Model Predictive Control for Inside Temperature of an Energy Efficient Building

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Abstract. The paper presents the development and implementation of a model predictive control (MPC) used for inside temperature control of a building. The inside temperature is tracking a prescribed reference inside a comfort zone define by the optimization problem implementing offset free control through a Kalman filter state estimator. The MPC is validated by simulation and experiment using a building thermal model, a 24 hour ahead predicted solar irradiance and ambient temperature and measured actual weather data and inside temperature for the closed loop simulation operation.

Keywords: Model predictive control, energy efficient building, Kalman filter, offset free control, quadratic program.

1 Introduction

As the building sector represents a major consumer in the world, with an estimated 26.5% of the total global consumption in 2009 according to [1], research on developing energy efficient buildings represents a large energy savings potential.

The model predictive control algorithm is well suited for energy efficient buildings and systems where both the control variables and state variables have physical constraints. The MPC is able to take action before a predicted event actually takes place; in this case, the system having large time constants, the electrical heaters are turned off prior to mid day, where the solar irradiance has a powerful effect on increasing the inside temperature.

In building temperature control there are two large approaches on MPC algorithms: the first are tracking a prescribed reference, minimizing the difference between the controlled variable and the reference inside temperature as in [2] and [3] and the economic MPC which minimizes the cost of energy in keeping the inside temperature inside the comfort zone [4].

In [5] is studied the impact of modeling accuracy on the MPC of the passive building thermal capacitance to minimize an objective function.

MPC algorithms for controlling the inside temperature were developed considering weather predictions as in [6] and stochastic disturbances in [2] or human presence [7].

The objective function of the PMC can be stated as a linear programming LP problem as [8] or as a quadratic problem QP as in [2].

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This paper presents a MPC for controlling the inside temperature of a building to track the prescribed reference formulated as a quadratic problem, with prediction on the two disturbances, the solar radiation and ambient temperature.

2 Model of the System

The temperature dynamics of a given space can be modeled with the help of an equivalent electric circuit. This paper considers the simplified thermal model of an eight room building from SYSLAB facility from DTU Elektro at RISO campus. The building has 125 squared meters and one electrical heater in each room, except two rooms which are equipped with two electrical heaters each. The parameter identification and validation tests of the mathematical model are presented in the work of Bacher and Thavlov[9].

The objective of this paper is to develop, implement, and test through simulations and experiment, a model predictive controller which considers the inside temperature of the building as the controlled variable. The objective is for the controlled variable to track the prescribed reference, minimizing the peak consumption and maintain the inside temperature inside the comfort zone considering the difference between the predicted and measured perturbation: the solar irradiance and ambient temperature. This paper uses offline 24 hours ahead prediction data for the ambient temperature and solar irradiance and online measured values of weather data taken from the local weather station.

The building can be estimated as having a single room, with an additional assumption that each electrical heater has the same power (1kW) and the same effect on the inside temperature on this one room model. The equivalent (RC) electric circuit of the building is shown in Fig.1.

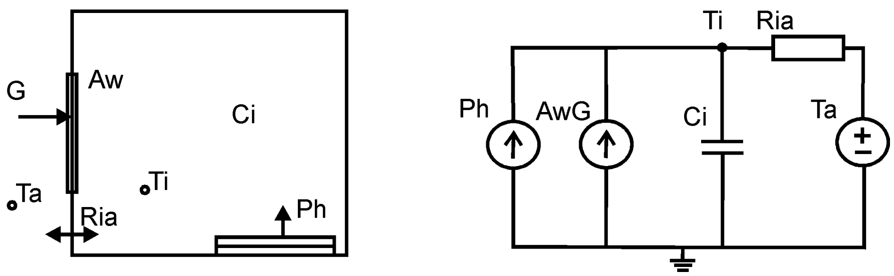

Fig. 1. Equivalent thermal model of the building

The mathematical thermal model of the one room equivalent building is shown in (1):

$$C_i \frac{dT_i}{dt} = \frac{1}{R_{ia}}(T_a - T_i) + A_w G + P_h \tag{1}$$

Where

C_i – thermal capacitance of the house

R_{ia} – thermal resistance of walls and windows isolating the house from the outside environment

A_w – window effective area
T_i – inside temperature
T_a – ambient temperature
G – solar irradiance on the house

For developing the MPC, the state-space model from (2) is needed. The parameters of the model are presented in Table 1.

$$\begin{cases} x_{k+1} = Ax_k + Bu_k + Ed_k \\ z_k = Cx_k \end{cases} \tag{2}$$

Table 1. Model parameters

C_i [kW/°C]	R_{ia} [°C/ kW]	A_W [m^2]	A	B	E	C
3.42	4,87	5,53	[0.99]	[0.0487]	[0.01, 0.2695]	[1]

3 MPC Algorithm

Model predictive control uses a mathematical model of the system, a discrete state space system in this paper, to estimate the process output for a fixed number of steps N into the future predicted horizon, according to the current internal state values, the predictions, and the reference trajectory. The MPC calculates the control sequence of the future N steps by minimizing an objective function. From this sequence only the first control output is considered and at the next time step the optimization is recalculated according to the new values available.

The MPC starts from a model, in this paper a state-space model, presented in (2), and an optimization function as seen in (3).

$$\min_{u \in R^n} f(u) = \frac{1}{2}\sum_{k=0}^{N} \left\| z_k - r_k \right\|_Q^2 + \frac{1}{2}\sum_{k=0}^{N-1} \left\| \Delta u_k \right\|_S^2 \tag{3}$$

This is a weighted quadratic optimization function with two objectives:

- The output variable (z_k) to track the reference (r_k), and
- doing so with small variations (Δu_k)

This optimization is realized for the next N steps interval, from which only the first term is considered for the controller output. At the next step, the optimization problem is recalculated, with the new values for the inputs, predictions, and measurements specific to that step.

Another important aspect of this control is the ability to alter the importance of the optimization terms in the optimization function by using weight coefficients, in this case Q for minimizing the trajectory difference and S for minimizing the variations of the manipulated variables.

The controller was developed in Matlab, using the *quadprog* solver, from the optimization toolbox. The optimization problem from (3) had to be rewritten in the form of (4), specific to this Matlab solver.

$$\min_{U} f = \frac{1}{2}U^{'}HU + g^{'}U \tag{4}$$

Subject to

$$A_q U \le b_q \tag{5}$$

From (2), the output values of the system can be estimated for the next prediction horizon, z_i , (i=1...N) considering the following values: x_0 - as the initial value for the state variable at each time step interval, D- the predicted disturbance vector over the predicted horizon and U - the system input, the command variable for the control algorithm, in this case the output power of the heaters. The predicted output values for the next N steps are:

$$\begin{aligned}
\underbrace{\begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ \vdots \\ z_N \end{bmatrix}}_{=Z} = \underbrace{\begin{bmatrix} CA \\ CA^2 \\ CA^3 \\ \vdots \\ CA^N \end{bmatrix}}_{=\Phi} x_0 + \underbrace{\begin{bmatrix} H_1 & 0 & 0 & \cdots & 0 \\ H_2 & H_1 & 0 & \cdots & 0 \\ H_3 & H_2 & H_1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & 0 \\ H_N & H_{N-1} & H_{N-2} & \cdots & H_1 \end{bmatrix}}_{=\Gamma} \begin{bmatrix} u_0 \\ u_1 \\ u_2 \\ \vdots \\ u_{N-1} \end{bmatrix} + \\
\underbrace{\begin{bmatrix} H_{1,d} & 0 & 0 & \cdots & 0 \\ H_{2,d} & H_{1,d} & 0 & \cdots & 0 \\ H_{3,d} & H_{2,d} & H_{1,d} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & 0 \\ H_{N,d} & H_{N-1,d} & H_{N-2,d} & \cdots & H_{1,d} \end{bmatrix}}_{=\Gamma_d} \begin{bmatrix} d_0 \\ d_1 \\ d_2 \\ \vdots \\ d_{N-1} \end{bmatrix}
\end{aligned} \tag{6}$$

where:

$$H_k = CA^{k-1}B \tag{7}$$

$$H_{k,d} = CA^{k-1}E \tag{8}$$

Equation (6) can be rewritten in a simpler way to handle, as shown in (9)

$$Z = \Phi x_0 + \Gamma U + \Gamma_d D \tag{9}$$

By developing the first term of (3) and using (9) , it is obtained:

$$f_z = \frac{1}{2}\sum_{k=0}^{N}\left\| z_k - r_k \right\|_Q^2 = \frac{1}{2}\sum_{k=0}^{N}\left\| Z - R \right\|_Q^2 = \frac{1}{2}\sum_{k=0}^{N}\left\| \Gamma U - b \right\|_Q^2 \tag{10}$$

Where

$$b = R - \Phi x_0 - \Gamma_d D \tag{11}$$

By considering the form of (4), (10) is written as (12):

$$\begin{aligned} f_z &= \frac{1}{2}\sum_{k=0}^{N}\left\| \Gamma U - b \right\|_Q^2 = \frac{1}{2}\left(\Gamma U - b\right)' Q \left(\Gamma U - b\right) = \\ &= \frac{1}{2} U' \Gamma' Q \Gamma U - \left(\Gamma' Q b\right)' U + \frac{1}{2} b' Q b \end{aligned} \tag{12}$$

For the second term of (3), the following form is obtained:

$$f_\Delta = \frac{1}{2}\sum_{k=0}^{N-1}\left\| \Delta u_k \right\|_S^2 = \frac{1}{2}\left\| \begin{bmatrix} u_0 \\ u_1 \\ \vdots \\ u_{N-1} \end{bmatrix} - \begin{bmatrix} u_{-1} \\ u_0 \\ \vdots \\ u_{N-2} \end{bmatrix} \right\|_S^2 \tag{13}$$

Which develops into (14):

$$f_\Delta = \frac{1}{2}\begin{bmatrix} u_0 \\ u_1 \\ u_2 \\ \vdots \\ u_{N-1} \end{bmatrix}' \underbrace{\begin{bmatrix} 2S & -S & 0 & \cdots & 0 \\ -S & 2S & -S & \cdots & 0 \\ 0 & -S & 2S & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & -S \\ 0 & 0 & 0 & -S & S \end{bmatrix}}_{=H_S} \begin{bmatrix} u_0 \\ u_1 \\ u_2 \\ \vdots \\ u_{N-1} \end{bmatrix} + \underbrace{\left(- \begin{bmatrix} S \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} u_{-1} \right)}_{=M_{u-1}} \begin{bmatrix} u_0 \\ u_1 \\ u_2 \\ \vdots \\ u_{N-1} \end{bmatrix} \tag{14}$$

And for the required form of (4), the second term of (3) is:

$$f_\Delta = \frac{1}{2} U' H_S U + M_{u_{-1}} u_{-1} U \tag{15}$$

From (12) and (15), the parameters H and g of (4) can be identified from the initial objective function of (3), in order to use the Matlab toolbox:

$$H = \Gamma' Q \Gamma + H_S \tag{16}$$

$$g = -\Gamma' Q\left(R - \Phi x_0 - \Gamma_d D\right) + M_{u_{-1}} u_{-1} \tag{17}$$

An advantage of the MPC is the ability to deal with constraints, both on the manipulated and controlled variables. The constraints on the two types of variables defined in (18) and (19) have to be written as a function of the manipulated variable and written in the form of variables A_q and b_q from (3).

$$U_{\min} \leq U \leq U_{\max}, k = 0 \ldots N-1 \tag{18}$$

$$Z_{\min} \leq Z \leq Z_{\max}, k = 0 \ldots N \tag{19}$$

As (19) have to be a function of manipulated variable U, the equation is rewritten as (20) and (21) as only the "smaller than" relation can be used for both the ends of the interval.

$$Z_{\min} - \Phi x_0 - \Gamma_d D \leq \Gamma U \tag{20}$$

$$-Z_{\max} + \Phi x_0 + \Gamma_d D \leq -\Gamma U \tag{21}$$

The parameters A_q and b_q from (4), for the considered model have the following relation:

$$A_q = \begin{bmatrix} I_N \\ -I_N \\ \Gamma \\ -\Gamma \end{bmatrix} \quad b_q = \begin{bmatrix} U_{\min} \\ -U_{\max} \\ Z_{\min} - \Phi x_0 - \Gamma_d D \\ -Z_{\max} + \Phi x_0 + \Gamma_d D \end{bmatrix} \tag{22}$$

Parameters H, g, A_q, and b_q from (16), (17), and (22) will be computed and used as call parameters for the *quadprog* solver:

$$U = \text{quadprog}(H, g, A_q, b_q) \tag{23}$$

The function returns the next N values of the manipulated variable U, which represents the output power of the electrical heaters for the N step prediction interval. From this, only the first value U_1 is used, and at the next step the optimization problem is recalculated.

4 Offset Free MPC

The system used in this paper and described in (3) uses two disturbance predictions which both have a large influence on the system output (inside temperature). These are the solar irradiance and the outside ambient temperature. These data are provided by the local weather station with 24 hours-ahead prediction. However, like in the general case, these predictions are not perfect as it is shown in the figures of the next section.

These differences between the actual and the predicted values have to be considered in the model, otherwise they will introduce an offset in the controlled variable.

One of the possibilities to incorporate an integrator in the system model is to extend the state-space model with a number of additional states equal to the disturbances, and using a Kalman filter for their estimation. The offset free control has been discussed in [10] and [11].

If stochastic perturbation on the state variables (ω_k), on the system output (v_k) and errors in prediction (ξ_k) are considered, the augmented system (3) becomes:

$$\begin{cases} x_{k+1} = Ax_k + Bu_k + E(d_k + \eta_k) + \omega_k \\ \eta_{k+1} = \eta_k + \xi_k \\ z_k = Cx_k + v_k \end{cases} \tag{24}$$

And in the state-space form of:

$$\begin{cases} \begin{bmatrix} x_{k+1} \\ \eta_{k+1} \end{bmatrix} = \underbrace{\begin{bmatrix} A & E \\ 0 & I \end{bmatrix}}_{=\tilde{A}} \begin{bmatrix} x_k \\ \eta_k \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u_k + \begin{bmatrix} E \\ 0 \end{bmatrix} d_k + \begin{bmatrix} \omega_k \\ \xi_k \end{bmatrix} \\ z_k = \underbrace{\begin{bmatrix} C & 0 \end{bmatrix}}_{=\tilde{C}} \begin{bmatrix} x_k \\ \eta_k \end{bmatrix} + v_k \end{cases} \tag{25}$$

where $\omega_k \approx N(0,Q_w)$; $\xi_k \approx N(0,Q_{xi})$; $v_k \approx N(0,R_v)$ are zero-mean white-noise

The system from (25) is used to resolve the MPC problem according to the equations presented in section 3.

For the off-set free control, the states are estimated as follows:

$$\begin{bmatrix} \hat{x}_{k|k} \\ \hat{\eta}_{k|k} \end{bmatrix} = \begin{bmatrix} \hat{x}_{k|k-1} \\ \hat{\eta}_{k|k-1} \end{bmatrix} + K\left(z_k - C\hat{x}_{k|k-1}\right) \tag{26}$$

And the prediction of future augmented states is obtained by:

$$\begin{bmatrix} \hat{x}_{k+1|k} \\ \hat{\eta}_{k+1|k} \end{bmatrix} = \begin{bmatrix} A & E \\ 0 & I \end{bmatrix} \begin{bmatrix} \hat{x}_{k|k} \\ \hat{\eta}_{k|k} \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u_k + \begin{bmatrix} E \\ 0 \end{bmatrix} d_k \tag{27}$$

Equations (26) and (27) are the measurement update and time update state of a Kalman filter used to introduce an integrator element into the control. The two equations can be written in a condensed form:

$$\begin{bmatrix} \hat{x}_{k+1|k} \\ \hat{\eta}_{k+1|k} \end{bmatrix} = \tilde{A} \begin{bmatrix} \hat{x}_{k|k-1} \\ \hat{\eta}_{k|k-1} \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u_k + \begin{bmatrix} E \\ 0 \end{bmatrix} d_k + \tilde{A} K \left(z_k - C \hat{x}_{k|k-1} \right) \tag{28}$$

Where the Kalman filter gain is computed offline according to:

$$K = P\tilde{C}' \left(\tilde{C} P \tilde{C}' + R_v \right)^{-1} \tag{29}$$

And P is the unique symmetric positive semidefinite solution of the discrete algebraic Riccati equation:

$$P = \tilde{A} P \tilde{A}^T + Q - \tilde{A} P \tilde{C}^T \left(\tilde{C} P \tilde{C}^T + R_V \right)^{-1} CPA^T \tag{30}$$

5 Simulation and Experiment

For the simulation and experiment, the augmented model from (24) is used with a prediction horizon of 50 steps of 10 minutes and parameters from Table 1.

For the Kalman filter implementation the following values were chosen for the covariance matrices: $R_v = [0.01]$, $Q_w = [0.015]$, $Q_{xi} = [1\ 0;\ 0\ 1]$.

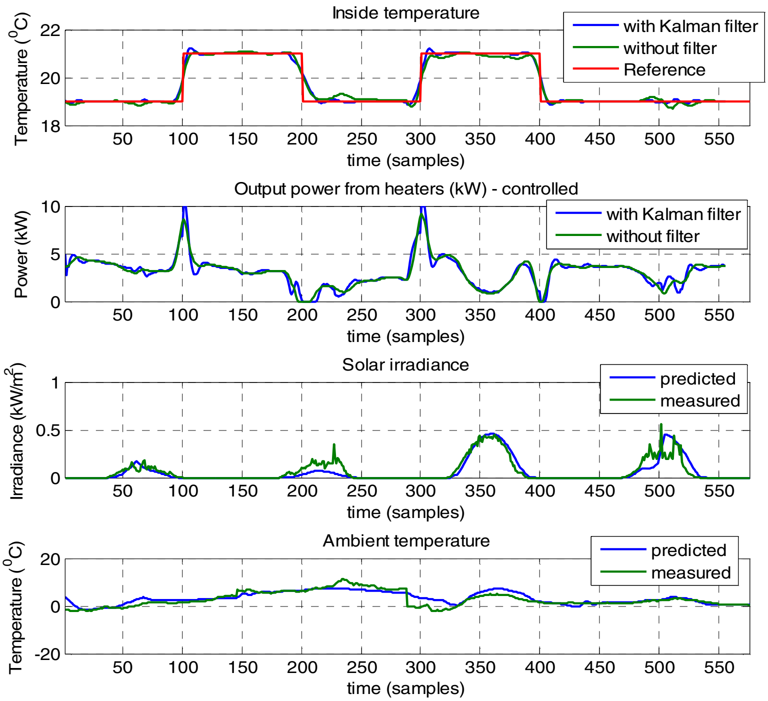

Fig. 2. Simulation results considering continuous consumption of the electrical heaters Φ_{normal}=0.0195 and $\Phi_{observer}$=0.0165

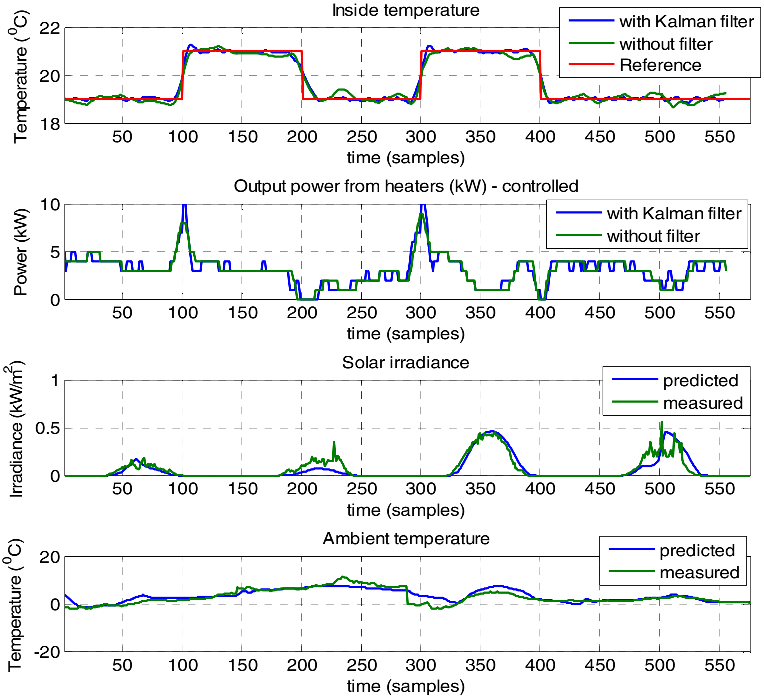

Fig. 3. Simulation results considering 1kW steps consumption of the electrical heaters; Φ_{normal}=0.0252 and $\Phi_{observer}$=0.0174

The simulation presented in Fig. 2 was performed by considering real weather data taken from the SYSLAB facility: historical 24 hour prediction data set for four days in May and the historical measured data for the same period of time.

The constraints on the manipulated variable, inside temperature are set to be 20° and 25° C. The limits on the manipulated variable, the electrical heaters output power, is set to U_{min}=0 and U_{max}=10.

In both simulation, as seen in the "Inside temperature" plot, are represented three variables: the prescribed reference, the inside temperature when using for the optimization problem the system from (2), which presents offset and the augmented system (24) for offset free solution by using the Kalman filter to estimate the initial states x_0. For comparison reasons, a cost function was introduced:

$$\overline{\Phi} = \frac{1}{2(T_f - T_0)} \sum_{t=T_0}^{T_f} (z_k - r_k)^2 \tag{31}$$

The solution of the optimization problem is a real number. In order to operate the ten 1kW electrical heaters, this solution was rounded to the closest integer to the actual solution. The results by using a more realistic 1kW step power consumption of the electrical heaters are presented in Fig. 3.

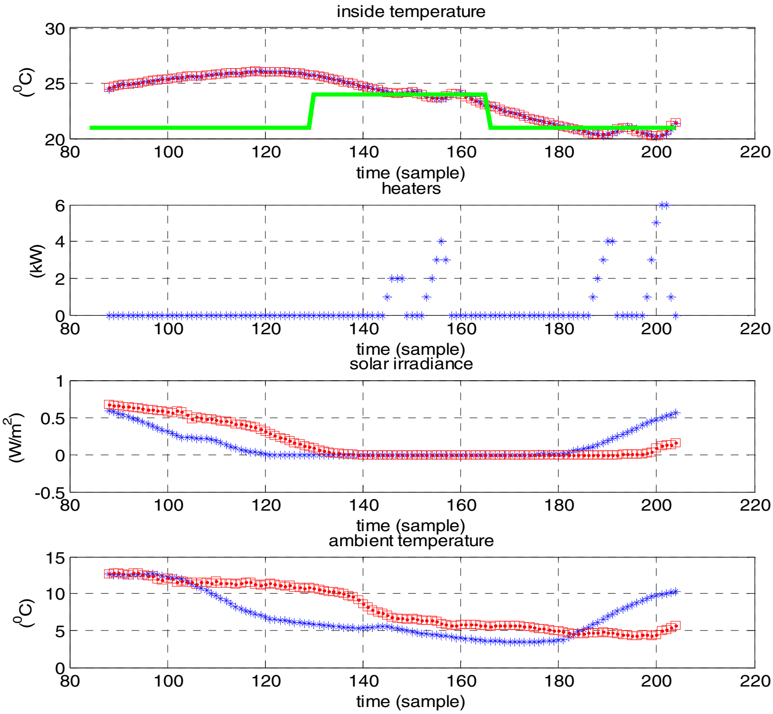

Fig. 4. Experimental results

Even if in the second simulation the performance is lower for both MPC solutions compared to the first simulation, the usage of the observer can be well justified as $\Delta\Phi_{normal}$= 0.0057 and $\Delta\Phi_{obsever}$= 0.0009 and will be more meaningful in the case additional unknown disturbances affect the system.

A validation experiment was conducted using the MPC developed in Matlab. Using the described MPC, and online measurements of the inside temperature, solar irradiance and ambient temperature the controlled variable (inside temperature) was tracking the prescribed trajectory, as seen in Fig. 4.

Some additional errors and perturbations were observed: the inside temperature was considered as the average of the temperature inside the eight rooms the electrical heaters had different output powers and the system had a very simplified model. Another issue was the relatively hot and sunny weather which interfered with the experiment, the amount of energy from the exterior being enough to heat the house to the desired temperature and even to exceed it.

The MPC developed in this paper can affect the controlled variable in a single way: it can only increase the inside temperature by increasing the output power of the heaters and not to cool down the temperature since it does not use any air conditioning unit. This phenomenon is clear in the first part of the experiment, when the inside of the building is overheated by the weather during the day.

As it can be seen from fig. 4, at the end of the first day (sample 126 is 9 PM), the temperature inside the house is 26 degrees and the MPC can operate only during the night to track the prescribed values.

The controller of this process can affect only one direction of the system, that is the MPC cab only increase the temperature by turning heaters on and cannot take any action in lowering it. The decrease in temperature must come naturally, and the MPC can only set all the heaters to off.

6 Conclusions

The controlled variable of the MPC presents offset free by using augmented state space model, considering an additional state for the two disturbances of the system, the solar irradiance and the ambient temperature.

The influence of using an observer to estimate the unmeasured disturbances that affect the process is reflected in the performance cost comparing the evolution of the system for the two cases: when using an observer and when the disturbance is not considered by the controller.

The simulated MPC algorithm was validated through experiments using a 125 square meter building and predicted and measured weather data.

The MPC presented in this paper it is a “one sided control” since it can influence only the increase of the inside temperature, by the use of the electric heaters and not the cooling of the building. The cooling of the building comes as a natural factor, influence only by the disturbances.

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References

1. European Commission eurostat - Energy Consumption Statistics, `http://epp.eurostat.ec.europa.eu/statistics_explained/index.php/Consumption_of_energy`
2. Oldewurtel, F., Parisio, A., Jones, C.N., Morari, M., Gyalistras, D., Gwerder, M., Stauch, V., Lehmann, B., Wirth, K.: Energy Efficient Building Climate Control using Stochastic Model Predictive Control and Weather Predictions. In: America Control Conference, pp. 5100–5105 (2010)
3. Balan, R., Stan, S.D., Lapusan, C.: A Model Based Predictive Control Algorithm for Building Temperature Control. In: 3rd IEEE International Conference on Digital Ecosystems and Technologies, pp. 540–545. IEEE Press (2009)
4. Halvgaard, R., Niels, K.P., Madsen, H., Jørgensen, J.B.: Economic Model Predictive Control for Building Climate Control in a Smart Grid. In: IEEE PES Innovative Smart Grid Technologies (ISGT). IEEE Press (2012)

5. Liu, S., Henze, G.P.: Impact of Modeling Accuracy on Predictive Optimal Control of Active and Passive Building Thermal Storage Inventory. ASHRAE Transactions, 151–163 (2004)
6. Prívara, S., Široký, J., Ferkl, L., Cigler, J.: Model predictive control of a building heating system: The first experience. Energy & Buildings 43(2-3), 564–572 (2011)
7. Erickson, V., Lin, Y., Kamthe, A., Rohini, B., Surana, A., Cerpa, A., Sohn, M., Narayanan, S.: Energy efficient building environment control strategies using real-time occupancy measurements. In: 1st ACM Workshop Embedded Sensing Systems for Energy Efficiency in Buildings, pp. 19–24 (2009)
8. Hazyuk, I., Ghiaus, C., Penhouet, D.: Optimal temperature control of intermittently heated buildings using Model Predictive Control: Part II Control Algorithm. Building and Environment Journal, 388–394 (2012)
9. Bacher, P., Thavlov, A., Madsen, H.: Models for Energy Performance Analysis. Technical report, Informatics and Mathematical Modeling Technical University of Denmark (2010)
10. Pannochia, G., Rawlings, J.: Disturbance Models for Offset-Free Model Predictive Control. AIChE Journal 49(2), 426–437 (2003)
11. Huusom, J.K., Poulsen, N.K., Jørgensen, S.B., Jørgensen, J.B.: Tuning of Methods for Offset Free MPC based on ARX Model Representations. In: Proceedings of the American Control Conference, pp. 2255–2360 (2010)