Timed CTL Model Checking in Real-Time Maude[*]

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Abstract. This paper presents a timed CTL model checker for Real-Time Maude and its semantic foundations. In particular, we give a timed CTL model checking procedure for that is sound and complete for closed-bound formulas under a *continuous* semantics for a fairly large class of systems. An important benefit of our model checker is that it also automatically provides a timed CTL model checker for subsets of modeling languages, like Ptolemy II and (Synchronous) AADL, which have Real-Time Maude model checking integrated into their tool environments.

1 Introduction

Real-Time Maude [30] extends Maude [14] to support the formal modeling and analysis of real-time systems in rewriting logic. Real-Time Maude is characterized by its expressiveness and generality, natural model of object-based distributed real-time systems, the possibility to define any computable data type, and a range of automated formal analysis such as simulation, reachability and temporal logic model checking. This has made it possible to successfully apply the tool to a wide range of real-time systems, including advanced state-of-the-art wireless sensor network algorithms [17,32], multicast protocols [31,21], scheduling algorithms requiring unbounded queues [26], and routing protocols [33].

Real-Time Maude's expressiveness and generality also make it a suitable semantic framework and analysis tool for modeling languages for real-time systems [24]. For example, the tool has been used to formalize (subsets of) the industrial avionics modeling standard AADL [25], a synchronous version of AADL [6], Ptolemy II discrete-event (DE) models [7], the web orchestration language Orc [2], different EMF-based timed model transformation frameworks [34,10], etc. Real-Time Maude formal analysis has been integrated into the tool environment of some of these languages, enabling a model engineering process that combines the convenience of an intuitive modeling language with formal analysis.

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In Real-Time Maude, the data types of the system are defined by an algebraic equational specification, and the system's instantaneous transitions are modeled by (instantaneous) rewrite rules. Time advance is modeled explicitly by so-called *tick (rewrite) rules* of the form `{`t`} => {`t'`} in time` u `if` *cond*, where `{_}` is an operator that encloses the entire global state, and the term u denotes the *duration* of the rewrite. Real-Time Maude is parametric in the time domain, which may be discrete or dense. For dense time (in particular), tick rules typically have the form `{`t`} => {`t'`} in time` x `if` x `<=` d `/\` *cond*, where x is a new variable not occurring in t, d, or *cond*. This form of the tick rules ensures that any moment in time (within time d) *can* be visited, also for a dense time domain.

Real-Time Maude extends Maude's rewriting, search, and linear temporal logic model checking features to the timed case. For dense time, it is of course not possible to execute all possible rewrite sequences. The fairly restrictive timed automaton formalism [3] trades expressiveness for decidability of key properties for dense/continuous time, since the state space can be divided into a finite number of "clock regions" so that any two states in the same region satisfy the same properties. Such a quotient seems hard to achieve for the much more expressive real-time rewrite theories. Instead, the general approach taken in Real-Time Maude is to use *time sampling strategies* to instantiate the new variable x in the tick rules, and to analyze the resulting specification instead of the original one. One such strategy advances time by a fixed amount Δ in each application of any tick rule. The *maximal* time sampling strategy advances time as much as possible in each application of a tick rule. Although the fixed-increment strategy can cover all possible behaviors in the original system when the time domain is discrete, the maximal time sampling typically only analyzes a *subset* of all the possible behaviors. However, in [28], it is shown that for a fairly large set of real-time systems appearing in practice, the maximal time sampling strategy yields sound and complete analyses for untimed LTL properties. For example, systems where events are triggered by the arrival of messages or by the expiration of some "timer," and where time elapse does not change the valuation of the atomic propositions (this requirement almost always holds, since time elapse typically only changes timers and clocks, whose values are rarely relevant for temporal logic properties) satisfy the requirements for maximal time sampling analyses to be sound and complete.

Until recently, Real-Time Maude could only analyze *untimed* temporal logic properties, but not quantitative properties such as "the airbag must deploy within 5 ms of a crash," or "the ventilator machine cannot be turned off more than once every 10 minutes." This paper presents a model checker for Real-Time Maude for the timed temporal logic TCTL [5], which is an extension of the branching time logic CTL in which the temporal operators are annotated with a time interval, so that, for example, the formula $E\ \varphi_1\ U_{[2,4]}\ \varphi_2$ holds if there is a path in which φ_2 holds after some time $2 \leq r \leq 4$ and where φ_1 holds in all states until then.

Going from untimed temporal logic to a timed temporal logic presents at least two significant challenges for Real-Time Maude:

1. What is the intended semantics of a Real-Time Maude specification with the above tick rule w.r.t. timed temporal logic properties? For example, given a tick rule `{`$f(y)$`} => {`$f(y + x)$`} in time` x `if` x `<=` $3 - y$, should the property $AF_{[1,2]}$ `true` (in all paths, a state satisfying `true` will be reached in some time between 1 and 2) hold for initial state `{`$f(0)$`}`? There are paths (e.g., jumping directly from `{`$f(0)$`}` to `{`$f(3)$`}`) where no state is visited in the desired time interval. On the other hand, the above rule could be seen as a natural way to specify a *continuous* process from `{`$f(0)$`}` to `{`$f(3)$`}` in Real-Time Maude, so that the *intended* semantics should satisfy the above property. We address this problem by presenting two different semantics for the satisfaction of a TCTL formula in Real-Time Maude: the *pointwise* semantics takes all paths into account, including the one where we jump directly from time 0 to time 3, whereas the *continuous* semantics allows us to break up longer ticks in smaller steps.
2. The previous soundness and completeness results for maximal time sampling analyses no longer hold. In the example above, maximal time sampling would not satisfy the existential formula $E\,F_{[1,2]}$ `true`, although the original model satisfies it in both the continuous and the pointwise semantics. To achieve sound and complete time sampling analyses for the continuous semantics and dense time, we always advance time by a time value $\frac{\bar{r}}{2}$, where $\bar{r}$ is the "greatest common divisor" (as axiomatized in Section 4) of all the (non-zero) time values appearing in the annotations in the TCTL formula, as well as all the time values of the maximal tick steps reached from the initial state. We have only implemented our model checker and proved its completeness for the continuous semantics; however, we conjecture that for the pointwise semantics, we should use this time increment, as well as any multiple of it.

This paper describes our model checker, its semantic foundations, and its implementation in Maude. Most importantly, we prove that our model checker provides sound and complete model checking under the continuous semantics for TCTL formulas where the intervals are *closed* intervals; i.e., have the forms $[r_1, r_2]$ and $[r_1, \infty)$.

An important benefit of our work is that a TCTL model checker for Real-Time Maude also gives us a TCTL model checker *for free* for Ptolemy II DE models, synchronous AADL models, and other modeling languages for which Real-Time Maude models can be generated. As shown in Section 6, our model checker has already been integrated into the Ptolemy II tool, allowing the user to model check TCTL properties of Ptolemy II models from within Ptolemy II.

Related Work. The tools Kronos [38], Redlib [36], and TSMV [22] implement TCTL model checkers for, respectively, timed automata, linear hybrid automata, and timed Kripke structures. The tool Uppaal [9] provides an efficient symbolic model checking procedure for timed automata for a subset of *non-nested* TCTL properties. The Roméo tool [16,11], based on a timed extension of Petri nets [1,23], has an integrated timed model checker for some *non-nested* TCTL modalities, with the addition of bounded response properties. These formalisms are significantly less expressive than real-time rewrite theories [27], which makes their model checking problems decidable. The first approaches to model checking timed temporal properties for Real-Time Maude are described in [20,37] and analyze important *specific* classes of timed temporal logic formulas (time-bounded response, time-bounded safety, and minimum separation), but only for *flat* object-based specifications. Unlike in [20,37], our new model checker is not limited to specific classes of temporal logic properties, but offers the *full* TCTL. The new model checker is also not limited to flat object-oriented systems, but can analyze any (sensible) Real-Time Maude model.

Paper Structure. Section 2 introduces real-time rewrite theories and Real-Time Maude. Section 3 describes our model checker and its semantics. Section 4 presents our soundness and completeness results. We discuss our model checker implementation in Section 5, and demonstrate its applicability on a Ptolemy II DE model in Section 6. Finally, concluding remarks are given in Section 7.

2 Real-Time Rewrite Theories and Real-Time Maude

A *rewrite theory* is a tuple (Σ, E, R), where (Σ, E) is a *membership equational logic theory* [14] that defines the state space of a system as an algebraic data type, with Σ a *signature* declaring sorts, subsorts, and function symbols, and E a set of *conditional equations* and *membership axioms*, and where R is a set of *labeled conditional rewrite rules* of the form $[l]: \ t \longrightarrow t'$ **if** *cond*, where l is a label, t, t' are Σ-terms, and *cond* is a conjunction of *rewrite conditions* $u \longrightarrow u'$, *equational conditions* $v = v'$, and *membership conditions* $w : s$, where u, u', v, v', w are Σ-terms and s is a sort in Σ. A rule is implicitly universally quantified by the variables appearing in t, t' and *cond*, and specifies a set of local *one-step transitions* in the system. Rules are applied modulo the equations E. The set $\mathbb{T}_{\Sigma/E,s}$ of *states* of sort s is defined by the E-equivalence classes of ground terms of sort s.

Real-time rewrite theories [27] are used to specify real-time systems in rewriting logic. Rules are divided into *tick rules*, that model time elapse in a system, and *instantaneous rules*, that model instantaneous change. Formally a *real-time rewrite theory* $\mathcal{R}$ is a tuple $(\Sigma, E, R, \phi, \tau)$ such that

- (Σ, E, R) is a rewrite theory, with a sort `System` and a sort `GlobalSystem` with no subsorts or supersorts and with only one operator `{_}` : `System` $\rightarrow$ `GlobalSystem` which satisfies no non-trivial equations; furthermore, for any $f : s_1 \dots s_n \rightarrow s$ in Σ, the sort `GlobalSystem` does not appear in $s_1 \dots s_n$.
- ϕ : *TIME* $\rightarrow (\Sigma, E)$ is an equational theory morphism which interprets *TIME* in $\mathcal{R}$; the theory *TIME* [27] defines time abstractly as an ordered commutative monoid $(\mathit{Time}, 0, +, <)$. We write $0, +, \dots$ instead of $\phi(0), \phi(+), \dots$ and use `Time` for $\phi(\mathit{Time})$.
- τ is an assignment of a term τ_l of sort `Time` to each rewrite rule in R of the form $[l]: $ `{`t`}` $\longrightarrow$ `{`t'`}` **if** *cond*. Such a rule is called a *tick rule* if $\tau_l \neq 0$; in this case τ_l denotes the duration of the step. Rules that are not tick rules are called *instantaneous rules* and are assumed to take zero time. Since the

initial state has the form {t}, the form of the tick rules ensures that time advances uniformly in the whole system.

We write $t \xrightarrow{r} t'$ when t can be rewritten into t' in time r by a *one-step rewrite*, and also write $t \xrightarrow{inst} t'$ for one-step rewrites applying an *instantaneous* rule.

A tick step $t \xrightarrow{r} t'$ is *maximal* if there is no $r' > r$ with $t \xrightarrow{r'} t''$ for some t''. A *timed path* in $\mathcal{R}$ is an infinite sequence $\pi = t_0 \xrightarrow{r_0} t_1 \xrightarrow{r_1} t_2 \xrightarrow{r_2} \cdots$ such that

– for all $i \in \mathbb{N}$, $t_i \xrightarrow{r_i} t_{i+1}$ is a one-step rewrite in $\mathcal{R}$; or
– there exists a $k \in \mathbb{N}$ such that $t_i \xrightarrow{r_i} t_{i+1}$ is a one-step rewrite in $\mathcal{R}$ for all $0 \leq i < k$, there is no one-step rewrite from t_k in $\mathcal{R}$, and $t_j = t_k$ and $r_j = 0$ for each $j \geq k$.

For paths π of the above form we define $d^\pi_m = \sum_{i=0}^{m-1} r_i$, $t^\pi_m = t_m$ and $r^\pi_m = r_m$. We call the timed path $\pi = t_0 \xrightarrow{r_0} t_1 \xrightarrow{r_1} t_2 \xrightarrow{r_2} \cdots$ a *timed fair path* if

– for any ground term Δ of sort `Time`, if there is a k such that for each $j > k$ there is a one-step tick rewrite $t_j \xrightarrow{r} t$ with $\Delta \leq d^\pi_j + r$ then there is an l with $\Delta \leq d^\pi_l$, and
– for each k, if for each $j > k$ both a maximal tick step with duration 0 and an instantaneous rule can be applied in t_j then $t_l \xrightarrow{inst} t_{l+1}$ is a one-step rewrite applying an instantaneous rule for some $l > k$.

We denote the set of all timed fair paths of $\mathcal{R}$ starting in t_0 by $tfPaths_{\mathcal{R}}(t_0)$. A term t is *reachable* from t_0 in $\mathcal{R}$ in time r iff there is a path $\pi \in tfPaths_{\mathcal{R}}(t_0)$ with $t^\pi_k = t$ and $d^\pi_k = r$ for some k. A path π is *time-divergent* iff for each time value $r \in$ `Time` there is an $i \in \mathbb{N}$ such that $d^\pi_i > r$.

The Real-Time Maude tool [29] extends the Maude system [14] to support the specification, simulation, and analysis of real-time rewrite theories. Real-Time Maude is parametric in the time domain, which may be discrete or dense, and defines a supersort `TimeInf` of `Time` which adds the infinity element `INF`. To cover all time instances in a dense time domain, tick rules often have one of the forms

```
crl [tick] : {t} => {t'} in time x if x <= u /\ cond [nonexec] .   (†),
crl [tick] : {t} => {t'} in time x if cond [nonexec] .             (∗), or
 rl [tick] : {t} => {t'} in time x [nonexec] .                     (§).
```

where x is a new variable of sort `Time` not occurring in {t} and *cond*. This ensures that the tick rules can advance time by *any* amount in rules of the form (∗) or (§) and *any* amount less than or equal to u in rules of the form (†). Rules of these forms are called *time-nondeterministic* and are not directly executable in general, since many choices are possible for instantiating the new variable x.

In contrast to, e.g., timed automata, where the restrictions in the formalism allow the abstraction of the dense time domain by "clock regions" containing bisimilar states [3], for the more complex systems expressible in Real-Time Maude there is not such a discrete "quotient". Instead, Real-Time Maude executes time-nondeterministic tick rules by offering a choice of different *time sampling strategies* [29], so that only some moments in the time domain are visited. For example, the *maximal* time sampling strategy advances time by the maximum possible time elapse u in rules of the form (†) (unless u equals `INF`), and tries to advance time by a user-given time value r in tick rules having other forms. In the *default* mode each application of a time-nondeterministic tick rule will try to advance time by a given time value r.

The paper [29] explains the semantics of Real-Time Maude in more detail. In particular, given a real-time rewrite theory $\mathcal{R}$ and a time sampling strategy σ, there is a real-time rewrite theory $\mathcal{R}^{\sigma}$ that has been obtained from $\mathcal{R}$ by applying a theory transformation corresponding to using the time sampling strategy σ when executing the tick rules. In particular, the real-time rewrite theory $\mathcal{R}^{maxDef(r)}$ denotes the real-time rewrite theory $\mathcal{R}$ where the tick rules are applied according to the maximal time sampling strategy, while $\mathcal{R}^{def(r)}$ denotes $\mathcal{R}$ where the tick rules are applied according to the default time sampling strategy (tick steps which advance time by 0 are not applied).

A real-time rewrite theory $\mathcal{R}$ is *time-robust* if the following hold for all ground terms t, t', t'' of sort `GlobalSystem` and all ground terms r, r', of sort `Time`:

– $t = t'$ holds in the underlying equational theory for any 0-time tick step $t \xrightarrow{0} t'$.
– $t \xrightarrow{r+r'} t''$ if and only if there is a t' of sort `Time` such that $t \xrightarrow{r} t'$ and $t' \xrightarrow{r'} t''$.
– If $t \xrightarrow{r} t'$ is a tick step with $r > 0$, and $t' \xrightarrow{inst} t''$ is an instantaneous one-step rewrite, then $t \xrightarrow{r} t''$ is a *maximal* tick step.
– for $M = \{r \mid \exists t'.\ t \xrightarrow{r} t'\}$ we have that either there is a maximal element in M or M is the whole domain of `Time`.

Real-Time Maude extends Maude's *linear temporal logic model checker* to check whether each behavior, possibly up to a certain time bound, satisfies an (untimed) LTL formula. *State propositions* are terms of sort `Prop`. The labeling of states with propositions can be specified by (possibly conditional) equations of the form

`{`*statePattern*`} |=` *prop* `=` *b*

for b a term of sort `Bool`, which defines the state proposition *prop* to evaluate to b in all states matching the given pattern. We say that a set of atomic propositions is *tick-invariant* in $\mathcal{R}$ if tick rules do not change their values.

Since the model checking commands execute time-nondeterministic tick rules according to a time sampling strategy, only a subset of all possible behaviors is analyzed. Therefore, Real-Time Maude analysis is in general *not sound and complete*. However, the reference [28] gives easily checkable sufficient conditions for soundness and completeness, which are satisfied by many large Real-Time Maude applications.

3 Timed CTL Model Checking for Real-Time Maude

In untimed temporal logics it is not possible to reason about the *duration* of/between events. There are many *timed* extensions of temporal logics [4,35,12]. In this paper we consider TCTL [5] with interval time constraints on temporal operators.

3.1 Timed CTL

In *computation tree logic (CTL)* [5], a *state formula* specifies a property over the computation tree corresponding to the system behavior rooted in a given state. State formulae are constructed by adding universal (A) and existential (E) path quantifiers in front of *path formulae* to specify whether the path formula must hold, respectively, on *each* path starting in the given state, or just on *some* path. Path formulae are built from state formulae using the temporal operators X ("next") and U ("until"), from which F ("finally") and G ("globally") can be derived.

Timed CTL (TCTL) is a quantitative extension of CTL [5], where the scope of the temporal operators can be limited in time by subscripting them with time constraints. In this paper we consider an interval-bound version of TCTL where the temporal operators are subscripted with a time interval. A *time interval* I is an interval of the form $[a, b]$, $(a, b]$, $[a, b_\infty)$ or (a, b_∞), where a and b are values of sort `Time` and b_∞ is a value of sort `TimeInf`.

Definition 1. *Given a set Π of atomic propositions, TCTL formulae are built using the following abstract syntax:*

$$\varphi \quad ::= \quad true \quad | \quad p \quad | \quad \neg\varphi \quad | \quad \varphi \wedge \varphi \quad | \quad E\,\varphi\, U_I\, \varphi \quad | \quad A\,\varphi\, U_I\, \varphi$$

where $p \in \Pi$ and I is a time interval.

We omit the bound $[0, \infty)$ as subscript and we write $\leq b$, $< b$, $\geq a$ and $> a$ for $[0, b]$, $[0, b)$, $[a, \infty)$ and (a, ∞), respectively. We denote by TCTL_{cb} the fragment of TCTL where all time bounds are of the form $[a, b]$ with $a < b$, or $[a, \infty)$.

3.2 Timed Kripke Structures and TCTL Semantics

The semantics of TCTL formulae is defined on Kripke structures. A *Kripke structure* is a transition system with an associated labeling function, which maps each state in the transition system to the set of atomic propositions that hold in that state.

A *timed Kripke structure* is a Kripke structure where each transition has the form $s \xrightarrow{r} s'$, where r denotes the duration of the transition step.

Definition 2. *Given a set of atomic propositions Π and a time domain $\mathcal{T}$, a* timed Kripke structure *is a triple $TK = (S, \xrightarrow{\mathcal{T}}, L)$ where S is a set of states, $\xrightarrow{\mathcal{T}} \subseteq S \times \mathcal{T} \times S$ is a* transition relation with duration*, and L is a labeling function $L : S \to \mathcal{P}(\Pi)$. The transition relation $\xrightarrow{\mathcal{T}}$ is* total,[1] *i.e., for each $s \in S$ there exist $r \in \mathcal{T}$, $s' \in S$ such that $(s, r, s') \in \xrightarrow{\mathcal{T}}$. We write $s \xrightarrow{r} s'$ if $(s, r, s') \in \xrightarrow{\mathcal{T}}$.*

[1] A transition relation $\xrightarrow{\mathcal{T}}$ can be made *total* by defining $(\xrightarrow{\mathcal{T}})^\bullet = \xrightarrow{\mathcal{T}} \cup \{(s, 0, s) \in S \times \mathcal{T} \times S \mid \neg\exists\, s' \in S, r \in \mathcal{T} \text{ s.t. } (s, r, s') \in \xrightarrow{\mathcal{T}}\}$.

We use a similar notation for timed paths in a timed Kripke structure $\mathcal{TK}$ as for real-time rewrite theories. Thus, a *timed path* is written $\pi = t_0 \xrightarrow{r_0} t_1 \xrightarrow{r_1} \ldots$, we define $d_m^\pi = \sum_{i=0}^{m-1} r_i$, $t_m^\pi = t_m$ and $r_m^\pi = r_m$, and the set of all timed fair paths originating in state t is denoted by $\mathit{tfPaths}_{\mathcal{TK}}(t)$.

The semantics of TCTL formulae is defined as follows:

Definition 3. *For timed Kripke structures $\mathcal{TK} = (S, \xrightarrow{\mathcal{T}}, L)$, states $t \in S$, and TCTL formulae φ, the* pointwise satisfaction relation *$\mathcal{TK}, t \models_p \varphi$ is defined inductively as follows:*

$\mathcal{TK}, t \models_p$ *true*		*always.*
$\mathcal{TK}, t \models_p p$	*iff*	$p \in L(t)$.
$\mathcal{TK}, t \models_p \neg\varphi_1$	*iff*	$\mathcal{TK}, t \not\models_p \varphi_1$.
$\mathcal{TK}, t \models_p \varphi_1 \wedge \varphi_2$	*iff*	$\mathcal{TK}, t \models_p \varphi_1$ *and* $\mathcal{TK}, t \models_p \varphi_2$.
$\mathcal{TK}, t \models_p E\ \varphi_1\ U_I\ \varphi_2$	*iff*	*there exists* $\pi \in \mathit{tfPaths}_{\mathcal{TK}}(t)$ *and an index k s.t.* $d_k^\pi \in I$, $\mathcal{TK}, t_k^\pi \models_p \varphi_2$, *and* $\mathcal{TK}, t_l^\pi \models_p \varphi_1$ *for all* $0 \le l < k$.
$\mathcal{TK}, t \models_p A\ \varphi_1\ U_I\ \varphi_2$	*iff*	*for each* $\pi \in \mathit{tfPaths}_{\mathcal{TK}}(t)$ *there is an index k s.t.* $d_k^\pi \in I$, $\mathcal{TK}, t_k^\pi \models_p \varphi_2$, *and* $\mathcal{TK}, t_l^\pi \models_p \varphi_1$ *for all* $0 \le l < k$.

For a timed Kripke structure $\mathcal{TK} = (S, \xrightarrow{\mathcal{T}}, L)$, a state $t \in S$ and paths $\pi, \pi' \in \mathit{tfPaths}_{\mathcal{TK}}(t)$ we say that π' is a *simple time refinement* of π if either $\pi = \pi'$ or π' can be obtained from π by replacing a transition $t_k \xrightarrow{r_k} t_{k+1}$, $r_k > 0$, by a sequence $t_k \xrightarrow{r'_k} t \xrightarrow{r''_k} t_{k+1}$ of transitions for some $t \in S$ and time values $r'_k, r''_k > 0$ with $r'_k + r''_k = r_k$. A path π' is a *time refinement* of another path π if π' can be obtained from π by applying a (possibly infinite) number of time refinements. We also say that π is a *time abstraction* of π'.

Definition 4. *The* continuous-time satisfaction relation *$\mathcal{TK}, t \models_c \varphi$ is defined as the pointwise one for the first four cases; for the last two cases we have:*

$\mathcal{TK}, t \models_c E\ \varphi_1\ U_I\ \varphi_2$	*iff*	*there is a path* $\pi \in \mathit{tfPaths}_{\mathcal{TK}}(t)$ *such that for each time refinement* $\pi' \in \mathit{tfPaths}_{\mathcal{TK}}(t)$ *of* π *there is an index k s.t.* $d_k^{\pi'} \in I$, $\mathcal{TK}, t_k^{\pi'} \models_c \varphi_2$, *and* $\mathcal{TK}, t_l^{\pi'} \models_c \varphi_1$ *for all* $0 \le l < k$.
$\mathcal{TK}, t \models_c A\ \varphi_1\ U_I\ \varphi_2$	*iff*	*for each path* $\pi \in \mathit{tfPaths}_{\mathcal{TK}}(t)$ *there is a time refinement* $\pi' \in \mathit{tfPaths}_{\mathcal{TK}}(t)$ *of* π *and an index k s.t.* $d_k^{\pi'} \in I$, $\mathcal{TK}, t_k^{\pi'} \models_c \varphi_2$, *and* $\mathcal{TK}, t_l^{\pi'} \models_c \varphi_1$ *for all* $0 \le l < k$.

3.3 Associating Timed Kripke Structures to Real-Time Rewrite Theories

To each real-time rewrite theory we associate a timed Kripke structure as follows:

Definition 5. *Given a real-time rewrite theory $\mathcal{R} = (\Sigma, E, R, \phi, \tau)$, a set of atomic propositions Π and a protecting extension $(\Sigma \cup \Pi, E \cup D) \supseteq (\Sigma, E)$, we define the associated timed Kripke structure*

$$\mathcal{TK}(\mathcal{R})_\Pi = (\mathbb{T}_{\Sigma/E,\mathtt{GlobalSystem}},\ (\overset{\mathcal{T}}{\longrightarrow}_{\mathcal{R}})^\bullet,\ L_\Pi),$$

where $(\overset{\mathcal{T}}{\longrightarrow}_{\mathcal{R}})^\bullet \subseteq \mathbb{T}_{\Sigma/E,\mathtt{GlobalSystem}} \times \mathbb{T}_{\Sigma/E,\phi(\mathit{Time})} \times \mathbb{T}_{\Sigma/E,\mathtt{GlobalSystem}}$ *contains all transitions of the kind* $t \xrightarrow{r} t'$ *which are also* one-step rewrites *in* $\mathcal{R}$ *and all transitions of the kind* $t \xrightarrow{0} t$ *for all those states* t *that cannot be further rewritten in* $\mathcal{R}$*, and for* $L_\Pi : \mathbb{T}_{\Sigma/E,\mathtt{GlobalSystem}} \to \mathcal{P}(\Pi)$ *we have that* $p \in L_\Pi(t)$ *if and only if* $E \cup D \vdash (t\ \mathtt{|=}\ p) = \mathtt{true}$*.*

We use this transformation to define $\mathcal{R}, L_\Pi, t_0 \models_c \varphi$ as $\mathcal{TK}(\mathcal{R})_\Pi, t_0 \models_c \varphi$, and similarly for the pointwise semantics. The model checking problems $\mathcal{TK}(\mathcal{R})_\Pi, t_0 \models_p \varphi$ and $\mathcal{TK}(\mathcal{R})_\Pi, t_0 \models_c \varphi$ are decidable if

- the equational specification in $\mathcal{R}$ is *Church-Rosser* and *terminating*,
- the set of states reachable from t_0 in the rewrite theory $\mathcal{R}$ is *finite*, and
- given a pair of reachable states t and t', the number of one-step rewrites of the kind $t \xrightarrow{r} t'$ in $\mathcal{R}$ is *finite.*

As mentioned above, real-time rewrite theories generally contain a time-non-deterministic tick rule, but since Real-Time Maude executes such theories by applying a *time sampling strategy* σ, our model checker does not analyze $\mathcal{R}$ but the executable theory $\mathcal{R}^\sigma$ in which the time sampling strategy transformation has been applied. Thus, we associate a timed Kripke structure not to $\mathcal{R}$, but to $\mathcal{R}^\sigma$, and hence the third requirement is satisfied by all but the most esoteric cases; indeed, the tick rules in all Real-Time Maude applications we have seen are deterministic, in the sense that there is at most one one-step tick rewrite $t \xrightarrow{r} t'$ from any state, when the time sampling strategy is taken into account.

We denote by $\mathcal{TK}(\mathcal{R}, t_0)_\Pi$ the timed Kripke structure associated to $\mathcal{R}$ which is restricted to states reachable from t_0, and for states t reachable from t_0 we write $\mathcal{R}, L_\Pi, t \models \varphi$ for $\mathcal{TK}(\mathcal{R}, t_0)_\Pi, t \models \varphi$.

4 Sound and Complete TCTL Model Checking for Real-Time Maude

As mentioned above, for dense time domains, Real-Time Maude only analyzes those behaviors obtained by applying the tick rules according to a selected time sampling strategy. The paper [28] specifies some conditions on a real-time rewrite theory $\mathcal{R}$ and on the atomic propositions that ensure that model checking $\mathcal{R}^{maxDef(r)}$, i.e., using the maximal time sampling strategy, is a sound and complete model checking procedure to check whether all behaviors in the original model $\mathcal{R}$ satisfy an *untimed* LTL formula without the next operator.

For example, if no application of a tick rule changes the valuation of the atomic propositions in a formula and instantaneous rewrite rules can only be applied after *maximal* tick steps or after applying an instantaneous rule, then model checking $\mathcal{R}^{maxDef(r)}$ gives a sound and complete model checking procedure for

$\mathcal{R}$.[2] This result yields a feasible sound and complete model checking procedure for many useful (dense-time) systems, that include many systems that cannot be modeled as, e.g., timed automata.

As explained in the introduction, this completeness result does not carry over to *timed* temporal logic properties. In the following we focus on dense time, since we can achieve sound and complete model checking for discrete time by exploring all possible tick steps in the pointwise semantics, and by advancing time by the smallest possible non-zero duration in the continuous semantics. Furthermore, as already mentioned, in this paper we restrict our treatment to TCTL$_{cb}$ formulas under the continuous semantics.[3]

Our goal is therefore to find a discrete abstraction of a real-time rewrite theory $\mathcal{R}$, so that model checking the abstraction (under the pointwise semantics) is equivalent to model checking $\mathcal{R}$ under the continuous semantics. One part of our solution is to make sure that time progress "stops" at any time point when a time bound in the formula could be reached. This can be achieved if we split any tick step by an amount that divides all possible maximal tick durations and all possible finite non-zero time bounds in the formula. Let $\bar{r}$ be the greatest common divisor of the durations of all maximal tick steps in $\mathcal{R}^{maxDef(r)}$ reachable from the initial state and each finite non-zero time bound in the formula; then "stopping" at each interesting time point should be acheieved if we divide each maximal tick step into smaller steps of duration $\bar{r}$.

However, the following example shows that it is not sufficient to always advance time by this greatest common divisor $\bar{r}$ to obtain a sound and complete abstraction under the continuous semantics. Consider a (dense-time) theory $\mathcal{R}$ that has only one behavior in terms of maximal tick steps, which we show here in terms of validity of the atomic proposition p in the corresponding states:

$$\pi = \neg p \xrightarrow{1} \neg p \xrightarrow{inst} p \xrightarrow{inst} \neg p \xrightarrow{1} \cdots (\neg p \text{ forever})$$

That is, a p-state is reachable in exactly time 1, and ticks do not change the valuations of the atomic propositions. In this model all maximal tick steps have duration 1. Let's consider the formula $\varphi = E\ \varphi_1\ U_{[1,1]}\ \mathit{true}$, where φ_1 is the formula $E\ F_{[1,1]}\ p$. The formula φ says that φ_1 must hold all the way until we reach time 1. The greatest common divisor of all maximal time increments and all time values in φ is still 1, so the "greatest common divisor" abstraction is equivalent to $\mathcal{R}^{maxDef(r)}$. In particular, this abstraction (i.e., the above behavior) satisfies φ w.r.t. the initial state $\pi(0)$. However, $\mathcal{R}, L_{\{p\}}, \pi(0) \models_c \varphi$ does *not* hold, since φ does not hold in the timed refinement (where the first tick has been split into two smaller ones)

$$\pi' = \neg p \xrightarrow{1/2} \neg p \xrightarrow{1/2} \neg p \xrightarrow{inst} p \xrightarrow{inst} \neg p \xrightarrow{1} \cdots (\neg p \text{ forever})$$

because φ_1 does not hold in the second state in the refinement.

[2] The requirements in [28] are weaker than described here.

[3] We are currently working on releasing the restriction to closed bounds. However, our proof for the completeness result cannot be directly extended to TCTL formulas with open bounds.

Our approach is therefore to capture all these "intermediate" states by further splitting the "gcd" tick steps into two smaller tick steps. In essence, we advance time not by $\bar{r}$, but by "half" the gcd $\bar{r}$ in each tick step.

To formalize this notion, let us first consider the time domain. Real-time rewrite theories are parametric in their time domain; the time domain must only satisfy some abstract properties given in some *functional theory* defined in [27] that defines the time domain abstractly as a commutative monoid $(0, \leq, Time)$ with some additional operators. The following theory states that there exist functions `gcd` and `half` on the non-zero time values with the expected properties.

```
fth GCD-TIME-DOMAIN is including LTIME-INF .
  sort NzTime .   subsort NzTime < Time .   cmb T:Time : NzTime if T:Time =/= 0 .
  op gcd : NzTime NzTime -> NzTime [assoc comm] .
  op _divides_ : NzTime NzTime -> Bool .
  op half : NzTime -> NzTime .
  vars T1 T2 T3 : NzTime . vars T T' : Time .
  eq T1 divides T1 = true .
  ceq T1 divides T2 = false if T2 < T1 .
  eq T1 divides (T1 + T2) =  T1 divides T2 .
  eq gcd(T1, T2) divides T1 = true .
  ceq gcd(T1, T2) >= T3  if T3 divides T1 /\ T3 divides T2 .
  eq half(NZT) + half(NZT) = NZT .
endfth
```

In the following we assume that all considered time domains satisfy the theory `GCD-TIME-DOMAIN`, and write *gcd* and *half* for the interpretation of `gcd` and `half`, respectively.

The real-time rewrite theory $\mathcal{R}^{gcd(t_0,r,\varphi)}$ is obtained from the tick-robust real-time rewrite theory $\mathcal{R}$, a state t_0 in $\mathcal{R}$, and a TCTL formula φ, by advancing time by "half" the greatest common divisor of all the following values:

– all tick step durations appearing in paths from $\mathit{tfPaths}_{\mathcal{R}^{maxDef(r)}}(t_0)$ and
– all finite non-zero lower and upper bounds of all temporal operators in φ.

Definition 6. *For a real-time rewrite theory $\mathcal{R}$ whose time domain satisfies the theory* `GCD-TIME-DOMAIN`*, a non-zero time value r, a TCTL formula φ and a state t_0 of $\mathcal{R}$ we define*

$$T_1(\mathcal{R}, t_0, r) = \{r' \in \texttt{NzTime} \mid \exists \pi \in \mathit{tfPaths}_{\mathcal{R}^{maxDef(r)}}(t_0).\ \exists i \geq 0.\ r' = r_i^\pi\}$$
$$T_2(\varphi) = \{r \in \texttt{NzTime} \mid \textit{there exists a subformula } E\ \varphi_1\ U_I\ \varphi_2 \textit{ or } A\ \varphi_1\ U_I\ \varphi_2 \textit{ of } \varphi \textit{ with } r \textit{ a non-zero finite lower or upper bound in } I\}$$
$$GCD(\mathcal{R}, r, \varphi, t_0) = gcd(T_1(\mathcal{R}, t_0, r) \cup T_2(\varphi)).$$

If $T_1(\mathcal{R}, t_0, r)$ and $T_2(\varphi)$ are finite then the *GCD* value is well-defined and we can define the real-time rewrite theory $\mathcal{R}^{gcd(t_0,r,\varphi)}$ as follows:

Definition 7. *Given a real-time rewrite theory $\mathcal{R}$ whose time domain satisfies the theory* `GCD-TIME-DOMAIN`*, a non-zero time value r, a TCTL formula φ, a* *state t_0 of $\mathcal{R}$, and assume that $\bar{r} = GCD(\mathcal{R}, t_0, r, \varphi)$ is a defined non-zero time value. Then $\mathcal{R}^{gcd(t_0,r,\varphi)}$ is defined as $\mathcal{R}$ but where each tick rule of the forms (†), (∗), and (§) is replaced by the respective tick rule:*

```
crl [tick] : {t} => {t'} in time x if x := half(r̄) /\ cond [nonexec] .
crl [tick] : {t} => {t'} in time x if x := half(r̄) /\ cond [nonexec] .
crl [tick] : {t} => {t'} in time x if x := half(r̄) [nonexec] .
```

The following lemma states that the evaluation of the formula φ and its subformulas does not change inside tick steps of $\mathcal{R}^{gcd(t_0,r,\varphi)}$.

Lemma 1. *Assume a time-robust real-time rewrite theory $\mathcal{R}$ whose time domain satisfies the theory* `GCD-TIME-DOMAIN`. *Let Π be a set of tick-invariant atomic propositions, and assume a protecting extension of $\mathcal{R}$ defining the atomic propositions in Π and inducing a labeling function L_Π. Let t_0 be a state of $\mathcal{R}$, r a non-zero time value of sort* `Time`, *φ a $TCTL_{cb}$ formula over Π, and assume that $\bar{r} = GCD(\mathcal{R}, t_0, r, \varphi)$ is a defined non-zero time value.*

Then for each subformula φ' of φ, each time-divergent path $\pi \in tfPaths_{\mathcal{R}}(t_0)$ and for all tick step sequences $t_i^\pi \xrightarrow{r_i^\pi} \dots \xrightarrow{r_{j-1}^\pi} t_j^\pi$ in π satisfying $n \cdot \bar{r} < d_i^\pi < d_j^\pi < (n+1) \cdot \bar{r}$ for some n we have that

$$\mathcal{R}, L_\Pi, t_i^\pi \models_c \varphi' \quad \textit{iff} \quad \mathcal{R}, L_\Pi, t_j^\pi \models_c \varphi' \ .$$

Proof. The proof by induction on the structure of φ' can be found in our technical report [19].

Based on the above lemma we gain our completeness result:

Theorem 1. *Let $\mathcal{R}$, L_Π, t_0, r, φ and $\bar{r}$ be as in Lemma 1. Then*

$$\mathcal{R}, L_\Pi, t \models_c \varphi \quad \iff \quad \mathcal{R}^{gcd(t_0,r,\varphi)}, L_\Pi, t \models_p \varphi$$

for all states t reachable in $\mathcal{R}^{gcd(t_0,r,\varphi)}$ from t_0.

Proof. Again, the proof is given in [19].

5 Implementation

Our model checker makes the natural and reasonable assumption that given a real-time rewrite theory $\mathcal{R}$, and an initial state t_0 on which we would like to check some TCTL formula φ, all behaviors starting from t_0 are *time-diverging* w.r.t. the selected time sampling strategy σ. This assumption also implies that the transition relation $\xrightarrow{\mathcal{T}}_{\mathcal{R}^\sigma}$ in the timed Kripke structure $\mathcal{TK}(\mathcal{R}^\sigma, t_0)_\Pi$ is *total.*

The current implementation of the model checker assumes that time values are either in `NAT-TIME-DOMAIN-WITH-INF` or `POSRAT-TIME-DOMAIN-WITH-INF`, and provides the user with two possible model-checking strategies:

(i) The *basic* strategy, which performs the model checking on the model obtained by applying the user-defined time sampling strategy on the original model.

(ii) The *gcd* strategy, which extends the maximal time sampling strategy with the "gcd" transformation to perform the model checking for the satisfaction problem $\mathcal{R}^{gcd(t_0,r,\varphi)}, L_\Pi, t_0 \models_p \varphi$.

Soundness and completeness of the *gcd* strategy might come at the cost of a larger state space due to the application of the *gcd* transformation. When the *gcd* strategy is impractical, the user can still perform model checking with the generally faster basic strategy, which does not increase the system state space and can still be very useful to discover potential bugs, as illustrated below.

Real-Time Maude, and hence our model checker, is implemented in Maude, making extensive use of Maude's meta-programming capabilities. The model checker first constructs the timed Kripke structure, according to the selected model checking strategy, by collecting all the reachable states and transitions. When the gcd strategy is selected, the timed Kripke structure is refined by "splitting" the transitions into smaller ones of duration equal to half the computed greatest common divisor. Then, the satisfaction sets of each subformula are recursively computed. Since the meta-representation of the states can be fairly large[4], performing the rest of the model checking procedure on the generated timed Kripke structure is fairly inefficient. In our current implementation, we assign a unique natural number to each (meta-represented) state in the generated timed Kripke structure, and construct a more compact timed Kripke structure, where all the occurrences of these meta-represented states are replaced by their respective identifiers. We then perform the recursive computation of the satisfaction set of φ on this compact representation. This optimization led to a large performance improvement and made it feasible to apply our model checker to a number of case studies in reasonable time, whereas working directly on meta-represented terms made model checking unfeasible even for simple case studies.

Our implementation of the TCTL model checker is based on the *explicit-state* CTL model checking approach [8] that, starting with the atomic propositions, recursively computes for each subformula of the desired TCTL formula the set of satisfying reachable states. We implemented specific procedures for a basic set of temporal modal operators and we expressed other formulas into this canonical form. The basic set consists of the CTL modal operators $E\ \varphi_1\ U\ \varphi_2$, $E\ G\ \varphi$, the TCTL$_{\leq\geq}$[5] modal operators $E\ \varphi_1\ U_{\sim r}\ \varphi_2$ with $\sim \in \{>, \geq\}$, $E\ \varphi_1\ U_{\sim r}\ \varphi_2$ with $\sim \in \{<, \leq\}$, $A\ \varphi_1\ U_{>0}\ \varphi_2$ and the TCTL$_{cb}$ modal operator $E\ \varphi_1\ U_{[a,b]}\ \varphi_2$. The procedures for CTL modalities follow the standard explicit algorithm [8]. For TCTL$_{\leq\geq}$ modalities, our implementation adapts the TCTL$_{\leq\geq}$ model checking procedure defined in [18] for time-interval structures and to timed Kripke structures with time-diverging paths.

The ease and flexibility of the Maude meta-level allowed us to implement the model checker reasonably quickly and easily. However, the convenience of operating at the meta-level comes at a certain cost in terms of computational

[4] For example, *each* state in the Maude representation of the Ptolemy II model in Section 6 "contains" the entire Ptolemy II model.

[5] We denote by TCTL$_{\leq\geq}$ the restricted TCTL logic with time constraints on the temporal modalities of the form $\sim r$, where $\sim \in \{<, \leq, \geq, >\}$,

efficiency, even with our optimizations. Therefore, the current Real-Time Maude model checker should be regarded as a *working prototype* for a C++ implementation that we plan to implement in the future.

Our model checker is available at `http://folk.uio.no/leprid/TCTL-RTM/` together with the technical report [19], the specifications and analysis commands of the case studies in this paper.

5.1 Using the Model Checker

In Real-Time Maude, the user is provided with two TCTL model checking commands, corresponding respectively to the basic and the gcd strategy, with syntax

(mc-tctl t |= φ .) *and* (mc-tctl-gcd t |= φ .)

for t the initial state and φ a TCTL formula. The syntax of TCTL formulas is fairly intuitive, with syntactic sugar for (untimed) CTL formulas, common abbreviations and boolean connectors such as `AF`, `EF`, `AG`, `EG`, `iff` and `implies`, etc. For example, $E\ true\ U_{\leq r}(\neg\varphi \wedge A\ G\ (E\ F_{[a,b]}\varphi'))$ is written[6]

E tt U[<= than r](not φ and AG (EF[c a, b c] φ'))

We do not support counter-example generation, since, in contrast to linear temporal logics, where counter-examples are just paths, it is generally more complex to generate counter-examples in branching-time temporal logics, where counterexamples are parts of computation trees (see, e.g. [13]). For example, a counter-example to the validity of the formula $E\ F\ p$, for p an atomic proposition, is the entire computation tree (where each state is a $\neg p$-state).

6 Model Checking a Ptolemy II Discrete-Event Model

Real-Time Maude provides a formal analysis tool for a set of modeling languages for embedded systems, including Ptolemy II discrete-event (DE) models that cannot be formalized by, say, timed automata. Ptolemy II [15] is a well-established modeling and simulation tool used in industry that provides a powerful yet intuitive graphical modeling language. Our model checker has been integrated into Ptolemy II by Kyungmin Bae, so that we can now model check TCTL properties of Ptolemy II DE models *from within Ptolemy*[7]. We show the TCTL analysis of a Ptolemy II model of a hierarchical traffic light system, in which our model checker has uncovered a previously unknown flaw. Notice that Ptolemy II DE models satisfy the requirements for having a sound and complete analysis when using the gcd strategy. The analysis has been performed on a 2.4GHz Intel® Core 2 Duo processor with 4 GB of RAM.

[6] The model checker syntax for TCTL formulas supports also open bounds, e.g. the user could write [c 0, b o] for $[0, b)$, which would be internally reduced to [< than b].

[7] Real-Time Maude verification commands can be entered into the dialog box that pops up when the blue button in Fig. 1 is clicked.

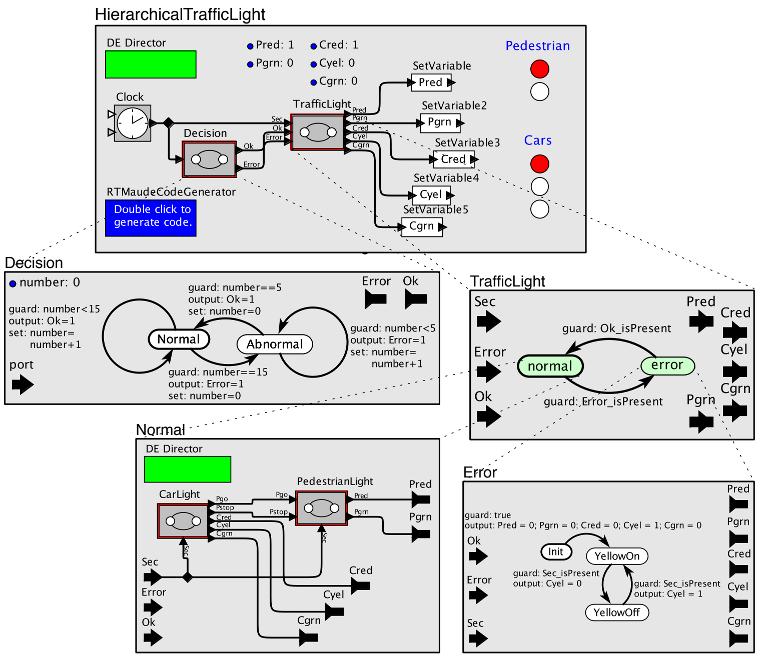


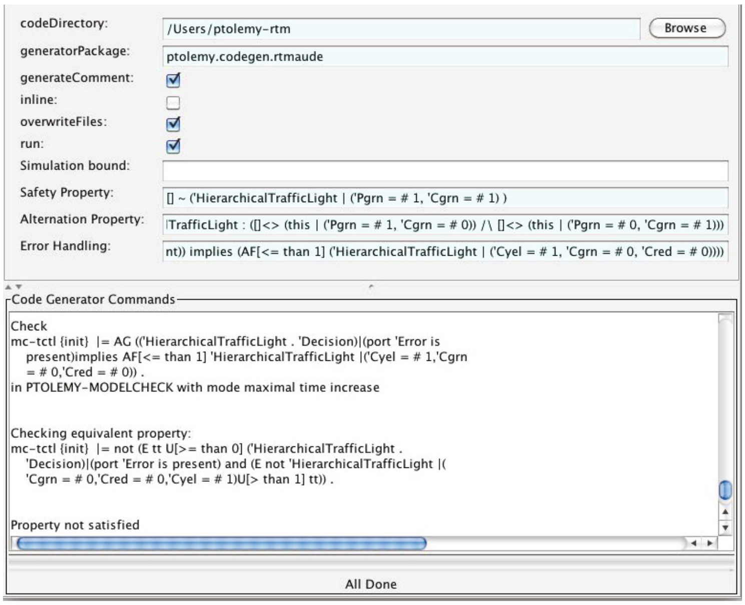
Fig. 1. A hierarchical fault-tolerant traffic light system in Ptolemy II

Figure 1 shows a hierarchical Ptolemy II model of a fault-tolerant traffic light system at a pedestrian crossing, consisting of one car light and one pedestrian light. Each light is represented by a set of *set variable* actors (`Pred` and `Pgrn` represent the pedestrian light, and `Cred`, `Cyel` and `Cgrn` represent the car light). A light is *on* iff the corresponding variable equals 1. The FSM actor `Decision` "generates" failures and repairs by alternating between staying in location `Normal` for 15 time units and staying in location `Abnormal` for 5 time units. Whenever the model operates in `error` mode, all lights are turned off, except for the yellow light of the car light, which is blinking. We refer to [7] for a thorough explanation of the model.

An important fault tolerance property is that the car light will turn yellow, *and only yellow*, within 1 time unit of a failure. We can model check this bounded response property with the command:

```
Maude> (mc-tctl {init} |=
  AG(('HierarchicalTrafficLight . 'Decision | (port 'Error is present))
    implies AF[<= than 1] ('HierarchicalTrafficLight |
                           ('Cyel = # 1, 'Cgrn = # 0, 'Cred = # 0)))) .)
```

In about 15 secs, the command returns that the property is not satisfied. This model checking uncovered a previously unknown scenario, which shows that,



codeDirectory:	/Users/ptolemy-rtm
generatorPackage:	ptolemy.codegen.rtmaude
generateComment:	✓
inline:	☐
overwriteFiles:	✓
run:	✓
Simulation bound:	
Safety Property:	[] ~ ('HierarchicalTrafficLight | ('Pgrn = # 1, 'Cgrn = # 1))
Alternation Property:	TrafficLight : ([]<> (this | ('Pgrn = # 1, 'Cgrn = # 0)) /\ []<> (this | ('Pgrn = # 0, 'Cgrn = # 1)))
Error Handling:	nt)) implies (AF[<= than 1] ('HierarchicalTrafficLight | ('Cyel = # 1, 'Cgrn = # 0, 'Cred = # 0))))

Code Generator Commands

```
Check
mc-tctl {init} |= AG (('HierarchicalTrafficLight . 'Decision)|(port 'Error is
   present)implies AF[<= than 1] 'HierarchicalTrafficLight |('Cyel = # 1,'Cgrn
   = # 0,'Cred = # 0)) .
in PTOLEMY-MODELCHECK with mode maximal time increase


Checking equivalent property:
mc-tctl {init} |= not (E tt U[>= than 0] ('HierarchicalTrafficLight .
   'Decision)|(port 'Error is present) and (E not 'HierarchicalTrafficLight |(
   'Cgrn = # 0,'Cred = # 0,'Cyel = # 1)U[> than 1] tt)) .


Property not satisfied
```

All Done

Fig. 2. Dialog window for the hierarchical traffic light code generation

after a failure, the car light may show red or green in addition to blinking yellow. Eleven of the 15 seconds used by the timed CTL model checker were used to generate the timed Kripke structure. Because of the large size of the system states in this case study, it was impossible to run the same analysis before implementing the optimization that mapped each state to an unique identifier. The same property can be model checked with the gcd strategy command `mc-tks-gcd` in about 22 secs.

Using the gcd strategy we can also determine a “minimal” time interval such that the above bounded response is satisfied in the system. In particular, we discovered that this interval is $[5, 12]$ by trying different values for a and b in the interval-bounded command

```
Maude> (mc-tctl {init} |=
  AG(('HierarchicalTrafficLight . 'Decision | (port 'Error is present))
    implies AF[c a, b c] ('HierarchicalTrafficLight |
                          ('Cyel = # 1, 'Cgrn = # 0, 'Cred = # 0)))) .)
```

Figure 2 shows the dialog window for the Real-Time Maude code generation of the hierarchical traffic light model: after entering the *error handling* property, a simple click on the `Generate` button will display the result of the model checking command execution in the “Code Generator Commands” box.

7 Conclusions and Future Work

We have described the semantic foundations of our TCTL model checker for Real-Time Maude. Our modeling formalism is more expressive than those of other timed model checkers, allowing us to analyze real-time systems which are beyond the scope of other verification tools. In particular, we have proved soundness and completeness of our model checker for a class of dense-time Real Time Maude specifications that contain many systems outside the scope of other real-time model checkers. Furthermore, the introduced TCTL model checker also provides for free a timed temporal logic model checker for interesting subsets of modeling languages widely used in industry, such as Ptolemy II and the avionics standard AADL.

So far, we have only proved soundness and completeness for formulas with closed intervals under the continuous semantics. We should also cover formulas with open time intervals and the pointwise semantics. The model checker should also provide counter-examples in a user-friendly way, when possible. We should also extend our model checker to *time-bounded* TCTL model checking to support the model checking of systems with infinite reachable state space. Finally, the current version of the tool is implemented at the Maude meta-level; for efficiency purposes, it should be implemented in C++ in the Maude engine.

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