Robot Colony Mobility in a Thermodynamics Frame

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Abstract. In the last decade the development of multirobot systems has come to maturation providing a lot of important results in many applicative domains. Many paradigms and approaches have been devised to this aim but one of them seems very promising for future applications: dense colony of robots where the large numbers of individuals is combined with a very small dimension for each of them. Here, the key point is a behavior-based paradigm embedded in the mobiligence framework as it appears particularly suitable to deal with the sensing activity where the physics of the interaction is made explicit to take advantage from it. Within this point of view the paper explores a design method to deal with sensor information which, avoiding any symbolic representation, is maintained at a somewhat physical level as a metaphor of the events observed in the environment. The idea of substratum is introduced as a convenient representation of the physical level currently in use. The key properties of the thermal metaphor are considered and implemented to trigger appropriately a colony of robots to execute a collective task. The temperature distribution, heat flux, diffusivity and dispersion are all discussed as different aspects of the stigmergy included as a key feature of the swarm which forces each individual to behave collectively.

Keywords: roboticle, mobiligence, robot coordination, stigmergy, thermal metaphor.

1 Introduction

Humans, animals and even insects show adaptive behaviors as a key feature of being living systems. To an external observer such individual and social behaviors appear like intelligent sensory-motor coordinations, most of them essential for their survival, where the coordination mechanism implements the adaptation to the environment in form of *pre-cognition* and *anticipatory behavior*.

Hence, the adaptive behavior is an emergent property of living systems which stems from the interaction of their bodies, brain included, and the environment itself whose information are acquired dynamically through locomotion, object grasping and manipulation, etc... Starting from these considerations, Asama [2], has coined the term **mobiligence** referring to the intelligence which emerges through the interaction between an agent and its environment because of its mobility.

In its seminal paper Asama supposes that such an intelligence stems from the distinguishing properties of *embodied plasticity*, *abduction* and *co-embodiment* with the environment. Within this approach he suggests to take into account both the behavior generated by perception and also the behavior generating knowledge which becomes the external and internal processing of flow of information through which agents can reason about. So we have location information, dynamical information and also experience, which refers more properly to the cognitive structure of the agent. From a different perspective, biological studies address physiological models useful to make hypotheses which can be integrated into engineered technologies and methodologies to grow up implementable dynamical systems. This approach is especially useful when a large number of robots must be coordinated to execute collective tasks where the interference for limited resource acquisition is critical.

To overcome this problem, instead of generating a specific model of the environment using local information, and considering that the model is only needed to deliver proper motor commands, one could claim the presence of a substratum which underlies both perception and action activities driving the flow of information accordingly.

For example, let us consider a colony of robot individuals performing the collective task of maintaining the group formation. Here, the main role of sensors is monitoring the task execution by suppling the control parameters with the right values to adjust the relative positions of individuals. The observed behavior can be interpreted as *gas diffusion process*, the substratum, where a number of robots move towards a region in the same fashion as a spreading gas.

Jantz et al.[13] and Kerr et al.[15] have used the kinetic theory of gases for analysing collision and realising sweep motion in a bounded region, Also, Kinoshita et al.[11] have defined and monitored suitable thermodynamic parameters of a robot group to identify its *macroscopic state*: random motion, periodic motion and deadlock. In both cases the substratum is more properly the *kinetic theory of gas*.

D'Angelo et al.[3] have used a thermodynamic approach to formulate the relationship between the effect of the behaviour of a single robot and both its diffusion and merging in a swarm, as it appears at macroscopic level. Each robot is interpreted as a gas particle, and the energy exchange is considered through thermodynamic equations. More recently, the fluid-dynamic-based model of roboticles [5] has been augmented with the convective motion [7] of the fluid driven by a temperature distribution which acts as stigmergic information and whose properties are discussed hereafter. The rest of the paper is organized as follow: Sect. 2 summarizes the key features of the roboticle model especially pointing out the autopoietic loop; Sect. 3 extends the model to a multirobot system whereas Sect. 4 discusses an example.

2 The Fluid Dynamic Metaphor

The behavior-based approach, including the roboticle model itself, is grounded on the so called *sensorimotor coordination* for the close connection between sensing and acting which drives the acquisition from the environment of all the relevant information to carry out robot tasks. Normally, data are processed at reactive level and only a small number of them cross the *deliberative fence* which converts the signal nature of sensing into an appropriate symbolic representation.

In fact, the role of sensor information is that of monitoring the physical objects through their relevant properties for the task execution. For example, the electrical metaphor is very popular and it is commonly used in many motion planning applications, i.e. Latombe [10]. Also Arkin [1] has used this metaphor to implement behaviors as a couple of perceptual and motor schemas.

Subsymbolic approaches [12], [9] are faced with processing a huge number of data [14], which come from many sources and which must deliver motor commands with a short latency time. Sensor information don't need to be converted into any symbolic representation but they can be maintained at physical level as a metaphor of observed events in the environment, depending on the physics of the interaction.

With this respect, the roboticle model suggests the *metaphor for the navigation task* by interpreting the motion as *it comes from the particle streaming in a fluid*. In this sense the roboticle model creates a substratum by fulfilling the operating environment with a fluid which flows from the source position to the target position and the robot follows one of its streamlines since they lap the obstacles without entering their contours.

Roboticle Model

The fluid dynamic metaphor ([4], [5], [6], [8]) implements the substratum as if sensor signals were coming from a really observed fluid. So, the substratum is the velocity field \mathbf{V} of a fluid whose streaming is generated by the scalar potential F and the vector potential \mathbf{M} termed, respectively, *dissipative function* and *internal momentum*, accordingly to the following equation

$$\mathbf{V} = -grad \ F - rot \ \mathbf{M} \tag{1}$$

where the vector potential $\mathbf{M} = M\hat{\mathbf{k}}$ has only one component, M, normal to the plain surface. Thus, roboticles are point-reduced robots whose dynamical model is based on the property to explore the environment where they are moving around. Their functioning stems from a simple and well-sound mechanism which triggers acquired information towards effectors and forces them along preset trajectories. Their behavior is completely described in terms of scalar and cross products between the current velocity \mathbf{V} and the short arc $d\mathbf{r}$.

In fact, the scalar product, termed *effort*, can be understood as the *elementary work* made by the vector field **V** along the elementary trajectory arc $d\mathbf{r}$ whereas the not null component of the cross product, termed *percept*, provides the rate of trajectory change when roboticle drives away from its nominal trajectory. If we express them in a cartesian frame of reference through the cartesian components u and v of the velocity **V** and the cartesian components dx and dy of the short arc $d\mathbf{r}$, it yields to

$$\mathbf{V} \cdot d\mathbf{r} = udx + vdy = Vds$$
$$\hat{\mathbf{k}} \cdot (d\mathbf{r} \wedge \mathbf{V}) = vdx - udy = \delta P$$
(2)

where δP accounts for the sensor information of the percept. If we make explicit the contributions of both potentials, we obtain

$$\delta P = \delta L + dM \qquad \delta L = g_{11}dM - g_{12}dF$$

$$Vds = \delta Q - dF \qquad \delta Q = g_{21}dM - g_{22}dF$$
(3)

where δL and δQ are two auxiliar quantities called *committing effort* and *committed perception*, respectively. The parameters g_{ij} describe, at abstract level, how the roboticle governor's unit really works.

Autopoietic Loop

The previous relations establishes the key feature of the roboticle model, known as *autopoietic loop*. In fact, by simple substitutions, we can easily obtain

$$\delta P = (1 + g_{11})dM - g_{12}dF$$

$$Vds = g_{21}dM - (1 + g_{22})dF$$
(4)

and, then, by introducing the quantities

$$\mathbf{K}_{f} = \frac{g_{21}}{1 + g_{11}} \qquad \mathbf{K}_{b} = -\frac{g_{12}}{1 + g_{22}} \tag{5}$$

called, respectively, *direct* and *reverse gains*, we can write the following more meaningful relations

$$\delta E_{ff} = \mathbf{K}_b \left[\delta P + \delta P_{erc} \right]$$

$$\delta P_{erc} = \mathbf{K}_f \left[V ds - \delta E_{ff} \right]$$
 (6)

where the quantities $\delta P_{erc} = g_{12}dF$ and $\delta E_{ff} = g_{21}dM$ have been introduced for reason of compactness. The graphical representation of the *autopoietic loop* is depicted in fig. 1 which stems directly from equations (6): all useful perceptual information enter the "black box" labelled \mathbf{K}_f where they are coherently amplified to feed effectors so that the "black box" labelled \mathbf{K}_b provides the necessary input information about effector functioning to be summed up with the sensor information coming from environment.

We conclude this short discussion by observing that the parameters g_{ij} are dimensionless because they are the ratio between quantities of the same type and, more important, they are only partially indipendent. In fact, the following identities hold

$$g_{11}g_{22} - g_{12}g_{21} = 1 \qquad \qquad g_{21} = g_{12} \tag{7}$$

and they justify that the autopoietic loop is completely described by assigning only the direct and reverse gains. Every instantiation of roboticle functioning requires a specific relation between these gains. For example, let us consider a robot moving around to detect meaningful beacons. We can design its control unit so that, if such a beacon is recognized, its behavior is regulated by the relation

$$\mathbf{K}_f = \frac{m\mathbf{K}_b}{\sqrt{1 + (1 - m^2)\mathbf{K}_b^2}} \tag{8}$$

and the observed behavior is depicted in fig. 2 where the highlighted robot trajectory is compared with the isothermal distribution detected around the obstacle. This topic will be discussed in some detail in the next sections.

3 The Thermodynamical Metaphor

If you look upon a colony of robots placed on a arena densely enough of individuals, you can regard the outermost robots as colony boundary and the internal dynamics shows

each individual is moving in a *brownian motion* fashion. Within this model we can regard the individuals of the colony as **gas particles** moving around in a 2-dimensional space where each robot has no active sensor and its motion is driven by direction changing due to the contact with other robots.

The fluid dynamical metaphor, depicted by eqns. (3) and presented in Sect. 2, can be also interpreted in term of the first law of thermodynamics: the roboticle formulation [8] suggests to understand δP as the *thermal* unordered energy entering the robot's governor unit in the form of *perceptual flow* whereas *Vds* is the *mechanical* ordered work driving effectors as *effort*. Really speaking, the roboticle model makes use of two such equations, the former referred to its sensors and the latter to its effectors. With the same respect, the *committing effort* δL and the *committed perception* δQ are a kind of internal work and internal heat, respectively.

Heat Diffusion

But the collisions among individuals can be also interpreted as the *temperature* of the colony so that it can be used to trigger its purely reactive group behaviour which stems from a number of individuals immersed in a fluid with well-specified thermal properties. The *motion engine* is the result of the fluid dynamic metaphor augmented with a suitable temperature distribution. It animates the hypothetical fluid by introducing a *convective motion* inside the fluid by the well-known formula

$$\mathbf{H} = -\rho \ grad \ T \tag{9}$$

where **H** is the *heat flux* due to a given *temperature gradient* inside the fluid. Its *diffusivity* ρ takes different values in different positions since it contrasts selectively heat diffusion. Thus, *grad* T acts as thermomotive force which maintains the *heat flux* **H** whereas ρ can be assimilated to the environmental response to the applied gradient and it takes into account the agent distribution inside the colony which is assumed dense in the environment.

From the point of view of a robot colony the interpretation of the thermomotive force *grad T* yields to a model of interaction among individuals where the *diffusivity* ρ provides the necessary regulation of the heat flux **H**. Because this mechanism works globally, namely, it is triggered by the collective task, the motion engine for each individual acts locally on each roboticle¹ through an appropriate velocity field which implements the fluid dynamic metaphor for the individual.

Thermomotive Force

The fluid dynamic extension to the convective streaming metaphor is a different way to implement the stigmergic coordination on colony behavior². The temperature level, which is a scalar quantity, can be interpreted as a *generalized pheromone* which stimulates the social behavior of the colony. For the aim of this paper we restrict the analysis to those behaviors which reduce interference among individuals.

If we review eqn. (9) from this point view, the interaction among individuals requires the thermomotive force to be bound to the dynamical sources of the fluid, namely, the dissipative function F and the internal momentum M. In our metaphor, the stationary

¹ The model we shall use for each individual of the robot colony.

² Namely, the implicit control of the collective task.

state of the convective motion should be responsible of the *maintainance property* of the behavior. This circumstance takes the form

$$\oint_{\mathscr{L}} \mathbf{H} \cdot d\mathbf{l} + \int_{\mathscr{S}} \mathbf{M} \cdot \hat{\mathbf{n}} \, dS = 0 \tag{10}$$

where the *heat flux* \mathbf{H} is selected as it were the *magnetic field* generated by the *current* \mathbf{M} . This assumption ensures that \mathbf{H} is fully compatible with the possibly steady state of the fluid dynamic of the roboticle velocity field.

A further property of the *heat flux* **H** relates the convective motion of the fluid to its attractors and repulsors. To this aim we start from the well-known principle of *heat flux conservation* by augmenting it with the *source density* F^3 to brings into equilibrium unbalanced gradient diffusion with the accumulated heat

$$\oint_{\mathscr{S}} \mathbf{H} \cdot \hat{\mathbf{n}} \, dS + \frac{d}{dt} \int_{\mathscr{V}} k_e T \, dV = \int_{\mathscr{V}} F \, dV \tag{11}$$

Now, let us transform the preceding integral relations into a more usable derivative form. To this aim we apply well-known mathematical theorems (such as Stokes) yield-ing to

$$\mathbf{M} = -rot \mathbf{H} \qquad F = div \mathbf{H} + k_e \frac{\partial T}{\partial t}$$
(12)

which bind the thermal flux to the internal momentum \mathbf{M} and the dissipative function F. The latter generalizes the thermal energy balancing by equating the sum of heat flux exiting a close region and its accumulation over the time (right side) with possible sources or sinks dislocated in the region itself (left side). Thus, the dissipative function F is just the mechanism which dissipates the thermal energy provided by the temperature gradient of the fluid inside the region.



Fig. 1. How the autopoietic loop controls a moving roboticle

Fig. 2. Temperature-based representation of a beacon driving a roboticle

³ The dissipative function previously discussed.

4 Stigmergic Heat Equation

Heat diffusion is central in the definition of the convective motion of roboticles. More precisely, in the following we shall show that \mathbf{H} obeys the non-homogenous heat equation. This statement comes easily from eqns. (1) and (12), using well-known vectorial identities, which yield to the non-homogenous heat equation

$$\nabla^2 \mathbf{H} - \frac{k_e}{\rho} \frac{\partial \mathbf{H}}{\partial t} = -\mathbf{V} \tag{13}$$

also by substituting the involved quantities appearing in (9). The time derivative term includes the scalar function ρ which only depends on coordinates since it characterizes the environment response to the heat flux. A similar equation can be obtained for the temperature *T* as it will be explained in the next subsection.

Diffusivity

The coordination of a dense colony of robots takes advantage of the thermal properties of the fluid⁴ in what it provides the designer with the explicit implementation of the stigmergic response of the robot colony. In fact, the temperature distribution triggers the streamline distribution within the fluid and this property strongly depends on the diffusivity ρ , namely, a scalar quantity which defines how the environment reacts to the convective motion.

Because the fluid dynamic motion of roboticles depends on the internal momentum and the dissipative function, let us make explicit such a relation. So, if we start from the internal momentum, we have

$$\mathbf{M} = -rot\mathbf{H} = rot(\rho\nabla T) = \nabla\rho \wedge \nabla T \tag{14}$$

namely, remembering that \mathbf{M} has only one component directed normal to the plain and multiplying both sides by ρ and rearrange the terms, we obtain

$$\rho M = H_1 \frac{\partial \rho}{\partial y} - H_2 \frac{\partial \rho}{\partial x} \tag{15}$$

where also eqn. (9) has been considered. With the same respect, starting from the latter equation appearing in (12), we can write

$$F = div(-\rho\nabla T) + k_e \frac{\partial T}{\partial t} = -\rho G - \nabla T \cdot \nabla \rho$$
(16)

where we have introduced the new scalar quantity G defining the non-homogenous heat equation for the temperature T,

$$\nabla^2 T - \frac{k_e}{\rho} \frac{\partial T}{\partial t} = G \tag{17}$$

Now, if we multiply both sides of the preceding relations by the diffusivity ρ and rearrange the terms, we obtain

$$\rho(F + \rho G) = H_1 \frac{\partial \rho}{\partial x} + H_2 \frac{\partial \rho}{\partial y}$$
(18)

⁴ Which hypothetically laps roboticles.

also remembering the heat flux definition given by eqn. (9). Thus, eqns. (15) and (18) show how the diffusivity influences the heat flux making evident the dependency of the internal momentum and the dissipative function of roboticles.

Temperature Distribution

The thermal properties of the fluid dynamic metaphor are really noteworthy. However, we are not interested to deal with the convective motion of the fluid in general but we want to find out temperature distributions which can solve easily the *problem of the stigmergic control* inside a group of robots. To this aim, let us review eqn. (9) by introducing the polar coordinates to compute the differential form. We have

$$-\rho dT = (H_1 x + H_2 y) \frac{dr}{r} + (H_2 x - H_1 y) d\varphi$$
(19)

so that, if we take the diffusivity in a given environmental point as the scalar product of the heat flux in that point and its distance **r** from the origin of the stigmergy, namely, $\rho = \mathbf{H} \cdot \mathbf{r} = H_1 x + H_2 y$, the preceding equation yields to

$$-dT = \frac{dr}{r} + \frac{tan\Psi - \tan\varphi}{1 + tan\Psi tan\varphi}d\varphi \qquad tan\Psi = \frac{H_2}{H_1}$$
(20)

having introduced the angle Ψ for the cartesian components of the heat flux. Now, the integration of the differential form expressing the temperature *T* is straightforward; let us rewrite it into a more compact form

$$-dT = \frac{dr}{r} + tan(\Psi - \varphi)d\varphi$$
(21)

and the temperature distribution is determined by the only directional component. This is a very interesting property because we want the convective motion of the fluid be interpreted as a suggestion for the roboticles to move on useful trajectories. To this aim we shall consider the temperature distributions built as follow.



Fig. 3. The obstacle representation as it is detected by a roboticle



Fig. 4. A group of roboticles forced by an obstacle to turn left

First of all we define the direction Ψ of the heat flux by the sum of the *basic directions* Ψ_i with *i* ranging form 1 to *N* and each Ψ_i bound to the radius direction φ with the simple relation

$$tan\Psi_i = k_i tan\varphi$$
 $tan\Psi_i = \frac{1}{k_i tan\varphi}$ $i = 1, ..., N$ (22)

so that the heat direction can be easily computed using the composition rule for the tangent, namely, $tan\Psi = tan(\Psi_1 + \Psi_2 + ... + \Psi_N)$. We shall term this kind of temperature allocation *direction driven distribution* because it highlights the response of a roboticle to the obstacle with the respect of assigned driving directions. For simplicity, we have positioned the obstacle on the center of the frame of reference.

This property provides some degree of assimilty for the roboticle motion approaching the obstacle. In such a way the environment around the obstacle appears anisotropic and Ψ_i is termed the *i*-th driving direction. In the following we shall consider the case of two driving directions at most. In fact, in all the simulations we have found that the stigmergy originating the isotropical, monopolar and dipolar temperature distributions can cover the most cases of interest.

Monopolar Source – When a monopolar source is assumed, only one driving direction is considered. In that case it is more convenient to take the constant k_1 with the form $k_1 = \frac{1-p}{1+p}$ so that the heat direction $tan\Psi$ is either $\frac{1-p}{1+p}s$ or $\frac{1+p}{1-p}\frac{1}{s}$ where, for convenience and compactness, we have introduced the auxiliar variable $s = tan\varphi$. In the former case the temperature distribution for an obstacle put on the center of the frame of reference, takes the form

$$a^{2}\exp(-2T) = r^{2}(1 + p\cos 2\phi)$$
(23)

An example of this distribution appears in fig. 2 where the obstacle is used as beacon by a roboticle whose autopoietic loop is triggered with the dissipative component greater than the conservative one (m < 1).

Dipolar Source – A more interesting situation stems from a dipolar source of stigmergy. For the following discussion we shell assume $tan\Psi_1 = \frac{1-n}{f}s$ and $tan\Psi_2 = -\frac{1-n}{f+n}s$ yielding to the direction component of the temperature appearing below

$$tan\Psi = \frac{ns}{p + (1-n)s^2} \qquad p = \frac{f+n}{1-n}f$$
(24)

where the auxiliar quantity p has been introduced for convenience. In this case, the temperature distribution takes the form

$$a^{2}\exp(-2T) = x^{2(1-n)} \left(px^{2} + y^{2}\right)^{n}$$
(25)

with the obstacle centered in the origin of the frame of reference. An example of a roboticle observing such an obstacle is shown in fig. 3.

Dispersion

Up to now we have considered only the temperature distribution designed to track actively the objects in the environment. Their recognition and interpretation is made on the basis of the stigmergic information associated with them. However, the collective behavior of a colony depends on both the single behavior of each individual and how each robot reacts to the behavior of the nearby individuals. The thermal metaphor deals with this key feature by factorizing the contribution of the diffusivity ρ by introducing a new scalar quantity W, termed *dispersion*, such that

$$\rho = W \exp(-kT) \tag{26}$$

where k is an auxiliar parameter useful to describe the relative weight of the temperature, namely, the stigmergic factor, with the respect to how the colony influences each robot individual.

In many cases we can assume k = 2 but in the avoiding obstacle example, briefly discussed, we shall take $k = \frac{2}{n}$ where *n* is a measure of the regulating parameter *q* which controls the autopoietic loop of each roboticle, reported below

$$\mathbf{K}_f = \frac{q\mathbf{K}_b}{q + \sqrt{1 + \mathbf{K}_b^2}} \qquad q = \frac{1 - n}{1 + n} \tag{27}$$

When a roboticle detects the obstacle, the autopoietic loop adapts its behavior by trying to match the parameter *n* with the same describing the obstacle as temperature distribution (see fig. 3). Now the obstacle acts as a cue with suggests the most appropriate trajectory for the roboticle. Such a situation is depicted in fig. 4 where a few roboticles show their motion around the obstacle. Each of them has modified its autopoietic loop choosing a different value⁵ for *m*, with $m = \sqrt{n}$.

However, as it appears from fig. 4, different roboticles provide different positions for the same obstacle; its real position is depicted by the green circle. The dispersion function used by this example allows roboticles to approach the obstacle uniformly except for a specified direction. It is the watershed which delimits the area beyond which roboticles cannot access. In this case the dispersion function W can be approximated by $W_0r^4(1 + \cos 2\varphi)$ where the distance r and the direction φ are referred to a frame of reference centered on the obstacle and the y-axis is taken as watershed. As it is not directly accessible from the colony, this fact motivates the different positions of the same obstacle detected by the roboticles.

5 Conclusion

In the paper we have presented a methodology design aimed to provide a colony of robots with a coherent collective behavior. The guideline has been the behavior-based paradigm of roboticles embedded in the more general framework of mobiligence in what sensor information, maintained at subsymbolic level, balance the behavior generated by perception with the behavior generating knowledge about the environment. This activity is carried out by the so called autopoietic loop which controls individual robots without a real interpretation of sensing at symbolic level.

By relating the autopoietic loop of each individual with the emergent collective behavior of the colony we have shown that its role can be fully understood through the

⁵ i.e., m=0.96 for the blue trajectory, m=0.39 for the red one.

stigmergic properties of the temperature distribution of the colony. In fact, this paradigm has been devised to extend coherently the roboticle model to a multirobot system: the information exchanging among individuals occur without considering explicitly the group because the perceptual interpretation is made from the point of view of the colony itself.

This kind of stigmergy has been obtained by assuming that each individual robot is acting under the influence of a thermomotive force, namely, a temperature gradient generated accordingly to the obstacles encountered during the roboticle motion. It defines indirectly the collective task of the group, provides an effective interpretation of the obstacles and motivates their presence. Moreover, we have argued some properties about the relationship between the diffusivity ρ , the temperature T and the dispersion W. A short example of a colony which encounters a meaningful obstacle, suggesting a change of direction, has been discussed. Simple simulations have shown that each individual robot provides approximatively the same interpretation of the thermal field and, moreover, intuitive properties of the diffusivity can be established. With the respect to the previous work we have found a general method to assign a thermal distribution to obstacles which, coherently to the required collective task, depends on their functional anisotropy.

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