

# Incentives and Security in Electricity Distribution Networks

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**Abstract.** We study incentive problems in electricity distribution when customer energy usage is imperfectly observable by the utility. Thus, we assume that each customer has private information about the amount of his consumed energy. Imperfect observability of individual user demand results in non-technical energy losses. In developing countries, these losses amount to 20 – 30% per year, and are largely attributed to theft by residential customers. Reducing these losses will allow a marked increase in efficiency of the electricity distribution. Usage of smart energy management devices enables new functionalities and brings the potential for such increased efficiency. However, employing smart energy management devices also entails a new set of problems. Typically, such devices are commercially produced, and employ off-the-shelf information technology (IT) solutions with inherent security vulnerabilities. Thus, due to technology limitations and cost constraints, smart devices are vulnerable to tampering and may enable systemic energy theft, threatening to reduce, or even erase the gains in efficiency. In this paper, we address incentives of utility company to combat theft (i.e., non-technical losses), when utility is subject to rate (tariff) regulation. From our analysis, such regulated utilities invest less than socially optimal in theft reduction. We suggest that regulators should include explicit targets for the allowable losses to remedy the problem of incentive misalignment.

## 1 Introduction

Energy theft in emerging economies has been a wide-spread practice. World Bank report [1] states that up to 50% of electricity consumed in certain parts of developing countries is acquired by means of theft. Here physical security considerations range from default on payment to stealing of energy through equipment manipulation. Second, cyber security threats to Advanced Metering Infrastructures (AMIs) are abundant. The AMI technology aims to cut cost of utilities and increase energy efficiency by providing new functionalities, including reducing unmetered and unbilled consumption. Yet, the AMI technology does not employ security-by-design principles [2],[3]. Unsurprisingly, a number of studies

have demonstrated that smart meters can be manipulated via tampering of physical and communication components as well as message spoofing [4],[5]. Also, [6] demonstrates the increased risk of energy theft which further justifies the importance of security considerations for AMIs. Finally, Anderson and Fuloria [7] point out that energy auditing and billing systems for AMIs suffer losses due to technical (e.g., transmission loss) and non-technical (e.g., fraud) reasons.

From the perspective of utility operator, the electricity losses in the distribution system are the amounts of electricity injected into the distribution network, which are not being paid by the users. These losses can be sub-divided on technical or non-technical. The resulting level of losses depends on the choices of utility operator and his customers. These choices are interdependent, and they also depend on technological and institutional environments.

Technical losses of the distributor occur due to the energy dissipation (i.e., current flowing) through resistive conductors and equipment used for power transmission, transformation, sub-transmission, and distribution. Non-technical losses occur due to the actions of (i) utility operator (for example, administrative losses due to errors in accounting and record keeping), (ii) customer theft (fraud or willful pilferage by bona-fide customers), (iii) customer non-payment (i.e., default), and lastly the theft by (iii) the outsiders (non-customers). In some cases, administrative errors are strategic, i.e., made with a purpose of assisting customer theft. Once the theft is detected, and the culprit is found, the losses (ii) and (iv) become subject of recovery, as in case (iii).

In this paper, we distinguish two main effects of the deployment of smart energy management devices. First, these devices permit to reduce the costs of utility operator via new functionalities, for example, the improved precision of dispatch, computerized metering and billing infrastructures. Second, these devices give customers new means for energy theft, for example via exploring IT insecurities. Our model could be straightforwardly modified to allow parametric assessment of these effects. And, while this paper focuses on the distribution system losses only, losses of the transmission system could be addressed in a similar manner.

The paper is organized as follows. In Section 2, we present an overview of non-technical losses in electricity distribution networks, and in Section 3 we briefly describe the regulatory regimes that are currently employed in different countries. Section 4 presents the model of interaction between consumers and a monopolist distributor. The consumers face a non-linear tariff for billed electricity, and are subject to an exogenous fine schedule for stolen/unbilled electricity (if detected). The distributor faces imperfect information about the consumer preferences and can partially recover the unbilled electricity using detection and enforcement mechanisms facilitated by Advanced Metering Infrastructures (AMIs). Under realistic assumptions on the probability of detection and fine schedule, we characterize the equilibrium consumption levels of billed and unbilled electricity, as well as the optimal tariff schedule and investment level of the distributor. In Section 5, we analyze the profit-maximizing tariff schedule and investment level when the distributor is subject to price cap

regulation. In particular, for average revenue regulation, we find that the investment level in AMIs can be sub-optimal relative to a perfectly informed regulator. In Section 6, we summarize our findings and conclude.

## 2 Non-technical Losses of Distribution System

The non-technical losses in electric distribution networks are due to theft, fraud, or uncollected (defaulted) payments. The consumers who fail to pay for electricity by acquiring it via stealing or defaulting on their bills, obtain the electricity at near zero prices. Effectively, electricity consumption of these non-paying parties is subsidized, because their consumption is paid by other members of the society. Specifically, the non-technical losses could be covered via (i) higher electricity tariffs for other consumers; (ii) the entire society (via taxes) if government subsidizes the distributor for these losses. In some cases, these losses remain uncovered for prolonged periods of time. Clearly, this situation negatively affects the efficiency of distribution system.

In most developed countries, the combined losses of transmission and distribution (T&D) systems do not exceed 10% [8]. First, the technical losses have been small due to historically adequate levels of investments in T&D, development and deployment of efficient transformers and other electric equipment, and transmission at higher voltages. For e.g., T&D losses in the US decreased from more than 16% in 1920 to less than 7% today. Only about half of these losses occur in the distribution system. Second, the non-technical losses in developed countries are also small, and in many countries even negligible. Industrialized countries have nearly 100% electrification, and for residential customers, expenses on electricity constitute a relatively low share of household incomes. For e.g., in today's US, electricity theft is considered unimportant. In comparison, the data for Italy suggests unusually high losses from theft. In the UK, the T&D losses are also high due to the aging grid infrastructure.

### 2.1 Losses in Developing Countries

In contrast with developed countries, many developing countries still experience high distribution system losses [9]. For South Asia (for example, India and Pakistan), and most sub-Saharan Africa countries, various official and unofficial estimates of T&D losses range from 15 – 50% [10]. Especially high levels of non-technical losses ultimately bear on the electricity rates (which are typically regulated), or higher taxes, or both.

We now briefly discuss the barriers in reducing these losses: Oftentimes, certain categories of consumers (e.g., agricultural, rural, or underprivileged consumers) are unmetered or pay a flat rate, i.e., the payment does not depend on quantity of consumed electricity. Such customers tend to increase their connected loads without obtaining the required sanctions for the increases of their loads. The under-payment by these consumers is often recovered from industrial or commercial customers who face higher tariffs. Such cross-subsidization,

combined with unreliability in supply (e.g., poor frequency control and irregular load-shedding during high-demand periods) encourages commercial enterprises to install their own local supply such as diesel generators. These locally generated electricity is expensive, and thus its usage introduces inefficiencies. Moreover, the distribution utility has incentives to fudge the consumption figures. Its reports to the regulator (e.g., public utility commissions) tend to provide higher estimates of unmetered consumption to under-report the actual losses.

Below we summarize the main channels of non-technical losses:

- Theft via availing unauthorized/unrecorded supply by tapping into conductors, feeders, and tampering service wires.
- Theft via willful pilferage by customers includes damaging or manipulating electric equipment and meters installed in their premises.
- Theft that is assisted by corrupt distribution utility's employees, who could make intentional billing errors in favor of certain consumers.
- Administrative losses including the errors in metering and billing of the actually consumed quantity, and errors in collecting billed amounts.

Combatting non-technical losses requires reducing the losses at each channel. This could be achieved via the implementation of measures at technological (e.g., detection tools) and organizational (e.g., enforcement capabilities) levels. We distinguish the following four categories of technological and regulatory measures that could be adopted to limit the non-technical losses:

- (1) *Technological (hardware) measures*: Installation of IT-supported meters at distribution transformers and feeders; Providing data-logging, remote monitoring and communication capabilities; Automated Meter Reading (AMR) and Advanced Metering Infrastructures (AMIs) to eliminate unmeasured and unbilled consumption.
- (2) *Technological (software) measures*: Management information systems equipped with data analytics tools to improve metering, billing and collection processes, and detection of fraud and unmetered connections.
- (3) *Regulatory measures*: Strengthening enforcement mechanisms (e.g., prosecution of theft); Publicizing theft cases for sharper public scrutiny (e.g., using the name and shame effect); Making consumers aware that electricity theft is a cognizable offense; Disconnection of customers related to fraud/debts and reconnection after clearance.
- (4) *Institutional measures*: Fixing the skewed tariff structures; Providing coordination and transparency in distribution operations; Investing in hardware and software upgrades.

## 2.2 Reforms in Distribution Sector

During past three decades, the power sector has experienced reforms. Overall, the reforms have resulted in unbundling of power sector operations, introduction of competitive wholesale electricity markets, and privatization of existing companies at the generation, transmission, and distribution levels. In this paper, we limit our attention to the distribution sector.

Before the reforms, the electricity distribution was largely provided by utilities, operating as state-owned enterprises (SOE). In general, these utilities tend to suffer from poor operational performance. The institutional environments with SOE typically also feature ill-defined and conflicting regulations, which distort managerial incentives, and could result in corrupt monitoring and enforcement practices. In addition, the state-owned utilities inherit other difficulties typical for non-market environments, such as sub-standard investment and overall poor managerial incentives. This translates in poor productivity, manifested by high losses and overall costs, and low service quality. In many cases, significant provision inefficiencies have resulted in widespread customer dissatisfaction.

Such non-performance of state-owned utilities necessitate the reforms of distribution system: drastic reorganizations of regulatory regime, and utility operator practices, including changes in ownership structures of utilities. Publicly available data about reforms is scarce, but there are clear indications that successful reforms of distribution system have resulted in substantial reduction of losses [11]. For e.g., independent regulatory commissions have been set up for licensing, regulating tariff structures, and promoting efficiency and competition. Indeed, distribution sector reforms have achieved increased efficiency levels by cutting technical and non-technical losses, and improving service quality. These reforms can be sustained over-time provided that properly designed institutional and regulatory framework to eliminates losses exists, and utilities have adequate incentives to improve their performance.

Even though post-reform losses of electricity distribution in developing countries are substantially lower than the respective pre-reform losses, *why these countries still face substantially higher losses in comparison to the developed countries?* Our stylized model suggests that imperfectly designed regulations, in particular, suboptimal levels of investment levels in monitoring and enforcement, could be responsible for that. In the next section, we present relevant insights from regulation of distribution sector.

### 3 Regulated Electricity Distribution

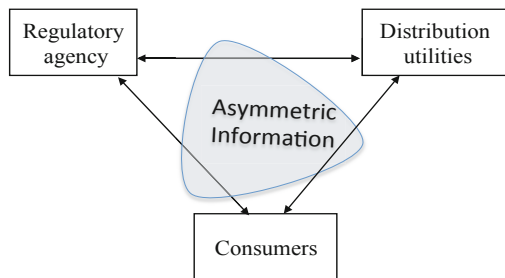
Electricity delivery to the end *consumers* is provided by utility companies (*distributors*), which operate as regulated monopolists. Each distributor can be viewed as an exclusive franchise subject to tariff and performance regulation. The entity responsible for overseeing the distributor is referred as the *regulator*. Thus, the actions of three types of entities are relevant for electric distribution: the regulator, the distribution utilities, and the consumers. The principles for regulating tariff structures are broadly similar across ownership structures of utilities (publicly owned and investor owned). Typically, the regulator's objective is to maximize consumer surplus, subject to a participation constraint for utility, and possibly other requirements, such as minimal level of service quality. The regulator's objectives can be summarized as [8]: operational efficiency to ensure reliably delivery at lowest reasonable cost, dynamic efficiency to meet

future demand, and consumption efficiency to ensure lowest prices subject to distributor's cost recovery and investment incentives. We also refer the reader to [12,13] for a modern treatment of regulatory principles in electricity distribution.

### 3.1 Asymmetric Information

The central issue in the design of regulatory policies in electricity distribution is the presence of *asymmetric information*; see Fig. 1. If regulator has perfect information about the distributor's costs and efficiency levels, and the consumer demand, designing regulatory requirements is straightforward [14]. If regulator's information is imperfect, and especially if hidden information is present, regulatory design becomes difficult, see [15]. The distributor has better knowledge of consumers' demand and its own technological capabilities (e.g., operational costs) in comparison to the regulator. There is a well-developed body of work on designing optimal regulatory policies of a monopoly distributor where he has privileged information about his technological capabilities and customers' demand and the regulator has well-defined inter-temporal commitment powers. However, such a normative analysis assumes that the regulator, although imperfectly informed about distributor's technological capabilities and customers' demand, perfectly knows the structure of regulated environment and has a formal model of information asymmetry between the regulator and the distributor.

Still, in practical situations, the precise nature of information asymmetry and the full set of relevant constraints on the regulator and the distributor are difficult to characterize a priori. Thus, design of regulatory policies should importantly take the robustness into account [16]. That is "well designed" regulatory policy must be robust, i.e., it must perform "reasonably well" under broad conditions, although such a policy may be sub-optimal in each particular setting. There are two main regulatory regimes that have been theoretically well-studied and adapted for a variety of practical settings: (i) rate of return (dominant regime in USA) and (ii) price cap (dominant regime in many parts of European Union and in some developing countries). Below we briefly outline each regime, but subsequently limit our analysis to price cap regulation.



**Fig. 1.** Players in regulated electricity distribution

### 3.2 Rate-of-Return versus Price Cap Regulation

Under *rate-of-return regulation*, the distribution utility is allowed a rate-of-return, and the rate structures for the electricity delivery are adjusted as the cost changes to ensure that the distributor has the opportunity to earn an authorized return. Here the regulator bears the onus of setting the prices and ensures that the realized rate of return does not deviate significantly from the target rate. Since the prices are directly linked to the realized costs, the distributor is unlikely to engage in cost-reducing activities. A classical example is the Averch-Johnson effect, which shows that the rate-of-return regulation deviates from cost minimization. However, since distributor faces limited risks of expropriation of his sunk investments by the regulator, upgrades of distribution network can be sustained in this form of regulation. The investment behavior of regulated distributor is especially important, since the infrastructure upgrades (e.g., capacity expansion) and modernization (e.g., AMI installations) require substantial costs.

Under *price cap regulation*, the tariffs provided to customers could increase, on average, at a specified rate during a pre-specified time. The specified rate is typically linked to the overall rate of inflation, and may fail to reflect the distributor's short-term realized costs and/ or profit. Typically, under a price cap regulatory regime, only average prices are controlled by the regulator, and the utility is given the flexibility to control the pattern of relative prices subject to pre-defined constraints. Since the tariffs are fixed and / or change according to a pre-specified rate for relatively long periods of time, the distributor has incentives to minimize its operating costs, and thus to operate efficiently.

Notice, that price cap does not directly provide good incentives for long-term investments in production, such as distribution network upgrades and reduction of non-technical losses. Similarly, price cap does not incentivize the distributor to choose optimal allocation of service quality. In this paper, we demonstrate that price cap regulation fails to incentivize the distributor to invest in monitoring and enforcement efforts to reduce unbilled electricity (e.g., consumer theft) at socially optimal levels.

When the pricing flexibility of price cap regulation is combined with the rewards (resp. punishments) for performance improvement (resp. deterioration) relative to the regulator's benchmark, the resulting regime is termed *performance-based* (or incentive) regulation. Indeed, in the face of rapidly changing technological environment and evolving customer preferences, the regulated electricity distribution industry is moving toward incentive regulation. The goal of incentive regulation is to improve distributor's incentives by decoupling regulated price structure from the need to know the exact operating / maintenance costs.

## 4 Consumer-Distributor Model

### 4.1 Consumer Preferences

We consider a population of consumers in which the individual tastes vary according to a type parameter  $\theta$ . Let  $\theta$  be distributed across the population

according to the density  $f^c(\theta)$  with cumulative distribution function  $F^c(\theta)$  on an interval  $[\underline{\theta}, \bar{\theta}]$  (where  $0 \leq \underline{\theta} < \bar{\theta}$ ). The electric distribution utility (a monopolist) cannot distinguish the type of given consumer, but knows the distribution  $F^c(\theta)$ .

Let us denote the billed and unbilled quantities for type- $\theta$  consumer as  $q_B(\theta)$  and  $q_U(\theta)$ , respectively. The total consumed quantity is  $q = q_B + q_U$ . The distribution utility (or distributor) offers a tariff (i.e., pricing schedule)  $T(q_B)$ , which specifies for each billed quantity  $q_B(\theta)$ , the total sum that the type- $\theta$  consumer should pay to the distributor. Special cases includes a linear pricing schedule corresponding to a single price, i.e.,  $T(q_B) = pq_B$ , and affine pricing schedule corresponding to a two-part tariff, i.e.,  $T(q_B) = A + pq_B$ . Here  $A$  is the fixed premium (or rental) and  $p$  is the charge varying with number of billed units. However, in general, the distributor can offer nonlinear tariff  $T(q_B)$ .

The unbilled quantity  $q_U$  constitute non-technical losses to the distributor and result from theft, fraud, or payment default by the consumers. If the distribution utility deploys Advanced Metering Infrastructures (AMIs), it improves its monitoring and billing efficiency, and thus reduces  $q_U$ . Let us denote the level of distributor's effort in deploying AMIs by  $e \in \mathbb{R}_+$ .<sup>1</sup> The efficiency of recovering unbilled electricity increases with  $e$ , and can be modeled as  $\rho : \mathbb{R}_+ \rightarrow (0, 1)$  which assigns for to each investment level  $e$ , a probability of detection. Thus, type- $\theta$  consumer's unbilled electricity is detected by the distributor with probability  $\rho(e)$ , and undetected with probability  $(1 - \rho(e))$ .

Let  $F^r(q_U)$  denote the fine schedule that is exogenously fixed by the regulator (and hence the superscript  $r$ ), and is known to consumers and the distributor. If the unbilled electricity  $q_U(\theta)$  were perfectly detected, a consumer of type  $\theta$  would pay  $F^r(q_U(\theta))$  to the distributor. However, under imperfect detection, the distributor only recovers payment for  $\rho(e)q_U < q_U$  via fines, and the remaining quantity,  $(1 - \rho(e))q_U$ , is considered stolen. In accordance with current detection technology and enforcement practices, we impose the following assumption:

**Assumption 1**  $\rho(\cdot)$  is concave increasing in  $e$ , and  $F^{r'}(\cdot)$  is nondecreasing in  $q_U$ .

Suppose that each consumer has the following utility function:

$$U = \begin{cases} \theta u(q_B + q_U) - T(q_B) - \rho(e)F^r(q_U) & \text{[AMIs deployed with effort } e] \\ \theta u(q_B + q_U) - T(q_B) & \text{[AMIs are not deployed],} \end{cases} \tag{1}$$

where the function  $u(\cdot)$  satisfies  $u(0) = 0$ ,  $u'(q) > 0$ , and  $u''(q) < 0$ , i.e., there is a decreasing marginal utility of electricity consumption. Also,  $u(\cdot)$  is assumed to be same for all consumers.

In our model, the unbilled electricity is undetectable when AMIs are not deployed. Then,  $q_B \equiv 0$  becomes a trivial solution, i.e., the consumers do not

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<sup>1</sup> The theory of regulation [17] has considered the distributor's cost reducing effort  $e$ . In this paper,  $e$  is specific to deployment of AMIs, and specifies the monitoring and enforcement effort of the distributor for reducing the quantity of unbilled electricity.



prefer a quantity dependent tariff (although they may still pay a flat rate). Hence, we only consider the case when the distributor deploys AMIs at effort level  $e$ . A type- $\theta$  consumer facing the distributor’s tariff schedule  $T(q_B)$  and a fine schedule  $F^r(q_U)$  obtains a net surplus  $v(\theta)$  given by

$$v(\theta) \equiv \max_{q_B \geq 0, q_U \geq 0} [\theta u(q_B + q_U) - T(q_B) - \rho(e)F^r(q_U)]. \tag{2}$$

The first-order-conditions (FOCs) are:

$$\theta u'(q_B + q_U) - T'(q_B) = 0, \quad \text{and} \quad \theta u'(q_B + q_U) - \rho(e)F^{r'}(q_U) = 0, \tag{3}$$

which implies

$$\theta u'(q(\theta)) = T'(q_B(\theta)) = \rho(e)F^{r'}(q_U(\theta)). \tag{4}$$

Hence, a small increase in the total quantity consumed by a type- $\theta$  consumer generates the marginal surplus  $\theta u'(q(\theta))$  equal to the marginal payment  $T'(q_B(\theta))$  (resp. *expected* marginal fine  $\rho(e)F^{r'}(q_U(\theta))$ ) for a small increase in the billed (resp. unbilled) quantity. Once the quantity functions  $q_B$  and  $q_U$  are known, the payment function can be obtained using (4). Also, since  $\rho < 1$ , we obtain

$$T'(q_B(\theta)) < F^{r'}(q_U(\theta)), \quad a.e.$$

Without loss of generality we assume that the tariff and fine schedules satisfy  $T(0) \leq 0$  and  $F^r(0) \leq 0$ , respectively (because the consumers have the option of consuming nothing at zero cost). Under our assumptions, the following holds:

**Lemma 1.** (i)  $v(\cdot)$  is non-negative, increasing, convex, and differentiable almost everywhere (a.e.); (ii) For a type- $\theta$  consumer, the chosen (optimal) quantity of electricity,  $q(\theta) \equiv q_B(\theta) + q_U(\theta)$ , is unique, increasing in  $\theta$ , and is given by  $v'(\theta) = u(q(\theta))$ ; (iii) the chosen billed  $q_B(\theta)$  and unbilled  $q_U(\theta)$  quantities are both unique, and satisfy

$$T'(q_B(\theta)) = \frac{\theta v''(\theta)}{(dq(\theta)/d\theta)}, \quad F^{r'}(q_U(\theta)) = \frac{\theta v''(\theta)}{\rho(e) (dq(\theta)/d\theta)}, \quad a.e. \tag{5}$$

*Remark 1.* The distributor’s collection efficiency can be defined as follows:

$$\eta \equiv 1 - \frac{\int_{\underline{\theta}}^{\bar{\theta}} (1 - \rho(e)) q_U(\theta) f^c(\theta) d\theta}{\int_{\underline{\theta}}^{\bar{\theta}} (q_B(\theta) + q_U(\theta)) f^c(\theta) d\theta}. \tag{6}$$

### 4.2 Distributor’s Revenue

Let us introduce the revenue function of the distributor. For a quantity  $Q$  of total electricity provisioned by the distributor, we define the revenue function  $R(Q)$  as his maximum revenue, when he offers a tariff schedule  $T(q_B)$  for billed

quantity  $q_B(\theta)$ , and implements a fine schedule  $F^r(q_U)$  to recover the unbilled quantity  $q_U(\theta)$  with probability  $\rho(e)$ :

$$R(Q) = \max \int_{\underline{\theta}}^{\bar{\theta}} [T(q_B(\theta)) + \rho(e)F^r(q_U(\theta))] f^c(\theta)d\theta, \text{ subject to} \tag{7}$$

$$\forall \theta, \quad [\theta u(q_B + q_U) - T(q_B) - \rho(e)F^r(q_U)] \geq 0, \tag{7a}$$

$$q_B(\theta), q_U(\theta) \text{ maximize } [\theta u(q_B + q_U) - T(q_B) - \rho(e)F^r(q_U)], \tag{7b}$$

$$Q \geq \int_{\underline{\theta}}^{\bar{\theta}} (q_B(\theta) + q_U(\theta)) f^c(\theta)d\theta, \tag{7c}$$

Here *individual-rationality* (IR) constraint (7a) ensures that all consumers are willing to purchase. Actually, it suffices to require that the lowest demand consumer (type- $\underline{\theta}$ ) is individually rational, i.e.,

$$[\underline{\theta}u(q_B(\underline{\theta}) + q_U(\underline{\theta})) - T(q_B(\underline{\theta})) - \rho(e)F^r(q_U(\underline{\theta}))] \geq 0.$$

The constraint (7b) ensures that the consumers do not exercise personal arbitrage. In other words, it requires that  $\forall \theta, \tilde{\theta}$

$$U(\theta) = \theta u(q_B(\theta) + q_U(\theta)) - T(q_B(\theta)) - \rho(e)F^r(q_U(\theta)) \\ \geq \theta u(q_B(\tilde{\theta}) + q_U(\tilde{\theta})) - T(q_B(\tilde{\theta})) - \rho(e)F^r(q_U(\tilde{\theta})),$$

i.e., the type- $\theta$  consumer must not choose the same quantity bundles as chosen by the type- $\tilde{\theta}$  consumer (where  $\tilde{\theta} \neq \theta$ ). These are known as *incentive compatibility* (IC) constraints. Finally, the constraint (7c) ensures that the billed plus unbilled quantity of electricity is no greater than  $Q$ .

We now seek an alternative representation of the revenue maximization problem (7). Let us write  $T(q_B(\theta)) + \rho(e)F^r(q_U(\theta)) = \theta u(q_B(\theta) + q_U(\theta)) - v(\theta)$ , and recall from Lemma 1 that  $v'(\theta) = u(q(\theta))$ . We can express the net surplus of type- $\theta$  consumer as

$$v(\theta) = \int_{\underline{\theta}}^{\theta} u(q_B(\zeta) + q_U(\zeta))d\zeta + v(\underline{\theta}) = \int_{\underline{\theta}}^{\theta} u(q_B(\zeta) + q_U(\zeta))d\zeta,$$

where the second equality uses the IR constraint ( $v(\underline{\theta}) = 0$ ). Then, the revenue maximization problem (7) can be re-written as:

$$R(Q) = \max \int_{\underline{\theta}}^{\bar{\theta}} \left[ \theta u(q_B(\theta) + q_U(\theta)) - \int_{\underline{\theta}}^{\theta} u(q_B(\zeta) + q_U(\zeta))d\zeta \right] f^c(\theta)d\theta,$$

subject to the constraints (7b), and (7c).

It is straightforward to see that the constraint (7c) is binding, because  $R(Q)$  can be increased by allocating larger quantities to high consumer types. Integrating by parts, and noting that constraint (7b) is equivalent to imposing that  $q(\cdot)$  is nondecreasing in  $\theta$  where  $q_B(\cdot)$  and  $q_U(\cdot)$  verify (5), we obtain

$$R(Q) = \max_{\theta} \int_{\theta}^{\bar{\theta}} [\theta f^c(\theta) - (1 - F^c(\theta))] u(q_B(\theta) + q_U(\theta)) d\theta \text{ subject to} \quad (8)$$

$$(i) \quad q(\theta) \equiv q_B(\cdot) + q_U(\cdot) \text{ nondecreasing, and (5) holds} \quad (8a)$$

$$(ii) \quad Q = \int_{\theta}^{\bar{\theta}} (q_B(\theta) + q_U(\theta)) f^c(\theta) d\theta. \quad (8b)$$

In solving the above optimization problem, we initially ignore the constraint (i) but verify it *ex post*. Let  $\lambda(Q)$  be the Lagrange multiplier associated with constraint (ii). From the Envelope Theorem, we obtain the following useful relation:

$$R'(Q) \equiv \lambda(Q). \quad (9)$$

Since  $R(Q)$  is non-decreasing in  $Q$ , we conclude that  $\lambda(Q)$  is non-negative. Moreover, the optimal response functions  $q^*(\theta) \equiv (q_B^*(\theta) + q_U^*(\theta))$  satisfy:

$$q^*(\theta) = \arg \max_{q \geq 0} \left[ \theta - \frac{1 - F^c(\theta)}{f^c(\theta)} \right] u(q) - \lambda(Q)q, \quad (10)$$

The FOC for pointwise maximization of (10):

$$\theta u'(q^*(\theta)) = \frac{\lambda(Q)}{\left[ 1 - \frac{1 - F^c(\theta)}{\theta f^c(\theta)} \right]}. \quad (11)$$

We make the following standard assumption about the *hazard rate* of the type distribution, which holds for many common distributions including uniform, normal, logistic, exponential, etc.

**Assumption 2** *The hazard rate of type distribution  $\frac{f^c(\theta)}{1 - F^c(\theta)}$  increases with  $\theta$ .*

From Assumption (2), we observe that the expression  $\left[ \theta - \frac{1 - F^c(\theta)}{f^c(\theta)} \right]$  increases with  $\theta$ . Then, from (11) and the fact that  $u$  is concave,  $q^*(\theta)$  is increasing in  $\theta$ . To complete checking the constraint (8a), see (12) below. The following lemma follows from [18]:

**Lemma 2.** *Under Assumption (2), the revenue function  $R(Q)$  is strictly concave.*

From (9) and Lemma 2, we observe that  $\lambda(Q)$  decreases with  $Q$ .

Now let  $p^*(q_B) \equiv (T^*)'(q_B)$  denote the marginal price for the billed quantity corresponding to the optimal tariff schedule  $T^*(q_B)$ . Similarly, let  $p_f^*(q_U) \equiv (F^r)'(q_U)$  denote the fine for an extra unit of unbilled electricity (if detected), when the consumer has an unbilled amount  $q_U$ . Equation (11), and the FOCs (3) for  $q_B^*(\theta) > 0$  and  $q_U^*(\theta) > 0$  to be optimal choices for type- $\theta$  consumer, imply the following result:

**Theorem 3.** *Let the assumption 2 hold. Then, for a quantity of total electricity Q and AMI investment level e by the distributor, the marginal price schedule and the marginal fine schedule satisfy*

$$p^*(q_B^*(\theta)) = \rho(e)p_f^r(q_U^*(\theta)) = \frac{\lambda(Q)}{\left[1 - \frac{1-F^c(\theta)}{\theta f^c(\theta)}\right]}, \tag{12}$$

where  $q_B^*(\theta) + q_U^*(\theta) = q^*(\theta)$ , with  $q^*(\cdot)$  given by (10).

Since we assume that  $p_f^r(\cdot)$  is nondecreasing in  $q_U$ , (12) implies that the optimal consumer choice of billed (resp. unbilled) electricity is increasing (resp. non-increasing) in  $\theta$ , i.e.,

**Corollary 1.** *Under assumptions 1 and 2,  $q_U^*(\cdot)$  (resp.  $q_B^*(\cdot)$ ) is non-increasing (resp. increasing) in  $\theta$ .*

We now deduce the shape of optimal tariff schedule. Since  $p^*(q_B^*(\theta))$  is decreasing in  $\theta$  (from (12)), and  $q_B^*(\cdot)$  is increasing in  $\theta$  (from Corollary 1), we conclude that  $p^*(q_B)$  is decreasing in  $q_B$ . Thus, under the assumption on non-decreasing marginal fine schedule, we obtain that  $T^*(\cdot)$  is concave in  $q_B$ . This is the classical quantity discount result for a revenue maximizing distributor [19]!

### 4.3 Unregulated Distributor

Consider an unregulated distributor with aggregate cost function  $C(\beta, e, Q)$  of provisioning the total quantity of electricity Q and effort level  $e \geq 0$  for detecting unbilled electricity via AMIs. The parameter  $\beta \in [\underline{\beta}, \bar{\beta}]$  signifies the distributor’s technological efficiency. Thus, a distributor of type  $\underline{\beta}$  (resp.  $\bar{\beta}$ ) is most (resp. least) efficient in reducing nontechnical losses (and hence, unbilled electricity). We assume the following standard assumptions:  $\partial_\beta C > 0, \partial_e C < 0, \partial_Q C > 0$ .

Let  $\psi(e)$  denote the distributor’s fixed cost of deploying AMIs at effort level e, where  $\psi'(e) > 0, \psi''(e) > 0$ . The problem of computing the profit maximizing quantity of electricity  $Q^*$  and optimal investment level  $e^*$  for an unregulated monopolist (who knows  $\beta$ ) can be written as

$$\pi^m(\beta) = \max_{Q \geq 0, e \geq 0} R(Q) - C(\beta, e, Q) - \psi(e), \tag{13}$$

Using (9), the FOCs for (13) involve setting  $Q^*$  and  $e^*$  to satisfy

$$\partial_Q C(\beta, e^*, Q^*) = \lambda(Q^*), \quad \text{and} \quad \partial_e C(\beta, e^*, Q^*) = -\psi'(e^*). \tag{14}$$

Then, from Theorem 3, the distributor chooses a tariff schedule  $T^*(q)$  and investment level  $e^*$  such that its profit from supplying the total quantity  $Q^*$  is maximized.

For simplicity, let us assume the following cost function:

$$C(\beta, e, Q) = (\beta - e)Q, \tag{15}$$

where marginal cost of distribution  $\beta - e > 0$  over the relevant range of operation. For cost function (15),  $e^*(\beta)$  and  $Q^*(\beta)$  satisfy:

$$e^*(\beta) = \beta - \lambda(Q^*(\beta)), \quad Q^*(\beta) = \psi'(e^*(\beta)).$$

Notice from (12) that the highest demand consumer pays the marginal *aggregate* cost for the billed electricity, i.e.,

$$p^*(q_B^*(\bar{\theta})) = \partial_Q C(e^*, Q^*) = \beta - e^*(\beta),$$

where we have used the fact that  $F^c(\bar{\theta}) = 1$ .

*Remark 2.* Following (6), the distributor’s collection efficiency becomes

$$\eta^*(e^*, Q^*) = 1 - \frac{(1 - \rho(e^*)) \int_{\underline{\theta}}^{\bar{\theta}} q_U^*(\theta) f^c(\theta) d\theta}{Q^*}, \quad \text{where } q_U^*(\theta) \text{ satisfies (12).}$$

To summarize, the interaction between consumers and distributor can be viewed as a non-zero sum *Stackelberg game*, where the distributor acts as leader and the consumers act as followers.<sup>2</sup> The fine schedule  $F^r(\cdot)$  and detection probability function  $\rho(\cdot)$  are common knowledge. Based on his prior belief of consumer types  $f^c(\cdot)$ , the monopolist distributor offers a tariff schedule  $T(\cdot)$ , and also selects output level  $Q$  and AMI investment level  $e$ . A type- $\theta$  consumer, knowing the strategy of the distributor, chooses his consumption levels of billed  $q_B^*(\theta)$  and unbilled  $q_U^*(\theta)$  quantities to maximize his individual utility; see Section 4.1. The distributor, knowing the consumers’ rationale, must choose optimal  $Q^*$ ,  $e^*$ , and  $T^*(\cdot)$  to maximize his profit.

## 5 Price Cap Regulation

We now analyze a form of price cap regulation in which the distributor faces an average revenue constraint. The distributor can offer tariff  $T$  and enforce penalty  $F^r$  with AMI investment level  $e$ , only if the induced consumer demand functions  $q_B(\theta)$  and  $q_U(\theta)$  permit an average revenue that is no more than a regulator-specified price cap. Two possible average revenue constraints are:

$$\int_{\underline{\theta}}^{\bar{\theta}} [T(q_B(\theta)) + \rho(e)F^r(q_U(\theta))] f^c(\theta) d\theta \leq \bar{p}Q \tag{15a}$$

$$\int_{\underline{\theta}}^{\bar{\theta}} [T(q_B(\theta)) + \rho(e)F^r(q_U(\theta))] f^c(\theta) d\theta \leq \bar{p}(Q - Q_S), \tag{15b}$$

where  $\bar{p}$  is the maximum permitted level of average revenue per unit of electricity determined by the regulator (i.e.,  $\bar{p}$  is the *price cap*),  $Q$  is the distributor’s total

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<sup>2</sup> Stackelberg games have been used in the context of incentive design in both engineering [20,21,22] and economics [14,15,16].

output ( $Q = \int_{\theta}^{\bar{\theta}} (q_B(\theta) + q_U(\theta)f^c(\theta)d\theta)$ ), and  $Q_S$  is the net quantity of stolen electricity ( $Q_S = (1 - \rho(e)) \int_{\theta}^{\bar{\theta}} q_U(\theta)f^c(\theta)d\theta$ ). Thus, (15b) is a stricter constraint in comparison to (15a) because it excludes the stolen electricity in computing the average revenue, and only accounts for the billed plus recovered quantities.

From the regulator’s viewpoint, the constraint (15b) is more desirable because he considers the consumer surplus for given tariff and fine schedules to be  $S(T, F^r) = \int_{\theta}^{\bar{\theta}} v(q_B(\theta + \rho(e)q_U))f^c(\theta)d\theta$ . In determining the price cap  $\bar{p}$ , the regulator will not account for the consumers’ surplus resulting from successfully stolen (undetected) electricity  $Q_S$ . From the distributor’s viewpoint, the constraint (15a) is more desirable because it eases the regulatory constraint.

We first suppose that at the tariff schedule chosen by the distributor, the average revenue constraint (15a) is imposed by the regulator and is binding.<sup>3</sup> Then, the distributor’s goal is to choose output level  $Q$  and AMI investment level  $e$  that solves the following maximization problem

$$\hat{\pi} = \max_{Q \geq 0, e \geq 0} R - C(\beta, e, Q) - \psi(e) \text{ subject to}$$

$$(i) \quad R = \bar{p}Q$$

$$(ii) \quad R \leq R(Q),$$
(16)

where the constraint (i) is the average revenue constraint, and (ii) specifies that  $R$  should indeed be attainable at total output  $Q$ . Now, (16) can be expressed as:

$$\max_{Q \geq 0, e \geq 0} \bar{p}Q - C(\beta, e, Q) - \psi(e), \quad \text{subject to } \bar{p}Q \leq R(Q).$$

From the concavity of  $R(Q)$  (see Lemma 2), we can conclude that there exists a unique  $\hat{Q} > 0$  such that the following two conditions hold: first,  $R(\hat{Q}) = \bar{p}\hat{Q}$ , and second,  $R(Q) \geq \bar{p}Q$  if and only if  $Q \leq \hat{Q}$ . Thus, (16) can be rewritten as

$$\max_{Q \geq 0, e \geq 0} \bar{p}Q - C(\beta, e, Q) - \psi(e), \quad \text{subject to } 0 \leq Q \leq \hat{Q}.$$

This observation leads to the following extension of the result by Armstrong, Cowan, and Vickers [18]:

**Theorem 4.** *Let  $\hat{Q}$  be the unique level of output level satisfying  $R(\hat{Q}) = \bar{p}\hat{Q}$ , and let  $\bar{p} \geq \partial_Q C(\beta, e, Q)$  for  $Q \leq \hat{Q}$ . Then, if the constraint (15a) binds, the distributor will choose output level  $\hat{Q} > Q^*$  and  $\hat{e} > e^*$ , where  $Q^*$  and  $e^*$  respectively denote the profit-maximising output and AMI investment level of the unregulated monopolist distributor, and  $\partial_e C(\beta, \hat{e}, \hat{Q}) = -\psi'(\hat{e})$ .*

Furthermore, the distributor will offer a nonlinear tariff:

$$\hat{p}(\hat{q}_B(\theta)) \equiv (\hat{T})'(q_B) = \frac{\lambda(\hat{Q})}{\left[1 - \frac{1 - F^c(\theta)}{\theta f^c(\theta)}\right]},$$
(17)

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<sup>3</sup> Our analysis is similar to [23,18].

where  $\hat{T}(\cdot)$  is the optimal tariff schedule under binding regulatory constraint (15a),  $\hat{p}(q_B) = \hat{T}'(q_B)$  is the marginal price, and  $\hat{q}_B(\theta)$  is the corresponding quantity of billed electricity chosen by type- $\theta$  consumer.

Thus, the marginal price schedule corresponding to the optimal tariff under binding regulatory constraint (15a) is of the same form as the corresponding marginal price schedule for unregulated monopoly. However, under binding regulatory constraint (15a), the total output and AMI investment level increases. Recall that  $\lambda(\cdot)$  is decreasing in  $Q$  and  $\partial_Q C(\beta, e^*, Q^*) = \lambda(Q^*)$ . If  $C$  is weakly convex in  $Q$  then we obtain

$$\lambda(\hat{Q}) < \partial_Q C(\beta, \hat{e}, \hat{Q}) \tag{18}$$

From (17) and (18), we conclude that for type- $\bar{\theta}$  consumer,  $\hat{p}(\hat{q}_B(\bar{\theta})) < \partial_Q C(\beta, \hat{e}, \hat{Q})$ . Thus, it is optimal for the distributor under average revenue regulation to set the marginal price schedule *below* the marginal cost for sufficiently high-demand consumers (higher  $\theta$ ). The pricing of billed electricity below marginal costs occurs because higher type  $\theta$  consumers have higher demand.

Next, suppose that at the tariff schedule chosen by the distributor, the average revenue constraint (15b) is binding. The distributor's choices of  $Q$  and  $e$  solve:

$$\begin{aligned} \tilde{\pi} \max_{Q \geq 0, e \geq 0} R - C(\beta, e, Q) - \psi(e) \quad & \text{subject to} \\ R = \bar{p}[Q - (1 - \rho(e)Q_U)] & \\ R \leq R(Q). & \end{aligned} \tag{19}$$

Rewriting, the problem (19) reduces to

$$\begin{aligned} \max_{Q \geq 0, e \geq 0} \bar{p}[Q - (1 - \rho(e)Q_U)] - C(\beta, e, Q) - \psi(e) \quad & \text{subject to} \\ R(Q) \geq \bar{p}[Q - (1 - \rho(e)Q_U)], & \end{aligned}$$

By using the definition of collection efficiency (6), (19) can be expressed as

$$\max_{Q \geq 0, e \geq 0} \bar{p}\eta(Q, e)Q - C(\beta, e, Q) - \psi(e) \quad \text{subject to} \quad R(Q) \geq \bar{p}\eta(Q, e)Q.$$

Again, from strict concavity of  $R(\cdot)$ , there exists a unique  $\tilde{Q} > 0$  and  $\tilde{e} > 0$  satisfying  $R(\tilde{Q}) = \bar{p}\eta(\tilde{Q}, \tilde{e})\tilde{Q}$ , and  $R(Q) \geq \bar{p}\eta(Q, e)$  if and only if  $Q \leq \tilde{Q}$ . The following result can be shown:

*Claim.* There exists a  $\bar{p}$ , such that  $\hat{\pi} > \tilde{\pi}$  and  $\hat{e} \leq \tilde{e}$ .

In this case, the distributor's preference is to induce the regulator in choosing (15a) as the binding regulatory constraint (since  $\hat{\pi} > \tilde{\pi}$ ). However, this regime also leads to a sub-optimal AMI investment level  $\hat{e}$  relative to the level achieved under when (15b) is binding ( $\hat{e} \leq \tilde{e}$ ), i.e., when the regulator is perfectly informed about  $Q_U$ .

## 6 Concluding Remarks

In this paper, we propose to study incentives of a regulated utility to invest in theft reduction via monitoring and enforcement. We have shown that utility under a price cap regulation will underinvest in monitoring customer theft relative to social planner, i.e., a perfectly informed regulator. Thus, in equilibrium, profit maximizing utility operator incurs higher aggregate losses, and has higher equilibrium theft than would be socially optimal. This effect is driven by the regulatory threat of lower price cap, which will be optimal with a higher monitoring level, and thus lower aggregate equilibrium theft.

Our results are consistent with published empirical evidence on electricity distribution losses. Indeed, successful reforms of financially inept state-owned utilities tend to be accompanied by strengthening of monitoring, and dramatic reduction of losses due to non-technical reasons (theft plus billing errors). A combination of technological and institutional means is used in Chile, Brazil, and Argentina; see [1],[5]. Our analysis could be modified to address the theft in transmission system as well. In addition, we argue that deployment of the AMI technology in developed or advanced industrial countries may result in resurgence of non-technical losses. The problem could be especially acute under a bleak economic conditions, when the theft traditionally raises.

We suggest that regulators should include explicit targets for the allowable losses to remedy the problem of incentive misalignment. While institutional and regulatory aspect of reforms are important to improve distribution sector performance, continual adaptation of information technology tools is also essential to maintain operational performance. Without regulatory, institutional, and technological structures in place, the poor operational performance and fiscal discipline will continue to mar the electricity distribution sector.

**Acknowledgement.** We are grateful to the GameSec'12 TPC Chairs for their help, and to anonymous reviewers for providing useful suggestions. This work was supported by MIT faculty start-up grant and NSF TRUST Science & Technology Center.

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