Chapter 18
Modelling Disequilibria and Equilibria

18.1 Introduction

Previous chapters presented models describing the activities of households and enterprises, where the main stress was placed on real processes. The models allowed generating the demand and supply of goods and services in the commodity markets, and demand and supply of labour force in the labour markets. In this chapter the equilibration mechanisms in these markets will be discussed. Firstly, the mechanisms of the quantitative, mainly short-term adjustments will be described in terms of changes in: (a) inventories, (b) the rates of capacity utilization (production factors), (c) foreign trade. Then the mechanisms of price and wage adjustments will be discussed, linked to the unemployment rate analysis.

The point of departure will be the basic national account identity that for the past behaviour has the following well known form:

$$X_t = H_t + E_t - M_t \tag{18.1}$$

and

$$H_t = C_t + G_t + J_t + \Delta R_t \tag{18.2}$$

where (all variables in constant prices):

C_t is personal consumption from disposable income,
E_t is exports,
G_t is consumption of public institutions,
H_t is absorption by domestic final users,
J_t is investment expenditures,
M_t is imports,
ΔR_t is changes in inventories and reserves,
X_t is GDP.

To analyse the potential states of equilibria (or disequilibria) the description of economic agents' behaviour must be complemented first by specifying the public sector equations (to generate its consumption G_t, employment and wage funds) and

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the foreign sector equations (to generate exports E_t and imports M_t). Secondly, the equations explaining producer prices must be supplemented using the equations for other agents, i.e. a comprehensive price system needs to be constructed. But most of all, it seems indispensable that adequate economic regimes be distinguished, so that the particular components of identities (18.1) and (18.2) can be alternatively identified either as the demand of economic agents or supply from them.

18.2 Modelling the Global Disequilibria

In macroeconometric models, global demand for domestic production (GDP) at the macroscale (X_t^D) is usually determined as a sum of its components, as in (18.1) and (18.2). In the disequilibrium models that do not assume that demand and the supplies delivered to final users (X_t^S) could be balanced even temporarily, i.e. assume that $X_t^D \neq X_t^S$, the supply equations are specified for X_t^S.

In the disequilibrium models the central role is played by potential output. Its volume may vary depending on the assumptions concerning the availability of production factors. If the availability is unconstrained, the final demand X_t^D can be met. Otherwise, if there are constraints on the availability of fixed capital, the potential output X_t^K can be generated; if there are scarcities in labour force availability, the potential output X_t^N will be determined (Barro and Grossman 1971). There may also exist deficits in energy or raw materials supplies, their most frequent causes being constraints in the availability of imports (or in the balance of payments) that generate the potential output X_t^M (Florczak and Welfe 2000; Welfe and Welfe 2004).

The generation of potential output implies the use of production functions (Welfe 2000). As mentioned in the previous chapter, the Cobb-Douglas and CES functions were used the most frequently (Dreze et al. 1990). For the sake of simplicity, the Cobb-Douglas production function will be used below.

Potential output generated with a fixed capital constraint, i.e. assuming its full utilization, will be obtained by using the following formula:

$$X_t^K = BA_t K_t^{\alpha} N_t^{K(1-\alpha)} \tag{18.3}$$

where N_t^K is the number of jobs, $N_t^K < N_t^S$.

Potential output generated under constrained availability of labour force, i.e. assuming full employment, will be obtained with the following formula:

$$X_t^N = BA_t K_t^{\alpha} N_t^{U(1-\alpha)} \tag{18.4}$$

where $N_t^U = N_t^S - U_t^N$ and N_t^S is the labour force supply, U_t^N is natural unemployment.

Full employment is understood here as the potential number of working-age people reduced by the amount of natural unemployment resulting from the conditions associated with the job-seeking process (Coen and Hickman 1976).

The generation of potential output constrained by inadequately available energy or raw materials implies that the production function should be extended by introducing additional explanatory variables. A disequilibrium indicator was used in most cases, showing the degree to which the demand for energy or raw materials was met. At the macrolevel these constraints mostly translated into constraints affecting imports or the current account in the balance of payments.

The potential output for the above variants established, the global supply of domestic products to the final users X_t^S can be obtained from the following equation:

$$X_t^S = \left(X_t^{K-\rho} + X_t^{N-\rho}\right)^{-1/\rho} \tag{18.5}$$

where $1/\rho$ is a mismatch coefficient or if the constraints in the supply of energy or raw materials are additionally considered:

$$X_t^S = \left(X_t^{K-\rho} + X_t^{N-\rho} + X^{M-\rho}\right)^{-1/\rho} \tag{18.5$'$}$$

Using the general assumptions concerning the aggregation of economic agents' activities, the total output provided for final users, i.e. GDP, can be determined from the following formula:

$$X_t = \left(X_t^{D-\rho} + X_t^{K-\rho} + X_t^{N-\rho}\right)^{-1/\rho} \tag{18.6}$$

For the West-European countries and Poland the mismatch coefficient estimates were close to 0.02 (Dreze et al. 1990; Welfe et al. 1996).

The empirical results of the presented investigations show that constraints in the availability of machinery and equipment played an important role in the 1970s and 1980s, but in the subsequent years the final demand constraints were decisive. The results obtained for Poland were approximately similar, however in the late 1970s and in the early 1980s the restrictions on the availability of raw materials, including imported ones, were strongly accentuated.

In the dynamic stochastic general equilibrium (DSGE) models the equilibrium conditions are determined for the major economic agents participating in final demand. The agents are households (private consumption), public institutions (social consumption), investors and exporters. The demand of these groups of economic agents is compared with the supply of products directed to them. The supply is represented by the output of the "final goods producers". This rather strange category may contain trading and logistic firms and exporter firms. This "output" is composed of domestic products (manufactured by domestic "producers of intermediate products") and imported products. As a rule, the supply flows are aggregated using the CES function. For example, the supplies of consumer goods directed to households, C_t^S, are obtained from:

$$C_t^S = \gamma\left(\delta X_t^{C-\rho} + (1-\delta) M_t^{C-\rho}\right)^{-V/\rho} \tag{18.7}$$

where:

X_t^C is supply of domestic products directed to households,
M_t^C is imports of consumer goods directed to households,
$\rho > 0$ is a parameter linked to the elasticity of substitution $\sigma_{X,M} : \sigma_{X,M} 1/(1+\rho)$.

This approach is particularly attractive from the perspective of analysing commodities flows going to final users. However, its implementation encountered serious informational barriers, especially in the case of the flows of imported commodities. The constructors of the supply-determined macromodels of the former CPE countries faced similar difficulties (Welfe 1992).

18.3 Modelling Demand of Public Institutions (Social Consumption)

For the description of the domestic final demand to be complete, the functions representing public institutions' demand for consumer and investment goods must also be specified. The most important here is the demand financed from the government budget.

In general, a simple hypothesis is proposed, assuming that the public institutions' demand for consumer goods (G_t) is a non-linear function of the real current expenditures of the government budget. The respective inertia must be introduced, recognizing the dependence on the existing infrastructure. Hence, for the long-run we have:

$$\ln G_t^* = b_0 + b_1 \ln(BCP_t/PG_t) + \varepsilon_t \tag{18.8}$$

where

BCP_t is the current expenditures of the government budget,
PG_t is the deflator of public consumption;

in the short-run we have:

$$\Delta \ln G_t = a_0 + a_1 \left(G_{t-1} G_{t-1}^*\right) + a_2 \Delta \ln(BCP_t/PG_t) + \alpha_3 \Delta \ln G_{t-1} + \varepsilon_t \tag{18.9}$$

In several models the explanatory variable is equal to the current expenditures adjusted for debt service and/or military expenditures, if the latter are ranked higher by the government.

Investment demand is usually assumed to be exogenous. Regarded as an important instrument of economic policy, it expresses the more or less arbitrary decisions of the government.

18.4 Modelling Inventories

In the demand-determined macromodels where demand, including final demand, is met, its component, i.e. the demand for an increase in inventories and reserves ΔR_t, is distinguished and special equations are built. In the disequilibrium models where the supply of commodities was directly specified, inventory changes were residual and played an equilibrating role. They retained the role also in the equations of the aforementioned demand-determined models.

The macroeconometric models of less-developed countries and the former CPE countries used production functions to generate output and, consequently, to allocate the supplies of commodities and services among final users. At the macro-scale, the global supply X_t^S was determined as a sum of value added in particular sections. Its volume usually did not match the volume of global demand—market equilibrium was ensured by appropriate inventory changes ΔR_t. In these models, the following identity was defined:

$$\Delta R_t = C_t^D + G_t^D + J_t^D + \left(E_t^D - M_t^D\right) - X_t^S \tag{18.10}$$

At present, it is used rather rarely. Other adjustments mechanisms predominate in macromodels. The accuracy of inventory changes calculated from an identity is non-satisfactory. This makes the attempts to explain the demand for an inventory increase more appealing.

In the specification of the equation explaining the demand for inventory increase the main stress must be placed on the major functions of inventories and reserves. They must ensure the continuity of production and trade. Hence, they should expand correspondingly to growing output and trade. The volume of the inventories of finished commodities depends i.a. on how precisely enterprises can predict the demand for output, as indicated in Chap. 17. Regarding raw materials and materials, their volume is determined by the frequency of their periodical supply. Moreover, they function as a buffer that protects production and trade processes against the possible demand shocks or commodity supply shocks (Lovell 1961, 1962).

The above premises were applied in macromodels to specify the inventory functions. The desirable level of inventories depends on the level of output (or sales). It is assumed that this adjustment process follows with a lag:

$$R_t^* = b_0 + b_1 X_t - b_2 R_{t-1} + \xi_t \tag{18.11}$$

where: R_t^* is the desired level of inventories at period end, $b_1 > 0$ and $0 < b_2 < 1$.

The impacts of shocks, whether they follow from changes in the factors determining demand $D_t(\cdot)$ or from changes in the factors determining supply $S_t(\cdot)$, have short-term implications. They are introduced into the short-term equation:

$$\begin{aligned}\Delta R_t = \alpha_0 + \alpha_1\left(R_{t-1} - R_{t-1}^*\right) + \alpha_2 \Delta X_t + \alpha_3 \Delta R_{t-1}\\ + \alpha_4\left(D_t(\cdot) - S_t(\cdot)\right) + \xi_t\end{aligned} \tag{18.12}$$

The above shocks are frequently characterised using the rate of capacity utilization WX_t. Its increase is associated with inventories decline and its decrease is followed by their expansion ($\alpha_4 < 0$).

In general, the prediction accuracy of an inventory increase is not high. Hence, only a few models treat the results of forecasts as important signals for enterprises, heralding the likely business cycle changes.

In the multisectoral models attempts were made to distinguish equations explaining particular kinds of inventories, such as finished commodities, work in progress, raw materials and materials (Juszczak 1982). The equations were specified for particular sections of the national economy, including the inventories of the trading organizations (cf. equations in the W-5 model of the Polish economy, Welfe 1992).

18.5 Modelling Adjustments in the Production Sector

In the demand-determined macroeconometric models output is determined by demand directed to enterprises (allowing for changes in the inventories of finished goods). This demand influences the demand for production factors that basically falls short of the full utilisation of their resources, so some or all of the factors remain in excess. The changes in this demand can be partly handled by adjusting appropriately capacity utilization and/or the utilization of equipment and employment.

Hence the obvious need that changes in enterprises' capacity utilization rates and/or in particular production factors be monitored, as mentioned in the previous chapter.

Solving this problem effectively poses several difficulties. Firstly, in many countries the information on the capacity utilization rate (WX_t) is limited. The Wharton Index used in the quarterly Wharton models of the US economy that was developed the earliest by comparing output with its values in the peak years was applied in the manufacturing industry (Klein and Summers 1966). In the European countries surveys making direct enquiries to enterprises about their capacity utilization rates were used (Grzęda-Latocha 2005). The application of these indicators is somewhat inconvenient, because to use them in forecasting equations explaining their dynamics must be built.

The concepts where the effective output (X_t) is compared with potential output derived from the production function seem to be the most appropriate theoretically and free of the above disadvantage. If the potential output were determined to ensure the full utilization of the fixed capital (X_t^K) as in (18.3), then the rate of capacity utilization will be obtained from:

$$WX_t^K = X_t/X_t^K \tag{18.13}$$

If the potential output were defined to ensure full employment (X_t^N), then the rate of capacity utilization will be calculated from:

$$WX_t^N = X_t/X_t^N \tag{18.14}$$

In this case, the precision of the estimates of the capacity utilization rates will depend on the computation accuracy of potential output. The above characteristics of the rate of potential output were broadly used in the macroeconometric models of the USA (Coen and Hickman 1976) and other countries, including Poland (Welfe 2009).

The rates of capacity utilization play a far-reaching role in macroeconometric models. They are introduced as explanatory variables representing market tensions not only into the equations explaining prices, but also into the foreign trade equations. Let us mention that in the equations explaining prices central banks often use a simplified procedure for capturing the effects of market tensions, which involves the construction of an indicator comparing effective GDP with GDP trajectory represented by an appropriate trend, which was mentioned in the previous chapter.

Macroeconometric models rather infrequently use the scarce information on the utilization rate of machinery and equipment as opposed to the use of information on employees' time worked. The latter allows analysing the utilization rate of employees' potential working time (Fair 2004), as well as the utilization rate of production potential by adjusting the number of shifts (Welfe 1992).

18.6 Modelling Unemployment

In the market economies labour market disequilibria are represented by unemployment, i.e. by the difference between labour force supply and employment. This understanding of unemployment can be found in the demand-determined macroeconometric models. In the supply-oriented models the excess demand for labour predominates, frequently related to hidden unemployment.

The level of unemployment is understood as registered unemployment or unemployment estimated through special surveys, such as LFS. Macromodels mostly use registered unemployment.

In macromodels, the level of unemployment UN_t and the rate of unemployment U_t are most frequently generated from respective identities:

$$UN_t = N_t^S(\cdot) - N_t^D(\cdot) \tag{18.15}$$

and

$$U_t = UN_t / N_t^S(\cdot) \tag{18.16}$$

where

$N_t^D(\cdot)$ is the demand function for employees,
$N_t^S(\cdot)$ is the labour supply function.

The above functions were specified for labour supply in Chap. 16 and for the demand of the enterprise sector in Chap. 17 (cf. Phillips 1958). The total demand for employees involves also public sector employees. Their number is either treated as exogenous or, preferably, generated from the equations that take into account the available infrastructure (frequently represented by lagged employment) and the likely changes in the level of total salaries assumed in the government budget. In this case, we have:

$$N_t^{PD} = \alpha_0 + \alpha_1 N_{t-1}^{PD} + \alpha_2 \Delta\left(F_t^{DP} / WP_t^P\right) + \varepsilon_t \tag{18.17}$$

where

N_t^{PD} is public sector' demand for employees,
FP_t^P is total salary fund for the public sector officers,
WP_t^P is average salary in the public sector.

In the long-run, average salaries paid in the public sector vary like average wages in the enterprise sector:

$$WP_t^P = AWP_t^{\alpha} e^{\xi_t} \tag{18.18}$$

Generating the rate of unemployment as a residual from identity (18.16) usually yields results that are not sufficiently accurate. The estimation (and prediction) errors of the demand and supply functions add up. Hence, attempts were made in several macromodels to estimate the changes in unemployment rates using the reduced specifications of the demand functions $N^D(\cdot)$ and of the labour supply functions $N^S(\cdot)$. It can be assumed in the short-run that labour supply changes slowly, so the impact of the changes can be either ignored or represented using a trend function. The most important are factors affecting employment demand, i.e. output and the effects of technological progress (Nickell 1984). This means that the rate of unemployment can be determined using the utilization rate of production capacities WX_t and the rate's fluctuations can be explained by the GDP fluctuations in the business cycle:

$$U_t = BWX_t^{\beta} e^{\mu t+\xi}$$

This approach is far from being excellent. As mentioned, the estimates of the capacity utilization rate can be obtained using competitive methods and are not highly accurate.

The observed unemployment rate represents the effects of changes in the labour market caused by employees changing employers (frictional unemployment) and the long-term tendencies arising from the permanent unavailability of jobs (structural unemployment). In the literature, Friedman (1968) and Phelps (1970) views were prevalent. They proposed distinguishing the category of natural unemployment, representing labour market equilibrium. They found that the economic policy instruments designed to reduce the rate of unemployment made it lower only temporarily and were followed by an increase in inflation (cf. Blanchard and Quah 1989).

The NAIRU (non-accelerating inflation rate of unemployment) concept derived from the analysis of the relations between entrepreneurs and employees (Layard et al. 1991) found more general applications. The NAIRU is an "equilibrium" unemployment rate such that exceeding it makes the rate of inflation go down and its decrease is followed by an increase in the rate of inflation. The values of the NAIRU computed for the USA oscillated around 6 % whereas for Western Europe were higher between 8–9 % (Elmeskov 1993).

To compute the NAIRU several simplifying assumptions concerning the equations explaining average wages and prices that serve as a point of departure must be introduced. For the sake of demonstration, let us derive the NAIRU using the long-run equations of average wages and prices. The equations will be shown by means of the logs of variables.[1]

The equation explaining the nominal average wages before tax reads as:

$$wp_t = \alpha_0 + \alpha_1 p_{t-1}^c + \alpha_2 wy_t - \alpha_3 u_t + \alpha_4 t_t^w \qquad (18.19)$$

where p^c is a consumption deflator, t^w is social contributions, wy_t is labour productivity.

[1] This example follows the modified derivation shown by Whitley (1994, pp. 102–103).

The long-run real average wages will be obtained assuming that the elasticities with respect to prices will be calibrated at levels equal to one and that no lags are present. Hence:

$$wp_t - p_t^c = \alpha_2 wy_t - \alpha_3 u_t + \alpha_4 t_t^w \tag{18.20}$$

Considering that the logarithm of the consumption deflator can be presented as the difference between the logarithms of producer prices (p_t) and indirect taxes (t_t^i):

$$p_t^c = p_t - t_t^i \tag{18.21}$$

after substituting to (18.20) we shall have:

$$wp_t - p_t = \alpha_2 wy_t - \alpha_3 u_t + \alpha_4 t_t^w + t_t^i \tag{18.20$'$}$$

The initial producer price equation, following (17.55), reads as follows:

$$p_t = \beta_0 + \beta_1 \left(wp_t - wy_t + t_t^w \right) + \beta_2 p_t^m + t_t^z \tag{18.22}$$

where

p_t^m is import deflator,
t_t^z is corporate income tax.

The long-term producer price equation will be obtained based on a simplifying assumption that price elasticity with respect to labour costs is one, $\beta_1 = 1$, and the import deflator can be obtained by multiplying the world prices (p_t^w) and the exchange rate (er_t): $p^m = p^w + er_t$.

As a result, we have:

$$p_t - wp_t = wy_t + t_t^w + \beta_2 \left(p_t^w + er_t \right) + t_t^z \tag{18.22$'$}$$

hence

$$wp_t - p_t = -wy_t - t_t^w - \beta_2 \left(p_t^w + er_t \right) - t_t^z \tag{18.22$''$}$$

The NAIRU defining equation will be obtained by comparing (18.20′) and (18.22″):

$$-\alpha_3 u_t = (\alpha_2 - 1) wy_t - (1 - \alpha_4) t_t^w - t_t^i - t_t^z - \beta_2 \left(p_t^w + er_t \right)$$

hence

$$\alpha_3 u_t = (1 - \alpha_2) wy_t (1 - \alpha_4) t_t^w + t_t^i + t_t^z + \beta \left(p_t^w + er_t \right) \tag{18.23}$$

This derivation of NAIRU shows that its level depends on the tax rates and the exchange rate, and on level of productivity.

Alternative methods were applied. The first uses the rates of growth of variables, the second is based on the reduced Phillips formula, utilizing the Kalman filter (Elmeskov and MacFarland 1993).

A simplified approach was elaborated similar to that applied in calculating NAWRU as shown below.

The NAWRU (non-accelerating wage rate of unemployment) is an alternative concept that has found a broad use. This unemployment rate is not followed by an increase in average wages. It was originally developed for the OECD models (Elmeskov and MacFarland 1993). Its estimates can be found in many European macromodels, including those for the OECD member countries (Bårdsen et al. 2005). It can be derived from the following equation:

$$U_t^{NAWRU} = U_t - \left(\Delta U_t / \Delta^3 w_t\right) \Delta^2 w_t \tag{18.24}$$

where

$$w_t = \ln WB_t$$

To avoid irregular shocks the HP filter was commonly applied.

18.7 Modelling Foreign Trade

18.7.1 Introduction

The equations explaining exports and imports have been present in all macroeconometric models since their beginning. Many models initially treated foreign demand as exogenous (in the world economy models it was generated for particular country by the system). Later on, endogenized foreign demand served as a characteristic of the links with the world economy that could be shaped by the improving competitiveness of the exporting country.

In the supply-determined models and the general equilibrium models the export supply functions were also specified.

The macroeconometric models' equations explaining domestic demand for imported commodities and services are specified in most cases. This demand depends on the level of domestic economic activity, as well as being sensitive to the level of imports' competitiveness vis-à-vis domestic production. In a few models attempts were made to explain the supply of imported commodities. A relevant example is the DSGE models, where the sources of imported products were established.

The foreign trade equations cover not only its total volumes. The most frequent disaggregation distinguishes commodities and services. The commodity imports are decomposed by their use, i.e. into consumer goods, investment goods, and intermediate commodities, with fuels being frequently distinguished. The world economy models frequently use the UN SITC classification of exports and imports.

In many macroeconometric models the foreign trade flows were decomposed using also the geographical and political criteria (Bożyk et al. 1973; Czyżewski and Welfe 1990).

The discussion in this section will concentrate on the presentation of the exports and imports equations, bearing in mind their role in balancing the economy; the review of the structure of the import and export price equations will be postponed until the next section and the role of the above equations in modelling the balance of payments will be presented in one of the next chapters.

18.7.2 Equations Explaining the Demand for Exported Goods

Foreign demand for commodities exported by a country is composed of the demand of particular countries represented by the appropriate fractions of their global imports. Macromodels distinguish the most significant importers or their groups, such as EU members, or use total world imports (equal to total world exports). Exports volumes are treated as exogenous, except that they are explained by the pertinent variables like in the multicountry models, first of all by GDP. The models that belong to the above class usually assume that the exports E_{it} of the i-th country can be obtained as a sum of the products of the components of the international trade matrix a_{ij} by the volumes of foreign countries' imports M_{jt}, i.e. $E_{it} = \sum_j a_{ij} \cdot M_{jt}$. The matrix components were often assumed to have constant values, which implied the necessity of updating them (Hickman and Lau 1973; Klein et al. 1999).

The exporters' reactions to foreign demand changes were not immediate, which seems to be due to the traditional conclusion of contracts with an appropriate lead. Hence the suitable lags in the adjustments to changes in foreign demand.

The empirical long-term relationships show certain stability, as well as revealing that growth tendencies are stronger for exports than for global imports as implied by the elasticity of exports with respect to the summary imports of foreign countries which is higher than one. This result does not seem justified, being rather an indication that some important factor has been omitted. It was therefore suggested that the long-term elasticity of exports (calibrated) should be equal to one, meaning a stable share of the given country's exports in global imports. The introduction of the structural changes and qualitative improvements could be represented through an exponential trend, the use of which could bring down the aforementioned elasticity to a level close to one (Whitley 1994).

Foreign demand is modified along with changing competitiveness of exported products. This competitiveness is most frequently expressed through the exporting country's relative prices, or the relative labour costs or relative unit costs which are used more rarely. Price competition has been recently giving way to quality-based competition, so the absolute values of estimates of the price export elasticities have been declining in the last years.

In the short-run, unexpected shocks in the domestic markets disturb export activities in several ways. Shocks, such as floods, that impose constraints on the supply to the domestic markets or induce increases domestic demand are compensated for by lower supplies for exports. To represent the disturbances the common characteristics of the market tensions are used, such as the utilization rate of productive capacities.

The above examination leads to the following long-run equation explaining the foreign demand function for exported commodities:

$$\ln E_t^{D^*} = \alpha_0 + \alpha_1 \ln M_t^W + \alpha_2 \ln\left(P_t^E/ER_t\right)/P_t^W + \varepsilon_t \tag{18.25}$$

The short-run foreign demand function is specified as follows:

$$\begin{aligned}\Delta \ln E_t^D = {} & \beta_0 + \beta_1\left(\ln E_{t-1}^D - \ln E_{t-1}^{D^*}\right) + \beta_2 \Delta \ln M_t^W + \beta_3 \Delta \ln\left[\left(P_t^E/ER_t\right)/P_t^W\right] \\ & + \beta_4 \Delta \ln WX_t + \beta_5 \Delta \ln E_{t-1}^D + \varepsilon_t \end{aligned} \tag{18.26}$$

where

ER_t is exchange rate—domestic currency/world currency,
M_t^W is imports of the distinguished territory,
P_t^E is export deflator in domestic currency,
P^W is import deflator of the distinguished territory,
WX_t is the utilization rate of productive capacity.

As mentioned, the estimate of the long-run elasticity α_1 exceeded one in many models (for Poland it was 1.2). Hence it was frequently calibrated, assuming $\alpha_1 = 1$. Then the exponential trend was introduced into the long-term equation (18.25). Notice, that the equation could be reduced by introducing the ratio between exports and global imports as the explained variable.

In the major UK models the long-term export elasticities with respect to relative prices were between −0.28 and −0.40 (Whitley 1994); for Poland they were substantially lower (−0.11) (Welfe 2002). The short-term elasticities with respect to global imports were usually smaller than one. However, the short-term export elasticities with respect to relative prices were higher in absolute terms than the long-term elasticities, thus displaying stronger short-run sensitivity of the world markets.

18.7.3 The Exports Supply Functions

Commodities intended for exports were modelled mainly in the supply-determined macromodels. Regarding the demand-determined models, their builders most frequently gave up the construction of separate exports supply functions, assuming that supply followed effective demand. The models built in the former CPE countries usually contained the supply functions that represented the export potential of the economy, without assuming that foreign demand would be met. More recently the exports supply functions could be found in the CGE models, and particularly in the DSGE models. The equations are special in that they decompose the supply of export commodities by its origin, i.e. into the supply of domestic products and of imported commodities (re-exports).

The initial specification of the exports supply function started with the potential domestic output, the aim being to estimate the amount by which the domestic production potential was larger than domestic demand. The domestic production potential was represented by either global output (Q_t) or global domestic output GDP (X_t).

The offer of commodities for exports depends also on the profitability of exports. Its changes are related to the changes in the relative export prices, i.e. in the relations between the prices paid for exports (P_t^E) and those obtained in the domestic markets (P_t).

Hence the long-run exports supply function (E_t^S) will be specified as follows:

$$\ln E_t^{S^*} = \alpha_0 + \alpha_1 \ln X_t + \alpha_2 \ln\left(P_t^E / P_t\right) + \xi_t \tag{18.27}$$

In the short-run, the effects of shocks produced by varying supply to the domestic users must be introduced. They are typically represented by the rate of capacity utilization (WX_t). Therefore, the short-run equation will have the following form:

$$\Delta \ln E_t^S = \beta_0 + \beta_1\left(E_{t-1}^S - E_{t-1}^{S^*}\right) + \beta_2 \Delta \ln X_t + \beta_3 \Delta \ln\left(P_t^E / P_t\right) + \beta_4 \Delta \ln WX_t + \xi_t \tag{18.28}$$

In the DSGE models the notion of exports potential is frequently extended by the explicit introduction of reexports into the supply equation. Let the global volume of the commodities for export be S_t^E; then, using the DSGE models terminology, the supply from "the producers of final goods" intended for export will equal:

$$S_t^E = \gamma\left(\delta X_t^{E^{-p}} + (1-\delta) M_t^{E^{-p}}\right)^{-v/p} \tag{18.29}$$

where

X_t^E is the output for exports from the "producers of intermediate goods",
M_t^E is imports for reexports.

The supply of commodities intended for exports redefined as above will substitute GDP in both the long-term equation (18.27)

$$\ln E_t^{S^*} = \alpha_0 + \alpha_1 \ln S_t^E + \alpha_2 \ln\left(P_t^E / P_t\right) + \xi_t \tag{18.27$'$}$$

and the short-term equation (18.28) explaining the supply of exports:

$$\Delta \ln E_t^S = \beta_0 + \beta_1\left(E_{t-1}^S - E_{t-1}^{S^*}\right) + \beta_2 \Delta \ln S_t^E + \beta_3 \Delta \ln\left(P_t^E / P_t\right) + \beta_4 \Delta \ln WX_t + \xi_t \tag{18.28$'$}$$

Assuming that the long-run elasticity of exports supply with respect to global supply S_t^E is one, i.e. that in the long-run $\alpha_1 = 1$, then the ratio of the exports supply E_t^S to the potential exports supply S_t^E can be treated as an explained variable. This ratio will be dependent on the relative prices (the Armington model, Armington 1969). For the long-term, we have:

$$\ln\left(E_t^S / S_t^E\right) = \alpha_0 + \alpha_2 \ln\left(P_t^E / P_t\right) + \xi_t \tag{18.30}$$

where the elasticity α_2 stands for the competitiveness of the commodities to be exported. It is worth knowing that special computations are needed to estimate the volumes of the commodities intended for exports.

18.7.4 The Imports Functions

The import of commodities and services performs two major functions. Firstly, it is to complement the range of commodities and services that are either not produced domestically or produced in insufficient amounts (complementary imports). Secondly, imports compete against the domestically produced goods, thus contributing to higher efficiency of domestic supplies (competitive imports).

In the simplest case, the imports equation describing the first function can be specified as an identity, with imports defined as a residual. At the macro-scale this will be:

$$M_t = H_t + E_t - X_t \tag{18.31}$$

In the multisectoral models this identity holds for the distinguished commodity groups. In *these equations* imports play a balancing role. This becomes particularly clear, when output represents potential domestic output.

However, macroeconometric models use most frequently imports functions that represent mechanisms generating demand for imported commodities and include competitive issues.

The competitive intensity is usually represented by relative import prices defined as the ratios between import prices (in domestic currency) P_t^m and domestic producer prices P_t. Hence, at the macro scale the long-run equation will have the following form:

$$\ln M_t^* = \alpha_0 + \alpha_1 \ln X_t + \alpha_2 \ln\left(P_t^M / P_t\right) + \xi_t \tag{18.32}$$

Many authors assume that the share of imports in GDP is constant in the long-run. Therefore, imports elasticity with respect to GDP (calibrated) can be assumed at a level equal to one ($\alpha_1 = 1$). With this assumption, the explained variable can be redefined as the share of imports in GDP, i.e. M_t / X_t. In this case, the relative prices will be the only explanatory variable:

$$\ln(M_t / X_t) = \alpha_0 + \alpha_2 \ln\left(P_t^M / P_t\right) + \xi_t \tag{18.33}$$

It must be added that this relationship represents the result of optimizing the behaviour of economic agents that choose from among the domestic and imported commodities (the Armington model).

The acceptance of the above assumption is sometimes associated with an inclination to allow for structural changes, which leads to an increase in the growth rates of imports. An exponential trend is then introduced (Whitley 1994).

In the short-run, the shocks likely to come from unexpected changes in domestic demand represented by changes in the capacity utilization rate (WX_t) or from changes in foreign direct investment (B_t^J) are introduced. Then the short-term equation will have the following form:

$$\begin{aligned}\Delta \ln M_t = \beta_0 + \beta_1\left(\ln M_{t-1} - \ln M_{t-1}^*\right) + \beta_2 \Delta \ln X_t + \beta_3 \Delta \ln\left(P_t^M / P_t\right) \\ + \beta_4 \Delta \ln WX_t + \beta_5 \Delta \ln B_t^J + \xi_t \end{aligned} \tag{18.34}$$

In many macroeconometric models commodity imports are decomposed according to their use. There are distinguished imports of intermediate commodities (M_t^Z), investment imports (M_t^J) and consumer goods imports (M_t^C). The long-term equations explaining the demand for the distinguished groups of imported commodities will be specified below.

It is generally assumed that the imports of intermediate commodities depend directly on gross output (Q_t):

$$\ln M_t^Z = \alpha_0 + \alpha_1 \ln Q_t + \alpha_2 \ln\left(P_t^M / P_t\right) \tag{18.35}$$

where the relative prices are defined as the ratios of intermediate commodity prices, if the appropriate information is available. The multisectoral models, including the input-output models, use the data on the import-output ratios showing the unit use of materials and fuels.

The demand for imported, mostly complementary, investment goods mainly depends on the expansion of the domestic investment activities. In the case of the emerging economies a special channel for the imports of investment goods is distinguished, namely foreign direct investments (FDI).

Hence we have:

$$\ln M_t^J = \alpha_0 + \alpha_1 \ln J_t^V + \alpha_2 \ln FDI_t + \alpha_3 \ln\left(P_t^{MJ}/P_t^{JV}\right) + \xi_t \tag{18.36}$$

where

J_t^V is investment expenditures on machinery and equipment (constant prices),
P_t^{MJ} is the deflator of imported investment goods,
P_t^{JV} is the deflator of domestic investment expenditures on machinery and equipment.

The demand for imported consumer goods is treated as competitive in most cases, which gives importance to the relative imports prices. The complementary imports help meet the domestic consumption. However, the major explanatory variable is the total consumption C_t, mainly because of restricted data availability. Hence we have:

$$\ln M_t^C = \alpha_0 + \alpha_1 \ln C_t + \alpha_2 \ln\left(P_t^{MC}/P_t^C\right) + \xi_t \tag{18.37}$$

where P_t^{MC} is the deflator of imported consumer goods and P_t^C is the deflator of individual consumption.

In the multisectoral models imports are decomposed further. The SITC classification is used most frequently to this end. Among the intermediate commodities the import of fuels is often distinguished.

As mentioned, the estimates of the long-term elasticities of imports with respect to GDP are either close to one or slightly exceed one. The long-term elasticities of consumer goods imports with respect to total private consumption are much higher, reaching 3 in the extreme cases. The price elasticities for the UK ranged between −0.5 and −0.8 (Whitley 1994) and for Poland between −0.3 and −0.8 (Welfe and Welfe 2004).

18.7.5 The Equations Explaining the Supply of Imported Commodities

The demand-determined models assume that the potential supply of commodities that importers may offer is unconstrained, hence the effective supply tends to match the demand for imported goods. The supply conditions are affected by import prices that will be discussed in the next section. Attempts have been made within the general equilibrium models and the DSGE models to estimate the supplies of imported commodities by looking at their sources.

18.8 Price Systems

18.8.1 Introduction

Prices perform many important functions in national economies. Changing, they clear the markets. They allow the flows of commodities and services to be transformed into the flows of incomes and expenditures (as deflators), i.e. they guarantee a transition from the real sphere to the sphere with financial flows (Courbis 1977; Tobin 1972). The most important role is played by the producer prices; the specification of the equations explaining these prices was shown in the previous chapter. In this section the discussion will be extended to include the equations explaining final goods prices and the prices of production factors that together constitute the price system functioning in the national economy. The discussion on how interest rates and exchange rates are generated will be postponed until the next chapters.

18.8.2 Equations Explaining the Prices of Domestic Final Goods

In specifying the equations explaining the prices of consumer or investment goods it is commonly assumed that the prices depend, as appropriate, on producer prices, import prices, trading mark-ups and indirect taxes (mainly VAT).

At the macro scale, the most frequent explanatory variables are GDP deflators; in the multisectoral models the deflators of gross output (or production sold) are applied in particular sections.

Like the retail price indices, the deflators of private consumption (CPI) are mostly computed as the functions of the geometric averages of GDP deflators, import deflators and indirect taxes. For the long-run, we have:

$$\ln P_t^{C^*} = \alpha_0 + \alpha_1 \ln P_t + (1 - \alpha_1) \ln P_t^{MC} + \alpha_2 \ln T_t^V + \varepsilon_t \qquad (18.38)$$

where T_t^V is the rate of indirect taxes (VAT).

In the short-run, we have:

$$\begin{aligned} \Delta \ln P_t^C = {} & \beta_0 + \beta_1 \left(\ln P_{t-1}^C - \ln P_{t-1}^{C^*}\right) + \beta_2 \Delta \ln P_t \\ & + (1 - \beta_2) \Delta \ln P_t^{MC} + \beta_3 \ln \Delta T_t^V + \varepsilon_t \end{aligned} \qquad (18.39)$$

The value of the parameter α_1 can be interpreted as the share of commodity flows of domestic origin. If the information on the decomposition of the consumer goods supplies is available, the value can be appropriately adjusted. Then Eq. (18.38) can be used to estimate price elasticity with respect to indirect taxes.

Equation (18.39) can be used to determine the rate of inflation and to make comparisons with the inflationary target assumed by the central bank.

Many macromodels decompose the deflators of consumer goods according to the groups of commodities and services, using the permanent components (core inflation) rather than the transitory components, such as the prices of agricultural products or fuel prices (those depend on the world business cycle).

The deflators of consumption of public institutions P_t^G are structured similarly to the deflators of private consumption. The deflators of inventories and their changes have a similar property.

The equations explaining the investment goods prices, i.e. the deflators of investment expenditures P_t^I, have a simple structure, because the prices usually do not include indirect taxes. Investment expenditures are frequently decomposed into amounts allocated to machines and equipment—here the deflators depend mainly on the prices in the electric machinery industry and on imports—and into amounts spent on buildings and constructions, where the deflators depend on the deflators in the building industry and transport. In general, the long-term equation reads:

$$\ln P_t^{J^*} = \alpha_0 + \alpha_1 \ln P_t^Q + (1 - \alpha_1) P_t^{MJ} + \varepsilon_t \tag{18.40}$$

and the short-term equation has the form:

$$\begin{aligned} \Delta \ln P_t^I = \beta_0 + \beta_1 \left(P_{t-1}^I - P_{t-1}^{I^*}\right) + \alpha_2 \Delta \ln P_t^Q \\ + (1 - \alpha_2) \Delta \ln P_t^{MJ} + \varepsilon_t \end{aligned} \tag{18.41}$$

where P_t^Q is the deflator of the appropriate section (or industry).

18.8.3 Equations Explaining Export and Import Prices

To specify the functions explaining export prices special assumptions about exporters' behaviour must be imposed. Following some Scandinavian models, it may be assumed that the exporters' role in price formation is meaningless, so the export prices are fully determined by the world prices:

$$P_t^{EW} = P_t^W \tag{18.42}$$

where

P_t^{EW} is the export prices of the exporting country in the world currency,
P_t^W is the world trade prices.

Many macroeconometric models assume, though, that exporters can efficiently compete in the world markets based on their export prices. Therefore, the effective prices of exported commodities will reflect the compromise between the world trade impacts (world prices) and exporters' activities (domestic prices converted into the world currency). When the homogeneity restriction is taken into account, the long-run equation will have the form:

$$\ln P_t^{EW^*} = \alpha_0 + \alpha_1 \ln P_t^W + (1-\alpha_1)\ln\bigl(P_t^E ER_t\bigr) + \varepsilon_t \tag{18.43}$$

where ER_t is the exchange rate, P_t^E is export prices in domestic currency.

Allowing for the impacts of the likely shocks in exporters' behaviour, in the short-run the equation will be formed as shown below:

$$\begin{aligned}\Delta \ln P_t^{EW} = {} & \beta_0 + \beta_1\bigl(\ln P_{t-1}^{EW} - \ln P_{t-1}^{EW^*}\bigr) + \beta_2 \Delta \ln P^W + (1-\beta_2)\Delta \ln(P_t/ER_t) \\ & + \beta_3 \Delta \ln U_t^E + \varepsilon_t \end{aligned} \tag{18.44}$$

where U_t^E is a variable representing the shocks in behaviour.

The estimate $\hat{\beta}_2 = 0.43$ was obtained for the stability period in Poland (Welfe 2002). Ignoring the likely impact of the 2008 shock would be misleading, though. The US dollar and euro exchange rates increasing suddenly by 20 % did not cause any decline in the export prices denominated in the two currencies, but contributed to higher domestic prices of domestic exports, i.e. to exporters' surplus. This effect could be captured by making an appropriate change to the variable $U_t^{E\cdot}$. Otherwise, assuming in this case (18.42), Eq. (18.44) could be solved for export prices in domestic currency P_t^E.

Imports deflators expressed in the world currency are commonly assumed exogenous. In the world multicountry models the deflators are computed as the weighted sums of the deflators of the exporting countries' exports to the country where the distinguished importers are based; the weights are obtained from the aforementioned world trade matrix. The deflators expressed in domestic currency are obtained from an identity that converts the world currency prices into domestic prices using the appropriate exchange rate:

$$P_t^M = P_t^{MW} \cdot ER_t \tag{18.45}$$

where P^{MW} is the import prices in the world currency.

18.9 The Economic Mechanisms in the Real Sphere

In economic analyses based on macroeconometric models it frequently becomes necessary to treat jointly the relations in the real sphere. In such cases the multiplier analysis is commonly used. It helps identify the joint impacts of a single shock by treating the real sphere as a subsystem of the national economy (Klein et al. 1999).

In the demand-determined models three major economic mechanisms entering into the simultaneous feedbacks are typically distinguished (Welfe 2005). They are represented by the following multipliers: consumer multiplier, accelerator, and inflationary spiral.

The consumer multiplier summarizes the impacts of a single shock increasing household consumption. A rise in consumer demand (C_t) is followed by larger domestic output (X_t), modified by appropriately expanding competitive imports of

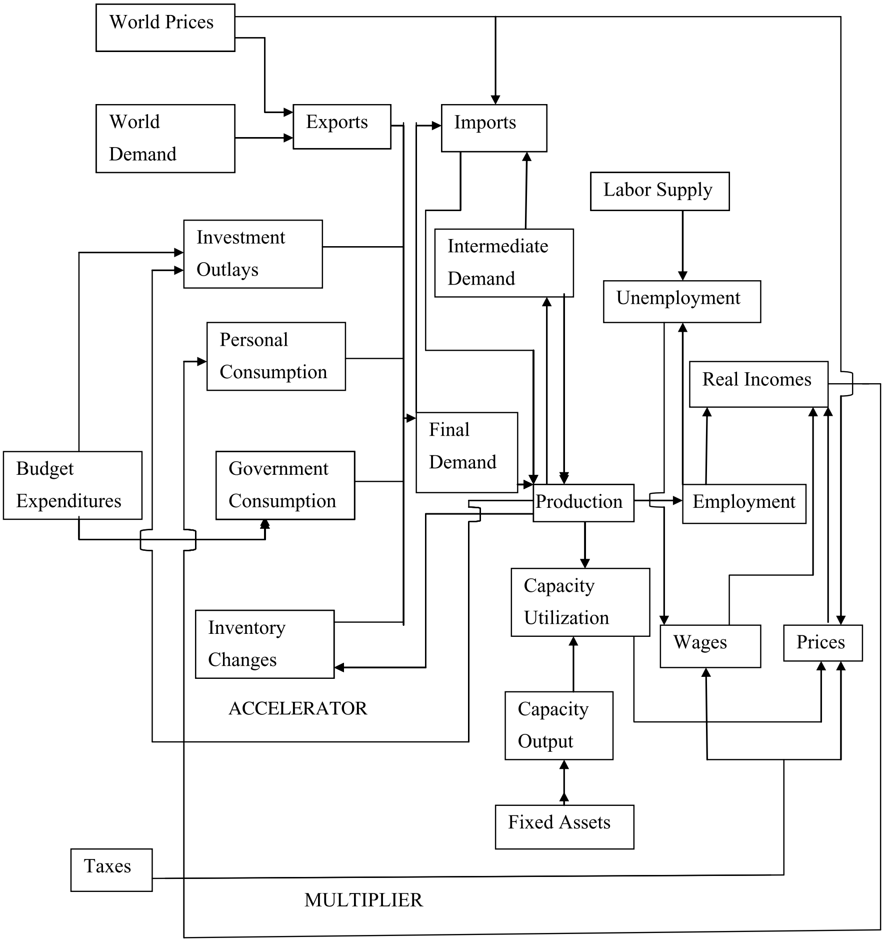

Fig. 18.1 Complete demand determined models. Real flows. Source: Klein et al. (1999, p. 113)

consumer goods. Growing output entails an increase in the number of working hours and, with a lag, in the number of employees (N_t) that is adjusted by the impact of technology changes. The wage bill and households' disposable incomes additionally grow and finally consumer demand expands.

The generalized consumer multiplier accounts for relationships containing financial flows. When the government budget increases its current expenditures the consumer expenditures of public institutions (G_t) also grow, as well as the wages bills related to public officers and old age and disability pensions, which increases households' incomes and stimulates their demand. The domestic output grows even more and the conditions for initiating the consumer multiplier are created.

The accelerator represents the relationships that control the investment process. An expected increase in production capacity triggers an rise in the demand for

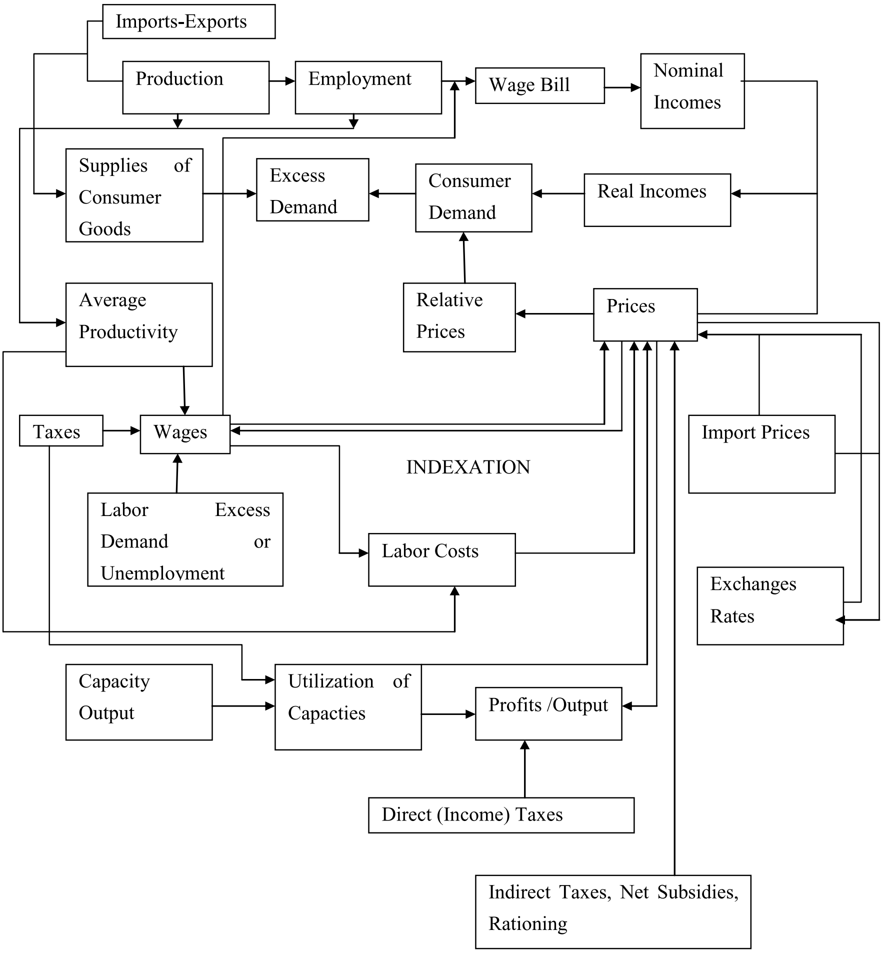

Fig. 18.2 Inflation. Source: Klein et al. (1999, p. 124)

investment goods. As a result, the domestic production of investment goods grows larger, constrained by additional imports of investment goods. Further increase takes place in the manufacturing industries. All this leads to stronger expectations of a production capacity increase.

These relationships are represented in Fig. 18.1.

The simplest version of the inflationary spiral is a summary of the relations between nominal wages and prices. An increase in the nominal average wages paid in the production sector will make labour costs grow, unless neutralized by increased labour productivity. The wage pressure will lead to an increase of producer prices and consequently of retail prices (CPI).

This has effect on the increase in nominal wages. It is worth noting that this feedback contains a mechanism reducing the rate of inflation, because the

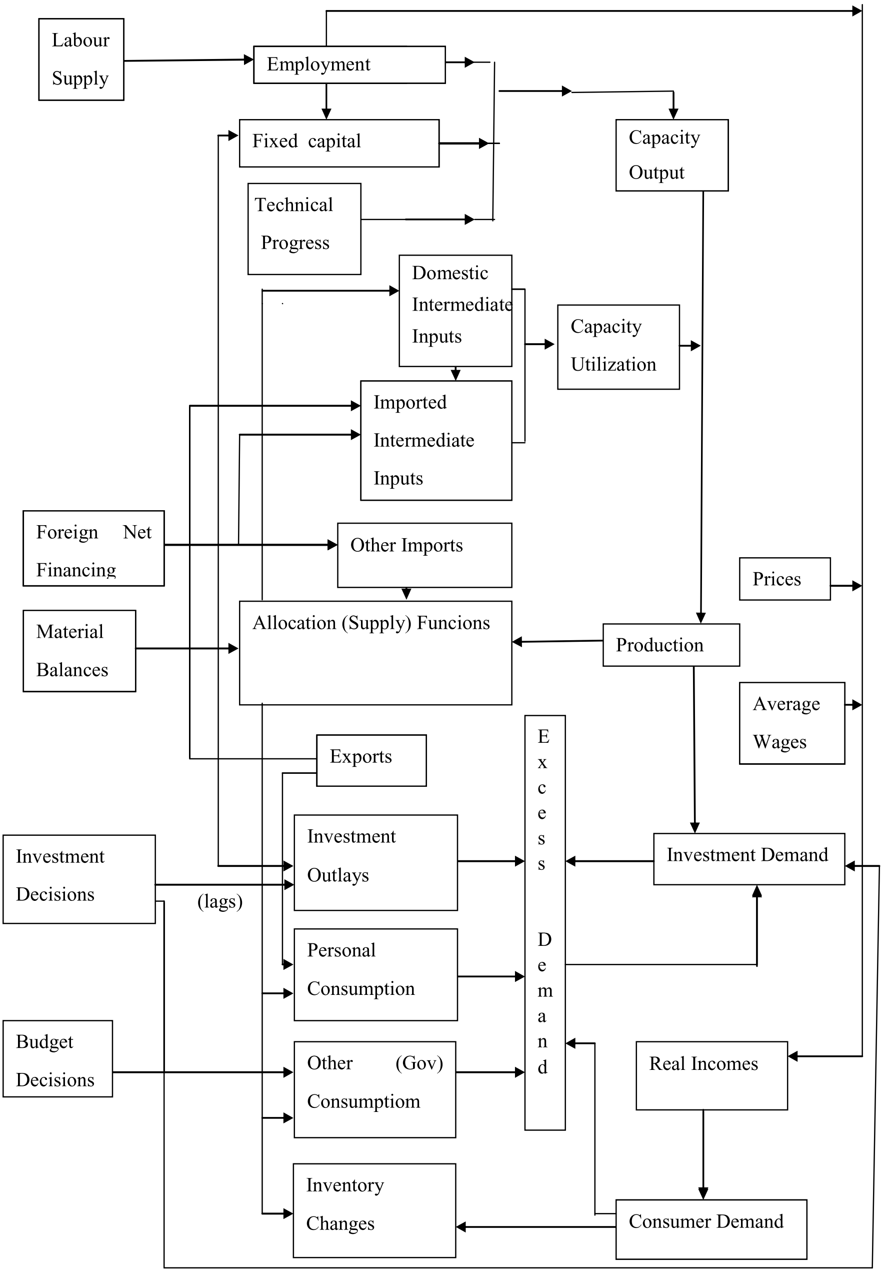

Fig. 18.3 Complete supply determined models. Real flows. Source: Klein et al. (1999, p. 112)

aforementioned increase in labour costs is generally unrelated to the increases in other items of production costs, such as import prices or interest rates. The likely impacts of changing interest rates and exchange rates will be discussed in the last chapter. Let us note that this system of wage and price equations, additionally enriched with the equation explaining the exchange rate, can be treated as a system of simultaneous equations. Hence it can be the subject of detailed cointegration analysis (Welfe 2002).

These relationships are shown in Fig. 18.2.

In the supply-determined models there is no room for the above economic mechanisms to function. Other feedbacks are entered instead.

In the case of consumer goods markets it is supply realized through market transactions. The changes in consumer demand are followed only by changes in excess demand. Hence, there are no conditions for changes related to the consumer multiplier to take place. A short-term supply-type consumer multiplier will appear instead (Barro and Grossman 1971). An increase in excess demand reduces labour force supply—this decreases output and commodity supply, so excess demand grows even larger. The decrease in the output can be partly offset, if the wage bill and household incomes become reduced at the same time. In the case of Poland this hypothesis was only partly confirmed, as the growing excess demand was followed by a declining number of hours worked, but not of employees (Welfe 1992).

In the above models the long-term supply accelerator plays an important role. An increase in the supply of investment goods augments (allowing for lags) the stock of fixed capital and subsequently of output and supplies, including the supplies of investment goods. This mechanism was a vital component of the policy of growth, particularly in the industrializing, developing countries (Welfe 2001).

In this class of models the short-term foreign trade multiplier can be additionally identified. An increase in exports determines the increase in aggregate imports and thus in production and exports.

The multisectoral models may have the bottleneck multiplier for chronic constraints in the supply of intermediate products. For instance, fuel delivery restrictions may constrain output as well as the energy industry, ultimately resulting to an energy crisis.

The above relationships are reproduced in Fig. 18.3.

References

Armington, P. (1969). *A theory of demand for products distinguished by place of production* (IMF Staff Papers XVI).

Bårdsen, G., Eitrheim, Ø., Jensen, E. S., & Nymoen, R. (2005). *The econometrics of macroeconomic modelling*. Oxford: Oxford University Press.

Barro, R. J., & Grossman, H. I. (1971). A general disequilibrium model of income and employment. *American Economic Review*, *61*, 82–93.

Blanchard, O., & Quah, D. (1989). The dynamic effects of aggregate supply and demand disturbances. *American Economic Review*, *79*, 635–673.

Bożyk, P., Maciejewski, W., & Piaszczyński, W. (1973). *Modele prognoz handlu międzynarodowego (Forecasting models of international trade)*. Warszawa: PWN.

Coen, R. M., & Hickman, B. G. (1976). Constrained joint estimation of factor demand and production function. *Review of Economics and Statistics*, *52*, 287–300.

Courbis, R. (1977). *Les modeles de prix. Pour la prevision et la planification etude comparative*. Paris: Dunod.

Czyżewski, B., & Welfe, W. (1990). Handel zagraniczny w ekonometrycznych modelach gospodarki narodowej (Foreign trade in macroeconometric models). In J. Kotyński (Ed.), *Kraje Europy Środkowo-Wschodniej w handlu zagranicznym* (pp. 179–198). Warszawa: PWN.

Dreze, J. H., Bean, C., Lambert, J. P., Metha, F., & Snessens, H. R. (Eds.) (1990). *Europe's unemployment problem*. Cambridge: MIT Press.

Elmeskov, J. (1993). *High and persistent unemployment: assessment of the problem and its causes* (OECD Economics Department Working Paper No. 132).

Elmeskov, J., & MacFarland, M. (1993) Unemployment persistence. *OECD Economic Studies 21*, 59–88.

Fair, R. C. (2004). *Estimating how the macroeconomy works*. Cambridge: Harvard University Press.

Florczak, W., & Welfe, W. (2000). Wyznaczanie potencjalnego PKB, łącznej produktywności czynników produkcji (Potential GDP and TFP determination). *Gospodarka Narodowa*, *11–12*, 10–55.

Friedman, M. (1968). The role of monetary policy. *American Economic Review*, *58*, 1–17.

Grzęda Latocha, R. (2005). Ekonometryczna analiza koniunktury gospodarczej w krajach strefy Euro (Econometric analysis of business cycle in the Euro-area). *Ekonomista*, *5*, 621–643.

Hickman, B. G., & Lau, L. J. (1973). Elasticities of substitution and export demand in a world trade model. *European Economic Review*, *4*, 347–380.

Juszczak, W. (1982). Ekonometryczne modele zapasów w gospodarce socjalistycznej (Econometric inventory models in socialist economies). *Studia Prawno-Ekonomiczne*, *29*, 131–146.

Klein, L. R., & Summers, R. (1966). *Studies in quantitative economics: Vol. 1. The Wharton index of capacity utilization*. Philadelphia: University of Pennsylvania.

Klein, L. R., Welfe, A., & Welfe, W. (1999). *Principles of macroeconometric modeling*. Amsterdam: North-Holland, Elsevier.

Layard, P. R. G., Nickell, S. J., & Jackman, R. (1991). *Unemployment: macroeconomic performance and the labour market*. Oxford: Oxford University Press.

Lovell, M. C. (1961). Manufactures inventories, sales, expectations and the acceleration principle. *Econometrica*, *29*.

Lovell, M. C. (1962). Buffer stocks, sales expectations and stability. A multisector analysis of the inventory cycle. *Econometrica*, *30*.

Nickell, S. J. (1984). The modelling of wages and employment. In D. H. Hendry & K. F. Wallis (Eds.), *Econometrics and quantitative economics*, Oxford: Basil Blackwell.

Phelps, E. S., et al. (1970). *Microeconomic foundations of employment and inflation theory*. New York: Norton.

Phillips, A. W. (1958). The relation between unemployment and the rate of change of money wage rates in the United Kingdom, 1961–1957. *Economica*, *25*(4), 283–299.

Tobin, J. (1972). The wage-price mechanism: overview of the conference. In O. Eckstein (Ed.), *The econometrics of price determination conference*. Washington: Federal Reserve System.

Welfe, W. (1992). *Ekonometryczne modele gospodarki narodowej Polski (Econometric models of Poland's national economy)*. Warszawa: PWE.

Welfe, W. (2000). Empiryczne modele wzrostu gospodarczego (Empirical models of economic growth). *Ekonomista*, *4*, 483–499.

Welfe, W. (Ed.) (2001). *Ekonometryczny model wzrostu gospodarczego (An econometric model of economic growth)*. Łódź: Wydawnictwo UŁ.

Welfe, W. (Ed.) (2002). Długookresowy, makroekonometryczny model W8D-2002 gospodarki polskiej (Long-term macroeconometric model W8D-2002 of the Polish economy). *Acta UŁ, Folia oeconomica, 172*. Łódź: Wydawnictwo UŁ.

Welfe, W. (2005). Stylized empirical model of economic growth. In W. Welfe & A. Welfe (Eds.), *Macromodels 2004, Problems of building and estimation of econometric models* (pp. 109–125). *Acta UŁ, Folia oeconomica, 190*. Łódź: Wydawnictwo UŁ.

Welfe, W. (Ed.) (2009). Makroekonometryczny model gospodarki polskiej opartej na wiedzy (A macroeconometric model of the knowledge economy for Poland). *Acta UŁ, Folia oeconomica, 229*. Łódź: Wydawnictwo UŁ.

Welfe, W., & Welfe, A. (2004). *Ekonometria stosowana (Applied econometrics)* (2nd ed.). Warszawa: PWE.

Welfe, W., Welfe, A., & Florczak, W. (1996). Makroekonomiczny roczny model gospodarki narodowej Polski (Macroeconomic annual model of the Polish economy) (Z Prac IRiSS No. 31). Warszawa.

Whitley, J. D. (1994). *A course in macroeconometric modelling and forecasting*. New York: Harvester Wheatsheaf.