Does History Matter? Empirical Analysis of Evolutionary Versus Stationary Equilibrium Views of the Economy

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Abstract The evolutionary vision in which history matters is of an evolving economy driven by bursts of technological change initiated by agents facing uncertainty and producing long term, path-dependent growth and shorter-term, non-random investment cycles. The alternative vision in which history does not matter is of a stationary, ergodic process driven by rational agents facing risk and producing stable trend growth and shorter term cycles caused by random disturbances. We use Carlaw and Lipsey's simulation model of non-stationary, sustained growth driven by endogenous, path-dependent technological change under uncertainty to generate artificial macro data. We match these data to the New Classical stylized growth facts. The raw simulation data pass standard tests for trend and difference stationarity, exhibiting unit roots and cointegrating processes of order one. Thus, contrary to current belief, these tests do not establish that the real data are generated by a stationary process. Real data are then used to estimate time-varying NAIRU's for six OECD countries. The estimates are shown to be highly sensitive to the time period over which they are made. They also fail to show any relation between the unemployment gap, actual unemployment *minus* estimated NAIRU and the acceleration of inflation. Thus there is no tendency for inflation to behave as required by the New Keynesian and earlier New Classical theory.

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We conclude by rejecting the existence of a well-defined a short-run, negatively sloped Philips curve, a NAIRU, a unique general equilibrium with its implication, a vertical long-run Phillips curve, and the long-run neutrality of money.

Economists face two conflicting visions of the market economy, visions that reflect two distinct paradigms, the Newtonian and the Darwinian. In the former, the behaviour of the economy is seen as the result of an equilibrium reached by the operation of opposing forces—such as market demanders and suppliers or competing oligopolists—that operate in markets characterised by negative feedback that returns the economy to its static equilibrium or its stationary equilibrium growth path. In the latter, the behaviour of the economy is seen as the result of many different forces—especially technological changes—that evolve endogenously over time, that are subject to many exogenous shocks, and that often operate in markets subject to positive feedback and in which agents operate under conditions of genuine uncertainty.[1]

One major characteristic that distinguishes the two visions is *stationarity* for Newtonian economics and *non-stationarity* for the Darwinian. In the stationary equilibrium of a static general equilibrium model and the equilibrium growth path of a Solow-type growth model, the path by which the equilibrium is reached has no effect on the equilibrium values themselves. *In short, history does not matter*. In contrast, an important characteristic of the Darwinian vision is path dependency: what happens now has important implications for what will happen in the future. *In short, history does matter*.

In this paper, we consider, and cast doubts on, the stationarity properties of models in the Newtonian tradition. These doubts, if sustained, have important implications for understanding virtually all aspects of macroeconomics, including of long term economic growth, shorter term business cycles, and stabilisation policy.

1 Two Worlds Views[2]

1.1 Views in Which History Does Not Matter

Virtually all mainline macro theories share a stationary equilibrium approach to understanding the economy. The old fashioned Keynesian model, expressed in its simplest form as IS-LM, had a short run equilibrium that did not necessarily

[1] The use of the terms Darwinian and Newtonian here is meant to highlight the significant difference in equilibrium concept employed in the two groups of theories that we contrast, the evolutionary and what we call equilibrium with deviations (EWD) theories. Not all evolutionary theories, including the one employed here, are strictly speaking Darwinian in the sense that they embody replication and selection. We use the term, Darwinian to highlight the critical equilibrium concept of a path dependent, non-ergodic, historical process employed in Darwinian and evolutionary theories and to draw the contrast between that and the negative feedback, usually unique, ergodic equilibrium concept employed in Newtonian and EWD theories.

[2] We have compared and contrasted many aspects of these two views in Lipsey et al. (2005: Chapter 2, hereafter LCB) and here we give only a brief outline to set the stage for what follows.

produce full employment. When it was subsequently closed by a simple Phillips curve, it had the property that, for any given money supply, a long run equilibrium emerged. Price level changes restored equilibrium income, Y^*, whenever actual income, Y, deviated from Y^* because of either expenditure or monetary shocks. In their critiques of the simple Phillips curve, Phelps and Freidman assumed a general equilibrium determination of Y^* and its corresponding equilibrium level of unemployment, U^*, the natural rates of national income and unemployment, deviations from which were caused by misperceptions of price signals. This treatment led to the expectations-augmented Phillips curve and the accelerationist hypothesis. According to the latter, any deviations from Y^* and U^* would set up price level changes that restored equilibrium or, if the monetary authorities insisted on validating the inflation with corresponding increase in the money supply (or 'validating' a deflation with corresponding reductions in the money supply), the inflation rate would accelerate in the face of a persistent positive output gap ($Y > Y^*$) or decelerate in the face of a persistent negative gap ($Y < Y^*$). The early New Classical models associated with Robert Lucas also used this concept of a general equilibrium in which markets were always cleared and were now inhabited by agents who had rational expectations and who maximized inter-temporally. These individuals did confuse relative and absolute price changes and were thus led to depart from equilibrium temporarily until the real market conditions were understood. Later, the new Keynesian models, and the so-called New Keynesian synthesis, followed New Classical economists in assuming rational inter-temporal maximisation and, since money wages were not sticky, a labour market that cleared continually. But output gaps still occurred because of assumed costs of changing goods prices. This implied that real marginal cost deviated temporarily from its full equilibrium value, and so output gaps continued to be part of this class of models. This branch of modern macroeconomic analysis uses the new Keynesian Phillips curve (as in Calvo) and despite its many NeoClassical features, including no involuntary unemployment, fully rational expectations and long run maximization, is referred to as 'New Keynesian.'

In all of these theories history does not matter (unless the system becomes unstable). There is a unique equilibrium which, if disturbed, is restored by an automatic adjustment mechanism and the path of the economy following on any disturbance and subsequent adjustment (if modelled at all) has no effect on the final outcome, which is to a restoration of the situation *ante bellum*. These a-historical theories all share the following characteristics: (1) there is an equilibrium or natural rate of national income, Y^*; (2) output gaps that are positive ($Y - Y^* > 0$) or negative ($Y - Y^* < 0$) can occur (for various reasons depending on the theory in question); (3) the rate of inflation is positively related to the output gap; (4) if the money supply is held constant (or changing at a slower rate than the price level is changing), output gaps of either sign will be removed by price level adjustments (possibly faster in the face of negative gaps than positive gaps); (5) if the money supply is changing at a rate that equals or exceeds the rate of change of the price level, the inflation rate will accelerate in the face of a positive gap and decelerate in the face of a negative gap; (6) in all but the New Keynesian theory, there is also a natural rate of unemployment, the NAIRU or U^*, deviations from which are a function of deviations of Y from its natural rate, Y^*. In New Keynesian theory, although employment changes as Y changes, the labour market clears continuously so that full employment is always maintained. (Very recently, a few new Keynesians have been extending this framework to admit unemployment.) It follows from these characteristics that there is only one level of income and of unemployment that are consistent with a constant, non-accelerating rate of inflation, the natural rates.[3] It is this implication of all of these equilibrium theories that we investigate in Sect. 3. In contrast, with evolutionary theories, which are all subject to constant not fully foreseeable changes and the latest New Classical Theories in which the economy is always in optimal equilibrium, these theories all have an equilibrium (either of the static or balanced growth variety), from which the economy can diverge, but to which it is returned by equilibrating forces. Since there is no collective name for the theories in this group, we name them *equilibrium with deviations*, or "EWD," theories.

The latest versions of New Classical macroeconomics do not contain income gaps nor Phillips curves of any form. Instead the behaviour of fully informed representative agents creates an equilibrium growth path by acting in response to an exogenous, stationary, stochastic, process that generates a constant long run trend of technological change. The level of output (the identical actual and natural levels) follows a cyclical pattern since there are persistence-generating mechanisms in the model. For example, the capital-stock accumulation identity makes technology shocks in one period matter for a number of future periods but not in the long run. Since all markets always clear, and all agents are farsighted and rational, all realised levels of income are equilibrium levels, representing optimal adjustments to the long term growth path and the disturbances around it. It follows that there are no output gaps and no role for policy to improve the behaviour of the whole economy. The proponents of this view regard the theory's ability to track the observed (and in some cases stylised) macroeconomic facts as a test of the theory, and it is this "test" that we investigate in Sect. 2 of our paper.

1.2 *The Evolutionary Theory in Which History Matters*

The assumptions concerning technology in evolutionary economics stand in sharp contrast to the stationarity assumptions of New Classical and EWD theories. Evolutionary economics accepts and builds on the understanding that continual but uneven endogenously induced technological changes are a fact of ordinary observation. These continually alter the structure of the economy, causing waves of

[3] U^* must be a NAIRU for reasons given in the text. However, in a model in which markets are allowed to be temporarily out of equilibrium, there may be another level of U that is a temporary NAIRU because of asymmetries in the speed of upward and downward adjustment to excess demands and excess supplies. See Tobin (1998).

serially correlated investment expenditure that are a major cause of cycles. These also drive long term growth in the sense that, without it, growth would eventually stop. In doing so, they continually transform our economic, social and political structures.

This is not the place to give an historical discussion of the origins of evolutionary economics. Suffice it to mention that the nineteenth century economist Rae (1905) saw that the existence of endogenous technological change upset many of the apparent policy implications of classical and neoclassical economics. Marx (1957) understood the transforming effects of technological changes on the social, economic and political structures of society. Veblen (1953) emphasised the importance of institutions and a deeper understanding of consumers' tastes beyond mere self-centred utility maximisation. Schumpeter (1934) made the entrepreneur-innovator the centrepiece of his dynamic view of the economy. Nelson and Winter (1982) wrote a seminal piece that pointed the way to the modern analysis of evolutionary change. Arthur (1994) and Lipsey et al. (2005) studied the scale effects that typically accompany technological developments, while Nathan Rosenberg (e.g., 1982) pioneered empirical research into the anatomy, causes and consequence of endogenous technological change.

Although evolutionary economics has no agreed canonical model, it's theorising has many common characteristics. The economy is seen as evolving continuously along path-dependent trajectories that are largely driven by technological changes generated endogenously by private-sector, profit-seeking agents competing in terms of new products, new processes and new forms of organisation and by public sector activities in such places as universities and government research laboratories. Because agents in both of these sectors make R&D decisions under conditions of genuine uncertainty (not just risk), there is no unique line of behaviour that maximises their expected profits. Thus agents are better understood as groping into an uncertain future in a purposeful, profit- or utility-seeking manner, rather than as maximizing their profits or utility.

When an economy is evolving under conditions of uncertainty, it cannot have a unique equilibrium balanced growth path (trend or difference stationary) along which agents wish to do the same thing period by period and to which it will return if disturbed. Such an equilibrium requires that the past be repeatable and that disturbances leave no trace once their effects have been worked out—history does not matter. In contrast, in evolutionary economics the trajectory of economic growth is non-unique because if agents could return to the same initial conditions, there is no guarantee that they would retrace their steps exactly since the outcome of successive actions subject to uncertainty may be different at each point in time. Technological changes are also path dependant. Scientific and technological advances build on themselves and those technological advances that firms decide to search for today depend on their current capabilities, and these in turn depend on what they have decided to search for in the past, and on how successful they were in these endeavours.[4] Thus, the concept of a unique stable equilibrium growth path is not applicable to an economy whose growth is being driven by endogenous technical change—history does matter.[5]

The discussion in this section goes a long way towards explaining why, in spite of much work both theoretical and empirical on the characteristics and behaviour of evolving economies, no generally agreed canonical model has been expounded. Canonical models, of theories such as the New Classical, the neoclassical and the New Keynesian, tend to be universal. Even when they contain random elements, they are deterministic at a quite abstract level in the sense that, given certain conditions, growth will always occur, while booms and slumps are always generated by the same disturbance mechanism and market disturbances are eliminated by a negative feedback mechanism. In short, the details of economic history do not matter for what we observe over all time periods. In contrast, the evolutionary view makes specific historical events matter. With growth, the Industrial Revolution happened when and where it did for very specific historical reasons. Although there is debate about the actual causes, most historians agree that these causes were specific to Europe at the time.[6] With cycles, although a major cause of cycles are successive waves of investment expenditure following on the innovation of new technologies, many other historical events can exert major influences. For example, major causes of the great recession that began in 2008 were the new financial innovation of derivates (enabled largely by the information handling capabilities of electronic computers) and a change in the regulatory structure followed, for example, by a change Wall Street partnerships becoming public corporations and in the process altering the incentive structure from concern with long term profitability to concern with short term volume. Agents often learn from transitory disturbances in ways that significantly affect their subsequent behaviour. For example, the exceptionally high interest rates in the early 1980s (short term rates of over 20 %) provided the incentive to learn how to manage previously idle transactions balances and because the fixed costs of such learning was then a bygone, the behaviour persisted when interest rates returned to more normal levels. No one-size-fits-all canonical model can handle such diverse, context-specific, current and historical events.

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[4] See LCB (2005: 77–82) for a discussion of the relevance of path dependence and a reply to those who doubt its importance.

[5] Most evolutionary economists accept that for many issues in micro economics, comparative static equilibrium models are useful. Also, there is nothing incompatible between the evolutionary world view and the use of Keynesian models—of which IS-LM closed by an expectations-augmented Phillips curve is the prototype—to study such short run phenomenon as stagflation and the impact effects of monetary and fiscal policy shocks. Problems arise, however, when such analyses are applied to situations in which technology is changing endogenously over time periods that are relevant to the issues being studied. Depending on the issue at hand, this might be as short as a few months.

[6] Pomeranz (2000) gives a dissenting view and we give our objections to it in LCB: 267.

2 Does History Matter for Growth and Cycles?

Nelson and Plosser's (1982) paper, and the subsequent voluminous time series empirical work on unit roots and cointegration, are generally taken to indicate that most macro time series are stationary, at least in differences (if not levels). These results are assumed to justify the assumptions of New Classical growth models and RBC theory in which growth takes place along a stationary trend or balanced (first difference stationary) path. The conclusion that the business cycle is stationary is then taken to support the classical dichotomy in which monetary and other shocks have no permanent effect on the equilibrium values of the real variables.

In this section we investigate these accepted propositions by conducting empirical analysis on data generated from a model whose structure we know. In this model, endogenous behaviour that determines the pattern of technological development and economic growth is explicitly non-stationary (trend and difference) and also contains significant elements of genuine uncertainty. Thus the model exhibits non-stationary behaviour and path-dependence because historical events and context have persistent effects—history matters. Following the practice of RBC theorists we analyse the business cycle properties of the simulated data generated by this model by matching its growth rates to actual Canadian data from the period 1961–2007 and find that their growth properties match the Canadian data. We then filter the simulated data and match it to the standard RBC properties. Then, following the practice of time series econometricians, we perform a time series econometric analysis of the unfiltered data.

2.1 The Simulation Model

The simulations performed in this paper utilize the model of Carlaw and Lipsey (2011), which is an elaboration of the model presented in Carlaw and Lipsey (2006). The following paragraphs outline the model whose details can be seen in Appendix. Italicised statements indicate alterations made to the model for purposes of the present paper. The model we now use has three sectors, each with several production activities and each containing many agents. Each has a production function that displays diminishing marginal returns to a fixed aggregate stock of a composite resource, R. Research labs in the pure knowledge sector produce a set of flows of pure knowledge concerning the various classes of technology such as power, organization, materials, transportation and information and communication: $g_t^x,\ x \in [1, X]$, where X is the number of such labs. The labs occasionally discover a new technology that has the potential to evolve into a GPT in one of these classes. The timing of these discoveries is determined by a random process that is not known by the labs *but that is influenced by the allocation of resources to both pure* *and applied R&D.*[7] *Increasing the resources to such R&D increases the likelihood of GPTs arriving in any period, making the distribution of the random arrival process for GPTs non-stationary.*

The existing stock of potentially useful pure knowledge is embodied in the new technology and then its efficiency slowly evolves according to a logistic function to become increasingly useful in applied research and in most cases to eventually become a fully fledged GPT. The Y research facilities in the applied R&D sector produce flows of knowledge, $a_t^y,\ y \in [1, Y]$, that are useful both in the consumption sector's I industries and the pure research sector's X labs, the latter being a feedback that is well established in the technology literature.[8] The consumption sector produces consumption goods that use the results of the various forms of applied research in their production functions. Technological structure is modelled in two ways. First, each sector has a number of production units, each with its own distinct production function that allows for variation in intra-sector technology.[9] Second, there is variation across the distinct characteristics embedded in the set of production functions for each of the three sectors—consumption, applied R&D and pure knowledge. *To simulate the technology shocks of the real business cycle model, we allow stationary random processes to influence the period by period realizations of investment and output by pre-multiplying the production functions within each sector by a normally distributed random variable with a mean of unity and a variance calibrated to match the stylized RBC facts*.The model contains many sources of uncertainty in invention and innovation with respect to any new technology including those that eventually become GPTs. In particular, the following things are uncertain: (1) how much potentially useful pure knowledge will be discovered by any given amount of research activity; (2) the timing of the discovery of new technologies; (3) just how productive a new technology will be over its lifetime *although the prior accumulation of GPTs within a given class positively influences the maximum productive potential of each subsequent potential GPT within that class, making the distribution of the potential impact of each non-stationary*; (4) how well the new technology will interact with technologies of other classes that are already in use; (5) how long a new technology that becomes a GPT will continue to evolve in usefulness; (6) when it will begin to be replaced by a new superior version of a GPT of the same class (7) how long that displacement will take and (8) if the displacement will be more or less complete (as were mechanical calculators) or if the older technology will remain entrenched in particular niches (as does steam that remains an important source of power for generating electricity).

As a result of these uncertainties the model displays considerable path dependency with both favourable and unfavourable occurrences affecting the future course of national income. Thus the model never settles down into a growth path that is stationary in its first differences.

[7] We allow the critical value of the arrival parameter λ^* in Carlaw and Lipsey (2011) to be a decreasing function of the accumulated amount of resources devoted to pure and applied R&D.

[8] See, for example, Rosenberg (1982: Chapter 7).

[9] For simplicity in the simulations reported below we let $X = Y = I = 3$.

2.2 Business Cycle Properties of the Simulated Data

We ran two classes of simulations of the model, calibrating it to produce annualized data. In Class 2 simulations we used all of the italicised additions to our 2011 JEE model listed above. In Class 1 simulations, we did not use the random disturbances on the production functions in the consumption and investment sectors designed to simulate the disturbances postulated in real business cycle theory. From each class of simulation we generated artificial time series data for (1) output, measured as consumption plus investment, (2) consumption, measured as the aggregate of all types of consumption goods, (3) labour, measured as the marginal product of labour times the total of all resources R,[10] (4) investment, measured as the flow output from all lines of applied R&D plus the input value of resources devoted to pure knowledge creation and (5) capital, measured as the stocks of useful accumulated knowledge from the pure and applied sectors. We ran hundreds of simulations in each of the two classes of simulation to ensure that the real growth properties that we use here were consistent with the average results produced by the model. Here we present a representative run from each class of the simulations, both containing 450 observations.

In Table 1 we compare the growth properties of the simulated data with those of the Canadian aggregate data for the period 1961–2007. For Canada, output is GDP, consumption is consumption of non-durables, semi-durables and services, investment is gross investment in non-residential capital, and labour is total hours worked. We find that the growth properties of the simulated data closely match the Canadian data, except for the very large Canadian figure of a 5.29 % annual investment growth over the last 25 years. In our simulation, the investment growth rate is only about 3.4 %.[11]

We then filtered each of the simulated time series using a Hodric–Prescott filter set for annual data and compared their properties to the filtered Canadian data. According to RBC theory the filtered Canadian data should exhibit the following properties when compared with output: investment should be about 2.5 times more volatile, consumption should be slightly less volatile, and labour should exhibit about the same volatility. All variables except capital should be highly correlated with output.

Table 2 shows the simulated data properties for Classes 1 and 2. Investment is about as volatile as output in Class 1 but slightly more than twice as volatile in Class 2. Consumption and labour are about as volatile as output in both cases. Investment,

[10] When we came to calculate an equivalent to labour in our model, we were forced to make some simplifying assumptions. First, we assumed that R is a composite of land and raw labour and that each unit of land is uniformly endowed to each unit of labour. Second, we assumed that labour will take out some of the value of its marginal product in consumption and some in reproduction that will expand the labour supply. For simplicity, we assumed a 50:50 split.

[11] The data used for these calculations are from the Canadian Socio-economic Information and Management System Database (CANSIM).

Table 1 Actual and simulated growth properties

Average growth rate %	Class 1 simulated data (450 annual periods)	Class 2 simulated data (450 annual periods)	Canada (1961–2007)
Output	3.44	3.32	3.85
Consumption	3.44	3.33	3.03
Investment	3.46	3.27	5.29
Labour	1.91	1.85	1.58

Table 2 Basic business cycle properties

	Class 1		Class 2	
Simulated data	Standard deviation (%)	Correlation with output	Standard deviation (%)	Correlation with output
Output	8.3724	1	9.2874	1
Consumption	6.9800	0.8329	7.7145	0.8730
Investment	8.1372	0.8851	18.6727	0.7033
Labour	7.3469	0.9893	7.2318	0.9598
Capital	7.1994	0.4955	6.8392	0.4594

consumption and labour are all highly correlated with output.[12] All of these comparisons indicate that our simulated data match well with the stylized RBC facts derived from the filtered Canadian data.

2.3 *Time Series Properties of the Simulated Data*

To analyze the time series properties, we first took logs of the simulated time series data, we then ran augmented Dickey–Fuller (ADF) tests on each individual time series for levels and first differences. In all cases we also ran the KPSS and Phillips-Peron unit root tests to confirm the ADF findings. These test all indicate that the testing results presented are consistent. The data are confirmed to be either levels or difference stationary by the tests.

For the first ADF test on the log of the levels we included an intercept but no trend because we believed that this is the case least likely to reject the null hypothesis of a unit root and therefore indicate that the data are non-stationary in the log of the levels. We found, as is shown in columns 2 and 3 of Table 3, that each series from both Class 1 and Class 2 rejected the null hypothesis of a unit root at the 5 % confidence level and all but investment in Class 2 rejected the null of a unit root at the 1 % confidence level. So according to this test, all of the series were stationary in the level!

[12] The simulated data are more volatile than the Canadian data and the usual RBC simulation models. Much of the additional volatility in our simulation comes from the arrivals of the major new technologies.

Table 3 Augmented Dickey–Fuller unit root test, levels, ADF, t-statistics

	Intercept, no trend		Intercept and trend	
Log of the time series	Class 1	Class 2	Class 1	Class 2
Output	−6.134979	−5.212397	−2.237547	−7.685560
Consumption	−6.377878	−5.853145	−2.117475	−8.022857
Investment	−6.689336	−3.221415	−2.452014	−0.204417
Labour	−6.716565	−3.729299	−1.972472	−0.165856
Capital	−7.923275	−5.055750	−0.493023	−5.331387

The critical t-statistic values for the ADF test are −2.570323 at the 10 % confidence level, −2.868089 at the 5 % confidence level and −3.445445 at the 1 % confidence level for this form of the ADF test

Next we ran the ADF test on the log of the levels but included a trend in the procedure. Columns 4 and 5 of Table 3 report these results for the two classes of data. In this case for Class 1 the null hypothesis that all of the series have a unit root cannot be rejected. However, for Class 2 the null is rejected for output, consumption and capital, indicating that these are stationary in the levels while investment and labour each exhibit a unit root, indicating non-stationarity in these variables. This is closer to what we expect given the non-stationary data generating process. However, there is still a puzzle with the Class 2 data in that output, consumption and capital from the simulation that most closely matches the RBC facts exhibit trend stationarity. This is what the RBC model predicts from its stationary data generating process but not what we would expect from our non-stationary data generating process.

Having discovered that all of the data in Class 1 and some of the data in Class 2 pass the tests for unit roots in the levels, we turned our testing to first differences of the data to see if the growth rates exhibit stationarity. In first differences we initially ran the ADF test including an intercept but no linear trend. These results are reported in the second and third columns of Table 4.

We next ran the ADF test on the first differences and included both an intercept and a trend. We report these results in the fourth and fifth columns of Table 4. According to the test Class 1 seems most likely to have a trend and no unit root. (This is because the null is most strongly rejected in the case where we run the unit root tests with the intercept and trend included). Class 2 appears to have no trend and no unit root. (This is because there is very little difference between the tests run with intercept and no trend those run with both trend and intercept.) Once again this is curious because the only difference between Class 1 and Class 2 is the addition of random noise on the production functions for consumption and applied R&D activities in the model.[13] In any case, the data appear either to be stationary in first differences or, in some cases, in levels. In Class 1 the data appear to exhibit stationarity in the first difference with a trend. This comes closest to what we would expect given the non-stationary data generating process, however, as we report in the last paragraph of this section even this result is somewhat misleading.

[13] The critical value for this ADF test is −3.445445 at the 1 % confidence level.

Table 4 Augmented Dickey–Fuller unit root test, first differences, ADF, t-statistics

	Intercept, no trend		Intercept and trend	
Log of the time series	Class 1	Class2	Class 1	Class 2
Output	−5.540376	−31.63073	−8.115891	−31.83105
Consumption	−4.806436	−15.75234	−8.752848	−15.81485
Investment	−12.78157	−15.04974	−14.37239	−15.55301
Labour	−6.390322	−14.97234	−8.733609	−15.49406
Capital	−17.46262	−19.20608	−19.86257	−15.75635

The critical value for this ADF test is −3.445445 at the 1 % confidence level

We wished to verify our interpretation of our analysis thus far: that the simulated data from a non-stationary data generating process appear to exhibit stationarity, in some cases in levels and in all cases in first differences. To do this, we ran a Johansen maximum likelihood-based cointegration test on both classes of simulated data. These tests are run on the simulated data in log form with a number of lags for the vector autoregression (VAR).[14] Tables 5 and 6 support the interpretation that the data are difference stationary and possibly stationary in levels for Class1 and Class 2.

These cointegration tests can be reported in a number of ways but in all of these it appears that Class 1 exhibits four cointegrating equations according to the trace test and two cointegrating equations according to the maximum eigenvalue test while Class 2 exhibits five cointegrating equations according to both the trace and the eigenvalue tests.[15] The cointegration tests appear to confirm that the Class 1 data are difference stationary. However, the Class 2 data appear to be levels stationary as indicated by the unit root tests presented in Table 4.

When we included a trend in the unit root estimations, they seemed to better detect the underlying data generating process. So for a final exercise we ran the cointegration tests with both an intercept and a trend. These results are reported in Tables 7 and 8. It appears from these results that the Class 1 data has three cointegrating equations and Class 2 has four cointegrating equations. Thus, each set of data appears to follow a difference stationary (I1) process but with a constant trend.

We make one final empirical observation. When we look at sub-periods of the Class 1 output growth rate series and fit trends using univariate regressions, we find significant negative trends in the growth rate for some subperiods while for others we find significant positive trends in the growth rate. This leads us to conclude that while the Unit Root and cointegration tests suggest that the data are at least difference stationary (if not levels stationary) with a constant trend, they are in

[14] We use the Eviews defaults of 1 through 4.

[15] This should not be surprising since the Class 2 data showed stationarity in the unit root test of the levels for each individual time series when run with no intercept and trend. So the cointegration test should show all series as being stationary. This is strictly speaking a slight abuse of the cointegration test because it is only valid for I(1) or higher orders of integration processes.

Table 5 Unrestricted cointegration rank test, class 1, variables in logs, intercept no trend

		Johansen trace				Maximum eigenvalue		
Hypothesised no. of CE(s)	Eigenvalue	Trace statistic	0.05 critical value	Prob.[a]	Hypothesised no. of CE(s)	Max-eigen statistic	0.05 critical value	Prob.[a]
None[b]	0.175839	160.7016	69.81889	0.0000	None[b]	82.19043	33.87687	0.0000
At most 1[b]	0.089460	78.51115	47.85613	0.0000	At most 1[b]	39.82990	27.58434	0.0008
At most 2[b]	0.044385	38.68125	29.79707	0.0037	At most 2	19.29485	21.13162	0.0886
At most 3[b]	0.038305	19.38639	15.49471	0.0123	At most 3[b]	16.59963	14.26460	0.0210
At most 4	0.006536	2.786762	3.841466	0.0950	At most 4	2.786762	3.841466	0.0950

[a]MacKinnon–Haug–Michelis (1999) p-values
[b]Denotes rejection of the hypothesis at the 0.05 level

Table 6 Unrestricted cointegration rank test, class 2, variables in logs, intercept no trend

		Johansen trace				Maximum eigenvalue		
Hypothesised no. of CE(s)	Eigenvalue	Trace statistic	0.05 critical value	Prob.[a]	Hypothesised no. of CE(s)	Max-eigen statistic	0.05 critical value	Prob.[a]
None[b]	0.303026	333.2193	69.81889	0.0001	None [b]	160.6483	33.87687	0.0001
At most 1[b]	0.177553	172.5711	47.85613	0.0000	At most 1[b]	86.98447	27.58434	0.0000
At most 2[b]	0.124996	85.58662	29.79707	0.0000	At most 2[b]	59.41944	21.13162	0.0000
At most 3[b]	0.046397	26.16718	15.49471	0.0009	At most 3[b]	21.14113	14.26460	0.0035
At most 4[b]	0.011231	5.026052	3.841466	0.0250	At most 4[b]	5.026052	3.841466	0.0250

[a]MacKinnon–Haug–Michelis (1999) p-values
[b]Denotes rejection of the hypothesis at the 0.05 level

Table 7 Unrestricted cointegration rank test, class 1, variables in logs, intercept and trend

		Johansen trace				Maximum eigenvalue		
Hypothesised no. of CE(s)	Eigenvalue	Trace statistic	0.05 critical value	Prob.[a]		Max-eigen statistic	0.05 critical value	Prob.[a]
None[b]	0.177238	175.0674	88.80380	0.0000	None[b]	82.91243	38.33101	0.0000
At most 1[b]	0.089540	92.15496	63.87610	0.0000	At most 1[b]	39.86719	32.11832	0.0046
At most 2[b]	0.071274	52.28777	42.91525	0.0045	At most 2[b]	31.42529	25.82321	0.0082
At most 3	0.041637	20.86248	25.87211	0.1853	At most 3	18.07476	19.38704	0.0767
At most 4	0.006538	2.787718	12.51798	0.9007	At most 4	2.787718	12.51798	0.9007

[a]MacKinnon–Haug–Michelis (1999) p-values
[b]Denotes rejection of the hypothesis at the 0.05 level

Table 8 Unrestricted cointegration rank test, class 2, variables in logs, intercept and trend

		Johansen trace				Maximum eigenvalue		
Hypothesised no. of CE(s)	Eigenvalue	Trace statistic	0.05 critical value	Prob.[a]		Max-eigen statistic	0.05 critical value	Prob.[a]
None[b]	0.303087	339.9031	88.80380	0.0000	None[b]	160.6874	38.33101	0.0000
At most 1[b]	0.184238	179.2157	63.87610	0.0000	At most 1[b]	90.61677	32.11832	0.0000
At most 2[b]	0.127701	88.59897	42.91525	0.0000	At most 2[b]	60.79735	25.82321	0.0000
At most 3[b]	0.048845	27.80153	25.87211	0.0284	At most 3[b]	22.82459	19.38704	0.0184
At most 4	0.012321	5.517037	12.51798	0.5240	At most 4	5.517037	12.51798	0.5240

[a]MacKinnon–Haug–Michelis (1999) p-values
[b]Denotes rejection of the hypothesis at the 0.05 level

fact not stationary. The trend in the growth rate is neither constant nor of the same sign throughout the data. Yet the time series econometrics would suggest that the growth rate is stationary with a constant (very small)[16] negative trend at least in our Class 1.[17]

2.4 Implications

Our findings are that the business cycle properties of the Canadian data for the period 1961–2007 when HP filtered can be closely replicated by data generated by the inherently non-stationary model in Carlaw and Lipsey (2011) when it has been HP filtered. This finding casts doubt on the implicit conclusion of New Classical theory that the macro-economy is stationary because the RBC model with its assumed stationary equilibrium fits the filtered data. In our analysis a clearly non-stationary data generating process, once filtered, also exhibits the RBC properties of the filtered real data for Canada.

Another important finding is that standard empirical time series analysis implies that the simulated data generated from our model is difference stationary, even though we know that the data generating process bears no resemblance to the theoretically stationary equilibrium of the New Classical RBC model and New Classical growth models. The unit root and cointegration tests indicate that the simulated data is at least difference stationary with a trend and in the Class 2 example appears to be levels stationary with a trend.

Our analysis casts serious doubt on the conclusion typically drawn by New Classical theorists that the passing of tests for stationarity by real time series data shows that they were generated by stationary processes in which history does not matter. Our data also pass these tests even though (1) they were generated by a model whose processes are non-stationary and in which history does matter and (2) the differing but significant trends in the sub-periods of generated Class 1 data show that its overall growth rate is not stationary.[18]

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[16] The coefficient on the trend for the ADF test (with intercept and trend) on the log difference of output in Class 1 is $-2.02e^{-05}$ with a t statistic of -5.742228.

[17] Both Class 1 and Class 2 output series exhibit a very small negative trend. This is likely due to the large initial growth rates that occur because of how the simulation is initially seeded with values.

[18] Further analysis to choose between these two interpretations will entail generating a number of simulated data sets from a model that is explicitly non-stationary to see under what conditions time series analysis will detect its non-stationary properties. For example, one stylised fact that emerges out of the historical analysis of general purpose technologies and economic growth is that sometimes the early stages of technologies that become transforming GPTs cause structural disruptions to the economy that lead to economic slowdowns for a period while they gestate and mature. This can be modelled explicitly within the Carlaw and Lipsey (2011) framework and can provide another source of non-stationarity (in terms of first differences) in the simulated data. Further analysis will reveal if the time series econometric techniques will detect these sources of

3 Does History Matter for the Economy's Output Gap/Inflation Behaviour?

We now come to the group of theories that we have termed EWD—equilibrium from which the economy can diverge temporarily. All of these theories are closed by one version or another of a Phillips curve that relates the rate of inflation to the output gap. Their key characteristics, as well as being found in the theories mentioned in the introduction, are incorporated in many econometric models of the economy. Belief in their relevance is also implicit in the behaviour of most central banks and treasury departments who measure output gaps, assume they can influence them by changes in fiscal and monetary policy, and worry about expanding the economy into the range of accelerating inflation.

In all of these theories, history does matter in the trivial sense that the economy's movement along a path towards equilibrium depends on where it was on the path yesterday. But history does not matter is the sense that the equilibrium to which the economy returns (either a static or a stationary balanced growth path) is the same as existed before it was disturbed by some shock. Because of this characteristic, most of these models display a long-run neutrality of money. A monetary disturbance can cause a gap-creating shock, but the equilibrium to which the economy returns after the effects of the shock have been worked out is not affected by the economy's behaviour during the adjustment process.

To investigate these theories empirically, we chose a key characteristic: the necessary existence of equilibrium values for output, Y^*, and unemployment, U^*. These are often called the natural rates of output and unemployment. They are the only values that are consistent with a constant level of prices and wages, or any fully anticipated, constant, non-zero rate of change of these variables. All other sustained values of Y and U must be associated either with an accelerating rate of inflation (a positive output gap) or a decelerating rate (a negative output gap). It is this basic accelerationist prediction of this group of models that we investigate in this section.

Empirical attempts to locate this required stable NAIRU over the last several decades have not been successful.[19] In response, more recent efforts have been directed at locating a time varying NAIRU, often by using a Kalman filter. In this section, we attempt the same and argue that our results cast serious doubt on the existence of a NAIRU that has any operative significance. We study data for five OECD countries, France, Italy, Spain, the UK and the US.[20] Space limitations allow

non-stationarity in the data. Until that time, we conclude that existing tests do not support the conclusion that the real data have been generated by stationary processes in which the details of history do not matter.

[19] The voluminous empirical work concerning the Phillips curve and the NAIRU is briefly discussed in the last section of this paper.

[20] The data are for the standardised unemployment rates and consumer prices provided by the OECD at http://oecd-stats.ingenta.com and accessed 1 August 2010. They begin at different years: France 1977, Italy 1978, Spain 1977, the UK 1970, and the US 1960. We use all the data available from that source since inter-country comparisons are not of major importance to our study.

us to present most of our graphs for only one country and we chose the UK. Most of the results for the other countries are given verbally or in tables.

3.1 The Kalman Filter Estimates of a Time-Varying NAIRU

We estimate a time-varying NAIRU using the Kalman filter which calculates the time series of the NAIRU through a recursive error adjustment mechanism:

$$U^{*}{}_{t} = U^{*}_{t-1} + \varepsilon_t \tag{1}$$

subject to the existence of some form of the accelerationist hypothesis, which is almost invariably imposed in linear form:

$$\dot{\pi}_t \equiv \pi_t - \pi_{t-1} = \beta(U_t - U^{*}{}_{t}) + \xi_t \tag{2}$$

The fitting procedure seeds the recursive process in (1) with some initial U, usually the U of the first period under consideration, and then uses a maximum likelihood procedure subject to (2). This procedure makes the estimated NAIRU vary from period to period so as to make it the best possible fit for the accelerationist hypothesis. It is meant, therefore, to account for shifts in the NAIRU caused by auto regressive processes in factors that influence it.

If we let $\pi^e_t = \pi_{t-1}$ and $\xi_t \sim N(0, \sigma)$ in (2), we get its implied Phillips curve:

$$\pi_t = \beta(U_t - U^*_t) + \pi^e_t, \quad \text{where}\, \beta < 0. \tag{3}$$

Although (2) is commonly used in Kalman filter estimates, the implied linear Phillips curve is not altogether satisfactory (1) because with $U \in [0, 100]$ the inflation rate approaches a maximum as U approaches zero, a maximum that is lower the lower is the value of U^* and (2), the Phillips curve is symmetric around U^* rather than being steeper when $U < U^*$ than when $U > U^*$.

A Phillips curve that has more desirable characteristics is:

$$\pi_t = b\left[\left(\frac{U^*}{U}\right) - 1\right] + \pi^e_t, \quad \text{where b} > 0. \tag{4}$$

This curve shows inflation increasing without limit as U approaches zero and deflation increasing at a diminishing rate as U approaches 100 %. However, it has a positive slope in contrast to the usual negative slope of the Phillips curve. This reversal is made solely because the Kalman filter that we use in EViews cannot handle non-linear values of the state variable U^*.

If we again let $\pi^e_t = \pi_{t-1}$, the acceleration equation for this curve becomes:

$$\dot{\pi}_t = \pi_t - \pi_{t-1} = b\left[\left(\frac{U^*}{U}\right) - 1\right] \tag{5}$$

In what follows, we estimate the time varying U^* using both the non-linear constraint of (5) and the more commonly used but less satisfactory linear constraint

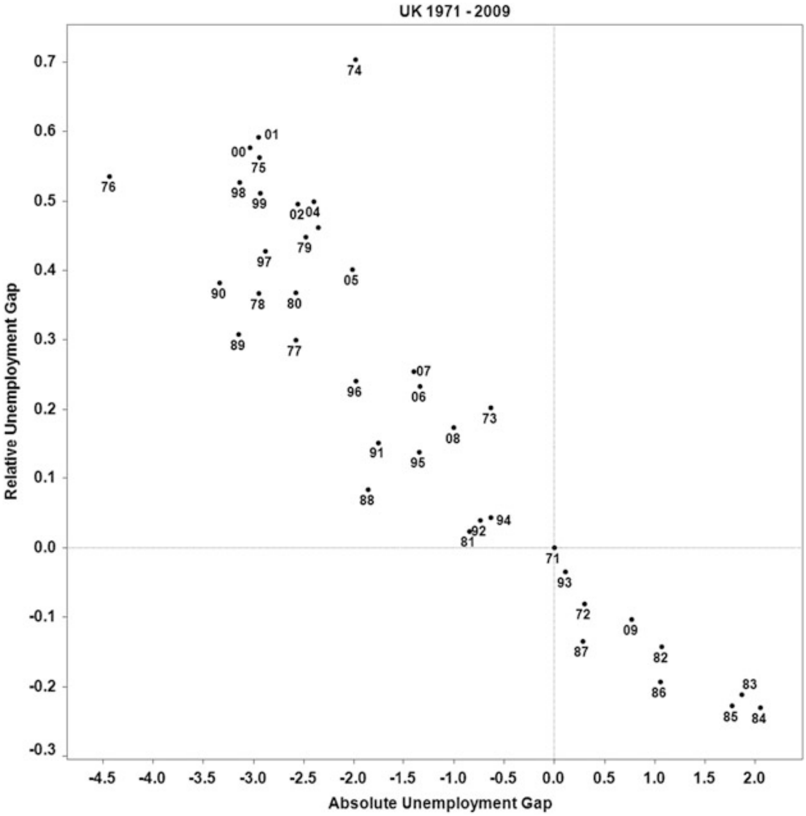


Fig. 1 The relative and absolute measure of the unemployment

of (2).[21] We refer to the NAIRU estimated using the linear constraint as U^{*1} and estimated by the nonlinear constraint as U^{*2}. We call $U - U^{*1}$ 'the absolute form of the unemployment gap', and $(U^{*2}/U) - 1$ the 'the relative form'. Figure 1 compares these two measures for the UK. As expected, the two are negatively related in all five countries with the dispersions being smaller the larger the absolute unemployment gap $(U - U^{*1})$.[22]

3.2 The Sensitivity of the U* *Estimates*

A little experimentation showed that the Kalman filter estimates of U^* for any one year are sensitive to the period over which the estimation is made. Figure 2 shows

[21] The data used in the following estimations can be obtained in an excel spreadsheet form from either author email: kenneth.carlaw@ubc.ca or rlipsey@sfu.ca.

[22] If each absolute gap is associated with the same U^*, the two measures will be perfectly correlated along a curved line. If some absolute gap's are associated with different U^{*s}, there will be a scatter of these relative gap values around their associated absolute gap values.

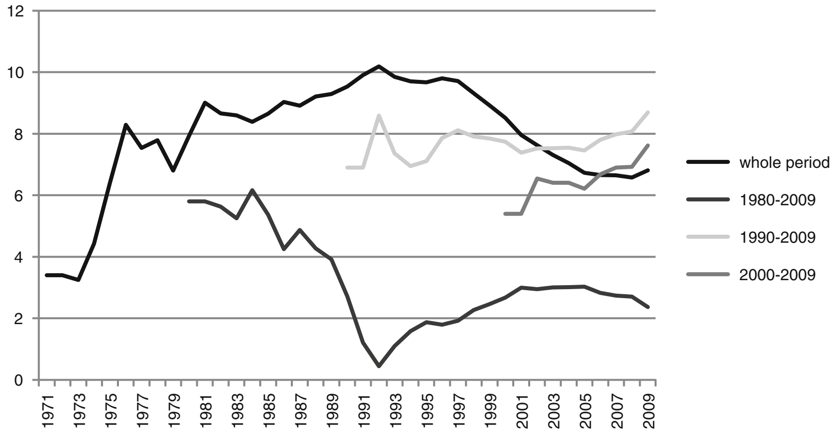


Fig. 2 UK NAIRU estimated over various time periods

Table 9 Estimated 2009 values for the U^{*2} when the estimation period begins in various years

Country	U^{*2f} (estimations begin in bracketed year)	U^{*2s} (all estimations begin in 1990)
France	10.3 (1978)	12.9
Italy	9.2 (1979)	9.6
Spain	5.4 (1978)	15.6
UK	6.8 (1971)	8.7
USA	5.8 (1960)	8.5

four different estimates for United Kingdom's U^{*2} that start in 1971, 1980, 1990 and 2000 respectively and all end in 2009. The value of U^* for the year 2009 estimated from each of these U^{*2} series is respectively 6.8, 2.4, 8.7 and 7.6.[23]

Inspection of the scatters for inflation and unemployment for the whole period suggested to us that there may have been a change in the relation somewhere around 1990. This is about the time that many central banks had got inflation more or less under control after a bout of deflation-inducing unemployment in the 1980s, after which expectations of a low and stable inflation rate became established. To give the NAIRU the best chance of doing what is expected of it in EWD theories, we estimated U^{*1} and U^{*2} over two periods, the full range over which we had data, which we termed U^{*1f} and U^{*2f}, and over the shorter period starting in 1990, which we termed U^{*1s} and U^{*2s}. The values of U^* for the year 2009 estimated from U^{*2f} and U^{*2s} are shown in Table 9. With the exception of Italy, the 2009 values for NAIRU are substantially different when they are estimated from a U^* fitted over the entire period and over the shorter period.

[23] The surprisingly low figure where the filter estimation starts in 1980 illustrates how sensitive U^* estimates are to the historical period over which they are made.

3.3 Does the Estimated Gap Explain Acceleration?

The Kalman filter will always provide estimates of a time varying U^* that is independent of the structure of any EWD model. So obtaining statically significant estimates of β in the linear version of the gap or a b in the non-linear version is not a test of the predicted existence of a NAIRU with the required properties. We consider two ways in which these estimated time-varying values can be used to make such a test.

The first way is to test some key prediction of the GE model that involves U^*. For this we use the accelerationist hypothesis that is basic to all equilibrium models that assume full rationality in the neoclassical sense. We relate the acceleration in the inflation rate to the unemployment gap measured as $U - U^{*2}$. In doing this, we are not just rediscovering the Kalman filter estimates. The filter estimates U^* as a value that varies in each time period so as to give the best fit to the acceleration hypothesis, the variations being assumed to be the result of the influences that cause U^* to shift. In our test, we use the estimated U^{*2} to calculate the relative unemployment gap and then relate this to the acceleration of inflation, forcing the regression line to pass through the origin in conformity with the prediction that zero acceleration should occur if and only if $U = U^*$.[24] This test has the advantage that it goes directly to the theoretical prediction that is of most concern to policymakers: that at any one time there is one and only one value for U (and correspondingly for Y) that is consistent with a stable inflation rate; for other values that rate either accelerates or decelerates continually.

We fitted the relation

$$\dot{\pi}_t = c\left[\left(\frac{U_t^{*2}}{U_t}\right) - 1\right] + \xi_t \qquad (6)$$

to the data for all five countries, first using U^{*2f} and then U^{*2s}, expecting a significant positive value for the slope coefficient c. We made this test over our two time periods. Because the series for U^{*s} seemed less volatile than U^{*f}, we thought U^{*s}, being less volatile than U^{*l} would give the hypothesis a better chance of passing test than U^{*f}. Figure 3 shows the results for the UK for both periods. The two relations have the right sign but are not statically significant.

The results for all the countries are reported in Table 10. The c coefficients estimated over the long and short periods for France and the long period for Italy have the wrong sign. Only the US and Spain over the long period show any a statistically significant relation. Over the shorter period, however, none of the c

[24] There is a possible problem in conducting this test since $U = U^*$ is predicted to be consistent with any stable inflation rate. For this to be a problem in practice we would have to have two or more successive years in which U stayed approximately equal to U^* (say $U = \pm\, 0.5U^*$ while the inflation rate stayed approximately constant over the period. However, such a situation has not arisen in any of our data.

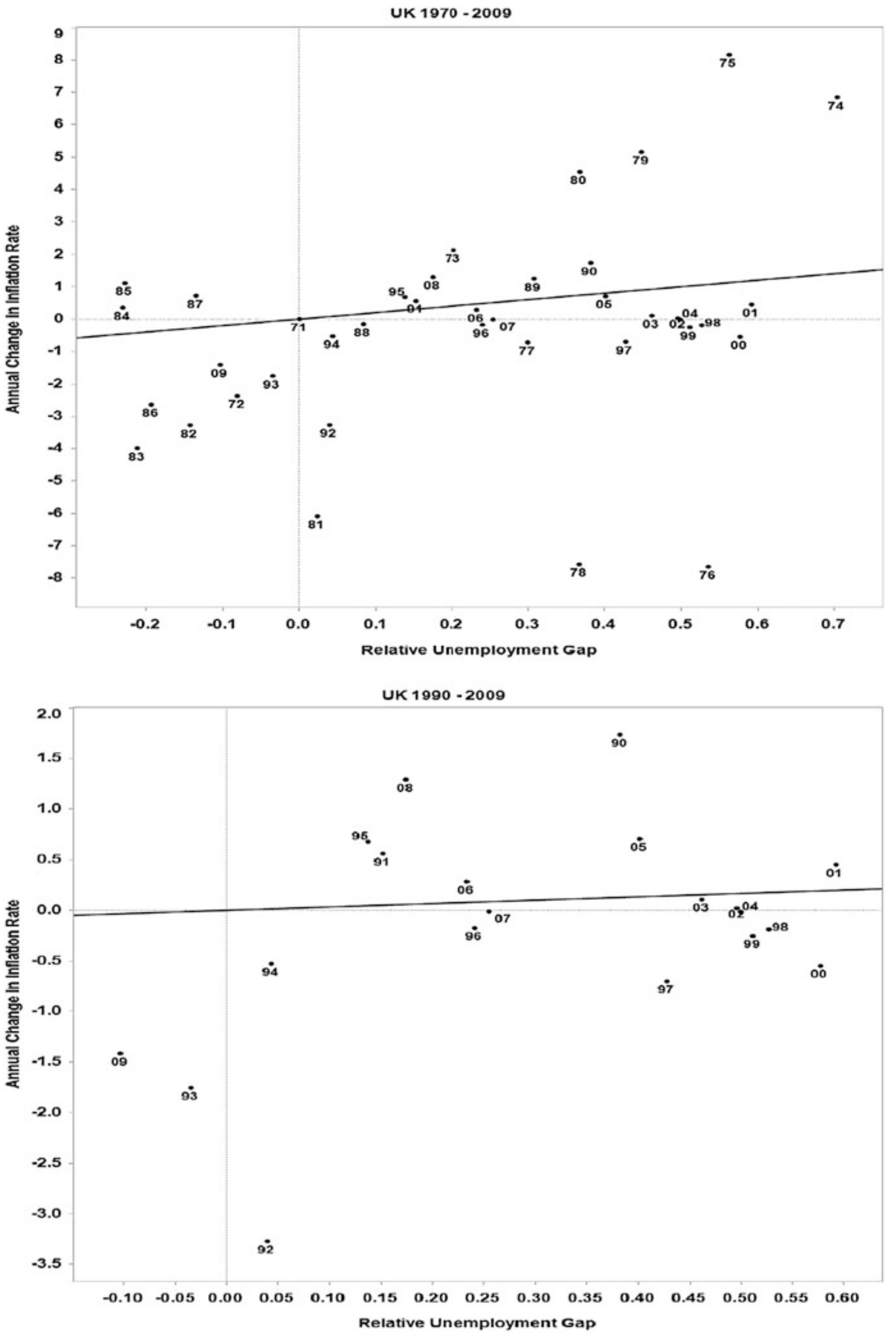


Fig. 3 Acceleration of inflation related to the relative unemployment gap

Table 10 Changes in the inflation rate related to two relative unemployment gap measures (estimates printed in Italic are significant)

Country	Whole period	Whole period	1990–2009	1990–2009
	Estimated *b*	Estimated *c*	Estimated *b*	Estimated *c*
France	*4.25445*	−0.199	1.811685	−0.087
	(*1.309119*)	(0.917)	(1.305114)	(0.707)
Italy	*4.854445*	−1.812	0.131584	0.233
	(*1.186975*)	(0.827)	(0.995747)	(1.081)
Spain	−0.084508	*1.205*	0.968721	0.479
	(0.631162)	(*0.464*)	(1.169691)	(0.668)
UK	3.134990	2.001	0.646443	0.539
	(2.104695)	(1.447)	(1.16911)	(0.670)
USA	*3.126144*	*2.519*	1.622797	0.134
	(*1.330744*)	(*0.941*)	(1.120825)	(0.449)

values are statistically significant, including those for the US and Spain. Indeed the *t* statistics are less than unity in all five cases.

Notice that the slope coefficient, *c*, differs from, and always has a lower significance coefficient than that of the *b* in the Kalman filter equation. The reason is that the Kalman filter provides the estimate of $U*$ in each year that makes the accelerationist hypothesis look as favourable as possible, while in our regressions we are testing the ability of the $U*$ so estimated for each year in conjunction with the actual U to predict the acceleration of inflation in that year. (Almost identical results were found when we related the absolute measure of the gap, $U*^{1f}$ and U^{*1s}, to the acceleration of inflation, $\pi = d\left(U_t^* - U_t\right) + \in_t$, the only qualitative difference being that the long-period coefficient for Italy had the correct sign.)[25]

A second method of testing this aspect of the EWD model is by relating changes in $U*$ to changes in the model itself rather than using a mechanistic filter to do the job. Strictly speaking, the EWD models, or any other model with a stationary equilibrium, implies that $U*$ and $Y*$ are constant. (When there is growth and an unchanged structure, $U*$ and $Y*/Y$ should be constant.) If they do change, this must be caused by changes in the model's exogenous variables and/or the parameters on one or more of its behavioural equations. For a direct test of the NAIRU theory, one would need to develop a formal theory of the determinants of the NAIRU's value—more formal, for example, than Friedman's statement that it was "the value ground out by the Walrasian equations". Then, when these determinants changed, alterations in the value of the NAIRU would be predicted. These predicted values could then be checked against the $U*^{s}$ estimated from the Kalman equation. To the best of our knowledge no one has attempted to do take this crucial second step.

[25] The estimated *d* coefficient values, this time expected to be negative, were for the short and long periods respectively, France: 0.046 (0.115), 0.024 (0.056); Italy: −0.136 (0.105), −0.064 (0.134); Spain: *−0.090 (0.031)*, 0.078 (0.065); UK: −0.116 (0.239), −0.079 (0.118); USA: *−0.746 (0.176)*, −0.097 (0.120).

In the absence of such a test, we can attempt to calculate what changes in U^* would have to occur from year to year to make the acceleration hypothesis fit the data. To do this fully would require a major study of its own. In the absence of such a study, we can make a rough approximation as follows. First, we use the absolute value of the unemployment gap. As shown in Fig. 1, this is not a bad approximation to the more satisfactory relative gap and it is the definition that has been used by those writers who have used the Kalman filter to estimate U^*. Thus using

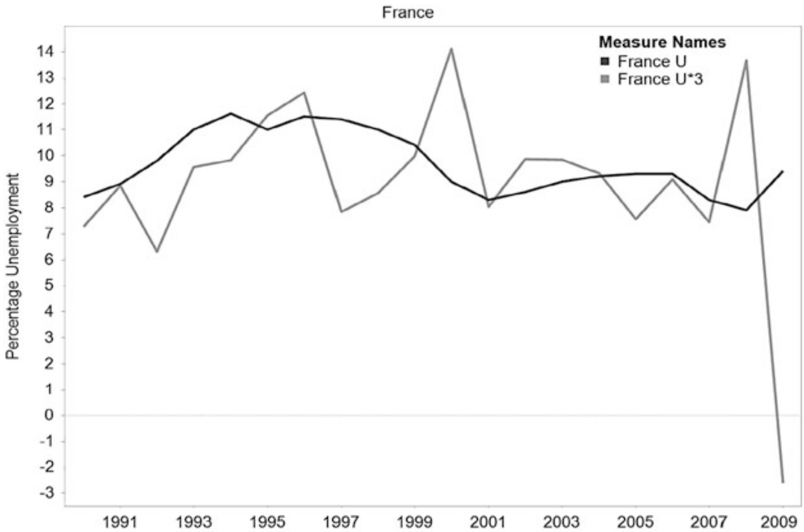
$$\dot{\pi}_t = e\left(U_t - U_t^*\right) \tag{7}$$

yields a new estimate for U^* which we term U^{*3}

$$U_t^{*3} = U_t - \frac{\dot{\pi}_t}{e} \tag{8}$$

The obvious way to obtain a value of e for each country is to use the β value from the Kalman filter in the linear form of the acceleration equation. We show the series for all our countries in Fig. 4 for the period 1990–2009. We use only this later period because all of the data show much less variability than they do over the earlier period so that the NAIRU would also be expected to be less variable than over the longer period. Nonetheless, an inspection of the four parts in Fig. 4 makes it clear that U^{*3} (shown as U^{*3} in the figure) is highly variable even over this more stable period. We summarise these results by calculating the ratio of the variance in U^{*3s} to U over the period. These values are 8.01 for France, 7.53 for Spain, 15.85 for the UK and 1.83 for the US.[26] So to explain the observed acceleration of inflation using a linear acceleration curve, the NAIRU would have to change nearly twice as much as the unemployment figures themselves changed in the US and many, many time more than twice in the other four countries. Thus, as a first approximation, the supporters of a time varying NAIRU that is explained from within any EWD model would have to show how changes in the model's parameters and exogenous variables, plus some random noise, combined to produce the highly variable time series of U^{*3s} as shown in Fig. 4. This seems to us to be a nearly impossible task and, even if it could be accomplished, it would spell the end of predictions based on a U^* that was changing only slowly or occasionally.

[26] Italy is omitted because the Kalman filter estimate of its β coefficient over the shorter period is almost zero and completely insignificant statistically. Thus massive variations in U^{*3} are required to create a sufficiently large unemployment gap to explain the observed variations in the acceleration of inflation. To check Italy, we estimated its coefficient e in (8) by the alternative method of fitting that equation to the data for U and $\dot{\pi}$. We then calculated its U^{*3} for each period and found it to be not dissimilar from those for the other countries, but still more variable with a ratio of the variance of U^{*3} to U of 84.57.



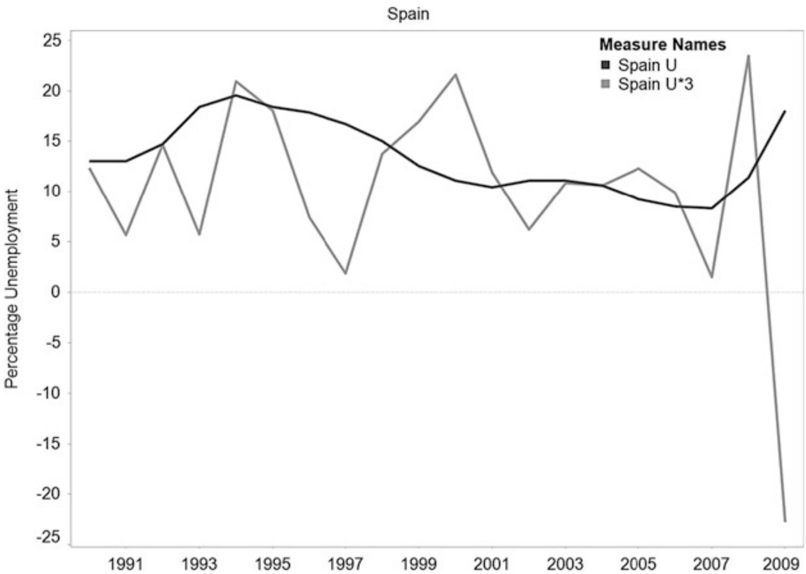


Fig. 4 (continued)

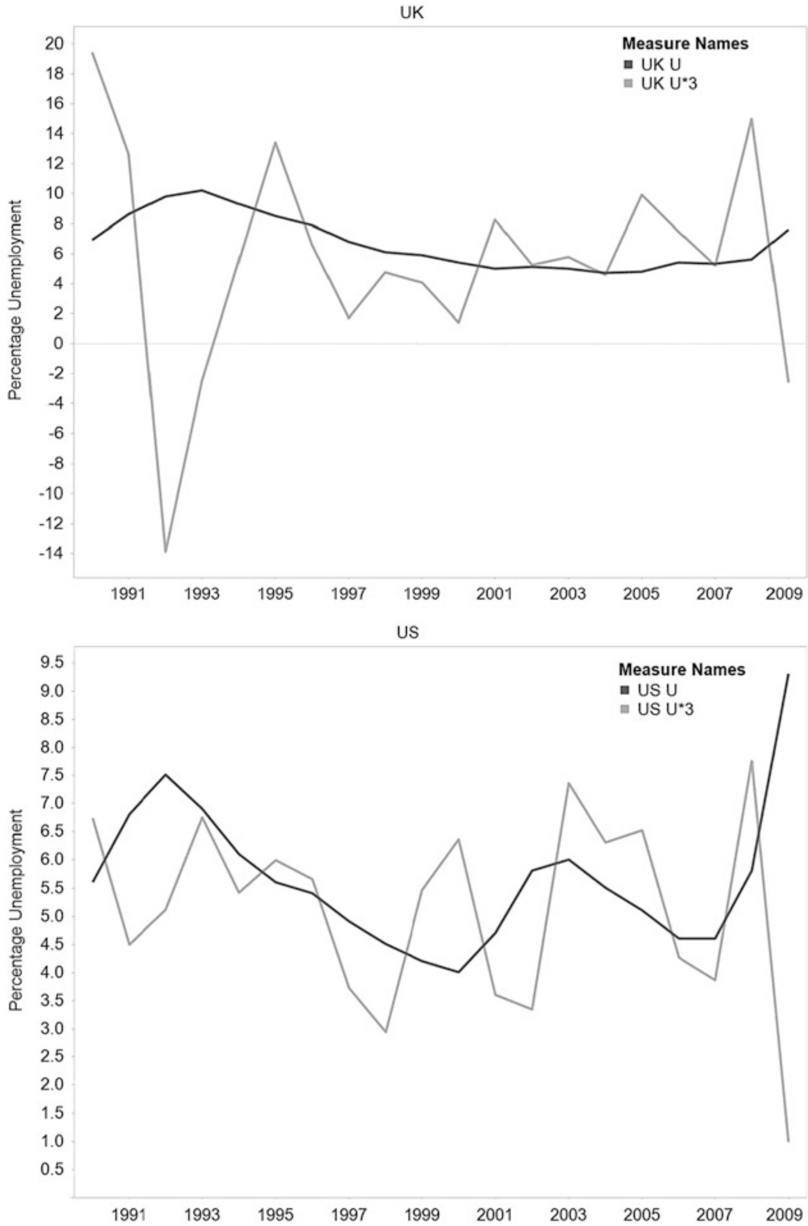


Fig. 4 U^{*3}: estimated value that U^* must take on to make the New Classical theory correctly predict the acceleration of inflation from 1990 to 2009

3.4 Implications

Much earlier econometric work has shown that no static NAIRU can be discerned in the data from most countries over the last 2–3 decades. Our Kalman filter estimates of the NAIRU for each of the six countries confirms this lack of structural stability over either the whole period for which comparable OECD data are available or the shorter one starting in 1990.

This leaves the possibility to save the accelerationist hypothesis and all the EWD models that require it, as a NAIRU that varies over time. When estimates are made of a varying NAIRU that give the best fit on the assumption that the accelerationist hypothesis does hold ((1) and (2) and (1) and (8)), the results do not provide any reliable data for dividing a range of accelerating inflation in which $U < U^*$ from a range of decelerating inflation in which $U > U^*$. Finally, if reasons why the NAIRU varies were to be specified from within EWD model, the reasons would have to vary substantially from year to year in order to explain the time series shown in Fig. 4. We conclude that our evidence conflicts with a key prediction of EWD models in which history does not matter in determining the short run behaviour of key macro variables.[27]

4 Conclusions

It is interesting to note that some of the concerns of those who accept the so called Neoclassical synthesis can be resolved by the evolutionary approach outlined in this paper.[28]

- The low and apparently trendless inflation rates that have prevailed in many countries since the early 1990s when their central banks accepted achieving such rates as their main goal requires EWD theorists to hold that each achieved level of unemployment and output are the natural rates, even though they have fluctuated considerably over the period. Structural changes that could cause these natural rates to fluctuate so widely from year to year are hard to imagine. In contrast, the obvious explanation, one that agrees with evolutionary economic theory, is that that there is no unique NAIRU so that the unemployment rate can vary over quite a wide range with no induced changes in the rate of either price or wage inflation (i.e., all unemployment rates within this range are NAIRUs).

[27] The NAIRU is not a merely part of what Imré Lakatos called a theory's protective belt. Instead it is part of the core of all EWD theories. Without it, the whole concept of a unique equilibrium for the economy, departure from which sets up equilibrating forces which can only be frustrated by agents making repeated errors, fails.

[28] The material in the bullet points that follow in the text are paraphrases of material in Lipsey and Scarth 2011, xxxii–xxiii).These authors give an extensive survey of the Phillip curve and NAIRU literature from the earlier times until the early twenty-first century.

For example Fortin (2001) makes this argument but without its application to evolutionary economics.

- In recent times, one common way of dealing with the empirical problems facing the new Keynesian versions of the Phillips curve and related concepts has been to assume that a subset of agents face such high decision-making costs that inter-temporal optimisation is not sensible for them. Instead, these agents follow a simple rule of thumb—they mimic the optimising agents with a one-period time lag. (See, for example, Gali and Gertler 1999.) In evolutionary theory, agents do look ahead but pervasive uncertainty implies that none can fully optimise over a very long time horizon, let alone the infinite one, as long as they are causing, or are being affected by, technological change (which applies to most producers as well as many workers and consumers). Of course, some turn out after the fact to have made good decisions and prosper while others turn out to have made bad decisions and do poorly. But this is groping behaviour based on knowledge, judgement, intuition and luck, not long-term optimisation and it does not appear to be well modelled by a dichotomy between long term maximizers and short term followers.
- There is strong evidence in the literature to support the proposition that the Phillips curve is better regarded as a band, not as a precise curve. For example, in the Federal Reserve Bank of Richmond's recent surveys on the Phillips curve, Nason and Smith (2008: i) conclude that "estimates of the slope of the NKPC (New Keynesian Phillips Curve) are imprecise and confidence intervals that are robust to weak identification are wide." In his overview essay for the Richmond Fed collection, Hornstein (2007: 305) indicates that this conclusion is "bad news for the NKPC as a model of inflation and for monetary policy." Be that as it may, it is good news for the evolutionary view of the economy.

As Lipsey and Scarth (2011, xxxiii) observe: "Today's prevailing paradigm involves the injunction that explicit dynamic optimisation is required as an under-pinning for a macro analysis to have pedigree." Evolutionary economists reject this injunction arguing that is its directing macroeconomics in the wrong direction because economic behaviour in the uncertain world in which endogenous technological change is a major factor cannot be understood as rational inter-temporal maximisation. Instead, it is a more empirically based, striving and groping into the fog of an uncertain future in which what is good, let alone optimal, can only be known after the event.

Few experienced economists are naïve enough to believe, however, that major paradigmatic theories die just because they have met with serious refutations of some of their predictions. Instead, repeated refutations, revealed contradictions, and inadequacies, plus some more attractive alternative are all needed before this happens. Nonetheless it is interesting to see just what is left of the theories we have criticized and what would be left behind were they to exit.

4.1 Goodbye To All That and Does it Matter?

First to go is the stable long run vertical aggregate supply curve, indicating a unique equilibrium level of national income, Y^*. Accepting that there are good reasons why the economy does not oscillate between hyperinflation and zero employment, is a long way from accepting the existence of a unique Y^* that persists for any length of time or that changes on a stable trend. Although the economy clearly does cycle, there has never been any serious evidence that it cycles around a stable equilibrium national income, Y^*, such that whenever current Y does not equal Y^* pressures will be clearly operating to return the economy to Y^*.

Second to go is the concept of a unique relation between the unemployment gap and wage and price inflation as shown by the Phillips curve. The original Phillips curve implied that *money* wage rates were highly sensitive to the state of demand in the labor market. It is one thing to say that the labor demand and supply will have some influence on wage changes, to which many would agree, and quite another thing to say that the rate of change in wages is uniquely and negatively related to the unemployment gap such that successive reductions in U will be reflected in ever higher rates of wage inflation. This auction-market view of the labor market denies the voluminous evidence that wages respond to many things other than just excess demand, or, as Hall (1980) put it many years ago, wages are more responsive to the economic climate than to the economic weather.

Next to go is the concept of the NAIRU, which puts the labor market on a fine edge equilibrium, any sustained departure from which causes the rate of wage changes to accelerate at an ever increasing rate (or decelerate at an ever falling rate).[29] Gone with it is the expectations-augmented Phillips curve which has the same properties as the NAIRU.

Note that the original Phillips curve and the NAIRU are distinct relations requiring separate refutations. The original Phillips curve implied only a negative relation between the unemployment gap and wage inflation. The NAIRU and the expectations-augmented Phillips curve required the existence of a *unique equilibrium level of unemployment*, departures from which could be sustained only if people were making repeated errors.

The original Phelps Friedman critique of the "naïve" Phillips curve that led to the concept of a NAIRU and of an expectations-augmented Phillips curve was based on an unquestioning acceptance of a unique general equilibrium of the economy. It is interesting that in the debate that followed the publications by these two economists, few questioned this basic assumption. However, once we abandon the concept of a unique general equilibrium for the economy and adopt the concept of an economy that is growing and constantly changing under the driving force of endogenous, path-dependent technological change, the theoretical

[29] At U^*, wages will be constant in a static model, or changing at the same rate as productivity is changing in a growth model. In either case, this results in the absence of any inflationary pressure emanating from the labour market.

justification for the NAIRU and the expectations-augmented Phillips curve disappears. Furthermore, as we have seen above, we find no empirical evidence for the existence of either of these as operational concepts.

Finally, what goes conclusively is the commonly held doctrine of the long run neutrality of money. There is no challenge to the proposition that the number of zeros on the monetary unit is of no economic significance, nor that changing all of them in unison, as in a comprehensive monetary reform will have no significant real economic effects. What is challenged by our results is the proposition that a monetary disturbance has real effects in the transition period but none in the long run. Since according to evolutionary economics there is no static, long-run equilibrium to which the economy returns after a disturbance, and since the response to any disturbance can alter the path taken by future technological change, there is nothing to support the theory that monetary disturbances are without long-term effects on the economy. The competing vision is of an economy whose parts and whole are changing constantly along paths that can be altered more or less permanently by such shocks as a sharp monetary expansion, a temporary oil shortage or an embargo that raises the price of oil to unprecedented heights for a long but not indefinite time.

What is seriously challenged, if not totally dismissed, is New Classical real business cycle theory. This theory which employs the ergodic axiom of an assumed stationary equilibrium never seemed reasonable to evolutionary theorists, and to many others. The critics see cycles as having many causes, some of which originate in the financial sector and others in the real sector, including serially correlated changes in the flows of investment and/or consumption expenditures. *Random* shifts in tastes and technology seem low on the list of potential and observed causes of cycles. We have shown that a model that bears no relation to the core theoretical model of RBC theory, one that is inherently non-stationary and exhibits path dependencies, generates data that when filtered using a Hodric–Prescott filter pass the same tests as are used by RBC theorists to match stylized RBC growth facts. RBC theory asserts that because the observed real data, once filtered, match the data generated by the stationary RBC model, the real data are generated by a stationary process in which history does not matter. While we have not refuted all of real business cycle theory, what we have done certainly puts this conclusion into question and calls for further critical investigation of that model.

Next to be seriously challenged, even if not totally dismissed, is the New Classical concept of growth being a process that is stationary in its first differences. Most growth models employ an explicitly stationary equilibrium concept. Many empirical tests of the real world data seem to verify this assumption. We employ the same empirical tests and find that they indicate that data generated from a model that is explicitly non-stationary appear to be stationary. At the very least this raises serious doubt about the belief that the stationary equilibrium assumption of most of growth theory has in fact been empirically verified. Our observation that the growth rate significantly changes sign in sub-periods even though the whole period passes stationarity tests, suggests that the power of these empirical tests may simply be too low to tell us if history does or does not matter in growth processes.

What seems to us to be overwhelming evidence shows that economic growth is not a stationary process. There are large differences in growth rates for any one country over time and among countries at any one time. More importantly, all growth models that are based on an aggregate production function contain nothing that would distinguish one country from another structurally, such as institutions, culture or past history. Yet economic historians and development economists are clear that country specific contexts have large effects on economic growth. This is attested to by such economic historians as Jacob (1997), Jones (1988), Landes (1969, 1998), Mokyr (1990, 2002), Musson and Robinson (1989), and North (1981) to mention just a few. Although they argue about the importance of various context-specific causes, they are clear that macro growth models based on a single aggregate production function are unable to explain why economic growth occurs at different periods in history in various countries at different rates (including zero). Also some evolutionary economists have provided historical and theoretical studies showing the importance of context specific issues including the evolution of key technologies. For example, Freeman and Louçã (2001) provide strong evidence that growth in the West over the last three centuries came in the kinds of long waves that Schumpeter hypothesized while Carlaw and Lipsey have built models of GPT driven economic growth, including the model used to generate the simulation data used in Part 2 of this paper.

Finally and more broadly, what must go is the GE theory of a perfectly or monopolistically competitive economy inhabited by representative agents who produce an equilibrium that is always the optimal response to whatever shocks are impinging on the economy and that carry no implication for the behaviour of the inflation rate (which is determined separately by a quantity theory equation). The New Classical model that supplanted the Keynesian model in most macro text books during the 1980s, swept into prominence on two main arguments. On the empirical side was the erroneous belief that the stagflation of the 1970s had refuted the Keynesian model. Lucas and Rapping spoke of “the spectacular failure of the Keynesian models in the 1970s” (1972: 54) and asked what could be salvaged from the “wreckage”. In fact, the stagflation of the 1970s and early 1980s was initially caused by a supply shock that raised prices but lowered unemployment. It was soon explained within the corpus of Keynesian economics by emphasising aggregate supply as well as aggregate demand (the text-book AD-AS model).[30] Also the Phillips curve was maintained as a short run adjustment equation by adding a price expectations term to produce what came to be called an ‘expectations-augmented Phillips curve’. On the theoretical side, was the argument that Keynesian economics lacked micro underpinnings, which the New Classical model supplied. In contrast, Lipsey (2000) has argued that Keynesian economics did have strong microeconomic underpinnings. However, because they captured the reality of small group competition in both product and labour markets, they could not be

[30] Robert Gordon’s triangle model is another approach that also does the same job.

formally aggregated into a single set of macro relations. The underpinnings of the New Classical model that replaced the Keynesian ones were typically based on atomistic competition and the aggregation problem was solved by assuming a representative consumer and representative firm each of which could be multiplied by the total number of such agents to represent the aggregation of that type of agent over the whole economy.

4.2 Hello to All This

What is left after all of these deletions? In the short term, the economy can exist with a range of Y and U and at various stable rates of inflation, provided that the central bank has a creditable policy to maintain the rate within a fairly narrow band that includes the present rate.[31] As a result, instead of the Phillips curve, there is a band shown by the broken lines in Fig. 5. The midpoint of the band is at the expected rate of inflation, shown by the solid line. The actual rate will vary around the expected rate depending on a number of variables including productivity and supply shocks, such as large changes in the price of oil and food, but not significantly on variations in U. At either end of this band, there may be something closer to a conventional Phillips curve. At the upper end U^u, a really major depression might cause changes in money wage rates and prices to fall to zero, or even become negative (the downward pointing arrow). At the lower end of U^l, a really major boom financed by money creation could cause wage and price inflation at very low levels of unemployment (the upward pointing arrow). Also anything that changes in the expected rate of inflation will shift the whole band.

In the medium and long term, the economy is evolving and constantly changing in structure, undergoing recessions and booms but not on a highly regular cycle, and growing on a non-stationary path that depends on many context-specific circumstances, some of the most important of which are technological changes generated endogenously at the micro economic level. Agents make decisions under conditions of Knightian uncertainty and some of these decisions may have consequences that persist for a very long time, perhaps to be latter displaced by future decisions made by other agents with consequences that in their turn persist for a very long time, and so on.

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[31] This lack of uniqueness is reinforced by two important characteristics. First, many firms (probably most) have short run cost curves that are flat, allowing a wide range of output fluctuations over the short run with little or no changes in product prices. Second, at some times, such as the last two decades, the nature of technological change creates a great deal of uncertainty in the labour market that puts strong pressure on labour to be fairly docile, not pushing aggressively for higher wages at the first sign of an economic expansion or even the onset of an output boom. See Lipsey 2010 for a full discussion of the importance of these two characteristics.

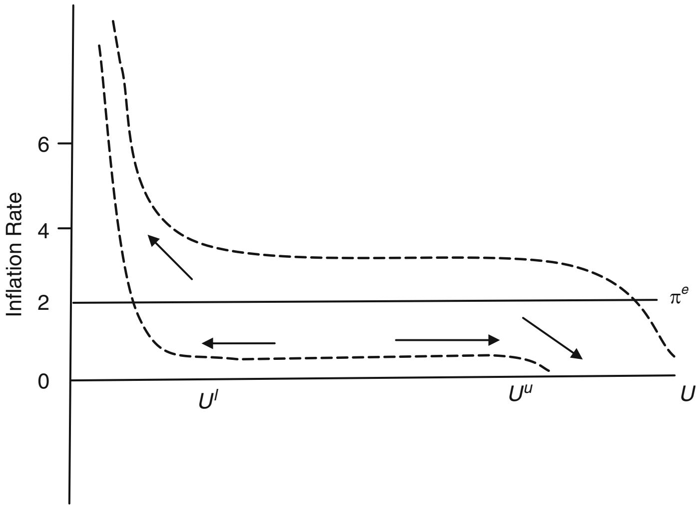

Fig. 5 The band of non-accelerating inflation: all unemployment rates between U^l and U^u are NAIRUs

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Appendix: Summary of Carlaw and Lipsey (2011) Model

The fixed supply of the composite resource, R, is allocated by private price-taking agents in the consumption and applied R&D sectors and by a government that taxes the applied R&D and consumption sectors to fund pure research at an exogenously determined level.

The constraint imposed by the composite resource is:

$$R_t = \sum_{i=1}^{I} r_t^i + \sum_{y=1}^{Y} r_t^y + \sum_{x=1}^{X} r_t^x \tag{9}$$

The Applied R&D and the Consumption Sectors

The output of applied knowledge from each applied R&D facility, y, depends on the amount of the resource it uses and its productivity coefficient, which is the geometric mean of each $(G_{n_x})_t$ term multiplied by its corresponding v term, as shown in (10).

$$a_t^y = z_t^y \left[\prod_{x=1}^{X} (\nu_{y,z}^{n_x} (G_{n_x})_{t-1})^{\beta_x} \right]^{\frac{1}{X}} (r_t^y)^{\beta_{X+1}}, \tag{10}$$

$$\beta_x \in (0,1] \ \ \forall x \in X, \ \ \beta_{X+1} \in (0,1)$$

where z_t^y is drawn from a Normal distribution with mean 1 and variance 0.2.

The stock of applied knowledge generated from each facility accumulates according to:

$$A_t^y = a_t^y + (1-\varepsilon)A_{t-1}^y, \tag{11}$$

where $\varepsilon \in (0,1)$ is a depreciation parameter.

In the consumption sector, we make the simplifying assumptions (1) that there are the same number of applied R&D facilities and consumption industries, $Y = I$, and (2) that the knowledge produced in each of the facilities, y, is useful only in the one corresponding consumption industry, i. The production function for each of the I industries in the consumption sector is then expressed as follows:

$$c_t^i = z_t^i (\mu A_{t-1}^y)^{\alpha_y} (r_t^i)^{\alpha_{Y+1}}, \ \alpha_y \in (0,1] \ \forall y \in Y, \ \alpha_{Y+1} \in (0,1) \text{ and } i = \mathrm{y} \tag{12}$$

where z_t^i is drawn from Normal distribution with mean 1 and variance 0.06

The Pure Knowledge Sector

There are X labs each producing one class of pure knowledge that leads to the occasional invention of a new version, n_x, of that class of GPT. The productivity coefficient in each lab is the geometric mean of the various amounts of the Y different kinds of applied knowledge that are useful in further pure research (one for each applied R&D facility and each raised to a power σ_y). The output of pure knowledge in lab x, g_t^x, is a function of the geometric mean of the various amounts of applied knowledge produced from the Y facilities doing applied R&D and the amount of the composite resource devoted to that lab.

$$g_t^x = \left[\prod_{y=1}^{Y} \left((1-\mu) A_{t-1}^y \right)^{\sigma_y} \right]^{\frac{1}{Y}} \left(\theta_t^x r_t^x \right)^{\sigma_{Y+1}}, \tag{13}$$

$$\sigma_y \in (0,1], \ \ \forall \ y \in Y \text{ and } \sigma_{Y+1} \in (0,1).$$

The stocks of *potentially useful* knowledge produced by each of the X labs accumulate according to:

$$\Omega_t^x = g_t^x + (1-\delta)\Omega_{t-1}^x \tag{14}$$

where $\delta \in (0,1)$ is a depreciation parameter.

New GPTs are invented infrequently in each of the X labs and their invention date is determined when the drawing of the random variable $\lambda_t^x \geq \lambda^{*x}$. For simplicity, we let the critical value of lambda for each of the X labs be the same: $\lambda^{*x} = \lambda^*$ $\forall\ x \in X$. When at any time, t, $\lambda_t^x \geq \lambda^*$, indicating that a new version of class-x GPT is invented, the index t_{n_x} is reset to equal the current t, and n_x is augmented by one.

Here we alter the arrival condition to make it a function of endogenous behaviour as follows. At any point in time, t, $\lambda_t^x \geq \frac{\lambda^*}{\left(\sum_{\tau=\tau last}^{t}\sum_{y=1}^{Y}(r_\tau^y)\right)}$,where $\tau last$ is the date that the last GPT of any class arrived in the economy.

Agents make their adoption decisions with incomplete information. In each applied R&D facility the only ν that agents expect to change is the one associated with the challenging x-class GPT, so, we can compare the productivities for any of the y facilities by simply comparing the $v_{y,z}^{(n-1)_x}\left(G_{(n-1)_x}\right)_{t_{n_x}}$ that would result if the incumbent were left in place with the $\bar{v}_{y,z}^{n_x}(G_{n_x})_{t_{n_x}}$ that is expected to result if the challenger were adopted. This comparison is made in each of the Y applied R&D facilities at time $t = t_{n_x}$ so the test, stated generally for all applied R&D facilities, is:

$$\left[\bar{v}_{y,z}^{n_x}(G_{n_x})_{t_{n_x}}\right] \geq \left[v_{y,z}^{(n-1)_x}\left(G_{(n-1)_x}\right)_{t_{n_x}}\right] \text{ for each y} \in [1,Y]. \tag{15}$$

If the test is passed, the new GPT is adopted in facility y.

The evolving efficiency with which the GPT delivers its services is shown in (16) below.

$$(G_{n_x})_t = \left(G_{(n-1)_x}\right)_{(t-1)_{n_x}} + \left(\frac{e^{\tau+\gamma(t-t_{n_x})}}{1+e^{\tau+\gamma(t-t_{n_x})}}\right)\left(\psi_t\Omega_{t_{n_x}}^x - \left(G_{(n-1)_x}\right)_{(t-1)_{n_x}}\right), \tag{16}$$

where

$$\psi_t = \frac{e^{n_t/X}}{10+e^{n_t/X}}$$

and n_t is the total number of GPT arrivals in the economy up to date t.

The equation shows the efficiency of the GPT, $(G_{n_x})_t$, increasing logistically as the full potential of the GPT is slowly realized. t_{n_x} is the invention date of the version n_x, of the class-x GPT, $\Omega_{t_{n_x}}^x$ is the full *potential* productivity of the new version of GPT x, $\left(G_{(n-1)_x}\right)_{t_{(n-1)_x}}$ is the *actual* productivity of the version that it replaced, evaluated at the time at which that earlier version was last used, $t_{(n-1)_x}$ and γ and τ are calibration parameters that control the rate of diffusion. The evolution of efficiency proceeds as follows. Initially, since $t_{n_x} = t$ (and because γ is very small, 0.07 in our simulations), the value of the efficiency coefficient is close to zero so that the initial productivity of the challenging GPT is close to that of the incumbent. As t increases over time the value of the efficiency coefficient approaches unity so that the GPT's productivity approaches its full potential.

In the subsequent periods, the test in (15) is modified to note the productivity changes that occur over time:

$$\left[\bar{v}_{y,z}^{n_x}\left(G_{n_x}\right)_t\right] \geq \left[v_{y,z}^{(n-1)_x}\left(G_{(n-1)_x}\right)_t\right] \tag{15'}$$

for each $y \in [1, Y]$ that has not yet adopted GPT G_{n_x}.

Resource Allocation

As we have already noted, in the pure knowledge sector the government pays for and allocates a fixed amount of the generic resource, R, to each of the pure knowledge producing labs. Producers in the applied R&D and consumption sectors maximize their profits each period taking prices as given.[32] The prices for output from the I consumption industries are derived from the maximization of an aggregate utility function, which we assume is additively separable across the I consumption goods.

$$U = \sum_{i=1}^{I} \left(c^i\right)^{\phi^i} \text{ and } \phi^i = \phi^{i'} = 1, i \neq i' \forall i, i' \in I \tag{17}$$

Maximizing this utility function and rearranging the first order conditions (FOCs) yields:

$$\frac{MU^{i=1}}{MU^{i\neq 1}} = \frac{P^{i=1}}{P^{i\neq 1}} = \frac{\phi^{i=1}\left(c^{i=1}\right)^{\phi^{i=1}-1}}{\phi^{i\neq 1}\left(c^{i\neq 1}\right)^{\phi^{i\neq 1}-1}} \tag{18}$$

Since $\phi^i = 1 \;\; \forall \;\; i \in I$ it follows that $P^{i=1} = P^{i\neq 1}$, i.e., the relative prices of all consumptions goods are unity.

[32] We suppress time subscripts in (17) through (24) because agents are not foresighted and are consequently performing a static maximization in each period.

We assume a competitive equilibrium in the market for the composite resource. This implies that it earns the same wage, w, regardless of where it is allocated.

Each consumption industry maximizes its profits taking the price of its consumption output, P^i, and the prices of its inputs, composite resource, w, and applied knowledge, P^y, as given. Profits are expressed as:

$$\pi^i = P^i c^i - wr^i - P^y A^y \tag{19}$$

Profit maximization yields the following FOCs in each of the I consumption industries:

$$\begin{aligned} P^i mpr^i - w &= 0 \\ P^i mpA^y - P^y &= 0 \end{aligned} \tag{20}$$

where mp represents marginal product. From the first FOC, the assumption the $P^i = 1$, and the definition of the production function for industry i we get:

$$r^{i*} = \left[\frac{\alpha_{Y+1}}{w}(\mu A^y)^{\alpha_y}\right]^{\frac{1}{1-\alpha_{Y+1}}}, \tag{21}$$

which is the reduced form expression for the demand for the composite resource in each consumption industry, i.

From the combination of both FOCs from the profit function for consumption industry i and the definition of the production function we get:

$$\frac{w}{P^y} = \frac{\alpha_{Y+1}}{\alpha_y}\frac{A^y}{r^i}$$

which implies that:

$$P^{y*} = \frac{\alpha_y w}{\alpha_{Y+1} A^y}\left[\frac{\alpha_{Y+1}}{w}(\mu A^y)^{\alpha_y}\right]^{\frac{1}{1-\alpha_{Y+1}}} \tag{22}$$

Each applied R&D facility maximizes profits taking the price of its applied knowledge output, P^y, and the composite resource, w, as given. The pure knowledge input in the form the currently adopted set of X GPTs is provided freely to the applied R&D facilities by the government financed labs. Profits are expressed as:

$$\pi^y = P^y a^y - wr^y \tag{23}$$

Maximization of the profit function and algebraic manipulation yields the following FOC:

$$P^y mpr^y - w = 0$$

The demand for the composite resource from each of the Y applied R&D facilities is thus:

$$r^{y*} = \left[\beta_{X+1}\left[\prod^{X}\left(v_{y.z}^{n_x}(G_{n_x})_t\right)^{\beta_x}\right]^{\frac{1}{X}}\frac{P^{y^*}}{w}\right]^{\frac{1}{1-\beta_{X+1}}} \tag{24}$$

With these resource demand equations we now have a complete description of the allocation of the composite resource across the three sectors.

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