Chapter 82
Algorithm for Estimating Mountainous Surface Area Based on GPS Data

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Abstract By combining global positioning system (GPS) device with carrier phase of tracking technology, the engineering staffs can obtain geological information more accurately. In this chapter, we did the estimation experiment of mountain surface acreage according to the algorithm created, analyzed the absolute accuracy and relative accuracy of the estimation results, and verified the reliability and accuracy of the algorithm. And then we further analyzed the stability of the algorithm , the estimate results obtained by the algorithm can maintain good stability in the case of a relatively small amount of data.

Keywords GPS · Algorithm · Triangulated irregular networks · Surface fitting

82.1 Introduction

82.1.1 GPS and Accurate Surveying

The global positioning system (GPS) is a space-based satellite navigation system that provides location and time information in all weather, anywhere on or near the Earth, where there is an unobstructed line of sight to four or more GPS satellites.

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It is maintained by the United States government and is freely accessible to anyone with a GPS receiver.[1] It became fully operational in 1994.

All satellites broadcast at the same two frequencies, 1.57542 GHz (L1 signal) and 1.2276 GHz (L2 signal). L1 contains Coarse-acquisition (C/A) code and encrypted precision $P(Y)$ codes, plus the L1 civilian (L1C) and military (M) codes on future Block III satellites. L2 contains $P(Y)$ code, plus the L2C and military codes on the Block IIR-M and newer satellites. The C/A code, for civilian use, transmits data at 1.023 million chips per second, whereas the P code, for U.S. military use, transmits at 10.23 million chips per second. The P code can be encrypted as a so-called $P(Y)$ code that is only available to military equipment with a proper decryption key. The L1 carrier is modulated by both the C/A and P codes, while the L2 carrier is only modulated by the P code [1].

Integrating external information into the calculation process can materially improve accuracy of positioning. A common method that is used in surveying applications is carrier phase tracking. The period of the carrier frequency multiplied by the speed of light gives the wavelength, which is about 0.19 m for the L1 carrier. Accuracy within 1 % of wavelength in detecting the leading edge reduces this component of pseudo range error to as little as 2 mm. This compares to 3 m for the C/A code and 0.3 m for the P code. However, the accuracy of 2 mm requires measuring the total phase—the number of waves multiplied by the wavelength plus the fractional wavelength, which requires specially equipped receivers. The method of carrier phase tracking has many surveying applications [2].

82.1.2 Methods for Surface Area Processing

Surface area is a fundamental parameter derived from terrain analysis. A variety of methods exist in the literature for measuring terrain irregularity. Hobson [3] described some early computational methods for estimating surface area and discussed the concept of surface area ratios. Beasom [4] described a method for estimating land surface ruggedness based on the intersections of sample points and contour lines on a contour map. Jenness [5] described a similar method based on measuring the density of contour lines in an area. Mandelbrot [6] described the concept of a "fractal dimension" in which the dimension of an irregular surface lies between 2 (representing a flat plain) and 3 (representing a surface that goes through every point within a volume). Calculating fractal dimension can be very computationally challenging. Polidori et al. [7], Lam and De Cola [8], and Lorimer et al. [9] discussed a variety of methods for estimating the fractal dimension for a landscape. Among these methods, two have been proved to be relatively simple and practicable and of the most widely use.

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[1] http://en.wikipedia.org/wiki/GPS.

Triangulated irregular networks (TINs) [10] has always been used to generate surface area statistics, especially when trying to analyze areas enclosed by vector-based polygons. It contains the simple notion of splitting the target area into small pieces and using the given data to estimate them each. This method can give a relatively more accurate result of estimation when given enough terrain information.

Raster-based [11] analyses offer several advantages that are difficult or impossible to achieve with TINs, including neighborhood analysis, faster processing speed, and more consistent output. Digital elevation models (DEM) has been researched mostly. A DEM is a continuous grid containing the elevation at its spatial location in each grid cell. The slope area for each pixel can be estimated based on the resolution and the slope for the pixel and then the surface area can be produced by summarizing the slope areas of all the pixels contained in the target region [12]. The surface area of each pixel can also be calculated directly from a DEM using elevation information from that pixel plus that of the eight adjacent pixels [13].

To estimate the surface area of a relatively small mountain region, method of TINs has its unique advantage. With adequate high-accuracy data of terrain information, TINs can give reliable result of estimation. In the situation of estimating small mountainous region, amount of calculation is less important comparing with accuracy. We consider introducing surface fitting methods to TINs in order to improve the accuracy and consistency of estimation.

82.2 TIN-Based Surface Fitting

Combined with triangulation theory and idea of least squares surface fitting, triangulate the plan according to the coordinate data points, then start the surface fitting by using each triangle and the ten points extended around the triangle. The strategy of taking point is shown in Fig. 82.1:

In order to obtain the approximate area of the triangle region, least squares fit of the binary function has been done by selecting the vertex of the triangle and the peripheral 10 points in area D, a total of 13 data points coordinates been selected. The specific way is as follows:
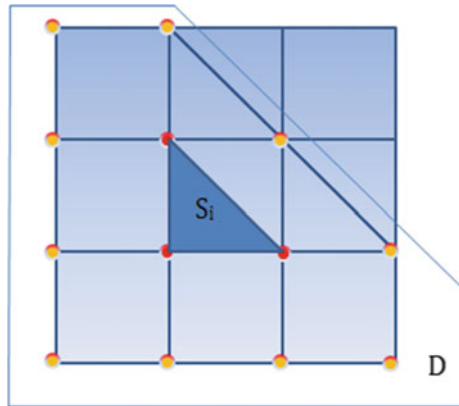
If $z = zx^2 + by^2 + cxy + dx + ey + f + \varepsilon_m$, a, b, c, d, e, f are constants.

$$Q = \sum_{m=1}^{n} \left(z_m - ax_m^2 - by_m^2 - cx_m y_m - dx_m - ey_m - f \right)^2$$

According to our model, $n = 13$.

$$\frac{\partial Q}{\partial a} = \frac{dQ}{db} = \frac{dQ}{dc} = \frac{dQ}{dd} = \frac{dQ}{de} = \frac{dQ}{df} = 0$$

Fig. 82.1 Surface fitting strategy

Results in:

$$
\begin{bmatrix}
\sum x_m^4 & \sum x_m^3 y_m & \sum x_m^2 y_m^2 & \sum x_m^3 & \sum x_m^2 y_m & \sum x_m^2 \\
\sum x_m^3 y_m & \sum x_m^2 y_m^2 & \sum x_m y_m^3 & \sum x_m^2 y_m & \sum x_m y_m^2 & \sum x_m y_m \\
\sum x_m^2 y_m^2 & \sum x_m y_m^3 & \sum y_m^4 & \sum x_m y_m^2 & \sum y_m^3 & \sum y_m^2 \\
\sum x_m^3 & \sum x_m^2 y_m & \sum x_m y_m^2 & \sum x_m^2 & \sum x_m y_m & \sum x_m \\
\sum x_m^2 y_m & \sum x_m y_m^2 & \sum y_m^3 & \sum x_m y_m & \sum y_m^2 & \sum y_m \\
\sum x_m^2 & \sum x_m y_m & \sum y_m^2 & \sum x_m & \sum y_m & n
\end{bmatrix}
\begin{bmatrix}
a \\ b \\ c \\ d \\ e \\ f
\end{bmatrix}
=
\begin{bmatrix}
\sum x_m^2 z_m \\
\sum y_m^2 z_m \\
\sum x_m y_m z_m \\
\sum x_m z_m \\
\sum y_m z_m \\
\sum z_m
\end{bmatrix}
\tag{82.1}
$$

According to Eq. (82.1) and the measured data in the domain D, we can calculate the values of a a, b, c, d, e, f.

Therefore, according to $z = ax^2 + by^2 + cxy + dx + ey + f$, we can get the least squares fitting surface of domain D, using the integral formula of the surface area:$S_i = \iint dS_i = \iint \sqrt{1 + z_x'^2 + z_y'^2} \, dxdy$, $z_x' = 2ax + cy + d$, $z_y' = 2by + cx + e$

Substitute them into the integral formula, we get:

$$
S_i = \iint \sqrt{1 + (2ax + cy + d)^2 + (2by + cx + e)^2} \, dxdy \tag{82.2}
$$

According to the integral (82.2), we can get each triangular unit area S_i, the sum will be the total surface area of the region $\sum S_i$.

This selection of data points and the approximate surface fitting, not only ensures the accuracy of the approximate fitting, but also links inner surface of the triangular unit with periphery of the triangular unit surfaces. Taking the advantages of these two ways, we can avoid the limitations and get better results in the approximate calculation of the surface area.

82.3 Experiment

82.3.1 Estimating Mountainous Surface Area

By the use of GPS devices combined with the carrier phase tracking technology, accurate mountainous terrain information and data can be obtained. The data can be converted into the form of three-dimensional vector (x, y, z), in which (x, y) denotes the horizontal coordinates, z denotes vertical elevation of that point. Below is a mountain contour map, the three-dimensional vector data of each evenly distributed point with five meters interval have been obtained during the experiment (Fig. 82.2).

According to the estimation algorithm of surface area, the surface area of the region which is 12 m above sea level can be estimated. When handling the boundary, take the characteristics of this algorithm into consideration and do the following process: intersect the plane whose altitude is 12 m with fitting surfaces in the triangular unit, re-determine the upper and lower limit of surface area integral in the triangular unit. That is: $z = ax^2 + by^2 + cxy + dx + ey + f = 12$.

The result is the triangular boundary unit area integral domain S.

$$S_i = \iint ds = \iint \sqrt{1 + (2ax + cy + d)^2 + (2by + cx + e)^2}\, dxdy \qquad (82.3)$$

Fig. 82.2 Contour map of the target area
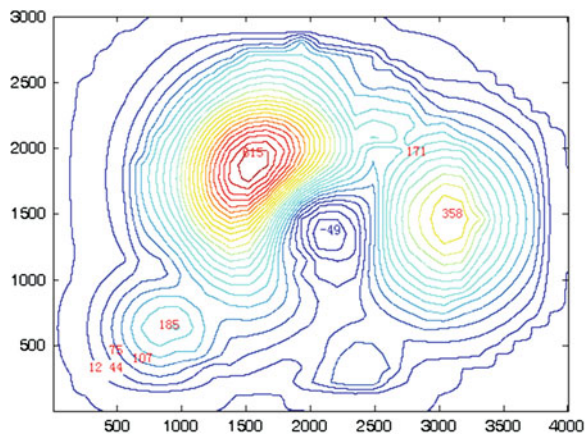
Obviously, the uncertainty of dealing with boundary situation in pure triangulation algorithm will not happen by using proposed algorithm. The determined limits of integration can be given during the calculation. This algorithm has unique advantages.

By calculating the fitting surface area of each triangular unit and summation, we finally get: $\sum S_i = 1.00631467599 * 10^7$.

82.3.2 Accuracy Analysis

According to the estimation model, further analyzing the accuracy of the estimation, we can get the following results based on the above derivation:

$$
\begin{aligned}
S_i &= \iint \sqrt{1 + z_x'^2 + z_x'^2}\,dxdy \\
z &= ax^2 + by^2 + cxy + dx + ey + f \\
S_i &= \iint \sqrt{1 + (2ax + cy + d)^2 + (2by + cx + e)^2}\,dxdy \\
u &= 2ax + cy + d, \quad v = 2by + cx + e
\end{aligned}
\tag{82.4}
$$

Estimation error is

$$
\Delta S_i = \frac{\partial S_i}{\partial a}\Delta a + \frac{\partial S_i}{\partial b}\Delta b + \frac{\partial S_i}{\partial c}\Delta c + \frac{\partial S_i}{\partial d}\Delta d + \frac{\partial S_i}{\partial e}\Delta e
\tag{82.5}
$$

According to the highest accuracy of GPS positioning, we can take data error as 10^{-4} m, then

$$
\Delta a = \Delta b = \Delta c = \Delta d = \Delta e = 10^{-4}
$$
$$
\Delta S_i = 10^{-4}\left(\frac{\partial S_i}{\partial a} + \frac{\partial S_i}{\partial b} + \frac{\partial S_i}{\partial c} + \frac{\partial S_i}{\partial d} + \frac{\partial S_i}{\partial e}\right)
\tag{82.6}
$$

We can do the transformation using the following formula.

$$
\frac{\partial S_i}{\partial a} = \frac{\partial S_i}{\partial u}\frac{\partial u}{\partial a} + \frac{\partial S_i}{\partial v}\frac{\partial v}{\partial a}
\tag{82.7}
$$

Do substitution, and then we can get.

$$
\begin{aligned}
\frac{\partial S_i}{\partial a} + \frac{\partial S_i}{\partial b} + \frac{\partial S_i}{\partial c} + \frac{\partial S_i}{\partial d} + \frac{\partial S_i}{\partial e} &= \frac{\partial S_i}{\partial u}(2x + y + 1) + \frac{\partial S_i}{\partial v}(2y + x + 1) \\
&= \iint \frac{u(2x + y + 1) + v(2y + x + 1)}{\sqrt{1 + u^2 + v^2}}\,dxdy
\end{aligned}
\tag{82.8}
$$

According to Cauchy inequality:

$$u(2x + y + 1) + v(2y + x + 1) \leq \sqrt{u^2 + v^2}\sqrt{(2x + y + 1)^2 + (2y + x + 1)^2}$$

(82.9)

Both sides divided by $\sqrt{1 + u^2 + v^2}$:

$$\frac{u(2x + y + 1) + v(2y + x + 1)}{\sqrt{1 + u^2 + v^2}} \leq \frac{\sqrt{u^2 + v^2}}{\sqrt{1 + u^2 + v^2}}\sqrt{(2x + y + 1)^2 + (2y + x + 1)^2}$$

$$\leq \sqrt{(2x + y + 1)^2 + (2y + x + 1)^2}$$

$$\Delta S_i \leq 10^{-4} \iint \sqrt{(2x + y + 1)^2 + (2y + x + 1)^2} \, dxdy$$

(82.10)

Then the absolute accuracy of the estimated results will be:

$$\sum \Delta Si \leq 10^{-4} \times 107 \times 960,000 \approx 10^3$$

(82.11)

According to the results we have got, the relative accuracy of the area estimation is

$$\frac{\sum \Delta S_i}{\sum S_i} \approx \frac{10^3}{10^7} = 10^{-4}$$

(82.12)

82.3.3 Stability Analysis of the Estimation

The relative error of this area estimation model can be controlled in 10^{-4} magnitude; we will further analyze the stability of established model.

The distance distribution of data points can be changed by reducing the quantity of the data collection point, estimate the area of the region respectively, compare the estimated results obtained in different amount of data, and the results obtained from triangulated plane area estimation were compared to the established algorithm. Under the condition of distance of data points are 5, 10, 15, 20, 25 m, the results are shown in Fig. 82.3.

It can be seen from the figure that with the gradual reduction in the amount of data, data-collected points (data points) distribution gradually sparse, description of terrain details gradually rough for estimated results, the accuracy of the two algorithms have different degrees of decline. But at the same time, we found that the algorithm used given terrain information for surface fitting, at the expense of
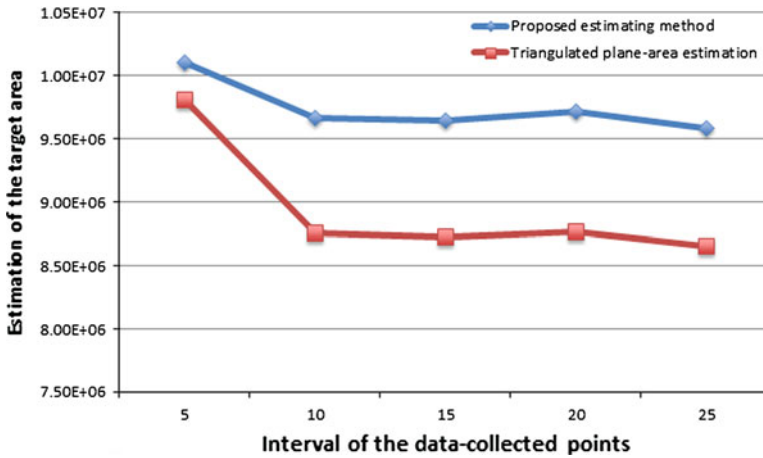
Fig. 82.3 Stability analysis of the estimating result comparing TIN-based surface fitting with triangulated plane-area estimation

some computational efficiency, but the results will much closer to the true value, and the amount of data has been reduced, data distribution also become thinning in the process, the estimated results can maintain to better stability.

82.4 Conclusion

GPS device combined with the carrier phase of tracking technology can help the engineering staff to obtain more accurate geological information, the use of surface data to infer surface characteristics of the geographical; can conduct more accurate surface area estimation. According to the combination of TINs methods and surface fitting methods, established the TIN-based Surface Fitting algorithm. This algorithm can obtain accurate surface information data to achieve higher estimation accuracy through the use of GPS equipment. Estimation experiment of the mountain surface area by using this algorithm verified the results of estimation has better estimation accuracy, so the algorithm has good reliability and accuracy. Besides, the estimates of this algorithm can also maintain good stability under the condition of the amount of data reduced and data-collected points gradually sparse. Finally, we still need to point out that the surface fitting algorithm for surface area estimates, making the calculation amount increased. It can get better estimation results, if the range of the surface area is small. However, for a wide range of surface area, the applicability of this algorithm will not do well and have certain limitations.

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