

Colour Image Segmentation with Integrated Left Truncated Bivariate Gaussian Mixture Model and Hierarchical Clustering

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Abstract. Image segmentation plays a dominant role in image analysis and image retrievals. Much work has been reported in literature regarding image segmentation based on Gaussian mixture model (GMM). The main drawback of GMM is regarding the assumption that each image region is characterized by Gaussian component, in which the feature vector is mesokurtic and having infinite range. But in colour images the feature vector is represented by Hue and Saturation which are non- negative and may not be symmetrically distributed. Hence the image segmentation can not be accurate unless the non-negative nature of the feature vector is included. In this paper an image segmentation method is developed and analyzed with the assumption that the bivariate feature vector consisting of Hue and Saturation of each pixel follows a left truncated bivariate Gaussian mixture model. In this method the number of components (Image regions) are determined by Hierarchical clustering. The segmentation algorithm is proposed under Bayesian frame with maximum likelihood. The experimentation with six images taken from Berkeley dataset reveals that the proposed image segmentation method outperforms the existing image segmentation method with GMM and finite left truncated bivariate Gaussian mixture model with K-means.

Keywords: Image Segmentation, Bivariate Gaussian Mixture model, Image Quality Metrics, Hierarchical clustering, EM- algorithm.

1 Introduction

Image retrieval and image segmentation become an important aspect in computer vision and machine learning with the evaluation of new information technology more effective and efficient methods are developed for human computer interaction [18]. Image segmentation is an important technology for image processing and

understanding. Over the last three decades, the interest of researchers around the world has been focused on image segmentation. Lot of research activities have been carried out on gray scale image segmentation. Colour features have been found to be effective in image segmentation [5] [6] [7][15] [17]. Choice of a colour space is the main aspect of colour feature extraction.

We can generate colour spaces such as HSI, CIE -Lab, and CIE-Luv by nonlinear transformation of the RGB space. The HSI offers the advantage that separate channels outline certain colour properties, namely Intensity (I), Hue (H), and Saturation (S). Since H and S are functions of I , we consider the feature vector for characterizing the colour image with Hue and Saturation. It is also supported by the arguments given in text book "Digital Image Processing" [12]. Hence, in this paper we consider the feature vector for characterizing the colour image is a bivariate vector consisting of Hue and Saturation.

In colour image segmentation, model based image segmentation methods are more efficient than edge based or threshold or region based methods [8]. In model based image segmentation it is customary to consider that the whole image is characterized by a finite Gaussian mixture model. That is, the feature vector of each image region follows a Gaussian distribution [3] [5] [6] [9] [10] [11] [13] [16] [19] [20] [21]. The image segmentation methods based on Gaussian mixture model work well only when the feature vector of the pixels are having infinite range and the distribution of the feature vector is symmetric and meso-kurtic. But in many colour images the feature vector represented by Hue and Saturation will have finite values (say nonnegative) and may not be mesokurtic and symmetric. Hence, to have an accurate image segmentation of these sorts of colour images it is needed to develop and analyze image segmentation methods based on truncated bivariate mixture distributions. With this motivation, in this paper some image segmentation technique based on truncated bivariate Gaussian mixture distribution are developed and analyzed.

Here, it is assumed that the feature vector in different image regions follows a left truncated bivariate Gaussian distribution and the feature vector of the whole image is characterized by a finite left truncated bivariate Gaussian mixture model. This assumption is made since the Hue and Saturation values of the pixel which represents the bivariate feature vector can take non negative values only and hence, the range of the Hue and Saturation values are to be left truncated at zero. The effect of the truncated nature of Hue and Saturation values cannot be ignored, since the leftover probability is significantly higher than zero in the left tail end of the distribution. This left truncated nature of the bivariate feature vector can approximate the pixels of the colour image more close to the reality.

The model parameters are estimated by using Expectation Maximization (EM) algorithm. The initialization of the model parameters for carrying the EM-algorithm is done through feature vector of the pixel intensities of the image regions obtained through Hierarchical clustering and moment method of estimation. An image segmentation algorithm with component likelihood maximization under Bayesian frame work is developed and analyzed.

The performance of the developed segmentation algorithm is compared with finite Gaussian mixture model with K -means and also with finite left truncated bivariate

Gaussian mixture distribution with K -means algorithm by obtaining segmentation performance measures. The performance of reconstructed images are studied by computing the image quality metrics [2].

2 Estimation of the Model Parameters by EM- Algorithm

In this section we discuss the estimates of the model parameters through EM-algorithm. Here, it is assumed that the feature vector of each image region follows a left truncated bivariate Gaussian distribution with joint probability density functions of the form

$$g_i(x_s, y_s; \theta) = \frac{\exp\left\{\frac{-1}{2(1-\rho_i^2)}\left[\left(\frac{x_s - \mu_{1i}}{\sigma_{1i}}\right)^2 - 2\rho_i\left(\frac{x_s - \mu_{1i}}{\sigma_{1i}}\right)\left(\frac{y_s - \mu_{2i}}{\sigma_{2i}}\right) + \left(\frac{y_s - \mu_{2i}}{\sigma_{2i}}\right)^2\right]\right\}}{2\pi\sqrt{1-\rho_i^2}\sigma_{1i}\sigma_{2i}\int_0^\infty\int_0^\infty f_i(x, y; \theta) dx dy} \tag{2.1}$$

where,

$$f_k(x_s, y_s; \theta) = \frac{1}{2\pi\sqrt{1-\rho_k^2}\sigma_{1k}\sigma_{2k}} \exp\left\{\frac{-1}{2(1-\rho_k^2)}\left[\left(\frac{x_s - \mu_{1k}}{\sigma_{1k}}\right)^2 - 2\rho_k\left(\frac{x_s - \mu_{1k}}{\sigma_{1k}}\right)\left(\frac{y_s - \mu_{2k}}{\sigma_{2k}}\right) + \left(\frac{y_s - \mu_{2k}}{\sigma_{2k}}\right)^2\right]\right\}$$

and $0 < x < \infty ; 0 < y < \infty$

As a result of this, the feature vector of the entire image follow a finite left truncated bivariate Gaussian distribution with probability density function

$$h(x, y; \theta) = \sum_{i=1}^K \alpha_i g_i(x, y; \theta) \tag{2.2}$$

where, $g_i(x_s, y_s; \theta)$ is as given in equation (2.1) and $0 < \alpha_i < 1, \sum_{i=1}^K \alpha_i = 1$.

The parameters $\alpha_k, \mu_{1k}, \mu_{2k}, \sigma_{1k}^2, \sigma_{2k}^2$, and ρ_k , for $k=1,2,\dots,K$ are obtained by using the EM-algorithm . The updated equations of the parameters in each image region are obtained for the EM-algorithm. The parameters $\alpha_k, \mu_{1k}, \mu_{2k}, \sigma_{1k}^2, \sigma_{2k}^2$, and ρ_k , for $k=1,2,\dots,K$ are taken as given in [14].

3 Initialization of the Parameters using Hierarchical Clustering

To utilize the EM-algorithm we have to initialize the parameter α_k and the model parameters $\mu_{1k}, \mu_{2k}, \sigma_{1k}^2, \sigma_{2k}^2$, and ρ_k which are usually considered as known apriori. The initial values of α_i can be taken as $\alpha_i = \frac{1}{K}$, where, K is the number of image regions obtained from the Hierarchical clustering algorithm [4]. After obtaining the final value for the number of regions K , we obtain the initial estimates of μ_{1k} ,

$\mu_{2k}, \sigma_{1k}^2, \sigma_{2k}^2$, and ρ_k for the k^{th} region using the segmented region values with the moment method of estimation given by [1] for truncated bivariate normal distribution with initial parameters. After getting these initial estimates for $\mu_{1k}, \mu_{2k}, \sigma_{1k}^2, \sigma_{2k}^2$, and ρ_k , we obtain the final refined estimates of the parameters through EM-algorithm given in section 2.

4 Segmentation Algorithm

In this section, we present the image segmentation algorithm. After refining the parameters the prime step is image segmentation by allocating the pixels to the segments. This operation is performed by segmentation algorithm. The image segmentation algorithm consists of four steps

Step 1) Obtain the number of image regions using hierarchical clustering algorithm.

Step 2) Obtain the initial estimates of the model parameters using hierarchical clustering and moment estimates for each image region as discussed in section 3.

Step 3) Obtain the refined estimates of the model parameters $\mu_{1k}, \mu_{2k}, \sigma_{1k}^2, \sigma_{2k}^2, \rho_k$ and α_k for $k=1,2,\dots,K$ by using the EM-algorithm with the updated equations given by in section 2.

Step 4) Assign each feature vector to the corresponding j^{th} region (segment) according to the maximum likelihood of the j^{th} component L_j .

That is, (x_s, y_s) is assigned to the j^{th} region for which L_j is maximum.

where,

$$L_j = \max_{j \in k} \left\{ \frac{\exp \left\{ \frac{-1}{2(1-\rho_k^2)} \left[\left(\frac{x_s - \mu_{1k}}{\sigma_{1k}} \right)^2 - 2\rho_k \left(\frac{x_s - \mu_{1k}}{\sigma_{1k}} \right) \left(\frac{y_s - \mu_{2k}}{\sigma_{2k}} \right) + \left(\frac{y_s - \mu_{2k}}{\sigma_{2k}} \right)^2 \right] \right\}}{2\pi\sigma_{1k}\sigma_{2k}\sqrt{1-\rho_k^2}} \int_0^\infty \int_0^\infty f_k(x, y, \theta) dx dy \right\}$$

5 Experimental Results and Performance Evaluation

To demonstrate the utility of the image segmentation algorithm developed in this section, an experiment is conducted with six images taken from Berkeley image data set (<http://www.eecs.berkeley.edu/Research/Projects/CS/Vision/bsds/BSDS300/html>). The images namely, OSTRICH, POT, TOWER, BEARS, DEER and BIRD are considered for image segmentation. The feature vector consisting of Hue and Saturation values of the whole image is assumed that it follows a mixture of left truncated bivariate Gaussian distribution. That is the whole image is a collection of K -components and the feature vectors in each component follows a left truncated bivariate Gaussian distribution. The number of image regions of each image considered for experimentation is determined by hierarchical clustering algorithm. The number of image regions for each image obtained through hierarchical clustering for the images under study are given in Table1.

Table 1. Estimated value of K (By Hierarchical Clustering)

IMAGE	OSTRICH	POT	TOWER	BEARS	DEER	BIRD
Estimate of K	2	3	4	3	3	2

From Table 1, it is observed that the images, OSTRICH and BIRD have two segments each, the images POT, BEARS and DEER have three segments each and the image TOWER has four segments. The initial values of the model parameters μ_{1i} , μ_{2i} , σ_{1i}^2 , σ_{2i}^2 , ρ_i and α_i , for $i = 1, 2, \dots, K$ for each image region are computed by using the method given in section 3. Using these initial estimates and the updated equations of the EM-algorithm given in section 2, the final estimates of the model parameters for each image are obtained. Using the estimated probability density function and image segmentation algorithm, the image segmentation is done for the six images under consideration. The original and segmented images are shown in Figure 1. The performance of the developed image segmentation method is studied by obtaining the image segmentation performance measures namely, probabilistic rand index (PRI), global consistency error (GCE) and the variation of information (VOI). A comparative study of the developed algorithm based on finite left truncated bivariate Gaussian mixture model with hierarchical clustering (FLTBMGM-H) with the image segmentation algorithms based on finite GMM with K -means algorithm and finite left truncated bivariate Gaussian mixture model with K -means is carried. The image segmentation performance measures are computed for the three methods and presented in Table 3.

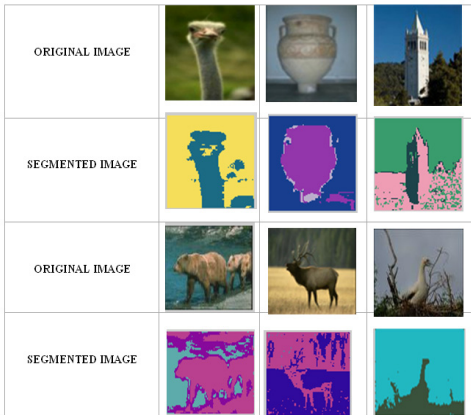


Fig. 1. Original and Segmented images

Table 2. Comparative Study of Image Quality Metrics

IMAGES	METHOD	PERFORMANCE MEASURES		
		PRI	GCE	VOI
OSTRICH	GMM-K	0.9234	0.4317	2.2761
	FLTBMGM -K	0.9782	0.4037	1.7611
	FLTBMGM -H	0.9810	0.3587	0.9481
POT	GMM-K	0.9456	0.4281	2.5973
	FLTBMGM -K	0.9796	0.4131	1.9263
	FLTBMGM -H	0.9801	0.3895	1.6415
TOWER	GMM-K	0.9615	0.4469	3.7121
	FLTBMGM -K	0.9816	0.4302	2.8194
	FLTBMGM -H	0.9821	0.3725	1.6554
BEARS	GMM-K	0.9121	0.4418	3.2693
	FLTBMGM -K	0.9831	0.4337	2.6421
	FLTBMGM -H	0.9834	0.4331	2.6386
DEER	GMM-K	0.9774	0.4829	2.2863
	FLTBMGM -K	0.9847	0.4030	1.3947
	FLTBMGM -H	0.9849	0.3995	1.2987
BIRD	GMM-K	0.9673	0.4671	2.7197
	FLTBMGM -K	0.9705	0.4226	2.3244
	FLTBMGM -H	0.9722	0.4170	2.3100

From the Table 3, it is observed that the PRI values of the proposed algorithm for the six images are more than that of the values from the segmentation algorithm based on finite Gaussian mixture model with K -means and finite left truncated bivariate Gaussian mixture model with K -means and close to 1. Similarly GCE and VOI values of the proposed algorithm for the images are less than that of finite GMM with K -means algorithm and finite left truncated bivariate Gaussian mixture model with K -means. This reveals that the proposed algorithm performs better than the existing algorithms based on the finite Gaussian mixture model and finite left truncated bivariate Gaussian mixture model.

Using the estimated probability density function of the images under consideration the images are retrieved and are shown in Figure 2. The image quality metrics with respect to the estimated models, the finite Gaussian mixture model with K -means and finite left truncated bivariate Gaussian mixture model with K -means and are presented in Table 4.

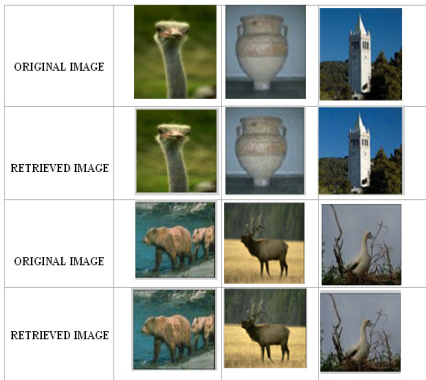


Fig. 2. The Original and Retrieved Images

Table 3. Table 4: Comparative Study of Image Quality Metrics

IMAGE	QUALITY METRICS	GMM- K	FLTRGMM- K	FLTRGMM-H
OSTRICH	Maximum Distance	0.4013	0.3067	0.5107
	Image Fidelity	0.7910	0.8076	0.9316
	Mean Square Error	0.0932	0.0793	0.0330
	Signal to Noise Ratio	13.3781	13.9959	15.1734
	Image Quality Index	0.8102	0.8302	0.8910
POT	Maximum Distance	0.3290	0.3957	0.3978
	Image Fidelity	0.6729	0.6786	0.6937
	Mean Square Error	0.0738	0.0467	0.0433
	Signal to Noise Ratio	11.7480	13.0240	13.1034
	Image Quality Index	0.6075	0.6174	0.6310
TOWER	Maximum Distance	0.8481	0.8757	0.9583
	Image Fidelity	0.5217	0.3854	0.7635
	Mean Square Error	0.2101	0.1792	0.0676
	Signal to Noise Ratio	8.8488	8.8724	10.9233
	Image Quality Index	0.5173	0.5271	0.7741
BEARS	Maximum Distance	0.5387	0.8765	0.8813
	Image Fidelity	0.4277	0.6386	0.6388
	Mean Square Error	0.0872	0.0484	0.0413
	Signal to Noise Ratio	9.1417	10.7590	10.7373
	Image Quality Index	0.5906	0.3951	0.6667
DEER	Maximum Distance	0.6217	0.6474	0.6592
	Image Fidelity	0.3992	0.4470	0.4640
	Mean Square Error	0.0828	0.0547	0.0510
	Signal to Noise Ratio	10.0629	11.8918	11.9536
	Image Quality Index	0.3703	0.3640	0.4131
BIRD	Maximum Distance	0.8429	0.9129	0.9321
	Image Fidelity	0.1920	0.2349	0.2352
	Mean Square Error	0.2013	0.0900	0.0894
	Signal to Noise Ratio	8.9231	9.3864	9.4108
	Image Quality Index	0.3481	0.4160	0.5479

From the Table 4, it is observed that all the image quality measures for the six images are meeting the standard criteria. This implies that using the proposed algorithm the images are retrieved accurately. A comparative study of the proposed algorithm with that of the algorithms based on finite Gaussian mixture model and finite left truncated bivariate Gaussian mixture model and K -means reveals that the proposed model in retrieving the images is better than the other models.

6 Conclusion

In this paper an image segmentation method based on finite left truncated bivariate Gaussian mixture model with hierarchical clustering is developed and analyzed. The model parameters are estimated by EM-algorithm and a segmentation algorithm with component maximum likelihood is developed. The performance of this algorithm is studied by conducting an experiment with six images. The probability density

functions of the images are also estimated. The image segmentation performance measures are computed for the six images. From a comparative study it is observed that the results obtained for the colour image segmentation method based on finite left truncated bivariate Gaussian mixture model with hierarchical clustering are better than the results obtained for image segmentation method based on finite left truncated bivariate Gaussian mixture model with K -means. It is further observed that the segmented images using the method discussed in this paper are having clear boundaries.

The performance analysis revealed that the hierarchical clustering used for the initial segmentation has significant influence on the performance of the image segmentation and image retrievals. This image segmentation method is much useful for segmentation and retrieval of the images in medical diagnosis, film and video production, remote sensing, robotics, security monitoring, etc., where, the colour image is characterized by Hue and Saturation values. This segmentation technique is also useful for denoising the image by filtering the background noise which is an important aspect of content based image retrieval. The integration of heuristic segmentation methods (K -means and Hierarchical) with model based image segmentation reduces the computational time and complexities in colour image segmentation.

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