Ostrogorski's paradox. It shows a distribution of support over two parties (X and Y) and three issues (issues $1, 2, 3$). Thus, for example, 20% of the electorate, denoted by group B, supports party X on issues 1 and 3 and party Y on issue 2. If all issues are of equal importance, then it may be assumed that they vote for that party which they support on more issues—cf. the last column.

Table 5. Ostrogorski's paradox

group	*issue 1*	*issue 2*	*issue 3*	*party supported*
A (20%)	X	X	Y	X
B (20%)	X	Y	X	X
C (20%)	Y	X	X	X
D (40%)	Y	Y	Y	Y

The result seems to be X since it is supported by 60% of the electorate. However, Ostrogorski argued that the legitimacy of X's victory could be challenged since, by voting on each issue separately, Y would win in each case by a majority of 60—40%.

In some countries where consultative non-binding referenda are being resorted to, a particular problem of great importance may be encountered, namely which result is more authoritative: the referendum outcome or the parliamentary voting outcome. This problem has certain similarities with Ostrogorski's paradox (see [41,42]).

Table 6. Referendum paradox

opinions	*MP's of A*	*MP's of B*	*vote total*
	1-6	*7-9*	
"yes"	5	11	63
"no"	6	0	36

Suppose that the parliament consists of 9 members and there are 99 voters. Assume, moreover, that the support for each elected member is the same, i.e., 11 votes for each member. Party A has 6 out of 9 or 2/3 of the parliament seats, while party B has 3 out of 9 or 1/3 of the seats. Suppose that the support of the parties corresponds to the seat distribution, that is, 2/3 of the electorate supports party A and 1/3 party B.

Now, suppose that a referendum is held in which the voters are asked to answer either *yes* or *no* to a question. Let the distribution of votes in both parliamentary elections and the referendum as shown in Table 6). Clearly, *yes* wins the referendum receiving 63 votes out of 99. Suppose now that the same issue is subjected to a parliamentary vote. Then, assuming that the members of parliament are aware of the distribution of opinions of their own supporters, it is plausible to predict that they vote in accordance with what they think is the opinion of the majority of their supporters. Thus, the members of party A would vote for *no* and those of party B for *yes*, and *no* wins by a handsome margin 6 to 3.

We have shown just a couple of voting paradoxes and in the next section we will show how one can eliminate them, or—better to say—alleviate them by using some elements of our approach to group decision making based on fuzzy preference relations and fuzzy majority.

4 Alleviating and Solving Some Voting Paradoxes

Now we will outline the essence of our approach on how to solve those voting paradoxes by using a setting of group decision making and voting with fuzzy preference relations. As an illustrative example we will consider Condorcet's and Borda's paradox only. A similar procedure, though more complicated, may be employed for other paradoxes.

We consider the set E of individuals and the set S of decision alternatives. Each individual $i \in E$ provides a fuzzy preference relation $R_i(x, y)$ over S. For each $x, y \in S$, the value $R_i(x, y)$ indicates the degree in which x is preferred to y by i with 1 indicating the strongest preference of x to y, 0.5 indifference between the two and value 0 the strongest preference of y to x.

To facilitate our discussion, let us briefly recall some issues related to fuzzy preference relations.

A fuzzy preference relation R is *connected* if and only if $R(x, y) + R(y, x) \geq 1, \forall x, y \in S$.

A fuzzy preference relation R is *reflexive* if and only if $R(x, x) = 1, \forall x \in S$.

A fuzzy connected and reflexive relation R is *max–min transitive* if and only if $R(x, z) \geq min[R(x, y), R(y, z)], \forall x, y, z \in S$.

For the case of Condorcet's paradox, a way out of cyclical collective preferences is to look at the sizes of majorities supporting various collective preferences. For example, if the number of voters preferring a to b is 5 out of 9, while that of voters preferring b to c is 7 out of 9, then, according to Condorcet, the latter preference is stronger than the former. By cutting the cycle of collective majority preferences at its weakest link, one ends up with a complete and transitive relation. Clearly, with a non-fuzzy preference relation, this method works only in cases where not all of the majorities supporting various links in the cycle are of same size.

With fuzzy preferences one can form the collective preference between any x and $y \in S$ using a variation of the average rule (cf. Intrilligator [13]), i.e.,

$$R(x, y) = \frac{\sum_i R_i(x, y)}{m} \qquad (22)$$

where $R(x, y)$ is the degree of collective fuzzy preference of x over y.

Now, if a preference cycle is formed on the basis of collective fuzzy preferences, one could simply ignore the link with weakest degree of preference and thus possibly end up with a ranking. In general, one can proceed by eliminating weakest links in collective preference cycles until a ranking occurs.

The above method of successive elimination of the weakest links in the preference cycles works with the fuzzy and nonfuzzy preferences. When the individual preferences are fuzzy, each voter provides the his/her preferences so that the following matrix can be formed:

$$R_i = \begin{pmatrix} - & r_{12} & \dots & r_{1n} \\ r_{21} & - & \dots & r_{2n} \\ \dots & \dots & \dots & \dots \\ r_{n1} & r_{n2} & \dots & - \end{pmatrix} \qquad (23)$$

where r_{ij} indicates the degree to which i prefers the i-th alternative to the j-th one.

By taking an average over the voters we obtain:

$$\bar{R} = \begin{pmatrix} - & \bar{r}_{12} & \dots & \bar{r}_{1n} \\ \bar{r}_{21} & - & \dots & \bar{r}_{2n} \\ \dots & \dots & \dots & \dots \\ \bar{r}_{n1} & \bar{r}_{n2} & \dots & - \end{pmatrix} \tag{24}$$

One can also compute first the row sums of the matrix: $\bar{r}_i = \sum_j \bar{r}_{ij}$ which represent the total fuzzy preference weight assigned to the i-th alternative in all pairwise preference comparisons, when the weight in each comparison is the average fuzzy preference value.

Let now $p_i = \frac{\bar{r}_i}{\sum_i \bar{r}_i}$, and, clearly $p_i \geq 0$ and $\sum_i p_i = 1$. Thus, p_i has the natural interpretation of a choice probability that can be used to form the collective preference ordering which is necessarily a complete and transitive relation.

For illustration, consider the example of Table 1 again and assume that each group consists of just one voter, and that the fuzzy preferences underlying the preference rankings are as shown in Table 7.

Table 7. Fuzzy Condorcet's paradox

voter 1				*voter 2*				*voter 3*			
	A	*B*	*C*		*A*	*B*	*C*		*A*	*B*	*C*
A	-	.6	.8	*A*	-	.9	.3	*A*	-	.6	.3
B	.4	-	.6	*B*	.1	-	.7	*B*	.4	-	.1
C	.2	.4	-	*C*	.7	.3	-	*C*	.7	.9	-

The $\bar{R}$- matrix is now: $\bar{R} = \begin{pmatrix} - & .7 & .5 \\ .3 & - & .5 \\ .5 & .5 & - \end{pmatrix}$ and $P_A = 0.4, P_B = 0.3, P_C = 0.3$.

Obviously, the solution is based on somewhat different fuzzy preference relations over the three alternatives. For identical, the preference relations we would necessarily end up with identical choice probabilities.

We can also resolve Borda's paradox by applying the same procedure. Suppose that Borda's paradox (exemplified by Table 2) in the fuzzy setting is as represented by the fuzzy preferences given in Table 8.

Table 8. A fuzzy Borda's paradox

4 voters				*3 voters*				*2 voters*			
	A	*B*	*C*		*A*	*B*	*C*		*A*	*B*	*C*
A	-	.6	.8	*A*	-	.9	.3	*A*	-	.2	.1
B	.4	-	.6	*B*	.1	-	.7	*B*	.8	-	.3
C	.2	.4	-	*C*	.7	.3	-	*C*	.9	.7	-

The matrix of average preference degrees is then: $\bar{R} = \begin{pmatrix} - & .6 & .5 \\ .4 & - & .6 \\ .5 & .4 & - \end{pmatrix}$. The choice probabilities of A, B and C are, thus, $0.37, 0.33, 0.30$. The choice probability of A is the largest. In a sense, then, the method does not solve Borda's paradox in the same way as the Borda count does since also the plurality method ends up with A being chosen instead of the Condorcet winner alternative B. Note, however, that the fuzzy preference relations give a richer picture of voter preferences than the ordinary preference rankings. In particular, A is strongly preferred to B and C by both the 4 and 3 voter groups, and its choice probability is the largest.

For additional information on voting paradoxes and some ways to solve them using fuzzy logic, we refer the reader to Nurmi and Kacprzyk [46].

5 Concluding Remarks

We have briefly outlined various ways to the derivation of group decision (voting) models under individual and social fuzzy preference relations and fuzzy majorities. In the first part we discussed issues related to their role as a tool to alleviate difficulties related to negative results in group decision making exemplified by Arrow's impossibility theorem. The second part has been focused on an important, sometimes dangerous phenomenon of so-called voting paradoxes which are basically intuitively implausible, surprising, counter-intuitive and generally unpleasant phenomena in voting contexts. We have provided some tools for finding a way of alleviating them. Our approach is based on the use of fuzzy preference relations. The results presented are of relevance for both social choice, voting, group decision making, etc. areas, but also for multi-agent systems in which some specific types of voting procedures are also employed.

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Research in Social Sciences: Fuzzy Regression and Causal Complexity

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Abstract. Social science research is now primarily divided into two types: qualitative, or case-oriented research, focused on individual cases, which reviews all aspects of a few case studies and quantitative, or variable-oriented research, which considers only some quantitative aspects (variables) of a large number of cases and is looking for correlations between these variables.

The first type of research is based primarily on evidence, the second on theoretical models. The fundamental criticism to case-oriented research is that it does not lead to general theoretical models, while the most important criticisms to the variable-oriented research are the assumption of a population a priori and the hypothesis that the elements of the population are homogeneous.

A compromise between the two points of view is the diversity-oriented research, which takes into account the variables and the diversity of individual cases.

The fundamental purpose of our paper is to study the possibilities provided by fuzzy sets and algebra of fuzzy numbers for the study of social phenomena. We deepen some aspects of the fuzzy regression, and we present some operations between fuzzy numbers that are efficient alternatives to those based on Zadeh extension principle. Finally, we present some critical remarks about the causal complexity and logical limits of the assumption of linear relationship between variables. A solution of these problems can be obtained by the fuzzy sets that play a key role in diversity-oriented research.

Keywords: social science research, fuzzy regression, alternative fuzzy operations, case-oriented research, causal complexity.

1 Introduction and Motivation

Social science research is now primarily divided into two types: *qualitative* or *case - oriented research* and *quantitative* or *variable-oriented research* [BR1], [BR2], [RA].

The first research strategy, also called *intensive*, focuses on complexity of social phenomena. It is based on an in-depth study of individual cases and analyzes all aspects of a very small number of case studies. The second line of research, also

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called *extensive*, considers some quantitative aspects (variables) of a large number of cases and is looking for correlations between these variables.

Recently, some authors have highlighted that usually there is a very sharp distinction between the two categories of studies (see e.g., [RA]). The majority of studies of social phenomena is based or on an *intensive* study of a few cases, approximately not more than a dozen, or on an *extensive* study for many cases, approximately not less than one hundred. There are few papers on a number of case studies ranging a dozen to a hundred.

The intensive research is based primarily on *evidence*, the extensive on *theoretical models*. In the transition from an intensive study of the case studies in an extensive research, as the number of cases increases, we lose the characteristics of individual phenomena and we come to assumptions of *homogeneity* of individual cases, which often are not suitable for the study of social phenomena.

The fundamental criticism to *case-oriented* research is that it does not lead to theoretical and general models, while the most important criticisms to the *variable-oriented* research are the assumption of a population a priori and the hypothesis that the elements of the population are homogeneous.

A compromise between the two points of view is the *diversity-oriented research*, which takes into account the variables and the diversity of individual cases [BR1], [BR2], [RA]. It is also based on an idea of population not fixed from the beginning, but changing during the research according to a critical examination of the intermediate results. This characteristic of the diversity-oriented research allows us also to consider an intermediate number of cases that do not necessarily reduce to the extremes of a handful of cases or many cases.

A powerful tool in diversity-oriented research is provided by the theory of fuzzy sets and the opportunities they offer to replace relations belonging to linear relationships between variables and probabilistic assumptions.

In this sense, fuzzy logic and algebra of fuzzy numbers (see e.g. [DP1], [GO], [ZA1], [ZA2], [ZA3], [ZA4]) are proposed as an alternative to the study of the linear relationship between the dependent variable, to be explained, and the explanatory variables [RA].

In other contexts, however, keeping to extensive research, the algebra of fuzzy numbers works in accordance with the hypothesis that there is, at least in a first approximation, a linear relationship between dependent and independent variables. This leads to the fuzzy regression, in which, unlike the classical regression where the conclusions are expressed in probabilistic terms, the conclusions must be formulated in terms of degree of membership of the values of the dependent variable to fuzzy numbers calculated by the model.

The paper is structured as follows. In Section 2, we introduce a brief review of basic concepts on fuzzy regression for further development of some themes. In Section 3, we introduce operations that are efficient and mathematical consistent alternatives to those based on the Zadeh extension principle [ZA1], [ZA2], [ZA3], [ZA4], [YA], [KY], [RO] and we analyze how they can make new fuzzy regression tools. Finally, in Section 4, we analyze some concepts on the study of social phenomena based on *diversity-oriented research* and present some critical remarks upon some implicit assumptions of the variable-oriented research as the homogeneity of the case studies and the linear relationship among the variables.

2 Fuzzy Numbers and Fuzzy Linear Regression: A Review and Critical Analysis of Some Fundamental Aspects

Fuzzy linear regression can be classified in "partially fuzzy" or "totally fuzzy" regression. In the first case, we have two possibilities: *fuzzy parameters with crisp data* or *fuzzy data with crisp parameters*. In the second case *data and parameters are both fuzzy* ([KY], [RO]).

The start point of a partially fuzzy linear regression is the individuation of an algebraic structure of fuzzy numbers (F, +, *) where F is a nonempty family of fuzzy numbers containing the set R of real numbers (e.g. the degenerate fuzzy numbers), + is an operation on F, called "addition", extension of the addition on R, and * is the multiplication of an element of F by a scalar, e.g., a function *: R×F →F, extension of the multiplication on R.

Usually, F is the set of triangular fuzzy numbers and the operations + and * are obtained by the "Zadeh extension principle". In this paper, we show that logical reasons and mathematical properties can lead to prefer other sets of fuzzy numbers or alternative fuzzy operations.

A total fuzzy linear regression needs a more complex algebraic structure of fuzzy numbers. Namely, we have to assign an algebraic structure (F, +, *, ·), where + is the addition, * is the multiplication of an element of F by a scalar, and · is a multiplication on F, extension to F×F of *.

Unfortunately, the multiplication defined by means of the Zadeh's extension principle presents some important drawbacks including the following:

(1) in general the Zadeh extension product of two triangular (resp. trapezoidal) fuzzy numbers is not a triangular (resp. trapezoidal) fuzzy number;
(2) if the supports of two triangular fuzzy numbers contain 0 the spread of the product of triangular fuzzy numbers does not have a simple expression;
(3) the distributive property applies only to particular triplets of fuzzy numbers.

For these reasons, in this section we consider especially the partial fuzzy linear regression. In the first subsection we recall and introduce some concepts and notations on fuzzy numbers necessary for the rest of the paper, in the second and third subsection we focus our interest on some fundamental aspects of partial fuzzy regression. Finally, in the fourth subsection we make a brief introduction and some critical comments on the totally fuzzy linear regression.

2.1 Fundamental Concepts and Notations on Fuzzy Numbers

Let us recall some fundamental concepts on fuzzy numbers and some related properties ([DP2], [KY], [YA], [RO], [ZA1], [ZA2], [ZA3], [ZA4], [YA], [MA1], [MA2], [CH]).

Definition 2.1. A fuzzy number is a function having as domain the set of real numbers and with values in [0, 1], u: R → [0, 1], such that:

(FN1) (*bounded support*) there are two real numbers a, b, with a ≤ b, called the *endpoints* of u, such that u(x) = 0 for x ∉ [a, b] and u(x) > 0 for x belonging to the open interval (a, b);

(FN2) (*normality*) there are two real numbers c, d, with a ≤ c ≤ d ≤ b such that u(x) = 1 if and only if x∈[c, d].

(FN3) (*convexity*) u(x) is a function increasing in the interval [a, c] and decreasing in the interval [d, b].

(FN4) (*compactness*) for every r∈(0, 1), the set {x∈R: u(x) ≥ r} is a closed interval.

The set of the real numbers x such that u(x) > 0 is said to be the *support* of u, denoted supp(u) or S(u), and the interval [c, d] is said to be the *core* or *central part* of u, noted core(u) or C(u). The intervals [a, c) and (d, b] are, respectively, the *left part* and the *right part* of u.

The fuzzy number u is said to be *simple* if c = d, i.e., C(u) is a singleton. Moreover, u is said to be *degenerate* if a = b, i.e., S(u) ={c}, c∈R.

The real numbers L(u) = c – a, M(u) = d – c, and R(u) = b – d are the *left, middle, and right spreads* of u, respectively. Their sum T(u) = b – a is the *total spread* of u.

For every r such that 0 ≤ r ≤ 1, the set of the x∈[a, b] such that u(x) ≥ r is denoted by $[u]^r$ and is said to be the *r-cut* of u. The left and right endpoints of $[u]^r$ are denoted, respectively, $u_\lambda{}^r$ and $u_\rho{}^r$. In particular, $[u]^0$ is the closure of the support of u and $[u]^1$ is the core of u.

Let us assume the following notations:

- (*endpoints notation*) u ~ (a, c, d, b) stands u is a fuzzy number with endpoints a, b, and core [c, d]; u ~ (a, c, b) for a simple u with endpoints a, b, and core {c};
- (*spreads notation*) u ~ [c, d, L, R] denotes that u is a fuzzy number with core [c, d] and left and right spreads L and R, respectively; u ~ [c, L, R] denotes a simple u with core {c};
- (*r-cut spreads notation*) the numbers $L^r(u) = (c - u_\lambda{}^r)$ and $R^r(u) = (u_\rho{}^r - d)$ are called the *r-cut left spread* and the *r-cut right spread* of u, then we can write $[u]^r = [c - L^r(u), d + R^r(u)]$;
- (*sign*) the fuzzy number u ~ (a, c, d, b) is said to be *positive*, *strictly positive*, *negative*, or *strictly negative*, if a ≥ 0, a > 0, b ≤ 0, or b < 0, respectively;
- (*c-sign*) the fuzzy number u ~ (a, c, d, b) is said to be *c-positive*, *strictly c-positive*, *c-negative*, or *strictly c-negative*, if c ≥ 0, c > 0, d ≤ 0, or d < 0, respectively.

Definition 2.2. Let C be the set of the compact intervals of R. For every pair of intervals [a, b] and [c, d] in C, we assume:

$$[a, b] + [c, d] = [a + c, b + d]; \tag{2.1}$$

$$[a, b] \cdot [c, d] = [\min\{ac, ad, bc, bd\}, \max\{ac, ad, bc, bd\}]; \tag{2.2}$$

$$[a, b] \leq [c, d] \text{ if and only if } a \leq c \text{ and } b \leq d. \tag{2.3}$$

The subtraction and division are also defined on C by the formulae:

$$[a, b] - [c, d] = [a, b] + [-d, -c]; \tag{2.4}$$

$$\text{if } 0 \notin [c, d],\ [a, b] / [c, d] = [a, b] \cdot [1/d, 1/c]. \tag{2.5}$$

Remark 2.3. [KY, 103] The addition + defined by (2.1) is commutative, associative, having 0 = [0, 0] as neutral element. The multiplication defined by (2.2) is commutative, associative, having 1 = [1, 1] as neutral element. Moreover, for every compact intervals [a, b], [c, d], [e, f], the following subdistributive property holds:

$$([a, b] + [c, d])\,[e, f] \subseteq [a, b]\,[e, f] + [c, d]\,[e, f], \quad \text{(subdistributivity)} \tag{2.6}$$

The distributivity holds iff $[a, b] \cdot [c, d] \geq 0$ or [e, f] is a degenerate interval.

Definition 2.4. We say that the fuzzy number u ~ (a, c, d, b) is a *trapezoidal fuzzy number*, denoted u = (a, c, d, b), if:

$$\forall x \in [a, c),\quad a < c \Rightarrow u(x) = (x-a)/(c-a), \tag{2.7}$$

$$\forall x \in (d, b],\quad d < b \Rightarrow u(x) = (b-x)/(b-d). \tag{2.8}$$

In spread notation, if u ~ [c, d, L, R] is a trapezoidal fuzzy number, we write u = [c, d, L, R]. A simple trapezoidal fuzzy number u = (a, c, c, b) is said to be a *triangular fuzzy number* denoted u = (a, c, b). In spread notation, if u ~ [c, L, R] is a triangular fuzzy number we write u = [c, L, R]. A trapezoidal fuzzy number u = (c, c, d, d), with support equal to the core is said to be a *rectangular fuzzy number* and is identified with the compact interval [c, d] of R.

The necessary and sufficient conditions for u ~ [c, d, L, R] in order to be a trapezoidal fuzzy number, in terms of r-cut left and right spreads, are:

$$L^r(u) = (1-r)\,(c-a) = (1-r)\,L, \qquad R^r(u) = (1-r)\,(b-d) = (1-r)\,R. \tag{2.9}$$

Remark 2.5. For every real number x, let us denote with $\alpha(x)$ the sign of x. For every fuzzy number u ~ [c, d, L, R], we define +L = -R = L and +R =-L = R. Let + e * be the Zadeh extension addition and multiplication of a real number by a fuzzy number. If x is a real number and u and v are fuzzy numbers, then the following properties of the left and right spreads hold [MA1]:

$$L(u + v) = L(u) + L(v),\ L(x*u) = |x|\,(\alpha(x)\,L(u)); \tag{2.10}$$

$$R(u + v) = R(u) + R(v),\ R(x*u) = |x|\,(\alpha(x)\,R(u)). \tag{2.11}$$

2.2 Partial Fuzzy Linear Regression with Fuzzy Parameters

Let $\{K_i \sim [c_i, d_i, L_i, R_i],\ i = 1, 2, \ldots, n\}$ be a set of fuzzy numbers, $\{x_i, i=1, 2, \ldots, n\}$ the set of the independent variables, and y the dependent variable. Equation (2.12) shows a general fuzzy linear regression model with K_i fuzzy parameters:

$$Y = K_1\,x_1 + K_2\,x_2 + \ldots + K_n\,x_n, \tag{2.12}$$

where the addition and the multiplication of a real number by a fuzzy number are the Zadeh's extension operations.

Suppose we have the following sample:

Table 1.

Sample number, j	Output values, y_j	Input values, x_{ij}
1	y_1	$x_{11}, x_{21}, \ldots, x_{n1}$
2	y_2	$x_{12}, x_{22}, \ldots, x_{n2}$
m	y_m	$x_{1m}, x_{2m}, \ldots, x_{nm}$

The first step of the fuzzy linear regression is to replace in (2.12) the numerical vector $(x_{1j}, x_{2j}, \ldots, x_{nj})$ to the vector of independent variables $(x_1, x_2, \ldots, x_n)$ and then to obtain a fuzzy number Y_j, for every $j \in \{1, 2, \ldots, m\}$.

The second step is to calculate the degree to which y_j belongs to Y_j. A possible conclusion is to consider the fuzzy coefficients K_i adequate if for each j the degree of belonging of y_j to Y_j is "*sufficiently high*".

A key aspect is the calculation of the spreads of Y. The larger are the spreads, the greater the degree to which y_j belongs to Y_j.

From formulae (2.10), (2.11) we have:

$$L(Y) = (\alpha(x_1)\, L(K_1))\, |x_1| + (\alpha(x_2)\, L(K_2))\, |x_2| + \ldots + (\alpha(x_n)\, L(K_n))\, |x_n| \qquad (2.13)$$

$$R(Y) = (\alpha(x_1)\, R(K_1))\, |x_1| + (\alpha(x_2)\, R(K_2))\, |x_2| + \ldots + (\alpha(x_n)\, R(K_n))\, |x_n| \qquad (2.14)$$

Moreover, the core of Y is obtained by the following formula:

$$C(Y) = (\alpha(x_1)\, C(K_1))\, |x_1| + (\alpha(x_2)\, C(K_2))\, |x_2| + \ldots + (\alpha(x_n)\, C(K_n))\, |x_n|, \qquad (2.15)$$

where, for every interval [a, b] of the real line +[a, b] = [a, b], -[a, b] = [-b, -a].

If the K_i are simple fuzzy numbers with symmetric spreads, let $s_i = L(K_i) = R(K_i)$ be the bilateral spread of K_i and let $s(Y) = L(Y) = R(Y)$ be the bilateral spread of Y. Then we have the simpler formulae:

$$s(Y) = s_1\, |x_1| + s_2\, |x_2| + \ldots + s_n\, |x_n|, \qquad (2.16)$$

$$C(Y) = c_1 x_1 + c_2 x_2 + \ldots + c_n x_n. \qquad (2.17)$$

Usually in the scientific literature (see, e.g. [KY], [RO]), the K_i are symmetric and triangular fuzzy numbers. It is well-known that the sum of two triangular fuzzy numbers and the product of a real number for a triangular fuzzy number are triangular fuzzy numbers, too ([KY], [RO], [MA1], [MA2]). Then also Y is a triangular fuzzy number.

If the spread s(Y) is not null, then the membership function of Y is given by the formula:

$$\mu(y) = 1 - |y - C(Y)|/s(Y), \qquad y \in (C(Y) - s(Y),\, C(Y) + s(Y)), \qquad (2.18)$$

and $\mu(y)$ is null otherwise. If the spread s(Y) is null, then Y reduces to the real number C(Y), i.e., a degenerate fuzzy number.

Let us suppose s(Y) is not null. Then, by considering the values of Table 1, we have:

$$\mu(y_j) = 1-|y_j-C(Y_j)|/s(Y_j) = 1-|y_j - c_1x_{1j}+c_2x_{2j}+\ldots+c_nx_{nj}|/(s_1|x_{1j}|+s_2|x_{2j}|+\ldots+s_n|x_{nj}|). \quad (2.19)$$

If $h\in(0, 1)$ is a number expressing (in the opinion of the decision maker) a sufficient degree of membership of y_j to Y_j, then the conditions $\mu(y_j) \geq h$, j=1, 2, …, m must be satisfied. Then from (2.19), we have the 2m constraints:

$$y_j \leq c_1x_{1j}+c_2x_{2j}+\ldots+c_nx_{nj} + (1-h)\,(s_1|x_{1j}|+s_2|x_{2j}|+\ldots+s_n|x_{nj}|), \quad j=1, 2, \ldots, m, \quad (2.20)$$

$$y_j \geq c_1x_{1j}+c_2x_{2j}+\ldots+c_nx_{nj} - (1-h)\,(s_1|x_{1j}|+s_2|x_{2j}|+\ldots+s_n|x_{nj}|), \quad j=1, 2, \ldots, m. \quad (2.21)$$

It is evident that whatever the sample (with the unique condition that $\forall j, \exists i: x_{ij} \neq 0$), and whatever the numbers c_i, conditions (2.20) and (2.21) are satisfied when the spreads s_i are sufficiently high. But high spreads mean an excessive vagueness, then it is necessary to introduce an objective function $u = f(s_1, s_2, \ldots, s_n)$, positive and increasing with respect to every variable, and seek a solution of the system (2.20), (2.21), with the unknowns c_i and s_i and minimizing u.

Two alternative objective functions are:

(1) *the sum of spreads of coefficients* [KY], e.g.,

$$f(s_1, s_2, \ldots, s_n) = s_1 + s_2 +\ldots+ s_n; \quad (2.22)$$

(2) *the sum of spreads of the sample* [RO], e.g.,

$$f(s_1, s_2, \ldots, s_n) = \Sigma_{i,j}\, |x_{ij}|\, s_{ij}. \quad (2.23)$$

2.3 Partial Fuzzy Linear Regression with Fuzzy Data

Equation (2.24) shows a general fuzzy linear regression model with fuzzy data:

$$Y = k_1 X_1 + k_2 X_2 + \ldots + k_n X_n, \quad (2.24)$$

where coefficients k_i are crisp numbers and the values of variables are fuzzy numbers.

Let us have the following sample:

Table 2.

Sample number, j	Output values, Y_j	Input values, X_{ij}
1	Y_1	$X_{11}, X_{21}, \ldots, X_{n1}$
2	Y_2	$X_{12}, X_{22}, \ldots, X_{n2}$
m	Y_m	$X_{1m}, X_{2m}, \ldots, X_{nm}$

For every j = 1, 2, …, m, we have to calculate the fuzzy number:

$$Y_j{}^* = k_1 X_{1j} + k_2 X_{2j} + \ldots + k_n X_{nj}, \qquad (2.25)$$

and to compare $Y_j{}^*$ with the sample value Y_j.

The logics of the algorithms used are fuzzy extensions of the previous case. The condition $\mu(y_j) \geq h$ is replaced by the condition of compatibility between $Y_j{}^*$ and Y_j:

$$\forall j \in \{1, 2, \ldots, m\}, \text{com}(Y_j{}^*, Y_j) = \sup_{y \in R} (\min(Y_j{}^*(y), Y_j(y)) \geq h. \qquad (2.26)$$

2.4 Total Fuzzy Linear Regression

Equation (2.27) shows a total fuzzy linear regression model:

$$Y = K_1 X_1 + K_2 X_2 + \ldots + K_n X_n, \qquad (\ 2.27)$$

where the coefficients K_i and the values of variables are fuzzy numbers.

Also for the algorithms of the total fuzzy linear regression formula (2.26) holds. But some problems arise because, in general:

(1) the Zadeh extension product of triangular fuzzy numbers is not a triangular fuzzy number;
(2) the left and right spreads of the product of symmetric fuzzy numbers are not equal;
(3) there is not a general simple formula for the spreads of Y.

In the next section we propose a way to overcome those difficulties by proposing alternative fuzzy operations to those based on the extension principle.

3 Alternative Fuzzy Operations and Fuzzy Regression

Let us consider the total fuzzy regression model with fuzzy coefficients and fuzzy parameters:

$$Y = K_1 X_1 + K_2 X_2 + \ldots + K_n X_n, \qquad (3.1)$$

where $K_i \sim [c_i, s_i, s'_i]$ and $X_i \sim [x_i, t_i, t'_i]$ are fuzzy numbers.

3.1 Some Problems and Drawbacks of the Zadeh Extension Fuzzy Regression

If, as usually happens, the addition and the multiplication are the Zadeh's extension operations, then ([BB], [BF], [BG], [DP2], [GM], [MA1], [MA2]) the left and right spreads of the Zadeh's extension product $u \cdot_z v$ of two fuzzy numbers u and v have simple formulae if the factors are positive fuzzy numbers (at most one of the factors can be c-positive). In this case the formulae of core and r-cut spreads of $u \cdot_z v$ are [MA1]:

$$c(u \cdot_z v) = c(u)\, c(v) \qquad (3.2)$$

$$\forall r \in [0, 1), L^r(u \cdot_z v) = u_\lambda{}^1 L^r(v) + v_\lambda{}^1 L^r(u) - L^r(u)L^r(v); \qquad (3.3)$$

$$\forall r \in [0, 1), R^r(u \cdot_z v) = u_\rho{}^1 R^r(v) + v_\rho{}^1 R^r(u) + R^r(u)R^r(v). \qquad (3.4)$$

In particular, if u = [c(u), L(u), R(u)], v = [c(v), L(v), R(v)] are triangular fuzzy numbers, previous formulae (3.3) and (3.4) reduce to:

$$\forall r \in [0, 1),\ L^r(u \cdot_z v) = c(u)\, L^r(v) + c(v)\, L^r(u) - L^r(u)L^r(v); \tag{3.5}$$

$$\forall r \in [0, 1),\ R^r(u \cdot_z v) = c(u)\, R^r(v) + c(v)\, R^r(u) + R^r(u)R^r(v). \tag{3.6}$$

Some consequences are;

(a) the product of two non-degenerate triangular fuzzy numbers is not a triangular fuzzy number;
(b) the product of two non-degenerate symmetric fuzzy numbers is not a symmetric fuzzy number;
(c) the left and right spreads of the product depends not only by the spreads of the factors, but they are strongly increasing with the increase of the cores of the factors;
(d) the Zadeh's extension multiplication is subdistributive with respect to the Zadeh extension addition, i.e., for every fuzzy numbers u, v, w, we have:

$$(u + v)\, w \subseteq u\, w + v\, w, \qquad \text{(subdistributivity)} \tag{3.7}$$

where $\subseteq$ denotes inclusion between fuzzy sets. Equality in (3.7) holds if and only if u and v are both positive or both negative fuzzy numbers or w is a degenerate fuzzy number.

From previous formulae it follows that if $K_i \sim [c_i, s_i, s_i]$ and $X_i \sim [x_i, t_i, t_i]$ are positive, simple, and symmetric fuzzy numbers, then the spreads of Y are:

$$L(Y) = \Sigma_i\, (c_i\, t_i + x_i\, s_i - s_i\, t_i);\ R(Y) = \Sigma_i\, (c_i\, t_i + x_i\, s_i + s_i\, t_i), \tag{3.8}$$

It is worth noting that formula (3.8) reduces to (2.16) if every t_i is null and every x_i is positive.

The drawback (a) can be overcome by replacing the Zadeh's extension multiplication $u \cdot_z v$ with the approximate multiplication $u \cdot_a v$, defined by formulae:

$$c(u \cdot_a v) = c(u)\, c(v) \tag{3.9}$$

$$\forall r \in [0, 1),\ L^r(u \cdot_a v) = u_\lambda{}^1\, L^r(v) + v_\lambda{}^1\, L^r(u) - L^r(u)L^r(v)/(1\text{-}r); \tag{3.10}$$

$$\forall r \in [0, 1),\ R^r(u \cdot_a v) = u_\rho{}^1\, R^r(v) + v_\rho{}^1\, R^r(u) + R^r(u)R^r(v)/(1\text{-}r). \tag{3.11}$$

If u and v are triangular fuzzy numbers, then $u \cdot_a v$ is a triangular fuzzy number having the same core and the same spreads of $u \cdot_z v$. The limits of the approximation can be highlighted by a comparison of formulae (3.5), (3.6) with (3.10), (3.11).

The drawback (c) deserves careful consideration. Suppose the values of the independent variables are positive real numbers. If we change the origin of axes, i.e., increasing the value assumed by each variable of a positive number h, it would seem logical to expect an increase in the core of Y but not of left and right spreads of Y. It might not be appropriate, at least in some cases, define an addition and a multiplication in which spreads only depend on spreads of factors and the multiplication is distributive with respect to the addition?

A solution is given by the "bounded operations". In terms of spreads notation, the b-addition is defined by the formulae:

$$C(u +_b v) = C(u) + C(v); \tag{3.12}$$

$$\forall r \in [0, 1),\ L^r(u +_b v) = \max\{L^r(u), L^r(v)\};\ \ R^r(u +_b v) = \max\{R^r(u), R^r(v)\}. \tag{3.13}$$

Moreover, the b-multiplication is defined by the formulae:

$$C(u \cdot_b v) = C(u) \cdot C(v); \tag{3.14}$$

$$\forall r \in [0, 1),\ L^r(u \cdot_b v) = \max\{L^r(u), L^r(v)\};\ \ R^r(u \cdot_b v) = \max\{R^r(u), R^r(v)\}. \tag{3.15}$$

Some important properties of b-addition and b-multiplication are [MA1]:

(B1) b-sum and b-product of two trapezoidal fuzzy numbers are trapezoidal fuzzy numbers. Moreover, b-sum and b-product of simple fuzzy numbers are simple fuzzy numbers.

(B2) b-addition and b-multiplication are associative, commutative, and have neutral elements 0 and 1, respectively.

(B3) b-multiplication is subdistributive with respect to the b-addition. That is, for every fuzzy numbers u, v, w, we have

$$(u +_b v) \cdot_b w \subseteq u \cdot_b w +_b v \cdot_b w. \tag{3.16}$$

The equality holds iff $C(u)\, C(v) \geq 0$ or $C(w)$ is a real number.

(B4) The set Δ of triangular fuzzy number is closed with respect to b-addition and b-multiplication. Moreover, in Δ b-multiplication is distributive with respect to b-addition.

(B5) (invariance for translation) for every real numbers (i.e., degenerate fuzzy numbers) h, k, if $u' = u + h$, $v' = v + k$, then:

$$\forall r \in [0, 1),\ L^r(u' +_b v') = L^r(u +_b v);\ \ R^r(u' +_b v') = R^r(u +_b v) \tag{3.17}$$

$$\forall r \in [0, 1),\ L^r(u' \cdot_b v') = L^r(u \cdot_b v);\ \ R^r(u' \cdot_b v') = R^r(u \cdot_b v) \tag{3.18}$$

Thus, unlike the Zadeh's extension operations, the b-product of triangular numbers is a triangular number, the distributive property of the b-multiplication w. r. to the b-addition holds in Δ and finally (3.17) and (3.18) imply that a change of the origin of axes do not change the spreads.

An extension of the bounded operations are the $\oplus$-operations, introduced in [MA1], where $\oplus$ is a t-conorm, i.e., an operation on the interval [0, 1], $\oplus$: $(a, b) \in [0, 1] \times [0, 1] \to a \oplus b \in [0, 1]$ associative, commutative, having 0 as neutral element and increasing with respect to every variable (see, e.g., [SU], [SV], [WE], [KY]).

We assume there exist two strictly positive real numbers, L_{max} and R_{max}, the maximum left and right spreads, respectively. Let S be the set of fuzzy numbers such that, for every $u \in S$, $L(u) \leq L_{max}$, and $R(u) \leq R_{max}$.

We define the ⊕-addition on S by formulae:

$$C(u +_{\oplus} v) = C(u) + C(v); \tag{3.19}$$

$$\forall r \in [0, 1),\ L^r(u +_{\oplus} v) = [(L^r(u)/L_{max}) \oplus (L^r(v)/L_{max})]\ L_{max}; \tag{3.20}$$

$$\forall r \in [0, 1),\ R^r(u +_{\oplus} v) = [(R^r(u)/R_{max}) \oplus (R^r(v)/R_{max})]\ R_{max}. \tag{3.21}$$

The ⊕-multiplication on S is defined by:

$$C(u \cdot_{\oplus} v) = C(u) \cdot C(v); \tag{3.22}$$

$$\forall r \in [0, 1),\ L^r(u \cdot_{\oplus} v) = [(L^r(u)/L_{max}) \oplus (L^r(v)/L_{max})]\ L_{max}; \tag{3.23}$$

$$\forall r \in [0, 1),\ R^r(u \cdot_{\oplus} v) = [(R^r(u)/R_{max}) \oplus (R^r(v)/R_{max})]\ R_{max}. \tag{3.24}$$

By previous definitions it follows:

(C1) The ⊕-addition and ⊕-multiplication are associative, commutative, having neutral elements 0 and 1, respectively.

(C2) If ⊕ is the fuzzy union, then the ⊕-addition and the ⊕-multiplication reduces to the bounded operations.

(C3) The left and right r-cut spreads of the sum $u +_{\oplus} v$ and the product $u \cdot_{\oplus} v$ are not greater than L_{max} and R_{max}, respectively.

(C4) the invariance for translations holds.

4 Remarks on Some Critical Points of the Variable-Oriented Research and Conclusions

The variable-oriented research presents some critical points (see e.g., [BR1], [BR2], [RA]). Among these are the following assumptions:

(1) homogeneity of the cases;
(2) a linear relationship among the variables;
(3) the additivity of the outcomes with respect to the variables input;
(4) the necessity and sufficiency of the causes for the outcomes.

The study of social phenomena based on *diversity-oriented research* put in evidence that these assumptions are often not justified by the evidence.

In fact, if the cases belong to different types (to define with suitable procedures) then the following circumstances may occur:

(a) the same causes give different outcomes;
(b) different causes may yield the same outcome;
(c) for some types a cause (or the intersection of a set of causes) is sufficient to produce an outcome, for other types it is not sufficient;
(d) for some types a cause (or the intersection of a set of causes) is necessary to produce an outcome, for other types it is not necessary;
(e) the aggregation of causes to obtain a sufficient condition is superadditive;
(f) the aggregation of necessary but not sufficient causes is subadditive.

Moreover, often a set of causes produces an outcome only if they exceed a certain level (or a real, positive, increasing w. r. to every variable, function of the levels of the causes is greater than a positive real number h). These situations can be formalized in terms of fuzzy sets. If $K = \{k_1, k_2, \ldots, k_p\}$ is a set of causes and S is a population, the level of the cause k_i on S can be defined as a fuzzy set $\mu_i : x \in S \rightarrow [0, 1]$. If $h_i \in (0, 1]$ is the level at which the cause produces effect, then we have outcomes only in the h_i-cut of μ_i.

It follows a fundamental role of fuzzy sets and their aggregation to obtain necessary and/or sufficient conditions for outcomes. This is an alternative approach to the linear regression. This approach takes into account the complexity of the phenomena, i.e., the various features of each case study. In this frame of reference the relations between causes and effects are not linear, dependent on the diversity of cases, the level at which the cause produces effect on the elements of the population and so on. Insights into these aspects are in [BR1], [BR2], [RA].

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Multiobjective Decision-Making, de Finetti Prevision and Fuzzy Prevision

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Abstract. We introduce an approach to multiobjective decision-making in the context of finite de Finetti random numbers. The objectives are events, the action of an alternative A with respect to an objective O is seen as a finite de Finetti conditional random number A/O and the global effect of A over O, i.e., the score of A with respect to O, is the de Finetti prevision of A/O. Coherence conditions are investigated. Furthermore criteria for aggregating scores are defined. Besides, more generally, some concepts on fuzzy prevision are introduced and coherence conditions are investigated.

Keywords: multiobjective decision-making, coherent prevision, fuzzy prevision, aggregation of scores.

1 Introduction and Motivation

A classical multiobjective decision-making model is based on a structure $(\mathbf{A}, \mathbf{O}, \mathbf{w}, \mathbf{s})$, where:

- $\mathbf{A}$ is the *set of the alternatives*;
- $\mathbf{O}$ is the *set of the objectives*;
- $\mathbf{w} : \mathbf{O} \to [0,1]$, the *weight function*, is a function that measures the weight, i.e., the importance, of the objectives;
- $\mathbf{s} : \mathbf{A} \times \mathbf{O} \to [0,1]$, the *score function*, is a function that, for every $A \in \mathbf{A}$ and $O \in \mathbf{O}$, measures the score of A with respect to O, i.e., to what extent the alternative A satisfies the objective O.

From now on we consider the case in which $\mathbf{A}$ and $\mathbf{O}$ are finite. Then we assume $\mathbf{A} = \{A_1, A_2, ..., A_m\}$, $\mathbf{O} = \{O_1, O_2, ..., O_n\}$. In this case $\mathbf{w}$ is represented by a vector $W = (w_1, w_2, ..., w_n)$ and $\mathbf{s}$ by a matrix $S = (s_{ij})$, where s_{ij} measures the degree in which the alternative A_i satisfies the objective O_j. The rows of S are vectors associated to the alternatives and the columns are associated to the objectives.

Many authors, especially when adopting AHP procedure [17], [6], [8], [9], [10] assume the normality conditions:

$$w_1 + w_2 + ... + w_n = 1, \tag{1}$$

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$$\forall j \in \{1,2,...,n\}, s_{1j} + s_{2j} + ... + s_{mj} = 1. \tag{2}$$

A classical procedure to obtain the global score $s(A_i)$ of the alternative A_i is as follows:

$$s(A_i) = w_1 s_{i1} + w_2 s_{i2} + ... + w_n s_{in}. \tag{3}$$

The preferred alternative is that having the maximum global score.

We here emphasize that the role of objectives and alternatives is similar to that of the events in the de Finetti subjective probability [4], [5], [3], [11]. Then let us extend the de Finetti terminology to the decision-making problem. In particular, a family of objectives (resp. alternatives) pairwise disjoint and exhaustive is said to be "a partition of the certain event Ω".

We assume there exist two finite partitions of the certain event Ω:

- U, called the set of microobjectives;
- V, called the set of microalternatives;

such that every objective O_j is a union of elements of U, and every alternative A_i is a union of elements of V.

But, as in the de Finetti subjective probability, every partition of the certain event is temporary, because every objective (resp. every alternative) can be partitioned into subobjectives (resp. subalternatives), i.e., for every partition of the certain event Ω we can consider a finer one.

In such a framework, an assessment of weights to a family of objectives (resp. alternatives) is similar to an assessment of probability to a set of events, and coherence conditions must be considered.

Then the weight function $\mathbf{w} : O_j \in \mathbf{O} \to w_j$ can be seen as a probability assessment on $\mathbf{O}$. The condition (1) follows by the hypothesis that $\mathbf{O}$ is a finite partition of the certain event Ω and $\mathbf{w}$ is a coherent probability assessment [4], [3].

Similarly, the function $\mathbf{s} : (A_i, O_j) \in \mathbf{A} \times \mathbf{O} \to s_{ij}$ plays the role of a conditional probability assessment, where s_{ij} is the probability of the conditional event A_i/O_j. The conditions (2) are consequences of the hypothesis that $\mathbf{A}$ is a finite partition of the certain event and $\mathbf{s}$ is coherent [4], [5], [3].

As a consequence, $s(A_i)$ plays the role of the probability of the event A_i and (3) is a well-known formula of the probability.

If $\mathbf{O}$ (or $\mathbf{A}$) is not a partition of the certain event, coherence conditions different from (1) (resp. (2)) hold, dependent on the logical conditions among the events O_j (resp. the alternatives A_i). These conditions reduce to the existence of solutions of suitable linear systems.

In this paper a different point of view about the scores of the alternatives with respect to the objectives is considered. The effect of an alternative A_i over an objective O_j is represented by a function Φ_{ij} defined on a finite partition of O_j, with values in the interval $[0,1]$ of the real numbers.

Using the de Finetti notation, if every subobjective is called "event", such a function Φ_{ij} can be seen as a finite conditional random number, where O_j is the conditioning event. Then the score s_{ij}, that measures to what extent the alternative A_i satisfies the objective O_j, is seen as the conditional prevision of the conditional random number Φ_{ij}.

The rest of this paper is organized as follows: Section 2 considers a reformulation of the multiobjective decision-making model in terms of finite random numbers and their previsions. In Section 3, coherence conditions of previsions are investigated. Section 4 introduces a fuzzy extension of the concept of prevision, and weaker coherence conditions. Finally, in Section 5 conclusions and recommendations are presented.

2 A Reformulation of the Multiobjective Decision-Making Model in Terms of de Finetti Previsions

Let us recall the de Finetti random number concept and some fundamental results.

Definition 1. *A* de Finetti random number *is a function $X : \Pi \to R$, where Π is a partition of the certain event Ω and R is the set of real numbers.*

Let $Im(X) = \{X(x), x \in \Pi\}$ be the *range* (or *image*) of X. For every $y \in Im(X)$, $X^{(-1)}(y)$ denotes the union of the events $x \in \Pi$ such that $X(x) = y$.

Let $\Pi^* = \{X^{(-1)}(y), y \in Im(X)\}$. The de Finetti random number $X^* : \Pi^* \to R$ such that, $\forall z \in \Pi^*$, $X^*(z) = y \Leftrightarrow X^{(-1)}(y) = z$, is said to be the *reduced form* of X.

From now on, with "random number" we intend a de Finetti random number.

If the range of X is contained in $\{0,1\}$, then X reduces to the characteristic function of the event $X^{(-1)}(1)$ and usually it is identified with this event.

Two random numbers X and Y are said to be *equivalent*, we write $X \sim Y$, if they have the same reduced form, i.e., $X^* = Y^*$. Usually two equivalent random numbers are considered to be equal, and then, in the sequel, we follow such a convention.

A random number $X : \Pi \to R$ is said to be *bounded* (resp. *finite*) if its range is bounded (resp. finite).

If X is finite, $\Pi = \{E_1, E_2, ..., E_n\}$, and $X(E_i) = a_i$, we write:

$$X = a_1E_1 + a_2E_2 + ... + a_nE_n. \quad (4)$$

Definition 2. *Let S be a set of bounded random numbers. A function $P : S \to R$ is said to be a* de Finetti prevision *(briefly a* prevision*) on S if the following conditions are satisfied:*

P1 *for every $X \in S$, $inf(X) \leq P(X) \leq sup(X)$;*
P2 *if $X, Y, X+Y \in S$, then $P(X+Y) = P(X) + P(Y)$;*
P3 *if $a \in R, X \in S, aX \in S$, then $P(aX) = aP(X)$.*

A prevision P on S is said to be coherent *if there exists a prevision P^* on $V(S)$, the vector space generated by S, extension of P, i.e., such that $\forall X \in S$, $P^*(X) = P(X)$.*

Previous definition reduces to the one of coherent probability if the elements of S are events. If $S = \{X_1, X_2, ..., X_m\}$ is finite and every X_i is finite, the coherence conditions on S can be obtained by extensions of the ones on the coherence conditions on a probability assessment given, e.g., in [3], as follows.

Let

$$X_i = \sum_{r_i=1}^{n_i} a_{ir_i} E_{ir_i},$$

where the events E_{ir_i} are the elements of the domain Π_i of X_i and the real numbers a_{ir_i} are their images.

The *atoms* (or *constituents*) of S are all the nonempty intersections $A_1 \cap A_2 \cap \ldots \cap A_m$, with $A_i \in \Pi_i$. Let $C = \{C_1, C_2, ..., C_k\}$ be the set of atoms. Then we have:

$$X_i = \sum_{j=1}^{k} b_{ij} C_j,$$

where

$$C_j \subseteq E_{ir_i} \Rightarrow b_{ij} = a_{ir_i}.$$

The following theorem holds:

Theorem 1. *The prevision P on S is coherent if and only if there exists a solution of the system:*

$$\begin{cases} \sum_{j=1}^{k} b_{ij} x_j = P(X_i), \forall i \in \{1, 2, ..., m\}; \\ \sum_{j=1}^{k} x_j = 1; \\ x_j \geq 0, \forall j \in \{1, 2, ..., k\}. \end{cases} \tag{5}$$

An extension of the concept of de Finetti random number is the one of conditional de Finetti random number.

Definition 3. *Let H be a non impossible event. A* random number conditioned *to H is a function $X : \Pi_H \to R$ where Π_H is a partition of the event H and R is the set of real numbers. If $\Pi_H = \{E_1, E_2, ..., E_n\}$ is finite and $X(E_i) = a_i$, we write:*

$$X = a_1 E_1/H + a_2 E_2/H + ... + a_n E_n/H. \tag{6}$$

Remark 1. If the range of X is contained in $\{0, 1\}$, then X reduces to the conditional event $X^{(-1)}(1)/H$. Moreover, if $H = \Omega$, then X reduces to a (unconditional) random number.

Definition 4. *Let S be a set of bounded random numbers conditioned to H. A function $P : S \to R$ is said to be a* conditional prevision *on S if the conditions P1, P2, and P3 hold. Moreover, a conditional prevision P on S is* coherent *if there exists a conditional prevision P^* on $V(S)$, the vector space generated by S, extension of P, i.e., such that $\forall X \in S$, $P^*(X) = P(X)$.*

Let $X = a_1 E_1/H + a_2 E_2/H + ... + a_n E_n/H$ be a random number conditioned to H. If $H \neq \Omega$, for every real number a we consider the (unconditional) random number:

$$X^a = a_1 E_1 + a_2 E_2 + ... + a_n E_n + a H^c,$$

where H^c is the contrary of H. Let us call (unconditional) *random number associated* to X the random number X^0.

Remark 2. If $p(H) > 0$, for every conditional event E_i/H, $p(E_i \cap H/H) = p(E_i \cap H)/p(H)$. Then

$$P(X) = P(X^0)/p(H). \tag{7}$$

Let us remark that if $p(H) = 0, H \neq \emptyset$, we can consider the conditional prevision $P(X)$ [4], [3], [5] but not previous formula.

Here we propose the following algorithm for a coherent reformulation of a multiobjective decision-making problem in terms of previsions.

step 1 the decision maker assesses strictly positive weights w_j to the events O_j, with the coherence condition (1);

step 2 for every pair $(A_i, O_j) \in \mathbf{A} \times \mathbf{O}$ the decision maker assigns a function Φ_{ij} defined on a finite partition $\Pi_{ij} = \{E_{ij}^r, r \in \{1,2,...,h_{ij}\}$ of O_j having values in the interval $[0,1]$ of the real numbers, where, for every $E \in \Pi_{ij}$, $\Phi_{ij}(E)$ denotes the extent to which the alternative A_i satisfies the aspect (facet, point of view) E of the objective O_j; every Φ_{ij} is seen as a random number conditioned to O_j and Φ_{ij}^0 is the associated (unconditional) random number;

step 3 the decision maker synthesizes Φ_{ij} with a real number $s_{ij} \in [0,1]$, that represents the prevision of A_i conditioned to O_j;

step 4 Φ_{ij}^0 is interpreted as the weighed score of the alternative A_i with respect to the various aspects of the objective O_j and null in O_j^c; from (7) its prevision $P(\Phi_{ij}^0)$ is the product $s_{ij}w_j$;

step 5 we verify whether the function $P : \{\Phi_{ij}^0,\ i \in \{1,...,m\}, j \in \{1,2,...,n\}\} \to s_{ij}w_j$, is a coherent prevision or not;

step 6 if the coherence conditions on P are satisfied, then formula (3) gives global scores of the alternatives, i.e., their previsions, otherwise, if P is not a coherent prevision, then the decision maker must change his/her conditional prevision assessment $\{s_{ij}, i = 1,2,...,m, j = 1,2,...,n\}$.

Since the events O_j are pairwise disjoint, for every $r,s \in \{1,2,...,n\}$, $r \neq s$, the vector space generated by de Finetti random numbers $\Phi_{ir}^0, i \in \{1,2,...,m\}$ is orthogonal to the one generated by the de Finetti random numbers Φ_{is}^0, $i \in \{1,2,...,m\}$. This implies that the prevision P is coherent if and only if for every $j \in \{1,2,...,n\}$ the restriction P_j of P to the set $\{\Phi_{ij}^0,\ i \in \{1,...,m\}\}$ is coherent.

For every $j \in \{1,2,...,n\}$, let $C^j = \{C_1^j, C_2^j, ..., C_{k_j}^j\}$ be the set of atoms of the de Finetti random numbers $\Phi_{ij}^0, i \in \{1,2,...,m\}$, i.e., the nonvoid intersections $\bigcap_{i=0}^m A_{ij}$, where $A_{ij} \in \Pi_{ij}^* = \Pi_{ij} \cup \{O_j^c\}$.

Then, for every $j \in \{1,2,...,n\}$ we have:

$$\Phi_{ij}^0 = \sum_{s=1}^{k_j} b_{ijs} C_s^j,$$

where

$$C_s^j \subseteq E \in \Pi_{ij}^* \Rightarrow b_{ijs} = \Phi_{ij}^0(E).$$

From theorem 1, for every $j \in \{1,2,...,n\}$, the assessment of previsions $s_{ij}w_j$ is coherent if there are solutions of the system:

$$\begin{cases} \forall i \in \{1,2,...,m\}, \sum_{s=1}^{k_j} b_{ijs} x_s = s_{ij} w_j; \\ \sum_{s=1}^{k_j} x_s = 1; \\ x_s \geq 0, \forall s \in \{1,2,...,k_j\}. \end{cases} \tag{8}$$

If every de Finetti random number Φ_{ij} conditioned to the objective O_j reduces to a conditional event E_{ij}/O_j and the set of the events $E_{ij}, i \in \{1,2,...,m\}$ is a partition of O_j, then the atoms are the events E_{ij} and the event $E_{m+1j} = O_j^c$. In this case the system (8) reduces to formula (2).

3 Decision-Making Models with General Logical Relations among the Objectives

Here we consider the general case in which the set of objectives $\mathbf{O} = \{O_1, O_2, ..., O_n\}$ does not form a partition of the certain event.

Then one of the following cases holds:

- the objective are not exhaustive, i.e., their union is not the certain event Ω;
- the objective are not pairwise disjoint.

In the first case let $O^* = \cup_{i=1}^n O_i$. We introduce as further objective, called the *residual objective*, the complement of O^*, i.e. the event $O_{n+1} = (O^*)^c$.

The set $\hat{\mathbf{O}} = \mathbf{O} \cup \{O_{n+1}\}$ is exhaustive. Then, if we replace the set $\mathbf{O}$ with the set $\hat{\mathbf{O}}$ we reduce to the case in which the objectives are exhaustive. If we do not want to attribute any importance to the residual objective, we simply assume that its weight is zero. This amounts to treating the set of the objectives $\mathbf{O}$ as if they were exhaustive. So whatever point of view considered, we can always confine ourselves to the case where the set $\mathbf{O}$ of objectives is exhaustive.

If the objectives belonging to $\mathbf{O}$ are pairwise disjoint, then the decision-making problem reduces to the one considered in the previous Sections. If the objectives are not disjoint, then the coherence conditions on the assessment of weights w_j are different from the formula (1).

Let $C = \{C_1, C_2, ..., C_k\}$ be the set of atoms of the objectives, i.e., the nonempty intersections $\cap_{j=1}^n A_j$, were $A_j \in \{O_j, O_j^c\}$.

The assessment of weights w_j is coherent if and only if there is a solution of the following system:

$$\begin{cases} \forall j \in \{1,2,...,n\}, \sum_{s=1}^k a_{js} x_s = w_j; \\ \sum_{s=1}^k x_s = 1; \\ x_s \geq 0, \forall s \in \{1,2,...,k\} \end{cases} \tag{9}$$

where:

$$a_{js} = \begin{cases} 1, \text{ if } C_s \subseteq O_j; \\ 0, \text{ otherwise.} \end{cases}$$

Once we have assigned weights to the objectives we have to assign the conditional previsions $s_{ij}, i \in \{1,...,m\}, j \in \{1,2,...,n\}$. Moreover, we must verify the coherence of the previsions $s_{ij} w_j$, $i \in \{1,...,m\}$, $j \in \{1,2,...,n\}$.

Let $C = \{C_1, C_2, ..., C_k\}$ be the set of atoms of all the de Finetti random numbers Φ_{ij}^0, i.e., the nonempty intersections $\bigcap_{ij} A_{ij}$ where $A_{ij} \in \Pi_{ij}^* = \Pi_{ij} \cup \{O_j^c\}$.

We have:

$$\Phi_{ij}^0 = \sum_{s=1}^{k} b_{ijs} C_s,$$

where

$$C_s \subseteq E \in \Pi_{ij}^* \Rightarrow b_{ijs} = \Phi_{ij}^0(E).$$

The assessment of previsions $s_{ij}w_j$ is coherent if there are solutions of the system:

$$\begin{cases} \sum_{s=1}^{k} b_{ijs} x_s = s_{ij} w_j, \\ \forall i \in \{1,2,...,m\}, j \in \{1,2,...,n\}; \\ \sum_{s=1}^{k} x_s = 1; \\ x_s \geq 0, \forall s \in \{1,2,...,k\}. \end{cases} \tag{10}$$

If the assessment of previsions $s_{ij}w_j$ is not coherent, then the decision maker must modify the numbers s_{ij} in order to obtain the coherence. If the assessment of previsions $s_{ij}w_j$ is coherent, then the next step is to decide the criterion to aggregate, for every alternative A_i, the previsions of the pairs $(A_i, O_j), j \in \{1,2,...,n\}$ in order to obtain the global score of A_i.

We must emphasize that, for each criterion that the decision maker wants to choose, there are more or less explicit assumptions that lead to prefer that over other possible criterion.

A usual choice is to aggregate the scores of the alternatives using the formula (3). This is acceptable if the decision maker is aware that, in this way, the score of each constituent is counted as many times as there are objectives in which the constituent is contained. If the decision maker believes that this assumption is correct for the decision problem under discussion, then it is right to use the formula (3).

We emphasize that from formula (3) and system (10) the global score of the alternative A_i is the number:

$$s(A_i) = \sum_{j=1}^{n} s_{ij} w_j = \sum_{s=1}^{k} [\sum_{j=1}^{n} b_{ijs}] x_s. \tag{11}$$

This means that the score assigned to the atom C_s is:

$$s(C_s) = \sum_{j=1}^{n} b_{ijs}, \tag{12}$$

i.e., it is the sum of the scores of C_s with respect to the alternative A_i in all the objectives containing C_s and

$$s(A_i) = \sum_{s=1}^{k} s(C_s) x_s. \tag{13}$$

There are many other criteria to assess the scores of atoms. For instance, if the decision maker wants the score of each constituent contained in at least an objective is counted only once in the aggregation of the scores of each alternative, he/she can assume the score of the atom C_s with respect to the alternative A_i is:

$$s(C_s) = \max_{j=1}^{n} b_{ijs}. \tag{14}$$

Of course there are many other possible formulae for $s(C_s)$. Precisely, we can assume:

$$s(C_s) = f(b_{i1s}, b_{i2s}, ..., b_{ins}), \tag{15}$$

where f is a non-negative real function, defined in $[0,1]^n$, null in $(0,0,...,0)$, continuous, symmetric respect to every pair of variables, and increasing respect to every argument. For instance, the operation of "sum" or of "max" can be replaced by an Archimedean t-conorm (for definition of t-conorm, see, e.g., [7]).

We emphasize that, if the formula (12) holds, then the value $s(A_i)$ in formula (13) is independent from the solution $(x_1, x_2, ..., x_k)$ considered of the system (10). On the contrary, if a different formula is adopted for $s(C_s)$, then the value $s(A_i)$ depends on $(x_1, x_2, ..., x_k)$, and the set of values $s(A_i)$ is a closed interval $[m_i, M_i]$ of the real line.

Of course m_i is obtained when $(x_1, x_2, ..., x_k)$ is a solution P_i of the mathematical programming problem:

$$\min s(A_i) = \sum_{s=1}^{k} f(b_{i1s}, b_{i2s}, ..., b_{ins})\, x_s, \tag{16}$$

with the constraints given by system (10).

Similarly M_i is obtained when $(x_1, x_2, ..., x_k)$ is a solution Q_i of the mathematical programming problem:

$$\max s(A_i) = \sum_{s=1}^{k} f(b_{i1s}, b_{i2s}, ..., b_{ins})\, x_s, \tag{17}$$

with the constraints given by system (10).

We propose, below, to assume that $s(A_i)$ is a suitable triangular fuzzy number. For definitions and results on fuzzy numbers, see, e.g., [21], [22], [7], [20].

It seems natural to assume the support of $s(A_i)$ is the closed interval $[m_i, M_i]$. In order to define the core $c(A_i)$ of the fuzzy number $s(A_i)$, we propose to consider the simplex $S = [P_i, Q_i, i \in \{1,2,...,m\}]$ generated by the vertices $P_i, Q_i, i \in \{1,2,...,m\}$. This simplex is contained in the set S^* of all the solutions of the system (10), that is also a simplex.

Let $G = (x_1^g, x_2^g, ..., x_k^g)$ be the barycenter of S. G belongs to S and it seems reasonable to assume that the core of $s(A_i)$ is the value of the formula (13) when $(x_1, x_2, ..., x_k) = G$. So we propose $s(A_i)$ is the triangular fuzzy number $(m_i, c(A_i), M_i)$ where

$$c(A_i) = \sum_{s=1}^{k} f(b_{i1s}, b_{i2s}, ..., b_{ins})\, x_s^g. \tag{18}$$

4 Decision-Making Model in Terms of Fuzzy Previsions

Let us introduce some possible extensions of the de Finetti prevision in the fuzzy ambit. The concept of prevision of de Finetti random numbers can be extended from many different points of view.

We consider two cases:

- *Random Numbers Extension* The de Finetti random numbers are replaced by random fuzzy numbers and a concept of fuzzy prevision is defined on sets of random fuzzy numbers (type RNE fuzzy prevision);
- *Fuzzy Measure Extension* The domain of a fuzzy measure considered, e.g., in [1], [2], [18], [19], [7] can be seen as a set of events. This point of view is considered in some papers of ours [12], [13]. Here we introduce a concept of fuzzy prevision as an extension of a fuzzy measure to a set of de Finetti random number (type FME fuzzy prevision).

The "random numbers extension" was pursued in some papers of ours (see, e.g., [14], [15], [16]). Here we limit ourself to the "fuzzy measure extension".

We introduce the following definition.

Definition 4: Let S be a family of de Finetti random numbers. We define *fuzzy prevision*, (of type FME), on S, any function $P : S \to R$ such that:

FP1 $\forall a, b \in R, \forall X \in S, a \leq X \leq b \Rightarrow a \leq P(X) \leq b$; (*mean property*)
FP2 $\forall X, Y \in S, X \leq Y \Rightarrow P(X) \leq P(Y)$. (*monotonicity*)

A fuzzy prevision P is said to be *coherent* if there exists a fuzzy prevision P^* extension of P on the vector space $V(S)$ generated by S .

Previous definition is very general and verifying coherence of an assessment of fuzzy previsions reduces to control the monotonicity.

A more interesting, and useful, point of view is considering a concept of decomposable fuzzy prevision as an extension of the one of decomposable fuzzy measure considered in [1], [2], [18] and in [19]. Some definitions, results, and applications to decision-making we have proposed in [13].

We introduce the following definition of additive generator on R, as a generalization of the concept of additive generator of an Archimedean t-conorm.

Definition 5: We define *additive generator* on R with base interval $[0, b_g]$ of $[0, +\infty]$ every function g defined in $[0, b_g]$, with codomain $[0, +\infty]$, and such that:

AG2 $g(0) = 0$;
AG3 g is strictly increasing and continuous.

Definition 5 permits us to extend the definition of pseudoinverse as follows:

Definition 6: Let g be an additive generator on R with base interval $[0, b_g]$. We define *pseudoinverse* of g the function $g^{(-1)}$, defined in $[0, +\infty]$, with codomain the base interval $[0, b_g]$ of g, and such that:

- $g^{(-1)}(y) = g^{-1}(y)$ if $y \in [g(0), g(b_g)]$;
- $g^{(-1)}(y) = b_g$ if $y > g(b_g)$.

Finally, we introduce the concept of g-operation as an extension of the one of Archimedean t-conorm.

Definition 7: Let g be an additive generator on R with base interval $I_g = [0, b_g]$. We define *operation generated* by g (or *associated* with g), we call it the *g-operation*, the operation $\oplus$ defined in the interval I_g as follows:

$$\forall a, b \in I_g,\ a \oplus b = g^{(-1)}(g(a) + g(b)). \tag{19}$$

We say that the g-operation $\oplus$ is:

- *strict*, if $g(b_g) = \infty$;
- *nonstrict*, if $g(b_g)$ is finite.

From (19) the following theorem follows:

Theorem 2: The g-operation $\oplus$ given by (19) is:

- increasing in each argument;
- associative;
- commutative;
- with 0 as neutral element.

In particular, if $b_g = 1$, then $\oplus$ reduces to an Archimedean t-conorm.

Now we can introduce a definition of *decomposable fuzzy prevision* as a generalization of the one of decomposable fuzzy measure.

Definition 8: Let g be an additive generator on R, with base interval $[0, b_g]$, and $\oplus$ the associate g-operation. Let S be a set of de Finetti random numbers with range contained in I_g and let P be a fuzzy prevision on S. We say that P is a $\oplus$-*decomposable fuzzy prevision* if

$$\forall X, Y \in S : XY = 0 \Rightarrow P(X+Y) = P(X) \oplus P(Y). \tag{20}$$

Remark: It is worth noting that if X and Y are incompatible events then $XY = 0 \Rightarrow X + Y = X \cup Y$; then the above definition reduces to that of Weber [19]. In our contest, the condition $XY = 0$ plays the role of the extension of the incompatibility between two events.

From previous definitions and theorems we can deduce the following theorem, that provide for an extension to the decomposable fuzzy previsions of the classification theorem 3.3 in [19] concerning decomposable fuzzy measures.

Theorem 3. Let g be an additive generator on R, with base interval $I_g = [0, b_g]$, and $\oplus$ the associate g-operation. Let S be a set of de Finetti random numbers with range contained in I_g and let P be a decomposable fuzzy prevision on S. Then for every pair (X,Y) of random numbers belonging to S such that $XY = 0$, we have:

A1 if $P(X+Y) < b_g$, then the following *additivity* holds:

$$g(P(X+Y)) = g(P(X)) + g(P(Y)); \tag{21}$$

A2 if $P(X+Y) = b_g$, then we have the *subadditivity*:

$$g(P(X+Y)) \leq g(P(X)) + g(P(Y)). \tag{22}$$

Let us use the notations introduced in Section 2 before Theorem 1. Then, from Theorem 3, we deduce the following coherence conditions for fuzzy previsions, that generalize Theorem 1.

Theorem 4: Let g be an additive generator on R, with base interval $I_g = [0, b_g]$, $b_g \geq 1$, and $g(1) \in R$, and let $\oplus$ be the associated g-operation. Let $S = \{X_1, X_2, ..., X_m\}$ be a set of random numbers with range contained in I_g. A $\oplus$-decomposable fuzzy prevision P

on S, such that $P(X_i) < b_g$ for all $X_i \in S$, is coherent if and only if there exists a solution of the system:

$$\begin{cases} \sum_{j=1}^{k} g(b_{ij}x_j) = g(P(X_i)), \forall i \in \{1,2,...,m\}; \\ b_g = 1 \Rightarrow \sum_{j=1}^{k} g(x_j) \geq g(1); \\ b_g > 1 \Rightarrow \sum_{j=1}^{k} g(x_j) = g(1); \\ x_j \geq 0, \forall j \in \{1,2,...,k\}. \end{cases} \tag{23}$$

If the function $P : \{\Phi_{ij}^0,\ i \in \{1,...,m\}, j \in \{1,2,...,n\}\} \rightarrow s_{ij}w_j$, is a coherent $\oplus$-decomposable fuzzy prevision, then it seems reasonable to assume the global scores of the alternatives is given by the formula:

$$s(A_i) = w_1 s_{i1} \oplus w_2 s_{i2} \oplus ... \oplus w_n s_{in}. \tag{24}$$

As in Sect. 3, if the objectives are not pairwise disjoint, formula (24) holds if and only if the decision makers want the score of each atom is counted as many times as there are objectives in which the atom is contained.

On the contrary, if the decision makers like a different aggregation criterion, formula (24) is to be replaced with a different formula, generalization of (13), where the addition is replaced by the operation $\oplus$. In this case, as we saw at the end of the previous section, in general, $s(A_i)$ is not a crisp number, but it is a triangular fuzzy number.

5 Conclusions

From the analysis undertaken in this work it follows that some possible benefits can be obtained using the models based on the de Finetti prevision or on the fuzzy prevision. In the first place, the assignment of estimates comes from a detailed analytical procedure, in which, for each pair (alternative A_i, objective O_j), all the different aspects of the objective O_j with respect to A_i are considered and the degree to which each aspect is satisfied if the alternative A_i is chosen.

In addition, the formulas considered simultaneously take into account the logical relations between the objectives, and the logical relationships between the alternatives. The consistency conditions ensure that the de Finetti or fuzzy previsions take into account the views and assessments of decision makers, the logical relations existing, and the rationale behind the aggregation criteria.

Finally, the consideration of fuzzy scores of alternatives appears to be a logical and inevitable consequence of the aggregation criteria. The uncertainty present in these scores is not a disadvantage, but, on the contrary, is a useful tool to highlight explicitly the uncertainty hidden and latent in the logic and procedure of the decision.

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Quantum Decision Making, Legal Complexity and Social Behavior

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Abstract. The relationship between Law (legal system) and Society can be and has been described in many different ways. Beyond the theoretical discussions, there lays reality, and it shows that decisions on legal policies are not neutral in terms of social impact. The legal system is composed by many instruments, institutes or institutions interacting among themselves—laws and other legal norms, including articles within them. It is clear that instruments, institutes or institutions, augment their number according to the appearance of new forms of relationship. Then, the Law is progressively composed by more and more instruments and there is no reason to expect them to be always working in a synchronized way. For this reason Law can be analyzed using complex systems tools, the 'Legal Complexity Notion'. Moreover, the idea of law is closely related to those of obedience and enforceability. However, a law is not only obeyed just because it is enforceable, but it is considered to express society's attitude toward a given situation. In other words, people are not likely to perform the behavior described in the law just because it is there. Leaving aside the area of compulsory rules—such as penal law-, where individuals cannot neglect the existence of such a norm, in the vast area of non-compulsory norms—such as contract law-, there are many laws in most legal systems that rule situations that are seldom chosen by the parties of a given business. In the present contribution; we analyze why individuals opt for some legal instruments which are not of compulsory application or do not choose them. One case is the kind of joint venture agreements—contract entailment among enterprises—businessmen decide to enter. Using the available data and a quantum decision-making model, we are able to describe, for the Argentinean case, why among the available typical joint venture regimes, the Temporary Union of Firms—UTE—is preferred by users rather than the Group of Collaborating Firms—ACE—or the Consortium of Cooperating Firms—CCE.

Keywords: Legal Complexity, Business Decision making, Nonlinear Spin Model.

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1 Introduction

The idea of law is closely related to those of obedience and enforceability. However, a law is not obeyed just because it is enforceable, but because it is considered to express society's attitude toward a given situation. In other words, people are not likely to perform the behavior described in the law just because it is there.

In the field of law, there are two different kinds of rules: a) Compulsory rules, such as administrative and penal law. In this area people are either forced to perform the prescribed conduct or punished if they do not so .b) Non-compulsory norms, such as contract law. In this area the individuals will preserve full autonomy. Parties can shape a voluntary system as they wish but, if they fail to forecast and rule any situation, then law comes in their help providing a subsidiary norm. We leave aside the area of compulsory rules—such as penal law—because individuals cannot neglect the existence of such a norm. In the vast area of non-compulsory norms; such as contract law—there are many laws in most legal systems that rule situations that are seldom chosen by the parties of a given business. Most of these laws are the answer lawmakers provided to rule some relationships they—lawmakers—either consider that need to be ruled or find suitable to solve problems they imagine businessmen face. Some of these are aimed to avoid the undesirable consequences in terms of liability that, in many legal systems, carry some 'atypical' contracts. The topic needs a brief explanation: everybody grants his debts with his whole personal assets. The limitation of liability—the most worthy wish of any individual next to make a contract—can only be declared by law and is applied in restricted situations. Therefore, sometimes people think that, being their contract very similar to another that does not carry joint and several liability, theirs shall work alike. And, usually too late, they are faced to the fact that a judge declares that it carries unlimited liability. In order to avoid this risk, some jurists develop magnificent legal engineering, which usually involves elaborated procedures on the side of the contract parties. In due time, these products of fine intellects are likely to become laws. And there they stay, remaining still for years if not forever. What has happened? There was a risk—the risk of acquiring joint and several liability; lawyers provided a mechanism designed to avoid it but those who were the alleged beneficiaries of that mechanism do not make use of it. This fact causes tremendous suffering to the specialists who write these legal bodies, such that sometimes they even acknowledge the fact in black and white [1] . Forty years later, the 'Fundamentals' of the Project of Civil and Commercial Code 2012 [2], which includes the contracts under analysis, shows that law-writers are tenacious in their arguments. Previous studies pose that the cause of this disregard of such allegedly beneficial rules lies on the discordance between the actual feelings and expectations of the prospective parties of businesses and those that are required to find satisfaction in providing one's businesses the shape such law offers [3], [4], [5]; shortly, actually making the contracts such law rules. Economy actors—businessmen (agents)- do not make contracts 'because' they are ruled in a law. In fact, we should rather say that they do not make contracts, they make businesses. And barely few of them actually know that those businesses are contracts. Fully exercising their free will, they make the businesses/contracts that suit their expectations and, in doing so, assume the consequent risk. Lawmakers should provide social and economic actors with laws that at the same time fulfill agents' expectations and avoid non-cooperative interactions with other laws within the legal system.

On the other side, the legal system—LAW—is composed by many instruments, institutes or institutions interacting among themselves. It is clear that instruments, institutes or institutions, augment their number. Then, the LAW is progressively composed by more and more instruments, generally defined at different social/economical/technological states, and for this reason they are not always working in a synchronized way. Each instrument/institute/institution is embedded in a more general assemblage of institutes that regulate a 'particular social activity' or social 'subsystem'. All these subsystems compose the legal frame of a given State. Let us call it the 'system'. Moreover, there are many conflictive situations that involve different legal subsystems (public law, private law, human rights). Moreover, most of them can only achieve a satisfactory solution when a final decision is taken by a third party, for example, an arbiter or a judge. This means that the different subsystems are not interacting in a simple way. From the physics point of view, when a set of subsystems interacting among themselves, lead to confused, unsolved scenarios, it is said that such interactions are nonlinear; therefore, we are facing a 'complex system'. Given the scenario described above, the introduction of the Legal Complexity notion has allowed us to prove that it is not merely by means of introducing new institutions in the existing legal system that the problems posed by social development shall find a satisfactory solution [6] .

In the present contribution; we analyze why individuals opt for some legal instruments which are not of compulsory application or do not choose them. One case is the kind of joint venture agreements—contract entailment among enterprises—businessmen decide to enter. Using the available data and a semiquantum model, we are able to describe, for the Argentinean case, why among the available typical joint venture regimes, the Temporary Union of Firms—UTE—is preferred by users rather than the Group of Collaborating Firms—ACE—or the Consortium of Cooperating Firms—CCE.

In order to model the above described situation, we consider appropriate to use a semiquantum nonlinear spin model, to fit the available data. We based our modelization in three facts: a) the necessity of having a yes-no description, given by the physical spin characteristics, b) to include the complexity of the legal system (included in the nonlinear nature of the model) and c) to include the uncertainty or risk in the decision process to adopt of a given contract.

1.1 Semiquantum Systems

The semiquantum approach is characterized by the description of a system that has quantum and classical degrees of freedom which are coupled [7,8,9,10]. A semiquantum system is composed by a quantum part, a classical part and a non-linear term in which a quantum and a classical variable are coupled. So, a semiquantum system may be described by a Hamiltonian which has the general form [7,9,10]

$$\hat{H} = \hat{H}_q + H_{cl} + H_{int}, \tag{1}$$

where $\hat{H}_q$ and H_{cl} represent the quantum and classical subsystems of the Hamiltonian, respectively, and H_{int} represents the interaction term. The purpose of the present model is to show that semiquantum, nonlinear spin systems, associated to the $SU(2)$ Lie algebra, are suitable to deal with the *yes-no* processes in opinion formation (see ref. [13] for more details). The quantum part of the system (the quantum subsystem) represents the "opinion-attitude-decision" taken by a group of individuals; meantime, the classical part of the system (the classical subsystem), which has a dissipative term, represents the "external conditions-social pressure-limit situation" under which the "opinion-attitude-decision" is taken.

Our main approach to the semiquantum systems problems is done by means of the Maximum Entropy Principle Approach (MEP) [7,8,11,12]. From MEP's point of view, the analysis of the semiquantum dynamics of a *yes-no* process in opinion formation is developed on a semiquantum phase space spanned by the variables $\langle\hat{\sigma}_x\rangle, \langle\hat{\sigma}_y\rangle, \langle\hat{\sigma}_z\rangle, q, p$, where the first 3 variables are the mean values of a complete set of quantum non-commuting observable ($CSNCO$) which close a partial semi Lie algebra under commutation with the Hamiltonian of the system and are the generators of the $SU(2)$ Lie algebra. These mean values correspond to the quantum degrees of freedom of the system; meanwhile, the q, p are the classical variables of the system [7]. It can be shown that all nonlinear spin Hamiltonians which close a Lie algerbra with the $SU(2)$ generators, exhibit a dynamical invariant: the generalized uncertainty principle (GUP) [10], which can be expressed in the very simple fashion

$$0 < \langle\hat{\sigma}_x\rangle^2 + \langle\hat{\sigma}_y\rangle^2 + \langle\hat{\sigma}_z\rangle^2 < 1, \tag{2}$$

this invariant enables us to fix properly the initial conditions of the system.

In our opinion, the MEP's point of view to tackle the semiquantum nonlinear $SU(2)$ systems, which mimics a *yes-no* process of opinion formation, takes advantage of three facts: 1) it is possible to describe the quantum degrees of freedom's dynamics in the dual space of Lagrange multipliers associated to the quantum observables [8] in a completely equivalent fashion as the Ehrenfest theorem does, 2) the GUP is an invariant of the motion (see Eq. (2)) for the $SU(2)$ semiquantum dynamic 3) the range of validity of this invariant (see Eq. (2)) together with the value of energy, $\left\langle\hat{H}\right\rangle$, at the time $t = 0$, will provide us with different sets of initial conditions $\{\langle\hat{\sigma}_x\rangle_{(0)}, \langle\hat{\sigma}_y\rangle_{(0)}, \langle\hat{\sigma}_z\rangle_{(0)}, q_{(0)}, p_{(0)}\}$ which allow for the elimination of any additional hypothesis for the wave function at $t = 0$. From here on, varying the value of the dynamic invariant (through the initial conditions) it will be possible to analyze the dynamics of the system in the process of opinion formation [13].

2 Contracts for Entrepreneurial Collaboration in Argentina

The designation of ‘Contracts for Entrepreneurial Collaboration’ is given in Argentina to a number of contracts that can easily be described as ‘joint ventures’. In fact these contracts are a family of the genre joint venture, the species being UTE (Unión Transitoria de Empresas, Temporary Union of Firms); ACE (Agrupación de Colaboración Empresaria, Group of Collaborating Firms) and CCE (Consorcios de Cooperación Empresaria, Consortium of Cooperating Firms). As it was said above, these contractual forms are exceptions of the ‘autonomy principle’, intended to avoid extensive liability. None of these contracts creates a legal entity—conventional person–. In 1983 a law introduced these contracts as a genre in the Argentinean legal system, which had been till that time reluctant to the inclusion of new typical contractual forms. The alleged reason for doing this was ‘the businessmen’s requirement’, as the Preliminary Recitals of Law 22903 express. It must be said that most of such solicitors were huge corporations that were actually signing joint ventures in which frame they risked having to pay for their co-contracting parts debts.

Among firms there are huge corporations but also small and medium enterprises—SME—, a vast universe of firms which sole common character is that they do not follow the logic of huge corporations, particularly, in the field of decision making. The lawmakers did not take into consideration the fact that the demand of a segment cannot be considered a requirement of the universe if the aim is to provide a tool that is suitable for all. These contracts demand either a high pay off or a collaborative attitude toward ‘colleagues’ and, particularly in those days—due to historical reasons regarding the sector’s formation—, SME were neither likely to make highly paid off businesses nor had the collaborative attitude. The direct consequence of this fact was that despite the quality of the regulation, SME were reluctant to make these contracts.

2.1 Contract Types

In the following paragraphs, we describe the different contract types regulated in Argentina.

Case 1: UTE (Unión Transitoria de Empresas, Temporary Union of Firms): Ruled by Law 22903 (1983), the Temporary Union of Firms, from now on UTE, is a contract firm whose aim is to join capacities to face some demand. Not a market demand, but a request of a given prospective contracting party. In the frame of this contract, each member—contracting part—provides his own part of the whole—either a service provision, works, delivery of goods, or whatever has been agreed. This particular form of typical joint venture requires little collaborative attitude.

Case 2: ACE (Agrupación de Colaboración Empresaria, Group of Collaborating Firms): The ACE (Group of Collaborating Firms) was also introduced in the legal system in 1983. The aim that underlies the celebration of this contract is, for the contracting firms, to do something in common, so as each one obtains it—either services, processes or goods–cheaper. The reduction of each firm's costs is caused by the sharing of fixed costs derived from this contract. The ACE demands a highly collaborative attitude.

Case 3: CCE (Consorcios de Cooperación Empresaria, Consortium of Cooperating Firms): This contractual alternative was introduced in the Argentinean legal system in January, 2005 by Law 26.005. The aim of the firms that come to sign this contract is to establish a common basis for their corporative behavior, in order to obtain better results for each participant's aim. Although there have been registered really scarce cases of this contract, the examples revised allow us to say that mayor corporations appear to be reluctant to sign CCE. This contract might be more suitable to grant its contracting parts the entrance in the market—something that in fact it is in a closely considered case—, than to work as a trust. In the Numerical Simulations Section, we plot the available data concerning the number of contracts signed on these three different modalities.

3 The Nonlinear Spin Model

As we said previously, we want to mimic a yes-no process in opinion formation by means of a $SU(2)$ nonlinear semiquantum Hamiltonian, so, our model given by Eq. (1) will always be able to be represented by a Hamiltonian of the form [13]

$$\hat{H} = B\ \hat{\sigma}_z + C\ q^n\ \hat{\sigma}_j + \frac{p^2}{2m} + V(q), \tag{3}$$

where n is a non-negative integer, $\hat{\sigma}_j$ stands for $\hat{\sigma}_x, \hat{\sigma}_y$ or $\hat{\sigma}_z$ and C is the coupling constant between the quantum and classical subsystems. The quantum subsystem has always the form $\hat{H}_q = B\ \hat{\sigma}_z$, where, $\hat{\sigma}_z$ is the $z-$component of a 1/2 spin particle and the B parameter is the external magnetic field (always parallel to $z-$direction) which obliges the $\hat{\sigma}_z$ spin's component to be aligned in the $z-$direction. So, in our model, the B parameter represents any mandatory statement which could come from authority or some "extreme event" and the quantum part of the system (quantum subsystem) $B\hat{\sigma}_z$ represents the "opinion-attitude-decision" assumed by a group of individuals, with respect to the alignment B parameter according to a given decision. The existence of two alternatives antagonist or not but, in any case, different, means that in order to change the decision yes-no (a mere convention, it could be no-yes) requires that the energy of the system relax (or dissipate) in order to some change in the supported position appears. This is logic as a permanent confrontation or social division in two positions cannot be sustained in time. The more natural way to introduce dissipation is by means of the classical subsystem [14] (via the parameter η) which has an *ad hoc* term, ηp, representing the "external-social pressure-limit situations" under which the "opinion-attitude-decision change" is taken. So, in our model, the yes-no process of collective opinion formation has to do with two kind of factors: **i)** the way in which the individuals interact between them (the potential $V(q)$) and **ii)** the way in which the individuals, as a whole, interact with their environment (external conditions, social pressure, limit situations).

On the other side, the classical part of the system (classical subsystem), $H_{cl} = \frac{p^2}{2m} + V(q)$, represents the collective of individuals which support another position different from that coming from B. In general, the term $\frac{p^2}{2m}$ represents individual motivations (individual interests, personal projects, etc.) and the potential $V(q)$ is the interaction or cohesion among individuals.

The main advantage of the MEP procedure (see refs. [7,8,9,10] and references therein) to deal with practical problems as in the present case, is its ability to translate a Hamiltonian model into a set of differential equations and, in this case, nonlinear differential equations of motion. The detailed analysis of semiclassical $SU(2)$ systems from the physics point of view is given in refs. [10,13] and references therein. Following the standard MEP procedure, the evolution equations of the classical variables q and p, are obtained through the mean value of the Hamiltonian $\left\langle \hat{H} \right\rangle$ [13] written as

$$\left\langle \hat{H} \right\rangle = B \ \langle \hat{\sigma}_z \rangle + C \ q^n \ \langle \hat{\sigma}_j \rangle + \frac{p^2}{2m} + V(q). \tag{4}$$

Evolution equations as well as dissipation is introduced in the standard way leading to [7,14]

$$\frac{dq}{dt} = \frac{p}{m}, \tag{5}$$

$$\frac{dp}{dt} = -\left(n \ C \ q^{n-1} \ \langle \hat{\sigma}_j \rangle + \frac{\partial V(q)}{\partial q} + \eta \ p \right). \tag{6}$$

In order to adjust the experimental data, we consider the following semiquantum Hamiltonian [9] which is a particular case of Eq. (3) (treated as example in [13])

$$\hat{H} = B \ \hat{\sigma}_z + C \ q \ \hat{\sigma}_x + \frac{p^2}{2m} + D \ \frac{q^4}{4}, \tag{7}$$

where q and p are canonically conjugate classical variables and $\hat{\sigma}_i$ are $\frac{1}{2}$ spin particles operators. The $B\hat{\sigma}_z$ term is the spin quantum part Hamiltonian, the $\frac{p^2}{2m} + D\frac{q^4}{4}$ term is the classical particle Hamiltonian, the $Cq\hat{\sigma}_x$ term represents the interaction between them and D is a constant necessary in order that the term $D\frac{q^4}{4}$ has energy units. By considering the Lie algebra $\{\hat{\sigma}_x, \hat{\sigma}_y, \hat{\sigma}_z\}$ as the relevant set [8], we see that, through MEP they define a complete set of non-commuting observable $CSNCO$ whose mean values are the quantal degrees of freedom of the system, while q and p are the classical ones. The temporal evolution of these spin components is given by (see [7,13] for more details)

$$\frac{d \langle \hat{\sigma}_x \rangle}{dt} = -2B \langle \hat{\sigma}_y \rangle, \tag{8}$$

$$\frac{d \langle \hat{\sigma}_y \rangle}{dt} = 2B \langle \hat{\sigma}_x \rangle - 2Cq \langle \hat{\sigma}_z \rangle, \tag{9}$$

$$\frac{d \langle \hat{\sigma}_z \rangle}{dt} = 2Cq \langle \hat{\sigma}_y \rangle, \tag{10}$$

The mean value of the Hamiltonian (7) is [13]

$$\left\langle \hat{H} \right\rangle = B \ \langle \hat{\sigma}_z \rangle + C \ q \ \langle \hat{\sigma}_x \rangle + \frac{p^2}{2m} + D\frac{q^4}{4}, \tag{11}$$

which, together with Eqs. (5) and (6) leads to the motion equations of the classical degrees of freedom

$$\frac{dq}{dt} = \frac{p}{m}, \tag{12}$$

$$\frac{dp}{dt} = -C \langle \hat{\sigma}_x \rangle - q^3 - \eta p. \tag{13}$$

4 Numerical Simulations

In our model, the $\langle \hat{\sigma}_z \rangle$ component of the spin is the physical magnitude which represents the 'decision' to be made by the contracts makers as time goes by. So, in our numerical simulations we have to solve the non-linear semiquantum equations of motion (8), (9), (10), (12) and (13) for proper initial conditions and parameters values (m, B, C, D) that are able to fit the experimental data.

The experimental data are shown in Fig. (1) (where we have depicted the normalized number of contracts signed under the UTE (upper) and ACE (bottom) modalities vs. year) and in Fig. (2) (where we have depicted the CCE normalized number of contracts vs. year). Both kind of data were taken from [15]. As the spin $\langle \hat{\sigma}_z \rangle$ component runs from +1 to -1, data have been accordingly normalized to 1 in those figures.

We have made several numerical simulations considering a wide range of initial conditions and parameter values in order to fit the experimental data (given by Figs. (1) and (2)) with our model and we have found that, as it can be seen from the original data, the only case that could be reasonably treated with our model is UTE case. This is due to data characteristics than due to model properties. For the CCE case, the existing data suggest that this contract is more suitable than ACE faced to SME demands, although the time horizon is too short to do so. For the ACE case possibly, the phenomena has stopped, as it demands a highly collaborative attitude. For sake of reliability, we restrict ourselves to the UTE case. So, we set the parameters of our model, and use Eqs. (8), (9), (10), (12) and (13) to get the temporary evolution according to the model. The obtained results are shown in Fig. (3). The $\langle \hat{\sigma}_z \rangle$ component evolution, shown in Fig.(3) (middle), was obtained for the following initial conditions for the set of differential equations: $\langle \hat{\sigma}_z \rangle_0 = 0.9$, $\langle \hat{\sigma}_x \rangle = \langle \hat{\sigma}_y \rangle = 0$, $q_0 = 0$, $p_0 = 1.2649112$ (obtained from the initial energy value, Eq. (11), see ref. [13] for more details), $\left\langle \hat{H} \right\rangle_0 = 0.5$, $B = 0.5$, $C = 1$, $\eta = 0.5$, $D = 1$ and $m = 16$. As it could be seen from Fig. (3), the model and data fit in a reasonable fashion till 2005. After that year both curves show discrepancies. This fact could be due to other factors which are not included in our model. Basically, around 2004, the number of UTE contracts is below the theoretical curve and after that point a bounce appears. Bounces are common in economic systems, so, the disagreements that appear around 2004 may be consequences of macroeconomic situations.

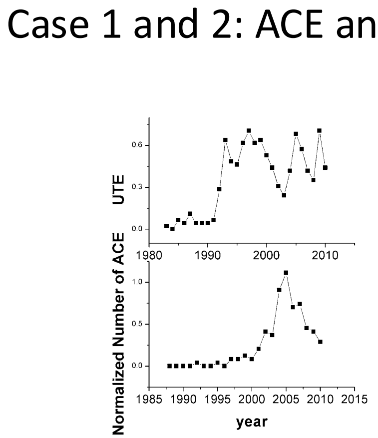

Fig. 1. Normalized number of contracts signed under the UTE (upper) and ACE (bottom) modalities

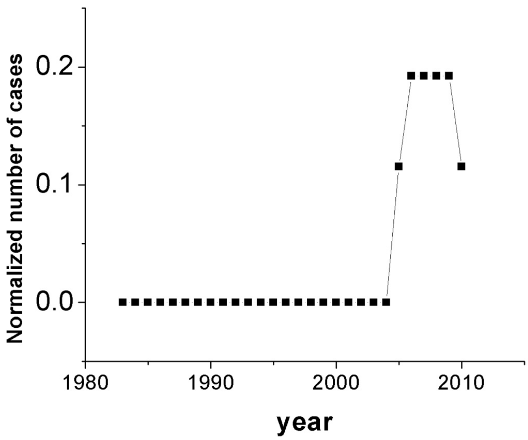

Fig. 2. CCE normalized number of contracts

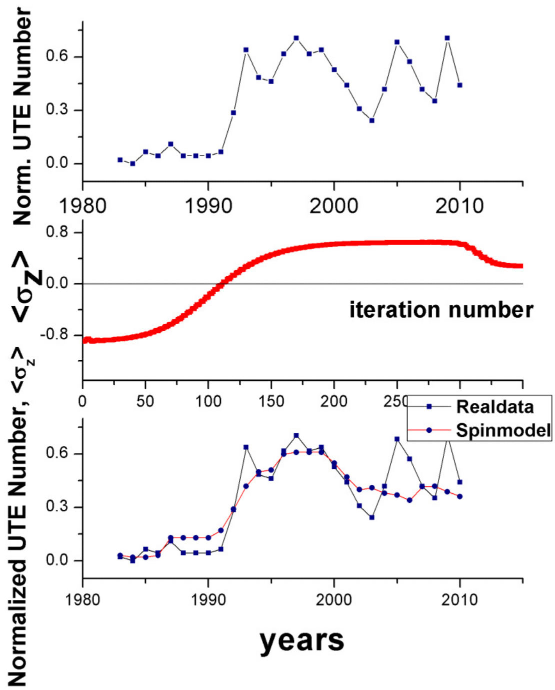

Fig. 3. Normalized original data (upper), the spin $\langle\hat{\sigma}_z\rangle$ component evolution (middle) and the supperposition of both curves (bottom) are shown. The $\langle\hat{\sigma}_z\rangle$ component evolution was shifted from -0,8 to zero in order to fit data.

5 Conclusions

We have presented a nonlinear semiquantum $SU(2)$ Hamiltonian, coming from quantum physics, to describe a decision-making process. If a dissipation term of the form $-\eta p$ form is included in the classical part, the $\langle\hat{\sigma}_z\rangle - spin$ component is able to mimic a binary decision-making process yes-no. Our main goal is to use this 1/2 spin Hamiltonian to model non-compulsory contracts by fitting the available experimental data. This kind of Hamiltonian fulfill the necessity of having a yes-no description, to include the complexity of the legal system (included in the nonlinear nature of the model) and to include the uncertainty or risk in the decision process to adopt for a given contract naturally contained into the quantum subsystem.

In Fig. (3) the final result of our work is shown. As it can be seen from the original data, only the UTE case is suitable to be modelized. This is due to data characteristics than due to model properties. For the CCE case, the existing data suggest that this contract is more suitable than ACE faced to SME demands, although the time horizon is too short to do so. For the ACE case possibly, the phenomena has stopped, as it demands a highly collaborative attitude. The CCE case could also be modelized using this model, as it can be seen from Fig. (2). However, the last point, as well as the novelty of the CCE regulation makes a little unrealistic to attempt to fit this data. For sake of reliability, we restrict ourselves to the UTE case.

The process of taken or not a given contract modality is a $yes - -no$ process, so, we need to appeal to some 1/2 spin model. Hamiltonian formalism is preferred in physics as it gives a deep understanding of the physical problem at hand. In order to mimic a yes – no process of decision-making; we take as the quantum subsystem of our Hamiltonian that of representing a 1/2 spin particle (largely considered in the literature [16,17]) because it is a well-known fact that in these kind of systems the observable (measurable physical quantity) $\hat{\sigma}_z$ component of the spin has two possible measurable values: $+\frac{1}{2}$ and $-\frac{1}{2}$ and then the system corresponds to a two states system: $|+\rangle$ and $|-\rangle$ [16] (these two states corresponding to a 1/2 spin particle are also known as "up" and "down" states or $|\uparrow\rangle$ and $|\downarrow\rangle$ states and, the two possible measurable values of $\hat{\sigma}_z$ component of the spin, are also known as $+1$ and -1) and they may be helpful to describe a process of decision making in which only two results are possible: "yes" or "no". To describe the 1/2 spin particle, we need the components: $\hat{\sigma}_x$, $\hat{\sigma}_y$ and $\hat{\sigma}_z$ given that it is considered a vectorial magnitude and it is a quantum degree of freedom of the particle that has no classic counterpart but, oversimplifying, we can say that it is equivalent to an intrinsic angular moment which means that if an external magnetic field B (in the Oz direction) is present, the $\hat{\sigma}_z$ spin component aligns with the field and as "it is quantized physical quantity, its discrete spectrum includes only two values (eigenvalues): $+\frac{1}{2}$ and $-\frac{1}{2}$" [16] (the interested readers can find an interesting discussion on ref. [16], Cap. 4). If the external field B changes its direction the spin follows it. So, if the field B is +B, we say (mere convention) that the $\hat{\sigma}_z$ spin component of the spin is "up" and if the field adopts the value -B, then the $\hat{\sigma}_z$ spin component is "down". In this sense, we said that the 1/2 spin quantum systems can describe yes-no (up-down; +1 or -1) situations. Even when the spin has three components, the yes-no situation is described following the behavior of the $\hat{\sigma}_z$ spin component on account it is the one used to be aligned (only by convention in the literature) with the external magnetic field B. For the purpose of describing a yes-no process for numerical simulations $+1$ and -1 values are taken [18].

As we have explained in the model description Section, the existence of different alternatives means that, in order to change the decision *yes* to *no*, (or zero) it requires that the energy of the system should dissipate so that some change in the supported position appears in the temporal evolution. It is a well-known fact that the more natural way to introduce dissipation in a semiquantum system is through an *ad hoc* term, ηp via the η parameter [14] in order not to violate any quantum rule (the uncertainty principle). In our case this η parameter represents the "external-social pressure-limit situations" under which the "opinion-attitude-decision change" is taken. Also, from Fig. (3) (middle), we can see that the $\langle\hat{\sigma}_z\rangle$ component evolves from -0.8 to almost +0.8, passing through zero value, making necessary to shift it in order to fit data. As we said the yes-no process is a mere convention related to the sign of the $B\langle\hat{\sigma}_z\rangle$ subsystem value at $t = 0$, and the shifting of the component is not relevant to the global process dynamics. Notice that we do not make any average on the original data which could improve the UTE case fitting.

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Analytic Hierarchy Process for Health Technology Assessment: A Case Study for Selecting a Maintenance Service Contract

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Abstract. Objective: In this study, the Analytic Hierarchic Process (AHP) was used to improve Health Technology Assessment (HTA) by: tracking the decision processes allowing stakeholders to understand the work done by decision-makers (DMs); weighting properly the most appropriate DM for each dimension of the problem; extending decision processes to DMs not skilled in complex mathematical methods. Moreover, our goal was to quantify qualitative knowledge, which affects HTA, using a scientific method. As a case study, we focused on the choice of a maintenance contract for a Computerized Tomography scanner.

Methods: The AHP was applied to support HTA for the need analysis and for the assessment of how each alternative fits each need. sixteen managers from eight hospitals were involved to assess the demand's needs. Managers of four leading manufacturers providing maintenance services were involved to analyze different offers.

Results: AHP allowed quantify the relative importance of each need in each Hospital, showing that the demand changes according to several factors as: technologic asset, mission, position, and environment. Moreover, the proposed method enabled to measure how each contract adhered the demand, without further features not strictly required, for which hospitals are not willing to pay. These results were achieved using a fully traceable method, allowing stakeholders to fully understand the decision process.

Conclusion: AHP allowed to model demand and offer in a coherent framework of decision making, including both qualitative and quantitative knowledge. This enabled to reach the objectives of this study, quantifying needs' relative importance and consequently the adherence of each contract.

Keywords: analytic hierarchy process, health technology assessment, maintenance service contract, user need elicitation.

1 Introduction

Health Technology Assessment (HTA) is a multidisciplinary and multidimensional decision-making process. Once stakeholders identify all the needs which have to be

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satisfied in order to solve a problem, HTA allows the scientific assessment of the technologies that best satisfy all the highlighted needs. Therefore, HTA is a decision-making process, which aims to identify useful technologies, alternative and competitive, and select the most effective and efficient one among these.

In healthcare service, this process is complicated by some peculiarities which characterize the environment. For instance, in democratic countries, in which the healthcare system is totally or partially supported by the public National Healthcare System (NHS), the Decision Makers (DMs) are ultimately responsible to the citizens for their decisions. In other words, it is not their own money they are spending. On the other hand, patient co-payment for products and services does not longer allow doctors, hospitals, and/or insurance companies to be the sole judges of which is the "right" technology, and the community of stakeholders extends all the way to patients and families [1]. For that reason, the use of scientific quantitative methods to support decision making is considered necessary in healthcare organizations, where the personnel are committed to follow only the best available evidence according to well-designed trials[2], meta-analyses [3] or network meta-analyses [4].

World Health Organization (WHO) made it clear that the term "health technology" must encompass all potential technical facets, including people and processes, not just medical devices. In this scenario, the choice of a maintenance contract can be considered to be within the scope of HTA, as it may influence the effectiveness and the efficacy of healthcare services.

Culture and values influence the HTA processes [5]. This influence should be measured as a variable of the decision problem. For instance, the needs to be satisfied by a new technology are so different that no one could be skilled, nor equally sensitive, in all the dimensions, which characterize the choice: some stakeholders could be more sensitive to clinical needs, while others might prioritize managerial or economic constraints. Therefore, the appropriateness of DMs' background and professional experience has to be taken into account in a scientific process of HTA. Finally, the complexities of health problems require a spectrum of qualitative and quantitative information [6]. Moreover, part of the information about the needs and their priority is based on subjective experience of health care professionals. This qualitative information has to be quantified to perform an analytic and scientific decision-making process, such as an HTA. Moreover, the outcome and the model provided by the decision-making process should be easy to understand, as the intelligibility is strongly appreciated in medical domain decision-making [7,8].

These considerations lead us to use the Analytic Hierarchy Process (AHP), as this method proved to be effective in quantifying personal experience and weighting properly DMs' background [9,10].

The AHP is based on the idea that judging the relative importance of risk factors, i.e., comparing pairs of them in a hierarchic structure is more reliable than judging their absolute importance. AHP is an analytic decision-making method, which aims to solve multi-factorial and multi-dimensional fuzzy problems [11]. Several studies have assessed the effectiveness of AHP as a method for medical and health care decision making [12,13]. A number of articles have highlighted the advantages in supporting hospital purchases with AHP, which is valuable for multidimensional and multifactor decisions [14], and is easy to use [15] and time saving [16].

In our study, we used the AHP to prioritize customer needs, by weighting them. Using these weights, we defined an objective function in order to assess the “best service contract”. The best contract is the one that fits the customer needs better, without unnecessary accessory services, for which the customer is not willing to pay.

The goal of this paper was to assess how AHP may improve HTA in identifying the best technology and in:

1. tracking the decision processes allowing citizens to check the job done by decision-makers;
2. considering properly DMs for each dimension of the problem, with respect to their area of work, experience and background, modelling DMs as a part of the decision process to highlight/avoid biases;
3. quantifying qualitative knowledge that is based on professional experience;
4. extending decision processes to stakeholder not experienced in mathematical methods.

As a case study we focused on service contract of Computed tomography (CT), because it is one of the medical devices that requires the most complex and expensive services. The evaluation was performed both at a national level and at a local level. The national level reflects the average demand of eight Italian hospitals located in different towns and regions, while the local level reflects the demand of a middle size hospital located in a remote area of southern Italy. Although the HTA was performed considering several dimensions of needs, here we present the results related to the technical dimension, which were considered the most important from the DMs involved and are sufficient to illustrate the benefits of the methods proposed.

2 Methods

The assessment of an health technology requires decision makers:

1. to define the problem to be solved, by:
   - identifying the needs that have to be satisfied;
   - classifying these needs into meaningful categories;
   - prioritizing them according to their relative importance;
2. to individuate all the technologies which can satisfy these needs;
3. to assess which technology satisfies the identified needs better (performance assessment);
4. to report and divulge results;
5. to observe the performance of the technologies in the short, medium, and the long term.

We used AHP in the points 1 (classification and prioritization), 3, and 4, following the steps in Figure 1 and described in the following sections.

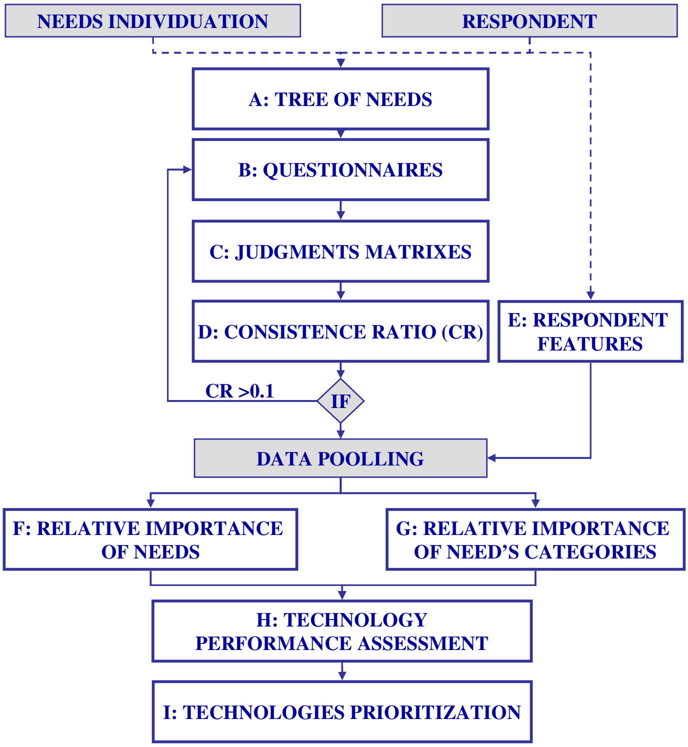

Fig. 1. Algorithm of the Analytic Hierarchy Process application

2.1 Tree of Needs (A)

With a focus group, we first identified a range of needs that a maintenance service contract has to satisfy, and then we grouped them into meaningful categories. Successively, we designed an oriented graph, a tree, in which the vertices were defined as following: each need was a leaf; each sub-category was a root; each category was a hyper-root. The relative importance of each need in a sub-category was used to weight the edge linking the leaf to its root. Iteratively, the relative importance of each sub-category of needs was the weight of the edge linking the root to its hyper-root. The same was done for roots and hyper-roots. The relative importance of each element of the hierarchy was assessed as further described.

2.2 Questionnaires (B)

For each pair of needs (i,j) of each category, respondents were asked the following question: "in your opinion is i, compared to j: much more important, moderately more important, equally important, moderately less important or much less important?" The respondents were required to choose one option. Similar questions were posed to compare categories of needs. In accordance with the scale of Saaty [17], a numerical value was given to each judgment. Although several scales have been proposed

[18-21] , we used the Saaty's natural scale, which consists in associating numerical values from 1 to 9 to the following judgments [22]: 1 if the need *i* is equal important as the need *j*, 3 if the need *i* is moderately more important than the need *j*, 5 if the need *i* is strongly more important to the need *j*, 7 if the need *i* is very strongly more important than the need *j*, or 9 if if the need *i* is extremely more important than the need *j*. Even numbers (2, 4, 6, or 8) were used for in between judgments [23].

2.3 Judgment Matrices (C)

For each category of needs, with the scores defined according to Saaty [17] scale, we constructed a judgment matrix A_{nxn}, where "n" was the number of needs in the same category, which have as the generic element (a_{ij}), the ratio between the relative importance of the need i (N_i) and the relative importance of the need j (N_j). Assuming reciprocity of judgment, the element a_{ji} is the reciprocal of aij and the diagonal elements aii are equal to one. These proprieties represent the assumption that: if N_i is 3 times more important than N_j, then the importance of N_j should be 1/3 compared to N_i; N_i is equally important to itself. It has been shown that, if the judgments are consistent with respect of the transitivity property explained in the following section, this matrix has only one eigenvalue (λ), which is equal to "n". The normalized components of the corresponding eigenvector represent the relative importance of each risk factor. This step was iterated for each category of needs. Finally, the same algorithm allowed determining the relative importance of each category of needs to be assessed.

2.4 Inconsistency (D)

From the judgments matrix, it is possible to estimate the consistency of the responses of each DM. AHP allows handling the natural inconsistencies of different DMs, by adding a redundant pair wise comparison and using the transitivity propriety, which states that if $N_i= a_{ij} * N_j$ and $N_j= a_{jk} * N_k$, then $N_i = a_{ik} * N_k$ where $a_{ik}=a_{ij}*a_{jk}$.

This property comes from the definition of a_{ij}, as shown in the following equation:

$$a_{ij} = \frac{N_i}{N_j} = \frac{N_i}{N_k} * \frac{N_k}{N_j} = a_{ik} * a_{kj} \quad (1)$$

If the judgments are not fully consistent, the matrix has more than one eigenvalue. The eigenvector corresponding to the major (λ_{max}) was used as described in paragraph C; this will generate an inconsistency, which can be estimated as following.

Considering three needs i,j, and k, the respondent is asked to perform the pair comparisons i-j and j-k, and then the (redundant) comparison i-k. The answer to the third question is compared with the ratio a_{ik} deduced from first two applying equation 1 and the difference represents the degree of inconsistency. Mathematically, the coherence of each response is measured with the error defined as: $error_{ij}=a_{ij}-a_{ik}*a_{kj}$. The global effect of these errors, which reflects the global inconsistency of the respondent, can be estimated measuring the difference of the major eigenvalue λ_{max} from "n". The error is zero when the framework is completely consistent. This error can be seen as a precision error and could be in part due to the scale adopted, which has only natural numbers.

From the operational point of view this error is estimated for each judgment matrix following these steps:

1) the Consistency Index (CI) is calculated using the following formula: CI = (λ_{max} –n) / (n–1):
2) the Random Index (RI), which is a tabled value growing with of the number of needs being considered in one node, is calculated as following: 0.58 with 3 needs: 0.90 with 4, 1.12 with 5, 1.24 with 6, 1.32with 7, 1.41with 8, and 1.45 with 9 needs
3) the Consistency Ratio (CR) is calculated as CI/CR.

A CR less than 0.1 is usually acceptable [17], altoght some outhors suggested to increase this value in more complex problems [24]. A higher CR would reflect a higher error that be considered too high for a reliable decision. Nonetheless, this kind of inconsistency is often due to distractions or loss of interest by the respondent, and not to a global incoherence of the respondent's opinion. For this reason, when responses are inconsistent, the questionnaire was re-submitted to the respondent, as shown in Figure 1.

2.5 Respondent Features and Data Pooling (E)

We integrated individuals' opinions, by applying the geometric mean [25,26] to the judgment matrices among respondents per each category of need. Each matrix was weighted according to the relative importance of each respondent. We weighted respondents according to their experience, taking into account the following features: years of experience, level of education, area of work. The weights were calculated according to Table 1. The outcome of this step was an average matrix (A_a) for each category of need.

Table 1. Weights assigned to the respondents according to their experience

Feature	Weight
Years of experience in your field	
>10	0.32
8-10	0.26
5-7	0.22
3-4	0.12
1-2	0.08
Education	
Ph.D. or equivalent	0.59
MD or equivalent	0.25
BCS or equivalent	0.11
Profess Qualification	0.05
Area of Work	
Pertinent with the category of needs	0.57
Partially pertinent with the category of needs	0.29
Other	0.14

2.6 Evaluation of Needs' Relative Importance (F)

From each matrix A_a, we calculated the relative importance of each need within each category, as the normalized components of the main eigenvector, as described above (C). We iterated the procedure described above (B, C, D, E, and F) for each category of needs. From this step, we calculated a vector for each category ($\underline{W}^C$), in which each element w_i^C represents the relative importance of the need "i" in category "C".

2.7 Evaluation of Need Category Relative Importance (G)

Once again, we iterated the procedure described above (B, C, D, E) by asking to each respondent to judge the relative importance of each category of needs. The outcome of this step was a vector ($\underline{C}$), in which each element represents the relative importance of a category of needs. Finally, the product of the weight of each need for its own category represents the global relative importance of each need. The output of this step is the vector of the weights (W), in which the element w_i, represents the global importance of the need "i" in the final decision.

2.8 Performance Assessment (H)

According to the need's hierarchy, we scheduled features of each contract. Then, for each need "i", each respondent was asked the following question: "how much does the contract C_i, compared to C_j fit the need i?". The DMs were asked to not consider features of the contract which are not required or overcoming a defined threshold. Repeating the steps C, D, and E, we calculated the vector of performance (P_j). In this vector, the element p_i^j represents how the technology "j" satisfies the need "i". Wherever possible, in order to harmonize the answers, we suggested a specific range of values for each identified need.

2.9 Technology Prioritization (I)

The technology which maximizes the scalar product of needs' weights per the effectiveness of each technology (W*P) is chosen as the "best technology". The same step was repeated for each category, in order to analyze how each technology satisfies each category of need (category performance).

3 Results

The tree of needs that we defined includes three main categories: economical, technical, and ethical. In this paper, we present and discuss the results focusing on the technical dimension, which is sufficient to illustrate the benefits of the methods proposed. In this category, we identified 15 customer needs. Within these needs, we defined a tree and prepared the questionnaire to assess their relative priorities. The questionnaires were submitted to 16 DMs of 8 hospitals, of which 4 are public and 4 are private. For each hospital, one DM was from the management and one was from the clinical engineering service. Similar questionnaires were submitted to managers of four leading manufacturers offering maintenance services in order to assess the correspondence with parameters required from customers. Moreover, from the questionnaires submitted to manufacturers, and by studying their standard services contracts, we assessed which kind of contracts they offer, before customization.

The five most important needs emerging from the customers‘ responses are the following:

1. Availability of a complete spare parts kit: to improve the FTFR (First Time Fix Rate) the service engineer will have a good chance to identify the problem and replace the defective part on the first visit to the site.
2. Lower intervention time: to increase the uptime (i.e., the machine’s availability for regular work) the service engineer must arrive on the site shortly after the failure notification.
3. Lower distance from the spare parts stock: in case the spare part kit is not sufficient to fix the machine, and further parts are necessary, the distance from the main spare part stock is an important issue.
4. Good standard services: each contract has the basic SLA (service level agreements), within which preventive maintenance, safety inspection, etc. are included.
5. Good additional services: such specialist services, e.g., X-ray tube included, remote diagnostic, proactive monitoring of the machine, virus protection, utilization management, and training. Not every company is able to provide **all these services and not all these services are generally required**.

The matrices in the equations 2 and 3 reflect the relative importance of the five aforesaid needs each other at national and local level.

$$\begin{bmatrix} 1.00 & 1.25 & 1.67 & 2.50 & 2.50 \\ 0.80 & 1.00 & 1.33 & 2.00 & 2.00 \\ 0.65 & 0.75 & 1.00 & 1.50 & 1.50 \\ 0.40 & 0.50 & 0.67 & 1.00 & 1.00 \\ 0.40 & 0.50 & 0.67 & 1.00 & 1.00 \end{bmatrix} \tag{2}$$

$$\begin{bmatrix} 1.00 & 2.50 & 1.67 & 2.50 & 1.25 \\ 0.40 & 1.00 & 0.67 & 1.00 & 0.50 \\ 0.60 & 1.50 & 1.00 & 1.50 & 0.75 \\ 0.40 & 1.00 & 0.67 & 1.00 & 0.50 \\ 0.80 & 2.00 & 1.33 & 2.00 & 1.00 \end{bmatrix} \tag{3}$$

For instance, the second element of the first row in the matrix in equation 2 means that the first need ("availability of a complete spare parts kit") is considered 1.25 times much more important than the second need ("lower intervention time") at national level. The second element of the first row in the matrix in equation 3 means that the first need ("availability of a complete spare parts kit") is considered 2.50 times much more important than the second need ("lower intervention time") at local level.

The relative importance of the five individuated needs is presented in Table 2.

Table 2. relative importance of customer needs, assessed via questionnaires

Needs	**National level**			**Local Level (Case Study)**	
	range and mean across the 8 hospitals involved			***Hospital "G. Moscati", Avellino***	
	Range	mean weight (Wm)	Wm in 1-5 scale	mean weight (Wm)	Wm in 1-5 scale
Spare parts kit	0.258-0.368	0.313	**5**	0.313	**5**
Intervention time	0.205-0.295	0.250	**4**	0.125	**2**
Distance from the spare parts stock	0.143-0.233	0.188	**3**	0.188	**3**
Standard services	0.090-0.160	0.125	**2**	0.125	**2**
Additional services	0.050-0.200	0.125	**2**	0.250	**4**

For each need, the second column shows the mean of the relative importance across the eight hospitals. The mean gives an idea of how each need is considered important at a "national level", as the hospitals involved are in different towns and regions. The third column shows values normalized on a scale of 1 to 5, which were adopted to communicate with stakeholders not skilled in mathematical methods. In the final two columns, the table shows the mean for the case study hospital in both scales.

Subsequently, for each hospital the performance of each contract in fitting each need was assessed. Wherever possible, in order to harmonize the answers, we categorized possible values for each need identified. Distances where categorized using 5 thresholds: 300 Km, 500 Km, 1000 Km, same continent, other continents. Intervention time was categorized using 5 thresholds: 4h, 8h, 12h, 24h, and 36h. DMs were asked to not consider features of the contract which are not required or overcoming a defined threshold. For instance, in our study, an intervention time of 4 hours was generally considered satisfactory. A lower intervention was not valuable, as it was under the sensibility of the reorganization process of Computer Tomographyscan activities. Therefore, since the minimum downtime was of 4 hours, because the patients of the next 4 hours after the breakdown of the CT were redirected to other units, an intervention of 2h was not considered an added value.

In Table 3, we present the results obtained analyzing the standard contracts (C1-C4) for the maintenance of a CT of the four manufacturers involved, with respect to each need at a national level and in the case study (local level).

Table 3. relative importance of each need and quantification of contract performance per need at national level and local level

NEEDS	National level				Local Level (Case Study)			
	P^1	P^2	P^3	P^4	P^1	P^2	P^3	P^4
Spare parts kit	*0.25*	*0.25*	*0.25*	*0.25*	*0.25*	*0.25*	*0.25*	*0.25*
Intervention time	*0.31*	*0.31*	*0.25*	*0.13*	*0.26*	*0.26*	*0.26*	*0.22*
Distance from the spare parts stock	*0.25*	*0.25*	*0.25*	*0.25*	*0.25*	*0.25*	*0.25*	*0.25*
Standard services	*0.25*	*0.25*	*0.25*	*0.25*	*0.25*	*0.25*	*0.25*	*0.25*
Additional services	*0.26*	*0.26*	*0.21*	*0.26*	*0.33*	*0.33*	*0.13*	*0.20*
Global Score	**0.27**	**0.27**	**0.25**	**0.22**	**0.27**	**0.27**	**0.22**	**0.23**

4 Discussion

In Table 2, the ranges reflect the difference in scoring each need among the hospitals. This may be due to the specific situation of each Hospital, depending from its own technological asset, from its position and from its mission. This is aligned with Kaplan and Shaw [27], which recommended that an Health Technology Assessment has to address how well a system works with particular users in a particular setting [28]. For instance, the “Intervention time” and “Standard Services” were not considered as crucial factors in the case study hospital, because in the same hospital another CT is available. This allows overloads to be faced in the case the first CT stop working. Although in the hospital there is another CT, the presence of a “spare parts kit” is still considered the most important parameter because it affects the FTFR in case of failure.

From the judgment matrices, such as the ones reported in equations 2 and 3, it is possible to fully understand clearly how each need has been considered important with respect to the others. For instance, it is possible to see that the main difference from the two levels (national vs local) is the relative importance of the additional services: additional services are considered two times more important than intervention time at the local level, while they are two times less important than intervention time at the national level, because of the assets of the case study hospital. This is an example of how the AHP makes the process fully traceable to citizens.

There are many methods used to facilitate group decision making when using the AHP [29,30] and these options can be combined to solve very complex group decision problems. The one we chose allows results to be combined without direct face to face discussion, which is costly when decision-makers reside in different places. The system proposed to score the DMs, although this requires further assessment, allows them to be prioritized according to their background, in relation to each dimension of the problem. Among the respondents, there were clinical engineers directly involved (also in the past) with CT scan maintenance, while others were DMs involved in the management of maintenance processes, but on other devices or without direct experience with maintenance of a CT scan. The former were scored with the maximum in this area of work.

Moreover, analyzing the answers of each respondent, it is possible to highlight biases among the decision-makers. For instance, as stated in [31], we observed that clinical engineers judged the technical dimension to be slightly more important than the economical aspect, while managers were much more sensitive to economic constraints. Moreover, industry overrides for profits; however, they clearly understand that competition focuses attention on the efficacy and efficiency of their technology. Nonetheless, further studies on different technologies, using the same method, are needed to highlight these biases.

The relative importance of the needs, which strongly affect the final decision, was not directly expressed in a quantified dimension. Similarly, the performance of some contracts in satisfying some needs, was not directly quantifiable, with the exception of times and distances. AHP allows this quantification. The main limit of this process is that paired comparisons do not give an absolute judgment, just relative ones. Nonetheless, choosing the best technology is a relative task and relative judgments fulfil this goal.

Regarding the specific performance of the four contracts analyzed (Table 3), with respect to the expressed needs, the second contract is equally effective as the first contract, because it satisfies the required needs, without extra services, which are not required. At the national level, the third and fourth contract does not fulfil all of the required needs. Nonetheless, the contract C_4 is less effective than the contract C_3, because it is less effective in the second need ("Intervention time"), which is considered quite important, although it is more effective in satisfying the fifth need ("Additional Services"), which is considered less important. This result is inverted in the case study hospital, in which "Intervention time" is less important than "Additional Services" because of the local asset.

5 Conclusions

In this article, we presented an application of Analytic Hierarchy Process to enhance Health Technology Assessment. As case study we focused on the choice of the most appropriate maintenance contract for a Computer Tomography scanner. We considered a national level, which reflects the average demand of eight Italian hospitals located in different towns and regions, and a local level, which reflects the demand of a middle size hospital located in a remote area of south of Italy. We presented in this paper only the results related to the technical dimensions, as these are sufficient to illustrate the method proposed.

The method we proposed, improved need analysis traceability, as each step provided intelligible outcomes. For instance the matrix we reported in equations 2 and 3 allows understanding, how each need is considered important compared to each other. The same matrix can be explored for each respondent, to identify and trace DMs' preferences. This process can be iterated within each need categories to explore and trace DMs' preferences end eventually any bias related their background. Nonetheless, differences in the opinions of decision-makers are meaningful because they highlight specific needs of different hospitals, according to their missions, assets, and the territories in which they work. The method presented allowed to quantify such differences.

Furthermore, AHP allows considering appropriately DMs for each dimension of the problem, with respect to their area of work, experience, and background. In fact, each feature of the DMs is weighted as shown in Table 1. This allows considering the DMs as a part of the decision problem. Moreover, this facilitates reaching a solution in an asynchronous group-decision making processes, also when participant does not reside in the same place.

Moreover, AHP allows quantifying qualitative knowledge, which is based on professional experience. In fact, both qualitative and quantitative factors affecting the final decisions are expressed in homogenous scale, as well as subjective and objective information. For instance the performances, which could be assessed using quantified objective data, e.g., distances and times, are expressed homogeneously among themselves (km and hours) and with needs importance (which has no intrinsic dimension).

Finally, none of the DMs was asked to use numbers during this HTA. The experts involved in this assessment were asked to participate to focus groups and to answer questions posed using just verbal expressions. This allowed us to include DMs not experienced in mathematical methods, and potentially to extend the decision processes to a higher number of stakeholders.

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Protocol ITACA: A Decision Tool for an Energetically Efficient Building Management

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Abstract. The building process is configured as an arranged sequence of steps, from the needs detection of clients/users for their satisfaction, to design, realization and management of human made capital. The level of performance of the object to the requirements defines the degree of quality that distinguishes it. Assessing, through the Protocol ITACA, the building performance during different phases of production and utilization, means to have a tool directing the design solutions to the environmental friendly behavior approach, and valuating weaknesses and potentiality, without losing the global effect of choices. This contribute illustrates the potentiality of the Protocol ITACA that must be not only an assessment tool finalized to control ex-post the energy-environmental performance of buildings, but also a method for designing innovative and efficient architecture and making appropriate decisions. Only when this happens, will it be really possible to change the design way.

Keywords: Multicriteria decision support system, Environmental performance, Energy requirements, Building process.

1 Rational Use of Energy, No Energy Sacrifice![1]

The rational use of energy is a technological operation whose objective consists in obtaining the same products or services (in quantity and quality) with a lower consumption of primary energy and possibly with a greater engagement of other resources (capital, work, materials, ...). This definition distinguishes the rational use of energy from the energy sacrifice, which is rather an economic and social operation finalized to stimulate users (by propaganda, using rates or rationing) to change their consumption habits, changing their needs so that the final result is a lower consumption of primary energy. In this case, the offered service has got a different quality.

But the rational use of energy by the construction industry is not the sole objective, as a building, in its operational life, leads to a series of impacts on the environment that do not depend only on the energy consumption.

[1] The Section is mainly due to Antonella Violano.

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The National Protocol ITACA[2] is a tool evaluating the interrelations established between building and surrounding environment. It is necessary starting from the analysis of the renewed design management models more and more linked with to respect of the cogent requirements, voluntary requirements such as non-compulsory specific techniques approved by an organization recognized to develop normative activity and recommendations (performance characteristics required by the current Best Practice).

Since April 2011 several Regions of Italy have been able to count upon a renewed (rather than new) tool to value the energy and environmental sustainability of the buildings, not only Residential, according to the Directive 2010/31/EC that foresees, within 2020, a "quite zero consumption" for buildings. Besides understanding new technical UNI norms about energy and comfort (UNI 11300) and the "National guidelines for the energy certification", the tool has improved the computing methodologies and modified some evaluation indicators.

The Protocol ITACA is an evaluation system that deals with the whole building inserted in his environmental context (urbanized area of relevance and supporting road network).

The method structure on which it is based is the Sustainable Building Method; it is the result of an international search known as Green Building Challenge. The evaluation process is coordinated by the International Initiative for Sustainable Built Environment (SBE), an international non-profit organization with the participation of more than 20 countries representing all continents.

The SBMethod is a *multicriteria* evaluation tool developed and managed in the international survey; the Green Building Challenge is constantly developing and revising a method that combines the advantage of using an international common standard with the possibility of a full contextualization in individual application areas.

2 Prospects for the Regional Development of the Protocol ITACA[3]

Italian Regions, such as Marche, Puglia, Umbria, Piemonte, Basilicata, Valle d'Aosta, Friuli Venezia Giulia, Veneto, Campania, Toscana, Lazio and Basilicata, have adopted the National Protocol ITACA according to climate differences, development needs and housing recovery.

Regulatory updates until 2010 fit the reference framework in order to give an immediate feedback to the current building practice, so the tendency is using it as a support for a sustainable recovery of the preexisting housing; therefore the Protocol ITACA development is in direct ratio to an economic will of revitalizing, as well as

[2] The National Protocol ITACA is a multicriteria decision support system of the Italian Institute for the Innovation and Trasparency of the Contracts and the Environmental Compatibility (the acronym ITACA in italian is for: Innovazione e Trasparenza degli Appalti e Compatibilità Ambientale).

[3] The Section is mainly due to Francesca Verde.

Piano Casa, starting from the bottom on the basis of environmental awareness of direct users.

Among the regional provisions, the approaches of Campania, Lombardia and Puglia stand out for different reasons: not because of the policy change or areas of assessment, as for the blatant desire to lead the design practice to the environmentally awareness.

The Region Campania, simultaneously with the approval of regional Protocol ITACA guidelines, with Resolution no. 572 of July 22, 2010, promotes and spreads knowledge about new regulations on sustainability by developing a *Vademecum* by which the basic principles of green building applied to social housing can convey, which is the original aim of Protocol ITACA 2004.

The attached resolution defines certain fields of application:

- the urban sustainable design (sustainable design, urban quality, urban rehabilitation);
- the requirements for quality and performance criteria of urban space (the guidelines for sustainability of urban space's design);
- the use of healthy materials and environmentally friendly.

By council resolution on 9th November 2011, no. IX/2477 the Region Lombardia illustrates how to identify positive experiences in terms of soil consumption and rehabilitation of existing buildings, writing: "Good practices are administrative actions that often depart from the territory, that is the place where all solutions are built with knowledge, sharing and collaboration, otherwise with the ideal components so that a measure of good will become effective".

So some aspects immediately effective for the ITACA evaluation are reported: ability to affect the containment of soil consumption and the rank of cultural and communicative impact.

Among the latest developments, the most interesting ideas can be attributed to the Region Puglia, which, with the approval of the GR no. 1562 of 31 July 2012 resolution, has taken up a program to finance interventions for primary and secondary urbanization according to the following types:

- recovery of public buildings relating to district services, namely cultural, social, sporting, recreational, etc., based on social and ecological sustainability criteria;
- new construction, rehabilitation and/or redevelopment of public areas (squares, open spaces for the collective clutch, public gardens, urban parks, green areas for play and recreation, public parking spaces);
- road infrastructures including measures to limit noise and air pollution;
- lighting, gas or telecommunications systems; energy efficiency lighting systems;
- construction or renovation of the water supply network and the network of wastewater treatment;
- bio-ecological interventions for the treatment of wastewater and for the reuse of rainwater;
- construction, rehabilitation or renovation of social, health, educational, cultural buildings and sports facilities;

- maintenance services, environmental retrieval or recovery and development of energy efficiency in public buildings and in urban areas;
- removal of architectural barriers in public buildings and in urban areas;
- improving road safety and promotion of sustainable urban mobility (cycle-cross ways and green ways);
- equipped spaces for the separate collection of urban rubbish.

The interventions incorporate the true aim of the Protocol ITACA drafting; it is a rating system that considers not only the building, but also the efficient technological solutions and innovative materials; it is still an element of a more complex urban system, in which the weight of the environmental loads and quality of the site location are equal to the specific energy performances.

The comparison of adopted regional Protocols ITACA allows to manifest how local needs can affect the national tool through decisions making. Each Region has approved the own Protocol ITACA including it in a broadest context of regional provision concerning to environmental sustainability and energy conservation. Below are listed many of Italian Regions that approved the national tool, picking out the relation with criteria of Protocol ITACA in 2009 version.

It can be noted how some requirements, for example Local Conditions; Eco friendly materials; Safe water and CO2 emissions are considered.

In the same way, other parameters like as services approachability, surrounding impact and building common areas are provided only for a few of regions. The Only two Regions, Puglia and Veneto, have an innovative approach.

The Regional experience has allowed to redistribute weights and adapt requirements based on the needs expressed in an application scale by defining four major changes: *Location Quality* becomes a single subject area in which conditions of the site and relationship with the urban location converge, it has been also introduced a requirement, suggested by the Region Puglia experience, about design of surrounding area.

Indeed, Region Puglia introduced an independent requirement about building performance because of many adjustments operated in the most of regional protocols in relation to primary energy.

This parameter now considers the energy needed for cooling and used for, the transmittance and the thermal inertia and the control of solar radiation.

The points about energy from renewable sources for DHW (Domestic Hot Water), energy incorporated in building materials in the thematic area 2 (Resources Consumption), and CO2 emissions from building materials have been deleted in the third area about environmental loads (they were removed in the regional experiences).

The thematic areas 3-4-5[4] have been simplified and combined in parameters, rather than a real exclusion of some of them, with the aim of being able to check broader aspects and, therefore, being more close to the common construction practice.

[4] Thematic areas are: 3. Environmental Loads, 4. Indoor Comfort, 5. Service Quality.

Table 1. Regional **evaluation tools adapt national criteria to local need**

EVALUATION CRITERIA OF PROTOCOL ITACA 2009	**Italian Regions with PROTOCOL ITACA**								
	Basilicata	**Campania**	**Lazio**	**Marche**	**Piemonte**	**Puglia**	**Toscana**	**Umbria**[5]	**Veneto**
1. Location Quality									
1.1 Location Conditions	×	×	×	×	×	×	×	×	+
1.2 Services approachability						+		×	×
2. Resources Consumption									
2.1 Non-renewable primary energy during lifecycle	-	-	×	-	×		×	×	-
2.2 Renewable energy	×	×	×	×	×		×	×	×
2.3 Eco-friendly materials	×	×	×	-	×	+	×	×	×
2.4 Safe water	×	×	×	×	×	×	×	×	×
3. Environmental Loads									
3.1 CO2 emissions	×	×	×	×	×	×	×	×	×
3.2 Waste water						+	×	×	+
3.3 Surrounding area impact						+			×
4 Indoor Comfort									
4.1 Air change			×			×	×	×	×
4.2 Hygrothermal comfort	×	×	×	×	×	×	×	×	
4.3 Visual comfort	×	×	×	×	×	×	×	×	
4.4 Acoustic comfort						×	×	×	
4.5 Electromagnetic pollution	×	×		×	×	×	×		
5. Service Quality									
5.1 Building control systems			×			×			×
5.2 Performance operating support	×	×		×	×	×	×	×	×
5.3 Building common areas						×			
5.4 Home automation					×	×		×	

× *= considered parameters*
+ *= particularly innovative parameters*
- *= very poor parameters*

3 Open Spaces Requirements[6]

We approach the evaluation of energy and environmental building system starting from its interactions with the surrounding environment.

The outdoor environment is not only an "environmental" heat sink, since the material and energy streams that are established between building organization and sur-

[5] The Region Umbria provides as an example also for Region Friuli Venezia Giulia and Region Valle d'Aosta independent evaluation systems.

[6] The Section is mainly due to Antonella Violano.

rounding environment influence – in some cases they produce impact – the energy-environmental behavior of the building-plant system, that is strongly conditioned by the type-morphological, building material and construction and performant characteristics of open spaces, particular planning attention must be paid to the definition of the qualities required to these spaces.

Protocol ITACA 2011 - Residence (as well as the P.I.- Offices, whose difference consists for some indicators of the performance scale) evaluate apparently the "Quality of the site" only through six indicators:

A.1.5 – Reuse of the Territory, in terms of reinsertion in the productive/housing cycle of areas temporarily or permanently denied to the use for damage, disposal, degradation; it becomes, however, necessary the preventive verification of the compatibility with its new appropriate use (residence, offices,..);

A.1.6 - Accessibility to public transport, according to which they prefer the sites better served by the public transport network, to reduce the use of the private means of transport without imposing a "sacrificed mobility" on the citizen;

A.1.8 - Functional mix of the area, according to which the quality is higher for those sites already endowed with infrastructures for business, education and health and sports facilities, leisure time and culture facilities: the urban areas!;

A.1.10 – Proximity to infrastructures, valued positively in relationship with the presence of primary (electricity, water, gas, sewerage,...) urban infrastructural networks, in order to reduce the need to carry out the new ones;

A.3.3. Well-equipped shared outdoor areas, appraising its equipment for rest, play and sports;

A.3.4 - Support to the use of bicycles, as much preferred as higher it is the relationship between the number of the bicycles parking places and the number of the prospective users of the area (Best Practice: guaranteeing 1 place out of every 5 residents).

Anyway, a careful reading of the tool makes it deduce that some of the proposed criteria involve the planning ways of the areas surrounding the building and therefore its environmental context:

B.3.3.- Energy produced in the site for electric uses, involves indirectly the planning of open spaces where the energy production system from renewable source is not "on building" but "on earth";

B.5.1 - Drinkable water for irrigation use, wishes the maximum cut of the use of drinkable water for the irrigation of green areas, which brings to a suitable planning of the use of the spaces and meteoric waters recovery and storage systems, not only on building scale, but also on open spaces scale;

C.3.2 - Solid waste produced in operational phase, must be interpreted, from the planning point of view, as the suitable and integrated presence/planning of collection centers to collect different typologies of waste on the base of their appropriate use;

C.4.2. Ground permeability, evaluate the closing of the water cycle in relationship with the typologies of surfaces, for which the design must prefer permeable, transpiring, eco-friendly finishes;

C.6.8. Heat island effect invites to maximize the shaded areas enjoyable in the summertime privileging green areas. So, it becomes useful the use of the "Matrix of site" that connects natural ventilation to sunny open spaces, in order to guarantee suitable comfort conditions all seasons.

Totally, out of 34 selected criteria, 11 concern the conformant and performant characteristics of open spaces at the service of the residence: nearly a third of the criteria!

This happens because the planning attention for open spaces is kept by the awareness that these are complex spaces with the role of a connective tissue of the urban system, not only for the mobility of both material (things, animals and people) and immaterial (energy, data, information,...) streams, but also for the psycho-physical comfort (health) and social comfort (cultural identity) of the modern man, who has got the city as his own prevailing habitat.

4 Indoor Requirements[7]

The Protocol ITACA 2011 focuses on the fulfillment of the Environmental Quality conditions for the indoor spaces, dedicating a specific subject area, further divided into 5 categories of requirements:

D.2 - Ventilation, it provides directions with respect to the control of the air healthiness while minimizing energy consumption due to air changes, in this aspect, especially for the offices application protocol, it is necessary to take into account the combination of natural ventilation and the one supported by mechanical means (Hybrid).

D.3 - Thermo-hygrometric comfort, it identifies levels of comfort for the summer period, the greater possibility of approximation to an ideal value, determined according to the location in question, will be the higher score achieved for the category.

D.4 - Visual comfort, it invites to maximize the percentage of natural light incoming in the occupied spaces, in order to reduce consumption relating to artificial lighting and heating systems.

D.5 - Acoustic comfort, it allows to identify the value of the acoustics class of the casing elements, by considering the individual acoustic requirements of the building, that are weighed in the global acoustic consequence.

D.6 - Electromagnetic pollution, it calls for action in order to minimize adverse effects from exposure to power-frequency magnetic fields, by taking into account both the possibility of pre-existing realities (substations, underground lines, etc.) and the contextual interventions (installation the electrical system).

The features relating to the performance of the housing and the supply of resources to guarantee comfort conditions are evaluated in the thematic area B. Consumption of resources:

B.1.2 - Primary energy for heating, it indicates, according to the parameters established by the standard no. 192/2005 and no. 311/2006, the percentage of non-renewable energy required for heating.

B.1.5 - Primary energy for domestic hot water, it invites the reduction of traditional sources consumption for the production of hot water for sanitary use.

B.5.2 - Safe water for indoor use, it focuses on the use of rain water, either through technological strategies by optimizing the use of water.

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[7] The Section is mainly due to Francesca Verde.

B.6.2 - Net energy for cooling, which allows to evaluate the performance of the building also during the summer period as expected from the application of the law DPR no. 59/2009.

B.6.3 - Thermal transmittance of the building, it introduces the rates of energy exchanges during the winter period between the building and the external environment, by identifying a single average value of the case elements.

B.6.4 - Control of solar radiation, it estimates the transmittance values of the transparent enclosures (understood as a package window / screen), that are calculated not only according to the exposure of the fronts, but also on the types of screens or obstructions.

B.6.5 - Thermal inertia of the building, it introduces a fundamental concept of dynamic capabilities of envelope (phase displacement and damping of thermal wave), these phenomena particularly happens in our latitudes during the summer.

It can be seen as the thermo-hygrometric comfort limits to observe only the summer condition, whereas the winter performance, needed for the building, is found in terms of compliance with the regulations and minimizing the use of non-renewable resources (the criteria listed above were an integral part of the evaluation of the indoor environment quality in the protocol version 2004).

5 Evaluation of Building Materials[8]

The Protocol ITACA also assesses the sustainability of a construction project during its full life cycle, from the insertion into the urban context, to the conception of systems for the maintenance and security of end users.

From the First Protocol 2004 to the most recent in 2011 it can be seen as the benchmark of eco-compatible materials (material consumption in 2004) remains included in the thematic area of resource consumption by dividing the number of criteria as follows:

B.4 - Eco-friendly materials

B.4.1 - Reuse of existing structures, the feature discourages the demolition and the knocking down of buildings in place of recovered structures. This evaluation item in the previous versions of the protocol referred to the site conditions.

B.4.6 - Recycled / recovered materials, it calls for reducing the consumption of raw materials, using materials from previous uses that reduce the environmental impact of the intervention.

B.4.7 - Materials from renewable sources, it considers the impact of the use of non-renewable raw materials.

B.4.9 - Local materials for finishes, it aims to promote the use of local materials in order to reduce the environmental impact of transportation and to promote the local economy.

B.4.10 - Removable and recyclable materials in reference to sections 4.6 and 4.7, it encourages to reduce consumption of raw materials, using recyclable materials and providing installation mode to enable selective demolition, by components and materials that are easy to separate, this also implies a reduction of demolition waste.

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[8] The Section is mainly due to Francesca Verde.

6 The Method[9]

The tool intends to assess the energetic efficiency of the buildings and provide technical choices with relevant support in the process through the many phases of the project. The wide range of supply options made available for project developers instruments fit the purpose of meeting the requirements and driving management choices[10].

In the number of those, it is worth mentioning the following:

- first generation systems – among which BREEAM Building Research Establishment Environmental Assessment Method and the LEED Leadership in Energy and Environmental Design– both characterized by the very same inherent limitations, that is their suitability just for the geographical region they are designed for.
- second generation systems – among which the German *Passivhaus* protocol and the SBC Sustainable Building Challenge, promoted by ITACA need to be recalled– show higher adaptability to local conditions and they better account for certain factors such as climate, economy and culture as well as environmental priorities, social context. Nevertheless, these systems are still able to keep the same reference framework[11].

The Protocol ITACA is an assessment method concerning the building as a whole and as a part of its own environmental surroundings (urban area and support infrastructure network). The foundations of this method (SBMethod) owe its structure to the international process coordinated by iiSBE, also called Green Building Challenge, a project which witnessed the participation of more than twenty nations from all the continents.

The SBMethod is an international multi-criteria evaluation method; the goal of the Green Building Challenge is to develop and continuously update a methodology that combines the advantage of using a common international standard with the possibility of fully framing the method in its very context.

The method is particularly efficient because its foundations rely on the need-performance approach and its criteria are no-prescriptive; however, it is fully pertinent to national law's assessment instruments/tools.

A reference matrix, based on hierarchy-centered areas, categories and criteria to be assessed, enables the performance analysis of the buildings.

Each thematic area accounts for the main environmental issues such as the quality of the location, the resource consumption, the environmental cost, the indoor environment and service quality. As a result, the performance assessment needs to be carried out through 34 criteria for housing applications.

The Protocol Office contains 35 criteria (thematic evaluation), roughly the same as residential, but the parameters are calculated and the performance scales for the scoring change. In addition, there are several specific criteria, such as the presence of Building Automation and Control System (BACS). Through the evaluation of the individual criteria, one particular aspect of the building is considered. It refers to a specific

[9] The Section is mainly due to Antonella Violano.

[10] Ref. D'Angelo A., Violano A., Strumenti per valutare le prestazioni energetico-ambientali degli edifici, in Cannaviello M., Violano A. (ed. by), La Certificazione Energetica degli edifici esistenti, Franco Angeli Editore, Milano, 2007.

[11] www.architetturaecosostenibile.it

theme – energy, water, materials, comfort, impact on the site, quality of service etc. – verifying both the proximity to the objective of sustainability required, as to the current construction practice. It is important to highlight that this is a not absolute assessment, but compared to engineering practice typical of the region in which the building is located.

In Protocol Ithaca 2009, the building for each criterion and sub-criterion receives a score that can range from -1 to +5. Zero represents the standard of comparison (benchmark), that must be considered the current construction practice, in compliance with laws or regulations; 3 is the best practice, 5 the excellence.

The weight can be assessed according to:

A the extent of the potential effect (3 = global or regional; 2 = urban or sub-urban; 1 = building or location);
B the intensity of the potential effect (3 = strong or direct; 2 = moderate or indirect; 1 = weak);
C the duration of the potential effect (3 $\geq$ 50 years; 2 $\geq$ 10 years; 1 $\leq$10 years).

The Protocol ITACA 2004 provided the option of a -2 score, intended for such a performance as those significantly lower than the average standards. The partial scores amount to each feature considered in the assessment, and they are supposed to be computed through a weighted sum intended to reach a final score still on a scale from -1 to +5.

The score is assigned according to the guidelines and the control method listed in the “description sheet” concerning each assessment criterion. The pieces of information available in each sheet are the following:

- the need, that is the objective of environmental quality to be pursued;
- the relative weight of the criterion accounting for the importance of that specific criterion in the overall assessment method;
- the performance indicator; that is the parameter applied to assess the performance level of the building with regard to the assessment criterion; the latter can be quantitative or qualitative;
- the unit of measurement, as far as the quantitative performance indicators are concerned.

The amount of the assessment scores can be reached through points concerning the quality of the location, thereby, regardless of both the choices of the project and the quality of the building. The combination of the two leads to the final score of the building, but the performance scale and the computing method can be varied according to the type of the intervention: new building-construction / requalification.

7 Conclusion[12]

Design only by complying with the requirements dictated by current standards in energy efficiency and reducing consumption of non-renewable sources, is no guarantee of physical and mental well-being and thermo-hygrometric comfort of

[12] The Section is mainly due to Antonella Violano.

users. In fact, as evidenced in the Environmental burden of disease associated with inadequate housing (EDB) relation, the priority objective in the design practice, both in the case of new construction that in the energy diagnosis of the existing building, is a more economic efficiency rather than the comfort and environmental protection. However, the effort to spread to the masses the principles of environmentally conscious behavior is gradually filling a sort of "collective deposit of ecological intelligence" as defined by Goleman, which will help us to make our own decisions – and here I refer specifically to the designers – increasingly oriented towards eco-friendliness.

Assessment tools must be used as methods controlling ex-post performance energy-environment of buildings, designing innovative and efficient architecture and making appropriate decisions. Only when this happens, will it be really possible to change the design way.

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Using Agent Importance to Combat Preference Manipulation in Group Decision Making

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Abstract. We consider a problem that can arise in group decision-making when the selection process is based upon a group preference function obtained by an aggregation of the individual preference functions of the group members. We observe the possibility of the individual agents strategically manipulating the information they provide so as to further their own goal of getting their most preferred alternative selected by the group. With this in mind we suggest ways of modifying the formulation of the group decision functions to discourage strategic manipulation by the participating agents.

Keywords: Group decision-making, Multi-Agent, Preference Manipulation, Preference Aggregation.

1 Introduction

Group or multi-agent decision-making focuses on situations in which multiple participants in some joint activity must agree upon one action, it has roots in social choice theory [1, 2]. Approaches based on fuzzy methods have been developed to aid in this type of group decision–making [3-7]. One approach to implementing group decision–making is to obtain a preference function from each of the participating agents, combine these individual preferences and then use this aggregated preference function to select the best alternative. Here the aggregated preference function represents a "group" preference function. The selected action is then the one that optimizes this group preference function. However, what must be kept in mind here is that each of the individual agent's real objective *is the maximization of their own individual preference function not the maximization of the group function*. With this in mind any individual participating agent may use any appropriate strategy to enhance their chances of optimizing their individual payoff resulting from the group selection. One method to accomplish this is a strategic manipulation of the preference information that they supply to the decision process [8]. Since each participating agent provides a preference function to the formulation of the group decision function, a participating agent can bias the information they provide in a way to enhance the chances of obtaining their most preferred alternative. One strategy that an agent can use is to diminish the value of alternatives that are not among their most preferred. This type of strategy is often used in labor negotiations.

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In this work we look at some procedures for formulating the group preference function from the individual agent's preference functions and suggest methods for modifying these procedures so as to reduce the ability of an individual agents to benefit from the strategic manipulation of the preference function they provide to the group decision mechanism.

2 Multi-agent Group Decision Making

Assume we have a group of *n* agents who must collaborate on the choice of an action from the set X of alternative possible actions. Here each agent can represent their preference in terms of a function A_j over the set X so that, $A_j(x_i) \in [0, 1]$ is the degree to which agent j is satisfied with alternative x_i Here we shall view A_j as a fuzzy subset over X. We assume each agent assigns a value of one to its most preferred alternatives. It is also assumed that each agent is unaware of the preference function of the other participating agents.

A group decision mechanism consists of a process for selecting one of the alternatives based upon the preferences of the individual agents making up the group. A non-discriminatory decision mechanism should treat all the participants in the same way, we call this requirement *impartiality*. A second required property of such a decision mechanism is that it should have a positive association between individual preferences and group preference, specifically an increase in any agents preference for some alternative should not result in a reduction of the groups preference for this alternative.

One commonly used approach for obtaining a group decision mechanism is to aggregate the individual agents preference functions to obtain the group preference function A. Once having this group preference function we can then choose, as the selected alternative, the one with the largest value in A.

In this type of approach to group decision making one issue becomes the choice of the function used for the aggregation of the individual preference functions. We shall call the function that combines the individual preference functions the **P**reference **A**ggregation **F**unction (PAF). One type of PAF, called a pointwise PAF, is a mapping $F: I^n \to I$ such that for any $x \in X$, the group value is $A(x) = F(A_1(x), A_2(x), ..., A_n(x))$. Here we note that the group evaluation of each x just depends on the individual agents evaluation of x.

A number of other basic properties can be required of F, these are:

1. Symmetry, the indexing of the participants is unimportant
2. Monotonicity, if $a_i \geq b_i$ then $F(a_i, a_2, ..., a_n) \geq F(b_1, ..., b_n)$
3. $F(1, 1,..., 1) = 1$
4. $F(0, 0, ...0) = 0$

The first condition assures us of impartiality, it requires that each of the participants be treated similarly. The second condition enforces the positive association. The third condition is that if all the participants are completely satisfied with a solution then the group is completely satisfied with the solution. The fourth condition is that if all the participants are completely dissatisfied by a solution then the group should be completely dissatisfied.

We note that these four requirements are not very restrictive and allow a large number of different possible manifestations for F. The actual selection of F must be a reflection of what *collaborative imperative* the group uses. Here by collaborative imperative we mean the rules that the members of the group agree to use in determining joint action.

One example of a group collaboration imperative is to require that the group's choice must be acceptable to *all* members of the group. This imperative is clearly the default group decision imperative. This default group imperative needs no prior agreement as it is naturally imposed by the autonomy of each of the agents. We shall call this the **primal** collaboration imperative. The main feature of this primal imperative is that any participating agent can dismiss an alternative if they don't like it. This type of group imperative can be implemented using a Min aggregation operator, $A(x) = Min_i[A_i(x)]$. If for any $x \in X$ there exists one agent i such that $A_i(x) = 0$ then $A(x) = 0$. Any agent can unilaterally dismiss an alternative, this is the essence of requirement that *all* agents accept the solution.

More generally we can model this type of primal imperative using any t-norm aggregation operator T as $A(x) = T_i(A_i(x))$. We recall that a t-norm operator is a mapping $T: I \times I \to I$ such that; 1. $T(a, b) = T(b, a)$, 2. $T(a, T(b, c)) = T(a, b), c)$, 3. $T(a, b) \geq T(c, d)$ if $a \geq c$ and $b \geq d$ and 4. $T(a, 1) = a$. The t-norm satisfies the primal condition of individual dismissiveness, $T_j(A_j(x)) = 0$ if $A_j(x) = 0$ for some j. In addition to the min other t-norms are $T(a, b) = a\, b$ (product) and $T(a, b) = Max(0, a + b - 1)$ (bounded difference). It is well known that for any arbitrary t-norm T we have $T(a, b) \leq Min[a, b]$

3 Preference Information Manipulation

Once having a group decision function the alternative chosen is the one that maximizes this function. However, what must be kept in mind is that each of the individual agents is not really interested in maximizing the group decision function but their goal is to maximize their own individual preference function. With this in mind any agent can be justified to use any possible legal way to affect the decision mechanism so as to optimize their ultimate payoff from the decision. One such avenue open to a participant is to bias the information which they provide to the decision process, their indicated preference function, so as to most benefit themselves.

One desirable feature of any group decision procedure is that it should not allow individual benefit from this type of strategic manipulation. Specifically, a good decision procedure should encourage participants to provide their true preferences and discourage potential benefits from strategically manipulating their preference information. Consider the j^{th} agent. Let A_j be his true preference function, $A_j(x_i)$ being his degree of satisfaction with x_i. Assume that when he gives this preference function to the decision process, the group decision mechanism selects as the best alternative x*, giving agent j satisfaction $A_j(x^*)$. Let $\widehat{A}_j$ be some preference function different from A_j. Assume that if agent j provides this preference function to the group decision procedure the resulting group choice is x**. If it is the case that $A_j(x^{**}) > A_j(x^*)$ then the decision procedure is said to encourage strategic preference manipulation.

We emphasize here that the motivation for strategic manipulation is based on the fact that while the "group" is using A(x) to measure its satisfaction the individual is measuring his satisfaction by A_j, his true concern is to maximize this function.

Let us look at the proposed group decision function based on t-norms and see if it rewards strategic manipulation. Consider the case in which we are using the product operator. Let us focus on agent A_n. For any alternative x the group preference function is

$$A(x) = \prod_{i=1}^{n} A_i(x) = \left(\prod_{i=1}^{n-1} A_i(x) \right) A_n(x)$$

Let us denote $\prod_{i=1}^{n-1} A_i(x)$ as Q(x), it is the portion attributed to the agents other then A_n. Thus $A(x) = Q(x)\,A_n(x)$. Assume agent A_n prefers alternative x*, $A_n(x^*) = 1$. His objective is to try to make this the group choice. One possible way for him to try to obtain this goal is to supply the manipulated preference function $\hat{A}_n$ such that $\hat{A}_n(x^*) = 1$ and $\hat{A}_n(x) = 0$ for all $x \neq x^*$. In this case $A(x) = 0$ for all $x \neq x^*$ and hence assures that $A(x^*) \geq A(x)$ for all $x \neq x^*$. We see that this decision mechanism can be made to reward the strategic manipulation of preference information.

Specifically by diminishing the value of alternatives that are not among its favorite, an agent can increase the possibility of getting a higher degree of satisfaction in the group decision.

More generally the use of any t - norm operator lends itself to this kind of strategic manipulation. Let us have *n* agents then $A(x) = T_j(A_1(x), A_2(x), ..., \hat{A}_n(x))$ because of associativity $A(x) = T_j(Q(x), \widehat{A}_n(n))$ here $Q(x) = T_j(A_1(x), A_2(x), ..., A_{n-1}(x)]$ and $\widehat{A}_n$ is the strategically manipulated preference information supplied by the n^{th} agent. Let $A_n(x)$ be the "true" preference function of the n^{th} agent and assume $A_n(x^*) = 1$, x* is the preferred alternative by agent n. Assume this agent doesn't know anything about the other players preference functions, the best strategy for this agent is to provide $\widehat{A}_n$ such that $\widehat{A}_n(x) = 0$ for $x \neq x^*$ and $A_n(x^*) = 1$. If $Q(x^*) \neq 0$ then under $\widehat{A}_n(x)$ we have

$$A(x^*) = T(Q(x^*), 1) = Q(x^*) \neq 0$$

$$A(x) = T(Q(x), 0) = 0 \quad \text{for } x \neq x^*,$$

hence x* is the decision choice and the payoff to agent n is $A_n(x^*)$ his maximal payoff.

We note that if $Q(x^*) = 0$ then $A(x^*) = 0$ and hence $A(x) = 0$ for all x. In this case all alternatives are tied. We see then that this strategy always guarantees that x* will be among the most preferred alternatives of the group.

It should be pointed out that the widespread use of this strategy by multiple agents ultimately leads to a situation in which the group preference function deteriorates to assigning a value zero for all alternatives and hence being useless.

We now suggest an approach to counter balance the type of strategic manipulation described in the preceding. The basic manipulation strategy used in the preceding was for an agent to diminish the scores of those alternatives he finds unacceptable. One way of counterbalancing this strategy of diminishing the score is to try to penalize an agent for doing this. Our method to accomplish this is to introduce some idea of importance associated with each agent and make this importance depend upon his overall preference function.

Consider now the multiple agent decision process with *n* agents. Assume each agent has a true preference function as expressed by the fuzzy subset A_j over the set of alternatives, hence $A_j(x_i)$ is his value for alternative x_i. Furthermore, let B_j be the preference function that the agent provides to the decision mechanism. Let $S_j = \sum_i B_j(x_i)$, it is the sum of the scores over all alternatives by the agent. Furthermore, let $S^* = \text{Max}_j[S_j]$. We shall now associate with each agent a degree of importance I_j such that $I_j = \frac{S_j}{S^*}$. We see that $I_j \in [0, 1]$ and each agent's importance in the process is directly related to his total score, the more total score he provides the more important he is in the aggregation process. We note the agent with the most score has importance one.

Let us now recall the method for including importances in the t-norm type aggregations [9-12]. Assume (a_i, α_i) is a collection of pairs in which $\alpha_i \in [0, 1]$ is the importance of object i and $a_i \in [0, 1]$ is the value of object i in the aggregation. The importance weighted t-norm aggregation is denoted $\mathbf{a} = \mathbf{T}_i(a_i, \alpha_i)$ where $\mathbf{T}$ is the t-norm used. In this case the objects we must aggregate are pairs. As discussed in [13, 12] the method used to calculate this weighted t-norm is a two-step process. The first step is to calculate the effective value of a_i under the importance α_i, $e_i = h(a_i, \alpha_i)$ The second step is to aggregate these effective values $\mathbf{a} = \mathbf{T}_i[e_i]$. The usual method of calculating the effective value is to use a t-conorm operator S, normally it is the dual of **T**, $e_i = S(\overline{\alpha}_i, a_i)$. That is e_i is the t-conorm of the negation of the importance, $\overline{\alpha}_i = 1 - \alpha_i$, and the score a_i.

Let us briefly comment on the appropriateness of the procedure for including importances. First we recall that the t-norm operator is such that the smaller values have more effect on the operation. This is clearly exhibited in the case of Min t-norm where the value is determined solely by the smallest value. We also see this in the case of the product t-norm, $T(x, y) = xy$. In this case $\frac{\partial}{\partial x}T(x, y) = y$ we see that the smaller argument has the larger derivative. Consider now $e_i = S(\overline{\alpha}_i, a_i)$. From the monotonicity of the t-conorm it follows that the smaller the importance, the larger $\overline{\alpha}_i$ the larger e_i, thus in turn reduces the effect of this object in the subsequent t-norm aggregation. Thus the smaller the importance the less effect the component has in the overall aggregation. As a matter of fact if $\alpha_i = 0$, then $\overline{\alpha}_i = 1$ and $e_i = S(1, a_i) = 1$. Since 1 is the neutral element in the t-norm aggregation a pair with zero importance plays no role in the t-norm aggregation. We also note then if $\alpha_i = 1$, then $\overline{\alpha}_i = 0$ and $e_i = S(0, a_i) = a_i$. Thus we see the effective value lies between 1 and a_i.

Normally in calculating $T_i(S(a_i, \overline{\alpha}_i))$ the T and S operators are chosen as duals, $S(a, b) = 1 - T(\overline{a}, \overline{b})$, though this is not necessary. Two special cases of operators have special practical significance as well as being useful in our discussions. The first case is where $T(x, y) = Min(x, y) = x \wedge y$ and $S(x, y) = Max(x, y) = x \vee y$. Here $\mathbf{T}_i(a_i, \alpha_i) = Min_i[(\overline{\alpha}_i \vee a_i)]$. The second case is one in which $T(x, y) = x\, y$ (product) and $S(x, y) = x + y - x\, y$ (bounded sum). In this case the effective value is $e_i = S(\overline{\alpha}_i, a_i) = \overline{\alpha}_i + a_i\, \alpha_i$. Using this we get $\mathbf{T}_i[(a_i, \alpha_i)] = \prod_i(\overline{\alpha}_i + a_i\, \alpha_i)$.

Having the machinery for evaluating the importance weighted t-norm operator let us now look at the effect of our proposal of associating an importance weight with an agent depending on their total scoring of the alternatives.

Consider the case in which we have two agents and three alternatives. Let agent one have the preference function $A_1 = \left\{\frac{1}{x_1}, \frac{a_2}{x_2}, \frac{a_3}{x_3}\right\}$ agent two have the preference function $A_2 = \left\{\frac{b_1}{x_1}, \frac{1}{x_2}, \frac{b_3}{x_3}\right\}$. Assume agent one is intent on strategically manipulating its supplied preference information to try to improve its overall payoff. It wants to increase the possibility of getting x_1 as the group's choice. Let this manipulated preference function be $\hat{A}_1 = \left\{\frac{1}{x_1}, \frac{\hat{a}_2}{x_2}, \frac{\hat{a}_3}{x_3}\right\}$. We assume agent one is unaware of the preference function supplied by agent two. Using the information supplied by the agents we have $S_1 = 1 + \hat{a}_2 + \hat{a}_3$ and $S_2 = b_1 + 1 + b_3$ and hence

$I_1 = \dfrac{S_1}{\mathrm{Max}(S_1, S_2)}$ $I_2 = \dfrac{S_2}{\mathrm{Max}(S_1, S_2)}$ In this situation $A(x_j) = T(S(\bar{I}_1, \hat{a}_j), S(\bar{I}_2, \hat{b}_j))$. Let us first assume the use of Min-Max type operation, $A(x_j) = (\bar{I}_1 \vee \hat{a}_j) \wedge (\bar{I}_2 \vee b_j)$. Here $A(x_1) = \bar{I}_2 \vee b_1$, $A(x_2) = \bar{I}_1 \vee \hat{a}_2$ and $A(x_3) = (\bar{I}_1 \vee \hat{a}_3) \wedge (\bar{I}_2 \vee \hat{b}_3)$.

Agent one's goal is to try to select the values $\hat{a}_2$ and $\hat{a}_3$ so as to maximize the possibility that both $A(x_2)$ and $A(x_3)$ are less than $A(x_1)$. Consider now the use of the previous strategy of making $\hat{a}_2 = \hat{a}_3 = 0$. Here

$$A(x_1) = \bar{I}_2 \vee b_1,\ A(x_2) = \bar{I}_1 \vee 0 = \bar{I}_1 \text{ and } A(x_3) = \bar{I}_1 \wedge (\bar{I}_2 \vee 0) \leq \bar{I}_1 \leq A(x_2)$$

In this case, where $\hat{a}_2 = \hat{a}_3 = 0$, we have $S_1 = 1 + \hat{a}_2 + \hat{a}_3 = 1$ and $S_2 = b_1 + 1 + b_2 > 1$, $S_2 \geq S_1$. From it follows that $I_2 = 1$ and $I_1 = \dfrac{S_1}{S_2} = \dfrac{1}{1 + b_1 + b_3}$ and therefore $\bar{I}_2 = 0$ and $\bar{I}_1 = \dfrac{b_1 + b_3}{1 + b_1 + b_3}$. This gives us $A(x_1) = b_1$ and $A(x_2) = \dfrac{b_1 + b_3}{1 + b_1 + b_3}$. In order to obtain the selection of x_1 agent one must attain $A(x_1) > A(x_2)$, this requires that $b_1 > \dfrac{b_1 + b_3}{1 + b_1 + b_3}$ which requires that $b_3 < \dfrac{(b_1)^2}{1 - b_1}$. Thus if $b_3 > \dfrac{(b_1)^2}{1 - b_1}$, the manipulating decision maker can't guarantee that the chosen alternative will be x_1, even if he inputs a manipulated preference function. Thus the use of inclusion of importance weights associated with the agent's total score is one method for preventing this type of manipulation.

Furthermore, it is possible for the manipulating agent to obtain less satisfaction using this type of strategy manipulation than he would obtain providing his true preference function [8].

4 Alternative Collaboration Imperatives

The requirement that all agents agree on the acceptability of a solution, as implicit in the use of the t-norm aggregation, is a very strong one. As we noted, this situation gives an agent a strong influence, it allows an agent to unilaterally dismiss an alternative even if all other agents desire it. This situation strongly contributed to the ability of an agent to benefit from strategic manipulation of its preferences.

There exist other possible rules that the group can use such as only requiring that *most* of the agents agree or 2/3 agree [14]. These rules reduce the influence of an individual agent. We shall call the rule used by the group its **collaboration imperative**. From a formal point of the structure of the aggregation function F used to combine the preference functions of the individual agents is determined by the collaboration imperative selected.

While a linguistic specification of the collaboration imperative such as *most* provides one way of capturing this information, a more quantified representation is needed in order to use this information to construct the aggregation function complying with it. In [15] Zadeh described a framework for transforming linguistic concepts into fuzzy subsets, the objects in our case being linguistic quantifiers, such as *all, most*, *some*

In the following we shall generalize on this idea and provide a formal definition for a collaboration imperative. This representation of a collaboration imperative will allow us to construct a unique formal function that can be used to combine the scores of the individual participants. In addition this representation can provide a bridge with which to connect linguistic or other less formal descriptions about the collaborative imperative into formal representations.

Formally by a collaborative imperative we shall mean a mapping $h: I \rightarrow I$ such that if an r proportion of the members of the group are satisfied with a solution, then h(r) is the degree to which the group is satisfied with a solution. A number of properties can be rationally associated with this function. The first is that if nobody in the group is satisfied by a solution the group has zero satisfaction, $h(0) = 0$. If everybody is satisfied then the group should be completely satisfied, $h(1) = 1$. Finally the more individual satisfaction the more the group satisfaction, $h(x) \geq h(y)$ if $x > y$. Thus $h: I \rightarrow I$ is a mapping such that: 1. $h(0) = 0$, 2. $h(1) = 1$ and 3. $h(x) \geq h(y)$ if $x > y$

It is interesting to note that this collaboration imperative which describes how the group translates individual satisfaction into group satisfaction is uncontaminated by reference to specific individuals. This indifference helps allow the group to formulate a group collaborative imperative.

Let us look at some examples of collaboration imperatives. One example, which we shall denote as h_*, is such that $h_*(x) = 0$ for all $x \neq 1$ and $h_*(1) = 1$. This is seen to be a characterization of the collaborative imperative of requiring **all** members of the group to agree on the preferred solution. Another example, h^*, is defined as $h^*(x) = 1$ for all $x \neq 0$ and $h^*(0) = 0$. This can be seen as formulation of the imperative "if any participant likes a solution it is acceptable to the group." These two can be seen as extreme opposite examples of collaboration imperatives. The duality of these two imperatives is formally expressed by the fact that these are the two bounding cases of collaborative imperatives, $h_*(x) \leq h(x) \leq h^*(x)$ for all $x \in [0,1]$. Other collaboration imperatives are linear $h(r) = r$ and the power case $h(r) = r^p$ where $p \in (-\infty, \infty)$.

In order to obtain the group preference function A(x) under the collaborative imperative h we can use an OWA aggregation [16, 17] in which the weights are determined by the collaborative imperative. Denoting $w_j = h(\frac{j}{n}) - h(\frac{j-1}{n})$ we get as our aggregation function

$$A(x) = \sum_{j=1}^{n} w_j \, b_j$$

when b_j is the j^{th} largest of the $A_i(x)$. Thus here $A(x) = OWA_h(A_1(x), \ldots, A_n(x))$

The aggregation resulting from some special cases of collaborative imperatives are worth pointing out. If h = h_*, h(1) = 1 and h(x) = 0, x ≠ 0, then our aggregation becomes the Min, A(x) = $Min_i(A_j(x)]$. If h = h*, h(0) = 0 and h(x) = 1 for x ≠ 0 then A(x) = $Max_i[A_j(x)]$. If h is linear, h(x) = x, then $w_j = h(\frac{j}{n}) - h(\frac{j-1}{n}) = \frac{1}{n}$ here then A(x) = it is the average of the individual agents preference for alternative x.

Assume h_1 and h_2 are two collaboration imperatives. Consider the situation in which h_1 and h_2 are such that $h_1(x) \geq h_2(x)$ for all x ∈ [0,1], we denote this as $h_1 > h_2$. In this case we see h_2 is more demanding in its requirements for group agreement. The smaller h the more unanimity is required for a solution to be acceptable. Viewed from a different perspective the smaller h the more power an individual has to dismiss a solution. For at the extreme, in the case of h_*, which is the smallest collaborative imperative, any individual can dismiss any alternative. Notice that on the other hand for the case h*, the largest collaborative imperative, the individual has the least ability to unilaterally dismiss an alternative. In order to capture this idea we shall associate with each collaborative imperative a measure we call the **V**alue **O**f **I**ndividual **D**isapproval, VOID, which is defined as

$$\text{VOID}(h) = 1 - \int_0^1 h(x)dx$$

We see $\int_0^1 h(x)dx$ is the area under h(x). It is easy to see that VOID(h_*) = 1 and VOID(h^*) = 0 Furthermore when h(x) = x, then VOID(h) = 0.5

In selecting a collaborative imperative a group may consider a number of issues. One is a desire to have unanimity, this means arriving at a solution acceptable to everyone. Here we are unwilling to sacrifice the good of an individual to benefit the others. This of course mediates an inclination toward using the collaboration imperative, h_* one that requires all participants agree.

While the use of a collaborative imperative other than the Min reduces the power of the individual agent and in turn reduces the effect of the type of strategic manipulation of preferences we discussed, it clearly doesn't eliminate it.

Consider, for example, the collaborative imperative, h(x) = x. This as we indicated leads to a situation in which A(x) = F($A_1(x)$, $A_2(x)$, ..., $A_n(x)$) $= \frac{1}{n}\sum_{i=1}^{n} A_i(x)$

Consider an agent q who has a preference function A_q such that $A_q(x_1) = 1$, his preference is for x_1. Using strategic manipulation to try to assure the selection of x_1 he can provide a preference function which reduces the satisfaction values attributed to the other alternatives, he can supply $\widehat{A}_q$ instead of A_q where $\widehat{A}_q(x_1) = 1$ and

$A_q(x_j) = 0$ for all $x_j \neq x_1$. This strategy clearly acts to increase the possibility of x_1 being the group choice.

As we have suggested a way to discourage an agent from engaging in this type of strategic manipulation is to penalize him for this type of manipulation. As already indicated one way to accomplish this is to make an agent's importance proportional to the total preferences he allocates to the all the alternatives. One way we can accomplish this in the case of a linear collaborative imperative is described in the following. Let $S_j = \sum_i A_j(x_i)$ and let $\mathcal{S} = \sum_{j=1}^{n} S_j$. Using this we obtain an importance weight for agent j, $I_j = \frac{S_j}{\mathcal{S}}$ We then can calculate the group satisfaction using a weighted average instead of the simple average,

$$A(x) = \sum_{j=1}^{n} I_j A_j(x) = \sum_{j=1}^{n} \frac{S_j}{\mathcal{S}} A_j(x)$$

Thus an agent's influence is made proportional to his score.

We easily see that the reduction of preferences by an agent diminishes his weight, influence in the aggregation. Let $S_j = \sum_i A_j(x_i)$ and let $\widehat{S}_j = \sum_i \widehat{A}_j(x_i)$ where $\hat{A}(x_i) \leq A(x_i)$. In this case $I_j = \frac{S_j}{\tilde{T} + S_j}$ and $\hat{I}_j = \frac{\hat{S}_j}{\tilde{T} + \hat{S}_j}$ where $\widetilde{T} = \mathcal{S} - S_j$. We see that then

$$\hat{I}_j - I_j = \frac{\widehat{S}_j}{\widetilde{T} + \widehat{S}_j} - \frac{S_j}{\widetilde{T} + S_j}$$

hence his influence decreases on all alternatives. This situation also means that his score is less effective on his most preferred one.

We now shall generalize the preceding approach for calculating the group preference function. We shall now introduce a procedure, based upon a modulation of an agent's importance, that can be used for penalizing an agent for strategic manipulation which is applicable for any collaboration imperative h,. This procedure uses a method for including importances in OWA aggregation developed in [18].

Assume we have *n* agents each providing a preference function in the form of a fuzzy subset A_j over the space of possible courses of action. Further we assume the agents have agreed to use the collaboration imperative h. The first step is to calculate an importance weight for each participating agent. We let $u_j = \frac{S_j}{\mathcal{S}}$ where as in the preceding $S_j = \Sigma_i A_j(x_i)$, the sum of preference scores allocated by agent A_j, and $\mathcal{S} = \sum_{j=1}^{n} S_j$, the total preference scores allocated by all the agents. Thus u_j is the importance weight associated with A_j. We note these u_j lie in the unit interval and sum to 1.

For any alternative x we now calculate the group score of x, A(x) as follows. Let b_i be the i^{th} largest of the $A_j(x)$ and let v_i be the importance value, associated with the agent which has the i^{th} largest preference for x. We then let $w_i = h(\sum_{k=1}^{i} v_k) - h(\sum_{k=1}^{i-1} v_k)$ and then we calculate $A(x) = \sum_{i=1}^{n} w_i b_i$ Here we again emphasize that b_i is the i^{th} largest satisfaction with x by the members of the group.

In general the use of this type of importance factor has the desired effect of penalizing agents for low overall scoring of the alternatives. Specifically we see this as follows. For simplicity assume the agents have been indexed such that $A_i(x) \geq A_j(x)$ if $i < j$, $A_1(x)$ is the largest and $A_n(x)$ is the smallest. In this case because of our indexing $b_i = A_i(x)$ and $v_i = u_i$. Here then $A(x) = \sum_{i=1}^{n} w_i A_i(x)$ where $w_i = h(\sum_{k=1}^{i} u_k) - h(\sum_{k=1}^{i-1} u_k)$. Let us denote $\sum_{k=1}^{i-1} u_k = g(i-1)$ and therefore $w_i = h(g(i-1) + u_i) - h(g(i-1))$. Since h is monotonic, essentially decreasing u_i decreases w_i and since $u_i = \frac{S_i}{\mathcal{S}}$ we see that decreasing S_i tends to decrease w_i the importance of alternative A_i in the formulation of the group preference function. Parenthetically it should be noted that while we have used $S_i = \Sigma_j A_i(x_j)$ other formulations, such as $S_i = \Sigma_j (A_i(x_j))^2$, can be used as long as they are monotonic with respect to the scores., ie if $S_i = R(A_i(x_1), A_i(x_2).......,)$ then R must be such that $A_i(x_j) \geq \widehat{A}_i(x_j)$ for all j then

$$R(A_i(x_1), A_i(x_2), A_i(x_3). ....,) \geq R(\widehat{A}_i(x_1), \widehat{A}_i(x_2), \widehat{A}_i(x_3). ....,)$$

Different formulations for R reflect different strengths for the penalization.

5 Conclusion

Multi-agent group decision-making is based upon the selection of some action that maximizes a group preference function where the group preference function is formulated by some type of aggregation of the individual participating agents preference functions. In this work it was pointed out while the selected alternative is the one that maximizes the group's preference function each individual member's goal is to try to attain a solution that maximizes its own individual preference function. Since the individual agent preference information used in the construction of the group preference function is provided by the agent itself, and since each agents goal is the selection its favorite alternative, this opens the possibility for strategic manipulation of the information provided by the agents. In this work we began to look at ways of addressing this issue. Fundamental to our approach is an attempt to penalize an agent for strategic manipulation by reducing their importance in the formulation of the group preference function. One type of strategic manipulation, one particularly effective in situations in which the collaborative imperative has a high value of individual disapproval (VOID), is for an agent to dramatically reduce their satisfaction with alternatives that are not among their most preferred. In this environment we indicated that an effective method for discouraging this type of manipulation is to make an agents importance in the formulation of the group decision function directly related to the total of the scores in its supplied preference function, the more satisfaction manifested the more importance.

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