Chapter 1
Cellular Automata: Models of the Physical World

Herbert W. Franke*

ZIB-Institute, Germany

Abstract. Cellular automata can be applied to simulate various natural processes, particularly those described by physics, and can also serve as an abstract model for all kinds of computers. This results in a intriguing linkage between physics and the theory of automata. Such connections prove to be suggestive in the experiment, to be described below, to apply cellular automata as models for mechanisms in the physical world. Based on such analogies, the properties of our world can be formulated in the simplest possible way. The primary focus lies not on the explicit simulation of certain laws of nature but on the general principle underlying their effects. By choice of suitable algorithms, local and causal conditions as well as random deviations can be visually rendered. In addition, the problem of determinism can be handled. Apart from the classification of computable and non-computable processes, a third category of phenomena arises, namely, mechanisms which are deterministic but not predictable. All of these characteristics of our world can be classified as aspects of some underlying structure. And, the laws of nature are apparently consistent with the evolution of a multiplicity of relatively well-defined structures.

The concept of cellular automata goes back originally to John von Neumann. The central proposition of his work was the concept of an abstract computer with universal capabilities, which could produce the blueprint of any possible computer as well as reproduce a copy of itself. The underlying question was whether, in this context, the possibility existed of self-reproduction of animate beings [28]. The idea of visualizing the distribution of instantaneous states on a graphical grid was introduced by the mathematician Stanislaw Ulam. John von Neumann's system (which contained a small error, corrected by his successors) was extremely intricate. Later, simpler solutions were discovered. For example, John Horton Conway's "Game of Life" [2] also turned out to be a cellular automaton.

The definitive advance is due to Stephen Wolfram, who proved that all of the systematic properties contained in a rectangular grid mirrored those which show up in a one-dimensional configuration, which can be represented along a single line. Wolfram had therefore identified the least complex type of cellular automaton [11, 12, 13, 14].

* I thank S.M. Blinder for help by the translation –HWF.

H. Zenil (Ed.): *Irreducibility and Computational Equivalence*, ECC 2, pp. 3–10.
DOI: 10.1007/978-3-642-35482-3_1

Based on their generic behavior, cellular automata can be categorized into four groups as follows.

Class 1: After a finite number of steps, a uniform homogeneous final state is reached, with all cells either empty or filled.

Class 2: Initially generated simple local patterns, sometimes changing into vertical stripes or continually recurring repetition of short cycles.

Class 3: Patterns spreading in an apparently irregular way, typical clusters evolving at intervals .

Class 4: Processes depending sensitively on a set of initial values. This might lead to behavior similar to one of the classes generated above. Sometimes, these structures are unstable and non-periodic. The automata belonging to this class will also generate laterally shifted patterns, that is oblique lines or stripes. This class possibly contains universal automata.

In all four cases, an infinite cell-space is necessary, so that the growth mechanism is unimpeded. Otherwise repetitions would necessarily be produced, sooner or later. This classification was based, more or less, on heuristic aspects; only later a parameter was found by Christopher G. Langton, which he labeled lambda, its value increasing with increasing class number. Lambda expresses quantitatively the possibility of a cell's survival in the transition to the next generation [7].

1 The Turing Machine and Gödel's Principle

Since all kinds of automata can be simulated by universal cellular automata, this also applies to Turing machines [5]. This raises the question of the connection with one of the deepest and most fundamental questions in mathematics: do unsolvable problems exist? Gödel had proved by complicated logical argumentation that there are indeed undecidable mathematical and logical problems. Now, that same proof can be carried out in a far more graphic way using Alan Turing's abstract automaton, which in its most general form also has the quality of universality. Every algorithm created to solve a problem of any kind can be simulated by a Turing automaton, and a problem turns out to be unsolvable if the output sequence does not terminate. There exist a number of deep analogies between the Gödel principle, the Turing machine, and cellular automata. Thus, fundamental principles of mathematics are equivalent to the functioning of automata and, by extension, to everything that can be simulated by them, including interactions among physical objects.

One special example is the predictability of questions that fall within the scope of logic or mathematics. There is no generally applicable procedure to determine whether a mathematical problem is solvable or not. The only way to find out is to actually construct a solution, by whatever creative means that can be applied. When you harness a Turing machine for such a problem, the sequence of steps will not be predictable in advance even if they follow one another in a deterministic manner.

2 Cellular World-Models

Cellular automata have been applied to all kinds of problems, including the elucidation of mathematical problems, the modeling of automata, and the simulation of scientific processes, such as evolutionary mechanisms [4, 1]. They have proven to be especially useful when applied to physical phenomena. Several attempts were directed towards a “digital mechanics”: Ed Fredkin suggests that classical-mechanical systems are equivalent to cellular automata [8]. Cellular automata later served to simulate various types of structure-generating processes, among others, diffusion processes in fluid mechanics. Furthermore they shed some light on the formation of symmetrical patterns in natural phenomena.

The first to introduce the concept of “Rechnender Raum”, or “Computational Space”, was Konrad Zuse. According to his ideas, elementary particles can behave as sub-microscopic computers, interacting among themselves and thereby somehow reproducing known physical phenomena [19]. In particular, those phenomena that can be represented by differential equations are well suited for the digital modeling via cellular automata [10].

Attempts to construct direct digital models of physical processes, e.g., the propagation of waves, might appear at first sight to be clumsy and unrealistic. More promising, however, is the exploration of the fundamental ordering principles in our universe, considering the analogy between physics with its mechanisms and cellular automata. The starting-point of the argument is this: if the physical world is describable at all in mathematical terms, then the entire sequence of intermediate steps must also be modeled as cellular automaton, although possibly in a rather complex and intricate way. Certain general properties that are valid for all cellular automata must then also apply for the world as a whole [6]. At least, all those possible structures that are also implemented in the smallest cellular automaton must be present. So that, while some processes can be simulated only within certain limitations, generally valid statements can be made about the whole system of laws of nature and their interrelationships, solely by comparison with the smallest possible devices which can simulate them.

3 Locality and Causality

The algorithms for the control of cellular automata can be considered to correspond to the basic laws of physics. These are embedded in a program that prescribes how they are applied [3]. The structure and design of this program is extremely simple, not only because of the rules for a minimal number of states and functional connections, but also by keeping these rules unchanged from start to finish in a program run. This principle corresponds to the widely-accepted presumption of physicists that the basic laws of nature have not changed since the beginning of our universe. Because at every step the newly-arising distribution of values is subject to the same set of rules, the sequence of states can conceptually be regarded as an iterative process.

Temporal continuity must be analogously true, corresponding to the usual assumption of a spatial continuum. It is taken for granted that the same laws of nature are valid everywhere in the universe. It would be quite easy to insert a local dependence into the program, but, as far as we know at present, that does not appear to be the case.

Two more evidently universal rules of physics have, from the outset, been included in the concept of the cellular automaton. By permitting only adjacent cells to influence the state of the next generation, we limit our considerations to behavior which obeys locality–there are no nonlocal effects, and each effect on one cell is mediated only by its immediate neighbors. It could be demonstrated that a kind of information transfer is feasible within cellular automata by freeing a cluster of cells from its surrounding group and setting it adrift in something like a round trip across space and time. This phenomenon corresponds to the emergence of diagonal stripes in cellular automata of the fourth type.

The same situation holds true for the time-dependent effects, which are of a strictly causal nature in the prototypical cellular automaton, and which are assumed to influence only immediately subsequent time intervals. Any effect transmitted from one cell to another thus needs the activation of all intermediate generations. These spatial and temporal adjacency rules demand that a certain cell can exert influence only within a certain limited space, and that an effect working on a certain cell can originate only within a limited space. This situation corresponds to Einstein's Light Cone, which degenerates in cellular automata into a triangle, the cell forming the starting or end point located at the top or bottom vertex. The time interval between the states, when the effects are handed on from one generation to the next, thereby behaves as an analogy to the finite speed of light.

Within classical mechanics, there arises the problem of the reversibility of events. As can easily be seen, this is normally not the case. State N+1 does not allow the reconstruction of the previous state N. In other words, different distributions in a generation can lead to exactly the same distribution in the next one. On the other hand, the algorithms can be designed so that the process will also run in the reverse direction. As Ed Fredkin has shown, this is the case if the principle of cellular automata is somewhat extended, such that not only the preceding generation, but, in addition, the antecedent of that generation are allowed to influence its successor. The simplest case is encoded by the following equations:

$$z(t) = f(t-1) - z(t-2) \tag{1}$$

Then there also exists an inverse algorithm:

$$z(t-2) = f(t-1) - z(t) \tag{2}$$

This leads to a correspondence with classical mechanics: information about the momentary place is not sufficient for calculating the subsequent state, additional information must be given about the rate of change (speed or impulse are normally used for this purpose). In this manner, by embedding the immediate as well as the remote past, the rate of change can be calculated.

4 Determinism or Randomness?

So far, we have discussed only purely deterministic examples, the course of events being immutably fixed by the starting conditions. As a result of the equivalences between cellular automata and the Turing machine, the process needs not necessarily be computable. It is conceivable that the physical processes described by the laws of nature never do come to an end, which means that we are simulating the behavior of a cellular automaton which runs deterministically, but is not computable. There are conflicting philosophical viewpoints that do not accept the influence of chance on what happens in the world. For them, determinism fulfills their belief that the world runs according to strict rules, embracing all creation and all apparent innovation, both expected and unexpected. Innovation, originating in this way, is the equivalent of chaos as understood in dynamical chaos theory, which, as we know, is based not on actual chance, but on noncomputability.

The type of randomness described above has to be distinguished from that encountered in quantum theory, which is non-deterministic on a fundamental level. Most theoretical physicists will accept that, despite some disagreements about details, a final definitive answer to this question remains to be formulated in the future. But it is quite possible to test this idea with cellular automata. This can be done by introducing randomly-induced modifications–“mutations”–into the algorithms. An easier way, however, of adding an irregular interference or disturbance would consist in randomly changing the states in various places;

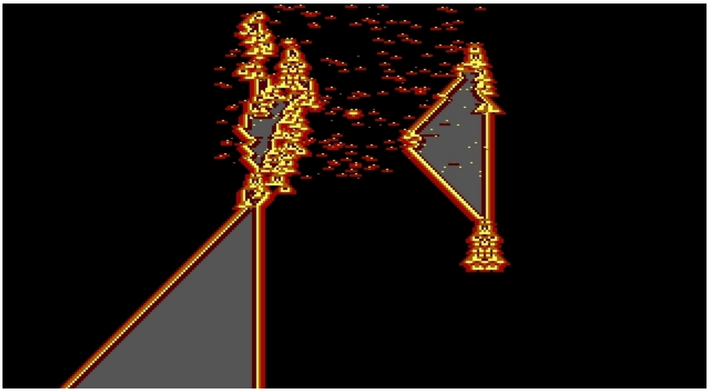
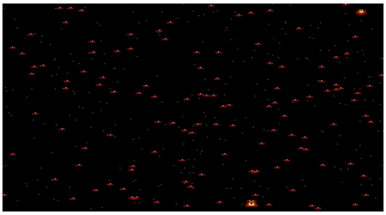
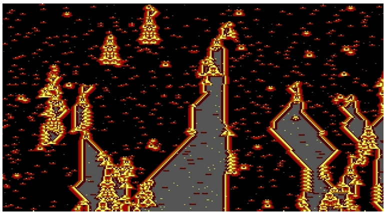


Fig. 1. For illustration, we use a cellular automaton with the two states and the transition code 0 1 2 3 4 5 / 1 1 0 1 0 0 probably of the class four type [11]. Scale of colors: 0 black, 1 bright brown, 2 bright blue, 3 yellow, 4 dark brown, 5 dark gray. The picture shows the origin of patterns on the begin of evolution, emerging of a locally and temporally limited field of chaotic distribution of initial states. As soon the evolution has reached the random free zone, the rules for the automaton produce no more shapes, but only emptiness or crystal-like order.

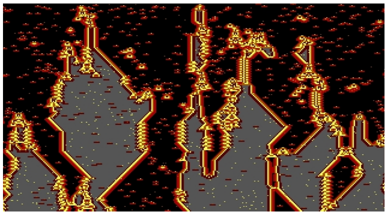


(a) 398

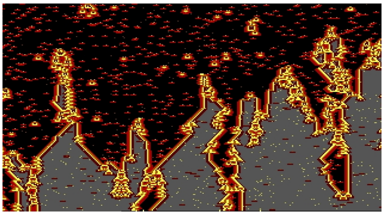
(b) 400



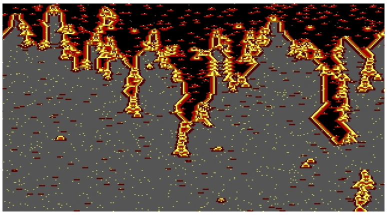
(c) 402



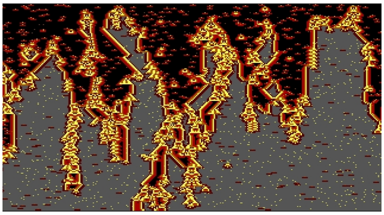
(d) 404



(e) 406



(f) 408



(g) 410

Fig. 2. Structures generated under an increasing influence of randomness (images followed by their density parameter). Some random-selected sites of the lines are occupied with the state 1. New shapes emerge there where such sites come in neighbourhood. By this series of pictures, the density of introduced randomness is expressed by a numerical parameter - the pictures show the situation for some parameters between 398 and 410. Randomness acts as a creative effect which counteracts against the trend towards order which the set of rules tries to maintain. A specific value of the mentioned parameter (approximately by 400) defines the status of balance between growing and destruction of structures. Such parameter gives a characteristic value for every cellular automaton.

we might say that chance could be interspersed, for example by adding a few extra lines of program code containing a randomizer.

Much information can be gained from a comparison between repeated runs of the same cellular automaton with and without disturbance (see figures). As can be seen, by the application of a disturbing element, an antagonism, a competition between order and disorder is triggered. There are cellular automata that obviously possess a strong trend towards expressing their repertoire of patterns, and thus easily suppressing all germs of chaos. On the other hand, there are others in which even a slight touch of randomness suffices to "lead them astray" or make them run out of control, so that a great multiplicity of different patterns is generated. A Class One linear cellular automaton requires a strong dose of randomness to get its regularity disturbed, but, all the same, the previous pattern will soon be re-established. In Class Three automata, by contrast, a minimal disturbance is enough to render impossible a return to a homogeneous pattern; the chances are that random effects generate nuclei of larger well-ordered clusters.

What is visually expressed in the illustrations can also be seen as aspects of information or complexity: irreversible and deterministic automata run in a manner in which complexity can never increase, but in most cases must inevitably decrease. As a consequence, the patterns get more and more simple, they degenerate into cyclic sequences that fill the whole available space or vanish completely. Only reversible processes retain their complexity, and innovation emerges, if at all, via re-ordering as understood by the deterministic modification of chaos theory. The formation of complexity then becomes possible only in stochastic models.

The structural variety of our world as we experience it might also spring from a deterministic model without the influence of chance if that model belongs to the category of undecidable mathematical problems. Since, however, a decisive answer on whether this is the case cannot possibly be given, since there would always exist the alternative that such a world will sooner or later turn into crystal-like rigidity or dissolve into chaos, possibly in the sense of the Heat Death of the universe. This kind of world is philosophically unsatisfying, but it is not our option to choose in what sort of world we actually live. It is quite informative to view it from a completely different point of view, asking ourselves: How must a universe be built that will keep its structure-creating capability forever and with certainty? The best solution is an endlessly running cellular automaton modified with that touch of randomness that conforms with its innate trend towards regularity.

5 Conclusions

To sum up, cellular automata turn out to be possible models to visualize the basic structure of our world. By reducing that structure to the least complex programs, they enable us to deal in a more definite way with various relevant problems, including those with philosophical implications–near and far effects, causality, determinism, and entropy. A new multiplicity of problems, triggered not in the least by chaos theory, is that of the formation of structures, as this capability inherent in nature is doubtlessly of fundamental relevance. Preeminent in this context is the role of randomness, an issue since the early days of the quantum theory. To identify our universe as a Class Four cellular automaton is at present no more than a speculation, but in future considerations involving this class of problems it will have to be considered as a promising candidate.

References

[1] Codd, E.F.: Cellular Automata. Academic Press, New York (1968)
[2] Berlekamp, E., Conway, J., Guy, R.: Gewinnen– Strategien für mathematische Spiele. Vieweg, Braunschweig (1985)
[3] Franke, H.W.: Die Welt als Programm. Naturwissenschaftliche Rundschau 45, 379 (1992)
[4] Gerhard, M., Schuster, H.: Das digitale Universu, Zelluläre Automaten als Systeme der Natur. Vieweg, Braunschweig (1995)
[5] Herken, R. (ed.): The Universal Turing Machine. A Half-Century Survey. Kammerer und Unverzagt, Hamburg (1988)
[6] Hedrich, R.: Komplexe und fundamentale Strukturen. BI Wissenschaftsverlag, Mannheim, Wien, Zürich (1990)
[7] Langton, C.G.: Life at the Edge of Chaos. In: Langton, C.G. (ed.) Artificial Life II. Addison-Wesley, Redwood City (1991)
[8] Margolus, N.: Physics-like Models of Computation. In: Farmer, D., Toffoli, T., Wolfram, S. (eds.) Cellular Automata, Physica D, vol. 10 (1984)
[9] von Neumann, J.: The Theory of Self-reproducing Automata. In: Burks, A.W. (ed.) Essays on Cellular Automata. University of Illinois Press, Urbana (1970)
[10] Wunsch, G.: Zellulare Systeme. Vieweg, Braunschweig (1977)
[11] Wolfram, S.: Cellular Automata as Models for Complexity. Nature 311, 419 (1984)
[12] Wolfram, S.: Universality and Complexity in Cellular Automata. In: Farmer, D., Toffoli, T., Wolfram, S. (eds.) Cellular Automata. North-Holland (1984)
[13] Wolfram, S.: Theory and Applications of Cellular Automata. World Scientific Publishing Company, Singapore (1986)
[14] Wolfram, S.: A New Kind of Science. Wolfram Media (2002)
[15] Zuse, K.: Rechnender Raum. Vieweg, Braunschweig (1969)