Parameters Sensitivity Analysis of Self-Excited Vibration of Tires

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Abstract To resolve the issues of multi-polygonal wear of tire with high-speed, a multi-dynamic model, considering the four inter-coupling degrees of toe angle, camber angle, vertical vibration and self-excited vibration of tread along with the lateral direction, is built. The model is motivated by the component velocity of vehicle driving speed along with the lateral direction of tire and the sensitivity analysis of parameters, affecting the bifurcation speeds of vehicle, is gained by a numerical simulation method. At the end of this paper, several control strategies with crucial engineering significance are concluded to suppress the self-excited vibration of tire, and then decrease the multi-polygonal ware of tread.

Keywords Multi-polygonal · Self-excited vibration · Friction coefficient · Bifurcation

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1 Introduction

With the development and popularization of expressway, vehicles running time on freeway increase significantly, as a result of that, a polygonal wear caused by self-excited vibrations often occurs on the tires. This wear can lead to tires' early retirement and cause some seriously traffic accident, which severely damage the vehicle manufacturers' image. There are some potential reasons responsible for this kind of uneven wear, such as vehicle's dynamic characteristics and suspension parameters. Other reasons could be tire structure parameters and tread pattern of tire.

Many researchers [1–4] have been spent much time focusing on micro and macro mechanism of interaction between tread and road to study tire's uneven wear, however, a reasonable theoretical method has not been found out yet. In Japan, Atsuo Sueoka [5] research team successfully finished an experiment that two rotating machinery can generate a polygon to the tire wear, however, the model took no account of dynamic toe angle and multiple degrees of vibration.

Based on the previous study of automotive college of Tongji University [6, 7], more researches are forced on studying the interaction mechanism between tread and pavement. A four degree-of-freedom dynamic vibration model of suspension-tire-tread, taking time-delay into account, is established to study the range of vehicle speed to motivate self-excited vibration and the relationship of vibration character and speed.

2 Choice of Friction Co-efficient

It is much difficult to clearly describe the friction characteristic in building a tire model that reflects the force changing between tread and pavement. The friction coefficient is consist of dynamic friction coefficient and static friction coefficient, and the switch between the two kinds of coefficients, which is very common in tire driving, is the most complicated problem to understand the mechanism. An over-simplified friction model can't describe the changing laws correctly while an over-complicated model is hard to get its numerical simulation results, therefore, it is a critical task to establish the right friction model to analyze the self-excited vibration motivated by friction force.

Friction coefficient is often considered as a constant or just an experiential expression increasing or decreasing with the variation of relative slip velocity in a tire-road contact model [8]. However, by means of a friction experiment of rubble block, it is confirmed that the friction coefficient is increasing linearly at first and then decreasing with the variation of relative slip velocity. The decreasing trend of friction coefficient is changing rapidly at first and then getting slow [9].

In 1995, C. Canudas de wit proposed a Lugre model [10], which was an improvement of bristle model. Compared with the numerous random elastic characteristic of bristle model, Lugre model provides a mean deformation of those
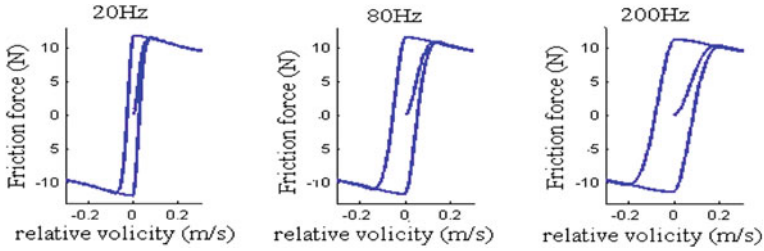
Fig. 1 Friction characteristic curves with different excitation frequencies

elastic characteristic, meanwhile, hysteresis characteristic and adherence friction between the two contact surfaces are also considered in Lugre model. The Lugre model equation can be written as expression (1).

$$
\begin{cases}
F = \sigma_0 z + \sigma_1 \dfrac{dz}{dt} + \sigma_2 v_r \\[2mm]
\dfrac{dz}{dt} = v_r - \dfrac{\sigma_0 |v_r|}{G(v_r)} \cdot z \\[2mm]
G(v_r) = F_c + (F_s - F_c) e^{-|v_r/v_s|^\delta}
\end{cases}
\tag{1}
$$

where σ_0 is the stiffness of bristle, σ_1 is the damping coefficient of bristle, σ_2 is the relative viscous damping coefficient, z is the mean elastic deformation of bristle, v_r is the relative velocity of the two objects contacting with each other, v_s is the Stribeck velocity, δ is the Stribeck exponent, F_s is the maximum static friction force, F_c is the sliding friction and F is the total friction force.

As shown in Fig. 1, the Lugre model has almost the same friction characteristics with static friction model while the variation frequency of relative velocity is low. With the increase of variation frequency of relative velocity, the dynamic hysteresis characteristics of Lugre model is becoming more and more obviously, which is accordant with the friction characteristics between tread and pavement.

3 Self-Excited Vibration and Analysis of Influencing Factors for Tire Wear

3.1 Classification of Self-Excited Vibration

Self-excited vibration is caused by negative slope of friction coefficient. In a classical vibration system, the vibration energy becomes lower and lower because of the inter consumption with a phenomenon that phase trajectory is converge to a stable point in the phase diagram. At this moment, friction force is determined by the positive slope section of friction coefficient. On the other hand, if the friction
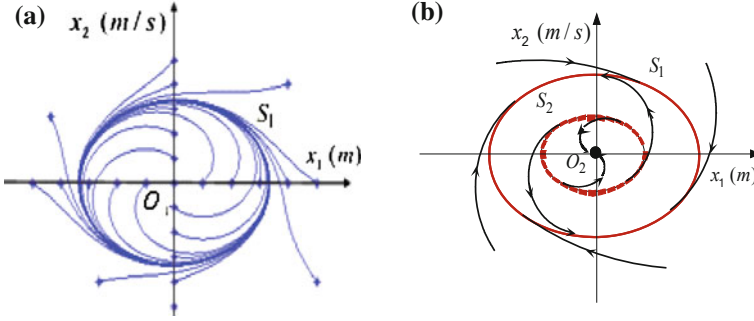
Fig. 2 Phase photogram of self-excited vibration. **a** Soft self-excited vibration. **b** Hard self-excited vibration

coefficient is at negative slope section, the vibration energy becomes higher and higher, which means that phase trajectory diverges more and more. Due to the existence of structure constrains, the friction coefficient swings alternately between positive slope section and negative slope section, that is to say, the vibration system balances between consuming energy and obtaining energy. Naturally, the self-excited vibration is generated.

Self-excited vibration usually consists of hard self-excited vibration and soft self-excited vibration. As shown in Fig. 2a, soft self-excited vibration is unstable. With any kind of exciting force, the vibration system will diverge to a stable state. It is obviously to understand the mechanism from the phase photogram that the phase trajectory diverges to the stable limit cycle S_1 at any initial situation. For a hard self-excited vibration, only if the initial exciting force is large enough to push the phase trajectory to pass the unstable limit cycle S_2 and to achieve the stable limit cycle S_1, otherwise, it will converge to the stable point O_2., which is shown in Fig. 2b. By analyzing the polygonal wear of tire, it is concluded that the tread's vibration is a typical hard self-excited vibration.

3.2 Factor Analysis of Influencing Tire Wear

Lupker [11] pointed out that the average wear rate of rubber would increase nonlinearly with the augment of vibration acceleration. As shown in Fig. 3, with the increase of vehicle speed, the normalized wear rate gets higher gradually, and then become smaller after the vehicle speed of 100 km/h. In another word, for a tire's self-excited vibration at the same frequency, different vibration accelerations can generate different vibration amplitudes, which cause different normalized wear rates. Therefore, it should be focused to control the tread's vibration amplitude or to move the vehicle speed that can cause hard self-excited vibration of tire out of the normal speed range.

**Fig. 3** Wear rate with different accelerations and speeds
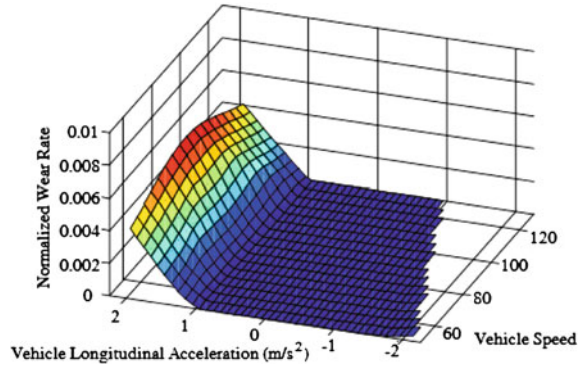
4 Multi-Body Model and Dynamic Equation

All the driving force is supplied by the friction force between tires and pavement except the wind drag while driving in an expressway. Hence, it is important to recognize the relationship between tire driving force and dynamic characteristic of vehicle. In order to represent the dynamic characteristics of tire driving force, a multi- body dynamic model, with a combination of tire and twist beam, driven by a rotary drum, is built. The schematic diagram is shown in Fig. 4 and the multi-body dynamic equation is given as expression (2).

$$
\begin{cases}
(J_1 \sin^2 \gamma + J_2 \sin^2 \beta \cos^2 \gamma + J_3 \cos^2 \beta \cos^2 \gamma)\ddot{\alpha} + J_1 \sin \gamma \cdot \ddot{\beta} \\
\quad + (J_3 - J_2) \sin \beta \cos \beta \cos \gamma \cdot \ddot{\gamma} = F_\alpha + (J_3 - J_2) \sin \beta \cos \beta \sin \gamma \cdot \dot{\gamma}^2 \\
\quad - (J_1 + (J_3 - J_2) \cos 2\beta) \cos \gamma \cdot \dot{\beta} \cdot \dot{\gamma} + (J_3 - J_2) \sin 2\beta \cos^2 \gamma \cdot \dot{\alpha} \cdot \dot{\beta} \\
\quad - (J_1 - J_2 \sin^2 \beta - J_3 \cos^2 \beta) \sin 2\gamma \cdot \dot{\alpha} \cdot \dot{\gamma} \\
J_1 \sin \gamma \cdot \ddot{\alpha} + J_1 \cdot \ddot{\beta} = F_\beta + (J_3 - J_2) \sin \beta \cos \beta (\dot{\gamma}^2 - \cos^2 \gamma \cdot \dot{\alpha}^2) \\
\quad - (J_1 - (J_3 - J_2) \cos 2\beta) \cos \gamma \cdot \dot{\alpha} \cdot \dot{\gamma} \\
(J_3 - J_2) \sin \beta \cos \beta \cos \gamma \cdot \ddot{\alpha} + (J_2 \cos^2 \beta + J_3 \sin^2 \beta)\ddot{\gamma} = F_\gamma \\
\quad - (J_3 - J_2) \sin 2\beta \cos^2 \gamma \cdot \dot{\beta} \cdot \dot{\gamma} - (J_2 \sin^2 \beta + J_3 \cos^2 \beta - J_1) \cdot \\
\quad \sin \gamma \cos \gamma \cdot \dot{\alpha} - ((J_3 - J_2) \cos 2\beta - J_1) \cos \gamma \cdot \dot{\alpha} \cdot \dot{\beta} \\
m_2 \ddot{x} = F_x
\end{cases}
\tag{2}
$$

where J1, J2 and J3 are the rotational inertias of rear axle body around x, y, z axles, respectively, m2 is the mass of tread, α, β and γ are the rotational angles of rear axle body around x, y, z axles, respectively, which reflect the toe angle, camber angel and vertical vibration, x is displacement of tread along with the lateral direction.

Fig. 4 Multi-body dynamic modal of suspension-tire-pavement
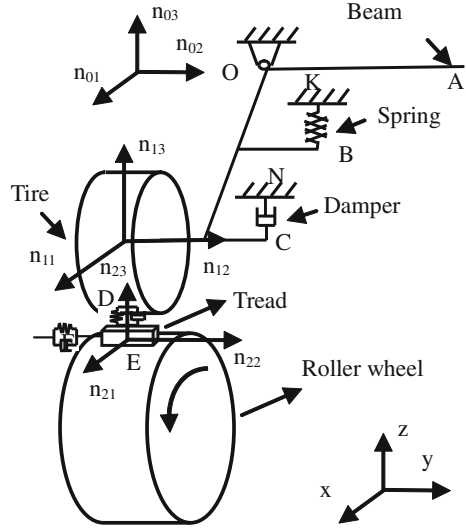
5 Numerical Simulation and Sensitivity Analysis

5.1 Bifurcation Simulation of Self-Excited Vibration of Tread Along with the Lateral Direction

Figure 5 shows the phase diagram of tread's self- excited at 110 km/h, simulated in MATLAB/SIMULINK. No matter what situation the initial condition is, the phase trajectory diverges or converges to a stable limit cycle at the end. The vibration amplitude can also be read from the phase diagram.

In order to describe the self-excited vibration of tread at different vehicle speeds, a bifurcation diagram of Eq. (2) is plotted in Fig. 6, in which the x axle is the vehicle speed and the y axle is the vibration amplitude of tread. The smaller bifurcation speed is called the first bifurcation speed and the bigger bifurcation speed is called the second bifurcation speed.

Thus it can be seen that the vibration energy, playing an important role in tire wear, becomes large while the self-excited vibration of tread occurring. To reduce the damage to a running vehicle, some control strategies should be done to decrease the tread's vibration energy or to move the vehicle speed that can cause hard self-excited vibration of tire out of the normal speed range. In order to accomplish the object, there is necessary to study how the parameters influence the characteristics of self-excited vibration.

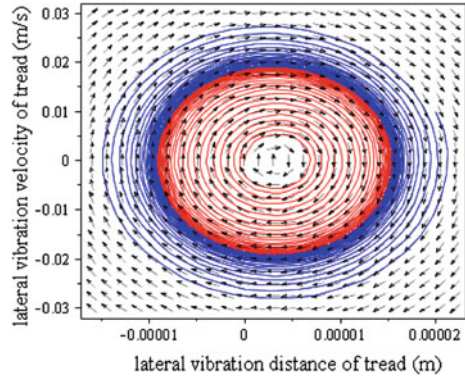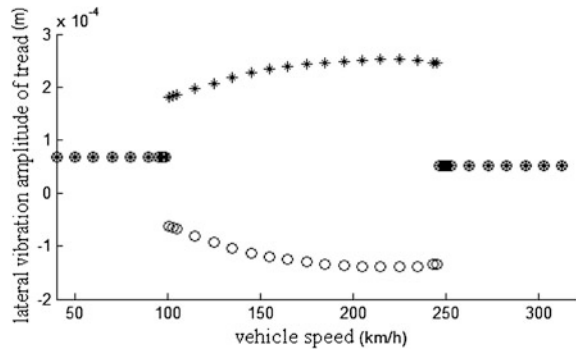**Fig. 5** Phase diagram of self-excited vibration at 110 km/h

Fig. 6 Bifurcation of vibration amplitude of tread with vehicle speed

5.2 Analysis of the Impact of Parameters' Changing on Self-Excited Vibration Characteristic of Model

Take the toe angle and camber angel as an example to analyze the impact of parameters' changing on self- excited vibration characteristic of multi-body dynamic model. Figures 7 and 8 show the changing trend of lateral vibration of tread, toe angle, camber angle and vertical vibration by numerical simulation in MATLAB/SIMULINK with different initial toe angles and cambers angles alternately. In the phase diagrams, x axles are vibration displacement and y axles are vibration velocity.

As seen in above figures, vibration amplitudes of the four parameters magnify with the increase of initial toe angle. It means that a larger vibration energy causes a worse wear of tire. On the other hand, the vibration energy of toe angle is magnified by a big initial camber angle, and the feedback from the toe angle will aggravate the wear of tire.
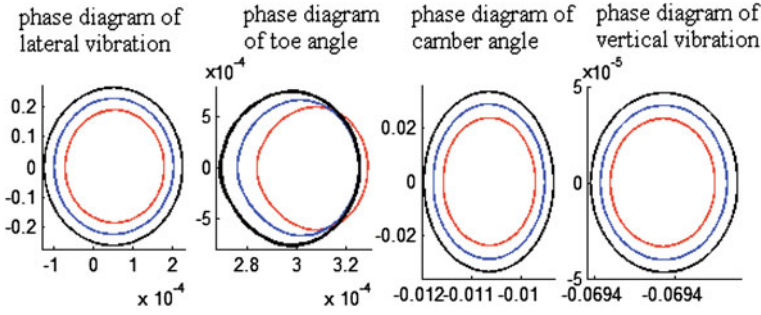
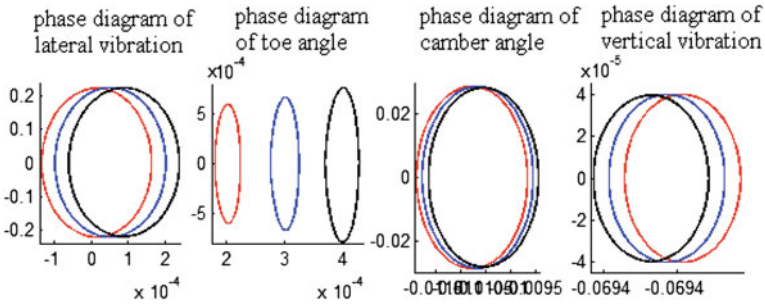**Fig. 7** Phase diagram with different toe angles

Fig. 8 Phase diagram with different camber angels

5.3 Calculation of Parameters' Sensitivity

A vibration model is often working with several affecting parameters. To optimize the vibration model, there is no need to analyze all the parameters, only a few of them are picked up to be optimized, and that is the core essence of accurate sensitivity analysis [12].

In engineering project, finite difference method is often used to calculate the system sensitivity [13]. It means that derivative method is approximately replaced by finite difference method while the design variable is changing just a little. The calculation expression of sensitivity analysis is given as

$$M = \frac{\partial f}{\partial \rho} = \frac{f(\rho^j) - f(\rho)}{\Delta \rho} \tag{3}$$

As shown in Figs. 9a and b, the sensitivity analysis of several parameters on the first bifurcation speed and the second bifurcation speed is calculated, where, parameters 1–7 are lateral stiffness and damping of tread, toe angel, camber angle, adhesion coefficient, tread mass and vertical load of tire.
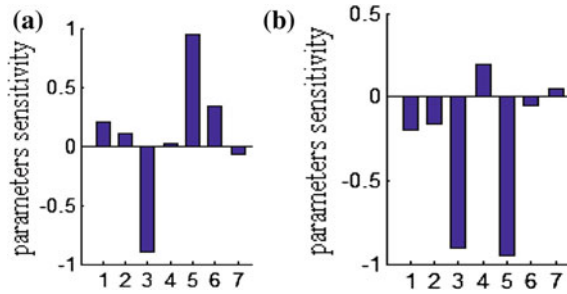
Fig. 9 Sensitivity analysis of different parameters. **a** Sensitivity analysis of the first bifurcation speed. **b** Sensitivity analysis of the second bifurcation speed

6 Control Strategies of Suppressing the Lateral Self-Excited Vibration of Tread

From the above analysis of parameter sensitivity, it is concluded that toe angel and adhesion coefficient are the most remarkable ones of all the parameters, including lateral stiffness and damping of tread, toe angel, camber angle, adhesion coefficient, tread mass and vertical load of tire.

Therefore, several control strategies are listed to reduce the polygonal wear of tire:

1. Wide radial tire, with the advantages of small slip rate, good adhesion performance and large contact area with road, should be chosen.

The friction coefficient depends on the material property contacting with each other. Big friction coefficient means good adhesion performance, and good adhesion performance results in small amplitude of self-excited vibration. It is well known that radial tire has the characteristics of small slip rate and good adhesion performance; wide tire can increase the contact area of road and suppress the lateral self-excited vibration of tire, which is in accordance with the Spinner [14]; moreover, tread patterns also has some influences on adhesion coefficient. The depth of pattern will change with the wear of tire, which leads to a bad adhesion performance. It explains why the self-excited vibration of tread often appears after running 250,000–40,000 km. The bifurcation comparison chart between original adhesion coefficient and optimized adhesion coefficient is given in Fig. 10.

2. Chose the proper toe angle and camber angel or it will aggravate the wear of tire. From the above analysis, it is known that toe angle should be adjusted firstly and then the camber angle.

Different OEM vehicle manufacturers have different positional parameters of tire. The positional parameters will change because of the looseness of wheel hub bearing, big clearance of rubber bushing and the deformation of rear axle. Hence, under the premise of meeting the request of auto control stability performance, it is necessary to adjust the toe angle and camber angle in order to reduce the vibration
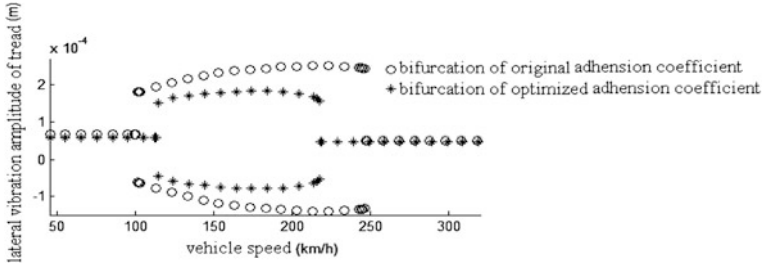
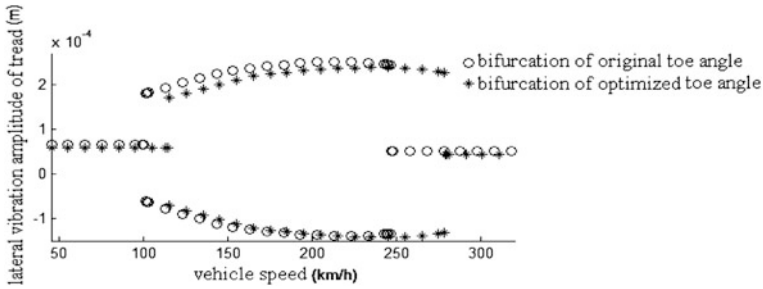**Fig. 10** Bifurcation comparison chart between original adhesion coefficient and optimized adhesion coefficient

Fig. 11 Bifurcation comparison chart between original toe angle and optimized toe angle
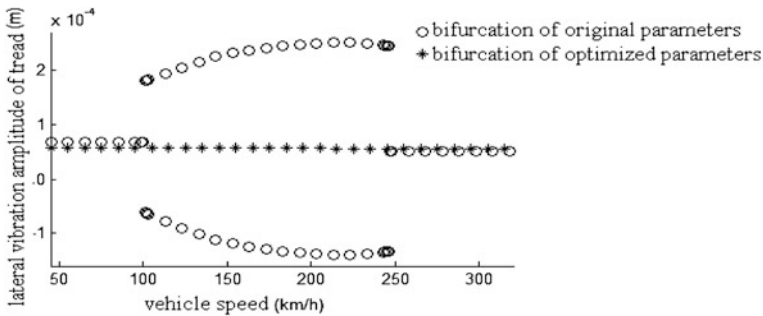

Fig. 12 Bifurcation comparison chart between original parameters and optimized parameters

amplitude of tread. The bifurcation comparison chart between original position parameters and optimized position parameters is given in Fig. 11.

3. Try not to drive an overloaded vehicle at low tire pressure for a long time.

Under the condition of overload, it is easily to increase the lateral friction force and to motivate the self-excited vibration of tire, and also it is easily to deform the rear axle while driving on a poor road and to destroy the normal positional

parameters. A low pressure tire can also motivate the self-excited vibration of tire because of the small lateral stiffness, which can be found out from the above parameters sensitivity analysis.

The range of vehicle bifurcation speed is narrowed or moved out of the normal one after optimizing the tire property parameters and position parameters, meanwhile, the vibration energy is lowered greatly, which reduces the tire average wear rate. As shown in Fig. 12, the polygonal wear of tire can be avoided completely if chosen the proper tire property parameters and position parameters theoretically.

7 Conclusions

1. The Lugre friction coefficient, taking time-delay into account, is used to research the form mechanism of polygonal wear of tire, and a multi-body dynamic mode that can reflect the hard self-excited vibration of tread is established in MATLAB/SIMULINK.
2. The influences of several parameters, including lateral stiffness and damping of tread, toe angel, camber angle, adhesion coefficient between tire and road, tread mass and vertical load of tire, on self-excited vibration of tread are analyzed by changing them alternatively, and the range of bifurcation vehicle speed is numerical simulated in MATLAB/SIMULINK.
3. It is concluded that toe angel and adhesion coefficient are the most remarkable ones of all the parameters, including lateral stiffness and damping of tread, toe angel, camber angle, adhesion coefficient, tread mass and vertical load of tire.
4. The range of vehicle bifurcation speed is lowered greatly or moved out of the normal one by adjusting the tire property parameters and position parameters, which can reduce the polygonal wear of tire remarkably. The control strategies of reducing the self-excited vibration of tread have an important engineering significance.

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