Chapter 1
Genesis of the Concept of Polarization Bremsstrahlung

1.1 Definition and Physics of Polarization Bremsstrahlung

Polarization bremsstrahlung (PBs) is a fundamental radiative process arising in scattering of a charged particle by a target with internal degrees of freedom [1]. An elementary example of PBs is radiation of an electron on an atom. In this case internal degrees of freedom correspond to bound electrons of an atom that can reradiate an electromagnetic eigenfield of an incident particle (in this case of an electron). This process is presented in Fig. 1.1 together with ordinary bremsstrahlung (OBs) that is also called static bremsstrahlung (SBs).

In Fig. 1.1 the dotted arrows show a real bremsstrahlung photon, the solid thick arrow shows a virtual photon of the eigenfield of an incident particle, the solid curve shows the trajectory of an incident particle.

It will be recalled that OBs results from acceleration of a charged particle in the field of a target. If an electron is scattered by a "bare" nucleus, bremsstrahlung (Bs) will proceed only by an ordinary (static) channel if nuclear degrees of freedom are neglected. In case of the presence of bound electrons, PBs arises together with OBs. To take into account the polarization channel, it is necessary to consider atomic electrons "on a par" with an incident electron, that is, as an independent dynamic degree of freedom able to participate "fully" in electromagnetic processes. At early stages of the theory of Bs on an atom an approximate model was used, within the framework of which bound electrons were replaced with the static distribution of electron charge. In other words, their role reduced exclusively to screening an atomic nucleus. So this model is called *the screening approximation*. In the screening approximation the polarization Bs channel is absent since atomic electrons are prevented from reradiating the electromagnetic field of an incident particle. But the fact is that atomic electrons are excited as a result of collision with a charged particle incident on an atom and can transform the energy of this excitation to the energy of a bremsstrahlung photon. It is significant that in this case the matter is so-called virtual excitation, when an electron is in a state with a

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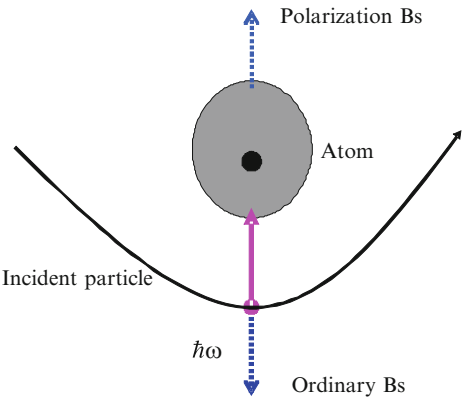

Fig. 1.1 Two channels of bremsstrahlung of an incident particle on an atom

higher energy that does not conform to the stationary state of an electron in the field of the atomic core.

It is possible to give PBs a double interpretation. First, this process can be represented as the conversion (reradiation) of a virtual photon making the eigenfield of an incident charge to a real photon on the electron subsystem of a target. This interpretation goes back to the method of equivalent photons that was for the first time applied by E. Fermi to calculate cross-sections of excitation of an atom by charged particles even before development of quantum mechanics. In the case under consideration PBs results from scattering of a virtual photon by atomic electrons to a real photon. This approach is especially descriptive in case of a relativistic incident particle, the electromagnetic field of which in its structure is close to the field of a free electromagnetic wave corresponding to radiation in an empty space. Another interpretation treats PBs as radiation arising as a result of induction of a variable dipole moment in the target core during scattering of a charged particle. The radiating dipole moment is proportional to the dynamic polarizability of a target at a specified frequency, so arising radiation is called polarization radiation. It is called bremsstrahlung since its energy, as in case of ordinary Bs, is drawn from the kinetic energy of an incident particle that is slowed down after emission of a photon.

Besides PBs on an atom, PBs on negative ions, positive ions with an electron core, on Debye spheres in plasma, on a solid body (amorphous, crystalline, polycrystalline), clusters, nanoparticles, etc. is possible.

In the presence of radiation an inverse bremsstrahlung effect by the polarization channel is also possible, when the energy of a radiation field is absorbed by an electron scattered by an atom (ion) as a result of "pumping" through the electron core of a target. Then the field energy decreases, and the electron is accelerated. In case of a reverse energy flow, on the contrary, the electron is slowed down, and the field is strengthened.

It should be emphasized that PBs and SBs arise in the same elementary radiative process. In terms of quantum mechanics this means that in obtaining the probability of total Bs it is necessary to sum their amplitudes. Hence it follows that the total

probability of the process contains an interference summand corresponding to interchannel interference during Bs. This summand is particularly important in case of bremsstrahlung of nonrelativistic electrons.

1.2 Analogs of Polarization Bremsstrahlung in a Medium

The term "polarization bremsstrahlung" was invented for radiation during scattering of a charged particle by an atom in 1985 [1]. Until then different scientific groups used three terms: dynamic Bs (V.M. Buimistrov's group), atomic Bs (M.Ya. Amus'ya with co-workers), and polarization Bs (B.A. Zon).

For radiation resulting from the conversion of the eigenfield of charged particles on Debye spheres in plasma A.V. Akopyan and V.N. Tsytovich [2] proposed the term "transient bremsstrahlung". The origin of this name is connected with the fact that PBs in plasma can be described as transient scattering of virtual photons to real photons by the Debye "coat". The correspondence between transient Bs and polarization (dynamic) Bs was established in the V.A. Astapenko's thesis [3], in which it was shown that transient Bs is nothing but polarization Bs on Debye coats surrounding ions in plasma.

Another analog of PBs is parametric X-radiation arising in scattering of a charged particle in a single crystal. This term was invented by Ya.B. Fainberg and N.A. Khizhnyak.

In works of N.N. Nasonov it was shown that parametric X-radiation represents a coherent part of PBs in a crystal in the spectral range, in which the high-frequency approximation for dynamic polarizability of crystal atoms is true.

Finally, another realization of PBs is emission of a charged particle on nonuniformities of dielectric permittivity of a medium that was studied theoretically by S.P. Kapitsa [4]. This emission results from scattering of the eigenfield of a charge in a substance that is caused by the nonuniformity of electron density of a target resulting in nonuniformity of dielectric permittivity.

From the above it is seen that the interpretation of PBs as the process of scattering of virtual photons is applicable to a wide range of phenomena both on atomic particles and on condensed media, in plasma, and in nanomaterials.

To summarize, the polarization mechanism is a basis for the consistent microscopic description of related radiative phenomena, in which the conversion of a virtual photon to a real photon on target electrons occurs (Table 1.1).

1.3 Main Properties and Characteristic Features of Polarization Bremsstrahlung

Since the mechanism of initiation of a polarization channel differs from the mechanism of initiation of ordinary Bs, PBs has a number of distinguishing characteristic features (Table 1.2).

Table 1.1 Synonyms of the term "PBs" and analogs of PBs in a medium

Radiation on an atom	Dynamic Bs (V.M. Buimistrov with co-workers)
	Atomic Bs (M.Ya. Amus'ya with co-workers)
	Polarization Bs (B.A. Zon)
Radiation in plasma	Transient Bs (V.N. Tsytovich, A.V. Akopyan)
Radiation in a condensed medium	Parametric X-radiation (Ya.B. Fainberg and N.A. Khizhnyak)
	Radiation on random nonuniformities (S.P. Kapitsa)

Table 1.2 Characteristic features of ordinary and polarization bremsstrahlung

Static Bs	Polarization Bs
Caused by acceleration of a charge in the field of a target	Caused by a variable dipole moment induced in the target core
Formed at short distances from a target	Formed at long distances from a target
The intensity is inversely proportional to the squared mass of an incident particle	The intensity does not depend on the mass of an incident particle
Has an acute angular distribution for relativistic incident particles	Has a dipole angular distribution for relativistic incident particles
The spectrum is defined by the potential of a target	The spectrum is defined by the dynamic polarizability of a target

The most essential distinction between these mechanisms of radiation is that the intensity of OBs is inversely proportional to the squared mass of an incident charged particle, and the intensity of PBs does not depend on this mass (as a first approximation).

Another distinction shows itself for relativistic charged particles: OBs has a narrow angular distribution along the velocity of a scattered charge, and the angular distribution of PBs is of a dipole nature.

In the relativistic case the PBs cross-section increases logarithmically with the energy of an incident particle, and the SBs cross-section does not depend on energy in the relativistic limit.

Another important distinction is that OBs is formed at short distances from a scattering center, and PBs is formed at large distances. So the polarization channel is more sensitive to the spatial structure of a target than the static mechanism of Bs. Based on this fact is the possibility to develop new methods of material diagnostics using PBs.

The OBs spectrum is defined by the potential of a target, while the spectral PBs cross-section is proportional to the squared dynamic polarizability of a target. This property of the polarization channel can be used to determine eigenfrequencies and strengths of oscillators of electron transitions of a target.

1.4 Theoretical Fundamentals Concerning Polarization Bremsstrahlung

Polarization bremsstrahlung results from virtual excitation of bound electrons of a target under the action of a variable electric field produced by an incident particle. Corresponding to the virtual excitation of a system in terms of the second-order perturbation theory is summation over intermediate energy states of the electron core of a target. Generally speaking, this summation covers the region of the discrete and continuous energy spectra. If the frequency of an emitted photon approaches one of eigenfrequencies corresponding to the transition of target electrons between bound states, the resonance increase of the PBs cross-section occurs. In this case *resonant polarization bremsstrahlung* takes place. The described situation arises in scattering of electrons by ions in the UV and X-ray (in case of multiply charged ions) ranges. Besides, it takes place in scattering of electrons by alkali and alkali-earth atoms in the visible spectrum. Corresponding to the resonance process is a resonance in the frequency dependence of the dynamic polarizability of a target. Such a resonance is absent in the X-ray frequency range for targets consisting of neutral atoms because of the low oscillator strength of bound-bound transitions in this case and influence of electron collective effects.

As already noted, an important characteristic feature of PBs is the independence of its cross-section from the mass of an incident particle, while ordinary bremsstrahlung (OBs) of heavy particles caused by acceleration of a charge in the electric field of the target is suppressed. The OBs cross-section is inversely proportional to the squared mass of an emitting particle. The independence of the PBs cross-section from the mass of an incident particle is connected with the nature itself of this phenomenon. The virtual photon of the charge eigenfield is defined by the velocity and the impact parameter and does not depend on mass.

Within the framework of the classical trajectory description it can be said that PBs is formed at distances exceeding the size of the electron core of a target since with smaller impact parameters the coherence of reradiation of a virtual photon of an incident particle eigenfield to a real photon is lost. On the contrary, OBs in scattering of an electron by a neutral atom corresponds to small impact parameters and occurs in the region of space where the scattering center field is the strongest. The trajectory approach loses its obviousness in charge scattering in an extended medium. Then it is better to use the conjugate space, in which the role of distance is played by a wave vector transferred from an incident particle to a target: the more is the transferred wave vector, the less is a corresponding spatial scale. In such an approach it can be asserted that the most contribution to the integrated cross-section of PBs on an isolated atomic particle is made by transferred wave vectors smaller than the inverse radius of a target.

In case of electron scattering in a condensed substance and in plasma, PBs with transfer of energy-momentum to collective excitations (phonons, plasmons, polaritons) as well as to the whole sample as a unit is possible. The latter case is realized in scattering of a charged particle in single-crystal and polycrystalline targets, when together with pair (incoherent) interaction processes with transfer of a

momentum (or a wave vector) to the whole crystal lattice are possible. In this situation the transferred wave vector is equal to one of the vectors of the reciprocal crystal lattice, and the yield of PBs photons is proportional to the squared concentration of medium atoms. Coherent PBs in a single crystal occurs at fixed frequencies defined by the reciprocal lattice vector and the incident particle velocity. It can be interpreted as the Bragg scattering of a virtual photon of the incident particle eigenfield by the crystal planes of a target. Thus the PBs spectrum in a regular structure has a set of sharp maxima (coherent peaks). When going to a polycrystal, it is necessary to average the PBs probability over possible orientations of microcrystallites. As a result, coherent peaks in the PBs spectrum disappear, and instead of them a stepped structure appears that is caused by sequential "turning-off" of the reciprocal lattice vector contribution to radiation as the bremsstrahlung photon energy growths.

Besides the coherent component of PBs, in crystalline targets at the nonzero temperature of a medium an incoherent radiation channel arises that is caused by lattice thermal vibrations. The incoherent component of PBs prevails in the high-frequency spectral range where the process is accompanied by transfer of reciprocal lattice vectors of high magnitudes from an incident particle to a target. Such processes are rather sensitive to the deviation of the lattice structure from the ideal structure.

PBs both of nonrelativistic and of relativistic electrons scattered by isolated centers of force has a dipole pattern, while OBs of a relativistic charge in this case is directed along its velocity. This is connected with the fact that a real photon during PBs is produced as a result of scattering of a virtual photon by nonrelativistic electrons of a target. If the radiation coherence length L_{coh} is introduced as a distance at which the detachment of a real photon from the eigenfield of a charged particle occurs, it can be shown that this value reaches macroscopic sizes for incident particles with the high Lorentz factor $\gamma = 1\Big/\sqrt{1-(\mathrm{v}/c)^2} >> 1$ (v is the velocity of an incident particle, c is the velocity of light in free space) $L_{coh} \approx \gamma^2 \lambda$ (λ is wavelength of emitted photon).

The coherence length of PBs in pair interaction remains being of the order of the radiation wavelength. This causes the absence of the density effect for PBs in disordered structures. It will be recalled that the density effect in OBs shows itself in reduction of process intensity as a result of increasing phase velocity of electromagnetic waves in a medium with the plasma dielectric permittivity (which is less than unity). This increase results in the fact that a real photon is detached from the incident particle eigenfield faster than it would occur in vacuum, which results in reduction of the OBs cross-section. This effect was for the first time predicted by M.L. Ter-Mikaelyan [5]. It is responsible for decreasing OBs intensity in the low-frequency range $\omega < \gamma\, \omega_{pe}$ (ω_{pe} is electronic the plasma frequency).

For applications of PBs spectroscopy as a physical basis for diagnostics of materials including nanostructured media, it should be noted that it is the formation of PBs at large impact parameters (or at low transferred momenta) that makes it sensitive to interatomic correlations. This ultimately makes it possible to obtain information on a target structure based on the analysis of spectral and angular dependences of PBs.

1.5 Polarization Bremsstrahlung in Atoms, Solids and Nanostructures

The consistent theory of PBs on an atom based on the quantum-mechanical approach has arisen in the works of V.M. Buimistrov, M.Ya. Amus'ya, and B.A. Zon [1, 6–10]. The historically first were the works of V.M. Buimistrov [6] and V.M. Buimistrov in co-author with L.I. Trakhtenberg [7]. In the first of the cited works PBs that was called dynamic Bs of a nonrelativistic electron was calculated in the resonant approximation, when the frequency of an emitted photon is close to one of the eigenfrequencies of an atom. In this case it was possible to retain one summand of the sum over virtual transitions of bound atomic electrons.

In the paper [7] a special method of summation of the perturbation theory series was used in calculation of the cross-section of PBs of an electron scattered by a hydrogen atom. As a result, it has been possible to describe the PBs spectrum in a wide frequency range, but not only in the region of resonance frequencies. In the work [10] the process was considered in the high-frequency approximation, when it is possible to carry out the approximate summation of the perturbation theory series. Within the framework of the consistent quantum-mechanical formalism in this work the static Bs channel was also taken into account. It was shown that in the high-frequency limit the cancellation of the contribution of the polarization channel and the screening effect of atomic electrons in the OBs amplitude occurs. As a result, the process proceeds as on a "bare" nucleus. This effect was called the effect of atom stripping during Bs (stripping approximation), or the descreening effect. Further it was shown that in case of multielectron atoms the "stripping" approximation gives a good conformity with the result of the more exact approach in a wide frequency range, but not only in the high-frequency limit. In the papers [6–10] an incident electron was assumed to be nonrelativistic, but fast enough, so it was possible to use the Born approximation for its interaction with an atom.

In the work of M.Ya. Amus'ya with co-authors [8] an incident electron, on the contrary, was assumed to be slow, and the inverse bremsstrahlung effect was calculated, that is, the absorption of a photon of the external electromagnetic field by an electron scattered by an atom. The contribution both of ordinary and of polarization Bs was taken into account, and in the latter case for calculation of an amplitude a special version of the many-particle perturbation theory was used: the random phase approximation with exchange. It was shown that the contribution of the polarization channel is rather essential and can exceed the contribution of the ordinary mechanism of the bremsstrahlung effect even far from resonance frequencies.

In the B.A. Zon's work [9] in the description of the amplitude of PBs on an atom in the Born-Bethe approximation the dynamic polarizability of an atom was used for the first time, which was an essential step in the development of the PBs theory since this has opened a possibility to use well-developed methods of calculation of atomic polarizability for description of the polarization channel.

The description of PBs in plasma in the approach of V.N. Tsytovich [2] was based on the use of the classical formalism for nonlinear current arising in scattering of charged particles in plasma. Nonlinear current radiation as a result of scattering processes of two types, Compton and transient, was considered. Compton scattering is accompanied by transfer of the momentum excess during radiation to plasma electrons and corresponds to ordinary Bs. Transient scattering is connected with transfer of the momentum excess to plasma ions, when a photon is emitted by a coherently vibrating charge of the Debye sphere, and corresponds to PBs. The consideration of PBs in plasma within the framework of the quantum-mechanical approach was given in the thesis [3] with the use of the method of the dynamic form factor of plasma components that was finally expressed in terms of different components of the plasma dielectric permittivity.

PBs on an amorphous target within the framework of the approach based on classical electrodynamics was studied theoretically in the paper [11]. A similar method was used in the work [12] in calculation of the spectrum of Bs of relativistic electrons in a polycrystal with account for of the polarization channel. In the cited paper it was shown that the PBs spectrum contains maxima corresponding to the fulfilment of the Bragg condition in scattering of a virtual photon to a real photon by a polycrystalline target. The position of the maximum is defined by the electron velocity and the vector of the reciprocal lattice of the polycrystal. With increasing velocity of an incident electron the spectral maximum is shifted towards higher photon energies. The PBs intensity at the maximum exceeds the intensity of ordinary Bs.

PBs of a nonrelativistic heavy charged particle on a thin polycrystalline target was calculated in the work [13] by the methods of quantum mechanics without considering photoabsorption and loss of energy of an incident particle in a substance. In this case the OBs intensity is negligible since it is inversely proportional to the squared charge mass. It was shown that in contrast to the relativistic case the significant contribution to the photon yield by the polarization channel is made by the incoherent summand in the PBs amplitude connected with transfer of the energy-momentum excess in binary collisions. The contribution of the coherent mechanism of PBs, when a momentum is transferred to the whole crystal lattice as a unit, is the most essential in the low-frequency part of the spectrum. An important feature of PBs of a nonrelativistic charge in a polycrystal is the presence of frequency steps in the spectrum connected with “turning-off” of the contribution of one of the reciprocal lattice vectors to the process. The position of the frequency step is defined by the reciprocal lattice vector and the velocity of an incident particle and thus carries information on the target properties.

PBs of a nonrelativistic electron in a thin polycrystalline target was calculated in the paper [14] in the stripping approximation with account for of the contributions of both channels to the process. It was shown that the spectrum of total Bs contains “frequency steps” arising due to the contribution of the coherent part of PBs to radiation.

As already noted, experiments on investigation of PBs in a condensed medium were carried out mainly with the use of relativistic electrons with an energy of

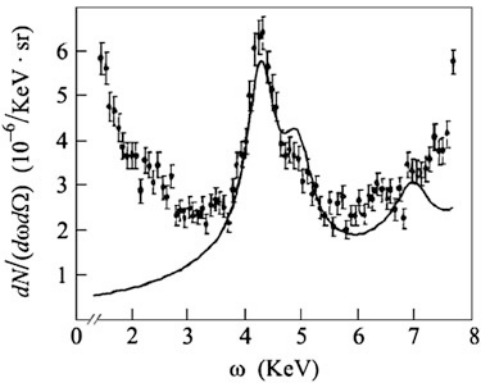

Fig. 1.2 The measured (*points*) and calculated (*solid curve*) spectra of PBs of an electron with an energy of 7 MeV from a copper foil [16]

several MeV. In this case the polarization channel shows itself as maxima on the spectral dependence of bremsstrahlung photon yield. For example, in the work [15] the spectrum of Bs of an electron with the energy of 2.4 MeV scattered in a polycrystalline aluminum target was recorded. The position of the spectral maximum of radiation falling on photon energy of 4 KeV was clearly located. The comparison with the calculation based on the approach [12] showed a good conformity between the experimental and theoretical data. In the process of this work in the paper [16] the measurements of the PBs spectra for electrons with the energy of 7 MeV scattered in polycrystalline films of aluminum, copper, and nickel were carried out, and the detailed quantitative comparison of the experimental and calculated data was presented. A good conformity between the theory and the experiment was obtained, which is demonstrated for a copper target by Fig. 1.2.

In the central part of this figure the major peak caused by the polarization channel is located. The low-frequency part of the spectrum is an exponential background. In the high-frequency wing there is a peak of the copper *K*-line with a maximum at 8.025 eV. In the spectrum there is also a background peak of the *K*-line of iron at 6.4 KeV. On the right side of the central peak of PBs there is a second peak. Fitting the spectrum with the Gauss distribution gives the position of peaks: 4.267 eV and 4.886 eV. The third peak of PBs should show itself according to calculations in a range of photon energies about 7 KeV. However, in this case it is difficult to discern this peak in the spectrum in view of its position at the rise of the copper *K*-line peak that surpasses PBs in intensity by more than two orders of magnitude.

Thus in case of scattering of relativistic electrons in polycrystalline targets a reliable experimental evidence of substantiality of the PBs contribution to the yield of X-ray photons is obtained, and the characteristic features of the polarization channel spectrum are reproduced by calculation within the framework of a corresponding theoretical approach [12].

The initial stage of theoretical and experimental investigations of PBs and a number of related processes was summed up in the review [17] and the monograph [18].

The book [18] (Chap. 7) contains also the theoretical description of PBs in collisions of electrons with metal clusters and fullerenes. The emphasis is on the spectra in the visible wavelength range, where radiation has characteristic maxima connected with collective excitations of the electron subsystem of a target. These collective excitations are caused by variations of the electron density of valence (in case of fullerenes) and delocalized (in case of metal clusters) electrons, by which a virtual photon of an incident particle is scattered to a real bremsstrahlung photon. The said variations occur in a thin near-surface layer of the target, the thickness a of which is of the order of the atom size and is supposed to be much lesser than the cluster radius R. In the interior of the cluster electron density is compensated by the positive charge of ions. The construction of the theory in the case under consideration is significantly simplified due to the presence of the small parameter a/R that makes it possible to use a simple model expression for the form-factor of the target $F(q)$.

The generalized polarizability of the target $\alpha(\omega, q)$ necessary for calculation of the PBs cross-section in the Born approximation, as a rule, is calculated in the multiplicative approximation, in which $\alpha(\omega, q) = \alpha(\omega)\,\tilde{F}(q)$, where $\alpha(\omega)$ is the dipole dynamic polarizability, $\tilde{F}(q)$ is the normalized form factor of the target representing a spatial Fourier transform of electron density distribution. The above equation was proved in the paper [19] with the use of calculations made within the framework of the quantum-mechanical random phase exchange approximation for a case of multielectron atoms.

The dynamic polarizability of metal clusters and fullerenes was calculated in the resonant approximation [18, 20], when the function $\alpha(\omega)$ has a resonance at the plasma frequency ω_p:

$$\alpha(\omega) = R^3\,\frac{\omega_p^2}{\omega_p^2 - \omega^2 - i\,\omega\,\Gamma}, \tag{1.1}$$

where Γ is the damping constant. The plasma frequency is defined by the number of delocalized electrons N_e and the target radius R. The formula for plasma frequency used in [18, 20] in case of a metal cluster in atomic units ($e = m_e = \hbar = 1$) looks like: $\omega_p = \sqrt{N_e/R^3}$, in case of a fullerene $\omega_p = \sqrt{2\,N_e/3\,R^3}$.

For metal clusters the photon energy at the plasma frequency depending on the size of a cluster changes from 2 to 5 eV. For the fullerene C_{60} this energy is much more and is 19 eV.

It should be noted that the radius of the fullerene C_{60} is 0.35 nm (about 6 a.u.), and the radius of a carbon atom is less than 0.1 nm. The form factor for cluster targets can be calculated in the general form in view of the fact that PBs results from scattering of a virtual photon of the incident particle eigenfield in a thin near-surface layer. Substituting the equation $|\mathbf{r}_a| \approx R$ ($\mathbf{r}_a$ is the radius vector of a cluster atom) in the expression for the form factor, we can find: $F(q) = \frac{3 j_1(qR)}{qR}$ ($j_1(x)$ is the first-order spherical Bessel function). This expression is true for the transferred vectors q of not too high magnitudes that satisfy the inequality: $q << 1/a$.

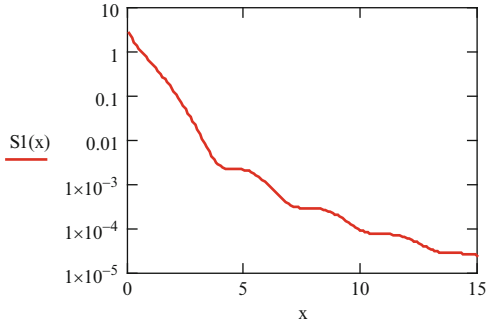

Fig. 1.3 The plot of the function $S_1(x)$ defining, together with the target polarizability, the dependence of the PBs cross-section on the size of a nanocluster

The expression for the spectral cross-section of PBs of a nonrelativistic electron on a nanocluster (in atomic units) looks like [18, 20]:

$$\omega \frac{d\sigma^{PB}}{d\omega} = \frac{16\,\omega^4}{3\,c^3\,\mathrm{v}^2}\,|\alpha(\omega)|^2\,[S_1(R\ q_{\min}) - S_1(R\ q_{\max})], \tag{1.2}$$

where $q_{\min} = \frac{m\mathrm{v}}{\hbar}\left(1 - \sqrt{1 - \frac{2\hbar\omega}{m\mathrm{v}^2}}\right), q_{\max} = \frac{m\mathrm{v}}{\hbar}\left(1 + \sqrt{1 - \frac{2\hbar\omega}{m\mathrm{v}^2}}\right)$ are the magnitudes of the minimum and maximum wave vectors transferred from an incident particle to the target, and

$$S_1(x) = \frac{1}{8\,x^6}\left[6 + 9\,x^2 - \left(12\,x + 2\,x^3 - 4\,x^5\right)\,\sin\left(2\,x\right) - \left(6 - 3\,x^2 + 2\,x^4\right)\right.$$
$$\left.\times\,\cos\left(2\,x\right) - 8\,x^6\,Ci\left(2\,x\right)\right] \tag{1.3}$$

is the function arising after integration with respect to the momentum transferred from an electron to the target, $Ci(x) = \gamma + \ln x + \int_0^x \frac{\cos t - 1}{t}\,dt$ is the integral cosine ($\gamma \cong 0.577$ is the Euler constant). According to the formula (1.2), the function $S_1(x)$, together with the target polarizability (1.1), defines the dependence of the PBs cross-section on the size of a nanocluster.

It should be noted that far from the kinematic boundary, when $\hbar\,\omega << m\,\mathrm{v}^2/2$, we have $q_{\min} \approx \omega/\mathrm{v}$ and $q_{\max} \approx 2\,m\,\mathrm{v}/\hbar$. In this case $q_{\min} << q_{\max}$ and according to the plot of Fig. 1.3, the inequality $S_1(R\ q_{\min}) >> S_1(R\ q_{\max})$ is true, so it is possible to retain only the first summand in the square brackets on the right of the Eq. 1.2. In the limit $R\ q_{\min} << 1$ the formula (1.3) gives $S_1(x) \approx -\ln(2\,R\,q_{\min}) \cong \ln(\mathrm{v}/2\,\omega\,R)$, and (1.2) and (1.3) give the result known from the theory of PBs on an atom.

Presented in the book [18] and the review [20] are the results of calculations (in the first Born approximation for interaction of an incident particle with a target) of spectral cross-sections of PBs arising in scattering of electrons by metal clusters and fullerenes.

For a case of collision of an electron with fullerenes it was shown that the maximum of the spectral PBs cross-section is reached in fulfilment of the condition

$v = v_{max} \approx R\,\omega_p$, where v is the initial velocity of a scattered electron. The value of the optimum velocity v_{max} for the fullerene C_{60} is about 3.5 a.u., which corresponds to the energy of 167 eV. At such an electron energy the maximum of the spectral cross-section of PBs on the fullerene C_{60} falls on a photon energy about 19 eV, and the calculated value of the cross-section $\omega\, d\sigma/d\omega$ at a corresponding frequency is about 10^{-19} cm^2 (about $3.6{\cdot}10^{-3}$ atomic cross-section units). The width of the maximum in the spectrum of PBs on the fullerene is defined by the value of the ratio $\Gamma/\omega_p \approx 0.6$. The high value of this ratio is caused by the fact that the energy of plasmon resonance in a fullerene is more than the potential of ionization of its atoms, so the ionization processes (bound-free transitions) make a substantial contribution to damping of collective oscillations in a fullerene.

With decreasing electron velocity the maximum of the PBs cross-section is shifted towards lower photon energies. Thus at an electron velocity of 1.5 a.u. (an energy of 30.6 eV) the cross-section maximum is reached at a bremsstrahlung photon energy of 11.5 eV and is about $1.8{\cdot}10^{-20}$ cm^2. With increasing electron velocity the cross-section maximum is shifted to the high-frequency range. In this case the right resonance arm is amplified since the suppression of the PBs spectrum (due to the influence of the target form factor) occurs then at higher frequencies. The calculation of the spectral cross-section of PBs on a fullerene for an electron velocity of 7 a.u. gives a maximum cross-section value of $7{\cdot}10^{-20}$ cm^2 at a bremsstrahlung photon energy of 20 eV. At low velocities the process cross-section as a function of the bremsstrahlung photon energy oscillates. These oscillations are connected with the diffraction of the incident particle eigenfield on the target and are seen on the plot of Fig. 1.3 for the function $S_1(x)$.

The calculation of the velocity dependence of the cross-section of PBs on a fullerene shows that the cross-section maximum at the plasma frequency is reached at an electron velocity of 3.5 a.u. A corresponding curve shows a sharp dip with decreasing velocity, which is caused by the influence of the target form factor. At high velocities rather smooth reduction of the cross-section occurs, which is connected with the presence of the squared electron velocity in the denominator of the expression for the PBs cross-section.

The numerical analysis presented in [18] and [20] shows that the angular distribution of PBs in scattering of an electron by a fullerene depends on electron velocity. At the maximum of the frequency and velocity dependences ($\hbar\,\omega = 19$ eV, $v = 3.5$ a.u.) this distribution has a maximum at a radiation angle of 90°. At low velocities including the optimum velocity PBs is defined mainly by plasmon oscillations occurring in parallel with the velocity of an incident electron. At high electron energies the main role is played by transferred momenta perpendicular to the direction of incident particle motion, which corresponds to the maximum of PBs along or antiparallel to the electron velocity (the radiation angle is 0° or 180°).

The results of calculations of spectral, angular, and velocity dependences of the cross-section of PBs of an electron scattered by a metal nanoparticle are presented in the publications [18] and [20]. In particular, a cluster was considered that

contained 40 sodium atoms (Na_{40}). In this case the relative width of resonance is $\Gamma/\omega_p \approx 0.2$. The small value of this ratio is caused by the fact that the energy of plasmon resonance in a metal cluster is less than the potential of ionization of its atoms. So bound-free transitions (ionization) do not make a contribution to the process of plasma oscillation damping. As a result, the frequency dependence of the cross-section of PBs on a metal cluster near the plasmon resonance is defined mainly by the dynamic polarizability of a target.

The numerical analysis [18, 20] shows that the optimum value of electron velocity to reach the maximum in the spectral cross-section of PBs on the cluster Na_{40} at the plasmon resonance frequency ($\hbar\omega_p = 2.85$ eV) is 1.5 a.u. (30.6 eV). In this case the value $\omega\, d\sigma/d\omega$ is $2.8{\cdot}10^{-19}$ cm^2 (0.01 atomic cross-section units). With changing velocity the position of the maximum practically does not change, and the cross-section value decreases. Thus in case of twofold decrease or increase of incident particle velocity the cross-section of PBs at the maximum of the spectral dependence decreases approximately by half. Also given in the cited works are the calculations of the cross-section of PBs of an electron with the energy of 13.6 eV scattered by clusters containing 10^2, 10^3, 10^4, and 10^5 atoms. The photon energy at the maximum is practically the same for all types of clusters, and the maximum cross-section grows from $8{\cdot}10^{-19}$ to $4{\cdot}10^{-17}$ cm^2.

From the presented results it follows that the number of spectral oscillations of the cross-section grows with increasing size of the cluster, and these oscillations are most manifested in the low-frequency wing. With increasing size of the cluster the optimum value of the velocity of an incident electron at which the cross-section has maximum also increases.

It should be noted that the use of the model expression for polarizability in the region of plasmon resonance (1.1) in description of PBs on metal clusters with a great number of atoms (10^3 and more) seems inadequate. In this case it is more preferable to use the Mi theory based on the introduction of the dielectric permittivity of a target substance.

The ratio of the PBs cross-section to the ordinary bremsstrahlung cross-section is given by the expression [18, 20]:

$$\frac{d\sigma^{PB}}{d\sigma^{OB}} \propto N_e \left(\frac{\omega_p}{\Gamma}\right)^2 S_1\left(\frac{\omega_p R}{\mathrm{v}}\right), \tag{1.4}$$

that is, is defined mainly by the number of delocalized electrons and by the ratio ω_p/Γ that for fullerenes is 1.67, and for metal clusters is five and more. The argument of the function $S_1(x)$ in case of optimum values of parameters for a fullerene is 1.3, and the value of the function itself is about 0.36. In case of metal nanoclusters the argument of $S_1(x)$ does not exceed 0.2, and accordingly the value of the function itself is $S_1(x) \geq 2$.

In view of the presented values of the magnitudes included in the formula (1.4), it is possible to conclude that PBs of electrons on fullerenes and metal nanoclusters in the region of plasmon resonance surpasses the contribution of ordinary

bremsstrahlung in order of magnitude. The proportionality of the ratio of the PBs and OBs cross-sections to the number of delocalized electrons following from Eq. 1.4 is a consequence of coherent reradiation of a virtual photon of an incident particle by fullerenes and nanoclusters in the wavelength range under consideration.

PBs as a means of diagnostics of the fullerene structure was discussed in the paper [21] on the basis of calculation of the form-factor defining the intensity of radiation by the polarization channel. In contrast to the publications [18, 20], the work [21] considered the kiloelectron-volt range of bremsstrahlung photon energies, where the oscillations of PBs intensity increasing with decreasing photon energy should also show themselves. These oscillations are explained by the interference of contributions to the process from different sites of a fullerene. The calculations carried out in the paper [21] are based on the approximate approach, the main disadvantage of which is the absence of integration with respect to the wave vector **q** transferred from an incident particle to a target as well as inexact recording the value of the latter: $q \approx 2\,(\omega/c)\,\sin(\theta/2)$, where θ is the radiation angle. Actually, in calculation of the total PBs intensity it is necessary to sum the contribution to the process from all angles of electron scattering by a target, which corresponds to integration with respect to the wave vector magnitude from ω/v to $m\,\mathrm{v}/\hbar$. Another assumption made in the cited work consists in neglecting the bond of fullerene electrons with the nuclei of carbon atoms. This assumption in the spectral range under consideration is quite correct if valence electrons of atoms making a fullerene are concerned. But it is not so obvious for electrons of inner shells (K-electrons).

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